Sensor Data Fusion

Exercise 12

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DATA FUSION

Today

- Homework 11
- Sample exam 1

Homework 11: Solution

Assume the setting from the lecture. We have three classes $X = \{x_1, x_2, x_3\}$ and three possible measurements $Z = \{z_1, z_2, z_3\}$ with measurement sets

- $\Sigma_{x_1} = \{z_1\}$
- $\Sigma_{x_2} = \{z_1, z_2\}$
- $\Sigma_{x_3} = \{z_1, z_2, z_3\}$

a) Implement a function to recursively track the class probability given the standard approach.

b) Use the function from a) to track the probabilities over 20 time steps. Let $z^{(k)}$ be the measurement at time step k. Assume you receive measurements $z^{(7)} = z_2$, $z^{(13)} = z_3$, and $z^{(k)} = z_1$ for $k \in \{1, \ldots, 20\} \setminus \{7, 13\}$. Plot the development of the class probabilities.

c) Now implement a function to recursively track the class probabilities based on the random set approach, defining and using a generalized likelihood function. Use the same measurements as in b) to track the class probabilities. Visualize the results.

Sample Exam 1

Sample Exam 1

Why do we need multiple sensors? Name and explain two sensor fusion concepts.

Sensors have advantages and disadvantages. Using multiple different sensors can balance these out.
Two concepts are competitive fusion (multiple sensors measure the same space to increase accuracy) and complementary fusion (mutliple sensors measure different spaces to increase completeness).

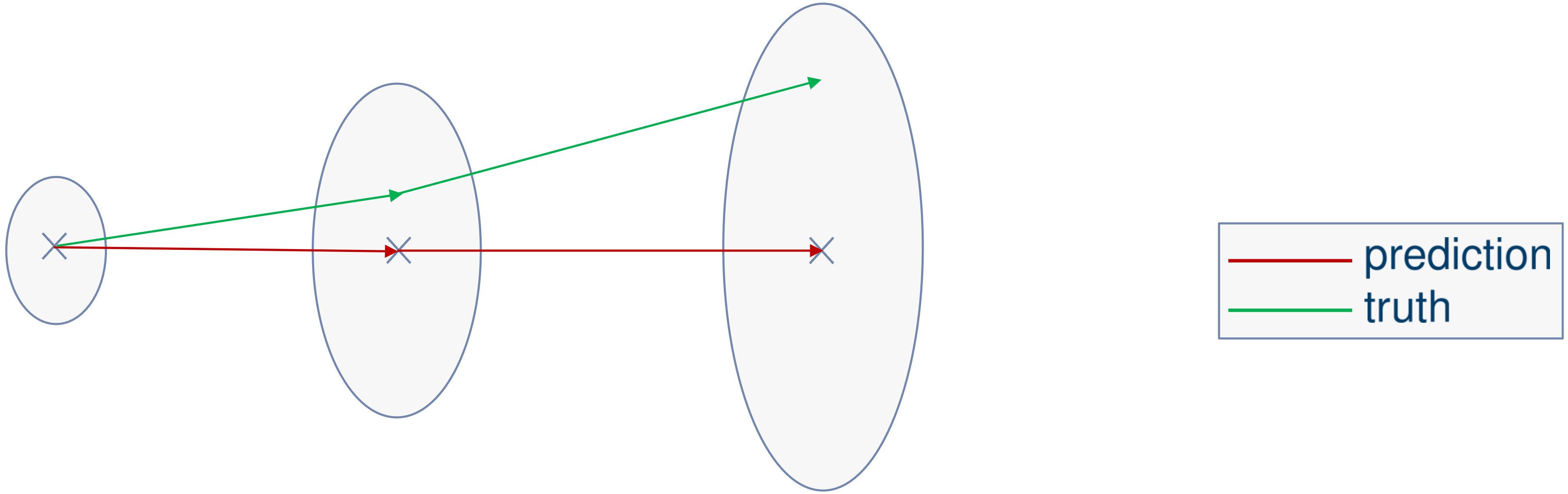
Sample Exam 1

Can you explain dead reckoning and its main problem?

Calculating your own position based on

- Previous position
- Internal sensors

Errors accumulate!



