How does the spatial scale of the T fluctuations affect the plane-of-sky variation?

TS4

Turbulence during the approach to IAH

If the line-of-sight crosses many regions of different T, then the variation in the average T is reduced.

Assume that a fraction f of the emissivity comes from gas with $T = T_0(1+\delta)$ whereas the remaining fraction $(1-f)$ comes from $T = T_0$

Better yet: assume that we have TS5

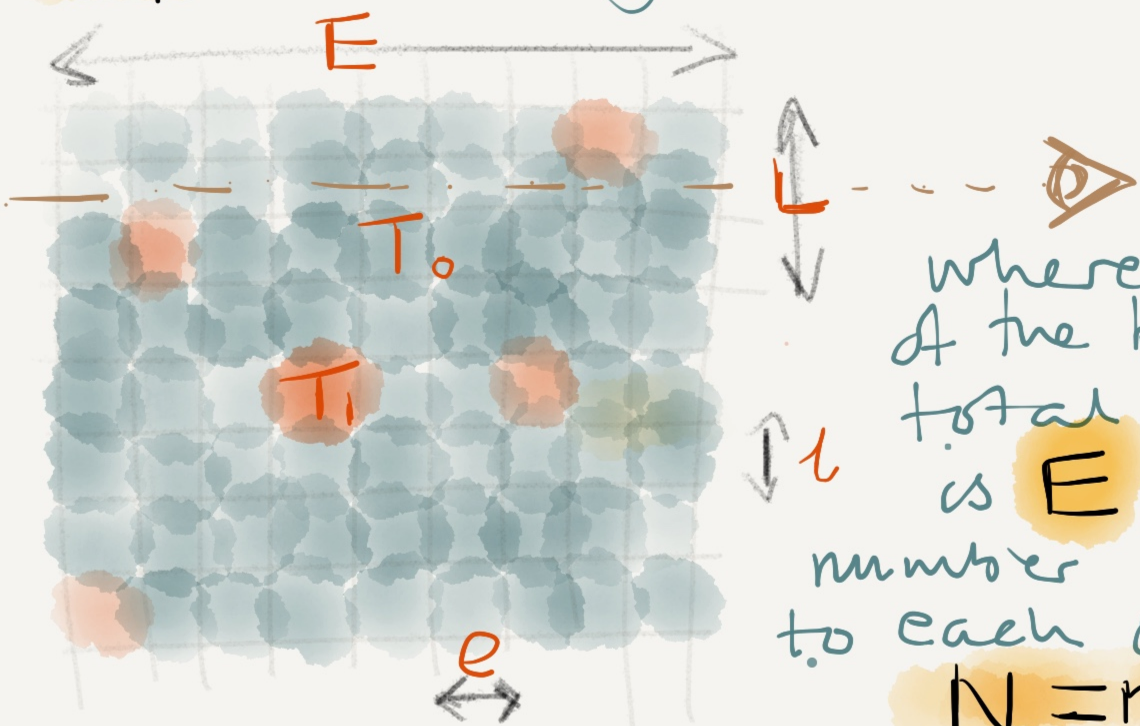
a large number of emission blocks,

each with exactly the same emiss measure e

Each block has a probability p of being

"anomalous" $T = T_1 = T_0(1+\delta)$ and a probability

$1-p$ of being "normal": $T = T_0$



We observe on an angular scale L whereas the angular scale of the blocks is l. The total emission measure is E. Therefore the number of blocks that contribute to each observation is

$$N = \max(1, EL^2/el^2).$$

Mean temperature: $\bar{T} = [1 - p + p(1+\delta)]\,T_0$ TS6

$$\bar{T} = (1 + p\delta)\,T_0$$

Fluctuations: $$t^2 = \frac{(T_0 - \bar{T})^2 (1-p) + (T_1 - \bar{T})^2 p}{\bar{T}^2}$$

$$(T_0 - \bar{T})^2 = p^2 \delta^2 T_0^2$$

$$(T_1 - \bar{T})^2 = (1-p)^2 \delta^2 T_0^2$$

$\Rightarrow$ numerator is

$$T_0^2 p(1-p)\delta^2 [p + (1-p)] = p(1-p)\delta^2 T_0^2$$

$$\Rightarrow t^2 = \frac{p(1-p)\delta^2}{(1+p\delta)^2}$$

If $p \ll 1$ and $p\delta \ll 1$, then $\boxed{t^2 \simeq p\delta^2}$ Note that we don't necessarily have δ being small — it can be large so long as $p \ll 1$ and t^2 will still be small. Also, we can have $\delta < 0$, corresponding to cool spots.

