Homework #06: Multivariate distribution

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Problem 1. Plot the Probability Density Function of a Multivariate Normal Distribution. Consider x and y as the random variables, $\mu_x = 0, \sigma_x^2 = 1$ and $\mu_y = 1, \sigma_y^2 = 2$.

From [1], the Bivariate Normal Distribution [1] is defined as

$$P(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left[-\frac{z}{2(1-\rho^2)}\right] \tag{1}$$

where

$$z \equiv \frac{(x_1-\mu_1)^2}{\sigma_1^2} - \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2} \tag{2}$$

$$\rho \equiv cor(x_1, x_2) = \frac{V_{12}}{\sigma_1\sigma_2} = \frac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\sigma_1\sigma_2} \tag{3}$$

Given that $\sigma_{ij} = 0, i \neq j$, then $\rho = 0$ and (1) can be rewritten as

$$P(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2} \exp\left[-\frac{z}{2}\right] \tag{4}$$

In a similar fashion, 2 can be rewritten as

$$z = \frac{(x_1-\mu_1)^2}{\sigma_1^2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2} \tag{5}$$

Figure 1 presents the PDF for the given values, employing the built-in function *mvnpdf* and an implementation of the equations developed previously.

[1] a multivariate distribution of two variables is known as a bivariate distribution

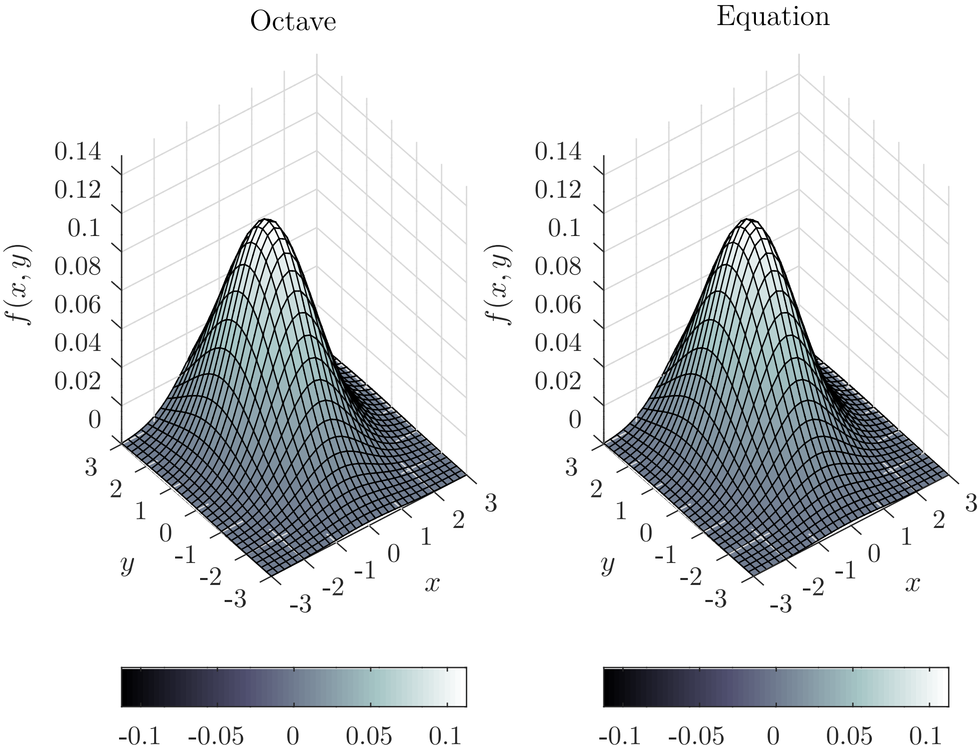

Figure 1: Multivariate Normal Distribution.

Problem 2 Plot the distributions F_X and F_Y of the function in problem 1.

Given that x and y are independent random variables with a normal distribution, both F_X and F_Y are obtained evaluating the normal pdf with their respective values. Figure 2 shows the plots for both of them.

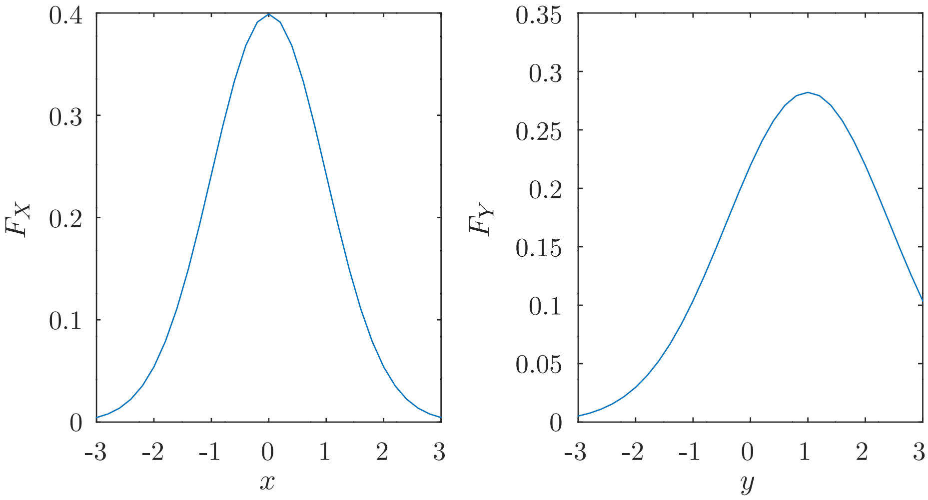

Figure 2: Distributions F_X and F_Y.

Problem 3 What would the plot of Multivariate Normal Distribution of 3 variables look like?