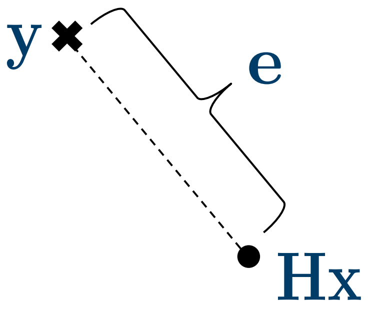
Sample Exam 1

What is a measurement equation? Could you write down the linear measurement equation in a general form and explain the components in it?

The measurement equation projects the state onto the measurement space and describes the measurement as the sum of the projected state and an error.

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{e}$$

with state $\mathbf{x} \in \mathcal{R}^n$, measurement $\mathbf{y} \in \mathcal{R}^m$, measurement matrix $\mathbf{H} \in \mathcal{R}^{m \times n}$, and error $\mathbf{e} \in \mathcal{R}^m$.



Sample Exam 1

What is the objective/cost function of least squares?

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{e}$$

$$\mathbf{x}^{LS} = \underset{\mathbf{x}}{argmin} \underbrace{||\mathbf{y} - \mathbf{H}\mathbf{x}||^2_{\mathbf{W}}}_{G(\mathbf{x})}$$

$$G(\mathbf{x}) = (\mathbf{y} - \mathbf{H}\mathbf{x})^{\mathrm{T}}\mathbf{W}(\mathbf{y} - \mathbf{H}\mathbf{x})$$

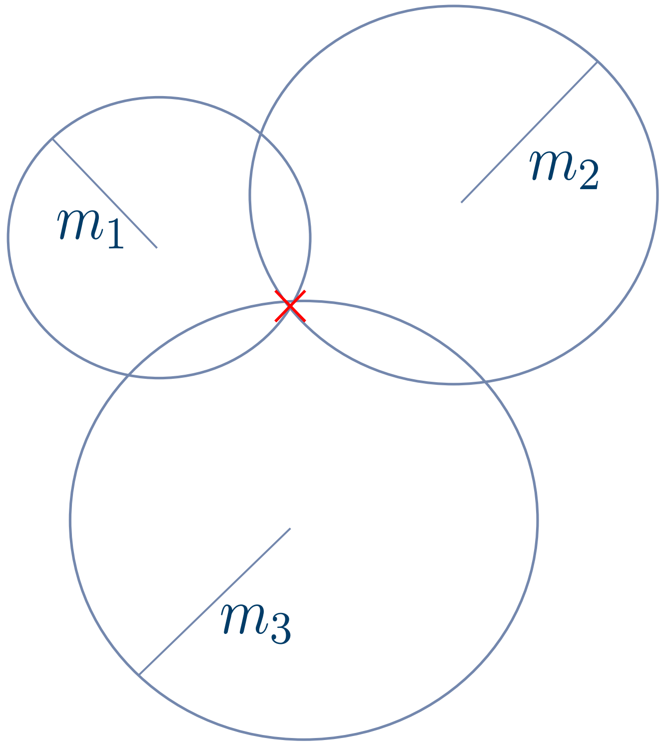
$$G'(\mathbf{x}^{LS}) \overset{!}{=} 0$$

$$\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{y} = \mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{H}\mathbf{x}^{LS}$$

$$\mathbf{x}^{LS} = (\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{y}$$

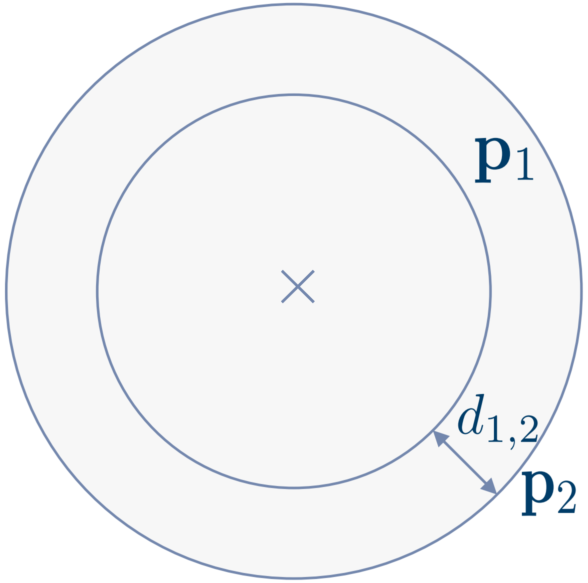
Sample Exam 1

How does triliteration work?



