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Subject: A Generic Curve Builder & Curve Construction under OIS Discounting

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This document describes a generic curve builder implemented in the RiskWatch system. The model can be used for both discount curve and forward curve construction, under the assumption of either OIS discounting or Libor discounting. The model can also be used for Treasury curve construction.

Under OIS discounting, the model is capable of curve building in the collateral currency, the non-collateral currencies, or the cheapest to deliver currency from a basket of deliverable collateral currencies.

The procedure for curve construction under OIS discounting is demonstrated with examples.

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1 OIS Discounting

Prior to mid-2007, it was assumed all banks could borrow at Libor. As a result, Libor was used as the standard discount rate for pricing cashflows in a swap. The credit crisis changed these assumptions dramatically. Capital became scarce and banks began to differentiate pricing methodologies between collateralized and uncollateralized trades [1].

There is a new standard for pricing collateralized trades. Collateral postings have the effect of funding for a derivatives desk. Under a collateral agreement (CSA), the firm receives collateral from the counter party when the present value of the contract is positive, needs to pay the interest on the outstanding collateral at a relevant overnight rate, and vice versa. This is why the future cashflows in collateralized trades are discounted at an OIS rate.

Although the details can differ from CSA to CSA, the most commonly used collateral is cash in a currency like USD, EUR, GBP or CAD, and the mark-to-market of the contracts is made daily. In the case of cash collateral, the overnight rate for the collateral currency, such as Fed-fund rate for USD, is usually used as the collateral rate.

Pricing of collateralized products with partially collateralization is quite complicated. The collateralized portion of the mark-to-market is default risk free and justified for the OIS discounting, while the not collateralized portion has default risk and remains under the Libor discounting. A non-linearity arises from switching from one discounting rule to another according to the presence or absence of the default risk.

In this document, we make the following three assumptions:

(1) the mark-to-market is made continuously and the collateral posting occurs instantaneously (i.e. at zero minimum transfer amount);

(2) the posted amount of collateral is 100% of the contract's present value (i.e. at zero collateral threshold);

(3) the required collateral is cash in a specified currency.

Under the above assumptions, we can neglect the counterparty default risk and recover the linearity among different payments. Therefore, the cashflow of a collateralized swap can be decomposed into a portfolio of independently collateralized strips of payments.

In this document, we will show the methodology to build a family of curves in the collateral currency based on the OIS discounting. Given a collateral currency, we first construct the OIS discount curve from the market quotes of OIS instruments in the collateral currency. Then, we build the Libor curves (3-month, 6-month tenor etc) as forward curves from the known OIS discount curve.

Then, we will also show how to build curves in currencies that are different from the collateral currency. For example, for a CAD interest rate swap whose collateral currency is USD, we will need in the discount curve to be consistent with the USD OIS based funding rate.

2 Curve Building Overview

The generic curve builder is a general purpose tool. It can be used for either discount curve or forward curve construction, under the assumption of either OIS discounting or legacy style Libor discounting. Like the current K2 curve builder [2], the generic curve builder can take a combination of cash, futures/FRA and swaps as curve instruments.

Below we will briefly compare our familiar K2 curve builder with the new generic curve builder.

2.1 Instrument Definition

Under the legacy Libor discounting, the floating Libor side of a swap plus notional is always valued to par regardless whether it resets quarterly pays quarterly, resets semi-annually pays semi-annually, or resets quarterly pays semi-annually (compounding at flat). This property was heavily exploited in the current K2 curve builder. However, it no longer holds under OIS discounting where the Libor forward curve is different from the OIS discount curve.

Therefore, unlike the current K2 curve builder which only takes the swap fixed side as input, the generic curve builder requires the user to provide details on both sides of a swap instrument.

2.2 Curve Dependency

In general, the valuation of a swap instrument can depend on four curves: a forward curve and a discount curve on each of the two sides. The generic curve builder is designed to construct one curve from up to three input curves, unlike the current K2 curve building [3] that may depend on only one input curve at most.

2.3 Curve Bootstrapping

The current K2 curve builder assumes all swap instruments share the same set of payment dates. Furthermore, the payment dates must coincide with the swap period dates. Essentially, periods in all swap instruments must coincide exactly with the periods in the longest swap instrument. The par swap rates are then linearly interpolated from the market quotes to all payment dates for bootstrapping. Consequently, the curve nodes are spaced at swap payment frequency. For computational efficiency, the swap pricing are replicated within the curve builder independently from the swap model for deal pricing.

The generic curve builder simply solves for the zero rate on a swap instrument’s last cashflow date, such that it yields a zero present value for the instrument, under the applicable interpolation rule. There are two main advantages with this approach: flexibility and consistency. The user now can easily customize the swap instrument to any specific market convention. In principal, the swap instruments for a curve can even be specified independently from one another. The same swap model is used in both curve building and deal pricing. This design ensures the market quotes always recovered as long as the curve instruments are properly specified.

3 Methodology Description

The generic curve builder can take a combination of cash deposits, futures/FRA and swaps as curve instruments. The following sections will describe the curve building methodology for each type of the curve instruments. We will pay particular attention to the swap instruments, since they are affected by the discounting rule assumed.

In the end of this section, we will also discuss how to build treasury curves.

3.1 Notation

We first define some commonly used notations.

- t_0 Instrument start date (issue date)
- t_n Instrument end date (maturity date)
- t_i Instrument period dates
- t_i^P Instrument payment dates, a number of business days lagged from t_i
- Δt_i $\Delta t_i = \Delta t(t_{i-1}, t_i, dc)$ period daycount multiplier from time t_{i-1} to t_i
- dc Daycount basis, e.g. A/360. A360, 30/360
- $df_{XYZ}(t_0, t_n)$ Discount factor from time t_0 to t_n on curve XYZ
- $F_{XYZ}(t_{i-1}, t_i)$ Forward rate from time t_{i-1} to t_i on curve XYZ
- $s_0 = 0$ Curve anchor day (today or valuation date)
- s_n Curve node date
- r_n Curve zero rate at node s_n
- df_n Curve discount factor at node s_n, $df_n = df(0, s_n) = e^{-r_n \Delta s_n}$
- Δs_n $\Delta s_n = \Delta t(0, s_n, A/365)$ period daycount multiplier at node s_n

3.2 Curve Instruments

3.2.1 Cash Instruments

Cash instrument n is defined by a start date t_0, end date t_n, market quoted cash rate $\overline{R}_n^{Cash}$ and daycount dc_n, satisfying

$$F_{Cash}(t_0,t_n)\Delta t_n df_{XYZ}(t_0,t_n^P) = \overline{R}_n^{Cash}\Delta t_n df_{XYZ}(t_0,t_n^P)$$

where $F_{Cash}(t_0,t_n)$ is the forward rate between the start date t_0 and end date t_n, $\Delta t_n = \Delta t(t_0,t_n,dc_n)$ the period daycount multiplier from time t_0 to t_n, and $df_{XYZ}(0,t_n^P)$ the discount factor from the payment date under a selected discounting "XYZ".

As we can see, the discounting is irrelevant for any bullet cashflow since it cancels out from both side of the equation,

$$F_{Cash}(t_0,t_n) = \overline{R}_n^{Cash}$$

Using the forward rate formula,

$$\overline{R}_n^{Cash} = F_{Cash}(t_0,t_n) = \frac{1}{\Delta t_n}\left(\frac{df(0,t_0)}{df(0,t_n)} - 1\right)$$

We can now build a curve node (s_n, r_n, df_n):

$$s_n = t_n,\ r_n = -\frac{\ln(df_n)}{s_n},\ df_n = df(0,t_n) = \frac{df(0,t_0)}{1+\overline{R}_n^{Cash}\Delta t_n}$$

For cash instruments, the same methodology is applicable to both discount curve and forward curve construction, independent of the assumption of OIS or Libor discounting.

3.2.2 Cash Spread Instruments

A cash spread instrument (e.g. 6m/3m spread) is defined with respect to a reference rate implied from a reference curve (e.g. 3m Libor curve). Given cash spread quote $\overline{S}_n^{Cash}$ and a reference curve, the corresponding all-in rate can be derived through

$$\overline{R}_n^{Cash} = F_{Cash}^{Ref}(t_0,t_n) + \overline{S}_n^{Cash} = \frac{1}{\Delta t_n}\left(\frac{df_{Ref}(0,t_0)}{df_{Ref}(0,t_n)} - 1\right) + \overline{S}_n^{Cash}$$

Then, the standard bootstrapping procedure described above can be applied.

3.2.3 Futures/FRA Instruments

FRA (or futures) instrument n is defined by a start date τ_n, end date t_n, market quoted forward rate $\overline{R}_n^{FRA}$ and daycount dc_n, satisfying

$$F_{FRA}(\tau_n, t_n)\Delta t_n df_{XYZ}(0, t_n^P) = \overline{R}_n^{FRA}\Delta t_n df_{XYZ}(0, t_n^P)$$

where $F_{FRA}(\tau_n, t_n)$ is the forward rate between the start date τ_n and end date t_n, $\Delta t_n = \Delta t(\tau_n, t_n, dc_n)$ the period daycount multiplier from time τ_n to t_n, and $df_{XYZ}(0, t_n^P)$ the discount factor from the payment date t_n^P under a selected discounting "XYZ".

The discounting is irrelevant here as the FRA instrument directly dictate a forward rate,

$$F_{FRA}(\tau_n, t_n) = \overline{R}_n^{FRA}$$

Using the forward rate formula,

$$\overline{R}_n^{FRA} = F_{FRA}(\tau_n, t_n) = \frac{1}{\Delta t_n}\left(\frac{df(0, \tau_n)}{df(0, t_n)} - 1\right)$$

If the curve has already built till the FRA start date, $df(0, \tau_n)$ is known. We can readily extend the curve to t_n

$$s_n = t_n,\ r_n = -\frac{\ln(df_n)}{s_n},\ df_n = df(0, t_n) = \frac{df(0, \tau_n)}{1 + \overline{R}_n^{FRA}\Delta t_n}$$

If the curve has not yet built till the FRA start date τ_n, $df(0, \tau_n)$ is not known. We need to involve a root finder to solve for the zero rate r_n at t_n, such that the market quoted forward rate is recovered by the curve,

$$\overline{R}_n^{FRA} = F_{FRA}(\tau_n, t_n) = \frac{1}{\Delta t_n}\left(\frac{df(0, \tau_n)}{df(0, t_n)} - 1\right)$$

The discount factor $df(0, \tau_n)$ is computed according to the interpolation rule between the curve nodes.

For futures/FRA instruments, the same methodology is applicable to both discount curve and forward curve construction, independent of the assumption of OIS or Libor discounting.

3.2.4 Futures/FRA Spread Instruments

A Futures/FRA spread instrument (e.g. 6m/3m spread) is defined with respect to a reference rate implied from a reference curve (e.g. 3m Libor curve). Given spread quote $\overline{S}_n^{FRA}$ and a reference curve, the corresponding all-in rate can be derived through

$$\overline{R}_n^{FRA} = F_{FRA}^{Ref}(\tau_n, t_n) + \overline{S}_n^{FRA} = \frac{1}{\Delta t_n}\left(\frac{df_{Ref}(0, \tau_n)}{df_{Ref}(0, t_n)} - 1\right) + \overline{S}_n^{FRA}$$

Then, the standard bootstrapping procedure described above can be applied.

3.2.5 Fed Fund Futures Average Rate – 1st Order Approximation

The payoff of US fed fund futures is based on the averaged daily OIS rate rates. BNS developed a method to accurately and efficiently compute this averaged rate (see later sessction on "USD Fed Fund Rates Approximation"),

$$\overline{R}_n^{FRA} = F_{FRA}(\tau_n, t_n) \approx \frac{1}{\Delta t_n} \ln\left(\frac{df(0,\tau_n)}{df(0,t_n)}\right)$$

Correspondingly, for fed fund futures spread instrument, we have

$$\overline{R}_n^{FRA} = F_{FRA}^{Ref}(\tau_n, t_n) + \overline{S}_n^{FRA} = \frac{1}{\Delta t_n} \ln\left(\frac{df_{Ref}(0,\tau_n)}{df_{Ref}(0,t_n)}\right) + \overline{S}_n^{FRA}$$

3.2.6 Instrument Override Logic

User could choose any number of futures/FRA instruments to build the mid-section of the curve. In case of any overlap, the mid-section (i.e. Futures) will override the short-section (i.e. cash instruments) from the start date or maturity date (per user's choice) of the first the futures/FRA instrument, while only those swap instruments that mature 7 calendar days[1] after the last futures/FRA instrument's end date will be used to build the long-section of the curve.

In the cross-currency curve construction, the curve builder will pre-check that the input swap instruments have their end dates in ascending order after the holiday adjustment. Any swap instrument with an adjusted end date equal or beyond the adjusted end date of the next swap instrument will not participate in the curve bootstrapping.

[1] A 7 calendar day buffer is inserted to prevent the first swap instrument end date from getting too close the last futures end date. For example, 12th futures instrument could end 1 day before the 3-year swap instrument in the USD 3m Libor curve. The two very closely spaced end dates could results in a jump in the tension spline.

3.2.7 Swap Instruments

In this section, we will present the generic curve construction methodology in the context of OIS discounting. The same methodology is also applicable to the Libor discounting (see Appendix C).

We will present various situations of the curve construction using swap instruments in the order of increasing complexity in the following three sub-sections,

In general, there are four curves involved in pricing a swap: the forward and discount curves for the pay side, as well as the forward and discount curves for the receive side. Among these four curves, some could be the same (e.g. discount curves in a single currency swap).

We first discuss the direct curve construction from one set of swap instruments, when there is only one unknown curve involved in the pricing. This category covers OIS discount curve construction from OIS swap quotes, as well as forward Libor curve construction from IR swap or tenor basis swap quotes.

Then, we present the single currency curve co-construction that is applicable to the OIS discount curve building in the collateral currency with the OIS/Libor basis swap instruments.

Finally, we describe the cross currency curve co-construction using the cross currency basis swap instruments. The methodology can be used to build the discount curve in a non-collateral currency consistent with the OIS discounting in the collateral currency.

3.3 Single Currency Curve Construction

In this section, we present some simpler situations where the curve can be readily built from a single set of the swap instruments. Generally, there is only one unknown curve involved the pricing of those swap instruments.

Unlike cash and FRA instruments, bootstrapping from swap instruments depends on the choice of discounting rule. In general, the valuation of a swap instrument can depend on four curves: a forward curve and a discount curve on each of the two sides. The generic curve builder is designed to construct one curve (either discount or forward curve) from up to three input curves. In this section, we will present the simpler curve construction cases when there is only one unknown curve in the valuation of the swap instruments. The subject of curve co-construction when there are multiple unknown curves in the valuation of the swap instruments will be presented in the next section.

The curve building from swap instruments generally takes the following steps:

First, define a swap instrument as specified by a start date, end date, and some attributes for each side of the swap, such as fixed rate/floating spread, daycount basis, reset frequency, payment frequency, payment lag. Interest rate swaps (IR swaps) have one floating side and one fixed side, while the basis swaps have two floating sides.

Then, specify the forward curve and discount curve for each side of the swap instrument. For the situations present in this section, there is only one unknown curve among them.

Finally, solve for the zero rate on each swap instrument’s last cashflow date, such that it yields a zero present value for the swap, under the applicable interpolation rule (only linear interpolation on zero rate between curve nodes is available currently). Denote t_n^P n-year swap instrument's last payment date. A curve node is then assigned on date $s_n = t_n^P$. The root finder solves for a zero rate r_n that yields a zero present value for the swap. The discount factor at the curve node is related to the zero rate through

$$df_n = df(0, s_n) = e^{-r_n \Delta s_n}$$

where $\Delta s_n = \Delta t(0, s_n, A/365)$ is the period daycount multiplier at node s_n.

The above steps are repeated from the swap instrument with the shortest maturity to the one with the longest maturity.

The same swap model [4] is used in both curve building and deal pricing. This design ensures the market quotes always recovered as long as the curve instruments are properly defined.

In following sections, we will illustrate the concept with concrete examples under the assumption of the OIS discounting. Our methodology closely follows the reference 5.

3.3.1 Discount Curve Construction from OIS Swaps

Consider a n-year OIS swap with a market quoted fixed rate $\overline{R}_n^{1d}$. The floating rate resets daily to the over night rate and compounds at flat. The payment period dates $t_i = t_{i-1} + 12m$, $i = 1,...,n$ are annually spaced. The pay days t_i^P are 1 or 2 business days lagged from t_i. Denote $F_{1d}(t_{i-1}, t_i)$ the forward rate for period (t_{i-1}, t_i), compounded from the daily over night rates. We have

$$\sum_{i=1}^{n} F_{1d}(t_{i-1}, t_i) \Delta t_i \, df_{1d}(t_0, t_i^P) = \overline{R}_n^{1d} \sum_{i=1}^{n} \Delta t_i \, df_{1d}(t_0, t_i^P)$$

The curves involved in pricing this swap are listed in the table below:

	Pay Side	Rec Side
Forward Curve	OIS Curve	N/A
Discount Curve	OIS Curve	OIS Curve
Input Curves	None	
Output Curve	OIS Curve	

With the OIS curve the only unknown in the above equation, it can be readily obtained through bootstrapping.

This methodology is applicable to the OIS discount curve construction in the collateral currency, when OIS/Libor basis swap instruments are not present.

3.3.2 Major Forward Curve Construction from IR Swap Quotes and the Discount Curve

Consider a n-year interest rate swap with a market quoted fixed rate $\overline{R}_n^{3m}$. The floating rate resets quarterly to the 3-month Libor. The payment period dates $t_i = t_{i-1} + 3m$, $i = 1,...,4n$ are quarterly spaced and there is no pay day lag. Denote $F_{3m}(t_{i-1}, t_i)$ the forward rate for period (t_{i-1}, t_i). We have

$$\sum_{i=1}^{4n} F_{3m}(t_{i-1}, t_i)\Delta t_i\, df_{1d}(t_0, t_i) = \overline{R}_n^{3m} \sum_{i=1}^{4n} \Delta t_i\, df_{1d}(t_0, t_i)$$

The curves involved in pricing this swap are listed in the table below:

	Pay Side	Rec Side
Forward Curve	3m Curve	N/A
Discount Curve	OIS Curve	OIS Curve
Input Curves	OIS Curve	
Output Curve	3m Curve	

With the 3-month Libor curve the only unknown in the above equation, it can be readily obtained through bootstrapping.

This methodology is applicable to the forward curve construction in either the collateral or non-collateral currency.

3.3.3 Minor Forward Curve Construction from Tenor Basis Swaps

Consider a n-year tenor basis swap with a market quoted spread $\overline{S}_n^{6m}$. The pay side floating rate resets semi-annually to the 6-month Libor and pays semi-annually. Its payment period dates $T_j = T_{j-1} + 6m$, $j = 1,...,2n$ are semi-annually spaced. The receive side floating rate resets quarterly to the 3-month Libor and pays quarterly. Its payment period dates $t_i = t_{i-1} + 3m$, $i = 1,...,4n$ are quarterly spaced. There is no pay day lag. Denote $F_{6m}(T_{j-1}, T_j)$ the forward rate for period (T_{j-1}, T_j). We have

$$\sum_{j=1}^{2n} \left[F_{6m}(T_{j-1}, T_j) + \overline{S}_n^{6m}\right]\Delta T_j\, df_{1d}(T_0, T_j) = \sum_{i=1}^{4n} F_{3m}(t_{i-1}, t_i)\Delta t_i\, df_{1d}(t_0, t_i)$$

The curves involved in pricing this swap are listed in the table below:

	Swap Side 1	Swap Side 2
Forward Curve	6m Curve	3m Curve
Discount Curve	OIS Curve	OIS Curve
Input Curves	OIS Curve, 3m Curve	
Output Curve	6m Curve	

With the 6-month Libor curve the only unknown in the above equation, it can be readily obtained through bootstrapping.

This methodology is applicable to the forward curve construction in either the collateral or non-collateral currency.

3.4 Single Currency Curve Co-construction

In this section, under the single currency setting, we will show how to use multiple sets of the swap instruments to bootstrap the required curves. Here, we assume that the single currency IR swaps are in the collateral currency.

The methodology is applicable to the OIS discount curve construction in the collateral currency, using OIS/Libor basis swap instruments.

3.4.1 Discount Curve Construction from Basis Swaps – One Forward Curve Known

In this example, we assume the OIS swap has been built to 1-year using the OIS swap instruments, while the 3-month Libor curve has been built to 3-year using cash and futures instruments. Our task here is to construction the 1-year to 3-year section of the OIS curve.

Consider a n-year USD fed fund vs Libor basis swap with a market quoted spread $\overline{S}_n^{1d}$. The pay side floating rate resets daily to the overnight rate and pays quarterly. The receive side floating rate resets quarterly to the 3-month Libor and pays quarterly. The payment period dates $t_i = t_{i-1} + 3m$, $i = 1,...,4n$ are quarterly spaced. There is no pay day lag. Denote $F_{1d}^{Avg}(t_{i-1}, t_i)$ the average overnight rate over period (t_{i-1}, t_i). We have

$$\sum_{i=1}^{4n} \left[F_{1d}^{Avg}(t_{i-1}, t_i) + \overline{S}_n^{1d} \right] \Delta t_i \, df_{1d}(t_0, t_i) = \sum_{i=1}^{4n} F_{3m}(t_{i-1}, t_i) \Delta t_i \, df_{1d}(t_0, t_i)$$

The curves involved in pricing this swap are listed in the table below:

	Swap Side 1	Swap Side 2
Forward Curve	OIS Curve	3m Curve
Discount Curve	OIS Curve	OIS Curve
Input Curves	3m Curve	
Output Curve	OIS Curve	

With the OIS curve the only unknown in the above equation, it can be readily obtained through bootstrapping from 1-year to 3-year.

3.4.2 Discount Curve Construction from Basis Swaps – One Forward Curve Not Known

Continuing from the example in the last section, both the OIS swap curve and the 3-month Libor now have been built to 3-year. Our task here is to extend the OIS curve further. In the fed fund vs Libor (OIS/3m) basis swap equation, both the OIS curve and 3m curve are not known,

$$\sum_{i=1}^{4n} \left[F_{1d}^{Avg}(t_{i-1}, t_i) + \overline{S}_n^{1d} \right] \Delta t_i \, df_{1d}(t_0, t_i) = \sum_{i=1}^{4n} F_{3m}(t_{i-1}, t_i) \Delta t_i \, df_{1d}(t_0, t_i)$$

We need to work jointly with another set of swap instruments in the market. From the previous section, we have the (3m) Libor IR swap equation,

$$\sum_{i=1}^{4n} F_{3m}(t_{i-1}, t_i) \Delta t_i \, df_{1d}(t_0, t_i) = \overline{R}_n^{3m} \sum_{i=1}^{4n} \Delta t_i \, df_{1d}(t_0, t_i)$$

The curves involved in pricing the two swaps are listed in the table below:

	OIS/3m Basis Swap		3m IR Swap	
	Pay Side	**Rec Side**	**Pay Side**	**Rec Side**
Forward Curve	OIS Curve	3m Curve	3m Curve	N/A
Discount Curve	OIS Curve	OIS Curve	OIS Curve	OIS Curve

Combining the two swap equations, we can cancel out the 3-month Libor terms,

$$\sum_{i=1}^{4n}\left[F_{1d}^{Avg}(t_{i-1},t_i)+\overline{S}_n^{1d}\right]\Delta t_i\, df_{1d}(t_0,t_i)=\overline{R}_n^{3m}\sum_{i=1}^{4n}\Delta t_i\, df_{1d}(t_0,t_i)$$

Now, the only curve involved is the OIS curve in the newly combined swap:

Curve \ Side	Pay Side	Rec Side
Forward Curve	OIS Curve	N/A
Discount Curve	OIS Curve	OIS Curve
Input Curves	None	
Output Curve	OIS Curve	

With the OIS curve the only unknown in the above equation, it can be readily obtained through bootstrapping.

We have just shown a “combining two swaps” approach to build an OIS curve from two types of swap instruments: OIS/3m basis swaps and 3m IR swaps. This approach requires the market quotes for both sets of the swap instruments available on a common set of maturities. To meet this condition, the curve builder will insert an additional swap instrument at any maturity where only one type of swaps is quoted, by linearly interpolating the par rates from market quotes of the other type of swap instruments. For example, if the 3m IR swap quote is available at 12-year maturity, while the OIS/3m basis swap is not. The curve build will insert the OIS/3m basis swap quote at 12-year maturity, by linearly interpolating the par rates from market quoted OIS/3m basis swaps at 10-year and 15-year maturities. Similarly, if the 3m IR swap quote is not available at 12-year maturity, while the OIS/3m basis swap is, the curve build will insert the 3m IR swap quote at 12-year maturity, by linearly interpolating the par rates from market quoted 3m IR swaps at 10-year and 15-year maturities.

3.4.3 Forward Curve Construction from IR Swaps

Once the OIS discount curve is constructed, the 3-month Libor curve can be obtained from the Libor IR swap quotes using the methodology described in the “Swap Part 1” section. Please note that here we must use the 3m IR quotes on a set of maturities common with the OIS/3m basis swap quotes, to be consistent with the OIS discount curve construction above.

3.4.4 USD Fed Fund Rates Approximation

The most liquid USD OIS market instruments are Fed Fund (FF) rate futures and FF/Libor basis swaps. The most liquid short term instruments are the monthly FF futures typically up to 2 years. The most liquid longer term instruments are FF/Libor basis swaps. For a given period (1m for FF futures and 3m for FF/Libor basis swaps), the FF rate is the average of the daily 1-day forward rates. However it is very costly to compute these daily forward rates.

Here we propose and develop an approximate method for computing the average directly without computing the daily forward rates.

Let $\tau_i : i = 0,1,2,...,N$ be the calendar days in the given period and $\Delta\tau = \tau_i - \tau_{i-1}$. The daily forward rates are computed as

$$F(\tau_i, \tau_{i+1}) = \frac{e^{r(\tau_{i+1})\tau_{i+1} - r(\tau_i)\tau_i} - 1}{\Delta\tau}$$

Where $r(\tau_i)$ is the zero rate at time τ_i.

The average forward rate for the period $[\tau_0, \tau_N]$ is then given by

$$F^{AVG}(\tau_0, \tau_N) = \frac{1}{N}\sum_{i=0}^{N-1} F(\tau_i, \tau_{i+1})$$

Applying Taylor expansion to $e^{r(\tau_{i+1})\tau_{i+1} - r(\tau_i)\tau_i}$ immediately produces the first order approximation of the average forward rate

$$F^{AVG}(\tau_0, \tau_N) = \frac{r(\tau_N)\tau_N - r(\tau_0)\tau_0}{N\Delta\tau}$$

This first order approximation is very accurate with accuracy within 0.05 bps for the current rates environment (the current longer term swap rates are around 5%). As it requires only one equivalent forward rate calculation instead of 90 forward rates (for the 3m case), it is much faster than the exact one.

3.5 Cross Currency Curve Construction

In this section, we will show how to build the cross-currency discount curve, consistent with the OIS discounting in the collateral currency, when the forward curve on the non-collateral currency side is known. The cross currency basis swap instruments are utilized to bridge the OIS discount curves between two currencies.

3.5.1 Discount Curve Construction from Currency Basis Swaps

Here, we use the USD collateralized CAD/USD cross-currency swap as a concrete example to demonstrate the methodology.

We assume that the USD OIS discount curve and USD 3m-Libor forward curve have already been built. Furthermore, we assume that CAD 3m curve already bootstrapped from the single currency 3m IR swaps collateralized in CAD.

Our goal is to construct a CAD discount curve that is consistent with the USD OIS discounting.

Consider a n-year CAD/USD currency basis swap with a market quoted spread $\overline{S}_n^{CadUsd}$. The pay side floating rate resets quarterly to the 3-month USD Libor $F_{3m}^{Usd}(t_{i-1}^{Usd}, t_i^{Usd})$ on the tenor period dates $t_i^{Usd} = t_{i-1}^{Usd} + 3m$, $i = 1,...,4n$. The receive side floating rate resets quarterly to the

3-month CAD Libor $F_{3m}^{Cad}(t_{i-1}^{Cad}, t_i^{Cad})$ on the tenor period dates $t_i^{Cad} = t_{i-1}^{Cad} + 3m$. Both pay and receiving sides pay on a common set of quarterly spaced payment period dates $t_i^{pCadUsd} = t_{i-1}^{pCadUsd} + 3m$ without a pay lag. The tenor period dates t_i^{Cad} and t_i^{Usd} are adjusted to the relevant CAD and USD reset holiday centers respectively, while the payment period dates $t_i^{pCadUsd}$ are adjusted to the union of CAD and USD payment holiday centers.

In the CAD/USD currency basis swap equation can be written as

$$FX_{Cad/Usd}(0)\left\{df_{Usd}^{Usd1d}(0, t_0^{pCadUSD}) - \sum_{i=1}^{4n} F_{3m}^{Usd}(t_{i-1}^{Usd}, t_i^{Usd})\Delta t_i^{pCadUsd} df_{Usd}^{Usd1d}(0, t_i^{pCadUSD}) - df_{Usd}^{Usd1d}(0, t_{4n}^{pCadUSD})\right\}$$

$$= df_{Cad}^{Usd1d}(0, t_0^{pCadUSD}) - \sum_{i=1}^{4n}\left[F_{3m}^{Cad}(t_{i-1}^{Cad}, t_i^{Cad}) + \overline{S}_n^{CadUsd}\right]\Delta t_i^{pCadUsd} df_{Cad}^{Usd1d}(0, t_i^{pCadUSD}) - df_{Cad}^{Usd1d}(0, t_{4n}^{pCadUSD})$$

where $FX_{Cad/Usd}(0)$ is the FX rate on the curve anchor date.

The curves involved in pricing this swap are listed in the table below:

	Swap Side 1	**Swap Side 2**
Forward Curve	USD 3m	CAD 3m
Discount Curve	USD discount df_{Usd}^{Usd1d}	CAD discount df_{Cad}^{Usd1d}
Input Curves	USD df_{Usd}^{Usd1d} curve, USD 3m Curve, CAD 3m Curve	
Output Curve	CAD df_{Cad}^{Usd1d} curve	

We added a superscript to reference the collateral currency. "df_{Usd}^{Usd1d} curve" denotes the USD OIS discount curve with USD as the collateral currency, which was previously constructed as "USD df_{1d}" in the single currency curve construction section. However, "df_{Cad}^{Usd1d} curve" here denotes the CAD discount curve under the USD OIS discounting. It is different from the CAD OIS curve (i.e. df_{Cad}^{Cad1d} curve) under the CAD OIS discounting.

With the CAD df_{Cad}^{Usd1d} discount curve the only unknown in the above equation, it can be readily obtained through bootstrapping.

This methodology is applicable to the case when the forward curve on the non-collateral currency side is known.

3.5.2 Currency Basis Swaps with FX Reset Notional

The market often quotes "currency basis swaps with FX reset notional" (or "FX reset swap" in short). Similarly to the traditional currency basis swaps, the parties exchange the Libor in one currency and the Libor plus spread in another currency with notional exchanges. However, in FX reset swaps, the notional on the currency paying Libor flat (usually USD per market convention) is adjusted at the every start of the Libor calculation period based on the spot FX, and the difference between the notional used in the previous period and the next one is also paid or received at the reset time. The notional for the other currency is kept constant throughout the contract.

For pricing in the curve construction, we can consider it as a portfolio of the strips of the one-period traditional currency basis swaps,

$$\sum_{i=1}^{4n} FX_{Cad/Usd}(t_{i-1}^{fxReset})\left[df_{Usd}^{Usd1d}(0,t_{i-1}^{pCadUSD}) - F_{3m}^{Usd}(t_{i-1}^{Usd},t_i^{Usd})\Delta t_i^{pCadUsd} df_{Usd}^{Usd1d}(0,t_i^{pCadUSD}) - df_{Usd}^{Usd1d}(0,t_i^{pCadUSD})\right]$$

$$= \sum_{i=1}^{4n}\left\{df_{Cad}^{Usd1d}(0,t_{i-1}^{pCadUSD}) - \left[F_{3m}^{Cad}(t_{i-1}^{Cad},t_i^{Cad}) + \overline{S}_n^{CadUsd}\right]\Delta t_i^{pCadUsd} df_{Cad}^{Usd1d}(0,t_i^{pCadUSD}) - df_{Cad}^{Usd1d}(0,t_i^{pCadUSD})\right\}$$

where the reset FX rate can be computed with the interest rate parity relationship

$$FX_{Cad/Usd}(t_{i-1}^{fxReset}) = FX_{Cad/Usd}(0)\frac{df_{Usd}^{Usd1d}(0,t_{i-1}^{fxReset})}{df_{Cad}^{Usd1d}(0,t_{i-1}^{fxReset})}$$

and $t_{i-1}^{fxReset}$ denotes the FX reset date at the start of each Libor period.

More details on the FX reset swap pricing are available the FX reset swap model description [6].

In term of curve construction, the FX reset swaps have the same curve dependency as the traditional currency basis swaps. Therefore, we could use the same curve construction procedure as described for the traditional currency basis swaps in the previous sections.

3.6 Swap Part 4. Cross Currency Curve Co-construction

In this section, we will show how to build the cross-currency discount curve, consistent with the OIS discounting in the collateral currency, when the forward curve on the non-collateral currency side is NOT known. The cross currency basis swap instruments are utilized to bridge the OIS discount curves between two currencies.

3.6.1 Discount Curve Construction from Currency Basis Swaps

Here, we use the USD collateralized MXN single currency swap and the USD collateralized MXN/USD cross-currency swap as a concrete example to demonstrate the methodology.

We assume that the USD OIS discount curve and USD 1m-Libor forward curve have already been built. Our goal is to construct a MXN discount curve that is consistent with the USD OIS discounting.

Consider a n-period MXN/USD currency basis swap with a market quoted spread $\overline{S}_n^{MxnUsd}$. The pay side floating rate resets every 28 days to the 1-month USD Libor $F_{1m}^{Usd}(t_{i-1}^{Usd},t_i^{Usd})$ on the tenor period dates $t_i^{Usd} = t_{i-1}^{Usd} + 1m$, $i = 1,...,n$. The receive side floating rate resets also every 28 days to the 28-day MXN Libor $F_{28d}^{Mxn}(t_{i-1}^{Mxn},t_i^{Mxn})$ on the tenor period dates $t_i^{Mxn} = t_{i-1}^{Mxn} + 28d$. Both pay and receiving sides pay on a common set of 28-day spaced payment period dates $t_i^{pMxnUsd} = t_{i-1}^{pMxnUsd} + 28d$ without a pay lag. The tenor period dates t_i^{Mxn} and t_i^{Usd} are adjusted to the relevant MXN and USD reset holiday centers respectively, while the payment period dates $t_i^{pMxnUsd}$ are adjusted to the union of MXN and USD payment holiday centers.

In the MXN/USD currency basis swap equation, both the MXN OIS curve and MXN 28d curve are not known,

$$FX_{Mxn/Usd}(0)\left\{df_{Usd}^{Usd1d}(0,t_0^{pMxnUSD})-\sum_{i=1}^{n}F_{1m}^{Usd}(t_{i-1}^{Usd},t_i^{Usd})\Delta t_i^{pMxnUsd}df_{Usd}^{Usd1d}(0,t_i^{pMxnUSD})-df_{Usd}^{Usd1d}(0,t_n^{pMxnUSD})\right\}$$

$$=df_{Mxn}^{Usd1d}(0,t_0^{pMxnUSD})-\sum_{i=1}^{n}\left[F_{28d}^{Mxn}(t_{i-1}^{Mxn},t_i^{Mxn})+\overline{S}_n^{MxnUsd}\right]\Delta t_i^{pMxnUsd}df_{Mxn}^{Usd1d}(0,t_i^{pMxnUSD})-df_{Mxn}^{Usd1d}(0,t_n^{pMxnUSD})$$

where $FX_{Mxn/Usd}(t_0)$ is the FX rate on the curve anchor date.

We need to work jointly with another set of swap instruments in the market. Consider a n-period MXN 28d Libor interest rate swap with a market quoted fixed rate $\overline{R}_n^{Mxn28d}$. Both sides pays every 28 days on period dates $t_j^{pMxn}=t_{j-1}^{pMxn}+28d$, $j=1,...,2n$ without a pay lag. The pay side floating rate resets every 28 days to the 28d MXN Libor $F_{28d}^{Mxn}(t_{i-1}^{Mxn},t_i^{Mxn})$ on the tenor period dates $t_i^{Mxn}=t_{i-1}^{Mxn}+28d$, $i=1,...,n$. The period dates t_i^{Mxn} and T_j^{pMxn} are adjusted to the relevant MXN holiday center.

$$\sum_{i=1}^{n}F_{28d}^{Mxn}(t_{i-1}^{Mxn},t_i^{Mxn})\Delta t_i^{pMxn}df_{Mxn}^{Usd1d}(0,t_i^{pMxn})=\overline{R}_n^{Mxn28d}\sum_{i=1}^{n}\Delta t_i^{pMxn}df_{Mxn}^{Usd1d}(0,t_i^{pMxn})$$

The curves involved in pricing the two swaps are listed in the table below:

	MXN/USD Basis Swap		**MXN 28d IR Swap**	
	Pay Side	**Rec Side**	**Pay Side**	**Rec Side**
Forward Curve	USD 1m	**MXN** 28d	**MXN** 28d	N/A
Discount Curve	USD df_{Usd}^{Usd1d}	**MXN** df_{Mxn}^{Usd1d}	**MXN** df_{Mxn}^{Usd1d}	**MXN** df_{Mxn}^{Usd1d}

"USD df_{Usd}^{Usd1d}" denotes the USD OIS discount curve with USD as the collateral currency. However, "MXN df_{Mxn}^{Usd1d}" here denotes the MXN discount curve under the USD OIS discounting. It is NOT the MXN OIS curve under the MXN OIS discounting (i.e. df_{Mxn}^{Mxn1d} curve).

The MXN 28d-Libor sides from the two swaps may not exactly cancel due to the potential payment date difference. The currency basis swap's payment dates $t_i^{pMxnUsd}$ are adjusted to the union of MXN and USD payment holiday centers, while the IR swap's payment dates T_j^{pMxn} are only adjusted to the relevant MXN holiday center. As a result, the "combining two swaps" approach described in the "Swap Part 2" section will not yield an exact "MXN df_{Mxn}^{Usd1d}" curve, but only a good estimate of it. We could use an iterative procedure to refine it to a desirable accuracy.

As a result, given the "USD df_{Usd}^{Usd1d}" and USD 1m curves, both the "MXN df_{Mxn}^{Usd1d}" and MXN 28d curves must be co-built together from the swaps jointly:

Curve \ Side	USD Curve	MXN Curve
Forward Curve	USD 1m	MXN 28d
Discount Curve	USD df_{Usd}^{Usd1d}	MXN df_{Mxn}^{Usd1d}
Input Curves	USD 1m, USD df_{Usd}^{Usd1d}	
Output Curve	MXN 28d, MXN df_{Mxn}^{Usd1d}	

3.6.1.1 Curve Co-construction Scheme

In practice, the following iterative procedure is used to build the OIS discount curve in a non-collateral currency (i.e. MXN in this example).

Step 1. Find an initial guess of the MXN 28d-Libor curve from the MXN IR swap quotes.

Curve \ Side	Pay Side	Rec Side
Forward Curve	MXN 28d	N/A
Discount Curve	MXN 28d	MXN 28d
Input Curves	None	
Output Curve	MXN 28d (forward curve)	

Here, we utilized the observation that the 28d-Libor curve under the USD OIS discounting is very close to the one under MXN Libor discounting.

Step 2. Given the MXN 28d Libor curve, solve for the "MXN df_{OIS}" curve from the MXN/USD cross currency swap quotes

Curve \ Side	Pay Side	Rec Side
Forward Curve	USD 1m	MXN 28d
Discount Curve	USD df_{Usd}^{Usd1d}	MXN df_{Mxn}^{Usd1d}
Input Curves	USD df_{Usd}^{Usd1d} USD 1m, MXN 28d	
Output Curve	MXN df_{Mxn}^{Usd1d} (discount curve)	

Step 3. Given the "MXN df_{Mxn}^{Usd1d}" curve, solve for the MXN 28d-Libor curve from the MXN 28d IR swap quotes.

Curve \ Side	Pay Side	Rec Side
Forward Curve	MXN 28d	N/R
Discount Curve	MXN df_{Mxn}^{Usd1d}	MXN df_{Mxn}^{Usd1d}
Input Curves	MXN df_{Mxn}^{Usd1d}	
Output Curve	MXN 28d (forward curve)	

Repeat step 2 and 3 till all market instruments price sufficiently close to zero.

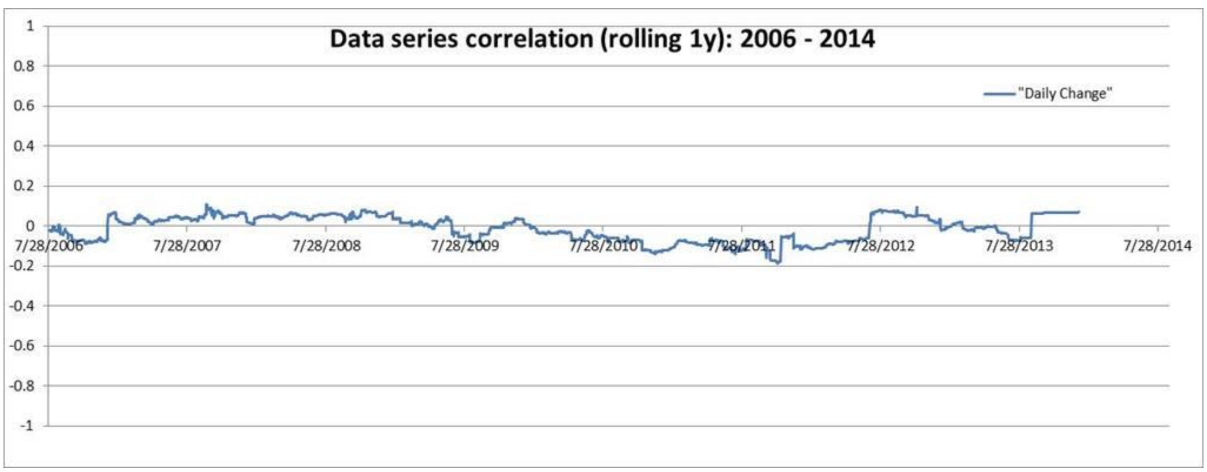
3.7 Other Discount Curve Construction

Other discount curve can be derived under the FX forward invariance principle from some known discount count curves through interest rate parity relationship. That is, the FX forward rate implied by the discount curves will not change with a change of the collateral currency or a switch from OIS to Libor discounting.

The FX forward invariance principle assumes that FX forwards do not depend on the collateral posted. This is the market standard assumption today.

Some literature [10] argued the possible existence of a collateral convexity driven by the dynamic relationship of the FX rate and the spread between the OIS rates of the relevant currencies. However, the author also recognized the parameters needed for the convexity estimate cannot be easily estimated from market series, there is no suitable hedge for this convexity, and even less a constructable arbitrage to exploit the adjustments. Thus, he recognized that it is debatable whether the convexity should be marked in pricing. Furthermore, his market evidence supports that the collateral convexity is NOT currently regarded as a contributing factor in pricing.

