A single quantum cannot be cloned

If a photon of definite polarization encounters an excited atom, there is typically some nonvanishing probability that the atom will emit a second photon by stimulated emission. Such a photon is guaranteed to have the same polarization as the original photon. But is it possible by thls or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? If it were, the amplifying process could be used to ascertain the exact state of a quantum system: in the case of a photon, one could determine its polarization by first producing a beam of identically polarized copies and then measuring the Stokes parameters1 We show here that the llnearity of quantum mechanics forbids such replication and that this conclusion holds for all quantum systems.

Note that if photons could be cloned, a plausible argument could be made for the possibility of faster-than-light communication2 . It is well known that for certain non-separably correlated Einstein-Podolsky-Rosen pairs of photons, once an observer has made a polarization measurement (say, vertical versus horizontal) on one member of the pair, the other one, which may be far away, can be for all purposes of prediction regarded as having the same polarization3 If this second photon could be replicated and its precise polarization measured as above, it would be possible to ascertain whether, for example, the first photon had been subjected to a measurement of linear or circular polarization. In this way the first observer would be able to transmit information faster than light by encoding his message into bis choice of measurement. The actual impossibil ity of cloning photons, shown below, thus prohibits super luminal communication by this scheme. That such a scheme must fail for some reason despite the well-established existence of long-range quantum correlations6 8 , is a general consequence of quantum mechanics 9

A perfect amplifying device would have the following effect on an incoming photon with polarization state $|s\rangle$:

$$|A_0\rangle|s\rangle \rightarrow |A_s\rangle|ss\rangle \qquad (1)$$

Here $|A_0\rangle$ is the 'ready' state of the apparatus, and $|A_s\rangle$ is its final state, which may or may not depend on the polarization of the original photon. The symbol $|ss\rangle$ refers to the state of the radiation field in which there are two photons each having the polarization $|s\rangle$. Let us suppose that such an amplification can in fact be accomplished for the vertical polarization $|V\rangle$ and for the horizontal polarization $|H\rangle$. That is,

$$|A_0\rangle|V\rangle \rightarrow |A_v\rangle|VV\rangle \qquad (2)$$

and

$$|A_0\rangle|H\rangle \rightarrow |A_h\rangle|HH\rangle \qquad (3)$$

According to quantum mechanics this transformation should be representable by a linear (in fact unitary) operator. It therefore follows that if the incoming photon has the polarization given by the linear combination $\alpha|V\rangle + \beta|H\rangle$ - for example, it could be linearly polarized in a direction 45° from the vertical, so that $\alpha = \beta = 2^{-1/2}$ the result of its interaction with the apparatus will be the superposition of equations (2) and (3):

$$|A_0\rangle(\alpha|V\rangle + \beta|H\rangle) \rightarrow \alpha|A_v\rangle|VV\rangle + \beta|A_h\rangle|HH\rangle \qquad (4)$$

If the apparatus states $|A_v\rangle$ and $|A_h\rangle$ are not identical, then the two photons emerging from the apparatus are in a mixed state of polarization. If these apparatus states are identical, then the two photons are in the pure state

$$\alpha|VV\rangle + \beta|HH\rangle \qquad (5)$$

In neither of these cases is the final state the same as the state with two photons both having the polarization $\alpha|V\rangle + \beta|H\rangle$. That state, the one which would be required if the apparatus were to be a perfect amplifier, can be written as

$$2^{-1/2}(\alpha a_v^\dagger + \beta a_h^\dagger)^2|0\rangle = \alpha^2|VV\rangle + 2^{1/2}\alpha\beta|VH\rangle + \beta^2|HH\rangle$$

which is a pure state different from the one obtained above by superposition [equation (5)). Thus no apparatus exists which will amplify an arbitrary polarization. The above argument does not rule out the possibility of a device which can amplify two special polarizations, such as vertical and horizontal. Indeed, any measuring device which distinguishes between these two polarizations, a Nicol prism for example, could be used to trigger such an amplification.

The same argument can be applied to any other kind of quantum system. As in the case of photons, linearity does not forbid the amplification of any given state by a device designed especially for that state, but it does rule out the existence of a device capable of amplifying an arbitrary state.

Milonni (unpublished work) has shown that the process of stimulated emission does not lead to quantum amplification, because if there isstimulated emission there must also be-with equal probability in the case of one incoming photon-spon taneous emission, and the polarization of a spontaneously emit ted photon is entirely independent of the polarization of the original.

It is conceivable that a more sophisticated amplifying apparatus could get around Milonni's argument. We have there fore presented the above simple argument, based on the iinearity of quantum mechanics, to show that no apparatus, however complicated, can amplify an arbitrary polarization.

We stress that the question of replicating individual photons is of practical interest. It is obviously closely related to the quantum limits on the noise in amplifiers (10 11). Moreover, an experiment devised to establish the extent to which polarization of single photons can be replicated through the process of stimulated emission is under way (A. Gozzini, personal communication; and see ref. 12). The quantum mechanical prediction is quite definite; for each perfect clone there is also one randomly polarized, spontaneously emitted, photon.

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