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基於總體成本與駕駛與乘客間公平性之最佳化共乘演算法

An Optimization Based Carpool Algorithm in Consideration of the Total Cost and Fairness among the Driver and All Passengers

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摘要

共享經濟是隨著網路以及行動裝置普及下逐漸興起的熱潮，傳統的計程車與共乘服務在共享經濟的風潮下產生出新的商業模式，不同於以往使用者必須到目的地才能知道價錢，或是只能針對特定路線如通勤、通學等才容易有共乘機會；透過線上叫車服務，使用者除了可以事先知道價錢，還可以透過共乘配對服務，自動配對有相近路線的乘客，共同分攤車資，提供使用者更便宜經濟的選擇。

目前的線上共乘配對服務，為共乘平台針對目前行徑中或是配對中的乘客透過運算，找尋最適合的路線與乘客。自動配對的共乘服務中，往往需要多繞路以同時滿足不同乘客間的載運需求，如何讓使用者之間繞路多寡符合公平性，便是重要的挑戰。本研究考慮共享經濟中共乘服務的多乘客路線規劃問題，將乘客可接受的抵達時間、繞多少路的可接受程度，車輛人數限制，以及載客的優先順序納入考量，以最小化最大乘客共乘後節費比例，透過拉格朗日鬆弛法以取得最佳解。

關鍵字：共乘、共享經濟、車輛路徑問題、公平性、斯坦納樹問題、拉格朗日鬆弛法

Abstract

Keywords: Carpooling, Sharing economy, Vehicle routing problem, Fairness, Steiner Tree Problem, Lagrangian relaxation

Contents

Page

摘要 **3**

Abstract **5**

Contents **7**

List of Figures **9**

List of Tables **11**

Chapter 1 Introduction **1**

1.1 Background 1

1.2 Motivation 2

1.3 Objective 3

1.4 Thesis Organization 3

Chapter 2 Literature Review **5**

2.1 Related Work on Carpooling 5

2.1.1 Dial-a-ride Problem 6

2.1.2 Steiner Tree Problem 7

2.2 Fairness 9

2.2.1 Jain's Fairness Index 9

Chapter 3 Problem Formulation **11**

3.1 Problem Description 11

3.1.1 Assumptions 15

3.1.2 Fairness of Vehicle Dispatching 15

3.2 Mathematical Model 17

References **21**

List of Figures

2.1 Example graph of a Steiner tree problem 8
2.2 Steiner minimal tree with cost = 5 8
3.1 Example road network with driving fare and points of passengers and their destinations 12
3.2 Best routing path when the driver only serving Passenger 1 13
3.3 Best routing path when the driver only serving Passenger 2 13
3.4 One of the best routing when both Passenger 1 and Passenger 2 carpool . 14
3.5 Another best routing when both Passenger 1 and Passenger 2 carpool . . . 14

List of Tables

3.1 Notations of given parameters 17
3.2 Notations of decision variables 18

Chapter 1　Introduction

1.1　Background

近年來，隨著網際網路的普及以及幾乎人手一支的行動裝置帶動之下，資訊分享無遠弗屆，開始有人將閒置資源放到網路上與人們共享與交換，然而與陌生人共享意味者風險，隨著使用者評比系統的出現，共享平台開始可以透過評比知道使用者好壞，大大降低了網路共享行為的風險 [1]，此後這樣的共享平台逐漸推廣到生活的各個層面，蔚為風潮，從食物外送、住宿、乘車都可以見到他的身影，而人們稱呼這樣的新模式為「共享經濟」。

如何定義共享經濟目前並沒有統一的說法，一般來說，共享經濟中的活動主要分為四大類型：商品的再循環、增加耐久財的使用率、服務交換與經營性資產的共享 [1]。這些活動本質上圍繞著「閒置資產」的重新分配 [2]，不論是透過資源的交換，或從中賺取報酬，只要透過共享經濟平台的媒合，低廉的交易成本使得閒置資產擁有者將資產發揮更大的價值，而需求者能以較低廉的成本運用該資產，如此一來便能增進整體社會資產的使用率，甚至可以「以租代買」，改變以往必須擁有資產才能使用的思維。

