Material from Bian (2014)

William Henney

November 22, 2015

Contents

- Models of acceleration of solar flares
  - Coronal loops have n = 1e11 pcc, T = 2e7K, length L = 1e9 cm
    - => Kn = 0.005 or so
    - Thermal electron energy is about 2 keV
    - Flares produce deka-keV electrons, so 10 times more energetic than thermal
    - X-ray spectra suggest $\kappa \simeq 5$
- Their Section 5: Spatial transport and escape
  - Isotropization of the distribution function on deflection timescale τ_D
    - They call this the "pitch-angle scattering timescale"
  - Then pitch-angle dependent diffusion along the field lines
- From their section 7
  - They derive a relationship for kappa:

$$\kappa = \frac{3}{2}\frac{\lambda_c}{\lambda}\left(\frac{E_D}{E_\parallel}\right)^2$$

  - where λ_c is the collisional mean free path
  - λ is the turbulent mean free path
    - we need to unpack this further, but it seems to be roughly equal to the scale of their system

- $E_D = kT/e\lambda_c$ is the Dreicer field, which is field required to accelerate an electron to the thermal velocity over one mean free path
- $E_\parallel$ is the accelerating electric field in the flare
- This has the bizarre property that κ is smaller when the collisional mean free path is smaller
    * *This is an illusion* (see below). There is a hidden factor of λ_c^{-2} in the Dreicer field
- All this requires that the turbulent pitch-angle scattering timescale is a decreasing function of v
    * Contrast with collisional pitch-angle scattering timescale $\lambda_c/v \sim v^3$
    * If turbulent mean free path $\lambda(v)$ is independent of v, then this leads to the acceleration time and collisional deacceleration term having the same v dependence: $\sim v^3$. This allows for convergence towards a stationary kappa distribution

• In the introduction they have the collision parameter:

$$\Gamma = \frac{4\pi e^4 \ln \Lambda n}{m_e^2}$$

- This is half the α_r from the plasma formulary
- In terms of which they have a collisional deceleration time:

$$\tau_c(v) \simeq v^3/\Gamma$$

    * More precisely, using the Plasma Formulary equations, we have

$$\tau_c(v) = 2.885 v^3/\Gamma$$

- Which would mean collisional mean free path

$$\lambda_c = 2.885 v^4/\Gamma$$

• **Recasting their equation in terms of Kn**

- They say in equation (76) that

$$\kappa = \frac{\Gamma}{2D_0}$$

- This is just repeating their equation (14), where they had it as

$$\kappa = \tau_{acc}/2\tau_c$$

  - where the acceleration time is $\tau_{acc} = v^2/D_{\rm turb}(v)$ and the turbulent diffusion coefficient has the form $D_{\rm turb}(v) = D_0/v$
    - At this point it is just "the diffusion coefficient in velocity space associated with an as yet unspecified stochastic acceleration mechanism."
  - This version makes sense because κ increases as collisions become more important ($\tau_c \to 0$)
- But then in section 7, they talk about the specific acceleration mechanism and we get

$$D_0 = \frac{e^2 E_\parallel^2 \lambda}{2m^2}$$

  where λ is turbulent mean free path
- So, subbing into the κ equation gives

$$\kappa = 2.885 v^4 m^2/\lambda_c e^2 E_\parallel^2 \lambda$$

- When subbing in the Dreicer field, this gives the equation I give above that seems to have $\kappa \propto \lambda_c$, but because of the hidden dependency of E_D^2 on λ_c^{-2} everything is OK and we really have $\kappa \propto \lambda_c^{-1}$
- A better way of presenting things would be to define a length scale: $z_E e E_\parallel = 10kT = 5mv^2$
  - so that z_E is the distance required for the field $E_\parallel$ to accelerate an electron to 10 times the thermal energy, as required by the flare observations
  - Self-consistency requires that the acceleration region has a size $L \approx z_E$
- With that, we get $\kappa = 0.1154(L/\lambda_c)(L/\lambda)$
  - It seems that turbulent mfp $\lambda \approx L$ so that we get $\kappa \approx 0.1/\mathrm{Kn}$
  - So $\kappa = 5$ requires $\mathrm{Kn} = 0.02$, which is not too different from the inferred value
  - If we rescale it to $\mathrm{Kn} = 0.005$ at $\kappa = 5$, then it becomes $\kappa \approx 0.025/\mathrm{Kn}$
  - Although if turbulent mean free path < size of acceleration region, then this would also make kappa larger for a given Kn