MA-101/1841

B.Tech. (Semester-I) Exam-2017 Mathematics-I

Time: Three Hours Maximum Marks: **100**

Note: Attempt questions from all the sections.

Section-A

(Short Answer Type Questions)

Note: Attempt any ten questions. Each question carries 4 marks. (4x10=40)

1. Find the rank of the matrix:

$$\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \end{bmatrix}$ find two non-singular matrices P and Q such that PAQ=I.

3. Find the characteristic equation of $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

4. If
$$x = \sin\left(\frac{\log y}{a}\right)$$
 then evaluate the value:

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0.$$

5. If
$$u = \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$$
 show that:

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0 .$$

- 6. Trace the curve: $9ay^2 = x(x-3a)^2$, a>0.
- Expand $e^x \sin y$ in powers of x < y, x = 0, y = 0 as far as terms of third degree by using Taylor's theorem.
- 8. If $x^2 + y^2 + u^2 v^2 = 0$ & uv + xy = 0 then prove that $\frac{\partial(u, v)}{\partial(x, y)} = \frac{x^2 y^2}{u^2 + v^2}.$
- 9. Evaluate: $\int_0^\infty \int_0^\infty e^{-x^{2(1+y^2)}} x dx dy$.
- 10. Evaluate: $\iint_A xy dx dy$ where A is the domain bounded by x-axis, ordinate x = 2a & the curve $x^2 = 4ay$.
- 11. Prove that $\beta(l,m) = \frac{\sqrt{l}\sqrt{m}}{\sqrt{m+l}}$.

- Find the directional derivative of $\varphi(x, y, z) = x^2yz + 4xz^2$ at (1, -2, 1) in the direction of $2\hat{i} \hat{j} 2\hat{k}$. Find the greatest rate of increase of φ .
- If $\overline{A} = (3x^2 + 6y)\hat{i} 14yz\hat{j} + 20xz^2\hat{k}$, Evaluate the line integral $\oint \vec{A} \cdot d\vec{r}$ from (0,0,0) to (1,1,1) along the curve C. x = t, $y = t^2$, $z = t^3$.
- 4. Find the divergence of: $\vec{V} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 y^2z)\hat{k}$ at (2,-1,1).
- In Estimating the number of bricks in a pile which is measured to be (5m×10m×5m), Count of bricks is taken as 100 bricks per m³. Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of bricks is ₹2000/- per thousand bricks.

Section-B (Long Answer Type Questions)

Note: Attempt any three questions. Each question carries (20x3=60)

For what values of K the set of equations:

$$2x - 3y + 6z - 5t = 3$$

$$y - 4z + t = 1$$

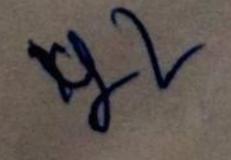
$$4x - 5y + 8z - 9t = K$$

has (i) no solution (ii) infinite number of solutions.

If
$$u = \log(x^3 + y^3 + z^3 - 3xyz)$$
, show that:

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}.$$

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3. Change the order of integration and evaluate:

$$\int_0^a \int_0^y \frac{x dx dy}{\sqrt{(a^2 - x^2)(a - y)(y - x)}}$$

- 4. Evaluate: $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ if the region of integration is bounded by the coordinate planes and the plane x+y+z=1.
- 5. Verify the Gauss divergence theorem for $\overline{F} = (x^2 yz)\hat{i} + (y^2 zx)\hat{j} + (z^2 xy)\hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.
- 6. Find the Eigen values and Eigen vectors of the matrix given below:

$$\begin{bmatrix}
1 & 0 & 0 \\
0 & 3 & -1 \\
0 & -1 & 3
\end{bmatrix}$$