共享經濟的閒置資產重新分配精神，為行之有年的計程車與共乘服務注入了活水，相對於傳統的大眾運輸服務，共享經濟下的新型態的運輸服務有以下的特

色：

1. 有效率的資源使用：傳統的計程車以往需要透過路邊攔車、電話叫車或是計程車隊的無線電派車模式，以台灣為例，根據交通部數據，巡迴攬客、招呼站等候以及定點排班 [3] 便佔了超過一半的比例，這樣的叫車模式，需要計程車在路上徘徊，或是在定點等候，容易增加計程車的時間空車率，造成資源浪費；透過叫車平台的派車服務，可以隨時媒合平台上的計程車，使用者也不須在路邊等候，只需要打開手機，便可明確知道計程車抵達時間，旅程時間，減少不確定性同時也可以提升計程車使用率。2. 動態的服務資源：叫車平台上的司機可彈性決定自己的工作時間，隨時可以透過登入與登出叫車 App，決定是否開放載客，因此相對於傳統無線電計程車有固定的派車資源，叫車平台的供給方有非常高度的動態性。3. 動態共乘：有些叫車 App 也提供共乘配對服務，如 UberPool 與 Lyft Line，只要開啟 App 中的共乘選項，便能及時配對有相近路線需求的乘客，不但可以一起分攤車資，還可以減少旅程中空位的浪費，然而對於平台設計者而言，如何在動態的路線需求中，規劃出能滿足各乘客的行車路線，並在媒合各乘客路線的同時，能有合理範圍的繞路。

從以上特性中，我們可以了解到，在新型態的叫車平台上，不論供需雙方都具有高度的動態性與不確定性，在叫車平台上的共乘服務，還需考量隨時從各地出現的路線需求中，找到合適的司機規劃出適合的路線，這對平台方來說，會是一大考驗。

1.2 Motivation

早期的共乘多半是要定點如計程車招呼站找人配對、由固定通勤路線長期配合，或是由互相認識的人有相近的路程 [4] 才比較有機會有共乘的可能。隨著行

動裝置的成熟與普及，使得行動裝置所能應用的領域呈現史無前例的廣泛，食衣住行育樂無所不包，任何將行動裝置拿出口袋時會想到的事情，都有可能成為 App 的題材；行動裝置上的唾手可及的定位服務，帶給了 App 開發者無限的想像空間，共乘就是一個很經典例子。在共享經濟時代下，只要隨手拿出手機，共乘服務就能將原本閒置的座位、車輛得到了更好地利用，司機透過共乘平台規劃，在原本的路線上多繞一點路，就能多賺取費用；乘客可以與司機及其他乘客一起分攤車資，用更划算的價格取得服務，使社會資源的到更良好的使用。

學術上，在目前針對共享經濟中行動裝置的動態共乘問題，已有許多探討，針對為了各個乘客的起訖而繞路的研究，目前來說只有乘客自行限制繞路的距離或由平台給定 [5]，至於乘客們之間對於自己起迄站最小路徑與實際上繞路的多寡，並沒有公平性機制的探討，換句話說，對於同車各個乘客之間的繞路多寡，是可能會有分配不均的情形在的，很可能造成一趟共乘中有人繞了很多路，而有人幾乎不繞路，這樣對於乘客共乘的權益，是很不公平的一件事，因此，如何在動態共乘問題中，考慮乘客間的公平性，是本文中所要套討的議題。

1.3 Objective

本研究目標是在不同乘客起迄站的服務要求中，有著駕駛資源與路線條件的限制下，最小化最大乘客繞路比例。(待補) 確保乘客對於旅程時間不會有過多的落差

1.4 Thesis Organization

Chapter 2 Literature Review

2.1 Related Work on Carpooling

This section will go through the related work and discuss the approaches for the carpooling problem. Before this section, we will make a brief throughout the history of carpooling. Carpooling has risen since World War II, as all the resources are for the war. At that time, carpooling is a tactic for rationing gasoline through car-sharing clubs, which match riders and drivers via a bulletin board. As technology progresses, the carpooling service starts to integrate the Internet and mobile phones in recent years. Inbuilt location-based service (LBS) on smartphones makes it easy to get a user's location in real-time. Making it possible to match drivers and riders on-demand at any time [4].

