國立臺灣大學管理學院資訊管理學系

碩士論文

Institute of Information Management

College of Management

National Taiwan University

Master Thesis

基於總體成本與駕駛與乘客間公平性之最佳化共乘演算法

An Optimization Based Carpool Algorithm in Consideration of the Total Cost and Fairness among the Driver and All Passengers

黃文鴻

Wen-Hong Huang

指導教授: 林永松 博士

Advisor: Frank Yeong-Sung Lin Ph.D.

中華民國 110 年 6 月

June, 2021

摘要

共享經濟是隨著網路以及行動裝置普及下逐漸興起的熱潮，傳統的計程車與共乘服務在共享經濟的風潮下產生出新的商業模式，不同於以往使用者必須到目的地才能知道價錢，或是只能針對特定路線如通勤、通學等才容易有共乘機會；透過線上叫車服務，使用者除了可以事先知道價錢，還可以透過共乘配對服務，自動配對有相近路線的乘客，共同分攤車資，提供使用者更便宜經濟的選擇。

在自動配對的共乘服務中，往往需要多繞路以同時滿足不同乘客間的載運需求，如何讓使用者之間的車資分配符合公平性，便是重要的挑戰。本研究考慮共享經濟中共乘服務的多乘客路線規劃問題，將乘客可接受的抵達時間、繞多少路的可接受程度，車輛人數限制，以及載客的優先順序納入考量，以最大化最小的乘客共乘節費比例，並透過拉格朗日鬆弛法以取得最佳解。

關鍵字：共乘、共享經濟、車輛路徑問題、公平性、斯坦納樹問題、拉格朗日鬆弛法

Abstract

The sharing economy is a boom with the rise of the Internet and mobile devices. Traditional taxis and ride-sharing services have produced new business models under the sharing economy. Unlike in the past, users can know the price only if they go to the destination. It is not easy to have shared ride opportunities if it is not the commutation route for work and school. Instead, in the online ride-hailing service, users can know the price in advance. The service can also automatically match passengers with similar routes in real-time. Sharing the fare provides users with a cheaper and more economical choice.

In the ride-sharing service with automatic matching, detours often happen when there are multiple riders with different needs at the same time. How to make it fair to charge the riders is a significant challenge. This research considers the problem of multi-riders route planning for ride-sharing services. Considering the acceptable arrival time, the acceptable degree of detours, vehicles' capacity, and passengers' priority order constraints. Maximize the minimal rider's savings ratio compared to self ride-hailing with carpooling. Use the

Lagrangian relaxation method to obtain the best solution.

Keywords: Carpooling, Sharing economy, Vehicle routing problem, Fairness, Steiner Tree Problem, Lagrangian relaxation

Contents

List of Figures

List of Tables

Chapter 1 Introduction

1.1 Background

In recent years, with the rising of the Internet and mobile devices, information sharing has become a daily thing. Some people began to put idle resources on the Internet to share and exchange with people. However, sharing with strangers means User risk. With the invention of user rating systems, sharing platforms can begin to know whether users are good or bad through the rating, which reduces the risk in online sharing[1]. As a trend, from food delivery, accommodation to transportation, the sharing platforms seem to be a part of daily life. People call this new concept a ”sharing economy.”

There is currently no clear definition of the sharing economy so far. Generally speaking, activities in the sharing economy contain four categories: recycling of goods, increasing the use rate of durable goods, exchange of services, and sharing of operating assets[1]. All of the activities are about the re-allocation of ”idle assets” [2]. No matter the exchange of resources or earning from the sharing platforms, low transaction costs make the idle assets more valuable. The demander can use the asset at a lower cost, increase the overall utilization rate of social assets. ”Renting rather than buying” changes the concept of assets.

The spirit of re-allocation of idle assets in the sharing economy has extended the taxis and ride-sharing services. Compared with traditional public transportation services, the new type of transportation services under the sharing economy have the following features:

1. Efficient use of resources: Hailing by the roadside, phone appointments are the traditional taxis method. Take Taiwan as an example. According to data from the Ministry of Transport, street hailing, taxi stands, and scheduling [3] are more than 50%, which can increase the time vacancy rate of taxis and cause waste of resource. Through the ride-hailing platform, a ride request can be matched on the platform at any time. Riders do not need to wait on the roadside. They only need to use their mobile phones to know the arrival time and journey time, reducing uncertainty and improving car usage rate. 2. Dynamic resources: Drivers on the ride-hailing service can decide their working hours with flexibility by log in and logout. Therefore, compared with traditional taxis with fixed resources, the ride-hailing platform's resources are highly dynamic. 3. Dynamic ride-sharing: Some ride-hailing apps also provide ride-sharing services, such as UberPool and Lyft Line. As long as drivers turned on the ride-sharing option, riders with similar routes can be matched in time. Not only can the fare be shared, but it also can reduce the waste of vacant seats in the journey.