TS7

So there are two different ways of getting a small t^2 value: $t^2 \ll 1$

A. A checkerboard ($p \sim 0.5$) of small fluctuations ($\delta^2 \ll 1$) $\Rightarrow$ $t^2 \simeq 0.25\,\delta^2$

B. A sprinkling ($p \ll 1$) of large fluctuations ($\delta \sim 1$) $\Longrightarrow$ $t^2 \approx p$

Note that it is not reasonable to have δ much larger than unity since collisional ionization would change the ionization state and the line spectrum.

When we measure the T of a region that contains N blocks (see TS5), then on average there will be $\bar{n} = pN$ anomalous blocks, but there will really be a spread of values $n = \bar{n} \pm \sqrt{\bar{n}}$ (if $\bar{n}$ is larger than a few).

So the mean T of a single patch of scale L will be TS8

$T(L) = (1 + f\delta) T_0$ where $f = \frac{n}{N}$ is the actual fraction of anomalous blocks in the patch and where n $P(n;\bar{n})$ is drawn from a Poisson distribution of mean $\bar{n} = pN$

$$\left(T(L) - \bar{T}\right)^2 = (f-p)^2 \delta^2 T_0^2$$

The plane-of-sky temp fluctuations are therefore

$$\tilde{t}^2(L) = \frac{1}{T_0^2} \left\langle \left(T(L) - \bar{T}\right)^2 \right\rangle$$

$$= \int_0^\infty dn\, P(n,\bar{n}) \left(\frac{n}{N} - p\right)^2 \delta^2$$

$$= \frac{\delta^2}{N^2} \mathrm{Var}(n) = \frac{p\,\delta^2}{N} = t^2/N$$

For the two phase model we had $t^2 = \frac{p(1-p)\delta^2}{(1+p\delta)^2} \simeq p\delta^2$ **TS9**

Whereas the extra heating or cooling can be found as

$$L = L_0(1-p) + pL_0(1+\delta)^a \qquad a \equiv \frac{d\ln L}{d\ln T}$$

In the case that $\delta \ll 1$ then we can simplify this $L = L_0(1 + ap\delta)$ where L_0 is the equilibrium cooling @ $T = T_0$

Using fact that $\delta \simeq (t^2/p)^{1/2}$

$$\Rightarrow L \simeq L_0\left(1 + ap^{1/2}(t^2)^{1/2}\right)$$

so assuming thermal equilibrium $H = L$ we have the fractional extra heating required as

$$\frac{H - H_0}{H_0} = \frac{L - L_0}{L_0} = ap^{1/2}(t^2)^{1/2} \equiv h$$

TS10

If $\delta \simeq 1$ then $t^2 \simeq p$ and we would have $h \simeq ap \simeq at^2$ in the above approx.

(of course we don't satisfy $\delta \ll 1$ so we need to do it properly).

On the other hand, if $p \simeq 0.5$ then $t^2 \simeq \frac{1}{4}\delta^2$ and $h \simeq \frac{1}{2}a\delta \simeq a(t^2)^{1/2}$

For the Ring Nebula, we have $t^2 \simeq 0.04$ from the ADF. Or $t_A^2 \simeq 0.005$ for the part at small/medium scales. The cooling sensitivity index $a \simeq 2$ I think, which would imply $h \simeq 0.4$ to reproduce the ADF t^2, if we had high filling factor small amplitude fluctuations. This is a lot of extra heating!

TS11

Going back to the rare, large amplitude case, we had $h = p[(1+\delta)^a - 1]$

If we take $\delta \simeq 1$ and $a \simeq 2$ this is $h \simeq 3p \simeq 3t^2$. This is much easier to satisfy since $t^2 = 0.04 \implies h \simeq 0.12$

Although this is still rather high.

If we only needed to explain the observed $t_A^2 \simeq 0.005$ then we would only require $h \simeq 1.5\%$, which is much more reasonable

What fraction of the total radiative energy absorbed by an HII region is retained as thermal energy?