Coja-Oghlan et al. [6] see the carpooling problem as a dial-a-ride problem (DARP) that allocates all resources through a centralized server. This paper treats the graph of the road network as a caterpillar tree, which is an undirected graph $G = (V, E)$ and all the vertices are on a central backbone. The self-developed Minimal Spanning Tree (MST) heuristic method can find an optimal solution in polynomial time with probability. By applying the bipartite Steiner tree, we can also find the optimal solution in polynomial time at random, which indicates that $NP = RP$. Overall, NP-hard is the worst cast, while on average is solvable efficiently.

Farin and Williams [7] propose a fair carpooling scheduling algorithm in different approaches focused on driving rotation on any given day when a given number of members form a carpooling club. There are the following factors to concern: who drives today and how many riders are on the carpool. This research also concerns the ”cost” of the trip as the least common multiple of 1,2,...,n, where n is the largest number of people participating in the carpool. The driver earns the ”cost” while the riders share it. Who has the least ” cost” is the driver of the next day.

Wolfson and Lin [8] indicate that ridesharing fairness is a new a problem in studies. The concept in this paper is related to the Nash Equilibrium. This paper proposes a payment scheme called Guaranteed-Ridesharing-Fairness (GRF) that guarantees each passenger/trip saves in an optimal ridesharing plan (RSP), which has no other RSP has saving greater than any RSP. In a GRF method, it combines both fair and optimal ridesharing queries.

2.1.1 Dial-a-ride Problem

When it comes to carpooling, the dial-a-ride problem could be one of the best discussions about this kind of scenario. Dial-a-rider problem (DARP) is a problem for finding vehicle routes to serve users with requests with specific origins and destinations, with objective to minimize cost or maximize satisfied demand [9]. DARP has been widely researched for about 50 years [10] and is considered as an NP-hard problem [11]. There are many variations in the discussion. Cordeau and Laporte [9] indicate that DARP mainly consists of the following factors:

- Vehicles: There might be 1 or n vehicles in the system. Most DARP studies assume that vehicles are with homogeneous capacity; some studies may use a heterogeneous fleet with different capacity.

- Requests: A request usually consists of these parts: passengers, origins, destinations, and time constraints. Static or dynamic is also a significant concern. For a static DARP, all the ride requests will be known in advance, and the central scenario is to allocate all the resources once. A dynamic DARP usually discusses ride request planning in a specific time window. For better understanding, we will use ride requests instead of requests.

- Routes: Routes of the road network usually consist of length and duration.

- Detour: Because a route of DARP is aggregate from many must-pass nodes proposed by ride requests. Rather than the shortest path, every passenger may have a detour.

A DARP is a kind of problem that consists of n must-pass nodes with the lowest cost. We will use a Steiner tree problem for finding routes.

2.1.2 Steiner Tree Problem

A Steiner tree problem is to find a minimum cost tree in given subset vertices of a given graph. When it comes to the Steiner tree, a minimum spanning tree problem can be mentioned together. While a minimum spanning tree problem is to find a minimum total weight that the links have to go through all the vertices in a given graph, a Steiner tree problem treats the vertices as terminal and non-terminal vertices. All terminals must go through, and the non-terminal vertices are allowed to contain to reduce cost [12]. Figure

2.1 as an example; this figure is an undirected graph $G = (V, E, w)$ with nonnegative weights. A set of vertices $L \subset V$ is the terminals. The minimal Steiner tree is to find a tree that passes all the terminals that can include other vertices in the graph. We could find a Steiner minimal tree shown in Figure 2.2 with cost = 5 [13]. According to Foulds and Graham [14], finding a Steiner minimal tree (SMT) is an NP-Complete problem, which has no deterministic algorithms to solve in the polynomial time. As a result, we tend to use heuristic approaches to this problem.