From the above features, we can understand that both the supplier and the demander have a high degree of dynamics and uncertainty on the new type of ride-sharing service. Finding a suitable route from many dynamic constraints will be a big challenge.

1.2 Motivation

In the early, ride-sharing was necessary to go carpooling by matching at a taxi station, a fixed commuter route or self-organized among people who knew each other [4]. With the maturity of mobile devices, they are widely used in every corner of life, such as food, clothing, housing, and transportation. Carpooling is a classic example. In the sharing economy era, when one takes out his/her mobile phone, the ride-sharing service can make better use of the "idle assets" such as seats and vehicles. Drivers can now earn more by detour s for ride-sharing. Riders can share the fare with others at a cost-effective price.

However, a trip may consist of many riders with different origins and destinations; charging riders with fairness is a big problem. There are detours and waiting times among the riders, which may be varied. Some people likely detour a lot in a shared ride, and some people hardly detour but have to pay a similar fare. It is very unfair for the riders in the same carpooling. Therefore, making it fair between passengers in the ride-sharing problem is the object of this article.

1.3 Objective

This research aims to maximize the minimal percentage of cost-saving between a rider choose to hail a ride on his/her own and go carpooling. In order to ensure there is no significant deviation from what a rider expects. Constraints are applied with a ride request of origin and destination, car capacity, and detour limitation.

1.4 Thesis Organization

The rest of the paper is organized as follows. We will go through the related work of the carpooling problem in Chapter 2. Chapter 3 will describe the carpooling problem in detail and formulate it into a mathematical model.

Chapter 2 Literature Review

2.1 Related Work on Carpooling

This section will go through the related work and discuss the approaches for the carpooling problem. Before this section, we will make a brief throughout the history of carpooling. Carpooling has risen since World War II, as all the resources are for the war. At that time, carpooling is a tactic for rationing gasoline through car-sharing clubs, which match riders and drivers via a bulletin board. As technology progresses, the carpooling service starts to integrate the Internet and mobile phones in recent years. Inbuilt location-based service (LBS) on smartphones makes it easy to get a user's location in real-time. Making it possible to match drivers and riders on-demand at any time [4].

Coja-Oghlan et al. [5] see the carpooling problem as a dial-a-ride problem (DARP) that allocates all resources through a centralized server. This paper treats the graph of the road network as a caterpillar tree, which is an undirected graph $G = (V, E)$ and all the vertices are on a central backbone. The self-developed Minimal Spanning Tree (MST) heuristic method can find an optimal solution in polynomial time with probability. By applying the bipartite Steiner tree, we can also find the optimal solution in polynomial time at random, which indicates that $NP = RP$. Overall, NP-hard is the worst cast, while on average is solvable efficiently.

Farin and Williams [6] propose a fair carpooling scheduling algorithm in different approaches focused on driving rotation on any given day when a given number of members form a carpooling club. There are the following factors to concern: who drives today and how many riders are on the carpool. This research also concerns the ”cost” of the trip as the least common multiple of 1,2,...,n, where n is the largest number of people participating in the carpool. The driver earns the ”cost” while the riders share it. Who has the least ” cost” is the driver of the next day.

Wolfson and Lin [7] indicate that ridesharing fairness is a new a problem in studies. The concept in this paper is related to the Nash Equilibrium. This paper proposes a payment scheme called Guaranteed-Ridesharing-Fairness (GRF) that guarantees each passenger/trip saves in an optimal ridesharing plan (RSP), which has no other RSP has saving greater than any RSP. In a GRF method, it combines both fair and optimal ridesharing queries.

