2^{nd} Chapter

Discrete-Time Signals and Systems.

problems

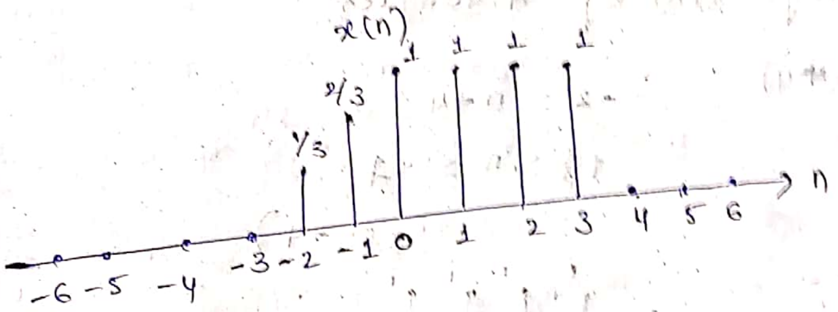
(1) A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1+\frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere.} \end{cases}$$

a) Determine its values and sketch the signal $x(n)$

sol:

n	-3	-2	-1	0	1	2	3	4
x(n)	0	1/3	2/3	1	1	1	1	0



$$x(n) = \{ \ldots 0, \tfrac{1}{3}, \tfrac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots \}$$

b) sketch the signals that result if we:

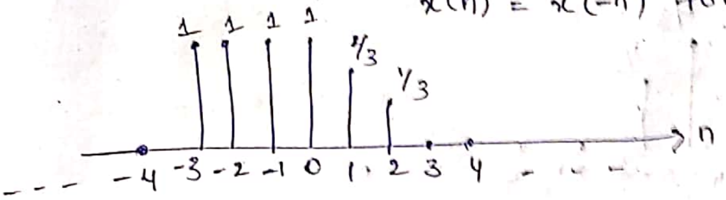
1. first fold $x(n)$ and then delay the resulting signal by four samples.
2. first delay $x(n)$ by four samples and then fold the resulting the signal.

sol:-

1) folding

$$x(-n) = \{ \ldots 0, 1, 1, 1, \underset{\uparrow}{1}, \tfrac{2}{3}, \tfrac{1}{3}, 0, 0, \ldots \}$$

$x(n) = x(-n)$ folding



delayed by 4 samples

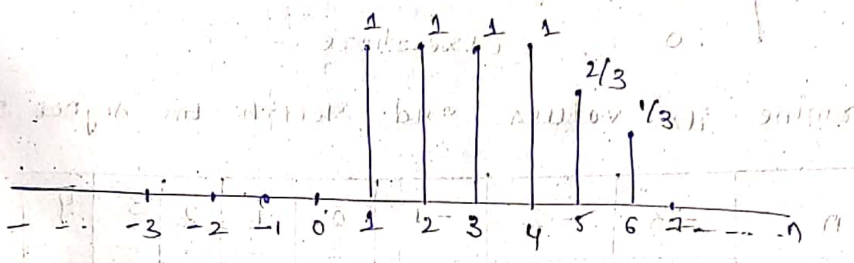
$x(-n+4) = \{ \ldots\, 0, \underset{\uparrow}{0}, 1, 1, 1, 1, 2/3, 1/3, 0, \ldots$