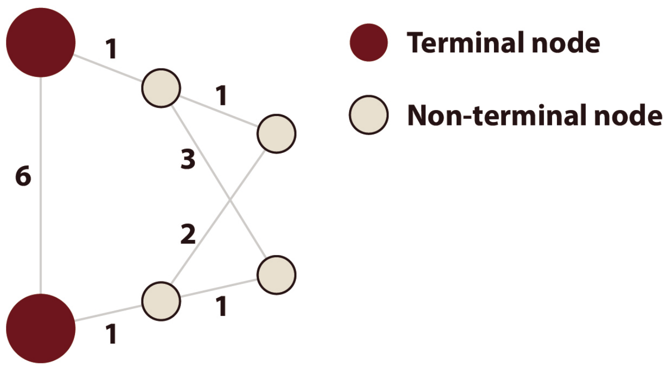

Figure 2.1: Example graph of a Steiner tree problem

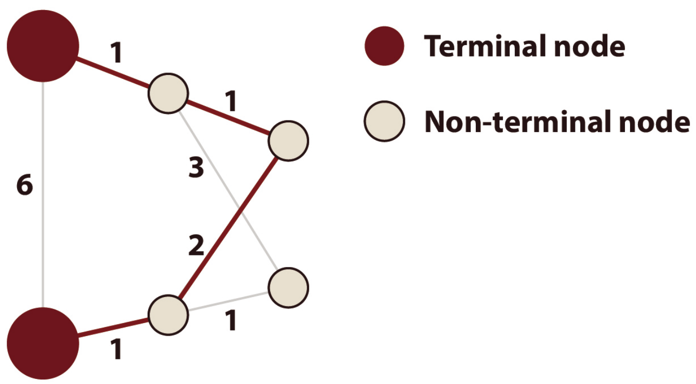

Figure 2.2: Steiner minimal tree with cost = 5

2.2 Fairness

Fairness is the central issue in this research. According to the Cambridge dictionary definition, fairness is ”the quality of treating people equally or in a way that is right or reasonable” [15]. There are many factors to be considered when it comes to carpooling. One of them is the mechanism of dispatching ride requests to drivers. Suppose a driver in the carpooling system is not treated equally, such as not receiving any ride request in the rush hour while others receive a lot. In that case, the driver might choose not to use the carpooling system. However, getting rid of unequal ride requests dispatching is not easy. There are many possibilities for drivers to think they are treated equally or unequally. For instance, one may think it is equal to receive the same ride requests as others. In contrast, another may think to be treated unequally for participating in carpooling for a long time. To get rid of the issue in such a resource allocation scenario, we define fairness in a mathematical approach to measure fairness. There are many indices to measure the fairness of resource allocation [16]. In this paper, we use Raj Jain’s approach, Jain’s fairness index [17], to know the status of ride request allocation by calculation.

2.2.1 Jain’s Fairness Index

Jain’s fairness index is a metric to measure the fairness of the resource allocation in a system, which is used in the networking engineering field. By definition, there are n users share resource in a system, a user $i \in \{1, 2, 3, ..., n\}$ allocates x_i resource [17]. The following is the formula:

$$Firness\ Index = f_A(x) = \frac{\left(\sum\limits_{i=1}^{n} x_i\right)^2}{n \sum\limits_{i=1}^{n} x_i^2}$$

The index ranges from $\frac{1}{n}$ to 1. When there is only one user monopoly to all resource, which means the others get no resource, the fairness index will be $\frac{1}{n}$. It is the most unfair scenario. When all the users get the same resource, the fairness index will be 1, which is the fairest scenario.

Chapter 3 Problem Formulation

In this chapter, we will introduce the scenario of the carpooling fairness problem. Then formulate the problem into a mathematical model with given parameters and decision variables.

3.1 Problem Description

The object of this research is to minimize the maximum percentage of cost saving when a passenger chooses to carpool, with consideration of the number of drivers, car capacity and routing limit constraints.

Take Figure 3-1 as an example scenario; Passenger 1 and Passenger 2 are requesting a trip. When Passenger 1 chooses to take a ride by himself/herself, called ”exclusive” riding shown as Figure 3-2, it would cost \$5. We can describe the scenario as a shortest path problem from S to D_1, and P_1 is a must-pass node. We use Steiner tree to solve this kind of shortest path problem with must-pass nodes. Passenger 2’s ”exclusive” riding would also cost \$5 in Figure 3-3. In this case, it would be a shortest path problem from S to D_2 with P_2 as a must-pass node.