2.1.1 Dial-a-ride Problem

When it comes to carpooling, the dial-a-ride problem could be one of the best discussions about this kind of scenario. Dial-a-rider problem (DARP) is a problem for finding vehicle routes to serve users with requests with specific origins and destinations, with objective to minimize cost or maximize satisfied demand [8]. DARP has been widely researched for about 50 years [9] and is considered as an NP-hard problem [10]. There are many variations in the discussion. Cordeau and Laporte [8] indicate that DARP mainly consists of the following factors:

- Vehicles: There might be 1 or n vehicles in the system. Most DARP studies assume that vehicles are with homogeneous capacity; some studies may use a heterogeneous fleet with different capacity.

- Requests: A request usually consists of these parts: passengers, origins, destinations, and time constraints. Static or dynamic is also a significant concern. For a static DARP, all the ride requests will be known in advance, and the central scenario is to allocate all the resources once. A dynamic DARP usually discusses ride request planning in a specific time window. For better understanding, we will use ride requests instead of requests.

- Routes: Routes of the road network usually consist of length and duration.

- Detour: Because a route of DARP is aggregate from many must-pass nodes proposed by ride requests. Rather than the shortest path, every passenger may have a detour.

A DARP is a kind of problem that consists of n must-pass nodes with the lowest cost. We will use a Steiner tree problem for finding routes.

2.1.2 Steiner Tree Problem

A Steiner tree problem is to find a minimum cost tree in given subset vertices of a given graph. When it comes to the Steiner tree, a minimum spanning tree problem can be mentioned together. While a minimum spanning tree problem is to find a minimum total weight that the links have to go through all the vertices in a given graph, a Steiner tree problem treats the vertices as terminal and non-terminal vertices. All terminals must go through, and the non-terminal vertices are allowed to contain to reduce cost [11]. Figure

2.1 as an example; this figure is an undirected graph $G = (V, E, w)$ with nonnegative weights. A set of vertices $L \subset V$ is the terminals. The minimal Steiner tree is to find a tree that passes all the terminals that can include other vertices in the graph. We could find a Steiner minimal tree shown in Figure 2.2 with cost = 5 [12]. According to Foulds and Graham [13], finding a Steiner minimal tree (SMT) is an NP-Complete problem, which has no deterministic algorithms to solve in the polynomial time. As a result, we tend to use heuristic approaches to this problem.

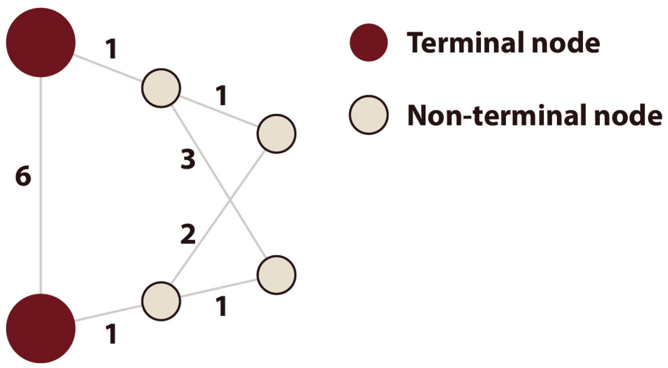

Figure 2.1: Example graph of a Steiner tree problem

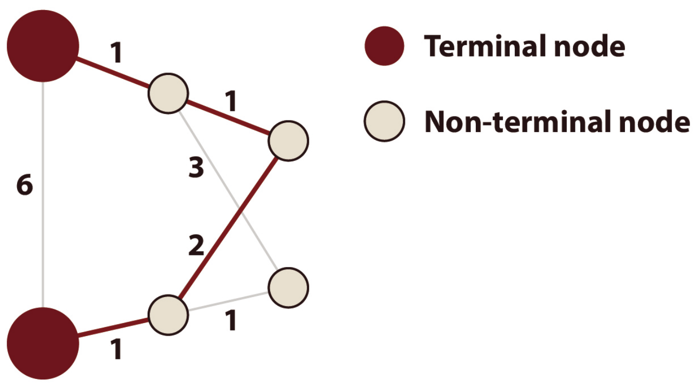

Figure 2.2: Steiner minimal tree with cost = 5

2.2 Fairness

Fairness is the central issue in this research. According to the Cambridge dictionary definition, fairness is ”the quality of treating people equally or in a way that is right or reasonable” [14]. There are many factors to be considered when it comes to carpooling. One of them is the mechanism of dispatching ride requests to drivers. Suppose a driver in the carpooling system is not treated equally, such as not receiving any ride request in the rush hour while others receive a lot. In that case, the driver might choose not to use the carpooling system. However, getting rid of unequal ride requests dispatching is not easy. There are many possibilities for drivers to think they are treated equally or unequally. For instance, one may think it is equal to receive the same ride requests as others. In contrast, another may think to be treated unequally for participating in carpooling for a long time. To get rid of the issue in such a resource allocation scenario, we define fairness in a mathematical approach to measure fairness. There are many indices to measure the fairness of resource allocation [15]. In this paper, we use Raj Jain’s approach, Jain’s fairness index [16], to know the status of ride request allocation by calculation.

