Sensor Data Fusion

Exercise 11

Prof. Dr.-Ing. Marcus Baum
M.Sc. Kolja Thormann

www.fusion.informatik.uni-goettingen.de

GEORG-AUGUST-UNIVERSITÄT
GÖTTINGEN

DATA
FUSION Lab

Today

- Lecture review
- Questions
- Homework 10 solution
- Example - Bayesian recursion for random sets
- Problem 11 - Random sets
- Homework 11 presentation

Questions

How can you classify uncertain information?

- Imprecise information, e.g., more than ..., an interval, ...
- Affected by random effect, e.g., probability funtion
- Vague information, e.g., approximate values
- Erroneous information, e.g., systematic errors

What are the axioms of probability for the Bayesian method?

We have the set of possible classes $x \in X$ and a probability mass function $p(x)$ with probability measure $P(A) = \sum_{x \in A} p(x)$ (with $A \subseteq X$). Also, let $A_1 \subseteq X$, $A_2 \subseteq X$, and $A_1 \cap A_2 = \emptyset$.

- $P(A_1) \geq 0$
- $P(X) = 1$
- $P(A_1 \cup A_2) = P(A_1) + P(A_2)$

Questions

What is the difference between the random set based method to the classical approach?

The random set based methods replace the likelihood by a generalized likelihood, which describes the probability of the measurement belonging to a set. Each state x is then mapped to a set in the measurement space.

Homework 10: solution

Consider the same motion model as in the last homework:

Assume a robot in 2D-space at position $\begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ moving with velocity v_1 in x_1 direction and v_2 in x_2 direction. Its state is defined as $\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & v_1 & v_2 \end{bmatrix}^T$.

a) This time, instead of measuring the position directly, we measure the distance to a landmark P_i. Formulate the measurement equation and write a function implementing it.

   For now, we will assume only a single landmark is used. In that case, the measurement noise will be a scalar.

$$h_i(\boldsymbol{x}) = \sqrt{(x_1 - P_{i,1})^2 + (x_2 - P_{i,2})^2}$$

Homework 10: solution

b) To use the measurement in the Kalman filter, we utilize the EKF formulas. Calculate the Jacobian of the measurement function from a) and implement the EKF measurement update.

Again, we assume that for now only one landmark exists, and $\mathbf{R}$ will be a scalar.

$$\mathbf{H}_i = \begin{bmatrix} \frac{x_1 - P_{i,1}}{\sqrt{(x_1 - P_{i,1})^2 + (x_2 - P_{i,2})^2}} & \frac{x_2 - P_{i,2}}{\sqrt{(x_1 - P_{i,1})^2 + (x_2 - P_{i,2})^2}} & 0 & 0 \end{bmatrix}$$

Homework 10: solution

c) Your implementation will be used to track the robot, getting measurements from a single landmark $P_1 = \begin{bmatrix}5 & 0\end{bmatrix}^T$.

The measured distance is noise corrupted with Gaussian noise having zero mean and variance 0.1.

For the initial state and the process covariance, we will use

$$\mathbf{C}_{\text{init}} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} 0.001 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0.02 & 0 \\ 0 & 0 & 0 & 0.02 \end{bmatrix}$$

For this, you will first need to implement the motion model of the robot.

GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN

d) To improve your estimate, use a second landmark $P_2 = \begin{bmatrix} 0 & 5 \end{bmatrix}^T$. Formulate a stacked measurement equation and calculate the corresponding Jacobian.

You will need to implement new versions of your `measurement_model` and your `update` functions.

Hints:

- since we are now dealing with a multi-dimensional scenario, you should consider using `np.random.multivariate_normal` instead of `np.random.normal`.

Example – Bayesian recursion for random sets

We have possbile classes $X = \{x_1, x_2, x_3\}$, which map to sets in the measurement space, which consists of $Z = \{z_1, z_2, z_3\}$, as

- $\Sigma_{x_1} = \{z_1\}$
- $\Sigma_{x_2} = \{z_1, z_2\}$
- $\Sigma_{x_3} = \{z_1, z_2, z_3\}$

The random set approach now uses a generalized likelihood function $\tilde{g}(z_i|x_j) = P(z_i \in \Sigma_{x_j})$, which describes the proability of the respective set containing a specific $z \in Z$., resulting in the confusion matrix

$\tilde{g}(z_i\|x_j)$	x_1	x_2	x_3
z_1	1	1	1
z_2	0	1	1
z_3	0	0	1

Example – Bayesian recursion for random sets

We can calculate the posterior use Bayes rule

$$p(x_i|z_i) = \frac{\tilde{g}(z_i|x_i)p(x_i)}{\sum_{x\in X} g(z_i|x)p(x)}$$

Assuming a uniformly distributed prior across all classes, we end up with a posterior depending on the generalized likelihood normalized over all possible classes, i.e.,

$p(x_j\|z_i)$	z_1	z_2	z_3
x_1	$\frac{1}{3}$	0	0
x_2	$\frac{1}{3}$	$\frac{1}{2}$	0
x_3	$\frac{1}{3}$	$\frac{1}{2}$	1