We performed independent study on the correlation between the FX rate and the spread between the OIS rates of the relevant currencies. The correlation is found to be small and largely within the +/-10% band, as shown in the figure below for EUR and USD pair. Therefore, if any, the amount convexity is expected to be small.



Note: The correlation was computed with one year rolling window. We observed the correlation oscillating around zero at higher frequency as the window size decreases.

3.7.1 Alternative Collateral Cross-Currency USD Discount Curve Construction

Given the USD discount curve df_{Usd}^{Usd1d} and CAD discount curve df_{Cad}^{Usd1d} under USD OIS discounting (i.e. USD cash collateral), we have the CAD/USD FX forward rate as

$$FX_{Cad/Usd}(t) = FX_{Cad/Usd}(0)\frac{df_{Usd}^{Usd1d}(0,t)}{df_{Cad}^{Usd1d}(0,t)}$$

Similarly, given the USD discount curve df_{Usd}^{Cad1d} and CAD discount curve df_{Cad}^{Cad1d} under CAD OIS discounting (i.e. CAD cash collateral), we have the CAD/USD FX forward rate as

$$FX_{Cad/Usd}(t) = FX_{Cad/Usd}(0)\frac{df_{Usd}^{Cad1d}(0,t)}{df_{Cad}^{Cad1d}(0,t)}$$

Equating the above two expressions for the FX forward gives

$$\frac{df_{Usd}^{Cad1d}(0,t)}{df_{Cad}^{Cad}(0,t)} = \frac{df_{Usd}^{Usd1d}(0,t)}{df_{Cad}^{Usd1d}(0,t)}$$

Therefore, knowing any three curves in the above equation, we can easily solve for the fourth curve. The USD discount curve df_{Usd}^{Cad1d} under the CAD OIS discounting can be constructed as

$$df_{Usd}^{Cad1d}(0,t) = \frac{df_{Usd}^{Usd1d}(0,t)}{df_{Cad}^{Usd1d}(0,t)} df_{Cad}^{Cad1d}(0,t)$$

Similarly, the USD discount curve df_{Usd}^{Eur1d} under the Eonia discounting can be constructed as

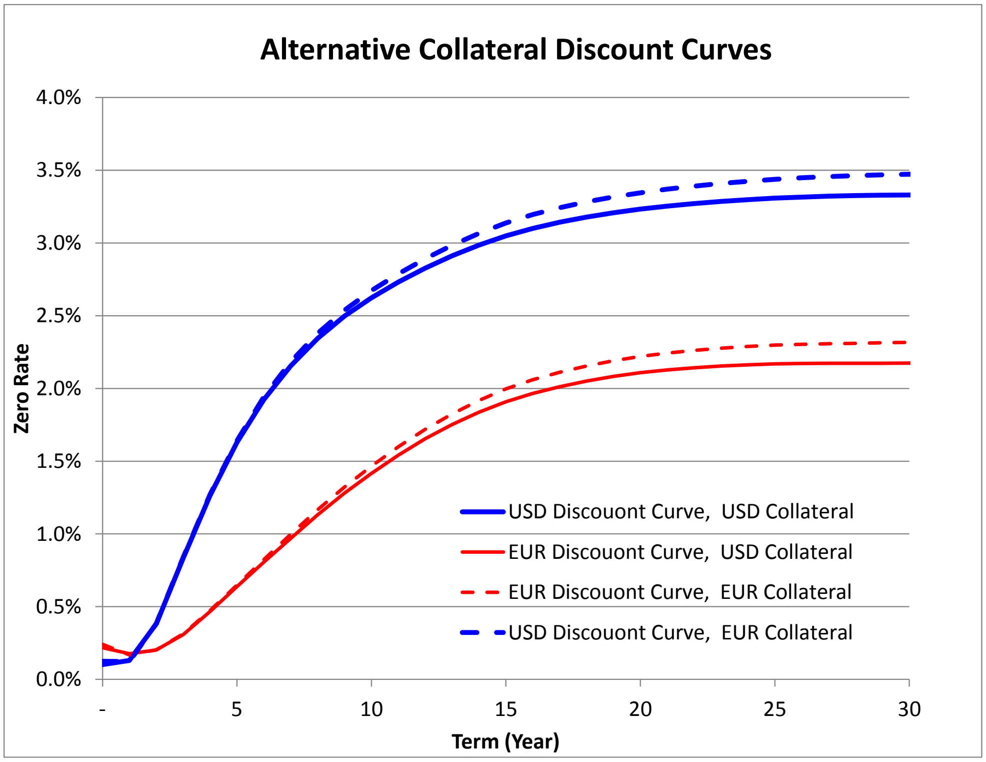
$$\frac{df_{Usd}^{Eur1d}(0,t)}{df_{Eur}^{Eur1d}(0,t)} = \frac{df_{Usd}^{Usd1d}(0,t)}{df_{Eur}^{Usd1d}(0,t)} \rightarrow df_{Usd}^{Eur1d}(0,t) = \frac{df_{Usd}^{Usd1d}(0,t)}{df_{Eur}^{Usd1d}(0,t)} df_{Eur}^{Eur1d}(0,t)$$

The USD discount curve df_{Usd}^{Gbp1d} under the Sonia discounting can be constructed as

$$\frac{df_{Usd}^{Gbp1d}(0,t)}{df_{Gbp}^{Gbp1d}(0,t)} = \frac{df_{Usd}^{Usd1d}(0,t)}{df_{Gbp}^{Usd1d}(0,t)} \rightarrow df_{Usd}^{Gbp1d}(0,t) = \frac{df_{Usd}^{Usd1d}(0,t)}{df_{Gbp}^{Usd1d}(0,t)} df_{Gbp}^{Gbp1d}(0,t)$$

As shown in the previous sections, the native currency OIS discount curves for df_{Usd}^{Usd1d}, df_{Cad}^{Cad1d}, df_{Eur}^{Eur1d} and df_{Gbp}^{Gbp1d} can be co-built from swap quotes in the native currency under the native currency OIS discounting. The cross-currency discount curves for df_{Cad}^{Usd1d}, df_{Eur}^{Usd1d} and df_{Gbp}^{Usd1d} can be built from the cross-currency basis swap quotes under the USD OIS discounting.

The graph below shows an example of constructed the USD discount curve df_{Usd}^{Eur1d} under the Eonia discounting, in relationship to the input curves. We can see the preservation of the interest rate parity relationship by design via the FX invariance principle: the EUR collateral curves (dashed lines) are above the corresponding USD collateral curves (solid line) by about the same proportion.



3.7.2 Alternative Collateral Cross-Currency Non-USD Discount Curve Construction

Given the USD discount curve df_{Usd}^{Usd1d} and CAD discount curve df_{Cad}^{Usd1d} under USD OIS discounting (i.e. USD cash collateral), we have the CAD/USD FX forward rate as

$$FX_{Cad/Usd}(t) = FX_{Cad/Usd}(0)\frac{df_{Usd}^{Usd1d}(0,t)}{df_{Cad}^{Usd1d}(0,t)}$$

Similarly, given the USD discount curve df_{Usd}^{Cad1d} and CAD discount curve df_{Cad}^{Eur1d} under EUR OIS discounting (i.e. EUR cash collateral), we have the CAD/USD FX forward rate as

$$FX_{Cad/Usd}(t) = FX_{Cad/Usd}(0)\frac{df_{Usd}^{Eur1d}(0,t)}{df_{Cad}^{Eur1d}(0,t)}$$

Equating the above two expressions for the FX forward gives

$$\frac{df_{Cad}^{Eur1d}(0,t)}{df_{Usd}^{Eur1d}(0,t)} = \frac{df_{Cad}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}$$

Therefore, knowing any three curves in the above equation, we can easily solve for the fourth curve. The CAD discount curve df_{Cad}^{Eur1d} under the Eonia discounting can be constructed as

$$df_{Cad}^{Eur1d}(0,t) = \frac{df_{Cad}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)} df_{Usd}^{Eur1d}(0,t)$$

Similarly, the CAD discount curve df_{Cad}^{Gbp1d} under the Sonia discounting can be constructed as

$$df_{Cad}^{Gbp1d}(0,t)=\frac{df_{Cad}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}df_{Usd}^{Gbp1d}(0,t)$$

As shown in the last section, the USD discount curves for df_{Usd}^{Eur1d} and df_{Usd}^{Gbp1d} can be built from the FX invariance principle.

The same approach can be used to build the EUR discount curve df_{Eur}^{Cad1d} and df_{Eur}^{Gbp1d} under the CAD OIS and Sonia discounting respectively,

$$\frac{df_{Eur}^{Cad1d}(0,t)}{df_{Usd}^{Cad1d}(0,t)}=\frac{df_{Eur}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}\rightarrow df_{Eur}^{Cad1d}(0,t)=\frac{df_{Eur}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}df_{Usd}^{Cad1d}(0,t)$$

$$\frac{df_{Eur}^{Gbp1d}(0,t)}{df_{Usd}^{Gbp1d}(0,t)}=\frac{df_{Eur}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}\rightarrow df_{Eur}^{Gbp1d}(0,t)=\frac{df_{Eur}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}df_{Usd}^{Gbp1d}(0,t)$$

as well as to build the GBP discount curve df_{Gbp}^{Cad1d} and df_{Gbp}^{Eur1d} under the CAD OIS and Eonia discounting respectively

$$\frac{df_{Gbp}^{Cad1d}(0,t)}{df_{Usd}^{Cad1d}(0,t)}=\frac{df_{Gbp}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}\rightarrow df_{Gbp}^{Cad1d}(0,t)=\frac{df_{Gbp}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}df_{Usd}^{Cad1d}(0,t)$$

$$\frac{df_{Gbp}^{Eur1d}(0,t)}{df_{Usd}^{Eur1d}(0,t)}=\frac{df_{Gbp}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}\rightarrow df_{Gbp}^{Eur1d}(0,t)=\frac{df_{Gbp}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}df_{Usd}^{Eur1d}(0,t)$$

Alternatively, we could also do the following

$$\frac{df_{Gbp}^{Eur1d}(0,t)}{df_{Eur}^{Eur1d}(0,t)}=\frac{df_{Gbp}^{Usd1d}(0,t)}{df_{Eur}^{Usd1d}(0,t)}\rightarrow df_{Gbp}^{Eur1d}(0,t)=\frac{df_{Gbp}^{Usd1d}(0,t)}{df_{Eur}^{Usd1d}(0,t)}df_{Eur}^{Eur1d}(0,t)$$

With this approach, we do not rely on non-USD collateralized USD discount curve $df_{Usd}^{Eur1d}(0,t)$ as an input. This is a more direct and arguably more efficient approach.

3.7.3 Collateral Currency and Discount Currency Arithmetic

The collateral currency and discount currency of an output curve can be implied from the collateral currencies and discount currencies of the three input curves. Taking the example from the last section,

$$df_{Gbp}^{Eur1d}(0,t)=\frac{df_{Gbp}^{Usd1d}(0,t)}{df_{Eur}^{Usd1d}(0,t)}df_{Eur}^{Eur1d}(0,t)$$

We can see that the collateral currencies in the superscripts are related by

$$\mathrm{Eur1d}=\frac{\mathrm{Usd1d}}{\mathrm{Usd1d}}\mathrm{Eur1d}$$

So are the discount currencies in the subscripts, by the same equation symbolically

$$\mathrm{Gbp}=\frac{\mathrm{Gbp}}{\mathrm{Eur}}\mathrm{Eur}$$

3.7.4 Alternative Collateral Cross-Currency Summary

The following table summarized the various discount curves under various collateral currencies, as well as the steps take to build them.

	USD Collateral $df^{Usd1d}_{???}$	CAD Collateral $df^{Cad1d}_{???}$	EUR Collateral $df^{Eur1d}_{???}$	GBP Collateral $df^{Gbp1d}_{???}$	USD Libor Funding $df^{Usd}_{???3m}$
USD Discount Curve $df^{?????}_{Usd}$	Step 1. Co-build df^{Usd1d}_{Usd}	Step 3. FX Invar df^{Cad1d}_{Usd}	Step 3. FX Invar df^{Eur1d}_{Usd}	Step 3. FX Invar df^{Gbp1d}_{Usd}	Step 1. Co-build df^{Usd3m}_{Usd}
CAD Discount Curve $df^{?????}_{Cad}$	Step 2. Xccy build df^{Usd1d}_{Cad}	Step 1. Co-build df^{Cad1d}_{Cad}	Step 4. FX Invar df^{Eur1d}_{Cad}	Step 4. FX Invar df^{Gbp1d}_{Cad}	Step 3a. FX Invar df^{Usd3m}_{Cad}
EUR Discount Curve $df^{?????}_{Eur}$	Step 2. Xccy build df^{Usd1d}_{Eur}	Step 4. FX Invar df^{Cad1d}_{Eur}	Step 1. Co-build df^{Eur1d}_{Eur}	Step 4. FX Invar df^{Gbp1d}_{Eur}	Step 3a. FX Invar df^{Usd3m}_{Eur}
GBP Discount Curve $df^{?????}_{Gbp}$	Step 2. Xccy build df^{Usd1d}_{Gbp}	Step 4. FX Invar df^{Cad1d}_{Gbp}	Step 4. FX Invar df^{Eur1d}_{Gbp}	Step 1. Co-build df^{Gbp1d}_{Gbp}	Step 3a. FX Invar df^{Usd3m}_{Gbp}

Notes

Step 1. Co-build: Co-build the 3m curve and OIS curve in the native currency under the native currency OIS discounting, from 3m swaps and OIS/3m swaps quoted in native currency collateral.

Step 2. Xccy build: Build the non-USD currency discount curves under the USD OIS discounting, from the cross-currency basis swaps quoted in USD collateral.

Step 3. FX Invar: infer the USD discount curves under the non-USD currency OIS discounting, using the FX invariance approached described in the last section.

Step 4. FX Invar: infer the non-USD discount curves under another non-USD currency OIS discounting, using the FX invariance approached described in this section.

Step 3a. FX Invar: infer the non-USD discount curves under the USD Libor discounting, using the FX invariance approached described in the next section.

3.7.5 Libor Discount Cross-Currency Discount Curve Construction

Given the USD discount curve df^{Usd1d}_{Usd} and CAD discount curve df^{Usd1d}_{Cad1d} under USD OIS discounting (i.e. USD cash collateral), we have the CAD/USD FX forward rate as

$$FX_{Cad/Usd}(t) = FX_{Cad/Usd}(0)\frac{df^{Usd1d}_{Usd}(0,t)}{df^{Usd1d}_{Cad}(0,t)}$$

Similarly, given the USD discount curve df^{Usd3m}_{Usd} and CAD discount curve df^{Usd3m}_{Cad} under USD Libor discounting (i.e. under USD Libor funding assumption), we have the CAD/USD FX forward rate as

$$FX_{Cad/Usd}(t) = FX_{Cad/Usd}(0)\frac{df^{Usd3m}_{Usd}(0,t)}{df^{Usd3m}_{Cad}(0,t)}$$

Equating the above two expressions for the FX forward gives

$$\frac{df_{Cad}^{Usd3m}(0,t)}{df_{Usd}^{Usd3m}(0,t)} = \frac{df_{Cad}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)}$$

Therefore, knowing any three curves in the above equation, we can easily solve for the fourth curve. The CAD discount curve df_{Cad}^{Usd3m} under the USD Libor funding can be constructed as

$$df_{Cad}^{Usd3m}(0,t) = \frac{df_{Cad}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)} df_{Usd}^{Usd3m}(0,t)$$

Similarly, the EUR discount curve df_{Eur3m}^{Usd} under the USD Libor funding can be built as

$$\frac{df_{Eur}^{Usd3m}(0,t)}{df_{Usd}^{Usd3m}(0,t)} = \frac{df_{Eur}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)} \rightarrow df_{Eur}^{Usd3m}(0,t) = \frac{df_{Eur}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)} df_{Usd}^{Usd3m}(0,t)$$

The GBP discount curve df_{Gbp}^{Usd3m} under the USD Libor funding can be constructed as

$$\frac{df_{Gbp}^{Usd3m}(0,t)}{df_{Usd}^{Usd3m}(0,t)} = \frac{df_{Gbp}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)} \rightarrow df_{Gbp}^{Usd3m}(0,t) = \frac{df_{Gbp}^{Usd1d}(0,t)}{df_{Usd}^{Usd1d}(0,t)} df_{Usd}^{Usd3m}(0,t)$$

As shown in the previous sections, the USD 3m curve df_{Usd}^{Usd3m} and OIS discounting curve for df_{Usd}^{Usd1d} can be co-built from USD swap quotes under the USD OIS discounting. The cross-currency discount curves for df_{Cad}^{Usd1d}, df_{Eur}^{Usd1d} and df_{Gbp}^{Usd1d} can be built from the cross-currency basis swap quotes under the USD OIS discounting.

3.7.6 Cheapest-to-Deliver Discount Curve Construction

Some CSA agreements have the option to choose from a basket of pre-specified collateral currencies. The collateral posting party will then deliver the collateral in the currency most favorable to it, i.e. in whichever currency that earns the highest 1-day OIS rate at the time of collateral posting.

This choice of the collateral currencies is an option that depends on the states of economy at collateral posting time over the life of the portfolio. A comprehensive treatment would embody a Monte Carlo simulation of the portfolio value under the relevant CSA on all risk factors relevant to the trades in the portfolio. Here, we adopt an industry standard practical approach that builds and uses a cheapest to deliver collateral discount curve to account for the collateral selection option over the life of a trade. This approach ignores the curve dynamics of curve and prices the option at its intrinsic value.

The cheapest-to-deliver (CTD) collateral discount curve is defined according to the maximum of instantaneous forward rate among the discount curves in the basket,

$$df_{CTD}(t) = e^{-\int_0^t f_{CTD}(\tau)\tau} = e^{-r_{CTD}(t)t}$$

$$f_{CTD}(t) = \max\{f_i(t), i = 1,..., n_{Curve}\}$$

We denote df_{CTD} and r_{CTD} the CTD curve's discount factor and zero rate respectively.

In practice, the 1-day forward rate can be used as the instantaneous forward rate.

$$f_i(t) = \frac{1}{\Delta t}\left(\frac{df_i(t)}{df_i(t+\Delta t)} - 1\right), \text{ where } \Delta t = \frac{1}{365}$$

The CTD instantaneous forward rate curve can be presented as piecewise sections from the individual discount curves. Let us denote $i(t_k)$ the index of CTD curve covering a time range of (t_{k-1}, t_k). That is

$$f_{CTD}(t) = f_{i(t_k)}(t),\ \ t_{k-1} < t \le t_k,\ \ k = 1,\dots,n_{Cross}$$

with $t_0 = 0$ being the curve anchor date. Then, the CTD discount factor is a piecewise integration over sections of the individual discount curves in the basket.

$$df_{CTD}(t) = e^{-\int_0^t f_{CTD}(\tau)\tau} = e^{-\sum_{k=1}^{m(t)}\int_{t_{k-1}}^{t_k} f_{i(t_k)}(\tau)\tau} = \prod_{k=1}^{m(t)} df_{i(t_k)}(t_{k-1}, t_k)$$

with $t_0 = 0$ and $t_{m(t)} = t$ being the two end points of the integration.

Applying $df(s,t) = df(t)/df(s)$, we finally arrived at the formula for the discount factor on any date on the CTD curve,

$$df_{CTD}(t) = \prod_{k=1}^{m(t)} \frac{df_{i(t_k)}(t_k)}{df_{i(t_k)}(t_{k-1})} = df_{i(t)}(t)\sum_{k=1}^{m(t)-1} \frac{df_{i(t_k)}(t_k)}{df_{i(t_{k+1})}(t_k)}$$

The methodology can be best demonstrated with a concrete example. Given a CSA agreement contains an option to post cash collateral in USD, CAD, GBP or EUR, we first build the "alternative collateral cross-currency discount curves" using the methodology presented in the previous section. To discount the USD cashflow, we need the USD discount curves under all four currencies' OIS discounting, df_{Usd}^{Cad1d}, df_{Usd}^{Usd1d}, df_{Usd}^{Gbp1d} and df_{Usd}^{Eur1d}.

The structure of a sample CTD curve is presented in the table below. The ratios of the discount factor are pre-computed at crossing points where CTD curve transits from one collateral currency to another.

k	Range	CTD Curve	Discount Ratio at Crossing
1	0y < t < 2y	CAD collateral	$\alpha_1 = 1$
2	2y < t < 5y	USD collateral	$\alpha_2 = \alpha_1 \times df_{Usd}^{Cad1d}(2y)/df_{Usd}^{Usd1d}(2y)$
3	5y < t < 10y	GBP collateral	$\alpha_3 = \alpha_2 \times df_{Usd}^{Usd1d}(5y)/df_{Usd}^{Gbp1d}(5y)$
4	10y < t < 30y	EUR collateral	$\alpha_4 = \alpha_3 \times df_{Usd}^{Gbp1d}(10y)/df_{Usd}^{Eur1d}(10y)$

Four representative discount factor calculations are provides below for dates located in the sections of the curve where CAD, USD, GBP or EUR is the cheapest to deliver.

$$df_{CTD}(1y) = \alpha_1 \times df_{Usd}^{Cad1d}(1y) = 1 \times df_{Usd}^{Cad1d}(1y) = df_{Usd}^{Cad1d}(1y)$$

$$df_{CTD}(3y) = \alpha_2 \times df_{Usd}^{Usd1d}(3y) = \frac{df_{Usd}^{Cad1d}(2y)}{df_{Usd}^{Usd1d}(2y)} \times df_{Usd}^{Usd1d}(3y) = df_{Usd}^{Cad1d}(2y) \times df_{Usd}^{Usd1d}(2y,3y)$$

$$\mathrm{df}_{\mathrm{CTD}}(7\mathrm{y}) = \alpha_3 \times \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Gbp1d}}(7\mathrm{y}) = \frac{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Cad1d}}(2\mathrm{y})}{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Usd1d}}(2\mathrm{y})} \times \frac{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Usd1d}}(5\mathrm{y})}{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Gbp1d}}(5\mathrm{y})} \times \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Gbp1d}}(7\mathrm{y})$$

$$= \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Cad1d}}(2\mathrm{y}) \times \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Usd1d}}(2\mathrm{y},5\mathrm{y}) \times \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Gbp1d}}(5\mathrm{y},7\mathrm{y})$$

$$\mathrm{df}_{\mathrm{CTD}}(15\mathrm{y}) = \alpha_4 \times \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Eur1d}}(15\mathrm{y}) = \frac{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Cad1d}}(2\mathrm{y})}{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Usd1d}}(2\mathrm{y})} \times \frac{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Usd1d}}(5\mathrm{y})}{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Gbp1d}}(5\mathrm{y})} \times \frac{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Gbp1d}}(10\mathrm{y})}{\mathrm{df}_{\mathrm{Usd}}^{\mathrm{Eur1d}}(10\mathrm{y})} \times \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Eur1d}}(15\mathrm{y})$$

$$= \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Cad1d}}(2\mathrm{y}) \times \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Usd1d}}(2\mathrm{y},5\mathrm{y}) \times \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Gbp1d}}(5\mathrm{y},10\mathrm{y}) \times \mathrm{df}_{\mathrm{Usd}}^{\mathrm{Eur1d}}(10\mathrm{y},15\mathrm{y})$$

The above algorithm can be implemented very efficiently. If we pre-calculated the discount factor ratios α_i at the crossing points, the effort is only slightly more than computing the discount factor from one individual curve in the basket.

4 Special Topics on Curve Building Methodology

This section present some special topics in the bootstrapping methodology, such as ways to treat serial futures in the 3m Libor curve, meeting date instruments in the OIS curve, and market instrument spreadizing for curve risks.

4.1 Building 3m Major Curves with Serial Futures

The benefit of including serial futures is to refine the 3m curve front end for more accurate pricing and more resolution in risk buckets.

This section is about building the major curve front end with serial futures under the "near linearly interpolated" forward rate. It is achieved mainly by 1) marking the 3m cash rate with a weighting scheme when close to a futures roll, 2) using instrument start date as zero curve nodes for heavily overlapped cash and futures, and 3) applying high tension to all futures. The features of this methodology are described in details below.

The features described in this section are accessible under the useFutures flag setting 1525, 1535 and 1545.

4.1.1 Curve Instruments

The front end of a 3-month curve typically includes one cash, a few serial futures, and some IMM futures.

For example, the USD 3m curve may have 1 cash, 2 serial futures and 12 IMM futures. The IMM futures are quarterly spaced in March, June, September and December, while the serial futures are monthly spaced in January, February, April, May, etc.

Using two serial futures, we will have three monthly spaced futures in the front as below,

- On April 1, we have April *serial* futures, May *serial* futures, and June IMM futures
- On May 1, we have May *serial* futures, June IMM futures, and July *serial* futures
- On June 1, we have June IMM futures, July *serial* futures, August *serial* futures

4.1.2 3m Cash Target Rate Marking

Libor fixing comes 11am London time while the futures quotes come at the end of a trading day. When the market had a big move during the day, the 3m cash rate (e.g. Libor fixing) could be materially different from the rate indicated by the futures. As the futures approaches the roll date, the 3m cash fixing tenor period becomes more and more overlapped with the first futures tenor period. Any discrepancy between the cash and futures rate will cause a leveraged jump or drop of the instantaneous forward rate in front of the futures. For example, when the cash and the first future is 90% overlapped, 2 bps higher cash fixing than the future quote will result in a 20 bps jump of the instantaneous forward rate on the zero curve node located at the first futures start date.

This problem could be fixed if we relax the fitting to the 3m cash instrument. We derive a "target" 3m cash rate weighted between the quoted cash rate and "expected" cash rate to be matched in the curve bootstrapping. We used the term "expected" cash rate to express the intention to have a 3m cash rate that is in line with the end of day futures quotes.

The target 3m cash rate marking could involve up to three instruments:

- 3m Cash rate F_0, start date t_0^{St}, end date t_0^{End}
- 1st futures rate F_1, start date t_1^{St}, end date t_1^{End}
- 2nd futures rate F_2, start date t_2^{St}, end date t_2^{End}

We first defined an "expected" cash rate F_0'. It could be a rate projected by the cash rate itself, by the first futures rate F_1, or by the linearly extrapolated rate from the first two futures F_1 and F_2,

Method A1: $F_0' = F_0$, for useFutures = 1525

Method A2: $F_0' = F_1$, for useFutures = 1535

Method A3: $F_0' = F_1 - \frac{t_1^{St} - t_0^{St}}{t_2^{St} - t_1^{St}}(F_2 - F_1)$, for useFutures = 1545

We then compute the "target" cash rate by linearly weighting between the quoted cash rate and the "expected" cash rate,

$$F_0^{Target} = \omega F_0 + (1 - \omega)F_0'$$

The weighting factor approach zeros on the futures roll date, when the "target" cash rate almost approaches 100% of the "expected" cash rate as indicated by the futures. The weighting scheme will kick in when today is close enough to the futures roll.

The weighting factor is computed as the number of days from the valuation date to one business day before the roll, plus 1[2], divide by the threshold number of days,

$$\omega = \frac{(\text{number of days to one business day before the roll}) + 1}{(Threshold\ Days)}$$

The threshold number of days is currently set to 10 days.

Example: USD 3m Libor curve on valuation date $Thu\ 12Apr2018$

- 3m Cash rate $F_0 = 2.34769\%$, start date $t_0^{St} = Mon\ 16Apr2018$
- 1st futures rate $F_1 = 2.35\%$, start date $t_1^{St} = Wed\ 18Apr2018$
- 2nd futures rate $F_2 = 2.345\%$, start date $t_2^{St} = Wed\ 16May2018$

Under useFutures = 1545, the "expected" cash rate is computed as

$$F_0' = 2.34769\% - \frac{2\ days}{28\ days}(2.345\% - 2.35\%) = 2.35036\%$$

For the April futures starting on $t_1^{St} = Wed\ 18Apr2018$, the roll date is $Mon\ 16Apr2018$ and the one day before the roll is $Fri\ 13Apr2018$, which is 1 day before the valuation date $Thu\ 12Apr2018$. Therefore, the weighting factor is

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[2] If we do not "plus 1" here, the "target" rate will 100% match the "expected" rate one business day before the roll. This will make the curve transition across the roll very smooth. However, the cash rate will no long have any influence to the bootstrapped curve if we use 0% of the cash rate in this situation. In the later section, we will need to adjust the cash rate such that the stub rate remains constant between the day1 curve and day1 Tom curve for the carry analysis. We have to keep the cash rate not 0%, so that it could influence the bootstrapping to produce a desirable stub rate.

$$\omega = \frac{(\text{number of days to one business day before the roll}) + 1}{(Threshold\ Days)} = \frac{1+1}{10} = 0.2$$

The "target" cash rate is,

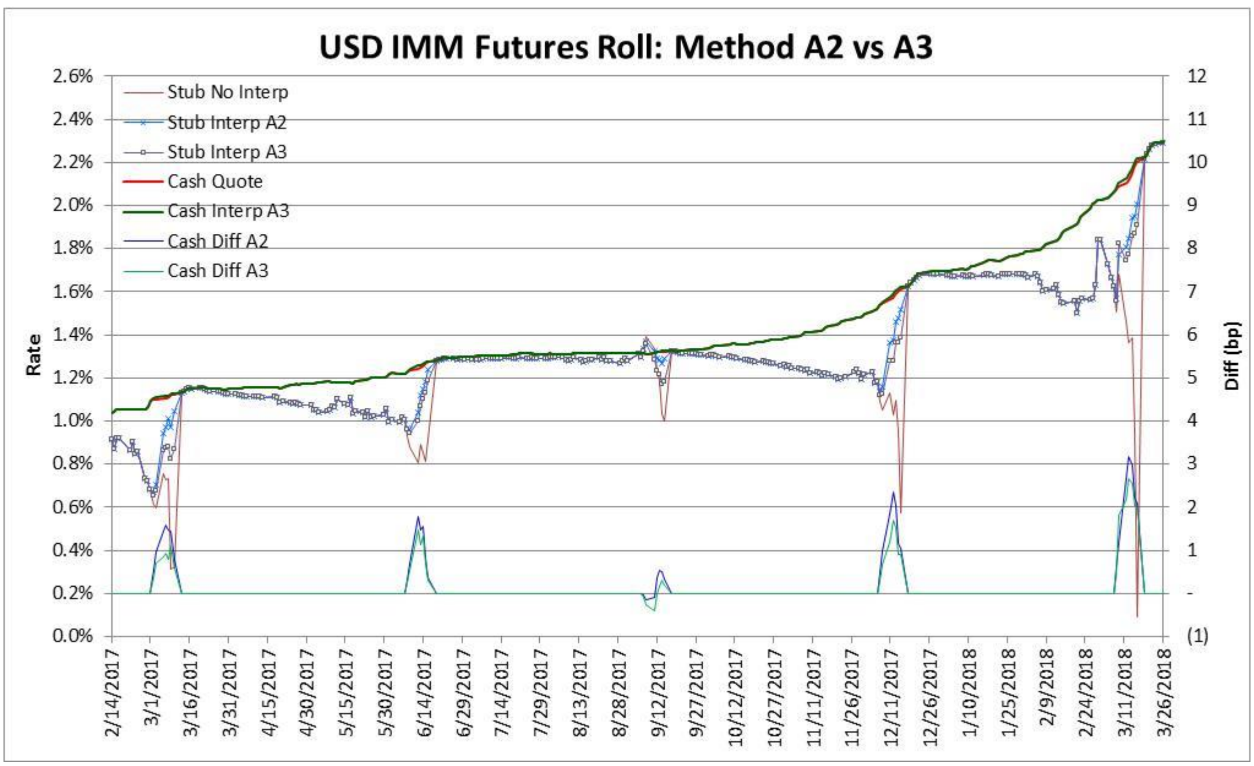
$$F_0^{Target} = \omega F_0 + (1-\omega)F_0' = 0.2 \times 2.34769\% + 0.8 \times 2.35036\% = 2.34982\%$$

Please note this weighting of 3m rate is only intended to treat the cash and futures overlapped situation. It will not be activated for curves built with non-overlapping instruments in the front end.

In summary, Method A2 marks the 3m cash rate by linear weighting between the quoted cash rate and the first futures rate when close to a futures roll. Method A3 marks the 3m cash rate by linear weighting between the quoted cash rate and an "expected" cash rate linearly extrapolated from the first two futures rates when close to a futures roll.

Among the weighting methods above, both A2 and A3 methods are good for a cash instrument overlapped with quarterly spaced IMM futures, where the futures are not heavily overlapped. With heavily overlapped monthly spaced serial futures, A3 is more suitable to provide a smooth transition across the roll. *It is important to maintain the front end slope as indicated by the first two futures to have a smooth transition across the rolls.*

The graph below shows the stub and cash rates implied in the curves built under various weighting methods across one year history, when only quarterly spaced IMM futures are used.



In the legend, we denote

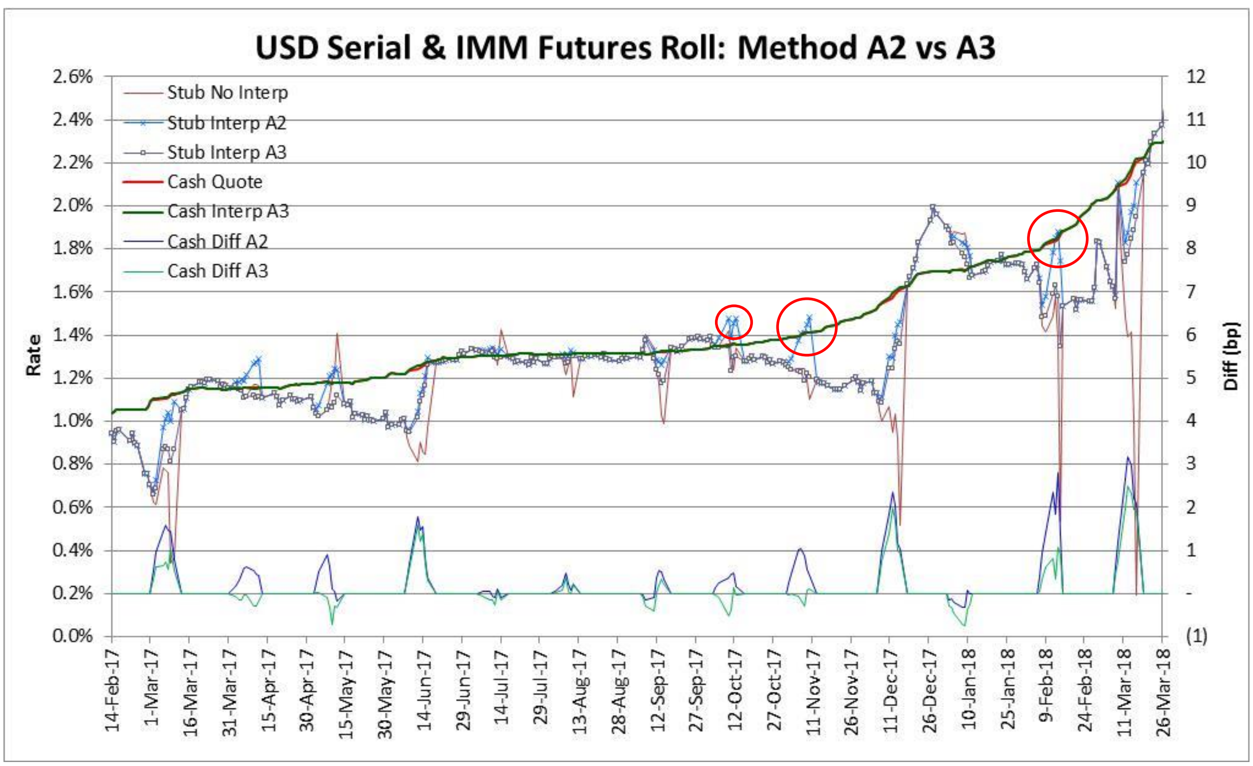
- "Stub No Interp" is the curve implied stub under Method A1 (i.e. no weighting)

- “Stub Interp A3” is the curve implied stub under Method A2 (with weighting)
- “Stub Interp A3” is the curve implied stub under Method A3 (with weighting)
- “Cash Quote” is the 3-month Libor fixing, or the target cash rate under Method A1. Curve built under Method A1 reproduces the “Cash Quote” exactly.
- “Cash Interp A3” is the target cash rate under Method A3 (i.e. with weighting). Curve built under Method A3 does not reproduce the “Cash Quote” exactly for the days within the threshold where the weighting is activated.
- “Cash Diff A2” = “Cash Interp A2” – “Cash Quote” in bps
- “Cash Diff A3” = “Cash Interp A3” – “Cash Quote” in bps

The stub rates, spanning from the cash start date to the first futures end date, are implied from the curves built under various methods.

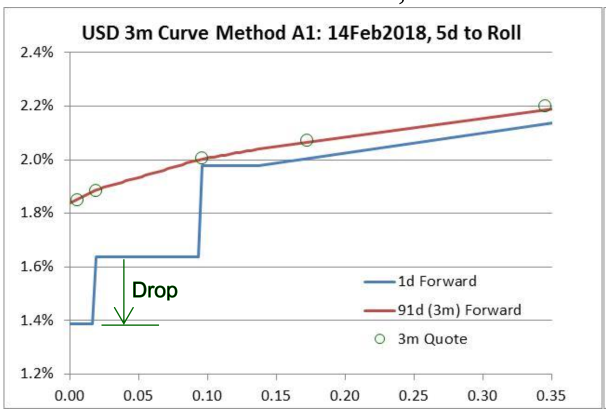
We can see that both methods A2 and A3 greatly stabilized the stub rate in comparison to the no weighting method A1. The performance of both methods A2 and A3 are similar. The adjustment from method A2 is a bit larger than method A3, such that the implied 3m cash rate from A2 curve is a bit further away from the 3m cash fixing quote that the implied 3m cash rate from A3 curve.

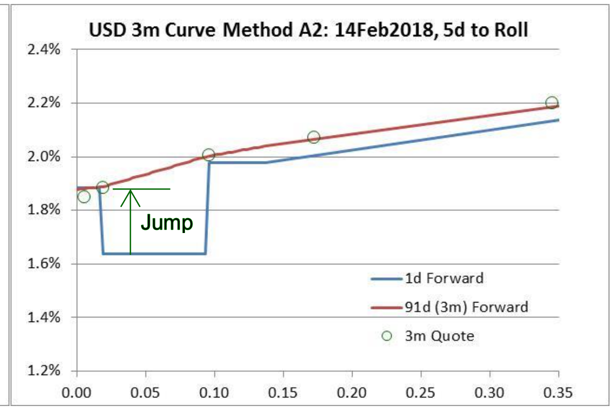
The graph below shows the stub and cash rates implied in the curves built under various weighting methods across one year history, when monthly spaced *serial* and IMM futures are used.



We can see that both methods A2 and A3 greatly stabilized the stub rate in comparison to the no weighting method A1. However, adjustment from method A2 is often larger than method A3, to the extent that it over-adjusted the stub drops into stub jumps (see the red circled peaks around roll in Oct, Nov2017, and Feb2018 for examples). Method A3 does not suffer this drawback. It consistently provides smooth transition near the serial and IMM futures rolls.

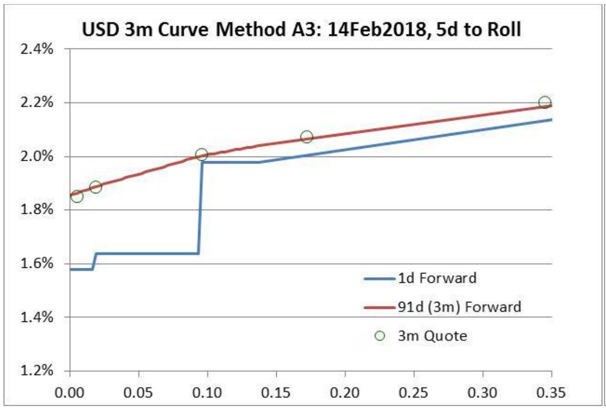
Method A2 weights the “target” cash rate towards the “expected” cash rate flat extrapolated from the first futures rate. In the upward sloping yield curve case, the first futures rate tends to overestimate the cash rate. Let us take a closer look at the before and after Feb2018 roll curves built under methods A1, A2 and A3.

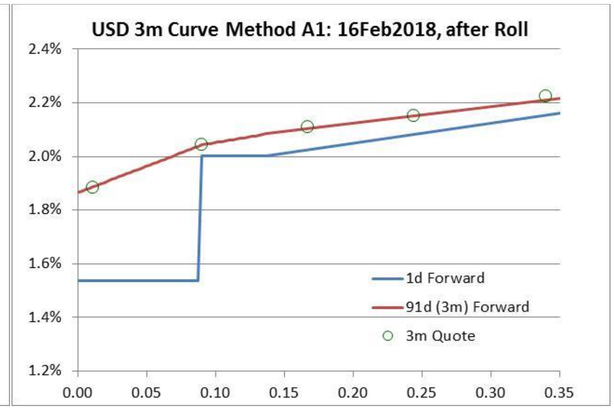




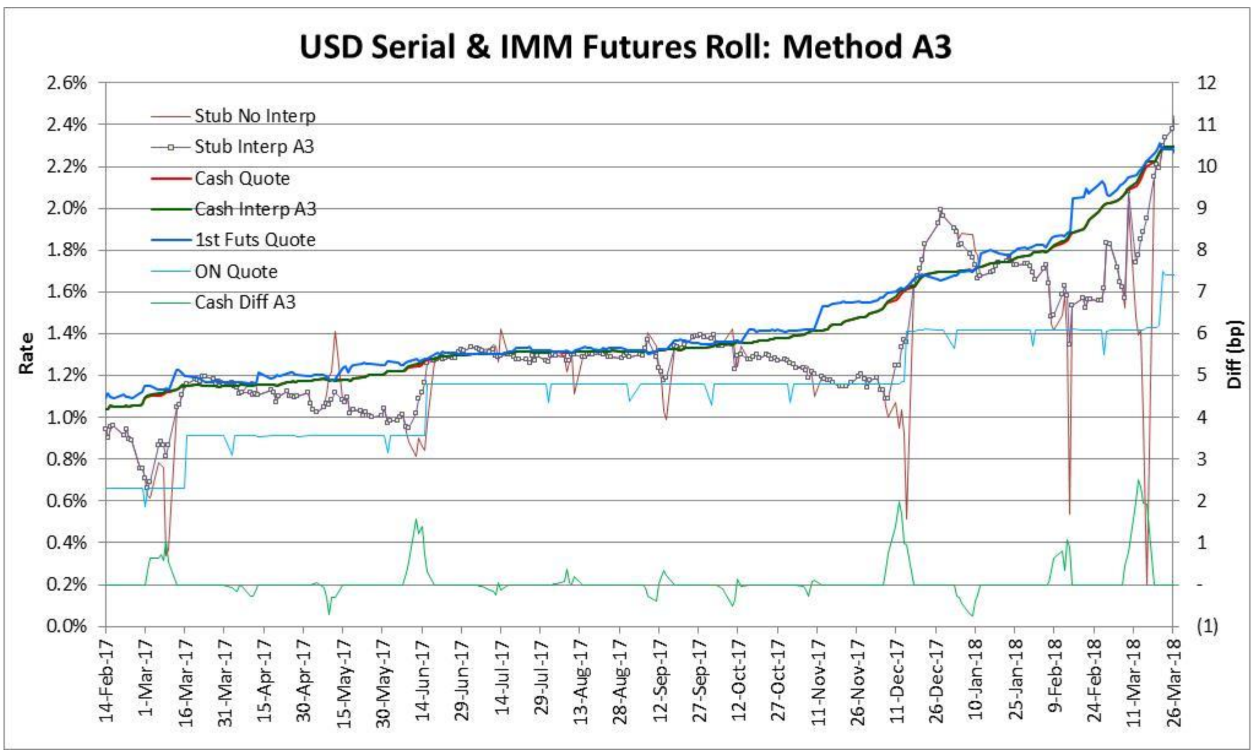
The graph above-left shows the 1-day (1d) and 3-month (91d) forward rates implied from the USD 3m Libor curve built under method A1 (no weight), five days before the roll. The graph above-right shows the same forward rates from the USD 3m Libor curve built under method A2. From the 1d forward curves, we can see that method A1 (no weighting) produced a stub rate “drop” in front of the first futures, while method A2 produced a stub rate “jump”, almost in the same magnitude. We can see a slight downward bend in the stub section of the 3m forward curve built under no weighting method A, as market by the actual 3m cash quote. Method M2 bends the stub section of the 3m forward curve upwards, away from the straight line between the first two futures quotes.