2.2.1 Jain’s Fairness Index

Jain’s fairness index is a metric to measure the fairness of the resource allocation in a system, which is used in the networking engineering field. By definition, there are n users share resource in a system, a user $i \in \{1, 2, 3, ..., n\}$ allocates x_i resource [16]. The following is the formula:

$$Firness\ Index = f_A(x) = \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n \sum_{i=1}^{n} x_i^2}$$

The index ranges from $\frac{1}{n}$ to 1. When there is only one user monopoly to all resource, which means the others get no resource, the fairness index will be $\frac{1}{n}$. It is the most unfair scenario. When all the users get the same resource, the fairness index will be 1, which is the fairest scenario.

Chapter 3 Problem Formulation

In this chapter, we will introduce the scenario of the carpooling fairness problem. Then formulate the problem into a mathematical model with given parameters and decision variables.

3.1 Problem Description

The object of this research is to minimize the maximum percentage of cost saving when a passenger chooses to carpool, with consideration of the number of drivers, car capacity and routing limit constraints.

Take Figure 3-1 as an example scenario; Passenger 1 and Passenger 2 are requesting a trip. When Passenger 1 chooses to take a ride by himself/herself, called "exclusive" riding shown as Figure 3-2, it would cost \$5. We can describe the scenario as a shortest path problem from S to D_1, and P_1 is a must-pass node. We use Steiner tree to solve this kind of shortest path problem with must-pass nodes. Passenger 2's "exclusive" riding would also cost \$5 in Figure 3-3. In this case, it would be a shortest path problem from S to D_2 with P_2 as a must-pass node.

When the passengers choose to take a carpool, which is called "sharing" riding. We can describe the scenario as a shortest problem from S to D_1 with P_1, P_2 and D_2 as must-pass nodes or S to D_2 with P_1, P_2 and D_1 as must-pass nodes. In Figure 3-4 is one of the best case, which would cost $8 for all the passengers, in Figure 3-5 would cost the same $8 and both of the fares they could share are $5 ($\overline{P_2D_1}$ and $\overline{P_2D_2}$ respectively); however, in Figure 3-4 would cost $\overset{\overline{P_1P_2}}{1} + \frac{1}{2} \times \overset{\overline{P_2D_1}}{5} = \$3.5$ for Passenger 1 and $\frac{1}{2} \times \overset{\overline{P_2D_1}}{5} + \overset{\overline{D_1D_2}}{1} = \$3.5$ for Passenger 2, in Figure 3-5 would cost $\overset{\overline{P_1P_2}}{1} + \frac{1}{2} \times \overset{\overline{P_2D_2}}{5} + \overset{\overline{D_2D_1}}{1} = \$4.5$ for Passenger 1 and $\frac{1}{2} \times \overset{\overline{P_2D_2}}{5} = \$2.5$ for Passenger 2.

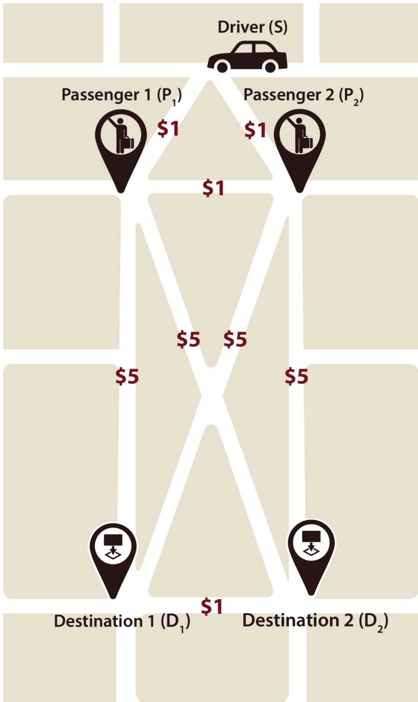

Figure 3.1: Example road network with driving fare and points of passengers and their destinations

