MA-101/1841

B. Tech. (Semester-I) Exam.–2018

Mathematics-I

Time: Three Hours

Maximum Marks: **100**

Note: **Attempt questions from all the sections.**

Section-A

(Short Answer Type Questions)

Note: **Attempt any ten questions. Each question carries 4 marks.** **(4x10=40)**

1. Find the rank of the matrix

   $$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

   page - 287

Scanned by TapScanner

6.

2. Reduce the matrix to a diagonal form

$$\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 2 \end{bmatrix}$$

3. Find the characteristics equation of the matrix

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

4. If $y = a\cos(\log x) + b\sin(\log x)$ then show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

5. If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$ the using Euler's theorem prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$

MA-101/1841-V-650

Trace the curve

$$y^2 = x^2(a - x)$$

Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ & $(y + 2)$ using Taylor's theorem.

If $x + y + z = 4$, $y + 3 = uv$, $z = uvw$, show that

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2v.$$

Evaluate

$$\int_0^1 \int_0^{x^2} e^{y/x} \, dy \, dx$$

Evaluate $\iint_R xy \, dx \, dy$, where R is the quadrant of the circle $x^2 + y^2 = a^2$ where $x \geq 0$, $y \geq 0$.

4

11. Show that:

$$\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta = \frac{1}{2}\left|\frac{1}{4}\right|\frac{3}{4}$$

12. Find the directional derivative of $\emptyset(x^2 + y^2 + z^2)^{-1/2}$ at the point $P(3,1,2)$ in the direction of the vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$.

13. A vector field is given by $\bar{F} = (\sin y)\hat{i} + x(1 + \cos y)\hat{j}$. Evaluate the line integral over a circular path $x^2 + y^2 = a^2, z = 0$.

4. If $\emptyset = 3x^2y - y^3z^2$, find grad $\emptyset$ at the point $(1, -2, 1)$.

A balloon is in the form of right circular cylinder of radius 1.5m and length 4m and is surrounded by hemispherical ends. If the radius is increased by 0.01m & the length by 0.05m, find the percentage change in the volume of the balloon.

Section-B

(Long Answer Type Questions)

e: Attempt any three questions. Each question carries 20 marks. (20x3=60)

Find for what values of λ and μ the system of linear equations:

$$x + y + z = 6; x + 2y + 5z = 10$$
$$2x + 3y + \lambda z = \mu$$

has

(i) a unique solution

(ii) no solution

(iii) infinite solution. Also find the solution for $\lambda = 2$ & $\mu = 8$.

2. If $x^x y^y z^z = c$ show that at $x = y = z$

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$$

page · 487

3. Change the order of integration and evaluate

$$\int_0^a \int_0^y \frac{dxdy}{\sqrt{(a^2 + x^2)(a - y)(y - x)}}$$

Evaluate *page - 567*

$$\int_0^{\log 2} \int_0^x \int_0^{x + \log y} e^{x+y+z} dzdydx$$

Verify Divergence theorem, given that $\bar{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

page - <u>358</u>

Find the Eigen Values and Eigen Vectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$\lambda =$