Example – Bayesian recursion for random sets

Let $z^{(k)}$ be the measurement at time step k. Assume we had $z^{(1)} = z_2$. The resulting posterior would be

$$p(x|z_2^{(1)}) = \begin{cases} 0 \text{ if } x = x_1 \\ 0.5 \text{ otherwise} \end{cases}$$

The normalization will be

$$\tilde{g}(z^{(2)}) = \begin{cases} 1 \cdot 0 + 1 \cdot 0.5 + 1 \cdot 0.5 \text{ if } z^{(2)} = z_1^{(2)} \\ 0 \cdot 0 + 1 \cdot 0.5 + 1 \cdot 0.5 \text{ if } z^{(2)} = z_2^{(2)} \\ 0 \cdot 0 + 0 \cdot 0.5 + 1 \cdot 0.5 \text{ if } z^{(2)} = z_3^{(2)} \end{cases}$$

which is essentially $\tilde{g}(z_1^{(2)}) = 1$, $\tilde{g}(z_2^{(2)}) = 1$, and $\tilde{g}(z_3^{(2)}) = 0.5$. Therefore, the posterior for the second time step is calculated as

$p(x_j \mid \{z_i^{(2)}, z_2^{(1)}\})$	$z_1^{(2)}$	$z_2^{(2)}$	$z_3^{(2)}$
x_1	0	0	0
x_2	$\frac{1 \cdot 0.5}{1}$	$\frac{1 \cdot 0.5}{1}$	$\frac{0 \cdot 0.5}{0.5} = 0$
x_3	$\frac{1 \cdot 0.5}{1}$	$\frac{1 \cdot 0.5}{1}$	$\frac{1 \cdot 0.5}{0.5} = 1$

Problem 11 – Random sets

Assume we have classes $X = \{x_1, x_2, x_3, x_4\}$ and measurements $Z = \{z_1, z_2, z_3, z_4\}$. x_1 can only produce measurements z_1 and z_2, x_2 can only produce measurements z_2 and z_3, x_3 can only produce measurement z_3, and x_4 can produce all measurements.

a) Determine the confusion matrix and calculate the posterior for the standard approach (assuming the likelihood is a probability distribution). Assume a uniform prior and a uniform distribution of the possible measurements for each class in the likelihood. Next, calculate the posterior matrix.

b) Now consider the random set theory. Identify the generalized likelihood function and calculate the confusion and posterior matrix.

Problem 11: Solution

Setting the likelihood to zero for all impossible measurement-target combinations and assuming a uniform distribution for the possible measurements regarding each target, we get the confusion matrix

$p(z_i \mid x_j)$	x_1	x_2	x_3	x_4
z_1	0.5	0	0	0.25
z_2	0.5	0.5	0	0.25
z_3	0	0.5	1	0.25
z_4	0	0	0	0.25

Given a uniform prior, we end up with the posterior

$p(x_j \mid z_i)$	z_1	z_2	z_3	z_4
x_1	$\frac{2}{3}$	$\frac{2}{5}$	0	0
x_2	0	$\frac{2}{5}$	$\frac{2}{7}$	0
x_3	0	0	$\frac{4}{7}$	0
x_4	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{7}$	1

Problem 11: Solution

We have the measurement sets

- $\Sigma_{x_1} = \{z_1, z_2\}$
- $\Sigma_{x_2} = \{z_2, z_3\}$
- $\Sigma_{x_3} = \{z_3\}$
- $\Sigma_{x_4} = \{z_1, z_2, z_3, z_4\}$

Using the generalized likelihood function $\tilde{g}(z_i|x_j) = P(z_i \in \Sigma_{x_j})$, we get the confusion matrix

$\tilde{g}(z_i\|x_j)$	x_1	x_2	x_3	x_4
z_1	1	0	0	1
z_2	1	1	0	1
z_3	0	1	1	1
z_4	0	0	0	1

Problem 11: Solution

Assuming a uniformly distributed prior across all classes, we end up with a posterior depending on the generalized likelihood normalized over all possible classes, i.e.,

$p(x_j \mid z_i)$	z_1	z_2	z_3	z_4
x_1	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{4}$
x_2	0	$\frac{1}{3}$	0	$\frac{1}{4}$
x_3	0	0	$\frac{1}{2}$	$\frac{1}{4}$
x_4	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{4}$

Homework 11

Assume the setting from the lecture. We have three classes $X = \{x_1, x_2, x_3\}$ and three possible measurements $Z = \{z_1, z_2, z_3\}$ with measurement sets

- $\Sigma_{x_1} = \{z_1\}$
- $\Sigma_{x_2} = \{z_1, z_2\}$
- $\Sigma_{x_3} = \{z_1, z_2, z_3\}$

a) Implement a function to recursively track the class probability given the standard approach.

b) Use the function from a) to track the probabilities over 20 time steps. Let $z^{(k)}$ be the measurement at time step k. Assume you receive measurements $z^{(7)} = z_2$, $z^{(13)} = z_3$, and $z^{(k)} = z_1$ for $k \in \{1, \dots, 20\} \setminus \{7, 13\}$. Plot the development of the class probabilities.

c) Now implement a function to recursively track the class probabilities based on the random set approach, defining and using a generalized likelihood function. Use the same measurements as in b) to track the class probabilities. Visualize the results.