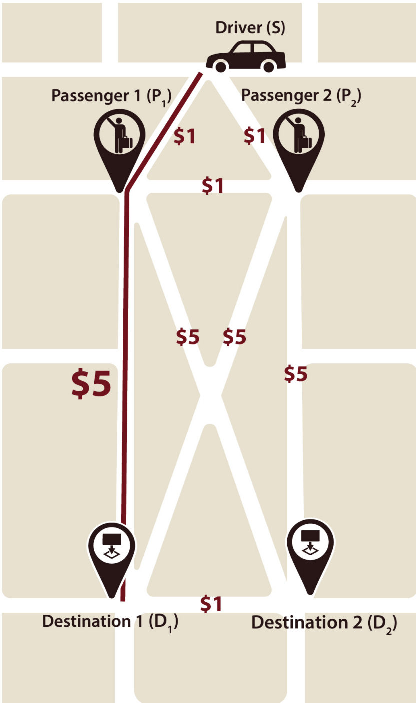

Figure 3.2: Best routing path when the driver only serving Passenger 1

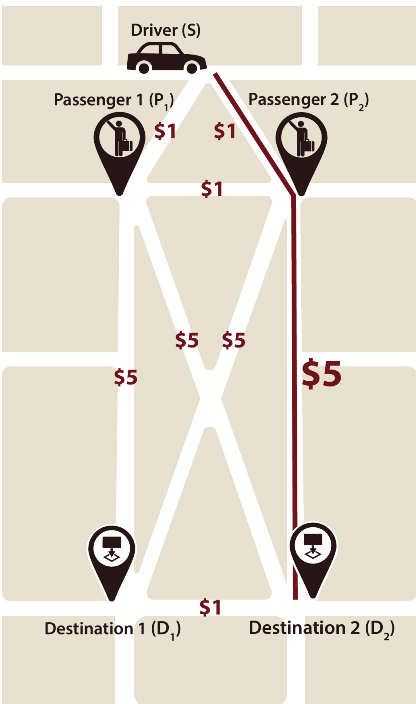

Figure 3.3: Best routing path when the driver only serving Passenger 2

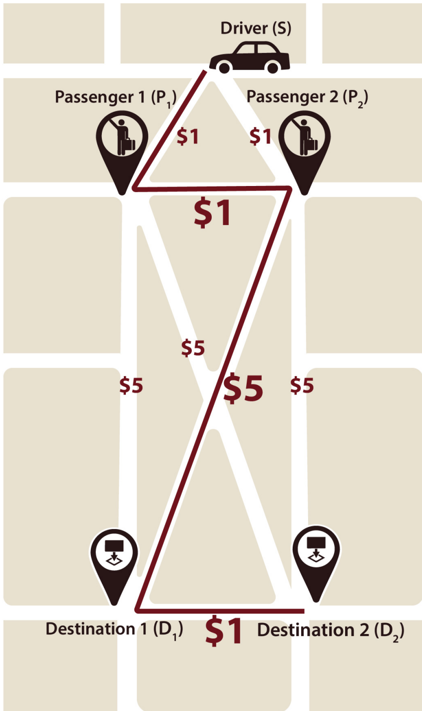

Figure 3.4: One of the best routing when both Passenger 1 and Passenger 2 carpool

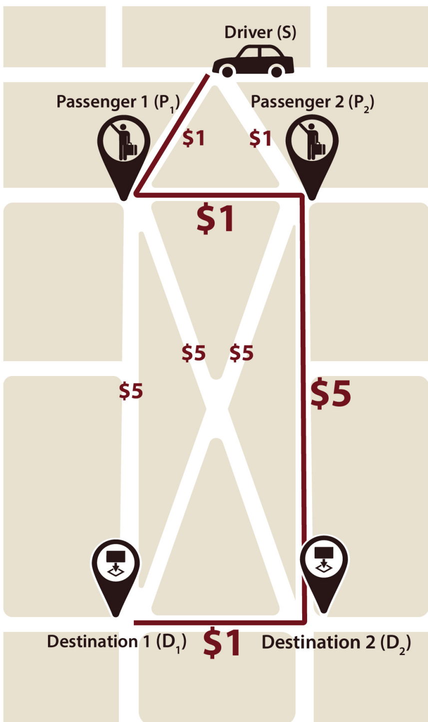

Figure 3.5: Another best routing when both Passenger 1 and Passenger 2 carpool

3.1.1 Assumptions

There are three assumption in out system:

1. System calculation time is not considered, that is, assume that drivers' location will still keep unchanged after the time of system calculation.