The graph below-left shows the forward rates implied from the USD 3m Libor curve built under method A3), five days before the roll. The graph below-right shows the same forward rates from the USD 3m Libor curve right after the roll. There is no difference right after the roll whether the curve is built under method A1, A2 or A3, since the weighting is not activated. From the 1d forward curves, we can see that method A3 produced a stub rate drop in front of the first futures, much smaller in magnitude than method A1. The small drop is due to fact that the weighted “target” cash rate does not match 100% the “expected” cash rate. If we set the “target” cash rate 100% matching the “expected” cash rate linearly extrapolated from the first two futures rates, the below-left (before roll) and below-right (after roll) graph will have the same flat shape (i.e. no stub drop). Consequently, the front section of the 3m forward curves remains linear between cash instrument and the first two futures, before and after the futures roll.





The graph below shows the stub and cash rates implied in the curve built under method A3 across one year history, when both the serial and IMM futures are used. This is the preferred method in the presence of serial futures.



In the legend, most of the items have been defined previously. The new items are

- “1st Futs Quote” is the quoted rate of the first futures, to mark the futures rolls
- “ON Quote” is the overnight rate, to mark the location of the rate hikes

Outside of the weighting periods (10 days away from the rolls), the weighting method A3 is the same as the no-weighting method A1. “Stub Interp A3” matches “Stub No Interp” outside the weighting periods, where the “Cash Interp A3” also matches the “Cash Quote” exactly.

As a roll approaches, the weighting scheme will kick in. The “Stub No Interp” from the no-weighting method A1 could become very unstable, while the “Stub Interp A3” from the weighting method A3 is much more stable. The large day-to-day moves of the stub rate are avoided. As a consequence of the weighting, the “Cash Interp 3A” no longer matches the

"Cash Quote" exactly. The peak difference between them could be 1.5-2.5 bps around some rolls.

Referencing to the jumps in "ON Quote", we can see larger deviation of stub rate away from the 3m cash rate before a rate hike. Correspondingly, the unstable periods of the stub rate span a larger number of days. The example periods are the dates leading to Mar, Jun, Dec2017 and Mar2018 IMM futures rolls over the graphed one year history. These dates coincide with those FOMC meeting dates with an expected rate change announcement. In practice, the rate changing FOMC meeting dates are not always located around the IMM futures rolls. They could also happen around a serial futures roll. Therefore, we set a uniform 10-day threshold for the weighting scheme across both the serial and IMM futures rolls.

4.1.3 Zero Curve Nodes

This section discusses the placement of the zero nodes corresponds to the curve instruments.

Use First Futures Start Date as Curve Node

The 3m cash end date is not used as a curve node. Rather the first future start date is used at its place to pin down the first futures forward rate start point, regardless whether the cash and futures instruments are overlapped or not. This approach provides a good representation in the forward rate space across the curve nodes on or near the futures start dates.

Use Futures Start Date as Curve Node When Heavily Overlapped

The 3m forward curve is commonly plotted as the forward rates against their dates. To create a linearly interpolated forward curve from a set of market quoted forward rate instruments, it is favorable to use the start dates of the instruments as the zero curve nodes.

Using start or end date does not matter much for the IMM futures, since the quarterly spaced 3-month serial futures are almost tiled head to tail. The end date of one IMM futures is very close to the start date of the next IMM futures. Taking futures end dates or taking futures start dates will end up with almost the same set of the sero curve nodes (except for the first and the last nodes). However, this is not the case for the serial futures, since the serial futures are monthly spaced.

We define the threshold for overlap as 12.5%. If the overlap between the two instruments is equal or greater than the threshold, it is considered to be heavily overlapped. This criterial is adequate for the serial futures, since the monthly spaced 3m futures are 33% overlapped.

In practice, we will process one curve instrument at a time. The instruments are ordered by end dates. Denote the current instrument start date is t_i^{St}, end date is t_i^{End}, the next instrument start date t_{i+1}^{St}, and end date t_{i+1}^{End}. The overlap ratio is calculated as

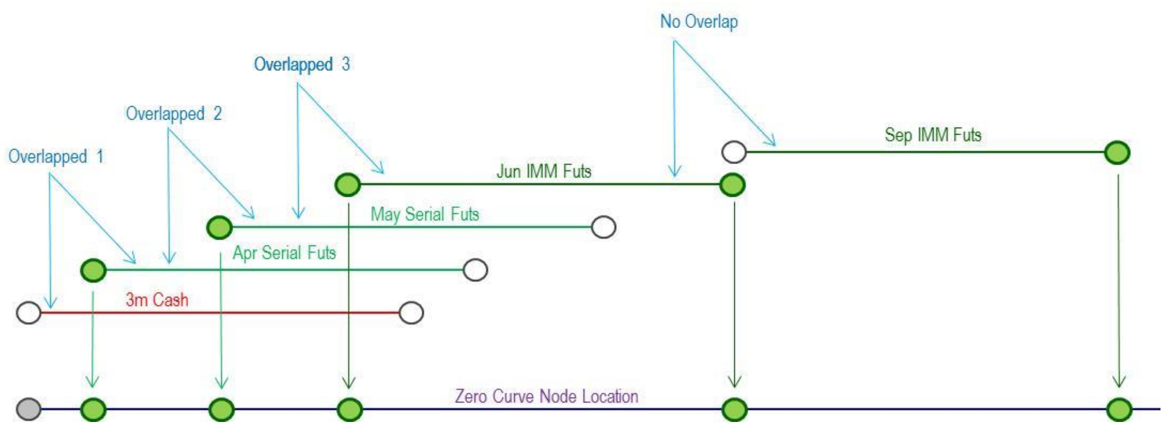
$$v_i = \frac{t_i^{End} - t_{i+1}^{St}}{t_i^{End} - t_i^{St}}.$$

If the overlap ratio $v_i \geq 12.5\%$, we take the next instrument start date t_{i+1}^{St} as a zero curve node. Otherwise, take this instrument start date t_i^{End} as a zero curve node.

Using two serial futures, we have three cases of monthly spaced serial and IMM futures, leading to three situations of zero curve node placements. We will illustrate this with examples.

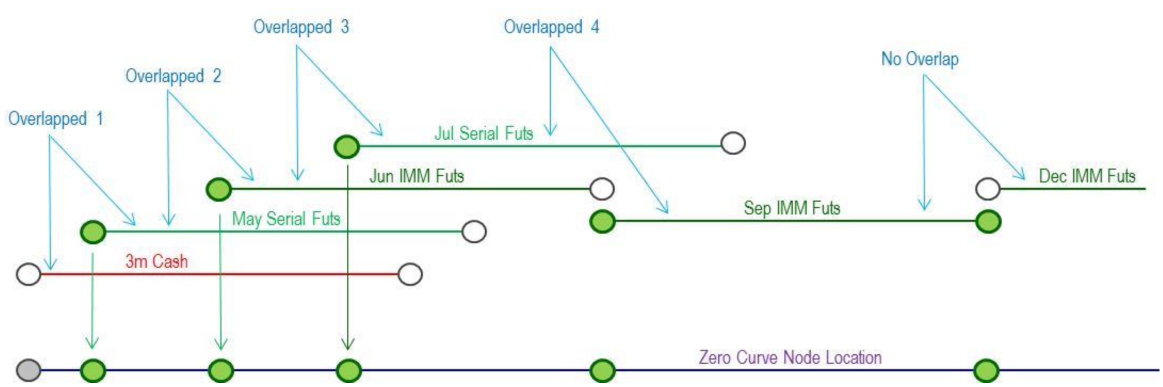
Case 1. Serial, Serial, and IMM Futures

On April 1, we have monthly spaced April *serial* futures, May *serial* futures, and June IMM futures in the front, as shown in the graph below. The 3m cash overlaps with April serial futures, so we use the April serial futures start date as a zero curve node. The April serial futures overlaps with May serial futures, so we use the May serial futures start date as a zero curve node. The May serial futures overlaps with June IMM futures, so we use the June IMM futures start date as a zero curve node. There is no overlap between June and September IMM futures, so we use the June IMM futures end date as a zero curve node. From this point on, there is no more overlap between the instruments, so we use the futures end dates as zero curve nodes.



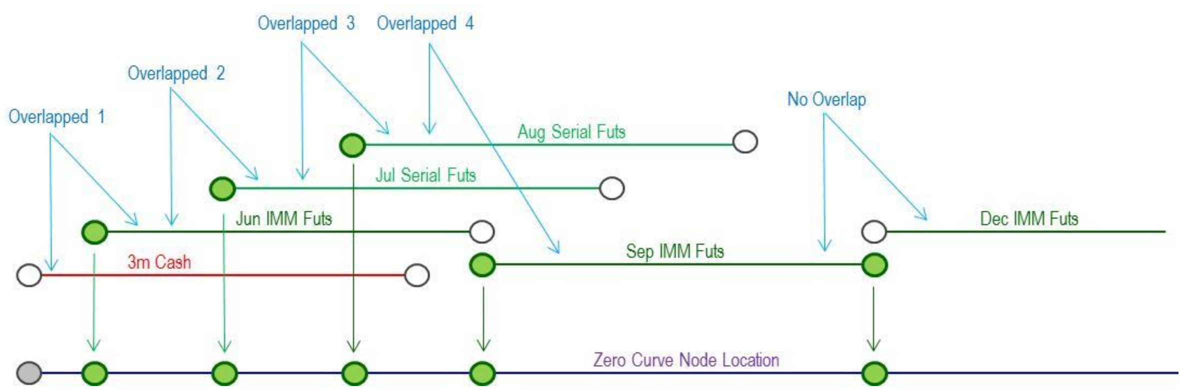
Case 2. Serial, IMM, and Serial Futures

On May 1, we have monthly spaced May *serial* futures, June IMM futures, and July *serial* futures in the front, as shown in the graph below. We have four overlapping pairs of the instruments in this case. The last overlapped pair is July serial futures and September IMM futures. Thus, we use the September IMM futures start date as a zero curve node. There is no overlap between September and December IMM futures, so we use the September IMM futures end date as a zero curve node. From this point on, there is no more overlap between the instruments, so we use the futures end dates as zero curve nodes.



Case 3. IMM, Serial, and Serial Futures

On June 1, we have monthly June IMM futures, July *serial* futures, and August *serial* futures, as shown in the graph below. The September IMM futures is also one month away from the August serial futures. We have four overlapping pairs of the instruments in this case. The last overlapped pair is August serial futures and September IMM futures. Thus, we use the September IMM futures start date as a zero curve node. There is no overlap between September and December IMM futures, so we use the September IMM futures end date as a zero curve node. From this point on, there is no more overlap between the instruments, so we use the futures end dates as zero curve nodes.



4.1.4 Stubdizing Overlapped Cash & Futures

The monthly spaced serial futures are heavily overlapped with each other as well the cash and IMM futures instruments. The overlapped instruments produce could large bucket delta risk in opposite directions. To have a cleaner risk representation of a portfolio, we prefer mapping the risk to a set of non-overlapping forward "stub" instruments.

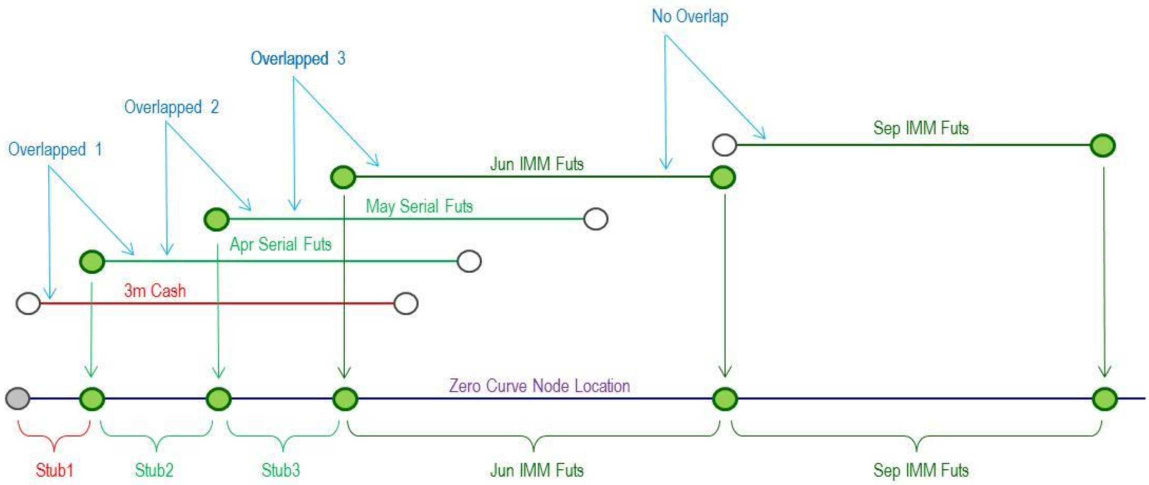
The "Stubdizing" process maps the overlapping cash and futures instruments to a set of non-overlapping forwards. We use the same the same overlapping criteria as defined in the previous section. The threshold for overlap is set to 12.5%. If the overlap between the two instruments is equal or greater than the threshold, it is considered to be heavily overlapped.

If one instrument is consider to be heavily overlapped with the next instrument, its end date will be replaced by the next instrument start date. This process turns the overlapped instruments into a set of non-overlapped instruments. Then, we use the curve bootstrapped from the overlapping instruments to imply the forward rates on the non-overlapping instruments, and used them as the "quotes" of the non-overlapping instruments. If we re-bootstrap the curve use these non-overlapping instruments, we will get the same zero curve as bootstrapped from the original set of overlapped instruments.

Using two serial futures, we have three cases of monthly spaced serial and IMM futures, leading to three situations of stub instrument creations. We will illustrate this with examples.

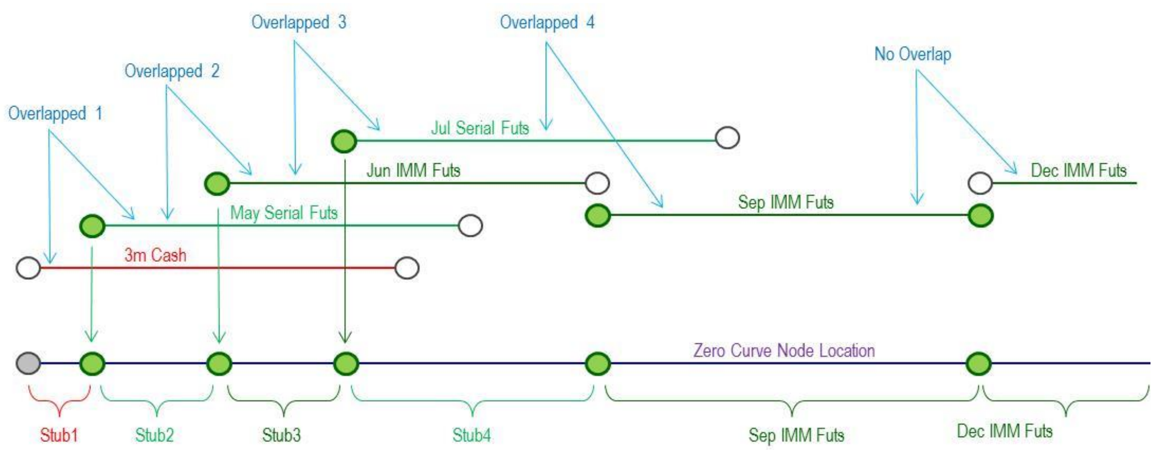
Case 1. Serial, Serial, and IMM Futures

On April 1, we have monthly spaced April *serial* futures, May *serial* futures, and June IMM futures in the front, as shown in the graph below. We have three overlapping pairs of the instruments. The 3m cash instrument overlaps with April serial futures, so its end date is replaced by the April serial futures start date. The April serial futures instrument overlaps with May serial futures, so its end date is replaced by the May serial futures start date. The May serial futures instrument overlaps with June IMM futures, so its end date is replaced by the June IMM futures start date. The three modified instruments are marked as stub 1, 2 and 3 in the graph below. As we can see, the stub instruments are non-overlapping. They are perfectly linked head-to-tail by design from one futures start date to the next futures start date. Most importantly, they produce the same zero curve nodes as the original heavily overlapped cash and futures instruments.



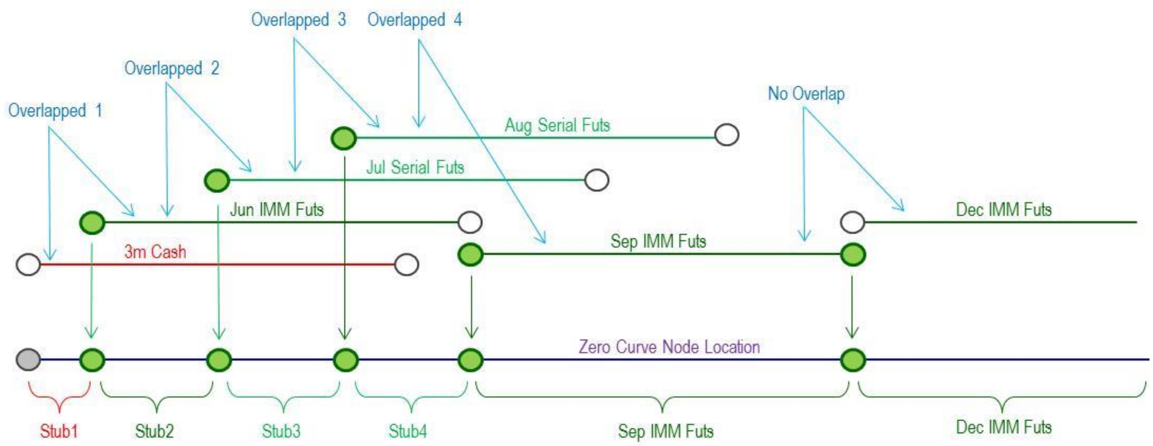
Case 2. Serial, IMM, and Serial Futures

On May 1, we have monthly spaced May *serial* futures, June IMM futures, and July *serial* futures in the front, as shown in the graph below. We have four overlapping pairs of the instruments in this case. The last overlapped pair is July serial futures and September IMM futures. Thus, the July serial futures instrument end date is replaced by the September IMM futures start date, resulting a 2-month long stub instrument. The four modified instruments are marked as stub 1, 2, 3 and 4 in the graph below. As we can see, the stub instruments are non-overlapping. They are perfectly linked head-to-tail by design from one futures start date to the next futures start date. They produce the same zero curve nodes as the original heavily overlapped cash and futures instruments.



Case 3. IMM, Serial, and Serial Futures

On June 1, we have monthly June IMM futures, July *serial* futures, and August *serial* futures in the front, as shown in the graph below. The September IMM futures is also one month away from the August serial futures. We have four overlapping pairs of the instruments in this case. The last overlapped pair is August serial futures and September IMM futures. Thus, the August serial futures instrument end date is replaced by the September IMM futures start date. The four modified instruments are marked as stub 1, 2, 3 and 4 in the graph below. As we can see, the stub instruments are non-overlapping. They are perfectly linked head-to-tail by design from one futures start date to the next futures start date.



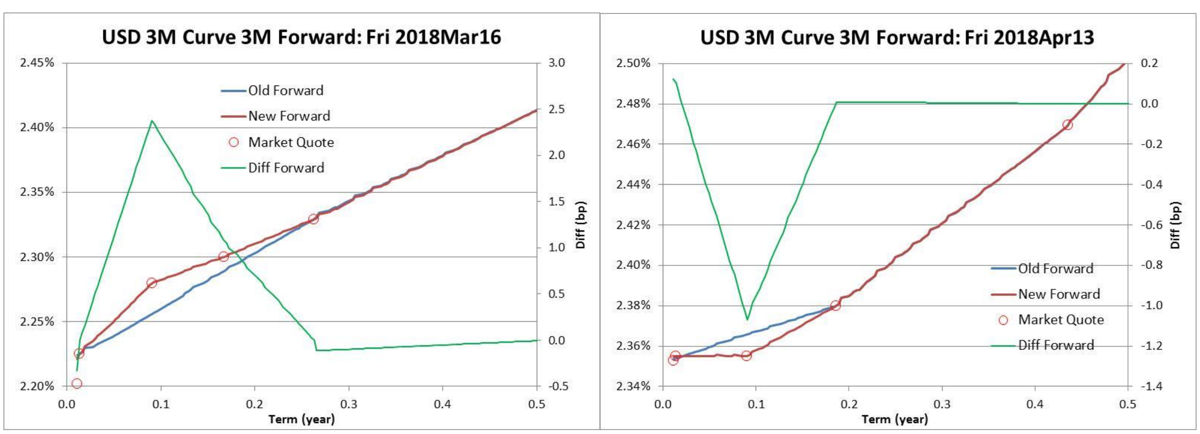
4.1.5 High Tension Front End

To provide near linearly interpolated forward rates, high tension can be applied on the log discount interpolation across a section of the market quoted forward rates.

The following is a sample BNS curve configuration for the nearly linear interpolated forward rates,

- Tension normalization flag = 2 for normalization under a short end high tension decaying scheme from a maximum tension level $A_{\infty} = 500$ (See equation (7) in [8] for the scheme details), with the tension decay starting from a specified future instrument.
- Combined with tension normalization flag = 2, setting useFutures flag 1525, 1535 and 1545 will apply high tension to all futures instruments.
- Interpolation axis yType = 2 for interpolation on log discount.

Sample 3-month Libor rates are provided below from the USD 3m curve on just before a IMM futures roll (16Mar2018), and just before a serial futures roll (13Apr2018).



Note: "New" curve was built with 2 serial futures and 12 IMM futures. "Old" curve has no serial futures.

4.1.6 Square Jacobian Matrix

The 3m cash start date is not used as a curve node. The curve anchor date node is forced to take the zero rate of the next curve node. Now, the number of free zero curve nodes matches the number of curve instruments, so the Jacobian matrix, that links the curve instrument rate changes to the curve node zero rate changes, is a square instead of a rectangle. This property results in a more stable bootstrapping.

All instruments (including cash) and their corresponding to the zero rates are included in the multi-dimensional root finding under the tension spline. This approach will yield a zero curve that matches all cash instruments exactly, even if not all cash instruments are connecting, and even if the linearly interpolated initial guess zero curve did not achieve exact match to all cash instruments.

4.1.7 Curve Re-anchoring & Carry

A portfolio's present value (PV) change from today (day1) to the next business day (day2) is called daily PL (profit and loss). The daily PL can be attributed to the carry and risk. The carry PL is the PV change due to the time change from day1 to day2 with the market being held "constant". The risk PL is the PV change from the market move within day2. For the linear products, the market move includes the interest curve and FX rate moves.

The carry consists of two components: the time carry and the curve re-anchoring carry. The time carry is the PV difference between day1 and day2 valuations, both valued using the day1 curves. The re-anchoring carry is the PV difference on day2 valuations, between the day1 curves and day1Tom curves.

To explain carry, let us introduce some terminologies to name the curves. We denote day1 curves as the curves anchored on day1 and built on day1 market quotes. We denote day2 curves as the curves anchored on day2 and built on day2 market quotes. We denote day1Tom curves as the curves re-anchored on day2 and built on the day1Tom market quotes. The spirit of the "day1Tom" quotes is to create a set of curves anchored on day2 under the assumption that the "market has not changed" from day1 to day2.

"Constant Quote" vs "Implied Quote" Re-anchoring

However, there is no unique view on the "market has not changed" from day1 to day2. The simplest approach is to keep the market quotes not changes between the day1 and day1Tom curves, i.e. to copy quotes from the day1 curves as is to the corresponding day1Tom curves. A more elaborate approach is to imply day1Tom curve quotes from the day1 curves, i.e. to assume that *the day1Tom market quotes are as predicted by the day1 curves*. We will show that the day1Tom zero curves built this way match well with the day1 zero curves, thus resulting in a cleaner (smaller) re-anchoring carry.

As described in the previous section, the serial futures are heavily overlapped with cash and IMM futures. To have a cleaner risk representation, we have a "stubdizing" process to map the overlapped cash and futures to a set of non-overlapping "stub" instruments. The "stubdized" day1 and day1Tom curves will be generated in K2 production environment. The day1 curves are generated first. Then, the day1Tom curves are created by re-anchoring the curve instrument dates to day2 and copying the day1 curve quotes to the day1Tom curves, under the assumption that the market quotes remains constant from day1 to day2. However, this approach causes the cash stub (located in front of the first futures) rate to change, leading to a quite different front end between day1 and day1Tom curves. This is issue can be fixed by using the "implied quote" carry methodology. We will show that by implying the day1Tom cash rate from the day1 curve, the stub rates in the day1Tom curve match the stub rates in the day1 curve.

"Implied Quote" Method for Overlapping Cash & Futures

The "implied quote" method assumes that *the day1Tom market quotes are as predicted by the day1 curves*. This method keeps the stub rates unchanged between the day1 and day1Tom curves.

Specifically, the "implied quote" method is only applied to the overlapping cash and futures instruments. For each cash and futures instruments on the day1Tom curve, we will check the same overlapping condition as in the "stubdizing" process. If the overlapping condition is met, the overlapping instrument's rate will be implied from the corresponding day1 curve. This process has two aspects:

1) imply the cash rate, on all dates
2) imply the newly rolled in serial futures rate, on the serial roll dates

The implied quote method is also applied to other overlapped futures, but the implied rates will stay the same as the day1 quotes in those cases.

In the new K2 process, the day1 curves are generated first. Then, the day1Tom curves are created by re-anchoring the curve instrument dates to day2 and copying the day1 curve quotes to the day1Tom curves. Then, the overlapping cash and futures rates are overridden by the "implied quotes".

Some Operational Considerations

If traders upload the stub rates directly into K2, K2 will treat them as the "original" market quotes, and keep them constant between day1 and day1Tom curve.

If traders upload the cash and futures rates into K2, K2 will keep the cash and futures rates constant between day1 and day1Tom curve. In this case, we need to insert an "implied quote" process into K2 to update the overlapped cash and futures quotes, so that the stub rates remain constant between day1 and da1Tom curves.

We currently use 8 quarterly spaced IMM futures in the CAD 3m curve. On the day before the futures rolls, there is a process in trader spreadsheets to mark the 9th futures rate as implied from the day1 curve. This 9th futures instrument will be used for the futures roll in the day1Tom curve. Using the implied rate for the 9th futures makes a cleaner re-anchoring carry.

With 2 monthly spaced serial futures, we would also like to have the same process on the day before the serial futures rolls for a cleaner re-anchoring carry. We could add a process in trader spreadsheets to mark the 3rd serial futures rate as implied from the day1 curve, to be used for the serial futures roll in the day1Tom curve. It is a simpler and more reliable alternative to leverage on the K2 "implied quote" process to imply the serial futures rates in the day1Tom curve automatically.

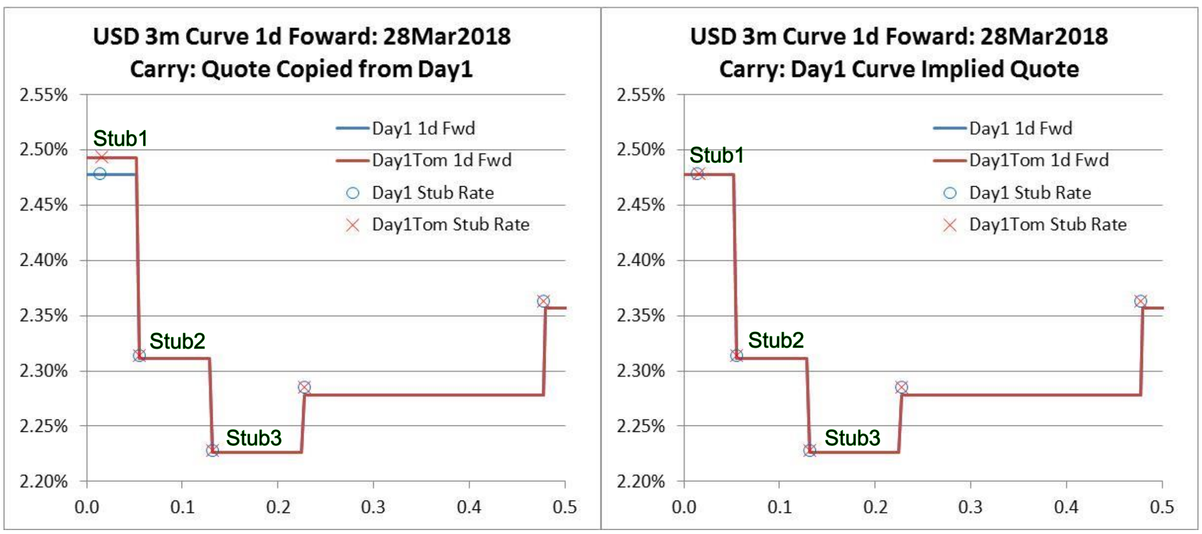
Re-anchoring Examples

The re-anchoring is demonstrated in the figures below on the USD 3m Libor curve. The see this problem in high resolution, we plotted the 1-day (1d) forward rate instead of the 3-month (3m) forward rate. Under the high tension, the 1d forward rate is constant within the stub periods between the futures start dates. The 3m forward rate can be viewed as weighted average of those 1d forward rates across a 3m tenor period covering 3-4 stub periods.

We use a pair of pictures to show the results from the "constant quote" and "implied quote" carry methods.

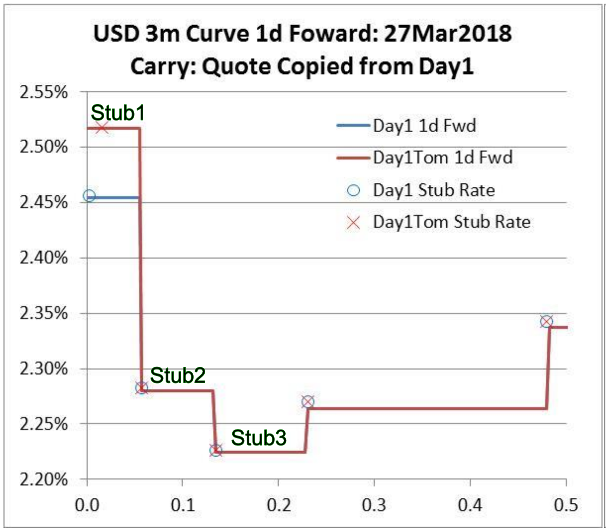
Example 1a. Across a Regular Day

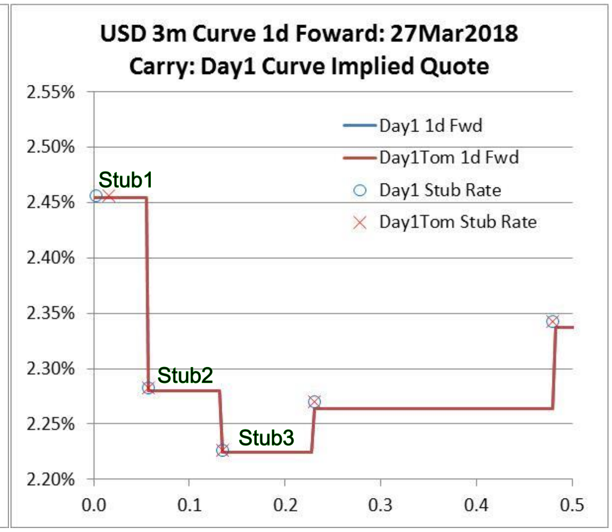
We first see a re-anchoring across a regular business day from day1 Wed 28Mar to day2 Thu 29Mar. The day1 curve cash starts on Tue 3Apr and day1Tom cash starts on Wed 4Apr, spaced with also one calendar day apart. Since the cash start date in day1Tom curve is one day after the cash start date in day1curve, its 3m tenor covers a small portion of stub1, and weighted less on the higher 1d rate in stub1. To maintain the day1 cash rate *2.3080%*, the day1Tom rate in stub1 has to go 1.53 bps higher (see below-left figure). Using Day1 curve implied cash rate *2.3056%* to build, the day1Tom curve matches well with the day1 curve (see below-right figure).



Example 1b. Across a Long Weekend

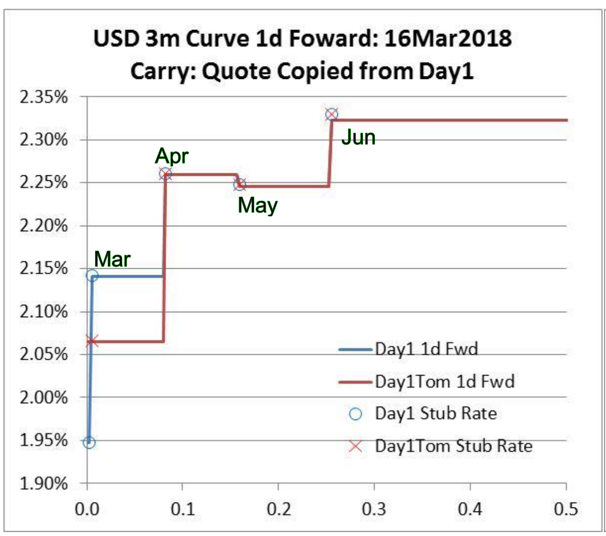
This effect will be magnified across a weekend and/or holidays. Re-anchoring from day1 Tue 27 Mar to day2 Wed 28Mar, the day1 curve cash starts on Wed 28Mar and day1Tom cash starts on Tue 3Apr, across the Easter long weekend. Since the cash start date in day1Tom curve is now 5 days after the cash start date in day1curve, its 3m tenor covers an even smaller portion of the stub1, and weighted much less on the higher 1d rate in stub1. To maintain the day1 cash rate *2.3020%*, the day1Tom rate in stub1 has to go 6.17 bps higher (see below-left figure). Using Day1 curve implied cash rate *2.2917%* to build, the day1Tom curve matches well with the day1 curve (see below-right figure).

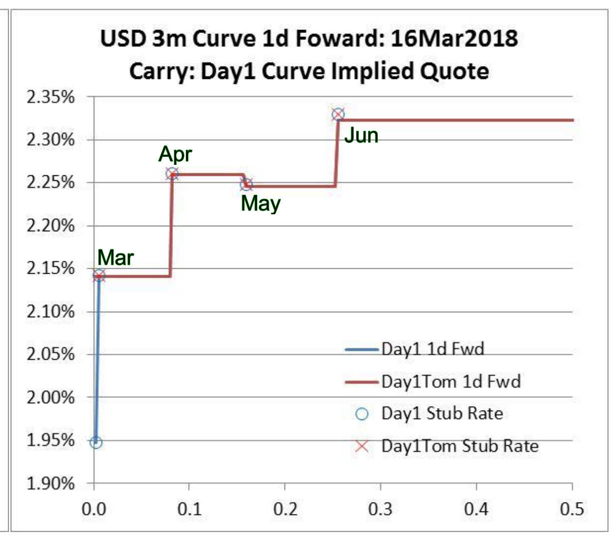




Example 2a. Across an IMM Futures Roll

In an IMM futures roll, the situation is complicated a bit by the fact that the front IMM futures is dropped out in the day1Tom curve. Re-anchoring from day1 Fri 16Mar to day2 Mon 19Mar, the Mar2018 contract rolled out and Mar2021 rolled in. The cash and Mar2018 futures rates are *2.2018%* and 2.2250% respectively. To maintain the same cash rate *2.2018%* in absence of the Mar futures, the rate in stub1 in day1Tom curve is set quite differently (see below-left figure). Using Day1 curve implied cash rate *2.2250%* to build, the day1Tom curve matches well with the day1 curve (see below-right figure).

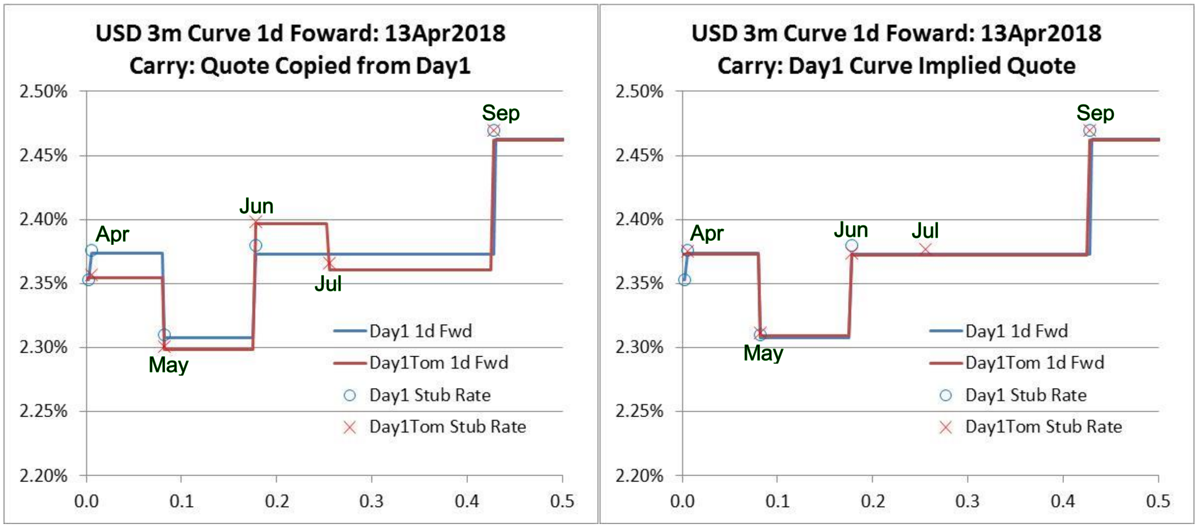




Note 1. Day1 curve stub rates are on cash stub, Mar IMM futs stub (to roll out), Apr serial futs stub, May serial futs stub.
Note 2. Day1Tom stub rates are on cash stub, Apr serial futs stub, May serial futs stub.
Note 3. Method A3 is applied for 3m cash target rate marking. The day1 curve is within 10-day weighting threshold range.

Example 2b. Across a Serial Futures Roll: Serial/Serial/IMM to Serial/IMM/Serial

In a serial futures roll, the situation is further complicated by one serial futures rolling in and another rolling out in the day1Tom curve. Re-anchoring from day1 Fri 13Apr to day2 Mon 16Apr, the Apr2018 contract rolled out and Jul2018 rolled in. The cash and Apr, May, Jun, Jul futures rates are *2.3528%*, 2.3550%, 2.3550%, 2.3800%, and *2.4000%* respectively. To maintain the same cash rate *2.3528%* in absence of the Apr futures, the rates in stubs in day1Tom curve have to set quite differently (see below-left figure). Using day1 curve implied cash rate *2.3550%* and day1 curve implied Jul2018 futures rate *2.4075%* to build, the day1Tom curve matches well with the day1 curve (see below-right figure).



Note 1. Day1 curve stub rates are on cash stub, Apr serial futs stub (to roll out), May serial futs stub, Jun IMM futs stub.
Note 2. Day1Tom stub rates are on cash stub, May serial futs stub, Jun IMM futs stub, and Jul serial futs stub just rolled in.
Note 3. Method A3 is applied for 3m cash target rate marking. The day1 curve is within 10-day weighting threshold range.

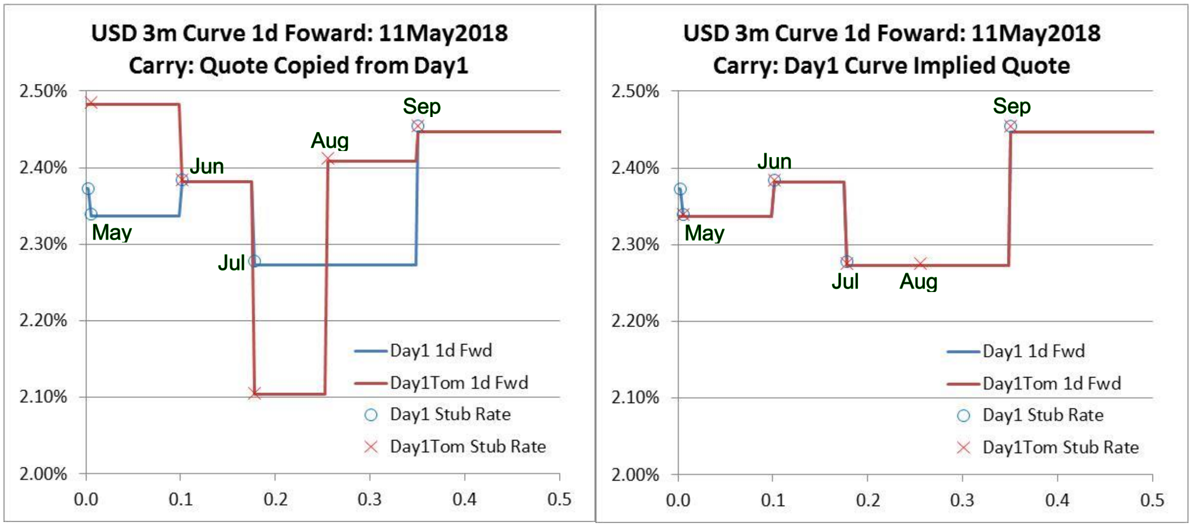
On above-right figure, we have small differences on the stub rates between the day1 and day1Tom curves on Apr (0.10 bp), May (0.16 bp) and Jun (0.06bp) futures start dates. This comes from 1-day misaligned zero curve nodes between the day1 and day1Tom curves. The day1 curve has a zero curve node on 20Sep2018 as the Jun futures end date, while the day1Tom curve has a zero curve node on 19Sep2018 as the Sep futures start date, when the futures contracts roll from day1 serial/serial/IMM sequence to day1Tom serial/IMM/serial sequence. If nodes are not aligned, the interpolation will be different. Implying a rate on a 3m day1Tom instrument from the day1 curve will not ensure the exact match of a stub rates between the day1 and day1Tom curves.