$-3 \le -n+4 \le +2$

$-7 \le -n \le -2$

$+7 \ge n \ge -2$

$x(-n+4) = x(-(n-4))$



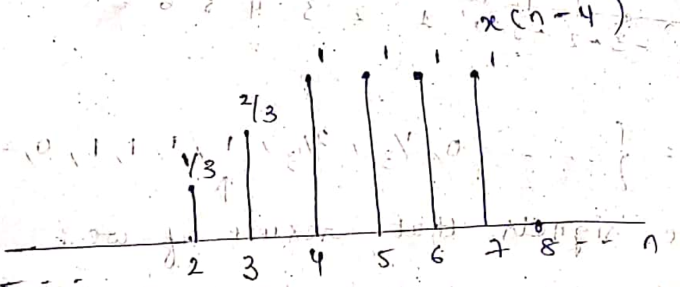
2) sol:-

delay $x(n)$ by 4 samples

$x(n-4)$ $\qquad -2 \le n-4 \le 3$

$+2 \le n \le +7$

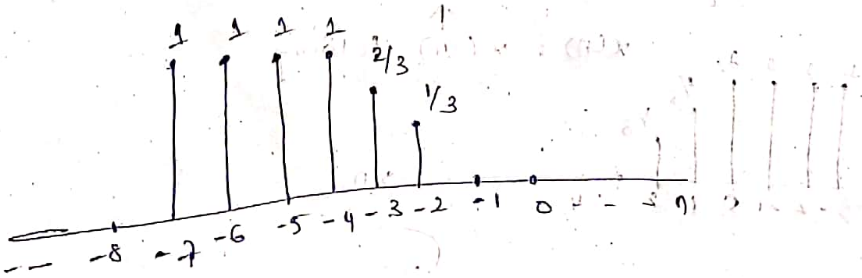
$x(n-4)$



$x(n-4) = \{ 0 \ldots, 0, \underset{\uparrow}{0}, 0, 1/3, 2/3, 1, 1, 1, 1, 0, 0, \ldots$

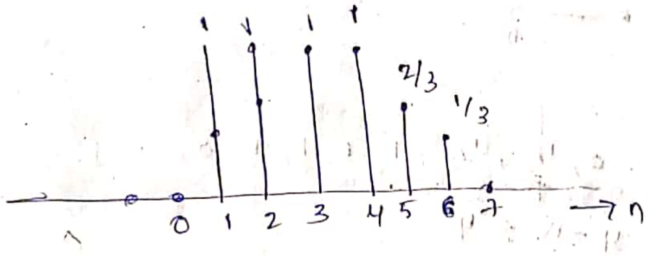
fold $x(n-4) \Rightarrow x(-n-4) = \{ \ldots 0, 1, 1, 1, 1, 2/3, 1/3, 0, \underset{\uparrow}{0}$

folding $x(n-4) \Rightarrow x(-n-4)$



c) Sketch the signal $x(-n+4)$.

$$x(-n+4) = \{ \ldots , \underset{\uparrow}{0}, 1, 1, 1, 1, 2/3, 1/3, 0, \ldots \}$$



$-2 \le -n+4 \le 3$

$-6 \le -n \le -1$

$6 \ge n \ge 1 \Rightarrow 1$

d) Compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$

To get $x(-n+k) \Rightarrow 1^{st}$ we need to fold $x(n)$, which results in $x(-n)$. then we need to shift by k samples to the right if $k > 0$

or

to the left if $k < 0$

(or)

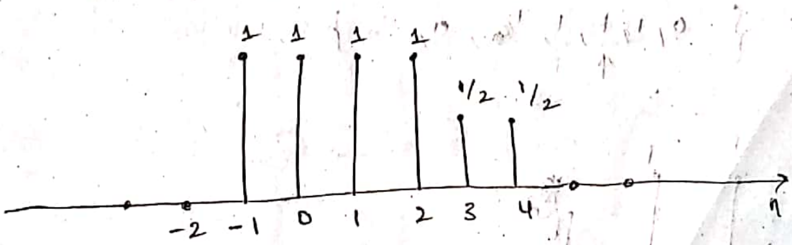
To get $x(-n+k)$

we need to shift by k samples to the right if $k > 0$ or to the left if $k < 0$ and then flip the resultant signal to get $x(-n+k)$.

e) can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$

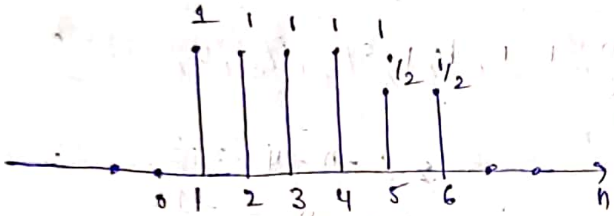
sol: Yes, $x(n) = 1/3\,\delta(n-2) + 2/3\,\delta(n-1) + u(n) - u(n-4)$

2) A discrete-time signal $x(n)$ is shown in fig. sketch and label carefully each of the following signals.



a) $x(n-2)$

$x(n-2) = \{ \ldots 0, \underset{\uparrow}{1}, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0 \ldots \}$ b)