A 3D plot would be insufficient to visualize the distribution, given that it would be in 4D. An approximation in 3D would be slices of the volume with a colormap on its surface to express the values of $P(x, y, z)$ at every coordinate, as shown in figure 3.

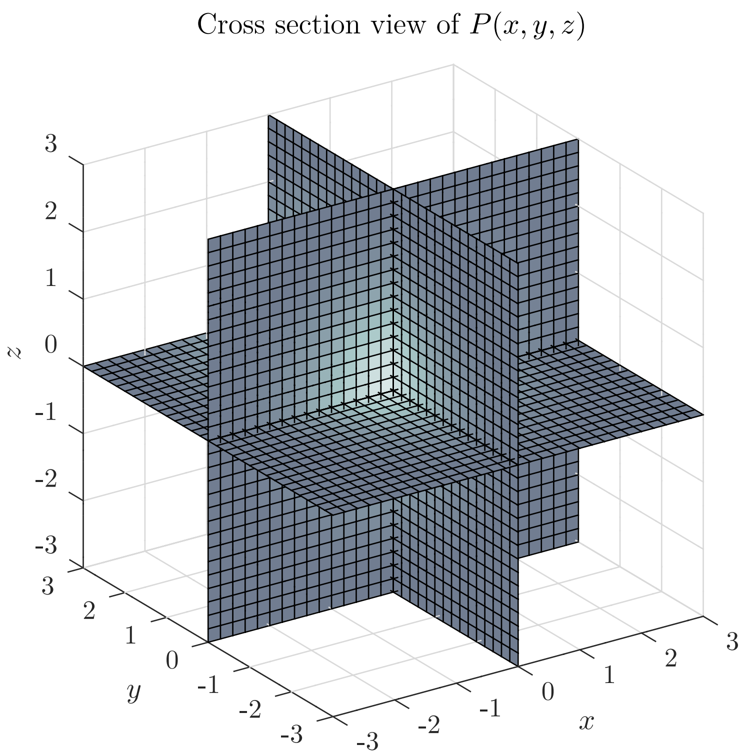

Figure 3: Slice visualization of $P(x, y, z)$ at $x, y, z = 0$.

References

[1] Wolfram MathWorld. *Bivariate Normal Distribution*. 2020. URL: `http://mathworld.wolfram.com/BivariateNormalDistribution.html` (visited on 02/10/2020).

A Octave Code

```
close all
clc
clear all
pkg load statistics

mu = [0 1]; % mu parameter for x and y
covarianceMatrix = [1 0; 0 2]; % covariance matrix for x and y
muX = 0;
muY = 1;
sigmaX = 1;
sigmaY = sqrt(2);

x = -3:0.2:3;
y = -3:0.2:3;
zz = zeros(31, 31);

[X,Y] = meshgrid(x,y);
Z = [X(:) Y(:)];
z = mvnpdf(Z,mu,covarianceMatrix);
z = reshape(z,length(y),length(x));
p = mvncdf(Z, mu, covarianceMatrix);
p = reshape(z,length(y),length(x));

for j= 1:31
  for i = 1:31
    zz(i,j) = ((x(i) - muX)^(2))/(sigmaX^2) + ((y(j) - muY)^(2))/(sigmaY^2);
  endfor
endfor
Pz = 1/(2*pi*sigmaX*sigmaY) * exp(-zz/2);

figure(1)
subplot(1,2,1)
surf(x,y,z)
colormap(bone)
caxis([min(z(:))-range(z(:)),max(z(:))])
axis([-3 3 -3 3 0 0.15])
```

```
xlabel('$x$','Interpreter','latex')
ylabel('$y$','Interpreter','latex')
zlabel('$f(x,y)$','Interpreter','latex')
title('Octave','Interpreter','latex')
colorbar('southoutside')
set(gcf,'Color',[1 1 1])

subplot(1,2,2)
surf(x,y,Pz')
colormap(bone)
caxis([min(z(:))-range(z(:)),max(z(:))])
axis([-3 3 -3 3 0 0.15])
xlabel('$x$','Interpreter','latex')
ylabel('$y$','Interpreter','latex')
zlabel('$f(x,y)$','Interpreter','latex')
title('Equation','Interpreter','latex')
colorbar('southoutside')

print('-dpdflatex', './img/hw06_multi.tex', '-S400,300');

FX = normpdf(x, muX, sigmaX);
FY = normpdf(y, muY, sigmaY);

figure(2)
subplot(1,2,1)
plot(x, FX);
xlabel('$x$','Interpreter','latex')
ylabel('$F_X$','Interpreter','latex')

subplot(1,2,2)
plot(y, FY);
xlabel('$y$','Interpreter','latex')
ylabel('$F_Y$','Interpreter','latex')

print('-dpdflatex', './img/hw06_normal.tex', '-S400,200');

clear all

x = -3:0.2:3;
```

```
y = -3:0.2:3;
z = -3:0.2:3;

mu = [0 1 0];
sigma = [1 0 0; 0 2 0; 0 0 1];
[X,Y,Z] = meshgrid(x,y, z);
P = [X(:) Y(:) Z(:)];
p = mvnpdf(P,mu,sigma);
p = reshape(p,length(y),length(x),length(z));

figure(3)
xslice = 0; % define the cross sections to view
yslice = 0;
zslice = 0;
slice(x, y, z, p, xslice, yslice, zslice)    % display the slices
ylim([-3 3])
view(-34,24)
colormap(bone)
caxis([min(p(:))-range(p(:)),max(p(:))])
xlabel('$x$','Interpreter','latex')
ylabel('$y$','Interpreter','latex')
zlabel('$z$','Interpreter','latex')
title('Cross section view of $P(x, y, z)$','Interpreter','latex')
print('-dpdflatex', './img/hw06_slice.tex', '-S300,300');

close all
```