The shift of zero curve nodes from one IMM futures end date to the next IMM futures start date between the day1 and day1Tom curves may only happen when the futures contracts roll from day1 serial/serial/IMM sequence to day1Tom serial/IMM/serial sequence. As shown by Case 1 and 2 in Section 4.1.3 “Zero Curve Nodes”, the start and end date of Jun futures (Case 1) are rolled into the start and end date of Sep futures (Case 2).

This would not happen when the futures contracts roll from serial/IMM/serial sequence to day1Tom IMM/serial/serial sequence. As shown by Case 2 and 3 in Section 4.1.3, the start and end date of Sep futures (Case 2) are still rolled into the start and end date of Sep futures (Case 3). It also would not happen when the futures contracts roll from day1 IMM/serial/serial to day1Tom serial/serial/IMM sequence. As shown by Case 3 and 1 in Section 4.1.3, the start and end date of Sep futures (Case 3) are rolled into the start and end date of Sep futures (Case 1 with “+3m” shift on all futures contracts in the graph, i.e. Jun futures in the graph becomes Sep futures, and so one).

Example 2c. Across a Serial Futures Roll: Serial/IMM/Serial to IMM/Serial/Serial

Similar to the previous example of the serial futures roll, re-anchoring from day1 Fri 11May to day2 Mon 14May, the May2018 contract rolled out and Aug2018 rolled in. The cash and May, Jun, Jul, Aug futures rates are *2.3425%*, 2.3375%, 2.3150%, 2.3350%, and *2.4544%* respectively. To maintain the same cash rate *2.3425%* in absence of the May futures, the rates in stubs in day1Tom curve have to set quite differently (see below-left figure). Using day1 curve implied cash rate *2.3375%* and day1 curve implied Jul2018 futures rate *2.3881%* to build, the day1Tom curve matches well with the day1 curve (see below-right figure).



Note 1. Day1 curve stub rates are on cash stub, May serial futs stub (to roll out), Jun IMM futs stub, Jul serial futs stub.
Note 2. Day1Tom stub rates are on cash stub, Jun IMM futs stub, Jul serial futs stub, and Aug serial futs stub just rolled in.
Note 3. Method A3 is applied for 3m cash target rate marking. The day1 curve is within 10-day weighting threshold range.

4.1.8 Curve Delta Risk

Curve delta risk can be used as a tool for hedging and PL prediction. It can be computed using a finite difference scheme, by “bump and run”. That is, bump one curve instrument by a small amount, rebuild the curves and re-value the portfolio.

The stubdizing process converts the overlapping cash and futures instruments into the stub instruments. The official risk calculation is performed by “bump and run” on the stub instruments. However, people may be interested in knowing what the risk will be with respect to the movement of the original market instruments (i.e. cash and futures). This could be achieved directly by “bump and run” on the original market instrument curve. Here, we present a Jacobian method alternative that converts the stub instrument risk to the original market instrument risk by a linear transformation. Since the system is quite linear under small bumps, this transformation is quite accurate.

We first introduce some notations.

- $R_i^F, i = 1, \dots, n$ — Instrument rates for a 3m curve whose front end is built with the original market instruments (cash and futures).
- $R_i^S, i = 1, \dots, n$ — Instrument rates for a 3m curve whose front end is built with stub instruments.
- t — Valuation date

- $V(t, Base)$ — Portfolio value at time t using the base curves. The base curve can be produced by not shocking the stub instrument rates R_i^S or the original market instrument rates R_i^F. As we described in the previous section, the stubdization is performed under the principal that the stubdized instruments produce the same zero curve as the original market instruments.
- $V(t, Shock\ R_i^F + \Delta R_i^F)$ — Portfolio value at time t using shocked curves. Only one original market instrument rate R_i^F is shocked by amount $+\Delta R_i^F$, while all other instrument rates remain the same.
- $V(t, Shock\ R_i^S + \Delta R_i^S)$ — Portfolio value at time t using shocked curves. Only one stub instrument rate R_i^S is shocked by amount $+\Delta R_i^S$, while all other instrument rates remain the same.

Typically, the risk is computed using the day1Tom curves, and the valuation date t is on the day1Tom curve anchor date. The day1 curves represent today's end of day (EOD) market. The day1Tom curves represent the next business day's start of day (SOD) market, when assuming the "market has not changed" from day1. Based on the market instrument moves from the SOD market to the EOD market, the risk can be utilized to make a PL prediction for the next business day's EOD market.

Stub Instrument Risk

For the 3m curve built with stub instruments in the front end, the delta risk can be computed by bumping the stub instruments one by one,

$$\frac{\Delta V}{\Delta R_i^S} = \frac{V\left(t, Shock\ R_i^S + \Delta R_i^S\right) - V(t, Base)}{\Delta R_i^S}, i = 1, \dots, n$$

Market Instrument Risk – Direct Method

For the 3m curve built with the original market instrument (cash and futures) in the front end, the delta risk can be computed by bumping the market instruments one by one,

$$\frac{\Delta V}{\Delta R_i^F} = \frac{V(t, ShockR_i^F + \Delta R_i^F) - V(t, Base)}{\Delta R_i^F}, i = 1, \dots, n$$

Market Instrument Risk – Jacobian Method

We first build the stub instrument curve and compute delta risk by bumping the stub instruments. Then, map the stub risk to cash/futures risk by a Jacobian transformation

$$\frac{\Delta V}{\Delta R_i^F} = \sum_{j=1}^{n} \frac{\Delta V}{\Delta R_j^S} \frac{\Delta R_j^S}{\Delta R_i^F}$$

The Jacobin matrix links the stub instrument rate changes to the market instrument rate changes. It can be computed by bumping the market instruments, rebuild the market instrument curve, imply the stub instrument rates from it, and then apply the formula below

$$\frac{\Delta R_j^S}{\Delta R_i^F} = \frac{R_j^S(t, Shock\ R_i^F + \Delta R_i^F) - R_j^S}{\Delta R_i^F}, where\ i = 1, \dots, n\ and\ j = 1, \dots, n$$

Jacobian Matrix Calculation Methodology

The following steps are required to compute the Jacobian matric from a 3m curve built from stub instruments.

- Build the 3m curve from the stub instruments.
- Extract the stub instruments from the 3m curve for later Jacobian calculation.
- Reverse stubdize to derive the original market instruments (cash and futures). If an instrument's end date lands exactly on the next instrument start date, and its tenor is more than 12.5% shorter than 3-month, it is considered as a stub instrument. We will convert the stub instrument back to a 3m month instrument by adding 3m to its start date per market convention for holiday adjustment. Then, imply the quote of this converted 3m market instruments R_i^F from the 3m curve.
- Re-build the 3m base curve using the reverse stubdized original market instruments. Then, imply all stub instrument rates as the base rates R_j^S.
- Shock the 3m market instruments R_i^F one by one. For each one shock $+\Delta R_i^F$, re-build the 3m curve using the 3m market instruments. Then, implied all stub instrument rates as $R_j^S(t, Shock\ R_i^F + \Delta R_i^F)$.
- Apply the above formula to compute the elements in the Jacobian matrix.

Since we applied very high tension in front of the curve, the cash and futures section of the curve behaves linearly. It has little dependency to both the swap instruments and the OIS curve. For performance reason, we only need to build the cash and futures section of 3m curve without co-bootstrapping with the OIS curve.

Furthermore, we only need to compute the Jacobian matrix to cover all instruments affected by the stubdization plus one extra instrument. The one extra instrument is affected by the risk re-allocation from its neighboring stub instrument in a linear system.

The shock size is typically set to 0.1 bp.

Test Results

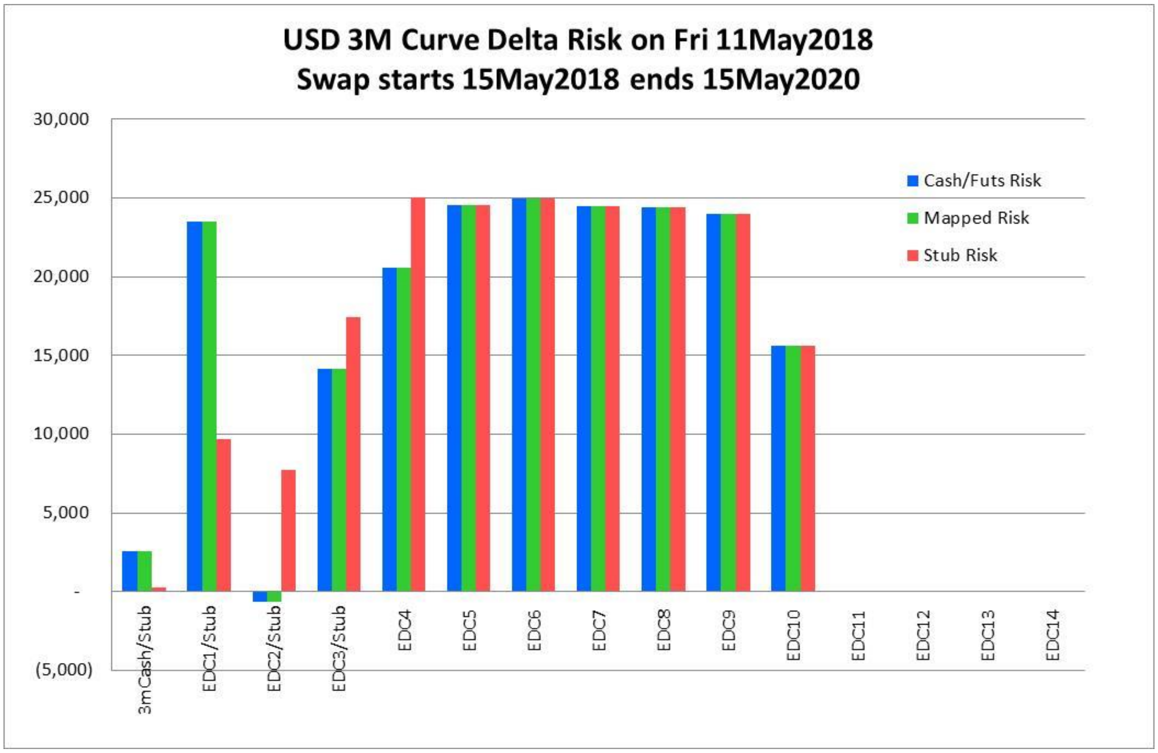
The figures below show the delta risk allocation of a 2-year spot starting fixed-float USD swap, before and after a serial futures roll, under 3 approaches:

- Stub Risk: Stub instrument delta from a 3m curve built with stub instruments, computed by bumping the stub instruments.
- Cash Futs Risk: 3m market instrument delta from a 3m curve built with cash and futures instruments, computed by bumping the cash and futures instruments.
- Mapped Risk: 3m market instrument delta computed by mapping "Stub Risk" to "Cash Futs Risk" using the Jacobian method.

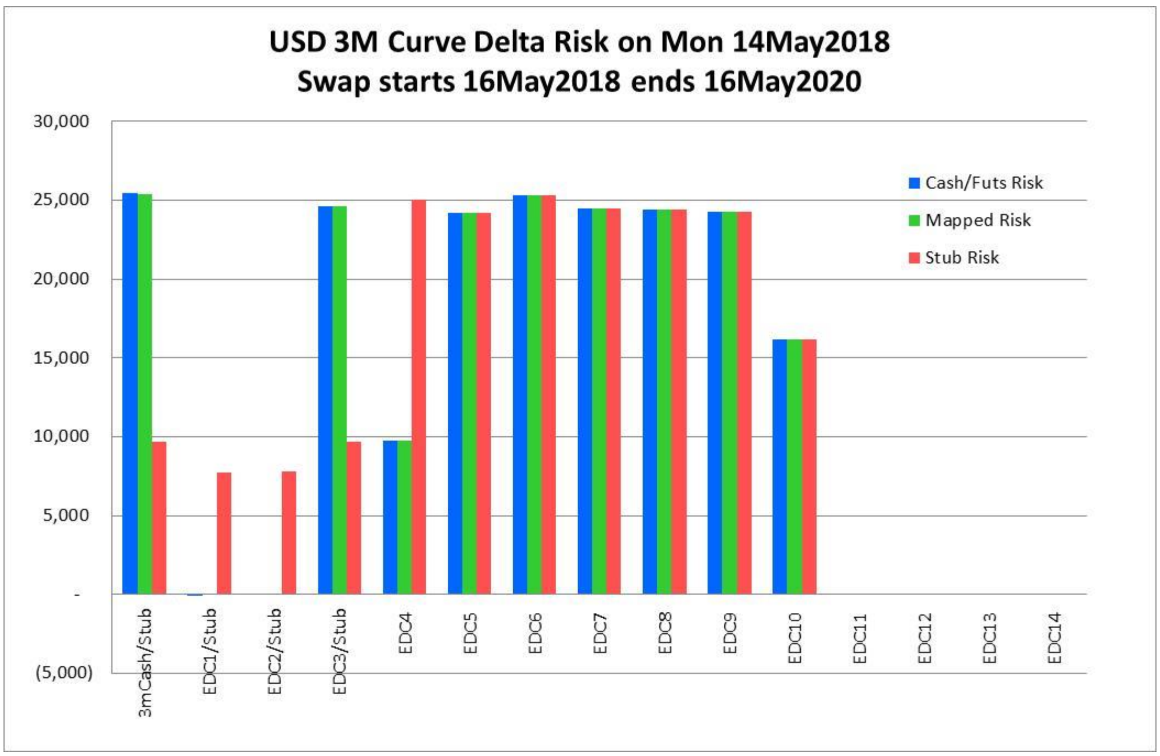
We can see the Jacobian "Mapped Risk" matches the directly calculated "Cash Futs Risk" very well. Furthermore, the "Stub Risk" and "Cash Futs Risk" only differs in the first five instruments, which include the four stubdized instruments plus one extra instrument as we expected.

The risk comparison is also provided on the entire USD swap portfolio, consisting of FRAs, swaps, basis swaps, as well as the USD leg of FX forward trades and cross-currency swaps.

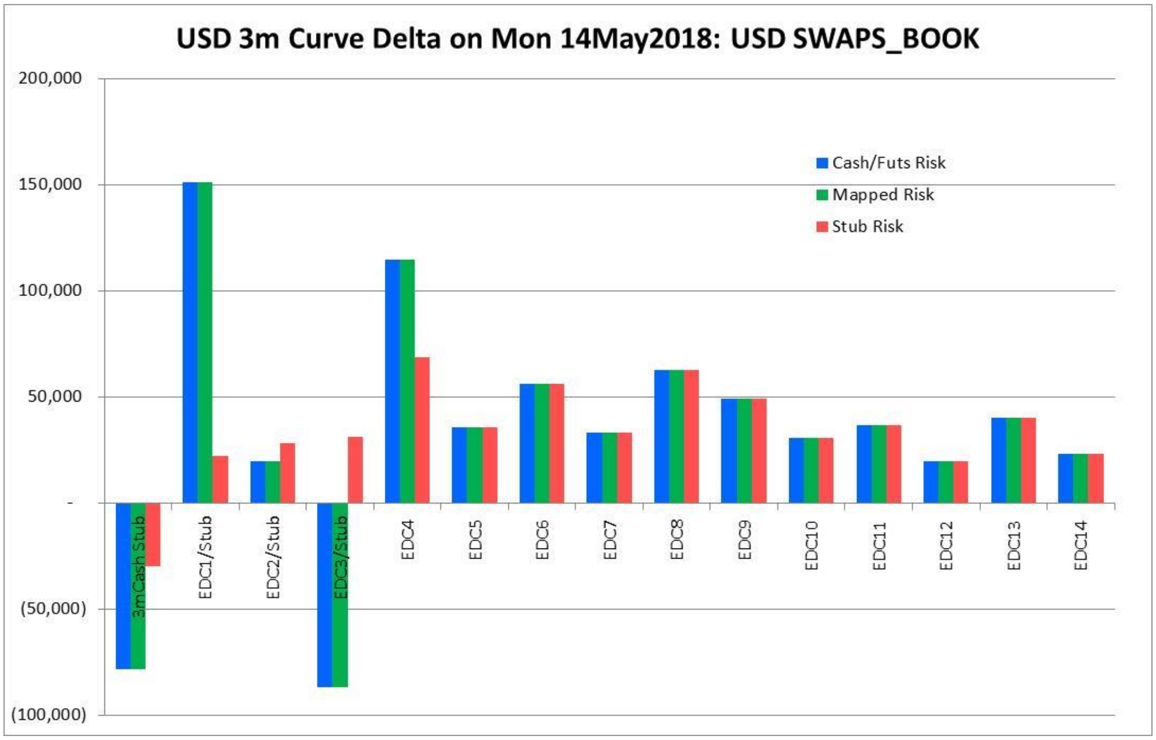
Delta risk on a 2-year swap – right before the May serial futures roll



Delta risk on a 2-year swap – right after the May serial futures roll



Delta risk on the USD swap book – right before the May serial futures roll



4.2 Building OIS Curves with Meeting Date Swaps

This section is about building the OIS curve with the OIS meeting date swap instruments.

We will provide detail description of the curve instrument selection, meeting date instrument rolling, stub meeting date swap quote marking, and the curve interpolation methodology.

The features described in this section are accessible under the useFutures flag setting 1515.

4.2.1 OIS Meeting Date Swaps

The OIS meeting date swaps and the regular OIS swaps are both traded liquidly in the major currencies, such as CAD, USD, EUR, and GBP markets. The OIS meeting date swaps have the same conventions the regular OIS swaps. The regular OIS swaps are spot starting with standard maturities 1-week, 2-week, 3-week, 1-month, 2-month, 3-month, etc., while the OIS meeting date swaps are forward starting instruments that start and end on the OIS meeting dates. The meeting dates are usually 6-8 months apart. The meeting date swaps are 6-8 month forward starting instruments.

As the time goes, the meeting date swaps roll closer and closer to today. Once the start date of a meeting date reaches the spot date, it will become a spot start swap, starting on the spot date and ending on the next meeting date. For convenience, we call it “stub” meeting date swap. The stub swap will live till its end date reaches the spot date.

The OIS rate target is set at the meetings. The market expects the rate stay flat until a meeting date. It is idea to model the front end of the OIS curve as piecewise flat 1-day forward with jumps only on the meeting dates. Using the OIS meeting date swaps as curve instruments will serve this purpose.

4.2.2 Instrument selection

We choose to use 5 meeting date swaps in front of the OIS curves, including a stub meeting date that starts on the spot date and ends on the next meeting date, to replace the regular OIS swaps with 1W, 2W, 3W, 1M, 2M, 3M, 4M, 5M, 6M maturity.

Over the rolling cycle, the 5 meeting date swaps will cover to 6 to 7.5 month term. Therefore, the 6-month regular OIS swap is always covered by the meeting date swaps.

The first regular OIS swap is at 9-month. The gap between the meeting date swaps and the regular OIS swaps varies between 1.5 to 3 months. This is provides a gradual transition from 1.5-month spaced meeting date swaps to the 3-month spaced regular OIS swaps.

TN Point Insertion

USD and EUR spot starting swaps start 2 business dates from today. The TN instrument is inserted into to bridge the gap between the ON and meeting date swaps to provide a rate jumping mechanism when the meeting date falls on the spot date (more details will follow). In addition, the TN instrument helps to localize the end of month turn effect to the ON point.

The OIS instrument selections for same major currencies are provided below.

<u>CAD OIS Curve</u>

Cash:	ON
Meeting Date OIS Swaps:	*MD Swap 1, 2, 3, 4, 5*
Regular OIS Swaps:	9M, 1Y, 15M, 18M, 2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y, 10Y. 12Y, 15Y, 20Y, 25Y, 30Y, 40Y

<u>USD OIS Curve</u>

Cash:	ON, *TN*
Meeting Date OIS Swaps:	*MD Swap 1, 2, 3, 4, 5*
Regular OIS Swaps:	9M, 1Y, 15M, 18M, 2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y, 10Y. 12Y, 15Y, 20Y, 25Y, 30Y, 40Y

<u>EUR OIS Curve</u>

Cash:	ON, *TN*
Meeting Date OIS Swaps:	*MD Swap 1, 2, 3, 4, 5, 6, 7, 8, 9, 10*
Regular OIS Swaps:	2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y, 10Y. 12Y, 15Y, 20Y, 25Y, 30Y, 40Y, 50Y

<u>GBP OIS Curve</u>

Cash:	ON
Meeting Date OIS Swaps:	*MD Swap 1, 2, 3, 4, 5, 6, 7, 8, 9, 10*
Regular OIS Swaps:	2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y, 10Y. 12Y, 15Y, 20Y, 25Y, 30Y, 40Y, 50Y

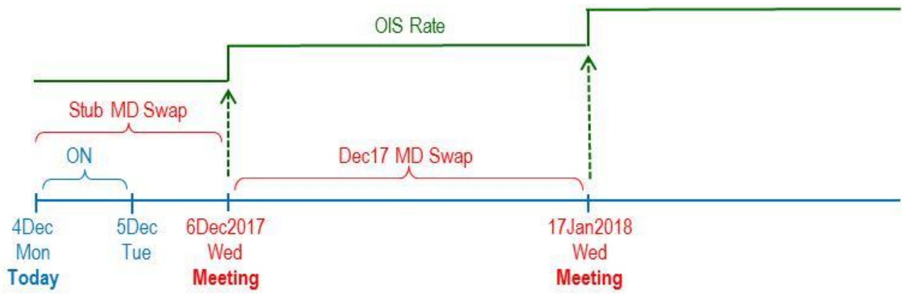
4.2.3 Meeting Date Swap Rolling

To provide a mechanism for the rate to jump on the next meeting date from today, we will keep the stub meeting date instrument as long as we could in the curve construction. The marking of this stub instrument is not an issue, since one would expect the OIS rate stay flat till the next meeting date.

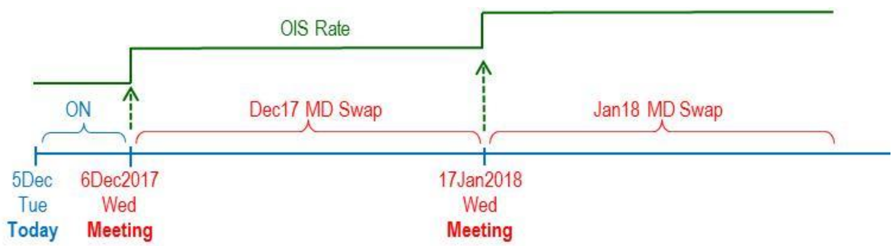
CAD & GBP Meeting Date Roll Sequence

CAD and GBP swaps spot start from today. The stub meeting date swap will stay as long as it does not collide with the ON cash instrument.

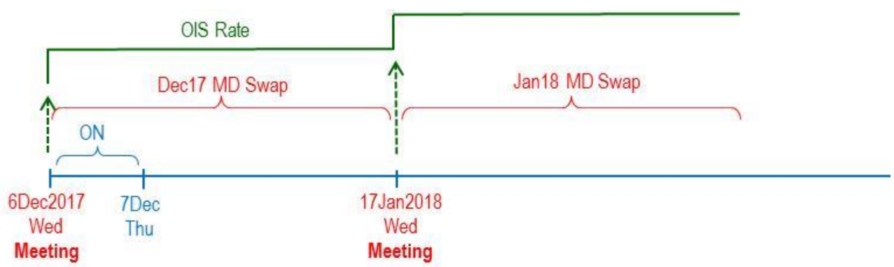
BoC meeting 2 days from today. The graph below shows the stub meeting date swap one day before the roll. At this point, it is a spot starting 2-day instrument from today to the next meeting date on 6Dec2017. It will collide with ON cash instrument on the next day.



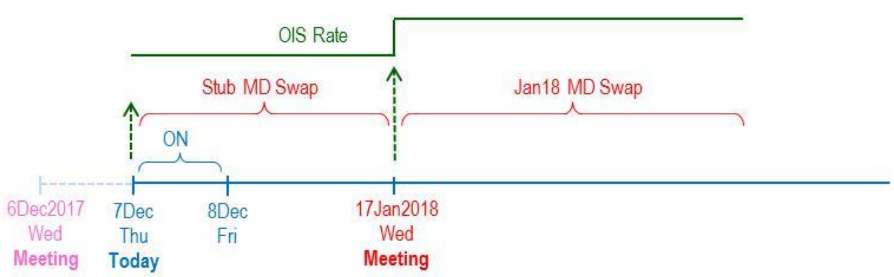
BoC meeting 1 day from today. This is the roll date. Dec17 meeting date swap becomes the front meeting date instrument. The ON cash instrument provides the rate jump on the meeting date.



BoC meeting today. One day after the roll, the front meeting date swap becomes spot starting. On this day, it still has the full length between the two meeting dates.



BoC meeting 1 day passed today. Two days after the roll, the front meeting date swap is one day shorter, thus becomes a stub swap. It no longer has the original full length between the two meeting dates.



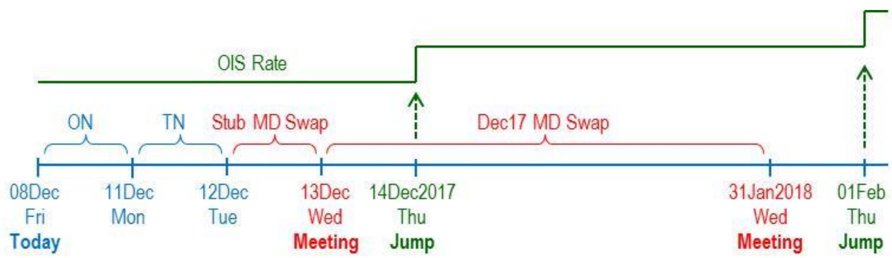
From this point on, with the passing of each day, the stub meeting date swap will be one day shorter. It will roll out once it collides with the ON cash instrument.

USD Meeting Date Roll Sequence

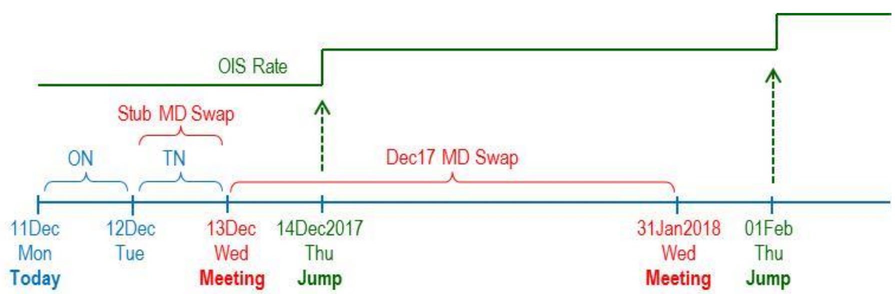
USD swaps spot starts 2 business days from today. The stub meeting date swap will stay as long as its term is not less than one day.

Unlike other markets, US OIS (feds fund) rate does not jump right on the meeting date. Instead, it jumps one day after the meeting date. Some special treatments are devised to accommodate this delayed jump.

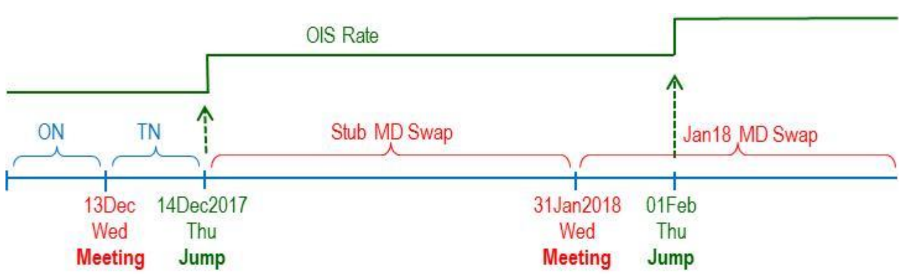
FOMC meeting 3 days from today. In the graph below, we are approaching a meeting date roll. The stub meeting date swap is a spot starting 1-day instrument from the spot date 12Dec2017 to the next meeting date on 13Dec2017. It provides a rate jump mechanism on the next day 14Dec2017.



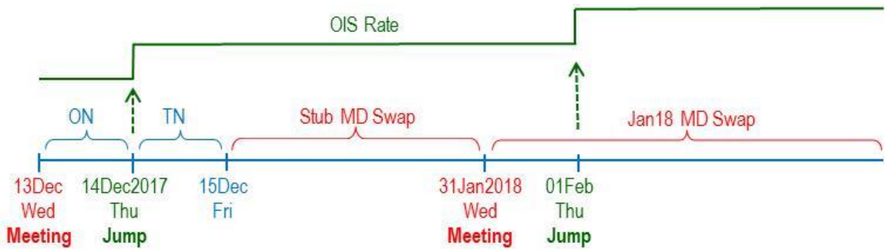
FOMC meeting 2 days from today. The meeting date now reaches the spot date. To provide a mechanism for the rate jump one day after the spot date, we have to keep the stub meeting date swap instrument on this day. In the graph below, we can see that the stub meeting date swap is sitting on top of the TN cash instrument. It will automatically override the TN cash instrument in the curve bootstrapping. Now, the second meeting date swap (Dec2017) becomes spot starting. It has the full length between the two meeting dates.



FOMC meeting 1 days from today. This is the roll date. Dec17 meeting date swap becomes the front meeting date instrument. It is one day shorter, thus becomes a stub swap. It no longer has the original full length between the two meeting dates. The TN cash instrument now provides the rate jump on the spot date 13Dec2017.



FOMC meeting today. The ON cash instrument now provides the rate jump on the spot date 13Dec2017.



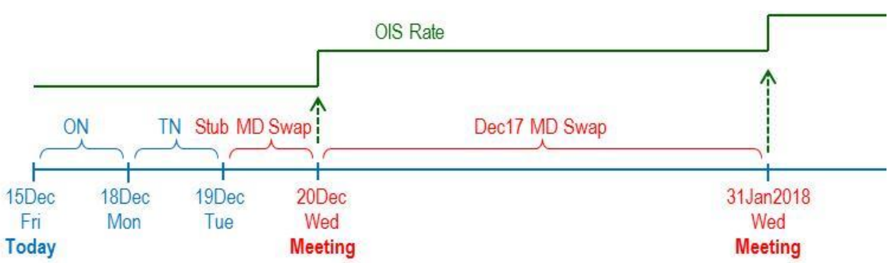
From this point on, with the passing of each day, the stub meeting date swap will be one day shorter. It will roll out once the meeting date passes the spot day again.

EUR Meeting Date Roll Sequence

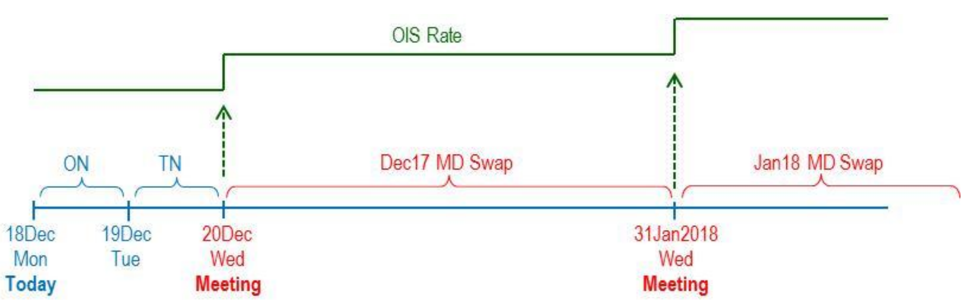
Similar to USD, EUR swaps spot starts 2 business days from today. The stub meeting date swap will stay as long as its term is not less than one day.

Same as other markets, EONIA rate jumps right on the meeting date.

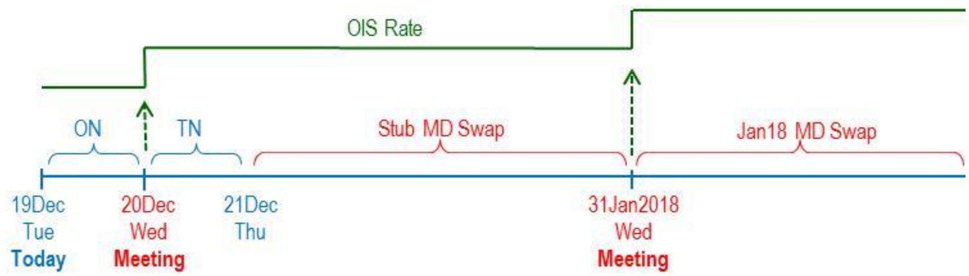
ECB meeting 3 days from today. In the graph below, we are approaching a meeting date roll. The stub meeting date swap is a spot starting 1-day instrument from the spot date 19Dec2017 to the next meeting date on 20Dec2017. It provides a rate jump mechanism on the meeting date 20Dec2017.



ECB meeting 2 days from today. The meeting date 20Dec2017 now reaches the spot date. The TN cash instrument now provides the rate jump on the spot date 20Dec2017. This is the roll date. Dec20 meeting date swap becomes the front meeting date instrument. It has the full length between the two meeting dates.



ECB meeting 1 day from today. The ON cash instrument now provides the rate jump on the spot date 20Dec2017. Dec2017 meeting date swap is one day shorter, thus becomes a stub swap. It no longer has the original full length between the two meeting dates.



From this point on, with the passing of each day, the stub meeting date swap will be one day shorter. It will roll out once the meeting date passes the spot day again.

4.2.4 Stub Meeting Date Swap Quote Marking

Forward starting meeting date swaps are liquidly traded in the major markets. Once the start date of the meeting date swap passes the spot date, it no longer has the original full length between the two meeting dates, and becomes a "stub" swap. The stub meeting date swap may not be liquidly traded when its term becomes very short. Its quote may not even be observable in some markets. This is not an issue from the quote marking perspective, since one expects that the OIS rate stay flat until the next meeting date.

This flat one day forward rate can be either implied from the ON cash instrument or the regular OIS swap instruments.

Imply 1D OIS rate from OIS Swap Quote

Right after a meeting date roll, the next meeting date is usually 6-7 weeks away from the spot date. The flat OIS rate till the next meeting date can be implied from 1W, 2W, 3W, or 1M regular OIS swaps. We could either do a best fit to all the above candidate OIS swaps, or simply fit to the one swap with longest maturity among all the candidates.

When the next meeting date is beyond 1M away, we could imply the rate from the 1M OIS swap. When the next meeting date is between 3W and 1M away, we could imply the rate from the 3W OIS swap. When the next meeting date is between 2W and 3W away, we could imply the rate from the 2W OIS swap. When the next meeting date is between 1W and 2W away, we could imply the rate from the 1W OIS swap. When the next meeting date is less than 1 week away, we could imply the rate from the ON cash rate.

Let t_{Swap}^{Start} and t_{Swap}^{End} be the start and end date of an OIS swap paying fixed rate F_{Swap}. We assume the OIS rate is flat between the swap's start and end date. The implied flat 1-day forward rate F_{1D} is then,

$$1 + F_{Swap}\Delta T_{Swap} = \left(1 + F_{1D}\Delta T_{1D}\right)^{t_{Swap}^{End} - t_{Swap}^{Start}}$$

or

$$F_{1D} = \frac{1}{\Delta T_{1D}}\left(\left(1 + F_{Swap}\Delta T_{Swap}\right)^{\frac{1}{t_{Swap}^{End} - t_{Swap}^{Start}}} - 1\right)$$

The daycount (dc) time fractions are

$$\Delta T_{Swap} = \Delta Time\left(t_{Swap}^{Start}, t_{Swap}^{End}, dcBasis\right) = \frac{t_{Swap}^{End} - t_{Swap}^{Start}}{dcDays}$$

$$\Delta T_{1D} = \frac{1}{dcDays}$$

For CAD and GBP, $dcBasis = act/365$ and $dcDays = 365$. For USD and EUR, $dcBasis = act/360$ and $dcDays = 360$.

Imply OIS Meeting Date Quote

Once we derived the 1-day rate F_{1D}, we could convert it to a meeting date swap rate by compounding.

Let t_{MDSwap}^{Start} and t_{MDSwap}^{End} be the start and end date of the next meeting date swap instrument paying fixed rate F_{MDSwap}. We assume the OIS rate is flat between the swap's start and end date. We have

$$1 + F_{MDSwap}\Delta T_{MDSwap} = \left(1 + F_{1D}\Delta T_{1D}\right)^{t_{MDSwap}^{End} - t_{MDSwap}^{Start}}$$

or

$$F_{MDSwap} = \frac{1}{\Delta T_{MDSwap}}\left(\left(1 + F_{1D}\Delta T_{1D}\right)^{t_{MDSwap}^{End} - t_{MDSwap}^{Start}} - 1\right)$$

The daycount time fraction is

$$\Delta T_{MDSwap} = \Delta Time\left(t_{MDSwap}^{Start}, t_{MDSwap}^{End}, dcBasis\right) = \frac{t_{MDSwap}^{End} - t_{MDSwap}^{Start}}{dcDays}$$

For CAD and GBP, $dcBasis = act/365$ and $dcDays = 365$. For USD and EUR, $dcBasis = act/360$ and $dcDays = 360$.

With some algebra, we could directly link an OIS swap quote to a meeting date swap quote through

$$F_{MDSwap} = \frac{1}{\Delta T_{MDSwap}}\left(\left(1 + F_{Swap}\Delta T_{Swap}\right)^{\frac{t_{MDSwap}^{End} - t_{MDSwap}^{Start}}{t_{Swap}^{End} - t_{Swap}^{Start}}} - 1\right)$$

4.2.5 Meeting Date Instrument & Interpolation Modeling

Meeting Date Instrument Modeling

The meeting date OIS swaps are like futures contacts many ways.

- Their start and end dates are fixed (fixed to the meeting dates). With each date passing by, a swap's start and end date move closer to today by one more day. The futures rolling logic could apply here.
- They are one period instrument, with one forward rate and one cash flow. Thus, they are independent of discounting for the purpose of curve bootstrapping.

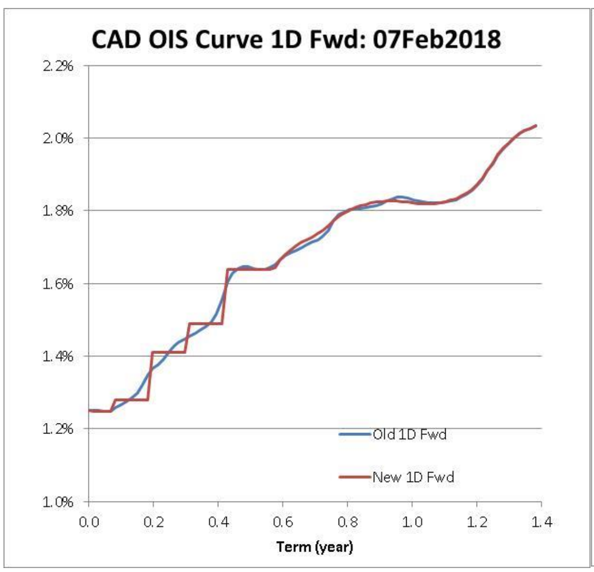
The floating leg of the meeting date swap pays the daily OIS rate and compounded on the same OIS index rate. Therefore, the daily compounded rate for the period can be efficiently computed as the period forward rate from the start and end date discount factors.

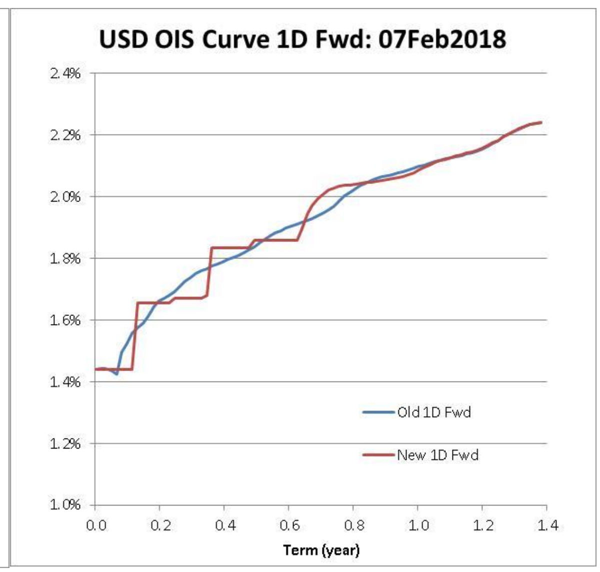
Interpolation Methodology

To create piecewise flat interpolation on the 1-day forward rate, with jumps only on the meeting dates, we applied very high tension to the tension spline curve across all the meeting date swap instruments. The high tension is gradual decreased in four steps as the approaches the last meeting date swap. With 5 meeting date swap instruments, this is achieved by the useFutures flag setting 1501 to apply high tension across all futures instruments.

For CAD, GBP and EUR, the OIS rate jumps exactly on the meeting dates, namely on the end dates of the OIS meeting date swaps. However, per USD OIS market convention, the OIS rate jumps one day after the meeting dates. To accommodate this USD convention, the curve model places the zero nodes one day after the end dates of the OIS meeting date swaps, so that the high tension curve will produces jumps one date after the meetings.

Sample CAD and USD OIS curves are plotted below. One day forward rates are plotted for the front of the OIS curves. The “Old” curves use the regular 1W, 2W, 3W, 1M, 2M, 3M, etc OIS swaps, while the “New” curves use 5 meeting date swaps with very high tension. We can see 5 flat steps stretched beyond 6 months.





4.3 Market Quote Spreadizing

Spreadizing is designed to facility curve delta calculation, in the situation when the curve instruments consist of a combination of both all-in and spread quotes.

For example, to build USD OIS curve, we could include 1-day cash, fed fund futures and OIS swap instruments as all-in quotes in the front section of the curve. Then, we have fed fund/3m Libor basis swap instruments as spread quotes in the back section of the curve. To build a homogeneous tenor 6m curve, we will have a 6m cash instrument as an all-in quote in the front, followed by 6m/3m basis swap instruments as spread quotes.

As we bump up the 3m curve instrument quotes, the section of 6m curve quoted as basis spreads will move up accordingly but the section of 6m curve quoted as all-in rates will be held. We could bump up the 6m all-in rate quotes as well if they are in alignment with the 3m quotes in bucket delta scenarios. However, this approach is only applicable in aligned situations, and the synchronized bump of instruments across difference curve demands more complicated to implementation.

Therefore, we need a Spreadizing methodology to convert all-in quotes for cash, futures, fed fund futures and swap instruments into corresponding basis spread quotes, such that minor curve zero rates and IR delta are preserved precisely before and after spreadizing.

For the purpose of spreadizing, we divide the market instruments into two categories: (1) single period instruments that make only one payment, such as cash, FRA, futures, fed fund and futures; (2) multi-period instruments that make multiple payments through its life, such as fixed-floating IR swaps

4.3.1 Single Period Instrument Spreadizing

A single period instrument can be generally defined by a start date τ_n, end date t_n, market quoted all-in forward rate $\overline{R}_n$ and daycount dc_n satisfying

$$F(\tau_n, t_n)\Delta t_n df_{XYZ}(0, t_n^P) = \overline{R}_n \Delta t_n df_{XYZ}(0, t_n^P)$$

where $F(\tau_n, t_n)$ is the forward rate between the start date τ_n and end date t_n, $\Delta t_n = \Delta t(\tau_n, t_n, dc_n)$ the period daycount multiplier from time τ_n to t_n, and $df_{XYZ}(0, t_n^P)$ the discount factor from the payment date t_n^P under a selected discounting "XYZ".