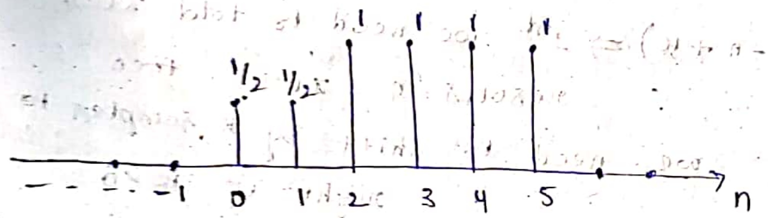
b) $x(4-n) = \{ \ldots 0, \underset{\uparrow}{\tfrac{1}{2}}, \tfrac{1}{2}, 1, 1, 1, 1, 0, \ldots \}$

$-1 \leq 4-n \leq 4$

$-5 \leq -n \leq 0$

$5 \geq n \geq 0$

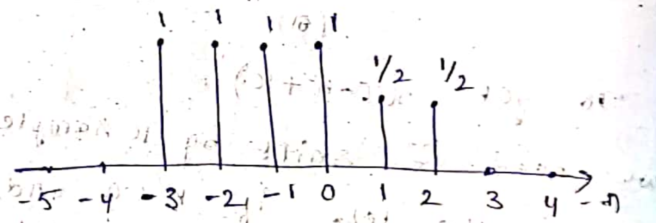
$x(4-n)$



(c) $x(n+2) = \{ \ldots 0, 1, 1, 1, \underset{\uparrow}{1}, \tfrac{1}{2}, \tfrac{1}{2}, 0, \ldots \}$

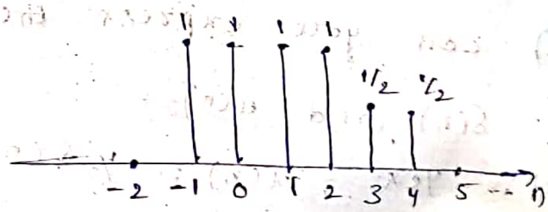
$-1 \leq n+2 \leq 4$

$-3 \leq n \leq 2$



d) $x(n)u(2-n)$

$x(n)$



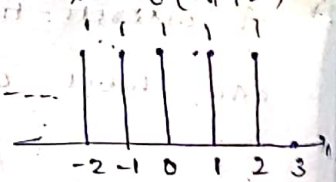
$u(n) = \{ \ldots 0, \underset{\uparrow}{1}, 1, 1, \ldots \}$

$u(n+2) = \{ \ldots 0, 1, 1, \underset{\uparrow}{1}, 1, 1, \ldots \}$

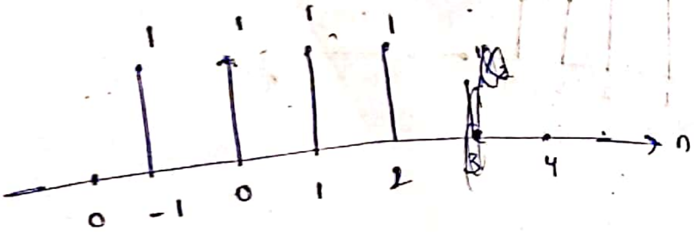
$u(-n+2) = \{ \ldots 1, 1, 1, \underset{\uparrow}{1}, 1, 1, 0, \ldots \}$

$x(n) = \{ \ldots 0, 1, \underset{\uparrow}{1}, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0, \ldots \}$

$x \quad u(-n+2)$

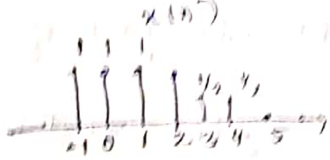


$x(n)u(2-n) = \{ 0, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, 0, \ldots \}$

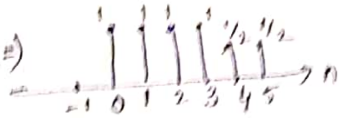


e) $x(n-1)\,\delta(n-3)$

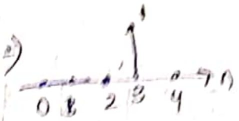
$x(n) = \{\ldots 0, 1, 1, 1, 1, 1/2, 1/2, 0 \ldots\}$ $\Rightarrow$