Sample Exam 1

Can you explain how Time-difference-of-Arrival works?



$$d_{1,2} = ||\mathbf{x} - \mathbf{p}_2|| - ||\mathbf{x} - \mathbf{p}_1|| + e$$

Sample Exam 1

Can you explain the steps of the Gauss-Newton algorithm?

$$\mathbf{x}^{LS} = \underset{\mathbf{x}}{argmin} || \underbrace{\mathbf{y} - h(\mathbf{x})}_{e(\mathbf{x})} ||^2 \qquad \mathbf{J} = e'(\hat{\mathbf{x}})$$

$$\begin{aligned} e(\mathbf{x}) &\approx e(\hat{\mathbf{x}}) + e'(\hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}}) \\ &= e(\hat{\mathbf{x}}) + \mathbf{J}\mathbf{x} - \mathbf{J}\hat{\mathbf{x}} \\ &= -(\mathbf{J}\hat{\mathbf{x}} - e(\hat{\mathbf{x}}) - \mathbf{J}\mathbf{x}) \end{aligned}$$

$$\mathbf{x}^{LS} \approx \underset{\mathbf{x}}{argmin} || \underbrace{\mathbf{J}\hat{\mathbf{x}} - e(\hat{\mathbf{x}})}_{\mathbf{y}} - \underbrace{\mathbf{J}}_{\mathbf{H}} \mathbf{x} ||^2$$

The resulting least squares estimate is based on an initial guess $\hat{\mathbf{x}}$. The result is then used in a next iteration step as the new guess. This is repeated until convergence.

Sample Exam 1

What is BLUE?

- Quality of an estimator measured via MSE
- Goal: Minimum MSE Estimator
- Problem: unknown $\mathbf{x}$
- Solution: assume unbiasedness

$$MSE(\theta_y) = \operatorname{Tr}\operatorname{Cov}[\theta_y] \cancel{+ ||\operatorname{E}[\theta_y] - \mathbf{x}||^2}$$

- New goal: Minimum Variance Unbiased (MVU) Estimator
- If MVU estimator is linear, we have the Best Linear Unbiased Estimator (BLUE)

Sample Exam 1

What is the difference between Bayesian and Fisher approach?

- We have $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{e}$
- Fisher: $\mathbf{x}$?, $\mathbf{e} \in \mathcal{N}(\mathbf{0}, \mathbf{C}_{ee})$
- Bayesian: $\mathbf{x} \in \mathcal{N}(\boldsymbol{\mu}_x, \mathbf{C}_{xx})$, $\mathbf{e} \in \mathcal{N}(\mathbf{0}, \mathbf{C}_{ee})$
- In general
  - Prior: $p(\mathbf{x})$
  - Likelihood: $p(\mathbf{y}|\mathbf{x})$
  - $p(\mathbf{x}, \mathbf{y}) = p(\mathbf{y}|\mathbf{x})p(\mathbf{x})$