It is obvious that discounting is irrelevant here since the FRA instrument directly dictate a forward rate,

$$\overline{R}_n = F(\tau_n, t_n) = \frac{1}{\Delta t_n}\left(\frac{df(0, \tau_n)}{df(0, t_n)} - 1\right)$$

For the purpose of spreadizing, we defined a hypothetical single period "spread" market instrument (e.g. 6m/3m cash spread) that pays a spread $\overline{S}_n$ with respect to a reference rate implied from a reference curve (e.g. 3m Libor curve), such that it is equivalent the corresponding all-in rate $\overline{R}_n$,

$$\overline{R}_n = F(\tau_n, t_n) = F_{Ref}(\tau_n, t_n) + \overline{S}_n$$

Once the reference curve (e.g. 3m Libor curve), and the target curve (e.g. 6m Libor curve) are built, an equivalent all-in forward rate of

$$\overline{S}_n = F(\tau_n, t_n) - F_{Ref}(\tau_n, t_n) = \frac{1}{\Delta t_n}\left(\frac{df(0,\tau_n)}{df(0,t_n)} - \frac{df_{Ref}(0,\tau_n)}{df_{Ref}(0,t_n)}\right)$$

Correspondingly, for fed fund futures spread instrument, we have

$$F(\tau_n, t_n) \approx \frac{1}{\Delta t_n}\ln\left(\frac{df(0,\tau_n)}{df(0,t_n)}\right),$$

$$\overline{S}_n = F(\tau_n, t_n) - F_{Ref}(\tau_n, t_n) = \frac{1}{\Delta t_n}\ln\left(\frac{df_{All-In}(0,\tau_n)}{df_{All-In}(0,t_n)} \Big/ \frac{df_{Ref}(0,\tau_n)}{df_{Ref}(0,t_n)}\right)$$

4.3.2 Single Period Instrument Spreadizing – Enhancement

Spreadizing under the above naïve approach for a single period instrument may result in a spread against a reference forward rate on a nonstandard tenor. As a result, the reference rate is not a market observable forward quote.

For example, spreadizing a 1-month USD OIS swap quote will result in a spread over a 1-month forward rate on the 3-month USD Libor curve. This is not an ideal setup for two reasons: (1) From the curve building methodology perspective, the USD 3-month Libor curve is built to provide good quality 3-month forward rates. The 1-month forward rates usually are not the primary quality objective for a 3-month curve. (2) From the market intuition perspective, when traders talk about a spread between the 1-month rate and 3 month rate, it is the 1-month rate on the 1-month curve against the 3-month rate on the 3-month curve. It is unlikely against the 1-month rate on the 3-month curve.

The solution to address this issue is to spreadize against the standard the tenor forward rate on the reference curve. For example, in the case of USD and CAD, the major curves are 3-month. The spreadizing will be against the 3-month reference rate. In the case of JPY, the major curve is 6-month. The spreadizing will be against the 6-month reference rate.

The single period all-in quotes could come in the form of cash or one period swap. The spreadization of both situations can be conveniently modeled under the basis swap convention. In the following subsections, typical samples of the spreadizing instruments are provided for both OIS and minor index instruments.

4.3.2.1 OIS Swaps to OIS/3m Libor Basis Swaps

The overnight rate (ON) and short term OIS swaps are observable in the major markets. They could be used as instruments to build the front section of the OIS curves. In the USD and CAD market, the major curve index tenors are 3-month. For any OIS instrument with a term shorter than 3-month, the spreadized spread will be defined with respect to the 3-month forward on the reference index curve.

Examples are provided below for both the cash and all-in swap instruments.

CAD OIS Curve: ON (1d) Cash Instrument

All-in swap quote convention:
Forward index start date: today +0 business days
Forward tenor: 1-day, A/365
Fixed rate: A/365

Basis swap quote convention for spreadization:
Forward index start date: today +0 business days
Forward tenor: 1-day, A/365
Reference forward index start date: today +0 business days
Reference forward tenor: 3-month, A/365
Spread is defined on the reference forward
In this case, the spread is the difference between the ON rate fixing and 3m cash fixing.

USD OIS Curve: ON (1d) Cash Instrument

All-in swap quote convention:
Forward index start date: today +0 business days
Forward tenor: 1-day, A/360
Fixed rate: A/360

Basis swap quote convention for spreadization:
Forward index start date: today +0 business days
Forward tenor: 1-day, A/360
Reference forward index start date: today +2 business days
Reference forward tenor: 3-month, A/360
Spread is defined on the reference forward
In this case, the spread is the difference between the ON rate fixing and 3m cash fixing. Please note that USD Libor fixing has 2 business day spot start, while the ON rate fixing is 0-day spot start.

USD OIS Curve: 1m Cash Instrument

All-in swap quote convention:
Forward index start date: today +2 business days
Forward tenor: 1-month, A/360 (It is equivalent to daily reset on 1d index, compounded on the same rate.)
Fixed rate: A/360

Basis swap quote convention for spreadization:
Forward index start date: today +2 business days
Forward tenor: 1-month, A/360
Reference forward index start date: today +2 business days
Reference forward tenor: 3-month, A/360
Spread is defined on the reference forward
In this case, the spread is the difference between the 1m compounded OIS rate and 3m cash fixing.

4.3.2.2 Tenor Swaps to Tenor Basis Swaps

The 1m cash fixing is usually used as the first instrument in the 1-month minor curve. In the USD and CAD market, the major curve index tenor is 3-month. For any minor curve instrument with a term shorter than 3-month, the spreadized spread will be defined with respect to 3-month tenor forward on the reference index curve.

USD 1m Curve: 1m Cash Instrument

All-in swap quote convention:
Forward index start date: today +2 business days
Forward tenor: 1-month, A/360
Fixed rate: A/360

Basis swap quote convention for spreadization:
Forward index start date: today +2 business days
Forward tenor: 1-month, A/360
Reference forward index start date: today +2 business days
Reference forward tenor: 3-month, A/360
Spread is defined on the reference forward

In this case, the spread is the difference between the 1m cash fixing and 3m cash fixing.

This approach can be easily extended to the short term swap instruments, even with multiple resets. For example, if a 2-month termed swap on the 1-month index is quoted as all-in, we could still spreadize the 1-month index leg (with two resets) versus the 3-month reference index, as long as the reference index leg only as one reset.

4.3.2.3 Currency Fixed-Float Swaps to Currency Basis Swaps

This feature will be developed in the future.

4.3.3 Multi-Period Instrument Spreadizing

Multi-currency speadizing involves with converting a fixed-floating swap instrument (quoted as an all-in rate) into a basis swap instrument (quoted as a spread).

The fixed-float swap instrument could be an OIS swap, tenor swap or even currency swap. We will show how speadizing can be performed in each situation.

4.3.3.1 OIS Swaps to OIS/3m Libor Basis Swaps

OIS swaps are quoted in the USD, EUR, GBP and CAD markets. They could be used as instruments to build the front section of the OIS curves.

Consider a n-year OIS swap with a market quoted fixed rate $\overline{R}_n^{1d}$. The floating rate resets daily to the over night rate and compounds at flat. The payment period dates $t_i = t_{i-1} + 12m$, $i = 1,...,n$ are annually spaced. The pay days t_i^P are 1 or 2 business days lagged from t_i. Denote $F_{OIS}(t_{i-1}, t_i)$ the forward rate for period (t_{i-1}, t_i), compounded from the daily over night rates. We have

$$\sum_{i=1}^{n} F_{1d}(t_{i-1}, t_i)\Delta t_i df_{1d}(t_0, t_i^P) = \overline{R}_n^{1d} \sum_{i=1}^{n} \Delta t_i df_{1d}(t_0, t_i^P)$$

Consider a n-year OIS/3m Libor basis swap with a hypothetically market quoted spread rate $\overline{S}_n^{1d}$. The floating OIS rate paying side is not at all changed, and the fixed rating paying side now pays 3m Libor rate plus a spread,

$$\sum_{i=1}^{n} F_{1d}(t_{i-1},t_i)\Delta t_i\, df_{1d}(t_0,t_i^P) = \sum_{i=1}^{n}\left[\overline{S}_n^{1d} + F_{3m}(t_{i-1},t_i)\right]\Delta t_i\, df_{1d}(t_0,t_i^P)$$

In the above two swap equations, it is important to keep all the payment period dates t_i and pay days t_i^P as well the period daycount multipliers Δt_i unchanged to ensure one basis point shift on the spread quote $\overline{S}_n^{1d}$ is equivalent to one basis point shift on the all-in quote $\overline{R}_n^{1d}$,

$$(\overline{R}_n^{1d} - \overline{S}_n^{1d})\sum_{i=1}^{n}\Delta t_i\, df_{1d}(t_0,t_i^P) = \sum_{i=1}^{n} F_{3m}(t_{i-1},t_i)\Delta t_i\, df_{1d}(t_0,t_i^P)$$

Once the OIS curve and 3m Libor curve are built, we can readily find the par spread $\overline{S}_n^{OIS}$.

4.3.3.2 Tenor Swaps to Tenor Basis Swaps

6m swaps are liquidly quoted in some markets (e.g. EUR, GBP and JPY). We may want to use them as instruments to build the 6m curve directly, instead of using the 6m/3m basis swap as instruments. In this case, our curve builder can convert the all-in tenor swap (e,g, 6m) quotes into the tenor basis spreads (e,g, 6m/3m) on demand.

Consider a n-year interest rate swap with a market quoted fixed rate $\overline{R}_n^{6m}$. The floating rate resets semi-annually to the 6-month Libor. The payment period dates $t_i = t_{i-1} + 6m$, $i = 1,...,2n$ are semi-annually spaced and there is no pay day lag. Denote $F_{6m}(t_{i-1},t_i)$ the forward rate for period (t_{i-1},t_i). We have

$$\sum_{i=1}^{2n} F_{6m}(t_{i-1},t_i)\Delta t_i\, df_{1d}(t_0,t_i) = \overline{R}_n^{6m}\sum_{i=1}^{2n}\Delta t_i\, df_{1d}(t_0,t_i)$$

Consider a n-year 6m/3m Libor basis swap with a hypothetically market quoted spread $\overline{S}_n^{6m3m}$. The floating 6m rate paying side is not at all changed, and the fixed rating paying side now pays 3m Libor rate plus a spread,

$$\sum_{i=1}^{2n} F_{6m}(t_{i-1},t_i)\Delta t_i\, df_{1d}(t_0,t_i) = \sum_{i=1}^{2n}\left[\overline{S}_n^{6m3m} + F_{3m}(t_{i-1},t_i)\right]\Delta t_i\, df_{1d}(t_0,t_i)$$

In the above two swap equations, it is important to keep all the payment period dates t_i and the period daycount multipliers Δt_i unchanged to ensure one basis point shift on the spread quote $\overline{S}_n^{6m3m}$ is equivalent to one basis point shift on the all-in quote $\overline{R}_n^{6m}$,

$$(\overline{R}_n^{6m} - \overline{S}_n^{6m3m})\sum_{i=1}^{2n}\Delta t_i\, df_{1d}(t_0,t_i^P) = \sum_{i=1}^{2n} F_{3m}(t_{i-1},t_i)\Delta t_i\, df_{1d}(t_0,t_i)$$

Once the OIS curve and 3m Libor curve as well as the 6m Libor curve are built, we can readily find the par spread $\overline{S}_n^{6m3m}$.

4.3.3.3 Currency Fixed-Float Swaps to Currency Basis Swaps

Consider a n-year XYZ/USD currency fixed-float swap with a market quoted rate $\overline{R}_n^{Xyz}$. The pay side floating rate resets quarterly to the 3-month USD Libor $F_{3m}^{Usd}(t_{i-1}^{Usd}, t_i^{Usd})$ on the tenor period dates $t_i^{Usd} = t_{i-1}^{Usd} + 3m$, $i = 1,...,4n$. The receive side floating rate resets quarterly to the 3-month XYZ Libor $F_{3m}^{Xyz}(t_{i-1}^{Xyz}, t_i^{Xyz})$ on the tenor period dates $t_i^{Xyz} = t_{i-1}^{Xyz} + 3m$. Usually, both pay and receiving sides pay on a common set of quarterly spaced payment period dates $t_i^{pXyzUsd} = t_{i-1}^{pXyzUsd} + 3m$ without a pay lag. The tenor period dates t_i^{Xyz} and t_i^{Usd} are adjusted to the relevant XYZ and USD reset holiday centers respectively, while the payment period dates $t_i^{pCadUsd}$ are adjusted to the union of XYZ and USD payment holiday centers. We have

$$FX_{Xyz/Usd}(0)\left\{df_{Usd1d}^{Usd}(0,t_0^{pXyzUSD}) - \sum_{i=1}^{4n} F_{3m}^{Usd}(t_{i-1}^{Usd}, t_i^{Usd})\Delta t_i^{pXyzUsd} df_{Usd1d}^{Usd}(0,t_i^{pXyzUSD}) - df_{Usd1d}^{Usd}(0,t_{4n}^{pXyzUSD})\right\}$$
$$= df_{Usd1d}^{Xyz}(0,t_0^{pCadUSD}) - \overline{R}_n^{Xyz}\sum_{i=1}^{4n}\Delta t_i^{pXyzUsd} df_{Usd1d}^{Xyz}(0,t_i^{pXyzUSD}) - df_{Usd1d}^{Xyz}(0,t_{4n}^{pXyzUSD})$$

where $FX_{Xyz/Usd}(t_0)$ is the FX rate on the curve anchor date.

Consider a n-year XYZ/USD currency basis swap with a hypothetically market quoted spread $\overline{S}_n^{XyzUsd}$. The floating USD paying side is not at all changed, and the fixed rating paying side now pays Xyz 3m Libor rate plus a spread,

$$FX_{Xyz/Usd}(0)\left\{df_{Usd1d}^{Usd}(0,t_0^{pXyzUSD}) - \sum_{i=1}^{4n} F_{3m}^{Usd}(t_{i-1}^{Usd}, t_i^{Usd})\Delta t_i^{pXyzUsd} df_{Usd1d}^{Usd}(0,t_i^{pXyzUSD}) - df_{Usd1d}^{Usd}(0,t_{4n}^{pXyzUSD})\right\}$$
$$= df_{Usd1d}^{Xyz}(0,t_0^{pCadUSD}) - \sum_{i=1}^{4n}\left[F_{3m}^{Xyz}(t_{i-1}^{Xyz}, t_i^{Xyz}) + \overline{S}_n^{XyzUsd}\right]\Delta t_i^{pXyzUsd} df_{Usd1d}^{Xyz}(0,t_i^{pXyzUSD}) - df_{Usd1d}^{Xyz}(0,t_{4n}^{pXyzUSD})$$

In the above two swap equations, it is important to keep all the dates and the period daycount multipliers unchanged to ensure one basis point shift on the spread quote $\overline{S}_n^{6m3m}$ is equivalent to one basis point shift on the all-in quote $\overline{R}_n^{6m}$,

$$(\overline{R}_n^{Xyz} - \overline{S}_n^{XyzUsd})\sum_{i=1}^{4n}\Delta t_i^{pXyzUsd} df_{Usd1d}^{Xyz}(0,t_i^{pXyzUSD}) = \sum_{i=1}^{4n} F_{3m}^{Xyz}(t_{i-1}^{Xyz}, t_i^{Xyz})\Delta t_i^{pXyzUsd} df_{Usd1d}^{Xyz}(0,t_i^{pXyzUSD})$$

Once the four curves involved in this cross currency basis swaps are built, we can readily find the par basis spread $\overline{S}_n^{XyzUsd}$.

4.4 Treasury Curve Bootstrapping

In this section we present the treasury curve building methodology. We model the zero rates on the treasury as tension spline. As result, treasury curve builder shares the same tension spline infrastructure and generic curve set up framework with swap curves. It is very similar to single swap curve bootstrapping in Libor discounting, as no curve dependency involved. We only highlight the difference as below. We will use US treasury curve as working example.

Depending on maturity, US treasury securities are quoted in yields in different convention in the market. For securities with less than 1 year maturity (“T-bills”) , they are quoted in discounted yields. For securities with more than 1 year maturity (“T-bonds”), they are quoted in semi-annual yields with ACT/365 daycount convention.

T-bills pay no explicit coupon interest. Instead, they are traded at a discount compared to par value. Discounted yield is defined as $y_{DR} = \left(1 - \frac{P}{100}\right) / \Delta t(t_{settle}, t_{end}; ACT360)$.

T-bonds pays semi-annual coupons. The yield is quoted in the market using semi-annual compounding and ACT/365 daycount (SABB yield).

We can convert the discount rate into SABB yield for a T-bill using the following formula.

$$y_{SABB} = 2 \times [\left(\frac{360}{360 - N y_{DR}}\right)^{365/2N} - 1]$$

In our implementation, we assume user at least provides one T-bill.

5 Model Implementation

The model is implemented in the FEDS library of C++ code, and linked in to RiskWatch. The model uses the standard suite of curve bootstrapping methods available through the FEDS library. The RiskWatch implementation gives the usual high level of risk management functionality we enjoy with RiskWatch.

The curve bootstrapping method is implemented as a state procedure in RiskWatch. Below are the details about the state procedure and the various examples how the procedure could be used.

5.1 Bootstrapping Methodology

In high level, the curve bootstrapping is an exercise to find a curve that matches the market quotes of all curve instruments.

The BNS curve is defined by a set of zero rates on the curve nodes and some interpolation rule. A curve node is usually located on the last payment date of each curve instrument, although sometimes it can also be located at the beginning of an instrument [9]. The curve bootstrapping is a root finding problem to find the set of zero rates such that the curve prices the curve instruments to the market quotes. For this reason, we often refer the bootstrapped curve as a zero curve.

5.1.1 Bootstrapping Under Linear Interpolation

Under the linear interpolation rule, the forward rates and discount factors required to value each instrument are fully determined by the zero curve built up to the last payment date of that instrument. Therefore, the curve can be bootstrapped progressively from the shortest maturity instrument to the longest maturity instrument through a sequence of one dimensional root finding to match one instrument quote each time.

5.1.2 Bootstrapping under Tension Spline Interpolation

Under the tension spline, however, the forward rates and discount factors required to value each instrument depends on the entire zero curve, including the section of zero curve beyond the last payment date of that instrument. Therefore, the curve needs to be built with a multi-dimensional dimensional root finding to determine the full set of zero rates to match all instrument quotes at the same time. For this purpose, we have implemented a multi-dimensional Newton-Raphson scheme, using the linearly interpolated zero curve as an initial guess.

5.1.3 Bootstrapping Method Configurations

The curve methodology can be configured with the useFutures flag in the BNS curve model. The flag can have the following settings:

0: Exclude all future instruments in the curve building.

1: For cash instruments ending on or after the first future start date, their corresponding nodes will be excluded from the zero curve. These cash instruments (including 3m cash) still participate in the bootstrapping (e.g. to determine the discount factor on the first future start date under linear interpolation). Just their end dates are not used as zero curve nodes. The first futures start date is used as a zero curve node instead.

2: Cash instruments ending on or after the first future end date will be excluded.

For the major curve bootstrapping, if user provides only one cash and at least three futures under the tension spline interpolation, the advanced homogenous tenor curve method will be activated (See [8] for details), where the front end slopping of the curve is dictated by the cash and the first three futures instruments. Very high tension is applied to the first 4 futures, such that the interpolation nearly linear in that region. Both the 3m cash end date and the first futures start date are used as a zero curve node Sample applications are the CAD and GBP major curves.

For non-major curve bootstrapping, only the instrument end dates (no first futures start date) are used as zero curve node. Very high tension is applied to the first 4 futures (if provided), such that the interpolation nearly linear in that region. Sample applications are the USD or CAD OIS curves with the FOMC/BoC meeting dates modeled as futures instruments.

1001: For OIS and minor curve building with spot date inserted as an extra zero curve node (See [9] for details). Sample applications are the USD OIS and 1m Libor curves.

1002: For major curve building with quad short end. User must provide one one cash and at least two futures to activate this mode under the tension spline interpolation. The front end slopping of the curve is dictated by the cash and the first two futures instruments. The 3m cash is included as a curve instrument, but its end date is not used as a zero curve node and the first futures start is used as a zero curve node instead. Sample application is the USD 3m Libor curve.

1003: For curve building with significantly overlapped instrument (See [9] for details). Depending on the overlapping condition, the instrument start date could be used instead of the instrument end date. Sample applications are the EUR 6m Libor curves where serial futures are significantly overlapped.

1012: The same as 1002 but use a simple swap model that is fast but only applicable to the market standard swaps. Currently, the simple swap model was only developed for USD convention, with the intended use for US swap desk real time pricing. Sample applications are USD OIS, major and minor curve.

Note. "useFutures" flag $n < 1500$, combined with tension normalization flag = 2, sets up a pre-defined tension decay scheme. It starts the high tension decay at the last cash instrument. The high tension decay always takes 4 additional instrument periods. Therefore, starting the high tension decay at the last cash instrument effectively extends the high tension region to cover the first 4 futures instruments. See equation (7) reference [8] for the high front end tension scheme.

"useFutures" flag $1499 < n < 1510$, combined with tension normalization flag = 2 sets up a user manually set tension decay scheme. For example, setting useFutures flag to 1500 will start the high tension decay at the last cash instrument; setting useFutures flag to 1501 will start the tension decay at the first futures instrument (effectively extending the high tension region to cover the first 5 futures instruments), and so on.

"useFutures" flag n=1515, 1525, 1535 or 1545, combined with tension normalization flag = 2 sets up a tension decay scheme automatically extending the high tension region to cover all the futures.

1500-1509: For the curve building with "near linearly interpolated" forward rate. This is achieved by applying high tension in the cash section and a part of the futures section. Sample application is the USD 3m Libor curve. For *3m curve building*, the main features of this methodology are: (1) User is allowed to provide other cash instruments shorter term than the 3m cash instrument (i.e. user inputs the 3m cash as the last cash instrument). The cash instruments spanning between the curve anchor date and the spot date (i.e. 3m cash start date) do not affect the forward rates implied by the curve. If the first cash instrument does not start on the curve anchor date, a cash instrument will be inserted to span the anchor date and the first cash instrument start date. The inserted cash instrument will have the same quoted rate as the first instrument. (2) The same as 1002, the 3m cash is included as a curve instrument, but its end date is not used as a zero curve node and the first futures start is used as a zero curve node instead. (See [9] for details) to pin down the first futures forward rate. (3) Apply high tension in the front of the curve to cover some or all futures.

This flag can also be used for *OIS curve building* to apply high tension on the OIS meeting date swap instruments modeled as futures. In the USD case, the zero curve nodes will be lagged one NYC business day from the end dates of OIS meeting date futures, to accommodate the market convention that the US rate hike comes one business day after the FOMC meeting.

This flag also provides near linearly interpolated forward rates, high tension can be applied on the log discount interpolation across a section of the market quoted forward rates, by setting the following parameters in the user input. This

- Tension normalization flag = 2 for normalization under a short end high tension decaying scheme from a maximum tension level $A_{\infty} = 500$ (See equation (7) in [7] for the scheme details), with the tension decay starting from a specified future instrument. Tension normalization flag = 1 for the standard tension with normalization (See in [7] for the details). This mode provides a smooth front end. The curve shape will still be close to linear, if the rates of cash and futures instruments are close to linearly positioned.
- "useFutures" flag to set n = or > 1500, combined with tension normalization flag = 2 sets up a user defined tension decay scheme, or combined with tension normalization flag = 1 sets up the standard tension scheme. For example, setting useFutures flag to 1500 will start the high tension decay at the last cash instrument; setting useFutures flag to 1501 will start the tension decay at the 1st futures instrument, and so on.
- Interpolation axis yType = 2 for interpolation on log discount.

1510-1519: 1515 applies high tension on all futures. 1514 applies high tension to one less instrument than all futures, while 1516 applies high tension to one more instrument than all futures, and so on.

In contrast to 1520-1549, this setting (1) marks the cash rate as is; (2) uses the futures start date as zero curve node, only when the last cash overlaps with the first futures; (3) does NOT use instrument start date as zero curve nodes for heavily overlapped cash and futures, (4) does NOT convert heavily overlapped cash and futures into non-overlapping forwards for risk calculation within the spreadizing function; (5) also uses a square Jacobian matrix for more stable bootstrapping. The curve anchor date node is forced to take the zero rate of the next curve node.

This configuration is intended for OIS curve building with the meeting date instruments. In the USD case, the zero curve nodes will be lagged one NYC business day from the end dates of OIS meeting date futures, to accommodate the market convention that the US rate hike comes one business day after the FOMC meeting.

1520-1529: 1525 applies high tension on all futures. 1524 applies high tension to one less instrument than all futures, while 1526 applies high tension to one more instrument than all futures, and so on. In addition, this setting triggers the following features: (1) Mark the 3m cash rate as is (i.e. weighting method A1). (2) Always use the futures start date as zero curve node, regardless whether the last cash and the first futures are overlapped or not. (3) Use instrument start date as zero curve nodes for heavily overlapped cash and futures. (4) Convert heavily overlapped cash and futures into non-overlapping forwards for risk calculation within the spreadizing function. (5) Use a square Jacobian matrix for more stable bootstrapping.

1530-1539: 1535 applies high tension on all futures. 1534 applies high tension to one less instrument than all futures, while 1536 applies high tension to one more instrument than all futures, and so on. In addition, this setting triggers the following features: (1) Mark the 3m cash rate by linearly weighting between the quoted cash rate and the first futures rate (i.e. weighting method A2) when close to a futures roll. All other features are the same as 1520-1529.

1540-1549: 1545 applies high tension on all futures. 1544 applies high tension to one less instrument than all futures, while1546 applies high tension to one more instrument than all futures, and so on. In addition, this setting triggers the following features: (1) Mark the 3m cash rate by linearly weighting between the quoted cash rate and an “expected” cash rate linearly extrapolated from the first two futures rates (i.e. weighting method A3) when close to a futures roll. All other features are the same as 1520-1529.

In contrast to 1500-1509, 1510-1549 do not insert a cash instrument to span the anchor date and the first cash instrument start date.

5.1.4 Two Curve Co-bootstrapping

The major curve and OIS curve co-bootstrapping are solved through an iteration procedure. The major curve is built first stand-alone under the Libor discounting (i.e. its own discounting). This often produces a very high quality initial guess. Then, the OIS curve is built with the given major curve, and the major curve is built given the OIS curve, and so on.

5.1.5 Multiple Interpolators in One Curve

There could be multiple interpolators in one curve. A zero curve could be presented by a base interpolator (usually the major curve), plus one or more spread interpolators. A discount factor of the curve at time t is simply the product of the discount factors from all the interpolators at time t. A zero of the curve at time t is simply the sum of the zero rates from all the interpolators at time t.

A minor or OIS curve could be built as a spread zero curve, over a major zero curve.

A minor curve could also be built as a spread zero curve, over another minor zero curve.

5.1.6 Treatment of Cash Instruments

A cash instrument is called “connecting”, if its start date lies on a previous cash instrument’s end date. For the first cash instrument, we need its start date on the curve anchor date. If all cash instruments are “connecting”, their bootstrapping does not rely on the interpolation rule. For example, ON cash spans from today to day 1, TN cash spans from day 1 to day 2 (spot date), and all other cash (e.g. 1w, 1m, 2, 3m) start from the spot date. In this case, the zero rates can be solved sequentially from the shortest to the longest cash instrument. Since the cash start date lies exactly on a previous built curve node, there is no interpolation needed to determine the discount factor on its start date. One can immediately imply the cash end date discount from the market quote and determine the end date zero rate. In this case, it will not make a difference in the result, whether we include the cash instruments and their corresponding zero rates in multi-dimensional root finding under the tension spline interpolation.

Under “useFutures” flags 0, 1 and 2, the cash instruments and their corresponding zero rates are not included in the multi-dimensional root finding under the tension spline. This approach will still yield a zero curve that matches the cash instruments exactly, if all cash instruments are “connecting” as described in the previous paragraph and are bootstrapped with the linear interpolation rule.

Under “useFutures” flags above 1000, the cash instruments and their corresponding zero rates are included in the multi-dimensional root finding under the tension spline. This approach will yield a zero curve that matches all cash instruments exactly, even if not all cash instruments are connecting, and even if the linearly interpolated initial guess zero curve did not achieve exact match to all cash instruments.

5.2 Exception Handling

5.2.1 Zero Term Swap Instrument

One day instrument is often used at the very front end of an interest rate curve. The treatment blow is applied to the first two curve instruments.

Overnight rate fixing instrument can be modeled as a 1-day basis swap. 1-day FX forward can also be modeled as a 1-day cross currency basis swap. For curve building on a holiday, user might place the start date of a 1-day instrument on the holiday. In this case, after the holiday adjustment, the start date could be collapsed on the end date, resulting a zero term swap instrument.

For a single or cross currency swap, if the swap start date collapsed on the swap end date after the holiday adjustment, the swap instrument will be ignored in the bootstrapping. This is a legacy exception treatment for cross-currency swaps.

5.3 Inputs of the Curve Building State Procedure

5.3.1 BNS Bootstrapping Generic

This is the main state procedure used for majority of the curve building purposes, for major curves, OIS curves, minor curves, cross-currency discount curves under either Libor or OIS discounting setting.

State Procedure Name: BNS Bootstrapping Generic

Function Parameters:

Function parameter 1: 0=no co-bootstrapping, or else co-bootstrapping (see details below)

Function parameter 2 (optional): Instrument input format: 0=G1 format, 1=G2 format. G1 format if parameters 2 and 3 are not provided.

Function parameter 3 (optional): File export flag: No export if the parameter=0 or not provided, 1=export the original curve, 2= export the curve after spreadizing and stubdizing, 3= export the curve after reverse spreadizing and reverse stubdizing. Both csv and noncsv format curve files will be exported.

Procedure Parameter: It takes up to four input curves. These curves can be either instrument curve (e.g. sUSD1d.Details) that contains the market instrument quotes (cash, futures, FRAs and swaps), or BNS curve that contains both instrument information and the bootstrapped zero curve. A BNS curve can be either discount curve (e.g. USDsUSD1d), or forward curve (e.g. USDsUSD3m), or both.

5.3.2 BNS Composite Curve Generic

State Procedure Name: BNS Composite Curve Generic

Function Parameters:

Function Parameter 1: 101 for building the cheapest to deliver curve

Function Parameter 2: The composite curve crossing index regeneration flag. The crossing index tracks where we switch from one input curve to another. For example, input curve 2 is the cheapest to deliver till day 365 then input curve 3 is cheapest till day 370, etc.

- User setting 0=regenerate the crossing index. It will auto set to 1 after curve building.
- Auto setting 1=not regenerate the crossing index. User should never manually input 1.
- User setting 2=always regenerate the crossing index. It will be still 2 after curve building.

Function Parameter 3 (optional): Debug printing flag. 0=no debug (default), 1=debug

Function Parameter 4 (optional): Curve file export flag, e.g. 1234 to print all files, 34 to print the csv and non-csv curve files.

Procedure Parameter: A list of alternative currency collateral discount curves.

5.4 RiskWatch Curve File Description

5.4.1 Build Discount Curve from IR Swap Quotes

Example: Build OIS curve from OIS swap quotes

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 0

Procedure Parameters: OIS instrument curve (e.g. sCAD1d.Details)

Output: OIS Curve (e.g. CADsCAD1d)

5.4.2 Build Major Forward Curve from IR Swap Quotes & Discount Curve

Example 1: Build CAD 3m Libor curve from 3m IR swap quotes, with CAD Collateral

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 0

Procedure Parameters: 3m instrument curve (e.g. sCAD3m.Details), OIS curve (for side 1, e.g. CADsCAD1d), OIS curve (for side 2, e.g. CADsCAD1d)

Output: 3m Curve (e.g. CADsCAD3m)

Example 2: Build USD 3m Libor curve from 3m IR swap quotes, with USD Collateral

Procedure Parameters: 3m instrument curve (e.g. sUSD3m.Details), OIS curve (for side 1, e.g. USDsUSD1d), OIS curve (for side 2, e.g. USDsUSD1d)

Output: 3m Curve (e.g. USDsUSD3m)

5.4.3 Build Minor Forward Curve from Tenor Basis Swap Quotes

Example 1: Build CAD 6m Libor curve from 6m/3m basis swap quotes, with CAD collateral

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 0

Procedure Parameters: 6m instrument curve (e.g. sCAD6m.Details), OIS curve (for side 1, e.g. CADsCAD1d), OIS curve (for side 2, e.g. CADsCAD1d), 3m Curve (for side 2, e.g. CADsCAD3m)

Output: 6m Curve (e.g. CADsCAD6m)

Example 2: Build USD 6m Libor curve from 6m/3m basis swap quotes, with USD collateral

Procedure Parameters: 6m instrument curve (e.g. sUSD6m.Details), OIS curve (for side 1, e.g. USDsUSD1d), OIS curve (for side 2, e.g. USDsUSD1d), 3m Curve (for side 2, e.g. USDsUSD3m)

Output: 6m Curve (e.g. USDsCAD6m)

5.4.4 Co-Build Forward Curve First from 2 Sets of Quotes – IR Swap in Collateral Currency

Step 1: Co-build 3m Libor curve from OIS/3m basis swap and 3m IR swap quotes

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 1 (for co-build)

Procedure Parameters: 3m instrument curve (e.g. sUSD3m.Details), OIS instrument curve (e.g. sUSD1d.Details)

Output: 3m Curve (e.g. USDsUSD3m)

Step 2: Build OIS curve from OIS/3m basis swap and 3m curve

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 0

Procedure Parameters: OIS instrument curve (e.g. sUSD1d.Details), 3m curve (for side 2, e.g. USDsUSD3m). *Please note that there is no co-build anymore as the 3m curve has been built in Step 1 above.*

Output: OIS Curve (e.g. USDsUSD1d)

5.4.5 Co-Build Discount Curve First from 2 Sets of Quotes – IR Swap in Collateral Currency

Step 1: Co-build OIS curve from OIS/3m basis swap and 3m IR swap quotes

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 2 (for co-build)

Procedure Parameters: OIS instrument curve (e.g. sUSD1d.Details), 3m instrument curve (e.g. sUSD3m.Details)

Output: OIS Curve (e.g. USDsUSD1d)

Step 2: Build 3m Libor curve from OIS curve and 3m IR swap quotes

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 0

Procedure Parameters: 3m instrument curve (e.g. sUSD3m.Details), OIS curve (for side 1, e.g. USDsUSD1d), OIS curve (for side 2, e.g. USDsUSD1d). *Please note that there is no co-build any more as the OIS curve has been built in Step 1 above.*

Output: 3m Curve (e.g. USDsUSD3m)

5.4.6 Direct-Build Cross-Currency Discount Curve – IR Swap in Collateral Currency

In this case, we assume that the forward curve has already been built (e.g. the CAD 3m curve in the has been built from the 3m CAD IR swaps collateralized in CAD). In this case, there is no longer a need for the co-construction procedure. The discount curve can be built directly.

Example 1: Build CAD cross-currency discount curve under USD collateral from CAD/USD cross currency basis swap quotes

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 0 (for no co-build)

Procedure Parameters: CAD/USD instrument curve (e.g. sCADUSD.Details), USD OIS curve (e.g. USDsUSD1d), USD 3m curve (e.g. USDsUSD3m), CAD 3m curve (e.g. CADsCAD3m)

Output: CAD cross-currency discount curve under USD OIS discounting (e.g. USDsCADUSD1d)

5.4.7 Co-Build Discount or Forward Curve First from 2 Sets of Quotes – IR Swap in Non-Collateral Currency

Example 1: Co-build MXN 28d curve first from 28d MXN/1m USD cross currency basis swap and MXN 28d IR swap quotes. Here we assumed the quotes for the single currency 28d MXN IR swaps are in USD.

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 3 (for co-build)

Procedure Parameters: MXN 28d instrument curve (e.g. sMXN28d.Details), MXN/USD instrument curve (e.g. sMXNUSD.Details), USD OIS curve (e.g. USDsUSD1d), USD 1m curve (e.g. USDsUSD1m)

Output: MXN 28d Curve under USD OIS discounting (e.g. USDsMXN28d)

Example 2: Co-build MXNUSD cross-currency OIS discount curve first from MXN/USD cross currency basis swap and MXN 28d IR swap quotes.

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 4 (for co-build)

Procedure Parameters: MXN/USD instrument curve (e.g. sMXNUSD.Details), MXN 28d instrument curve (e.g. sMXN28d.Details), USD OIS curve (e.g. USDsUSD1d), USD 1m curve (e.g. USDsUSD1m)

Output: MXNUSD cross-currency discount Curve under USD OIS discounting (e.g. USDsMXNUSD1d)

5.4.8 Build Alternative Collateral Discount Curve from 3 Other Discount Curves

In this case, we use the FX invariance principle to build the alternative discount curve from three other discount curves.

General Input Paramters

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 11 (for building under FX invariance principle), 1 (for G2 format. Note this feature is ONLY available in G2 format.)

Procedure Parameters: A dummy instrument curve (e.g. dummy.Details, a placeholder only. It is not used), plus input zero curves 1, 2 and 3. The output curve is built as

df(output curve) = df(input curve2) x df(input curve3) / df(input curve1)

Example 1: Alternative collateral cross-currency USD discount curve construction

Build the USD discount curve under the Eonia discounting from three other relevant discount curves.

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 11 (for building under FX invariance principle), 1 (for G2 format)

Procedure Parameters: dummy instrument curve (e.g. dummy.Details), EUR discount curve under USD OIS discounting (e.g. USDsEURUSD1d), USD discount curve under USD OIS discounting (e.g. USDsUSD1d), and the EUR discount curve under Eonia discounting (e.g. EURsEUR1d)

Output: USD discount curve under the Eonia discounting (e.g. EURsUSDEUR1d)

Example 2: Alternative collateral cross-currency non-USD discount curve construction

Build the CAD discount curve under the Eonia discounting from three other relevant discount curves.

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 11 (for building under FX invariance principle), 1 (for G2 format)

Procedure Parameters: dummy instrument curve (e.g. dummy.Details), USD discount curve under USD OIS discounting (e.g. USDsUSD1d), CAD discount curve under USD OIS discounting (e.g. USDsCADUSD1d), and the USD discount curve under Eonia discounting (e.g. EURsUSDEUR1d)

Output: CAD discount curve under the Eonia discounting (e.g. EURsCADEUR1d)

Example 3: Cross-currency Libor discount curve construction

Build the CAD discount curve under the USD Libor funding from three other relevant discount curves.

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 11 (for building under FX invariance principle), 1 (for G2 format)

Procedure Parameters: dummy instrument curve (e.g. dummy.Details), USD discount curve under USD OIS discounting (e.g. USDsUSD1d), CAD discount curve under USD OIS discounting (e.g. USDsCADUSD1d), and the USD 3m Libor curve under USD OIS discounting (e.g. USDsUSD3m)

Output: CAD discount curve under USD 3m Libor funding (e.g. USDsCADUSD3m)

5.4.9 Build the Cheapest to Deliver Discount Curve

In this case, we build the cheapest to deliver discount curve from a basket of discount curves on the deliverable collateral currencies.

Example 1: Build USD discount curve based the cheapest to deliver from a basket of deliverable currencies: USD, CAD, EUD and GBP.

State Procedure Name: BNS Bootstrapping Generic

Function Parameters: 101 (for cheapest to deliver curve build), 0 (for crossing index regeneration at the initial run only)

Procedure Parameters: USD discount curves under USD OIS discounting (e.g. USDsUSD1d), under CAD OIS discounting (e.g. CADsUSDCAD1d), under Eonia discounting (e.g. EURsUSDEUR1d), and under Sonia discounting (e.g. GBPsUSDGBP1d),

Output: USD discount curve under cheapest to deliver of USD, CAD, EUR and GBP collateral (e.g. CTDsUSDCTD1d)

Note that the CTDsUSDCTD1d can be more concisely denoted as simply CTDsUSD1d. For the same reason, the EURsUSDEUR1d can be more concisely (and perhaps less confusingly) denoted simply as EURsUSD1d. The first EUR indicates the collateral currency and the USD indicates the discount currency. The second EUR in the curve is redundant and was only there for historical reasons.

5.5 RiskWatch Instrument Curve Description

5.5.1 General Curve Parameters

The general curve parameters are listed on the top row of the RiskWatch input matrix.

- Column 0 Curve type, e.g. 15=SWAP, 434=CPI
- Column 1 Currency for side 1, e.g. 63=CAD, 64=USD, 74=GBP, 89=EUR
- Column 2 Currency for side 2, e.g. 63=CAD, 64=USD, 74=GBP, 89=EUR
- Column 3 Interpolation method. 0=Linear interpolation on zero rate, 2=Tension spline.
- Column 4 Lag that identifies the spot date, e.g. 2 for USD
- Column 5 Daycount basis for zero rate/discount factor conversion. Only 3=A/365 or 402=Bus/252 allowed.
- Column 6 Date adjustment type for non-swaps (not in use)
- Column 7 Date adjustment type for swaps, e.g. 7= "Modified Following"
- Column 8 "useFutures" flag. 0=no, 1=use the first futures start date to override cash instruments, 2=use first futures maturity to override cash instruments (see [8]), 1002=with quad short end (See [9]), 1500-1539=use "nearly linear" forward short end for the 3m Libor vurve (See [9]).
- Column 9 Use FRA flag, 1=yes, 0=no
- Column 10 Debug print flag, 1=yes, 0=no
- Column 11 Interpolation method between curve nodes. This parameter is only applicable to CPI curves. 3=Linear interpolation on CPI levels, 4=step interpolation on CPI levels with EOM roll, 5=step (i.e. piecewise constant) interpolation on CPI levels. See [7] for further details.
- Column 12 Side where basis swap spreads are quoted: 1=side 1, 2=side 2.
- Column 13 Tension parameter A (tension level): example settings are 3 and 18.
- Column 14 Tension parameter B (tension decay): example settings are 0 and 3.
- Column 15 Interpolation axis (or called yType): 2 for interpolation on log discount, 1 for interpolation on zero rate.
- Column 16 Tension normalization flag: 0 for no normalization, 1 for normalization, 2 for normalization with high tension at the short end.
- Column 17 A parameter which is not currently in use
- Column 18 The flag for determining the curve building method, 0 for all in zero all-in rate method and 1 for zero spread method
- Column 19 The anchor date of the curve. If it is 0, the start date of the first instrument will be used as the curve anchor date

5.5.2 G1 Format Par Instrument Input

Following the top row, each row of the RiskWatch input matrix defines the specifics of an underlying instrument. For any swap instrument, user must provide standard input parameters from columns 0 to 17. Input columns beyond this are optional.

- Column 0 Instrument type, e.g. 1=Cash, 2=Futures, 4=Swap, 5=FX reset swap, 11=Seasonal adjustments for CPI ("11" is a special "non-instrument".