2. The carpooling fare is shared equally by the time passengers in the vehicle. For a specific link in the route with n passengers, the fare of the link for a passenger will be divided by n.

3. All the passengers on the trip will accept the offer of carpooling.

4. All the drivers will accept any riding request when system dispatches the riding request to the driver.

5. Total driving time is equal to sum of time spending of links that pass by.

6. The number of drivers is sufficient that all the riding requests will be accepted in the decision round.

3.1.2 Fairness of Vehicle Dispatching

In order to consider the fairness when dispatching driver, we introduce Jain's fairness index to avoid unfair situation:

$$Firness\ Index = f_A(x) = \frac{\left(\sum\limits_{i=1}^{n} x_i\right)^2}{n \sum\limits_{i=1}^{n} x_i^2}$$

We take accumulate driving requests that a driver has received as the metrics for fairness. That is, when a driver receives more driving requests in the past, he/she should give his/her chance of receiving a new driving request to the one who takes fewer driving requests.

As a new driving request comes, we need to ensure our system must get more or at least keep the same fairness. Hence, we form a constraint that the fairness index can not decrease compared to the previous fairness index.

However, when the driving resource changes, such as a new driver joins our system, it will violate our constraint for fairness. Because the newcomer has not taken any driving request before, the fairness will decrease. Even if a driver leaves our system, the fairness will increase or decrease due to the driver's situation. As a result, we need to reset the fairness index to the initial state and reset all the drivers' accumulated driving requests to 0. Nevertheless, it will make the fairness index's denominator to 0, which can not be divided. To avoid this situation, we assign a virtual driver with one accumulated driving request, which will make the fairness index to $\frac{1}{(n+1)}$, where n is the number of drivers in our system. Moreover, for any situation that the fairness index will decrease when dispatching a new driving request, the fairness will reset.

3.2 Mathematical Model

We will go through the detailed given parameters and decision variables in the mathematical model before describing our mathematical model.

The road network is represented as a weighted graph $G = (N, L)$, where N is the nodes and L is the links in the graph.

A riding request consists with following parameters: origin node, destination node, number of passengers, start time and deadline for driver to pick up.

Table 3.1: Notations of given parameters

Notation	Description
C	Set of cab drivers in the system
R	Set of riding requests in the system (take i as index)
R'	Set of riding requests which are not dispatched in current decision round, $R' \in R$
O	Set of origin nodes of riding requests
D	Set of destination nodes of riding requests
N	Set of nodes on the road network, which include nodes of cab drivers, origin and destination nodes of riding requests, $N \in C \cup O \cup D$
L	Set of links on the road network. A link can connect with any two nodes in the road network.
o_i	Origin node of riding request $i \in R$, $o_i \in O$
d_i	Destination node of riding request $i \in R$, $d_i \in D$
q_i	Number of passengers associates with riding request $i \in R$
h_i	Maximum passengers limit of carpooling associates with riding request $i \in R$

Table 3.1: Notations of given parameters

Notation	Description
t_i	Remaining time to the start time of riding requesting $i \in R$ for driver to pick up in current decision round.
u_i	Remaining time to the deadline of riding requesting $i \in R$ for driver to pick up in current decision round.
T	Maximum additional buffer time to wait for pick up in the system.
Q_c	Maximum load capacity for driver, $c \in C$
L_O	Set of artificial links of origin nodes
L_D	Set of artificial links of destination nodes
P_{co_i}	Set of paths from current location of driver $c \in C$ to origin node of riding request $i \in R$
P_{cd_i}	Set of paths from current location of driver $c \in C$ to destination node of riding request $i \in R$
P_c	Set of available paths from current location of driver $c \in C$ to a virtual node out of the road network. A virtual node is connected with all of nodes in the road network, with 0 weight links.
t_{cl}	The driving time for driver $c \in C$ on the link $l \in L$
t_{ci}	The driving time of exclusive riding request $i \in R$ for driver $c \in C$
f_{cl}	The fare rate for driver $c \in C$ on the link $l \in L$
f_{ci}	The fare rate of exclusive riding request $i \in R$ for driver $c \in C$
E	Maximum detour time ratio of hitchhiking for passengers in a riding trip.
δ_{pl}	Indicator function which is 1 if link $l \in L$ on the route $p \in P_c \cup P_{co_i} \cup P_{cd_i}$
r_c	Total fare that the driver $c \in C$ has earned before current decision round.