When the passengers choose to take a carpool, which is called ”sharing” riding. We can describe the scenario as a shortest problem from S to D_1 with P_1, P_2 and D_2 as must-pass nodes or S to D_2 with P_1, P_2 and D_1 as must-pass nodes. In Figure 3-4 is one of the best case, which would cost \$8 for all the passengers, in Figure 3-5 would cost the same \$8 and both of the fares they could share are \$5 ($\overline{P_2D_1}$ and $\overline{P_2D_2}$ respectively); however, in Figure 3-4 would cost $\overset{\overline{P_1P_2}}{1} + \frac{1}{2} \times \overset{\overline{P_2D_1}}{5} = \$3.5$ for Passenger 1 and $\frac{1}{2} \times \overset{\overline{P_2D_1}}{5} + \overset{\overline{D_1D_2}}{1} = \$3.5$ for Passenger 2, in Figure 3-5 would cost $\overset{\overline{P_1P_2}}{1} + \frac{1}{2} \times \overset{\overline{P_2D_2}}{5} + \overset{\overline{D_2D_1}}{1} = \$4.5$ for Passenger 1 and $\frac{1}{2} \times \overset{\overline{P_2D_2}}{5} = \$2.5$ for Passenger 2.

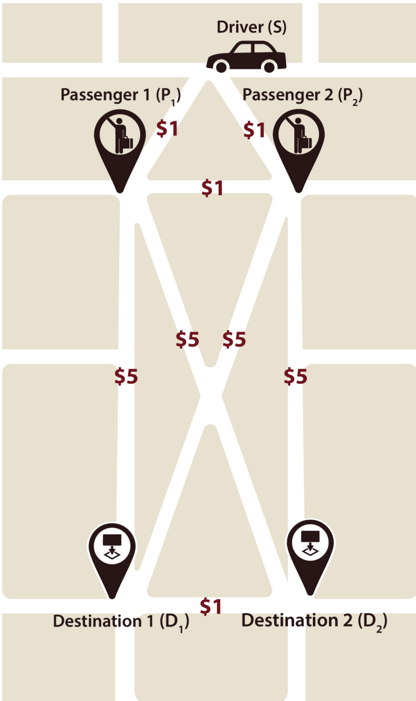

Figure 3.1: Example road network with driving fare and points of passengers and their destinations