Column 1 contains the number of CPI publications in a year, say 12. The following columns contain 12 seasonal adjustments)

• Column 1	Instrument maturity in years
• Column 2	Instrument start date (issue date)
• Column 3	Instrument end date (maturity date)
• Column 4	Pay fixed/float flag for swap side 1. 21=fixed, 22=float
• Column 5	Rate for cash/futures, or fixed rate/float spread for swap side 1
• Column 6	Daycount for cash, futures, or swap side 1, e.g. 2=A/360, 3=A/365.
• Column 7	Reset frequency for swap side 1
• Column 8	Reset frequency unit for swap side 1, e.g. 27=day, 29=month, 30=year
• Column 9	pay frequency for swap side 1
• Column 10	pay frequency unit for swap side 1, e.g. 27=day, 29=month, 30=year
• Column 11	Pay fixed/float flag for swap side 2. 21=fixed, 22=float
• Column 12	Fixed rate/float spread for swap side 2
• Column 13	Daycount for swap side 2, e.g. 2=A/360, 3=A/365.
• Column14	Reset frequency for swap side 2
• Column 15	Reset frequency unit for swap side 2, e.g. 27=day, 29=month, 30=year
• Column 16	pay frequency for swap side 2
• Column 17	pay frequency unit for swap side 2, e.g. 27=day, 29=month, 30=year

The following input columns are optional:

• Column 18	pay lag in number of business days for swap side 1. Default value = 0.
• Column 19	pay lag in number of business days for swap side 2. Default value = 0.

5.5.3 G2 Format Par Instrument Input

Following the top row, each row of the RiskWatch input matrix defines the specifics of an underlying instrument. For each instrument, user must provide standard input parameters from columns 0 to 6.

• Column 0	Instrument type: 1=Cash, 2=Futures, 4=Swap, 12=Bond
• Column 1	Instrument maturity in years (e.g. 10)
• Column 2	Instrument start date (issue date)
• Column 3	Instrument end date (maturity date)
• Column 4	Instrument quote type: 0=Default set according to the convention table, 1=All-in rate, 2=Spread. This can be used to override the “FixedFlag” in the instrument convention details to turn an all-in (i.e. fixed-float) instrument into a spread (i.e. float-float) instrument.
• Column 5	Quoted rate or spread (e.g. 0.01234 for 1.234%)
• Column 6	Instrument convention ID (integer). The convention details are defined in a configuration file (XML format).
• Column 7	Optional info. Depending on the instrument “type” specified in Column 0: Column 7 represents the swap rate over bond yield spread if the “type” is “Swap”; bond coupon rate if the “type” is “Bond”; or average reset to date if the “type” is “Futures” (to capture the daily resets of the prompt fed fund future.).

5.5.3.1 Instrument Convention Table

The instrument conventions are stored in a XML, which will be loaded only once when a RiskWatch session is loaded. An instrument convention is provided an example at the end of this section.

Each convention may contain the following attributes:

• ConventionGroup name	String, name for K2 and FedWin, e.g. “xGBPUSD”. It will be used to prefix the “convention name”
• Convention Name	String, name used in K2 and FedWin to be combined with the convention group name, e.g. “Sample1” yields “xGBPUSD/Sample1”.
• Convention UID	Integer, ID that uniquely identify the convention.
• InfoCurrency	String, currency used for K2 only, e.g. “USD”. This field will not be used in the bootstrapping.
• Type	String. This field is only used to note “Cash” or “Swap” only. This field will not be used in the bootstrapping.
• Quoted Side	Integer, to indicate which side of a swap the market quote will be applied to. 1=Side 1, else=Side2. All-in rates are always needs to be applied to Side2.
• BusDayConv	String, payment date adjustment type. It is only has impact when DateAdjust is set to “UNADJUST”. If not provided, it will default to “MODIFIED_FOLLOWING”. Other possible settings are “UNADJUST”, “PRECEEDING”, “PRECEEDING_FOLLOWING”, “FOLLOWING” and “MONTH_END”.
• PrincipalExchange	String, principal exchange flag (e.g. “NO” and “YES”). If not provided, it will default to “NO” for single currency swaps and to “YES” for cross currency swaps according to the side currencies set in the general curve parameters listed on the top row of the RiskWatch input matrix.
• FxResetTrade	Integer, FX reset flag: 0=No FX reset, 1=pay side FX reset, 2=receive side FX reset. If not provided, it will default to 0.
• FxResetLag	Integer, index reset lag (e.g. -2 for -2 business days). If not provided, it will take default according to default reset lag of the side currency set in the general curve parameters listed on the top row of the RiskWatch input matrix.
• FxResetHoliday	String, FX reset holiday centers (e.g. “LON;NYC;TOR”). If not provided, it will take default to the union of both sides’ payment holiday centers according to the side currencies set in the general curve parameters listed on the top row of the RiskWatch input matrix.
• Side1	It could contain many attributes of side 1
• Side2	It could contain many attributes of side 2

Each side may contain the following attributes:

• FixedFlag	String, “float” or “fixed” to indicate this side pays float or fixed
• InfoCurrency	String, currency used for K2 only, e.g. “USD”. This field will not be used in the bootstrapping.

- PmtDayCount String, payment daycount (e.g. "ACT/360"). If not provided, it will take default according to the side currency set in the general curve parameters listed on the top row of the RiskWatch input matrix. (e.g. "ACT/360" for USD and EUR, "ACT/365" for CAD and GBP)
- ResetFreq String, reset frequency (e.g. "3M" for quarterly, "1D" for daily)
- PayFreq String, payment frequency (e.g. "3M" for quarterly, "1D" for daily)
- CompundingMethod String, compounding method used when reset more frequently than payment. If not provide, it defaults to "ADJUST_FLAT". Other available settings are "NO", "FLAT_FLAT", "FLAT_ADJUST", "ADJUST_ADJUST", "FLAT_FLAT_GEOM", and "AVERAGE_ORDER1"
- ResetLag Integer, index reset lag (e.g. -2 for -2 business days). If not provided, it will take default according to the side currency set in the general curve parameters listed on the top row of the RiskWatch input matrix. (e.g. -2 for USD and EUR, 0 for CAD and GBP)
- IndexEffLag Integer, index effective lag (e.g. 2 for 2 business days). If not provided, it will take default according to the side currency set in the general curve parameters listed on the top row of the RiskWatch input matrix. (e.g. 2 for USD and EUR, 0 for CAD and GBP)
- PaymentHoliday String, payment holiday centers (e.g. "LON;NYC"). If not provided, it will take default according to the side currency set in the general curve parameters listed on the top row of the RiskWatch input matrix. (e.g. "LON;NYC" for USD, "TAR" for EUR, "LON" for GBP, "TOR"). For currency swaps, it will default to the union of both sides' holiday centers.
- ResetHoliday String, reset holiday centers (e.g. "LON"). If not provided, it will take default according to the side currency set in the general curve parameters listed on the top row of the RiskWatch input matrix. (e.g. "LON" for USD, "TAR" for EUR, "LON" for GBP, "TOR")
- PeriodHoliday String, payment holiday centers (e.g. "LON;NYC"). If not provided, it will default to the payment holiday center.
- AdjustType Sting, period date adjustment flag (e.g. "YES", "NO"). If not provided, it will default to "YES".
- DateAdjust String, period date adjustment type. If not provided, it will default to "MODIFIED_FOLLOWING". Other possible settings are "UNADJUST", "PRECEEDING", "PRECEEDING_FOLLOWING", "FOLLOWING" and "MONTH_END".
- PayLagConvention String, payment date adjustment type. If not provided, it will default to "PAY_LAG_BUSINESS". Another possible setting is "NO", "PAY_LAG_CALENDAR"
- StubMode String, stub mode. If not provided, it will default to "SHORT_FIRST". Other possible settings are "SHORT_LAST", "LONG_FIRST", and "LONG_LAST"
- PercentOfIndex Double, index leverage multiplier (e.g. 1.5 for 1.5 times of Libor). If not provided, it will default to 1.0

Note: Default settings are provided for user convenience. We should always exercise caution when use default settings. In case of any uncertainty, we always recommend to use the manually settings to ensure the desirable setting is actually be selected.

Sample Instrument Convention

The following is a sample of one instrument convention, copied from a XML file that contains a collection of instrument conventions.


```xml
<ConventionGroup name="xGBPUSD">
    <InfoCurrency>USD</InfoCurrency>
    <Convention name="Sample1" UID="9064500">
        <Type>Swap</Type>
        <QuotedSide type="int">2</QuotedSide>
        <Side1>
            <FixedFlag>float</FixedFlag>
            <InfoCurrency>USD</InfoCurrency>
            <PmtDayCount>ACT/360</PmtDayCount>
            <ResetFreq>3M</ResetFreq>
            <PayFreq>3M</PayFreq>
            <PmtLag type="int">0</PmtLag>
            <CompoundingMethod>ADJUST_FLAT</CompoundingMethod>
            <ResetLag type="int">-2</ResetLag>
            <IndexEffLag type="int">2</IndexEffLag>
            <PaymentHoliday>LON;NYC</PaymentHoliday>
            <ResetHoliday>LON</ResetHoliday>
            <PeriodHoliday>LON;NYC</PeriodHoliday>
            <AdjustType>YES</AdjustType>
            <DateAdjust>MODIFIED_FOLLOWING</DateAdjust>
            <payLagConvention>PAY_LAG_BUSINESS</payLagConvention>
            <StubMode>SHORT_FIRST</StubMode>
            <PercentOfIndex type="double">1</PercentOfIndex>
        </Side1>
        <Side2>
            <FixedFlag>float</FixedFlag>
            <InfoCurrency>GBP</InfoCurrency>
            <PmtDayCount>ACT/365</PmtDayCount>
            <ResetFreq>3M</ResetFreq>
            <PayFreq>3M</PayFreq>
            <PmtLag type="int">0</PmtLag>
            <CompoundingMethod>ADJUST_FLAT</CompoundingMethod>
            <ResetLag type="int">0</ResetLag>
            <IndexEffLag type="int">0</IndexEffLag>
            <PaymentHoliday>LON</PaymentHoliday>
            <ResetHoliday>LON</ResetHoliday>
            <PeriodHoliday>LON</PeriodHoliday>
            <AdjustType>YES</AdjustType>
            <DateAdjust>MODIFIED_FOLLOWING</DateAdjust>
            <payLagConvention>PAY_LAG_BUSINESS</payLagConvention>
            <StubMode>SHORT_FIRST</StubMode>
            <PercentOfIndex type="double">1</PercentOfIndex>
        </Side2>
        <FxResetTrade type="int">1</FxResetTrade>
        <FxResetLag type="int">-2</FxResetLag>
        <FxResetHoliday>LON;NYC</FxResetHoliday>
        <PrincipalExchange>Yes</PrincipalExchange>
        <BusDayConv>MODIFIED_FOLLOWING</BusDayConv>
    </Convention>
</ConventionGroup>
```

6 Impact Test Results

In this section, comparisons are made on curves and swap values under the Libor discounting and the OIS discounting. The results will be presented in three parts:

- Discount curve and 3m Libor curve in the collateral currency, as well as the resulting valuation difference on (3m Libor) interest rate swaps
- 1m and 6m Libor curves in the collateral currency, as well as the resulting valuation difference on 1m/3m, 3m/6m tenor basis swaps
- Discount curve and 3m Libor curve in the non-collateral currency, as well as the resulting valuation difference on (3m Libor) interest rate swaps and cross currency basis swaps

6.1 Impact Test 1 : OIS and 3m Libor Curves in the Collateral Currency

We have done impact test for CAD, USD, EUR and GBP. The input market quotes, the bootstrapped OIS and Libor curves and the impact to the valuation of fixed-floating interest rate swaps will be presented in the sequel.

6.1.1 Market Quotes for OIS & 3m Libor

We first present the market quotes used for the various curve bootstrapping. For all the currencies, the market quotes are

- OIS instrument quotes: OIS swaps, OIS/3m basis swaps
- 3-month Libor quotes: cash, futures and 3m IR swaps

The OIS instrument quotes are taken from Bloomberg. The screen shots of the market quotes are provided in Appendix B for reference. The 3-month Libor quotes are extracted from the K2 curves.

6.1.1.1 CAD Market Quotes

Table 1A. CAD OIS Instruments Quotes

Inst Type	Term	Start	End	Rate (in bps)
ON	1d	17-Aug-10	18-Aug-10	91.8
OIS Swap	1w	17-Aug-10	24-Aug-10	74.3
OIS Swap	2w	17-Aug-10	31-Aug-10	74.3
OIS Swap	3w	17-Aug-10	7-Sep-10	75.9
OIS Swap	1m	17-Aug-10	17-Sep-10	81.4
OIS Swap	2m	17-Aug-10	17-Oct-10	86.9
OIS Swap	3m	17-Aug-10	17-Nov-10	90.5
OIS Swap	4m	17-Aug-10	17-Dec-10	92.6
OIS Swap	5m	17-Aug-10	17-Jan-11	94.5
OIS Swap	6m	17-Aug-10	17-Feb-11	96.5
OIS Swap	7m	17-Aug-10	17-Mar-11	97.9
OIS Swap	8m	17-Aug-10	17-Apr-11	99.6
OIS Swap	9m	17-Aug-10	17-May-11	101.1
OIS Swap	10m	17-Aug-10	17-Jun-11	102.9
OIS Swap	11m	17-Aug-10	17-Jul-11	104.5
OIS Swap	1y	17-Aug-10	17-Aug-11	106.5
OIS Swap	2y	17-Aug-10	17-Aug-12	131.9
OIS Swap	3y	17-Aug-10	17-Aug-13	157.7
OIS Swap	4y	17-Aug-10	17-Aug-14	185.1
OIS Swap	5y	17-Aug-10	17-Aug-15	207.8

Table 1B. CAD 3m Instruments Quotes

Inst Type	Term	Start	End	Rate (in bps)
ON	1d	17-Aug-10	18-Aug-10	91.8
CASH	1m	17-Aug-10	17-Sep-10	91.8
CASH	2m	17-Aug-10	18-Oct-10	97.5
CASH	3m	17-Aug-10	17-Nov-10	106.8
CASH	6m	17-Aug-10	17-Feb-11	113.6
CASH	9m	17-Aug-10	17-May-11	119.1
CASH	12m	17-Aug-10	17-Aug-11	124.4
FUT	3m	13-Sep-10	13-Dec-10	113.0
FUT	3m	13-Dec-10	14-Mar-11	123.0
FUT	3m	14-Mar-11	13-Jun-11	132.0
FUT	3m	13-Jun-11	19-Sep-11	142.0
3m Swap	2y	17-Aug-10	17-Aug-12	150.2
3m Swap	3y	17-Aug-10	17-Aug-13	179.6
3m Swap	4y	17-Aug-10	17-Aug-14	207.0
3m Swap	5y	17-Aug-10	17-Aug-15	230.3
3m Swap	6y	17-Aug-10	17-Aug-16	249.5
3m Swap	7y	17-Aug-10	17-Aug-17	266.5
3m Swap	8y	17-Aug-10	17-Aug-18	283.2
3m Swap	9y	17-Aug-10	17-Aug-19	298.4
3m Swap	10y	17-Aug-10	17-Aug-20	312.7
3m Swap	12y	17-Aug-10	17-Aug-22	338.7
3m Swap	15y	17-Aug-10	17-Aug-25	367.4
3m Swap	20y	17-Aug-10	17-Aug-30	388.3
3m Swap	30y	17-Aug-10	17-Aug-40	376.3
3m Swap	40y	17-Aug-10	17-Aug-50	372.3

<u>CAD Curve Construction Sequence:</u>

- Build the OIS discount curve from OIS swap quotes
- Build the 3m Libor curve from cash and futures instruments, independent of the OIS discount curve.
- Extend the 3m Libor curve from swap instruments, using the already built OIS discount curve.

6.1.1.2 USD Market Quotes

Table 2A. USD OIS/3m Instruments Quotes

Inst Type	Term	Start	End	Rate (in bps)
ON	1d	17-Aug-10	18-Aug-10	22.66
TN	1d	18-Aug-10	19-Aug-10	22.66
OIS Swap	1m	19-Aug-10	19-Sep-10	17.70
OIS Swap	2m	19-Aug-10	19-Oct-10	17.20
OIS Swap	3m	19-Aug-10	19-Nov-10	17.10
OIS Swap	6m	19-Aug-10	19-Feb-11	16.90
OIS Swap	9m	19-Aug-10	19-May-11	18.60
OIS Swap	12m	19-Aug-10	19-Aug-11	21.30
FF/3m Swap	1.5y	19-Aug-10	19-Feb-12	24.38
FF/3m Swap	2y	19-Aug-10	19-Aug-12	25.00
FF/3m Swap	3y	19-Aug-10	19-Aug-13	25.88
FF/3m Swap	4y	19-Aug-10	19-Aug-14	26.13
FF/3m Swap	5y	19-Aug-10	19-Aug-15	25.88
FF/3m Swap	7y	19-Aug-10	19-Aug-17	25.00
FF/3m Swap	10y	19-Aug-10	19-Aug-20	23.75
FF/3m Swap	12y	19-Aug-10	19-Aug-22	22.88
FF/3m Swap	15y	19-Aug-10	19-Aug-25	21.88
FF/3m Swap	20y	19-Aug-10	19-Aug-30	20.75
FF/3m Swap	25y	19-Aug-10	19-Aug-35	20.00
FF/3m Swap	30y	19-Aug-10	19-Aug-40	19.38

Table 2B. USD 3m Instruments Quotes

Inst Type	Term	Start	End	Rate (in bps)
ON	1d	17-Aug-10	18-Aug-10	22.66
TN	1d	18-Aug-10	19-Aug-10	22.66
CASH	1m	19-Aug-10	26-Aug-10	25.88
CASH	2m	19-Aug-10	20-Sep-10	26.66
CASH	3m	19-Aug-10	19-Oct-10	30.34
FUT	3m	15-Sep-10	15-Dec-10	35.73
FUT	3m	15-Dec-10	16-Mar-11	41.89
FUT	3m	16-Mar-11	15-Jun-11	48.25
FUT	3m	15-Jun-11	21-Sep-11	58.55
FUT	3m	21-Sep-11	21-Dec-11	72.32
FUT	3m	21-Dec-11	21-Mar-12	89.54
FUT	3m	21-Mar-12	20-Jun-12	106.22
FUT	3m	20-Jun-12	19-Sep-12	123.86
FUT	3m	19-Sep-12	19-Dec-12	140.96
FUT	3m	19-Dec-12	20-Mar-13	160.01
FUT	3m	20-Mar-13	19-Jun-13	176.04
FUT	3m	19-Jun-13	18-Sep-13	194.01
3m Swap	2y	19-Aug-10	19-Aug-12	69.41
3m Swap	3y	19-Aug-10	19-Aug-13	100.68
3m Swap	4y	19-Aug-10	19-Aug-14	133.09
3m Swap	5y	19-Aug-10	19-Aug-15	164.51
3m Swap	6y	19-Aug-10	19-Aug-16	192.90
3m Swap	7y	19-Aug-10	19-Aug-17	216.03
3m Swap	8y	19-Aug-10	19-Aug-18	234.68
3m Swap	9y	19-Aug-10	19-Aug-19	250.57
3m Swap	10y	19-Aug-10	19-Aug-20	263.72
3m Swap	12y	19-Aug-10	19-Aug-22	284.72
3m Swap	15y	19-Aug-10	19-Aug-25	307.10
3m Swap	20y	19-Aug-10	19-Aug-30	325.24
3m Swap	30y	19-Aug-10	19-Aug-40	339.01
3m Swap	40y	19-Aug-10	19-Aug-50	340.51

Note: Spreads instead of rates are presented in the “Rate (in bps)” column for OIS/3m basis swaps.

<u>USD Curve Construction Sequence</u>:

- Build the OIS discount curve from OIS swap quotes to 1-year
- Build the 3m Libor curve from cash and futures instruments to 3-year, independent of the OIS discount curve.
- Extend the OIS discount curve from FF/3m basis swap instruments to 3-year, using the already built 3m Libor curve to 3-year.
- Co-build the OIS discount and 3m Libor curves beyond 3-year using both the FF/3m basis swap instrument and 3m IR swap instruments.

6.1.1.3 EUR Market Quotes

Table 3A. EUR OIS Instruments Quotes

Inst Type	Term	Start	End	Rate (in bps)
ON	1d	17-Aug-10	18-Aug-10	54.0
TN	1d	18-Aug-10	19-Aug-10	54.0
OIS Swap	1w	19-Aug-10	26-Aug-10	43.0
OIS Swap	2w	19-Aug-10	2-Sep-10	42.1
OIS Swap	3w	19-Aug-10	9-Sep-10	43.2
OIS Swap	1m	19-Aug-10	19-Sep-10	44.8
OIS Swap	2m	19-Aug-10	19-Oct-10	48.0
OIS Swap	3m	19-Aug-10	19-Nov-10	51.3
OIS Swap	4m	19-Aug-10	19-Dec-10	53.2
OIS Swap	5m	19-Aug-10	19-Jan-11	55.0
OIS Swap	6m	19-Aug-10	19-Feb-11	57.2
OIS Swap	7m	19-Aug-10	19-Mar-11	58.8
OIS Swap	8m	19-Aug-10	19-Apr-11	60.2
OIS Swap	9m	19-Aug-10	19-May-11	61.8
OIS Swap	10m	19-Aug-10	19-Jun-11	63.1
OIS Swap	11m	19-Aug-10	19-Jul-11	64.0
OIS Swap	1y	19-Aug-10	19-Aug-11	65.4
OIS Swap	2y	19-Aug-10	19-Aug-12	78.2
OIS Swap	3y	19-Aug-10	19-Aug-13	98.6
OIS Swap	4y	19-Aug-10	19-Aug-14	120.7
OIS Swap	5y	19-Aug-10	19-Aug-15	142.2
OIS Swap	6y	19-Aug-10	19-Aug-16	162.1
OIS Swap	7y	19-Aug-10	19-Aug-17	179.8
OIS Swap	8y	19-Aug-10	19-Aug-18	195.3
OIS Swap	9y	19-Aug-10	19-Aug-19	208.3
OIS Swap	10y	19-Aug-10	19-Aug-20	220.0
OIS Swap	11y	19-Aug-10	19-Aug-21	230.4
OIS Swap	12y	19-Aug-10	19-Aug-22	239.3
OIS Swap	13y	19-Aug-10	19-Aug-23	246.6
OIS Swap	14y	19-Aug-10	19-Aug-24	252.9
OIS Swap	15y	19-Aug-10	19-Aug-25	258.0
OIS Swap	20y	19-Aug-10	19-Aug-30	272.0
OIS Swap	30y	19-Aug-10	19-Aug-40	262.2

Table 3B. EUR 3m Instruments Quotes

Inst Type	Term	Start	End	Rate (in bps)
ON	1d	17-Aug-10	18-Aug-10	54.0
TN	1d	18-Aug-10	19-Aug-10	54.0
CASH	1w	19-Aug-10	26-Aug-10	54.0
CASH	1m	19-Aug-10	20-Sep-10	64.2
CASH	2m	19-Aug-10	19-Oct-10	72.7
CASH	3m	19-Aug-10	19-Nov-10	89.5
CASH	6m	19-Aug-10	21-Feb-11	114.9
CASH	9m	19-Aug-10	19-May-11	129.1
CASH	12m	19-Aug-10	19-Aug-11	142.1
FUT	3m	15-Sep-10	15-Dec-10	90.5
FUT	3m	15-Dec-10	16-Mar-11	96.0
FUT	3m	16-Mar-11	15-Jun-11	100.5
FUT	3m	15-Jun-11	21-Sep-11	105.4
FUT	3m	21-Sep-11	21-Dec-11	111.9
FUT	3m	21-Dec-11	21-Mar-12	122.3
FUT	3m	21-Mar-12	20-Jun-12	130.8
FUT	3m	20-Jun-12	19-Sep-12	141.7
FUT	3m	19-Sep-12	19-Dec-12	153.1
FUT	3m	19-Dec-12	20-Mar-13	167.6
FUT	3m	20-Mar-13	19-Jun-13	179.0
FUT	3m	19-Jun-13	18-Sep-13	192.9
3m Swap	2y	19-Aug-10	19-Aug-12	112.2
3m Swap	3y	19-Aug-10	19-Aug-13	131.0
3m Swap	4y	19-Aug-10	19-Aug-14	152.3
3m Swap	5y	19-Aug-10	19-Aug-15	173.5
3m Swap	6y	19-Aug-10	19-Aug-16	193.0
3m Swap	7y	19-Aug-10	19-Aug-17	210.1
3m Swap	8y	19-Aug-10	19-Aug-18	224.9
3m Swap	9y	19-Aug-10	19-Aug-19	237.4
3m Swap	10y	19-Aug-10	19-Aug-20	248.4
3m Swap	12y	19-Aug-10	19-Aug-22	267.0
3m Swap	15y	19-Aug-10	19-Aug-25	285.1
3m Swap	20y	19-Aug-10	19-Aug-30	297.8
3m Swap	30y	19-Aug-10	19-Aug-40	286.9

<u>EUR Curve Construction Sequence:</u>

- Build the OIS discount curve from OIS swap quotes
- Build the 3m Libor curve from cash and futures instruments, independent of the OIS discount curve.
- Extend the 3m Libor curve from swap instruments, using the OIS discount curve.

6.1.1.4 GBP Markets Quotes

Table 4A. GBP OIS Instruments Quotes

Inst Type	Term	Start	End	Rate (in bps)
ON	1d	17-Aug-10	18-Aug-10	55.0
OIS Swap	1w	17-Aug-10	24-Aug-10	49.0
OIS Swap	1m	17-Aug-10	17-Sep-10	49.0
OIS Swap	2m	17-Aug-10	17-Oct-10	49.0
OIS Swap	3m	17-Aug-10	17-Nov-10	49.1
OIS Swap	4m	17-Aug-10	17-Dec-10	49.4
OIS Swap	5m	17-Aug-10	17-Jan-11	49.6
OIS Swap	6m	17-Aug-10	17-Feb-11	49.9
OIS Swap	7m	17-Aug-10	17-Mar-11	50.3
OIS Swap	8m	17-Aug-10	17-Apr-11	50.9
OIS Swap	9m	17-Aug-10	17-May-11	51.6
OIS Swap	10m	17-Aug-10	17-Jun-11	52.6
OIS Swap	11m	17-Aug-10	17-Jul-11	53.7
OIS Swap	1y	17-Aug-10	17-Aug-11	54.6
OIS Swap	2y	17-Aug-10	17-Aug-12	76.2
OIS Swap	3y	17-Aug-10	17-Aug-13	108.6
OIS Swap	4y	17-Aug-10	17-Aug-14	139.6
OIS Swap	5y	17-Aug-10	17-Aug-15	167.8
OIS Swap	6y	17-Aug-10	17-Aug-16	193.8
OIS Swap	7y	17-Aug-10	17-Aug-17	217.0
OIS Swap	8y	17-Aug-10	17-Aug-18	237.5
OIS Swap	9y	17-Aug-10	17-Aug-19	255.4
OIS Swap	10y	17-Aug-10	17-Aug-20	270.8
OIS Swap	12y	17-Aug-10	17-Aug-22	294.7
OIS Swap	15y	17-Aug-10	17-Aug-25	318.0
OIS Swap	20y	17-Aug-10	17-Aug-30	335.6
OIS Swap	25y	17-Aug-10	17-Aug-35	343.7
OIS Swap	30y	17-Aug-10	17-Aug-40	348.0

Table 4B. GBP 3m Instruments Quotes

Inst Type	Term	Start	End	Rate (in bps)
ON	1d	17-Aug-10	18-Aug-10	55.0
CASH	1w	17-Aug-10	24-Aug-10	55.0
CASH	1m	17-Aug-10	17-Sep-10	55.0
CASH	2m	17-Aug-10	18-Oct-10	56.9
CASH	3m	17-Aug-10	17-Nov-10	62.2
CASH	6m	17-Aug-10	17-Feb-11	72.9
CASH	9m	17-Aug-10	17-May-11	102.1
CASH	12m	17-Aug-10	17-Aug-11	124.8
FUT	3m	15-Sep-10	15-Dec-10	73.0
FUT	3m	15-Dec-10	16-Mar-11	79.0
FUT	3m	16-Mar-11	15-Jun-11	86.0
FUT	3m	15-Jun-11	21-Sep-11	95.9
FUT	3m	21-Sep-11	21-Dec-11	108.9
FUT	3m	21-Dec-11	21-Mar-12	125.9
FUT	3m	21-Mar-12	20-Jun-12	143.8
FUT	3m	20-Jun-12	19-Sep-12	162.7
FUT	3m	19-Sep-12	19-Dec-12	182.7
FUT	3m	19-Dec-12	20-Mar-13	203.6
FUT	3m	20-Mar-13	19-Jun-13	220.5
FUT	3m	19-Jun-13	18-Sep-13	238.5
3m Swap	2y	17-Aug-10	17-Aug-12	103.5
3m Swap	3y	17-Aug-10	17-Aug-13	135.7
3m Swap	4y	17-Aug-10	17-Aug-14	167.3
3m Swap	5y	17-Aug-10	17-Aug-15	196.4
3m Swap	6y	17-Aug-10	17-Aug-16	222.3
3m Swap	7y	17-Aug-10	17-Aug-17	245.1
3m Swap	8y	17-Aug-10	17-Aug-18	265.3
3m Swap	9y	17-Aug-10	17-Aug-19	282.2
3m Swap	10y	17-Aug-10	17-Aug-20	297.1
3m Swap	12y	17-Aug-10	17-Aug-22	320.6
3m Swap	15y	17-Aug-10	17-Aug-25	342.8
3m Swap	20y	17-Aug-10	17-Aug-30	358.9
3m Swap	30y	17-Aug-10	17-Aug-40	367.6

GBP Curve Construction Sequence:

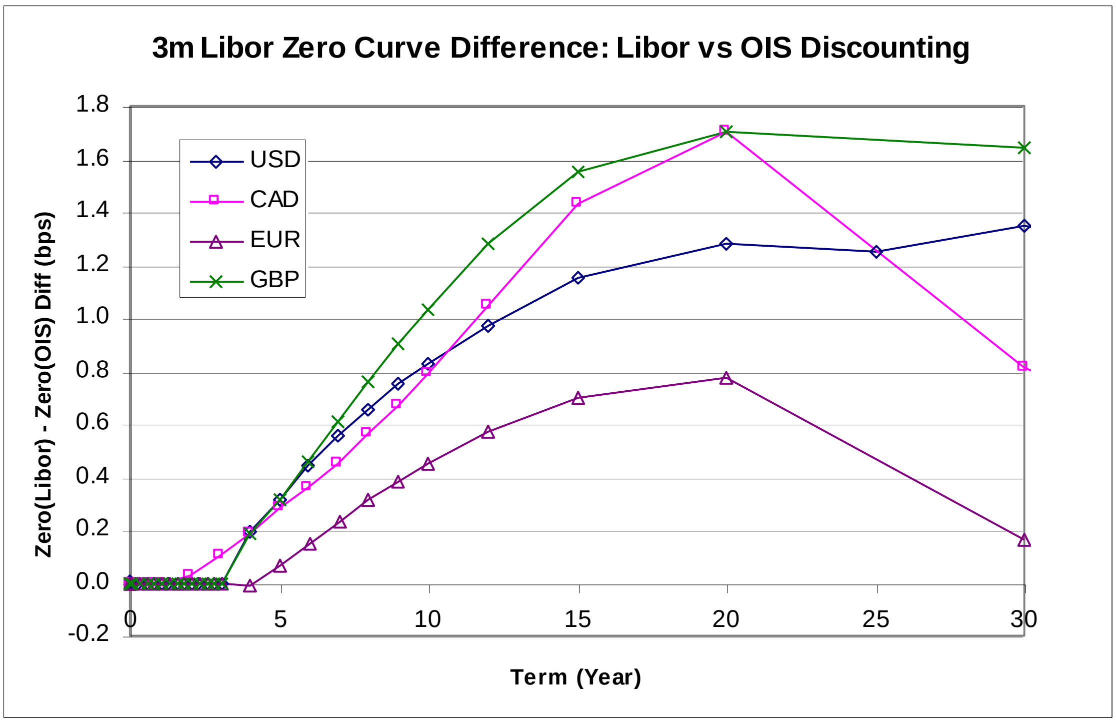
- Build the OIS discount curve from OIS swap quotes
- Build the 3m Libor curve from cash and futures instruments, independent of the OIS discount curve.
- Extend the 3m Libor curve from swap instruments, using the OIS discount curve.

6.1.2 Comparison of Zero Rates

In this section we present the curves bootstrapped from the above market quotes. Three curves described below are considered

- 3-month Libor curve under Libor discounting (forward and discount curve)
- 3-month Libor curve under OIS discounting (forward curve)
- OIS curve under OIS discounting (discount curve)

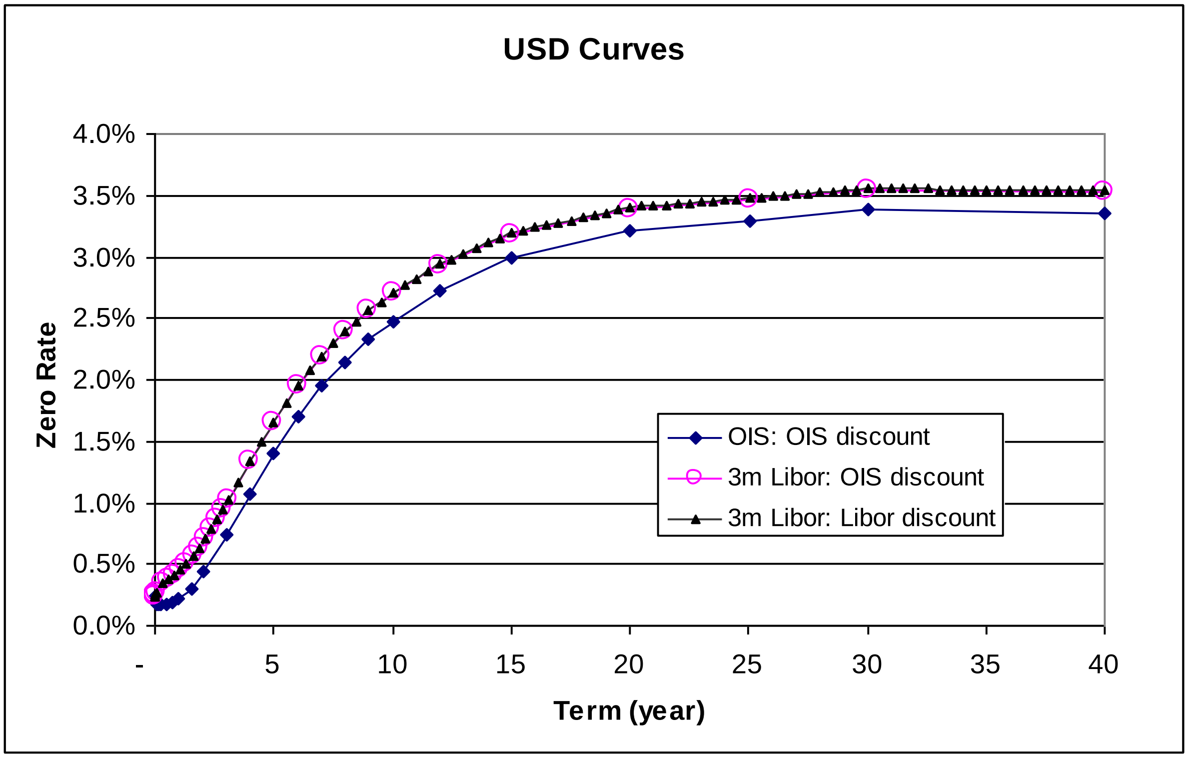
We found that the difference on the Libor curve between our current Libor discounting and the new OIS discounting is very small. There is little difference for shorter terms and under 2bp difference for longer terms, as shown in the figure below.

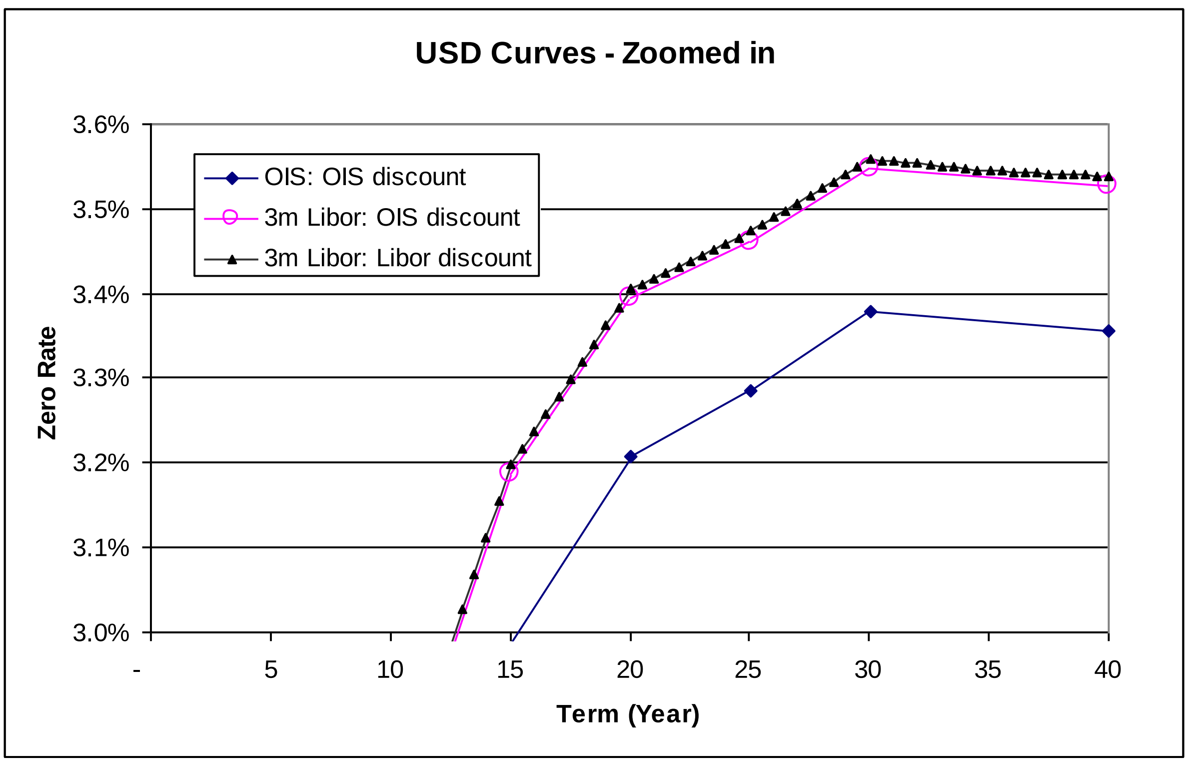


Note: In USD, EUR and GBP, the quoted OIS/3m spreads all monotonically decrease after year 5. The market only quotes the CAD OIS till 5-year. The CAD OIS quotes beyond 5-year were extrapolated using the shape of USD OIS/3m spread connecting to the CAD OIS/3m spread at year 5.

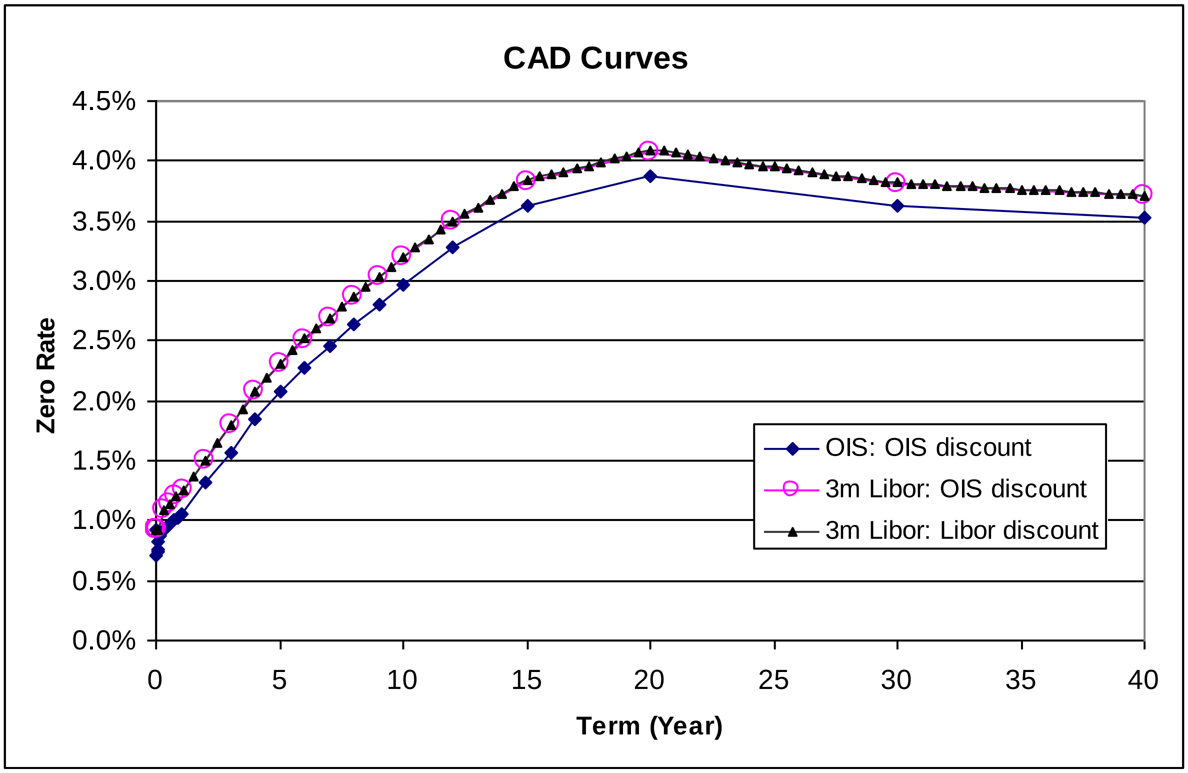
For each currency, we presented two graphs. The first one shows the above mentioned three curves. The line labeled LiborNew is the 3-month Libor curve built under the OIS discounting and the one labeled LiborOld is the 3-month Libor curve built under the current Libor discounting. The second graph is a zoomed-in version of the first one on a specific region to show the small difference between the new Libor curve and the old Libor curve.