Table 3.1: Notations of given parameters

Notation	Description
α	The fairness index in the previous decision round.

Table 3.2: Notations of decision variables

Notation	Description
x_p	Binary variable, 1 if route $p \in P_{co_i}$ is chosen for driver $c \in C$; 0 otherwise.
y_p	Binary variable, 1 if route $p \in P_{cd_i}$ is chosen for driver $c \in C$; 0 otherwise.
z_p	Binary variable, 1 if route $p \in P_c$ is chosen for driver $c \in C$; 0 otherwise.
w_{ci}	Binary variable, 1 if riding request $i \in R$ is chosen for driver $c \in C$; 0 otherwise.
s_{cl}	Binary variable, 1 if link $l \in L \cup L_O \cup L_D$ is passed by driver $c \in C$ in carpooling; 0 otherwise.

Objective Function

The objective function is to maximize the minimum discount percentage when applying carpooling among passengers in one trip.

$$\max_{i\in R}\min_{c\in C}\frac{w_{ci}f_{ci}-carpool\ cost_c}{w_{ci}f_{ci}} \tag{IP1}$$

Constraints

Constraint (3.1), (3.2) ensures the pick up time is suit for the riding request.

$$\sum_{p\in P_{co_i}}\sum_{l\in L}x_p\delta_{pl}t_{cl}\geq t_i-T \qquad \forall c\in C, i\in R' \tag{3.1}$$

$$\sum_{p\in P_{co_i}}\sum_{l\in L}x_p\delta_{pl}t_{cl}\leq u_i+T \qquad \forall c\in C, i\in R' \tag{3.2}$$

Constraints (3.3), (3.4), (3.5) ensure there is at most one path selected for a driver.

$$\sum_{p\in P_{co_i}}x_p\leq 1 \qquad \forall c\in C, i\in R' \tag{3.3}$$

$$\sum_{p\in P_{cd_i}}y_p\leq 1 \qquad \forall c\in C, i\in R' \tag{3.4}$$

$$\sum_{p\in P_c}z_p\leq 1 \qquad \forall c\in C \tag{3.5}$$

Constraint (3.6) ensures a driver $c \in C$ have one path when the driver accepts any riding requests, where M is an arbitrarily large number.

$$\sum_{i \in R'} w_{ci} \leq (\sum_{p \in P_c} z_p) M \qquad \forall c \in C \qquad (3.6)$$

Constraint (3.7) ensures a driver $c \in C$ have no path when the driver does not accept any riding requests, where ϵ is an arbitrarily small number.

$$\sum_{p \in P_c} z_p \leq \sum_{i \in R'} w_{ci} + \epsilon \qquad \forall c \in C \qquad (3.7)$$

Constraint (3.8) ensures a riding request only be assigned by at most one driver.

$$\sum_{c \in C} w_{ci} \leq 1 \qquad \forall i \in R' \qquad (3.8)$$

Constraint (3.9) ensures a riding request be picked up before dropping off.

$$\sum_{p \in P_{co_i}} \sum_{l \in L} x_p \delta_{pl} t_{cl} \leq \sum_{p \in P_{cd_i}} \sum_{l \in L} y_p \delta_{pl} t_{cl} \qquad \forall c \in C, i \in R' \qquad (3.9)$$

Constraints (3.10), (3.11), (3.12) ensure paths to origin or destination is overlapped with the carpooling path.

$$\sum_{p \in P_c} z_p \delta_{pl} = s_{cl} \qquad \forall l \in L \cup L_O \cup L_D, c \in C \tag{3.10}$$

$$\sum_{p \in P_{co_i}} x_p \delta_{pl} \leq s_{cl} \qquad \forall l \in L \cup L_O \cup L_D, i \in R', c \in C \tag{3.11}$$

$$\sum_{p \in P_{cd_i}} y_p \delta_{pl} \leq s_{cl} \qquad \forall l \in L \cup L_O \cup L_D, i \in R', c \in C \tag{3.12}$$

Constraint (3.13) ensures that riding requests not exceed capacity of the cab

$$\sum_{i \in R'} \sum_{p \in P_{cd_i}} y_p \delta_{pl} w_{ci} q_i - \sum_{i \in R'} \sum_{p \in P_{co_i}} x_p \delta_{pl} w_{ci} q_i \leq Q_c \quad \forall l \in L, c \in C \tag{3.13}$$