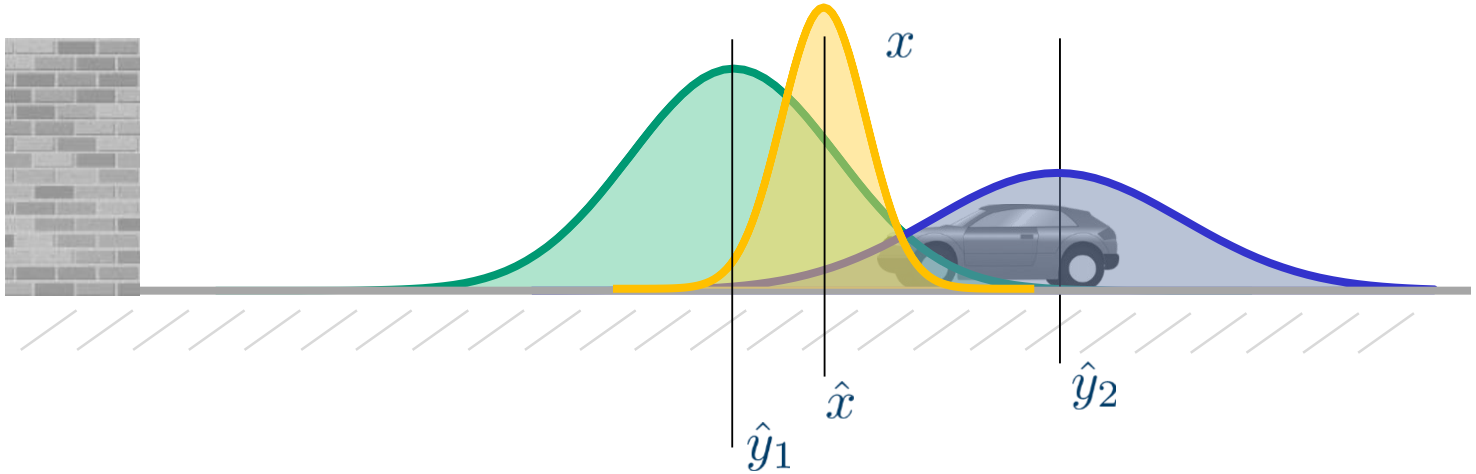
Sample Exam 1

Why is it difficult to obtain the optimal Bayesian estimator?

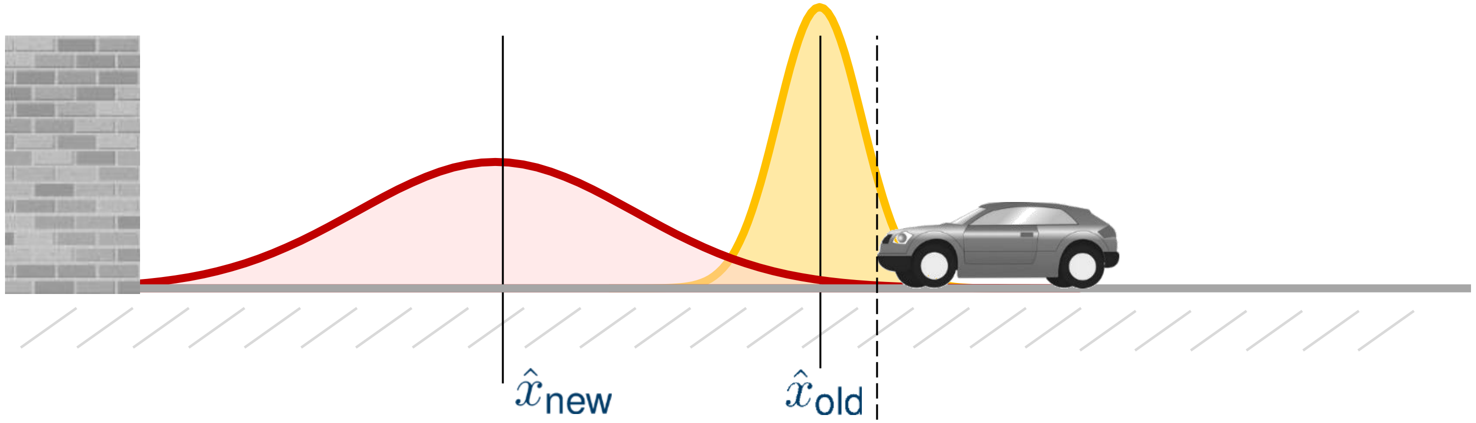
- No closed-form solution (in general)
- complete prior and likelihood information required
- Kalman filter is optimal Bayesian filter if
  - Linearity is given
  - All distributions are Gaussian

Sample Exam 1

In the Kalman filter, what happens to the state variance during measurement update? What happens during time update?



measurement update:
variance decreases



time update:
variance increases

Sample Exam 1

Assume during the time update, the state moves with a noise corrupted velocity. What Would happen to the state covariance if we want to estimate the state for $T \to \infty$?

We have $\sigma^2_{x_{k+1}} = \sigma^2_{x_k} + T^2\sigma^2_v$.

For $T \to \infty$, we get $\sigma^2_{x_{k+1}} \to \infty$.

Sample Exam 1

How can unknown correlation between estimates be handled?

Find uncorrelated ellipse encapsulating correlated one.

$$\tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}^{xx} & \mathbf{C}^{xy} \\ \mathbf{C}^{yx} & \mathbf{C}^{yy} \end{bmatrix}$$

$$\mathbf{C} > \tilde{\mathbf{C}}$$

$$\mathbf{C} = \begin{bmatrix} \frac{1}{0.5-\alpha}\mathbf{C}^{xx} & 0 \\ 0 & \frac{1}{0.5+\alpha}\mathbf{C}^{yy} \end{bmatrix}$$

Sample Exam 1

What assumptions does the discrete time Nearly Constant Velocity model make about the noise?

The acceleration is assumed to be zero-mean white noise which is constant during each sampling period.