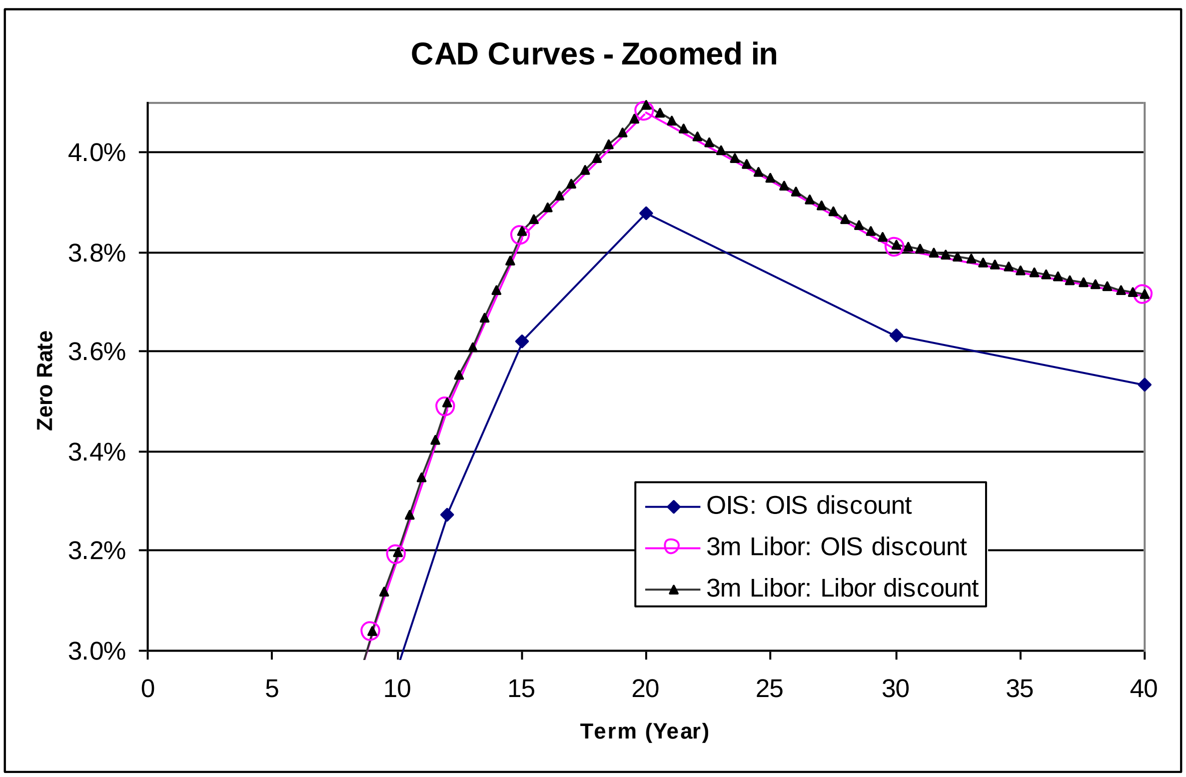
6.1.2.1 USD Curves – Comparison of Zero Rates



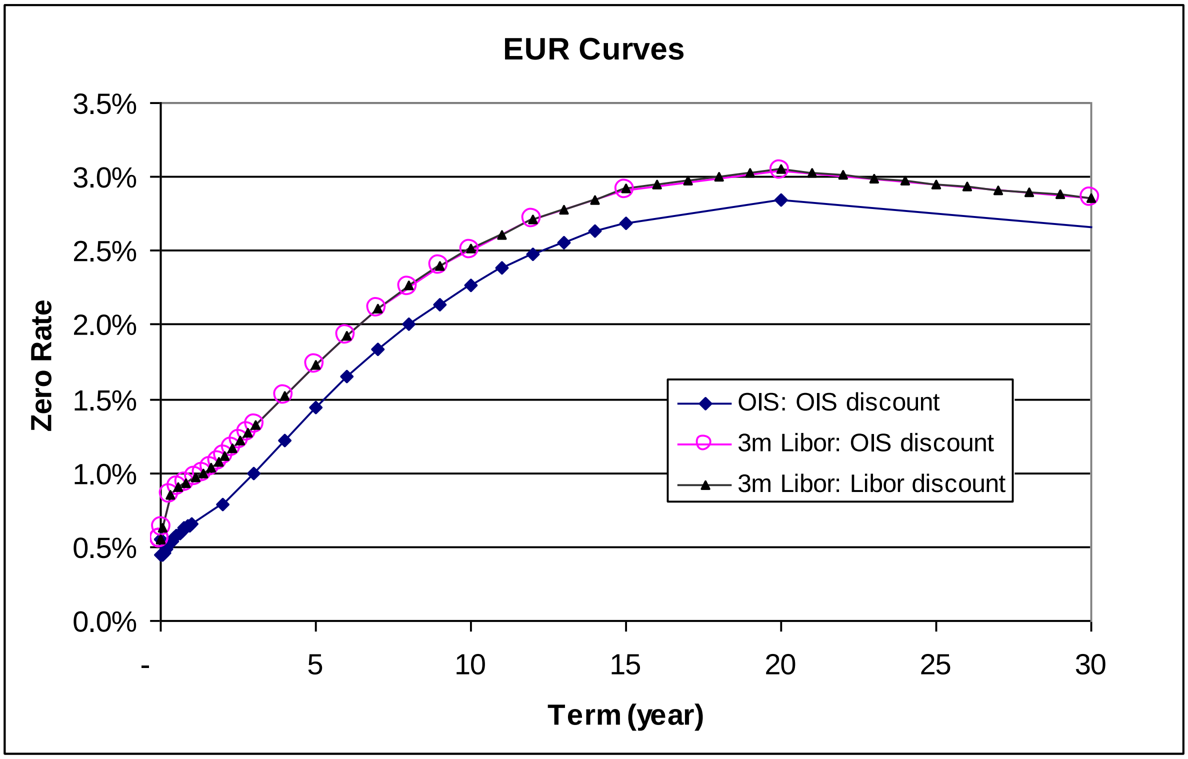


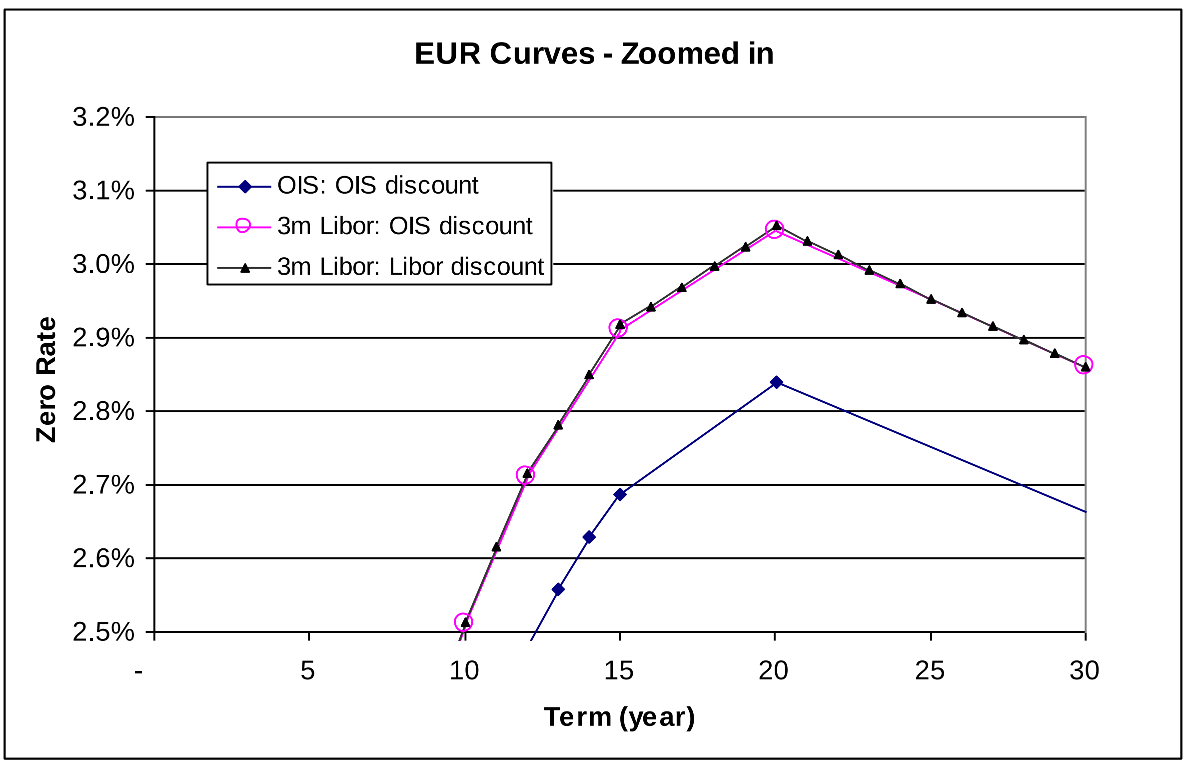
6.1.2.2 CAD Curves – Comparison of Zero Rates



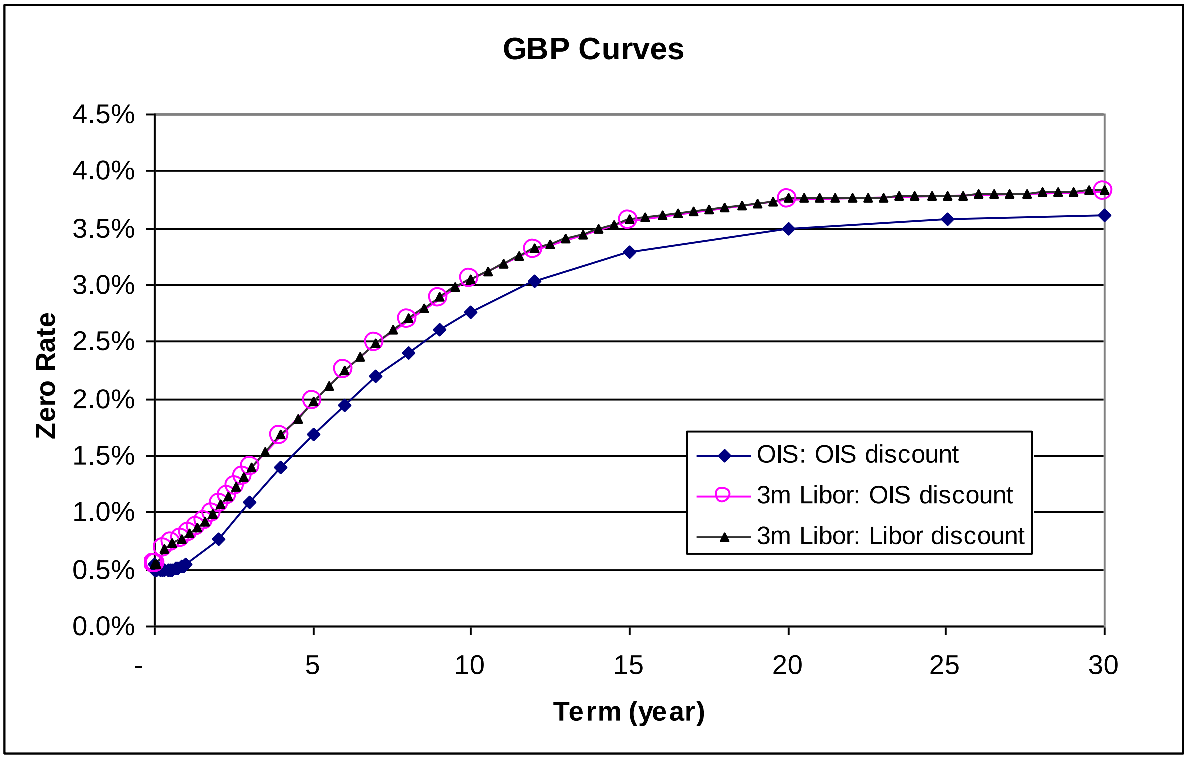


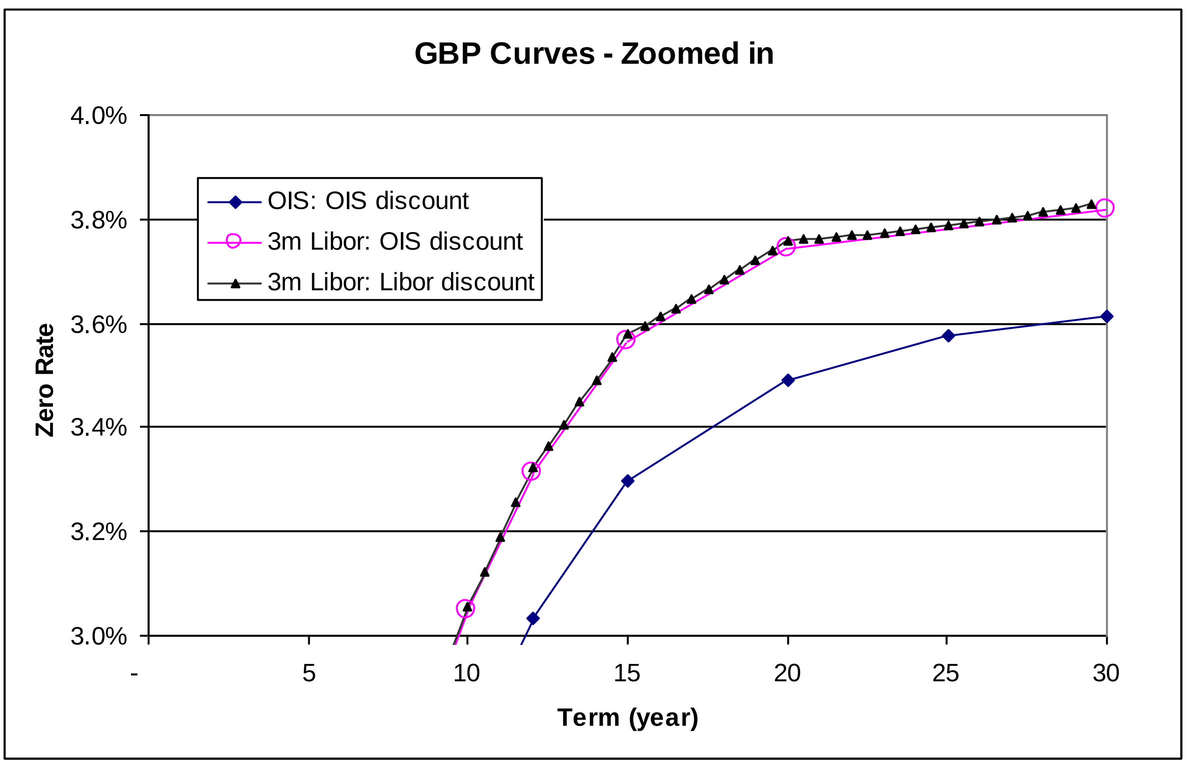
6.1.2.3 EUR Curves – Comparison of Zero Rates





6.1.2.4 GBP Curves – Comparison of Zero Rates





6.1.3 Comparison of Swap Pricing under Libor & OIS discounting

For each currency, we consider the 3-month tenor fixed-float interest rate swaps with 1, 5, 10, 20 and 30-year terms. By design, all at-the-money swaps have zero value whether under the OIS discounting or the current Libor discounting. However, there will be valuation difference to non-ATM swaps under the different discounting rules.

The valuation impact is first presented in terms of PV01, which is defined as the present value of 1bp coupon on 10,000 notional. The PV01 difference represents the valuation impacts to non ATM swaps. Consider a USD 10-year swap with notional 100,000,000 and a fixed rate 10 bps above the ATM rate. Its present value difference between the OIS discounting and Libor discounting is

$$[PV01(OIS) – PV01(Libor)] \times 10 \times 100{,}000{,}000/10{,}000$$

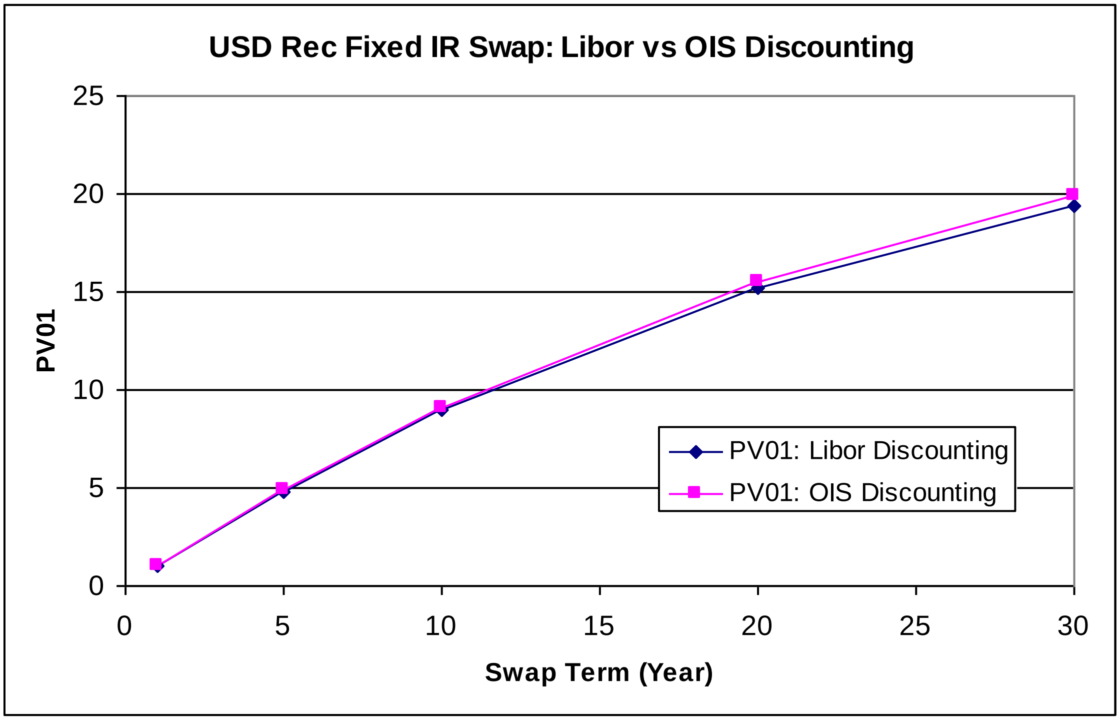
$$= 0.113 \times 10 \times 1{,}000{,}000/10{,}000 = 11{,}300$$

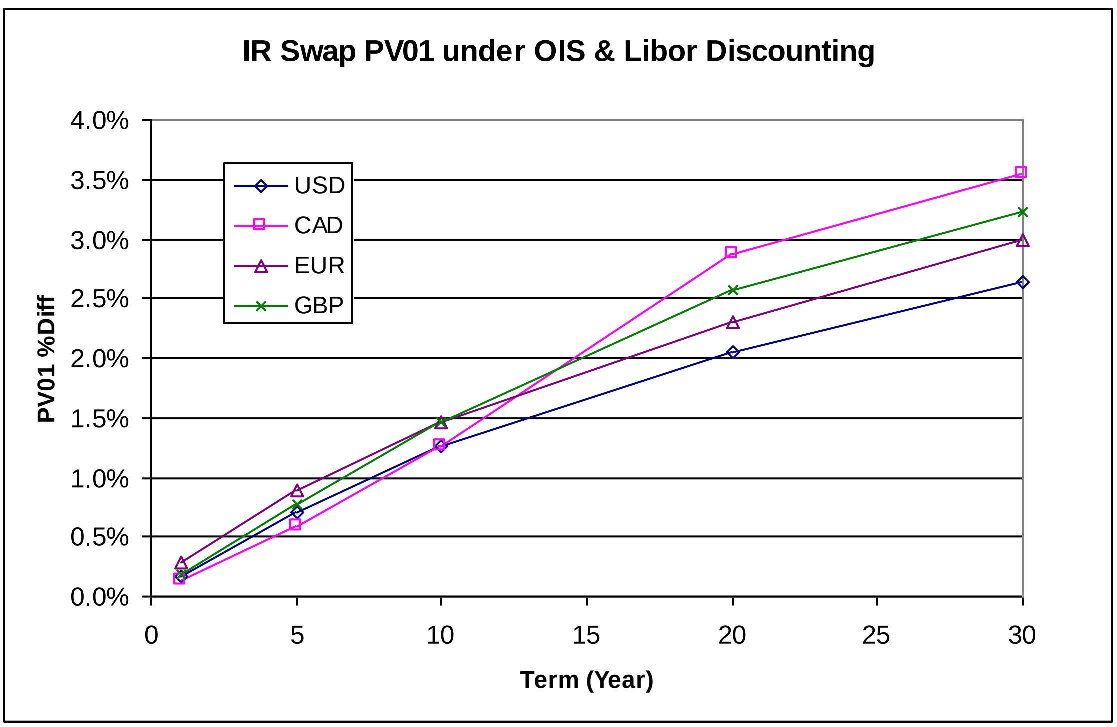
or 1.3% of the present value under Libor discounting. The PV01 under Libor discounting and OIS discounting are presented in Table 5 below for various currencies and terms. The PV01s of USD and PV01 differences for various currencies are graphed in the figures below.

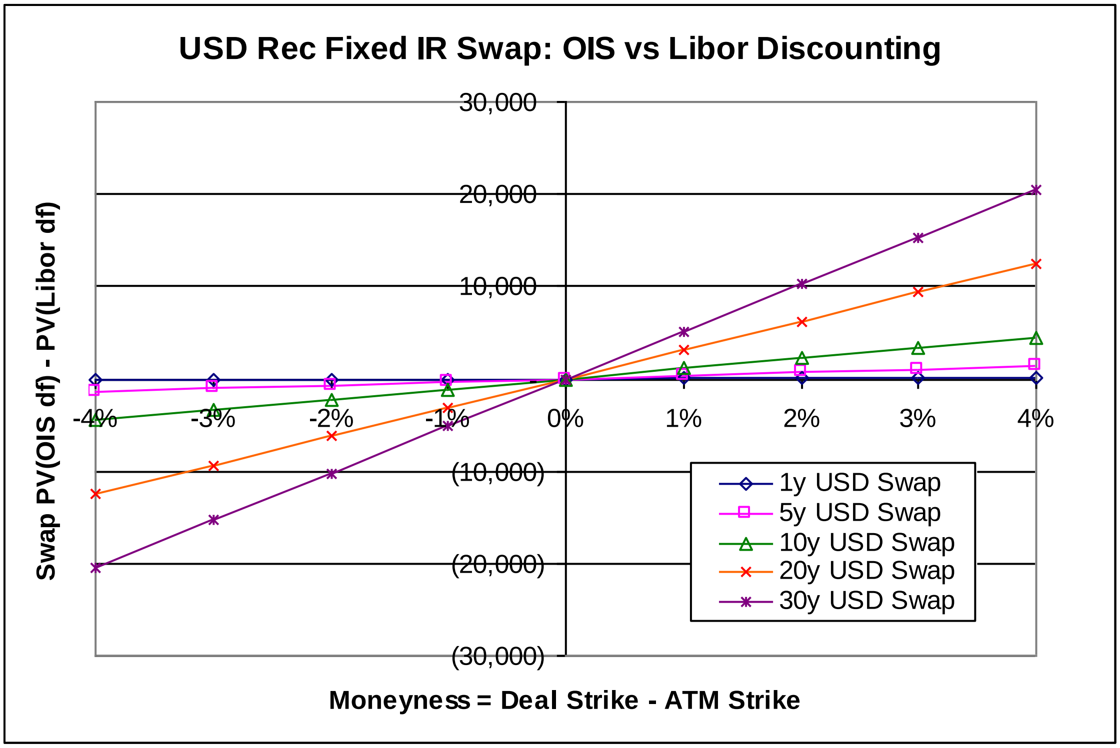
The valuation impact is generally small for short term swaps (below 1%). However, it could be as high as 3.6% for longer term swaps. This is mainly due to the steep upward slope observed in the zero rates as seen in the previous section. The absolute dollar impact is presented in the last four figures for received-fixed swaps at -4% to 4% moneyness in various currencies (1,000,000 notional).

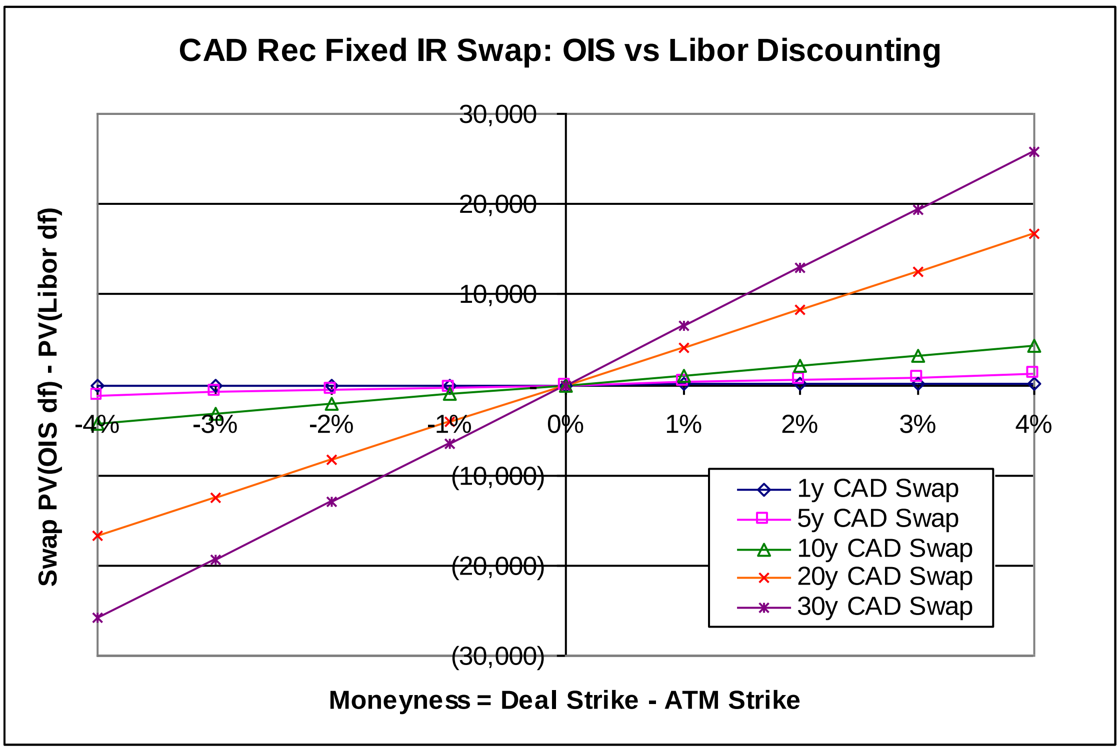
Table 5. PV01 comparison - Libor discounting vs OIS discounting (on 10,000 notional)

CCY	Swap Term	PV01 (Libor)	PV01 (OIS)	Diff	Diff in %
USD	1	0.997	0.999	0.002	0.2%
	5	4.842	4.876	0.034	0.7%
	10	9.010	9.124	0.113	1.3%
	20	15.200	15.511	0.311	2.0%
	30	19.370	19.881	0.511	2.6%
CAD	1	0.996	0.998	0.001	0.1%
	5	4.753	4.781	0.028	0.6%
	10	8.767	8.877	0.110	1.3%
	20	14.409	14.824	0.415	2.9%
	30	18.128	18.772	0.644	3.6%
EUR	1	0.996	0.999	0.003	0.3%
	5	4.794	4.836	0.043	0.9%
	10	8.964	9.095	0.131	1.5%
	20	15.363	15.716	0.353	2.3%
	30	20.099	20.701	0.602	3.0%
GBP	1	1.000	1.001	0.002	0.2%
	5	4.805	4.842	0.037	0.8%
	10	8.877	9.007	0.130	1.5%
	20	14.737	15.115	0.378	2.6%
	30	18.599	19.199	0.600	3.2%

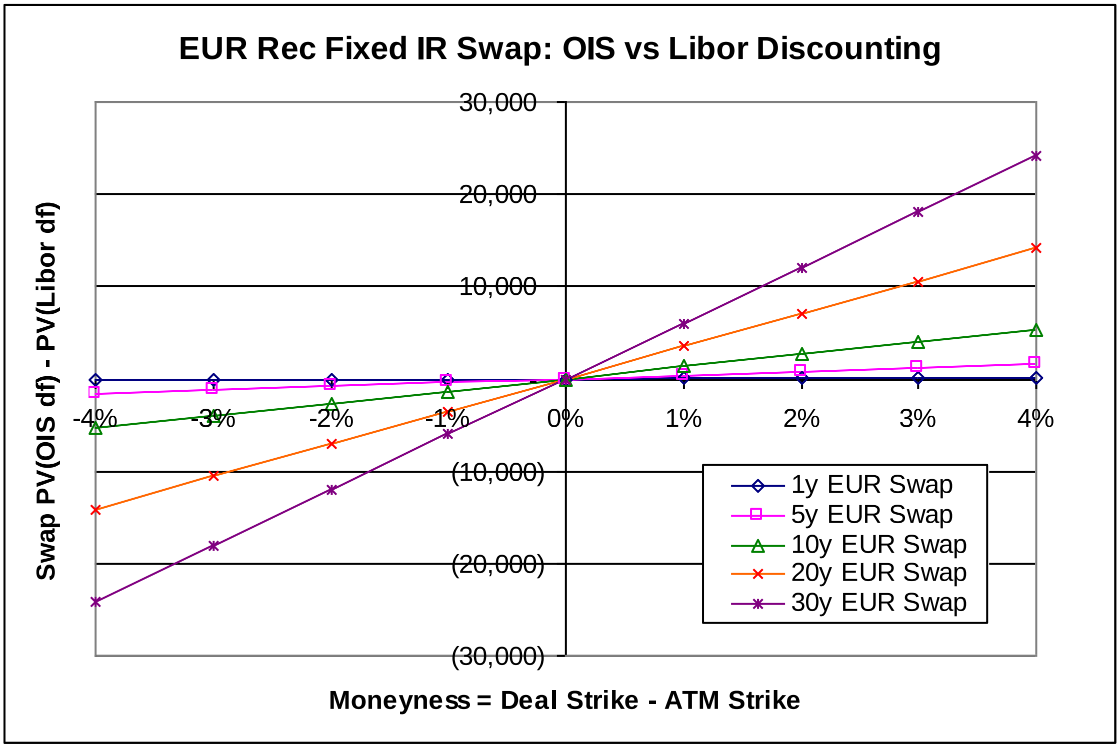

USD Rec Fixed IR Swap: Libor vs OIS Discounting
PV01
Swap Term (Year)
PV01: Libor Discounting
PV01: OIS Discounting
0
5
10
15
20
25
30

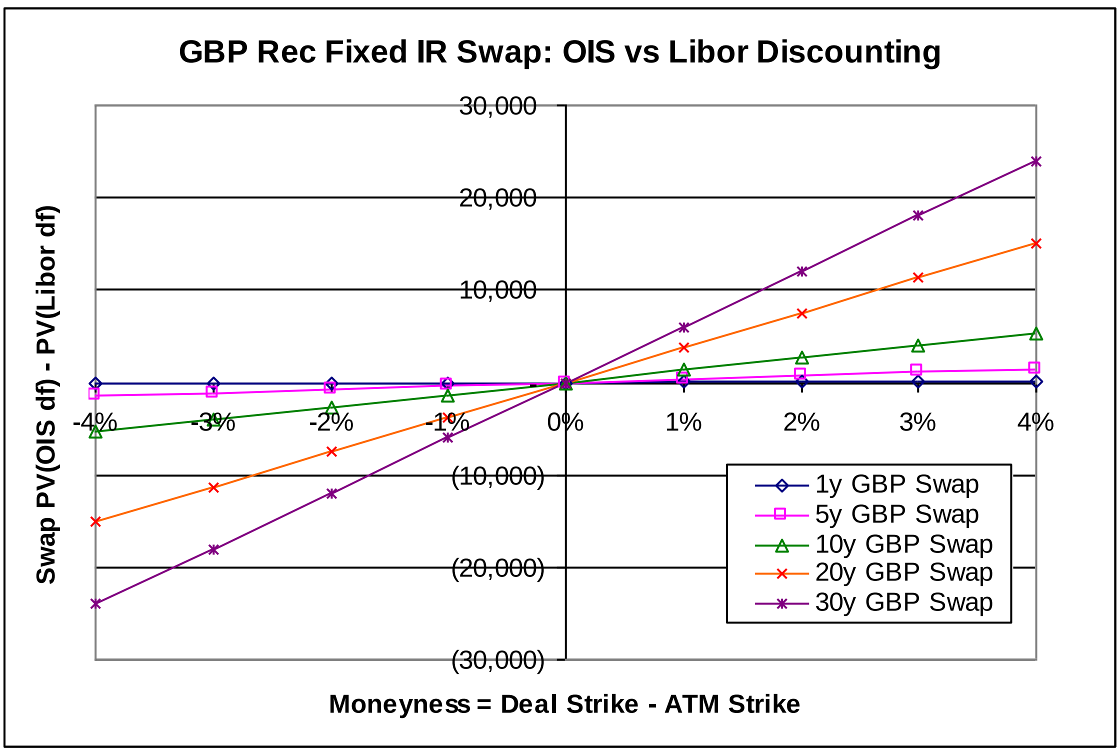

IR Swap PV01 under OIS & Libor Discounting
PV01 %Diff
Term (Year)
USD
CAD
EUR
GBP
0.0%
0.5%
1.0%
1.5%
2.0%
2.5%
3.0%
3.5%
4.0%





Note: The swap notional is 1,000,000 in the above two figures.





Note: The swap notional is 1,000,000 in the above two figures.

6.2 Impact Test 2 : 1m and 6m Libor Curves in the Collateral Currency

Previously, we have presented OIS and 3m Libor curves in CAD, USD, EUR and GBP, as well as the impact on the fixed-floating interest rate swaps. In this section, we consider tenor basis swaps such as 1m/3m and 3m/6m basis swaps. As we already bootstrapped the 3m Libor curves, we will use them together with the OIS discount curves to derive the 1m and 6m Libor curves using the basis swap instruments.

As the impact of OIS discounting on the 1m and 6m Libor curves for all the currencies are very similar, we will only present the USD curves here as an illustration.

6.2.1 Input Market Quotes for 1m & 6m Libor

We first present the market quotes used for the 1m and 6m Libor curves bootstrapping.

Table 6A. USD 1m/3m Basis Swaps Quotes

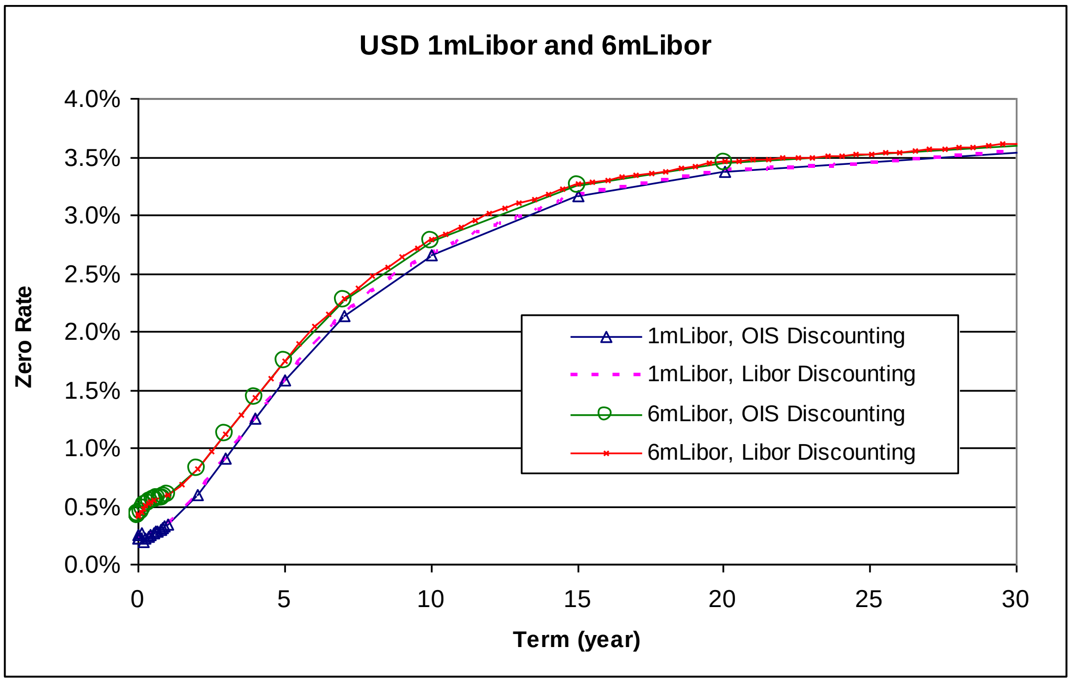
Inst Type	Term	Start	End	Rate or Spread (in bps)
ON	0	17-Aug-10	18-Aug-10	22.66
TN	0	18-Aug-10	19-Aug-10	22.66
1m/3m swap	1w	19-Aug-10	26-Aug-10	-
1m/3m swap	1m	19-Aug-10	20-Sep-10	-
1m/3m swap	2m	19-Aug-10	19-Oct-10	(10.00)
1m/3m swap	3m	19-Aug-10	19-Nov-10	(10.00)
1m/3m swap	4m	19-Aug-10	19-Dec-10	(10.00)
1m/3m swap	5m	19-Aug-10	19-Jan-11	(10.00)
1m/3m swap	6m	19-Aug-10	19-Feb-11	(10.00)
1m/3m swap	7m	19-Aug-10	19-Mar-11	(10.00)
1m/3m swap	8m	19-Aug-10	19-Apr-11	(10.00)
1m/3m swap	9m	19-Aug-10	19-May-11	(10.00)
1m/3m swap	10m	19-Aug-10	19-Jun-11	(10.00)
1m/3m swap	11m	19-Aug-10	19-Jul-11	(10.00)
1m/3m swap	1y	19-Aug-10	19-Aug-11	(10.00)
1m/3m swap	2y	19-Aug-10	19-Aug-12	(9.25)
1m/3m swap	3y	19-Aug-10	19-Aug-13	(8.75)
1m/3m swap	4y	19-Aug-10	19-Aug-14	(8.00)
1m/3m swap	5y	19-Aug-10	19-Aug-15	(7.50)
1m/3m swap	7y	19-Aug-10	19-Aug-17	(6.00)
1m/3m swap	10y	19-Aug-10	19-Aug-20	(5.00)
1m/3m swap	15y	19-Aug-10	19-Aug-25	(3.50)
1m/3m swap	20y	19-Aug-10	19-Aug-30	(3.00)
1m/3m swap	30y	19-Aug-10	19-Aug-40	(2.00)

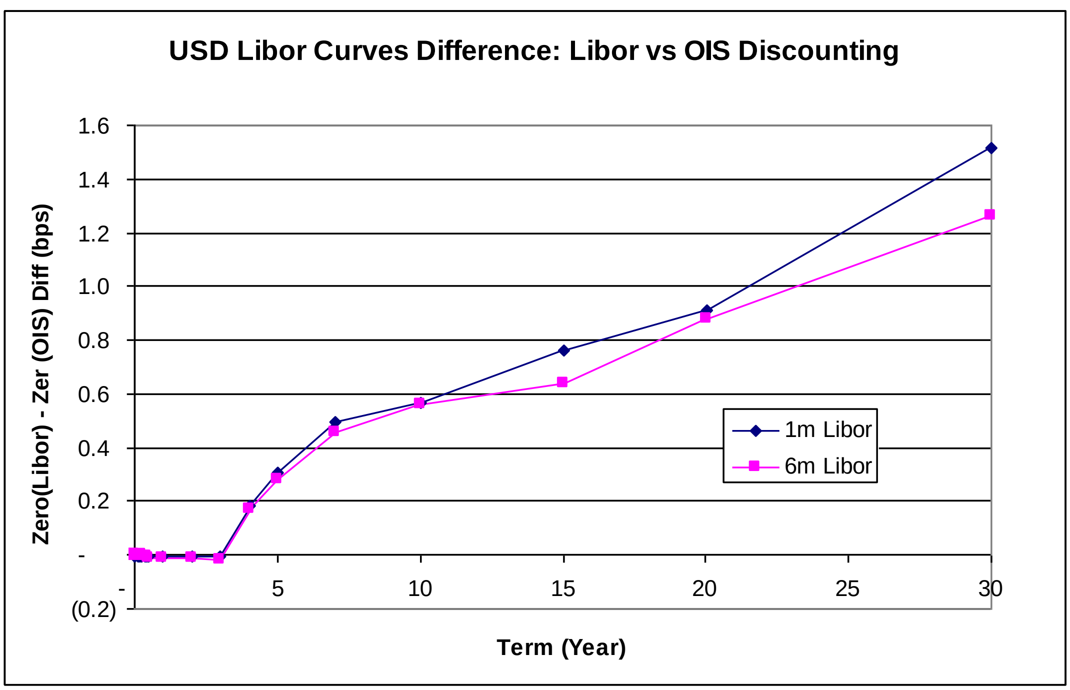
Table 6B. USD 3m/6m Basis Swaps Quotes

Inst Type	Term	Start	End	Rate or Spread (in bps)
ON	0	17-Aug-10	18-Aug-10	40.66
TN	0	18-Aug-10	19-Aug-10	40.66
3m/6m swap	1w	19-Aug-10	26-Aug-10	18.00
3m/6m swap	1m	19-Aug-10	20-Sep-10	18.00
3m/6m swap	2m	19-Aug-10	19-Oct-10	18.00
3m/6m swap	3m	19-Aug-10	19-Nov-10	18.00
3m/6m swap	4m	19-Aug-10	20-Dec-10	18.00
3m/6m swap	5m	19-Aug-10	19-Jan-11	18.00
3m/6m swap	6m	19-Aug-10	22-Feb-11	18.00
3m/6m swap	7m	19-Aug-10	19-Mar-11	17.50
3m/6m swap	8m	19-Aug-10	19-Apr-11	17.00
3m/6m swap	9m	19-Aug-10	19-May-11	16.50
3m/6m swap	10m	19-Aug-10	19-Jun-11	16.00
3m/6m swap	11m	19-Aug-10	19-Jul-11	15.50
3m/6m swap	1y	19-Aug-10	19-Aug-11	15.00
3m/6m swap	2y	19-Aug-10	19-Aug-12	13.25
3m/6m swap	3y	19-Aug-10	19-Aug-13	11.50
3m/6m swap	4y	19-Aug-10	19-Aug-14	10.25
3m/6m swap	5y	19-Aug-10	19-Aug-15	9.25
3m/6m swap	7y	19-Aug-10	19-Aug-17	8.25
3m/6m swap	10y	19-Aug-10	19-Aug-20	7.75
3m/6m swap	15y	19-Aug-10	19-Aug-25	7.00
3m/6m swap	20y	19-Aug-10	19-Aug-30	6.25
3m/6m swap	30y	19-Aug-10	19-Aug-40	6.00

6.2.2 Comparison of Zero Rates

In this section we present the 1m Libor and 6m Libor curves bootstrapped from the above market quotes. As we observed before for the 3m Libor curve, the impact on the 1m and 6m Libor curves between our current Libor discounting and the new OIS discounting is very small. There is little difference for shorter terms and under 2bp difference for longer terms, as shown in the figure below.





6.2.3 Comparison of Basis Swap Pricing under Libor and OIS discounting

As tenor basis swaps are float vs float with some spread applied to one side of the swap, the risk factor is the spread variation. If today's spread is the same as the one at deal inception, there should have no valuation impact from the choice of the curves set due to calibration. However, if the moneyness changed after the deal inception, the choice of the curves set will result valuation impact which should be the same (if the spread is on the 3m side) or very close to (if the spread is on the 1m or 6m side) the impact we observed in the 3m swap case (Please see Table 5 in the previous section). Since the spread is typically small, the resulting absolute dollar amount impact should be very small as well (it should have a similar percentage difference as the numbers in Table 5 in the previous section).

7 Model Limitations

Below is a list of model limitations:

- The end of a yield curve is defined by the location of the last curve node. Extrapolation beyond the curve end date is not reliable. Extrapolation can always be avoided by adding a curve instrument with a longer maturity. When building a curve with one or more dependency curves, the reliable length of the newly build curve is also driven by the reliable lengths of its dependency curves.

8 References

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[10] McCloud, P. (2013): "Collateral Convexity Complexity", Risk Magazine, May 2013, page 60-64.