Constraint (3.14) ensures detour time ratio of carpooling to exclusive riding for every riding requests not be larger than limit of the system.

$$\frac{\sum_{p \in P_{cd_i}} \sum_{l \in L} y_p \delta_{pl} w_{ci} t_{cl} - \sum_{p \in P_{co_i}} \sum_{l \in L} x_p \delta_{pl} w_{ci} t_{cl}}{w_{ci} t_{ci}} \leq E \quad \forall c \in C, i \in R' \tag{3.14}$$

Constraint (3.15) ensures that riding requests not exceed capacity limit of the riding request

$$\sum_{i \in R'} \sum_{p \in P_{cd_i}} y_p \delta_{pl} w_{ci} q_i - \sum_{i \in R'} \sum_{p \in P_{co_i}} x_p \delta_{pl} w_{ci} q_i \leq h_i \quad \forall l \in L, c \in C \tag{3.15}$$

Constraint (3.16), (3.17), (3.18), (3.19) ensure the upper bounds and the lower bounds of decision variables x_p, y_p, z_p and w_{ci} respectively.

$$x_p \in \{0,1\} \qquad \forall p \in P_{co_i}, c \in C, i \in R' \tag{3.16}$$

$$y_p \in \{0,1\} \qquad \forall p \in P_{cd_i}, c \in C, i \in R' \tag{3.17}$$

$$z_p \in \{0,1\} \qquad \forall p \in P_c, c \in C \tag{3.18}$$

$$w_{ci} \in \{0,1\} \qquad \forall c \in C, i \in R' \tag{3.19}$$

References

[1] J. Schor, “Debating the Sharing Economy,” https://greattransition.org/publication/debating-the-sharing-economy, May 2014.

[2] K. Frenken, T. Meelen, M. Arets, and P. van de Glind, “Smarter regulation for the sharing economy,” The Guardian, May 2015.

[3] “100 年計程車營運狀況調查報告,” 2012.

[4] N. D. Chan and S. A. Shaheen, “Ridesharing in North America: Past, Present, and Future,” Transport Reviews, vol. 32, no. 1, pp. 93–112, Jan. 2012.

[5] A. Coja-Oghlan, S. Krumke, and T. Nierhoff, “Average Case Analysis of a Hard Dial-a-Ride Problem,” May 2003.

[6] R. Fagin and J. H. Williams, “A Fair Carpool Scheduling Algorithm,” IBM Journal of Research and Development, vol. 27, no. 2, pp. 133–139, Mar. 1983.

[7] O. Wolfson and J. Lin, “Fairness versus Optimality in Ridesharing,” in 2017 18th IEEE International Conference on Mobile Data Management (MDM), May 2017, pp. 118–123.

[8] J.-F. Cordeau and G. Laporte, “The dial-a-ride problem: Models and algorithms,” Annals of Operations Research, vol. 153, no. 1, pp. 29–46, Sep. 2007.

[9] N. H. Wilson, J. M. Sussman, H.-K. Wong, and T. Higonnet, Scheduling Algorithms for a Dial-a-Ride System. Massachusetts Institute of Technology. Urban Systems Laboratory, 1971.

[10] J.-F. Cordeau, “A Branch-and-Cut Algorithm for the Dial-a-Ride Problem,” Operations Research, vol. 54, no. 3, pp. 573–586, 2006.

[11] B. Y. Wu and K.-M. Chao, “Steiner Minimal Trees$_*$.”

[12] 吳 志 峰, “The Steiner Tree Probelm,” http:// par.cse.nsysu.edu.tw/ ~homework/ algo01/9034811/Report/index.htm.

[13] L. R. Foulds and R. L. Graham, “The steiner problem in phylogeny is NP-complete,” Advances in Applied Mathematics, vol. 3, no. 1, pp. 43–49, Mar. 1982.

[14] “FAIRNESS | definition in the Cambridge English Dictionary,” https:// dictionary.cambridge.org/us/dictionary/english/fairness.

[15] R. Jain, “99-0045 Throughput Fairness Index: An Explaination,” p. 13.

[16] R. Jain, D. Chiu, and W. Hawe, “A Quantitative Measure Of Fairness And Discrimination For Resource Allocation In Shared Computer Systems,” arXiv:cs/9809099, Sep. 1998.