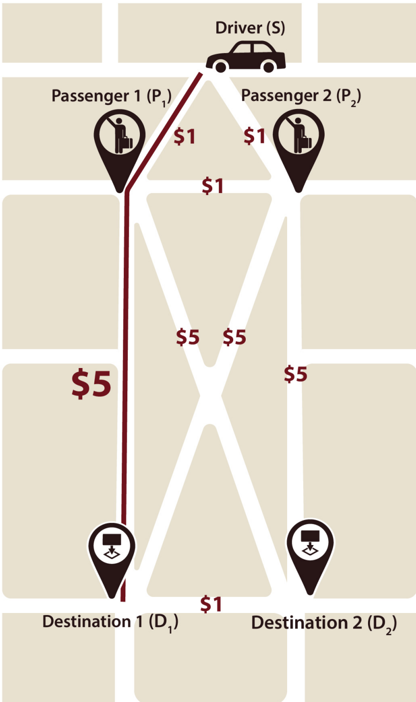

Figure 3.2: Best routing path when the driver only serving Passenger 1

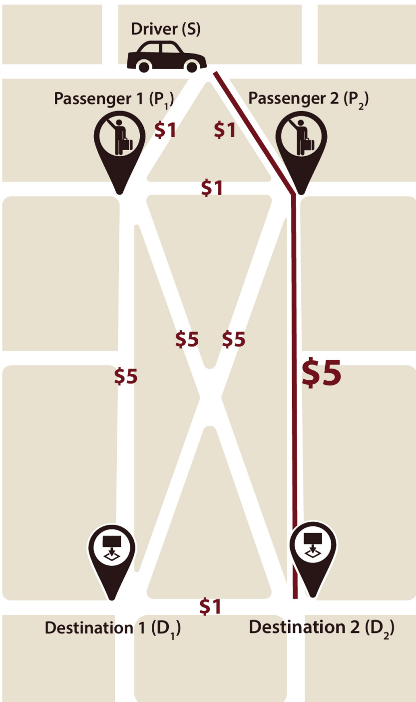

Figure 3.3: Best routing path when the driver only serving Passenger 2

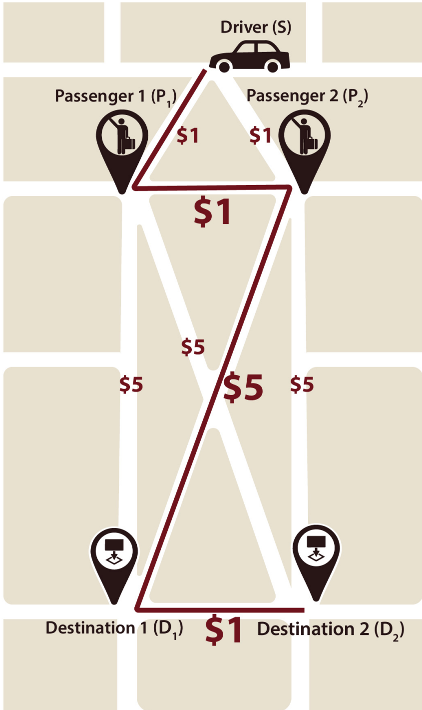

Figure 3.4: One of the best routing when both Passenger 1 and Passenger 2 carpool

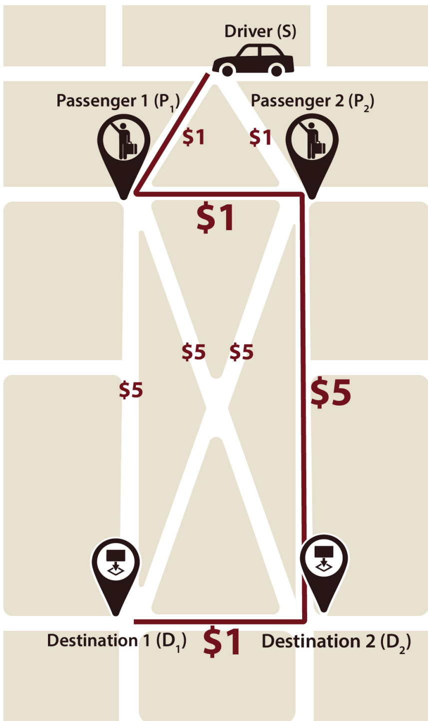

Figure 3.5: Another best routing when both Passenger 1 and Passenger 2 carpool

3.1.1 Assumptions

There are three assumption in out system:

1. Every driving request only includes one passenger. This driving request will be split into multiple one-person driving request for a driving request with multiple people.
2. System calculation time is not considered, that is, assume that drivers' location will still keep unchanged after the time of system calculation.
3. The carpooling fare is shared equally by the time passengers in the vehicle. For a specific link in the route with n passengers, the fare of the link for a passenger will be divided by n.

3.1.2 Fairness of Vehicle Dispatching

In order to consider the fairness when dispatching driver, we introduce Jain's fairness index to avoid unfair situation:

$$Firness\ Index = f_A(x) = \frac{\left(\sum\limits_{i=1}^{n} x_i\right)^2}{n \sum\limits_{i=1}^{n} x_i^2}$$

We take accumulate driving requests that a driver has received as the metrics for fairness. That is, when a driver receives more driving requests in the past, he/she should give his/her chance of receiving a new driving request to the one who takes fewer driving requests.

As a new driving request comes, we need to ensure our system must get more or at least keep the same fairness. Hence, we form a constraint that the fairness index can not decrease compared to the previous fairness index.

However, when the driving resource changes, such as a new driver joins our system, it will violate our constraint for fairness. Because the newcomer has not taken any driving request before, the fairness will decrease. Even if a driver leaves our system, the fairness will increase or decrease due to the driver's situation. As a result, we need to reset the fairness index to the initial state and reset all the drivers' accumulated driving requests to 0. Nevertheless, it will make the fairness index's denominator to 0, which can not be divided. To avoid this situation, we assign a virtual driver with one accumulated driving request, which will make the fairness index to $\frac{1}{(n+1)}$, where n is the number of drivers in our system. Moreover, for any situation that the fairness index will decrease when dispatching a new driving request, the fairness will reset.

3.2 Mathematical Model

We will go through the detailed given parameters and decision variables in the mathematical model before describing our mathematical model.