Appendix A. Sample RiskWatch Input & Output

8.1 Instrument Curve – USD 3m Libor

Name	sUSDMid.Details	State Procedure		Time Evolution	@Constant
ID	SampleCurves:sUSDMid.Details	Procedure Parameter		Interpolate Surface	
Type	Matrix Parameter Curve	Curve Function		Interpolate Axis 1	@None()
Datum	1996/04/01	Function Parameters		Interpolate Axis 2	@None()
Relative Curve	False	Function Identity	True	Extrapolate Axis 1	True
Origin Offset	0	Curve Unit	% CONT actual/365	Extrapolate Axis 2	True
		RiskMetrics Link	None	Default Interpolation Order	True

Axis 1/Axis 2

	0.0000	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000	10.0
0.0000	15.0000	64.0000	64.0000	2.0000	2.0000	3.0000	7.0000	7.0000	1,002.0000	0.0000	0.0
1.0000	1.0000	0.0000	41,668.0000	41,758.0000	1.0000	0.0024	1,000,102.0000	0.0000	0.0000	0.0000	0.0
2.0000	2.0000	0.0000	41,717.0000	41,809.0000	1.0000	0.0025	1,000,102.0000	0.0000	0.0000	0.0000	0.0
3.0000	2.0000	0.0000	41,808.0000	41,900.0000	1.0000	0.0029	1,000,102.0000	0.0000	0.0000	0.0000	0.0
4.0000	2.0000	0.0000	41,899.0000	41,990.0000	1.0000	0.0033	1,000,102.0000	0.0000	0.0000	0.0000	0.0
5.0000	2.0000	0.0000	41,990.0000	42,080.0000	1.0000	0.0040	1,000,102.0000	0.0000	0.0000	0.0000	0.0
6.0000	2.0000	0.0000	42,081.0000	42,173.0000	1.0000	0.0052	1,000,102.0000	0.0000	0.0000	0.0000	0.0
7.0000	2.0000	0.0000	42,172.0000	42,264.0000	1.0000	0.0067	1,000,102.0000	0.0000	0.0000	0.0000	0.0
8.0000	2.0000	0.0000	42,263.0000	42,354.0000	1.0000	0.0086	1,000,102.0000	0.0000	0.0000	0.0000	0.0
9.0000	2.0000	0.0000	42,354.0000	42,445.0000	1.0000	0.0109	1,000,102.0000	0.0000	0.0000	0.0000	0.0
10.0000	2.0000	0.0000	42,445.0000	42,537.0000	1.0000	0.0136	1,000,102.0000	0.0000	0.0000	0.0000	0.0
11.0000	2.0000	0.0000	42,536.0000	42,628.0000	1.0000	0.0163	1,000,102.0000	0.0000	0.0000	0.0000	0.0
12.0000	2.0000	0.0000	42,634.0000	42,725.0000	1.0000	0.0190	1,000,102.0000	0.0000	0.0000	0.0000	0.0
13.0000	2.0000	0.0000	42,725.0000	42,815.0000	1.0000	0.0216	1,000,102.0000	0.0000	0.0000	0.0000	0.0
14.0000	4.0000	2.0000	41,668.0000	42,398.0000	1.0000	0.0050	1,064,001.0000	0.0015	0.0000	0.0000	0.0
15.0000	4.0000	3.0000	41,668.0000	42,764.0000	1.0000	0.0088	1,064,001.0000	0.0013	0.0000	0.0000	0.0
16.0000	4.0000	4.0000	41,668.0000	43,129.0000	1.0000	0.0131	1,064,001.0000	0.0014	0.0000	0.0000	0.0
17.0000	4.0000	5.0000	41,668.0000	43,494.0000	1.0000	0.0171	1,064,001.0000	0.0012	0.0000	0.0000	0.0
18.0000	4.0000	6.0000	41,668.0000	43,859.0000	1.0000	0.0205	1,064,001.0000	0.0014	0.0000	0.0000	0.0
19.0000	4.0000	7.0000	41,668.0000	44,225.0000	1.0000	0.0233	1,064,001.0000	0.0010	0.0000	0.0000	0.0
20.0000	4.0000	8.0000	41,668.0000	44,590.0000	1.0000	0.0255	1,064,001.0000	0.0015	0.0000	0.0000	0.0
21.0000	4.0000	9.0000	41,668.0000	44,955.0000	1.0000	0.0273	1,064,001.0000	0.0015	0.0000	0.0000	0.0
22.0000	4.0000	10.0000	41,668.0000	45,320.0000	1.0000	0.0289	1,064,001.0000	0.0012	0.0000	0.0000	0.0
23.0000	4.0000	12.0000	41,668.0000	46,051.0000	1.0000	0.0313	1,064,001.0000	0.0027	0.0000	0.0000	0.0
24.0000	4.0000	15.0000	41,668.0000	47,147.0000	1.0000	0.0335	1,064,001.0000	0.0036	0.0000	0.0000	0.0
25.0000	4.0000	20.0000	41,668.0000	48,973.0000	1.0000	0.0354	1,064,001.0000	0.0032	0.0000	0.0000	0.0
26.0000	4.0000	30.0000	41,668.0000	52,625.0000	1.0000	0.0366	1,064,001.0000	-0.0001	0.0000	0.0000	0.0
27.0000	4.0000	40.0000	41,668.0000	56,278.0000	1.0000	0.0368	1,064,001.0000	0.0000	0.0000	0.0000	0.0

8.2 Instrument Curve – USD OIS

Name	USDsUSD1dMid.Details	State Procedure		Time Evolution	@Constant
ID	SampleCurves:USDsUSD1dMid.De	Procedure Parameter		Interpolate Surface	
Type	Matrix Parameter Curve	Curve Function		Interpolate Axis 1	@None()
Datum	1996/04/01	Function Parameters		Interpolate Axis 2	@None()
Relative Curve	False	Function Identity	True	Extrapolate Axis 1	True
Origin Offset	0	Curve Unit	% CONT actual/365	Extrapolate Axis 2	True
		RiskMetrics Link	None	Default Interpolation Order	True

Axis 1/Axis 2

	0.0000	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000	10.0
0.0000	391.0000	64.0000	64.0000	2.0000	2.0000	3.0000	7.0000	7.0000	1,001.0000	0.0000	0.00
1.0000	1.0000	0.0000	41,666.0000	41,667.0000	2.0000	-0.0015	1,000,102.0000	0.0000	0.0000	0.0000	0.00
2.0000	4.0000	0.0000	41,668.0000	41,758.0000	2.0000	-0.0015	1,064,002.0000	0.0000	0.0000	0.0000	0.00
3.0000	4.0000	0.0000	41,668.0000	41,849.0000	2.0000	-0.0016	1,064,002.0000	0.0000	0.0000	0.0000	0.00
4.0000	4.0000	0.0000	41,668.0000	41,941.0000	2.0000	-0.0017	1,064,002.0000	0.0000	0.0000	0.0000	0.00
5.0000	4.0000	1.0000	41,668.0000	42,033.0000	2.0000	-0.0018	1,064,002.0000	0.0000	0.0000	0.0000	0.00
6.0000	4.0000	0.0000	41,668.0000	42,123.0000	2.0000	-0.0019	1,064,002.0000	0.0000	0.0000	0.0000	0.00
7.0000	4.0000	0.0000	41,668.0000	42,214.0000	2.0000	-0.0019	1,064,002.0000	0.0000	0.0000	0.0000	0.00
8.0000	4.0000	2.0000	41,668.0000	42,398.0000	2.0000	0.0021	1,064,011.0000	0.0000	0.0000	0.0000	0.00
9.0000	4.0000	3.0000	41,668.0000	42,764.0000	2.0000	0.0023	1,064,011.0000	0.0000	0.0000	0.0000	0.00
10.0000	4.0000	4.0000	41,668.0000	43,129.0000	2.0000	0.0025	1,064,011.0000	0.0000	0.0000	0.0000	0.00
11.0000	4.0000	5.0000	41,668.0000	43,494.0000	2.0000	0.0026	1,064,011.0000	0.0000	0.0000	0.0000	0.00
12.0000	4.0000	6.0000	41,668.0000	43,859.0000	2.0000	0.0027	1,064,011.0000	0.0000	0.0000	0.0000	0.00
13.0000	4.0000	7.0000	41,668.0000	44,225.0000	2.0000	0.0027	1,064,011.0000	0.0000	0.0000	0.0000	0.00
14.0000	4.0000	8.0000	41,668.0000	44,590.0000	2.0000	0.0028	1,064,011.0000	0.0000	0.0000	0.0000	0.00
15.0000	4.0000	9.0000	41,668.0000	44,955.0000	2.0000	0.0028	1,064,011.0000	0.0000	0.0000	0.0000	0.00
16.0000	4.0000	10.0000	41,668.0000	45,320.0000	2.0000	0.0028	1,064,011.0000	0.0000	0.0000	0.0000	0.00
17.0000	4.0000	12.0000	41,668.0000	46,051.0000	2.0000	0.0028	1,064,011.0000	0.0000	0.0000	0.0000	0.00
18.0000	4.0000	15.0000	41,668.0000	47,147.0000	2.0000	0.0028	1,064,011.0000	0.0000	0.0000	0.0000	0.00
19.0000	4.0000	20.0000	41,668.0000	48,973.0000	2.0000	0.0028	1,064,011.0000	0.0000	0.0000	0.0000	0.00
20.0000	4.0000	30.0000	41,668.0000	52,625.0000	2.0000	0.0027	1,064,011.0000	0.0000	0.0000	0.0000	0.00
21.0000	4.0000	40.0000	41,668.0000	56,278.0000	2.0000	0.0027	1,064,011.0000	0.0000	0.0000	0.0000	0.00

8.3 Instrument Curve – USD 1m Libor

Name	sUSD1mMid.Details	State Procedure		Time Evolution	@Constant
ID	SampleCurves:sUSDx1mMid.Details	Procedure Parameter		Interpolate Surface	
Type	Matrix Parameter Curve	Curve Function		Interpolate Axis 1	@None()
Datum	1996/04/01	Function Parameters		Interpolate Axis 2	@None()
Relative Curve	False	Function Identity	True	Extrapolate Axis 1	True
Origin Offset	0	Curve Unit	% CONT actual/365	Extrapolate Axis 2	True
		RiskMetrics Link	None	Default Interpolation Order	True

Axis 1/Axis 2

	0.0000	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000	10.0000
0.0000	391.0000	64.0000	64.0000	2.0000	2.0000	3.0000	7.0000	7.0000	1.0000	0.0000	0.0000
1.0000	1.0000	0.0000	40,407.0000	40,408.0000	0.0000	0.0023	1,000,102.0000	0.0000	0.0000	0.0000	0.0000
2.0000	1.0000	0.0000	40,408.0000	40,409.0000	0.0000	0.0023	1,000,102.0000	0.0000	0.0000	0.0000	0.0000
3.0000	4.0000	0.0000	40,409.0000	40,416.0000	0.0000	0.0000	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
4.0000	4.0000	0.0000	40,409.0000	40,441.0000	0.0000	0.0000	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
5.0000	4.0000	0.0000	40,409.0000	40,470.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
6.0000	4.0000	0.0000	40,409.0000	40,501.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
7.0000	4.0000	0.0000	40,409.0000	40,531.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
8.0000	4.0000	0.0000	40,409.0000	40,562.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
9.0000	4.0000	0.0000	40,409.0000	40,593.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
10.0000	4.0000	0.0000	40,409.0000	40,621.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
11.0000	4.0000	0.0000	40,409.0000	40,652.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
12.0000	4.0000	0.0000	40,409.0000	40,682.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
13.0000	4.0000	0.0000	40,409.0000	40,713.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
14.0000	4.0000	0.0000	40,409.0000	40,743.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
15.0000	4.0000	1.0000	40,409.0000	40,774.0000	0.0000	0.0010	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
16.0000	4.0000	2.0000	40,409.0000	41,140.0000	0.0000	0.0009	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
17.0000	4.0000	3.0000	40,409.0000	41,505.0000	0.0000	0.0009	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
18.0000	4.0000	4.0000	40,409.0000	41,870.0000	0.0000	0.0008	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
19.0000	4.0000	5.0000	40,409.0000	42,235.0000	1.0000	0.0157	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
20.0000	4.0000	7.0000	40,409.0000	42,966.0000	0.0000	0.0006	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
21.0000	4.0000	10.0000	40,409.0000	44,062.0000	0.0000	0.0005	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
22.0000	4.0000	15.0000	40,409.0000	45,888.0000	0.0000	0.0003	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
23.0000	4.0000	20.0000	40,409.0000	47,714.0000	0.0000	0.0003	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
24.0000	4.0000	30.0000	40,409.0000	51,367.0000	0.0000	0.0002	1,064,004.0000	0.0000	0.0000	0.0000	0.0000
25.0000	4.0000	40.0000	40,409.0000	55,019.0000	0.0000	0.0002	1,064,004.0000	0.0000	0.0000	0.0000	0.0000

8.4 Instrument Curve – CAD/USD Cross Currency

Name	sCADUSD1dMid.Details	State Procedure		Time Evolution	@Constant
ID	SampleCurves:sCADUSDx1dMid.Details	Procedure Parameter		Interpolate Surface	
Type	Matrix Parameter Curve	Curve Function		Interpolate Axis 1	@None()
Datum	1996/04/01	Function Parameters		Interpolate Axis 2	@None()
Relative Curve	False	Function Identity	True	Extrapolate Axis 1	True
Origin Offset	0	Curve Unit	% CONT actual/365	Extrapolate Axis 2	True
		RiskMetrics Link	None	Default Interpolation Order	True

Axis 1/Axis 2

	0.0000	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000	10.0000
0.0000	391.0000	64.0000	63.0000	2.0000	0.0000	3.0000	7.0000	7.0000	1.0000	0.0000	0.0000
1.0000	1.0000	0.0000	40,407.0000	40,409.0000	0.0000	0.0027	1,000,103.0000	0.0000	0.0000	0.0000	0.0000
2.0000	1.0000	0.0000	40,409.0000	40,440.0000	0.0000	0.0078	1,000,103.0000	0.0000	0.0000	0.0000	0.0000
3.0000	1.0000	0.0000	40,409.0000	40,471.0000	0.0000	0.0088	1,000,103.0000	0.0000	0.0000	0.0000	0.0000
4.0000	4.0000	0.0000	40,409.0000	40,501.0000	0.0000	-0.0007	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
5.0000	4.0000	0.0000	40,409.0000	40,593.0000	0.0000	-0.0002	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
6.0000	4.0000	0.0000	40,409.0000	40,682.0000	0.0000	0.0002	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
7.0000	4.0000	1.0000	40,409.0000	40,774.0000	0.0000	0.0003	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
8.0000	4.0000	2.0000	40,409.0000	41,140.0000	0.0000	0.0008	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
9.0000	4.0000	3.0000	40,409.0000	41,505.0000	0.0000	0.0009	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
10.0000	4.0000	4.0000	40,409.0000	41,870.0000	0.0000	0.0011	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
11.0000	4.0000	5.0000	40,409.0000	42,235.0000	0.0000	0.0014	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
12.0000	4.0000	6.0000	40,409.0000	42,601.0000	0.0000	0.0014	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
13.0000	4.0000	7.0000	40,409.0000	42,966.0000	0.0000	0.0014	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
14.0000	4.0000	8.0000	40,409.0000	43,331.0000	0.0000	0.0015	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
15.0000	4.0000	9.0000	40,409.0000	43,696.0000	0.0000	0.0015	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
16.0000	4.0000	10.0000	40,409.0000	44,062.0000	0.0000	0.0016	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
17.0000	4.0000	12.0000	40,409.0000	44,792.0000	0.0000	0.0014	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
18.0000	4.0000	15.0000	40,409.0000	45,888.0000	0.0000	0.0011	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
19.0000	4.0000	20.0000	40,409.0000	47,714.0000	0.0000	0.0006	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
20.0000	4.0000	30.0000	40,409.0000	51,367.0000	0.0000	-0.0005	1,063,006.0000	0.0000	0.0000	0.0000	0.0000
21.0000	4.0000	40.0000	40,409.0000	55,019.0000	0.0000	-0.0005	1,063,006.0000	0.0000	0.0000	0.0000	0.0000

8.5 CAD 3m Curve Bootstrapping under CAD Libor Discounting

sCAD3mMid.Details

Name	sCAD3mMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:sCADx3mMid	Procedure Parameter	sCAD3mMid.Details	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	2010/08/17	Function Parameters	0.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	0	Reanchor Datum	False	Credit State	Undefined

8.6 CAD 1m Curve Bootstrapping under CAD Libor Discounting

sCAD1mMid.Details, sCAD3mMid, sCAD3mMid, sCAD3mMid

Name	sCADmMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:sCADx1mMid	Procedure Parameter	sCAD1mMid.Details, sCAD3mMid,	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	1996/04/01	Function Parameters	0.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	0	Reanchor Datum	False	Credit State	Undefined

8.7 CAD OIS Curve Bootstrapping under CAD Libor Discounting

sCAD1dMid.Details, sCAD3mMid, sCAD3mMid, sCAD3mMid

Name	sCAD1dMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:sCADx1dMid	Procedure Parameter	sCAD1dMid.Details, sCAD3mMid,	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	2010/08/17	Function Parameters	0.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	0	Reanchor Datum	False	Credit State	Undefined

8.8 USD 3m Libor Curve Co-construction under USD OIS Discounting

sUSDMid.Details, USDsUSD1dMid.Details

Name	sUSDMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:sUSDMid	Procedure Parameter	sUSDMid.Details, USDsUSD1dMid.Details	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	2014/01/27	Function Parameters	1.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	1,259	Reanchor Datum	False	Credit State	Undefined

8.9 USD OIS Curve Construction under USD OIS Discounting, Given USD 3m Curve

USDsUSD1dMid.Details, sUSDMid

Name	USDsUSD1dMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:USDsUSD1dMid	Procedure Parameter	USDsUSD1dMid.Details, sUSDMid	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	2014/01/27	Function Parameters	0.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	1,259	Reanchor Datum	False	Credit State	Undefined

8.10 USD 1m Libor Curve Bootstrapping under USD OIS Discounting

USDsUSD1mMid.Details, USDsUSD1dMid, USDsUSD1dMid, sUSDMid

Name	USDsUSD1mMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:USDsUSD1mMid	Procedure Parameter	USDsUSD1mMid.Details, USDsUSD1dMid,	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	2014/01/27	Function Parameters	0.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	1,259	Reanchor Datum	False	Credit State	Undefined

8.11 MXN 28d Forward Curve Co-construction under USD OIS Discounting

sMXNMid.Details, USDsMXNUSDMid.Details, USDsUSD1dMid, USDsUSD1mMid

Name	sMXN28dMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:sMXNMid	Procedure Parameter	sMXNMid.Details, USDsMXNUSDMid.Details,	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	2014/01/27	Function Parameters	3.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	1,259	Reanchor Datum	False	Credit State	Undefined

8.12 MXN Cross-Currency Discount Curve Construction under USD OIS Discounting

USDsMXNUSDMid.Details, USDsUSD1dMid, USDsUSD1mMid, sMXN28dMid

Name	USDsMXNUSD1d	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:sMXNUSDx1d_	Procedure Parameter	USDsMXNUSDMid.Details, USDsUSD1dMid,	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	1996/04/01	Function Parameters	0.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	1,259	Reanchor Datum	False	Credit State	Undefined

8.13 CAD Cross-Currency Discount Curve Construction under USD OIS Discounting

sCADUSD1dMid.Details, sUSD1dMid, sUSD3mMid, sCAD3mMid

Name	USDsCADUSD1dMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:sCADUSDx1dMid	Procedure Parameter	sCADUSD1dMid.Details, sUSD1dMid,	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	2010/08/17	Function Parameters	0.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	0	Reanchor Datum	False	Credit State	Undefined

8.14 CAD Cross-Currency Discount Curve Construction under USD Libor Discounting

sUSDMid.Details, USDsUSD1dMid, USDsCADUSD1dMid, sUSDMid

Name	USDsCADUSD3mMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	USDsCADUSD3mMid:	Procedure Parameter	sUSDMid.Details, USDsUSD1dMid,	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	1996/04/01	Function Parameters	11.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	0	Reanchor Datum	False	Credit State	Undefined

8.15 USD Cross-Currency Discount Curve Construction under CAD OIS Discounting

dummy.Details, USDsCADUSD1dMid, sUSD1dMid, sCAD1dMid

Name	CADsUSDCAD1dMid	State Procedure	@BNS Bootstrapping Generic	Time Evolution	@Constant
ID	SampleCurves:CADsUSDCAD1dMid	Procedure Parameter	dummy.Details, USDsCADUSD1dMid,	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	2010/08/17	Function Parameters	11.0000, 1.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	0	Reanchor Datum	False	Credit State	Undefined

8.16 USD Discount Curve under Cheapest to Deliver USD, CAD, EUR and GBP Collateral

sUSD1dMid, EURsUSDEUR1dMid, CADsUSDCAD1dMid, GBPsUSDGBP1dMid

Name	CTDsUSDCTDMid	State Procedure	@BNS Composite Curve Generic	Time Evolution	@Constant
ID	SampleCurves:sUSDCTDMid	Procedure Parameter	sUSD1dMid, CADsUSDCAD1dMid, EUR:	Interpolate Term	@Linear()
Type	Zero	Curve Function		Extrapolate Term	True
Datum	2010/08/17	Function Parameters	101.0000, 1.0000, 0.0000, 1,234.0000	Curve Unit	% CONT actual/365
Relative Curve	False	Function Identity	True	RiskMetrics Link	None
Origin Offset	0	Reanchor Datum	False	Credit State	Undefined

9 Appendix B. Bloomberg OIS Quotes

9.1 CAD Quotes

<u>OIS swap quotes: maturity 1-week to 3-year</u>

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Page 1/ World Currency Value

CAD (Canadian Dollar) Overnight Index (OIS) Swaps

Priced within: last month | Term: All | Quoted as: All

27 of 28 Tickers		Symbol	Value
1) CAD SWAP OIS	1 WK	CDSO1Z	0.7425
2) CAD SWAP OIS	2 WK	CDSO2Z	0.7425
3) CAD SWAP OIS	3 WK	CDSO3Z	0.7591
4) CAD SWAP OIS	1 MO	CDSOA	0.8135
5) CAD SWAP OIS	1ST BRM	CDSF1A	0.9120
6) CAD SWAP OIS	3RD BRM	CDSF3A	1.0125
7) CAD SWAP OIS	1MO 2ND BRM	CDSF2A	0.9755
8) CAD SWAP OIS	1MO 4TH BRM	CDSF4A	1.0485
9) CAD SWAP OIS	1MO 5TH BRM	CDSF5A	1.0915
10) CAD SWAP OIS	1MO 6TH BRM	CDSF6A	1.1420
11) CAD SWAP OIS	2 MO	CDSOB	0.8685
12) CAD SWAP OIS	3 MO	CDSOC	0.9050
13) CAD SWAP OIS	4 MO	CDSOD	0.9260
14) CAD SWAP OIS	5 MO	CDSOE	0.9450
15) CAD SWAP OIS	6 MO	CDSOF	0.9650
16) CAD SWAP OIS	9 MO	CDSOI	1.0105
17) CAD SWAP OIS	2 YR	CDSO2	1.3185
18) CAD SWAP OIS	3 YR	CDSO3	1.5770

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Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2010 Bloomberg Finance L.P.
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Page 2/ World Currency Value

CAD (Canadian Dollar) Overnight Index (OIS) Swaps

Priced within: last month | Term: All | Quoted as: All

27 of 28 Tickers		Symbol	Value
1) CAD SWAP OIS	4 YR	CDSO4	1.8505
2) CAD SWAP OIS	5 YR	CDSO5	2.0775
3) CAD SWAP OIS	7 MO	CDSOG	0.9785
4) CAD SWAP OIS	8 MO	CDSOH	0.9955
5) CAD SWAP OIS	10 MO	CDSOJ	1.0285
6) CAD SWAP OIS	11 MO	CDSOK	1.0445
7) CAD SWAP OIS	1 YR	CDSO1	1.0650
8) CAD SWAP OIS	15 MO	CDSO1C	1.1225
9) CAD SWAP OIS	18 MO	CDSO1F	1.1870

9.2 USD Quotes

OIS swap quotes: maturity 1-month to 10-year

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USD OVERNIGHT INDEX SWAPS FIXING

TERM	11:00AM LON		11:00AM NYC	15:00PM NYC	
1 Month	1) 0.177	08/16	19) 0.181	37) 0.186	08/17
2 Month	2) 0.172	08/17	20) 0.179	38) 0.182	08/17
3 Month	3) 0.171	08/17	21) 0.178	39) 0.180	08/17
4 Month	4) 0.170	08/17	22) 0.178	40) 0.180	08/17
5 Month	5) 0.168	08/17	23) 0.177	41) 0.177	08/17
6 Month	6) 0.169	08/17	24) 0.179	42) 0.178	08/17
7 Month	7) 0.172	08/17	25) 0.181	43) 0.181	08/17
8 Month	8) 0.178	08/17	26) 0.186	44) 0.185	08/17
9 Month	9) 0.186	08/17	27) 0.193	45) 0.192	08/17
10 Month	10) 0.192	08/17	28) 0.203	46) 0.202	08/17
11 Month	11) 0.205	08/17	29) 0.215	47) 0.213	08/17
1 Year	12) 0.213	08/17	30) 0.223	48) 0.221	08/17
18 Month	13) 0.310	08/17	31) 0.318	49) 0.312	
2 Year	14) 0.440	08/17	32) 0.446	50) 0.437	08/17
3 Year	15) 0.750	08/17	33) 0.751	51) 0.741	08/17
4 Year	16) 1.100	08/17	34) 1.101	52) 1.088	08/17
5 Year	17) 1.391	08/17	35) 1.390	53) 1.379	08/17
10 Year	18) 2.358	08/17	36) 2.376	54) 2.388	08/17

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Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
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Fed fund/3m Libor basis swap quotes: maturity 3-month to 30-year

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Fed Funds vs 3 Month Libor Basis Swaps

Term	Bid	Ask	Time
1) 3M	15.13	19.13	14:38
2) 6M	18.13	22.13	16:17
3) 9M	19.25	23.25	16:17
4) 1Y	20.25	24.25	16:33
5) 18M	22.38	26.38	16:08
6) 2Y	23.00	27.00	16:08
7) 3Y	23.38	28.38	15:14
8) 4Y	23.63	28.63	16:08
9) 5Y	23.38	28.38	15:06
10) 7Y	22.50	27.50	15:14
11) 10Y	21.25	26.25	9:57
12) 12Y	20.38	25.38	15:14
13) 15Y	19.38	24.38	12:35
14) 20Y	18.25	23.25	8:59
15) 25Y	17.50	22.50	8:56
16) 30Y	16.88	21.88	7:55

Libor Set For Tommorrow

Term	Bid	Ask	Time
17) 1M	0.26	0.27	6:28
18) 3M	0.34	0.34	16:17
19) 6M	0.56	0.56	16:17

Gerry Gaudet/Greg Joseph
Tel: 201 557 5363

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SN 872966 17-Aug-2010 16:36:48

9.3 EUR Quotes

Eonia swap quotes: maturity 1-week to 1-year

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EUR (Euro) Overnight Index (OIS) Swaps

Priced within	Forward	Term	Quoted as
last month	All	All	All

90 of 104 Tickers	Symbol	Value
1) EUR SWAP (EONIA) 1 WK	EUSWE1Z	0.4300
2) EUR SWAP (EONIA) 2 WK	EUSWE2Z	0.4210
3) EUR SWAP (EONIA) 3 WK	EUSWE3Z	0.4320
4) EUR SWAP (EONIA) 1 MO	EUSOA	0.4475
5) EUR SWAP (EONIA) 2 MO	EUSOB	0.4800
6) EUR SWAP (EONIA) 3 MO	EUSOC	0.5130
7) EUR SWAP (EONIA) 4 MO	EUSOD	0.5320
8) EUR SWAP (EONIA) 5 MO	EUSOE	0.5500
9) EUR SWAP (EONIA) 6 MO	EUSOF	0.5720
10) EUR SWP EONIA 6M 2ND CON	EUSE0103	0.6930
11) EUR SWP EONIA 6M 4TH CON	EUSE0305	0.7930
12) EUR SWAP (EONIA) 7 MO	EUSOG	0.5880
13) EUR SWAP (EONIA) 30 MO	EUSWE2F	0.8725
14) EUR SWAP (EONIA) 8 MO	EUSOH	0.6020
15) EUR SWAP (EONIA) 9 MO	EUSOI	0.6180
16) EUR SWAP (EONIA) 10 MO	EUSOJ	0.6305
17) EUR SWAP (EONIA) 11 MO	EUSOK	0.6400
18) EUR SWAP (EONIA) 1 YR	EUSO1	0.6540

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SN 872966 G793-1119-0 18-Aug-2010 09:12:27

Eonia swap and Eonia/3m Euribor basis swap quotes: maturity 1 to 30-year

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TPEONIA (c) 2010 Tullett Prebon Information 17-Aug-2010 20:22 GMT

	EURIBOR AB/6M	EONIA Act/360	EONIA EURIBOR 3M		EURIBOR Basis Swaps 1M/3M	3M/6M	6M/12M
1Y	1.190	0.616-0.666	29.50-34.50	1Y	20.00-24.00	20.80	25.70
2Y	1.296	0.757-0.807	28.20-33.20	2Y	18.70-22.70	18.30	19.00
3Y	1.474	0.961-1.011	26.90-31.90	3Y	18.00-22.00	17.00	15.50
4Y	1.678	1.182-1.232	25.90-30.90	4Y	17.30-21.30	15.90	13.55
5Y	1.879	1.397-1.447	25.20-30.20	5Y	16.70-20.70	15.10	12.25
6Y	2.066	1.596-1.646	24.20-29.20	6Y	16.20-20.20	14.40	11.20
7Y	2.230	1.773-1.823	23.50-28.50	7Y	15.80-19.80	13.70	10.35
8Y	2.374	1.928-1.978	22.90-27.90	8Y	15.40-19.40	13.10	9.65
9Y	2.495	2.058-2.108	22.20-27.20	9Y	15.00-19.00	12.50	9.00
10Y	2.600	2.175-2.225	21.60-26.60	10Y	14.65-18.65	11.85	8.45
11Y	2.694	2.279-2.329	21.10-26.10	11Y	14.30-18.30	11.35	7.95
12Y	2.774	2.368-2.418	20.60-25.60	12Y	14.00-18.00	10.85	7.50
13Y	2.842	2.441-2.491	20.30-25.30	13Y	13.75-17.75	10.45	7.10
14Y	2.898	2.504-2.554	19.90-24.90	14Y	13.50-17.50	10.05	6.75
15Y	2.943	2.555-2.605	19.50-24.50	15Y	13.30-17.30	9.65	6.45
20Y	3.058	2.695-2.745	18.40-23.40	20Y	12.50-16.50	8.20	5.00
30Y	2.933	2.597-2.647	17.20-22.20	30Y	11.10-15.10	6.80	3.80

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SN 872966 [illegible] 15:22:48

In the above screen, Eonia swap quotes are provided side by side to the equivalent Eonia/3m basis swap quotes.

9.4 GBP Quotes

Sonia swap quotes: maturity 1-week to 3-year

Sonia swap and Eonia/3m Libor basis swap quotes: maturity 6-month to 30-year

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GBP OIS Composite

SONIA	MPC SONIA	3m IMM FRA/OIS Spread	1y IMM IRS/OIS Spread	SONIA Ann	IRS Ann 3	SON/3S QQ
W 0.490	Sep 0.489	Sep 10 23.75	Sep 10 27.03	6M 0.499	0.750	25.00
1M 0.490	Oct 0.491	Dec 10 27.00	Dec 10 28.64	9M 0.516	0.777	26.00
2M 0.490	Nov 0.498	Mar 11 28.00	Mar 11 29.65	1Y 0.546	0.814	26.60
3M 0.491	Dec 0.505	Jun 11 29.00	Jun 11 29.93	2Y 0.762	1.051	28.60
4M 0.494	Jan 0.507	Sep 11 30.00	Sep 11 29.76	3Y 1.086	1.374	28.80
5M 0.496	Feb 0.524	Dec 11 31.00	Dec 11 29.05	4Y 1.396	1.684	28.60
6M 0.499	Mar 0.543	Mar 12 29.00	Mar 12 28.32	5Y 1.678	1.967	28.60
7M 0.503	Apr 0.556	Jun 12 28.00	--------------	6Y 1.938	2.227	28.50
8M 0.509	May 0.599	Sep 12 27.00	BBA LIBOR	7Y 2.170	2.459	28.37
9M 0.516	Jun 0.640	Dec 12 28.00	Tue 17-Aug	8Y 2.375	2.660	28.00
10M 0.526	Jul 0.640	Mar 13 27.00	1M 0.56875	9Y 2.554	2.835	27.50
11M 0.537		Jun 13 27.00	2M 0.62188	10Y 2.708	2.983	27.00
1Y 0.546			3M 0.72850	12Y 2.947	3.216	26.25
2Y 0.762			6M 1.02063	15Y 3.180	3.439	25.25
3Y 1.086			1Y 1.45750	20Y 3.356	3.597	23.50
				25Y 3.437	3.658	21.50
				30Y 3.480	3.685	20.00

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In the above screen, Sonia swap quotes are provided side by side to the equivalent Sonia/3m basis swap quotes.

10 Appendix C. Curve Construction under Libor Discounting

10.1 Introduction

In this appendix, we will demonstrate how to apply the generic curve builder designed for the OIS discounting framework, as described in the main body of this document, to build the current K2 style curves under the Libor discounting.

10.2 3m Libor Discount/Forward Curve Construction from IR Swaps

Under the Libor discounting, the 3-month Libor curve is used as both discount curve and 3m forward curves.

Consider a n-year interest rate swap with a market quoted fixed rate $\overline{R}_n^{3m}$. The floating rate resets quarterly to the 3-month Libor. The payment period dates $t_i = t_{i-1} + 3m$, $i = 1,...,4n$ are quarterly spaced and there is no pay day lag. Denote $F_{3m}(t_{i-1}, t_i)$ the forward rate for period (t_{i-1}, t_i). We have

$$\sum_{i=1}^{4n} F_{3m}(t_{i-1}, t_i)\Delta t_i \, df_{3m}(t_0, t_i) = \overline{R}_n^{3m} \sum_{i=1}^{4n} \Delta t_i \, df_{3m}(t_0, t_i)$$

The curves involved in pricing this swap are listed in the table below:

	Pay Side	Rec Side
Forward Curve	3m Curve	N/A
Discount Curve	3m Curve	3m Curve
Input Curves	None	
Output Curve	3m Curve	

With the 3-month Libor curve the only unknown in the above equation, it can be readily obtained through bootstrapping.

10.3 Forward Curve Construction from Tenor Basis Swaps

Consider a n-year tenor basis swap with a market quoted spread $\overline{S}_n^{6m}$. The pay side floating rate resets semi-annually to the 6-month Libor and pays semi-annually. Its payment period dates $T_j = T_{j-1} + 6m$, $j = 1,...,2n$ are semi-annually spaced. The receive side floating rate resets quarterly to the 3-month Libor and pays quarterly. Its payment period dates $t_i = t_{i-1} + 3m$, $i = 1,...,4n$ are quarterly spaced. There is no pay day lag. Denote $F_{6m}(T_{j-1}, T_j)$ the forward rate for period (T_{j-1}, T_j). We have

$$\sum_{j=1}^{2n} \left[F_{6m}(T_{j-1}, T_j) + \overline{S}_n^{6m}\right]\Delta T_j \, df_{3m}(T_0, T_j) = \sum_{i=1}^{4n} F_{3m}(t_{i-1}, t_i)\Delta t_i \, df_{3m}(t_0, t_i)$$

The curves involved in pricing this swap are listed in the table below:

	Swap Side 1	Swap Side 2
Forward Curve	6m Curve	3m Curve
Discount Curve	3m Curve	3m Curve
Input Curves	3m Curve	
Output Curve	6m Curve	

With the 6-month Libor curve the only unknown in the above equation, it can be readily obtained through bootstrapping.

10.4 OIS Forward Curve Construction from OIS Swaps

Consider a n-year OIS swap with a market quoted fixed rate $\overline{R}_n^{OIS}$. The floating rate resets daily to the over night rate and compounds at flat. The payment period dates $t_i = t_{i-1} + 12m$, $i = 1,...,n$ are annually spaced. The pay days t_i^P are 1 or 2 business days lagged from t_i. Denote $F_{OIS}(t_{i-1}, t_i)$ the forward rate for period (t_{i-1}, t_i), compounded from the daily over night rates. We have

$$\sum_{i=1}^{n} F_{OIS}(t_{i-1}, t_i)\Delta t_i df_{3m}(t_0, t_i^P) = \overline{R}_n^{OIS} \sum_{i=1}^{n} \Delta t_i df_{3m}(t_0, t_i^P)$$

The curves involved in pricing this swap are listed in the table below:

	Pay Side	Rec Side
Forward Curve	OIS Curve	N/A
Discount Curve	3m Curve	3m Curve
Input Curves	3m Curve	
Output Curve	OIS Curve	

With the OIS curve the only unknown in the above equation, it can be readily obtained through bootstrapping.

10.5 OIS Forward Curve Construction from OIS/3m Basis Swaps

Consider a n-year USD fed fund vs Libor basis swap with a market quoted spread $\overline{S}_n^{OIS}$. The pay side floating rate resets daily to the over night rate and pays quarterly. The receive side floating rate resets quarterly to the 3-month Libor and pays quarterly. The payment period dates $t_i = t_{i-1} + 3m$, $i = 1,...,4n$ are quarterly spaced. There is no pay day lag. Denote $F_{OIS}^{Avg}(t_{i-1}, t_i)$ the average over night rate over period (t_{i-1}, t_i). We have

$$\sum_{i=1}^{4n} \left[F_{OIS}^{Avg}(t_{i-1}, t_i) + \overline{S}_n^{OIS}\right]\Delta t_i df_{3m}(t_0, t_i) = \sum_{i=1}^{4n} F_{3m}(t_{i-1}, t_i)\Delta t_i df_{3m}(t_0, t_i)$$

The curves involved in pricing this swap are listed in the table below:

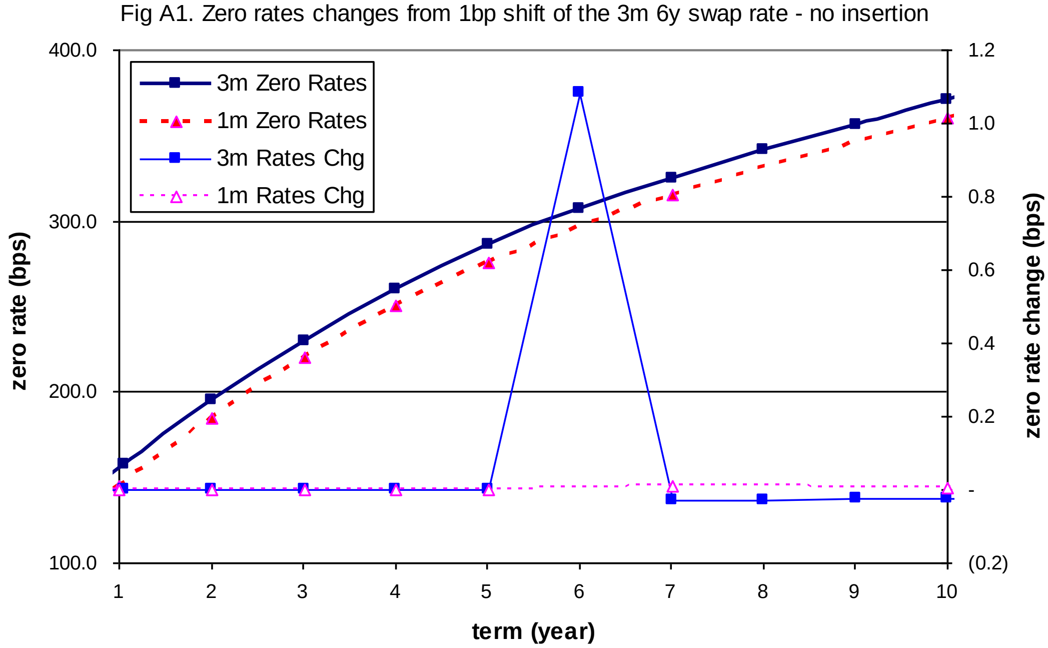
	Swap Side 1	Swap Side 2
Forward Curve	OIS Curve	3m Curve
Discount Curve	3m Curve	3m Curve
Input Curves	3m Curve	
Output Curve	OIS Curve	

With the OIS curve the only unknown in the above equation, it can be readily obtained through bootstrapping.

10.6 Instruments Insertion in the Construction of Forward Curves from Tenor Basis Swaps and Discount Curves from Currency Basis Swaps

Typically, tenor basis swaps (for example CAD 1m/3m basis swaps) and cross currency basis swaps (for example CAD/USD cross currency swaps) have fewer liquidly traded terms in the market than those of interest rate swaps (for example 3m fixed/floating swaps). Hereafter, for the ease of description, we call these more liquid interest rate swap curves as the major curves and the less liquid basis swap curves as the minor curves. As described above, under Libor discounting, both major curves and minor curves are bootstrapped as all-in zero rate curves. Due to liquidity, the term points on the major curve and the related minor curves are different with the minor curves having fewer number of term points. This misaligns in term points between the major curve and the related minor curves cause non-intuitive major curve bucket sensitivities for basis swaps. Below, we use the CAD 3m curve and the CAD 1m curve to illustrate the problem. From our K2 curve system, the 3m curve has yearly swap term points from 2-year to 10-year. However, the 1m curve doesn't have the 6-year, 8-year and 9-year points. Suppose that we want to calculate the 1bps 6-year bucket sensitivity of the 3m curve for a 6-year 1m/3m basis swap by shifting the 3m 6-year swap rate up 1bps . The 6-year zero rate of the 3m curve increases roughly 1bps with almost no changes to all other term points. Since the 6-year point is not on the 1m curve, the 1m zero rate curve is almost of no change (See Fig 1 below) and thus there is almost no change to the interpolated 1m 6-year zero rate. This is counter-intuitive as we do expect the 1m all-in zero rate to move up with the 3m curve together by 1bps at the 6-year point as well. This counter-intuitive zero rate movement of the 1m curve results large 6-year bucket sensitivity of the 3m curve for the 6-year basis swap.

Fig A1. Zero rates changes from 1bp shift of the 3m 6y swap rate - no insertion



One way to solve this issue is to build the 1m curve as a zero spread curve on the 3m curve instead of an all-in zero rate curve. However, this approach requires much more work and much more extensive test. We will consider this approach when we move to the OIS discounting methodology. Presently, we will take a much simple and straight forward approach by inserting the misaligned term points into the 1m curve. In the above CAD 1m and 3m example, we insert the missing 6-year, 8-year and 9-year 1m/3m basis swaps into the 1m instrument curve. The corresponding spreads of these missing terms are linearly interpolated from the quoted 5-year, 7-year and 10-year points. After the insertion, the 1bps shift of the 3m 6-year swap results approximately 1bps shift of the 1m 6-year zero rate as well as expected (See Fig 2 below).

This instrument insertion method will be applied to all our minor curves (tenor and cross currency basis swaps). Currently we only consider terms of swap instruments of the corresponding major curve. Terms of cash and future instruments of the major curve are not considered. In the case that the missing term point is beyond the last term point of the minor curve, flat extrapolated spread is applied.

Fig A2. Zero rates changes from 1bp shift of the 3m 6y swap rate with insertion