Table 3.1: Notations of given parameters

Notation	Description
R	Set of ride request
C	Set of passengers in the system (take j as index)
D	Set of destinations in the system (take k as index)
R	Set of drivers in the system (take i as index)
W	Set of passengers or destinations in the system, where $W = P \cup D$
B_r	Set of passengers that driver r has picked up, where $r \in R$
L	Set of links on the road network
L_c	Set of links on the road network that pass by the passenger $c \in C$
L_d	Set of links on the road network that pass by the destination $d \in D$
P_r	Set of all paths for driver r, where $r \in R$
P_{rw}	Set of all paths for driver r from initial location to a passenger or a destination, where $r \in R$ and $w \in W$
P_{rc_i}	Set of all paths for driver r from initial location to passenger c_i, where $r \in R$ and $c_i \in C$
P_{rd_i}	Set of all paths for driver r from initial location to destination d_i, where $r \in R$ and $d_i \in D$
Q_r	Maximum load capacity for driver r, where $r \in R$
f_l	The fare for the link l, where $l \in L$
g_r	The capacity that the driver $r \in R$ has loaded in the previous decision round.
α	The fairness index in the previous decision round.
δ_{pl}	Indicator function which is 1 if link $l \in L$ on the route $p \in P_r$ when driver $r \in R$ goes carpooling; 0 otherwise
ζ_{pl}	Indicator function which is 1 if link $l \in L$ on the route $p \in P_{rd_i}, i \in \{1, 2, 3, ..., n\}$ is selected when passenger $c_i \in C$ with index i; 0 otherwise

Table 3.2: Notations of decision variables

Notation	Description
u_l	Binary variable, 1 if link $l \in L$ is on the shortest path; 0 otherwise.
v_{wp}	Binary variable, 1 if route $p \in P_r$ is chosen; 0 otherwise.
x_p	Binary variable, 1 if route $p \in P_r$ is chosen when driver $r \in R$ goes carpooling; 0 otherwise.
y_p	Binary variable, 1 if route $p \in P_{rd_i}, i \in \{1, 2, 3, ...n\}$ is chosen when passenger $c_i \in C$ with index i does not take a carpooling; 0 otherwise.
z_p	Binary variable, 1 if route $p \in P_{rw}$ is chosen; 0 otherwise.
a_r	Total fare that the driver $r \in R$ has earned before current decision round.
b_r	Total fare that the driver $r \in R$ has earned after current decision round.

Objective Function

The objective function is to maximize the minimum discount percentage when applying carpooling among passengers in one trip.

$$\max_r \min \frac{Z_i - carpool\ cost_i}{Z_i} \tag{IP1}$$

$$where\ Z_i = \sum_{p \in P_{rd_i}} \sum_{l \in L} y_p \zeta_{pl} a_l$$

Constraints

Constraint 3.1 ensures there is only one path selected for a driver.

$$\sum_{p \in P_r} x_p \leq 1 \qquad (3.1)$$

Constraint 3.2 ensures the selected link on the route we chose is on the shortest path.

$$\sum_{p \in P_r} x_p \delta_{pl} \leq u_l \qquad \forall l \in L \qquad (3.2)$$

Constraint 3.3 ensures all the links pass by the node of passenger or destination we chose is the shortest path.

$$u_l \leq 1 \qquad \forall l \in L_c \cup L_d \qquad (3.3)$$

Constraint 3.4 ensures that all the routes to passengers or destinations we chose is on the shortest path and overlap the route we chose.

$$\sum_{p \in P_w} z_p \delta_{pl} \leq u_l \qquad \forall w \in W \qquad (3.4)$$

Constraint 3.5 ensures the route we chose will only pass by the node of passenger or destination once.

$$\sum_{p \in P_w} z_p \leq 1 \qquad \forall w \in W \qquad (3.5)$$

Constraint 3.6 ensures that we must pick up the passenger before we drop off the passenger.

$$\sum_{l \in L} \sum_{p \in P_{rc_i}} z_p \delta_{pl} f_l \leq \sum_{l \in L} \sum_{p \in P_{rd_i}} z_p \delta_{pl} f_l \qquad \forall i \in \{1, 2, , ..., n\}, r \in D \qquad (3.6)$$

Constraint 3.7 ensures the fairness index after current decision round will not less than the previous decision round.

$$\frac{\left(\sum_{r\in R} b_r\right)^2}{\sum_{r\in R} 1 \sum_{r\in R} b_r^2} \geq \alpha \tag{3.7}$$

Constraint 3.8 is the fairness index in the previous decision round.

$$\frac{\left(\sum_{r\in R} a_r\right)^2}{\sum_{r\in R} 1 \sum_{r\in R} a_r^2} \leq \alpha \tag{3.8}$$

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