$x(n-1) = \{\ldots 0, 1, 1, 1, 1, 1/2, 1/2, 0 \ldots\}$ $\Rightarrow$



$\delta(n-3) = \{\ldots 0, 0, 0, 1, 0, 0, \ldots\}$ $\Rightarrow$



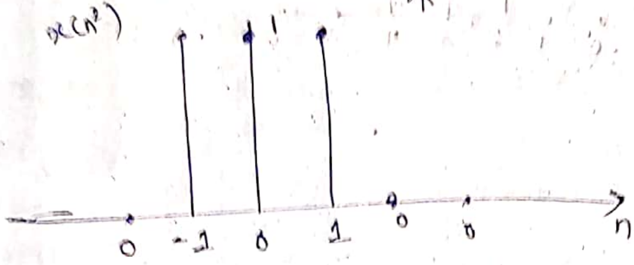
$x(n-1)\,\delta(n-3) = \{\ldots 0, 0, 0, 1, 0, 0 \ldots\}$

f) $x(n^2)$

$x(n) = \{\ldots x(-2), x(-1), x(0), x(1), x(2), x(3), x(4), \ldots\}$

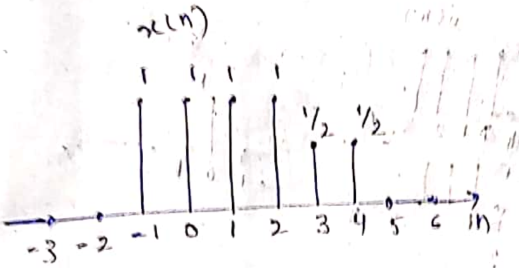
$x(n^2) = \{\ldots x(4), x(1), x(0), x(1), x(4), x(9), x(16), \ldots\}$

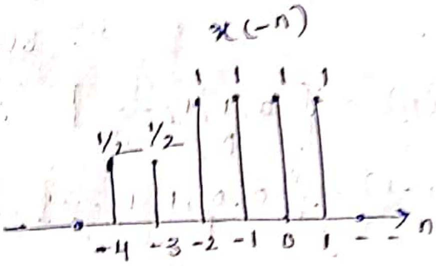
$= \{\ldots 0, 1, 1, 1, 0, 0, \ldots\}$

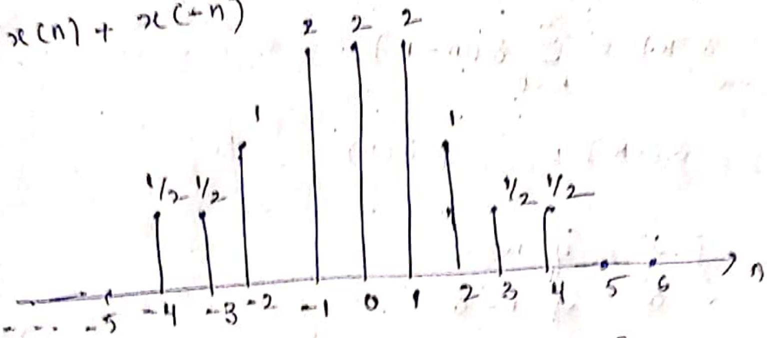


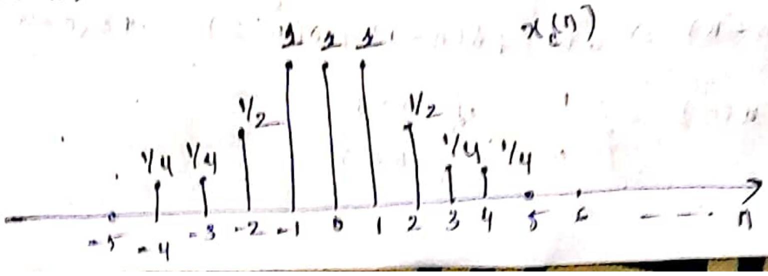
g) even part of $x(n)$

$$x_e(n) = \frac{x(n) + x(-n)}{2} = \{\ldots 0, 1/4, 1/4, 1/2, 1, 1, 1, 1/2, 1/4, 1/4, 0\}$$





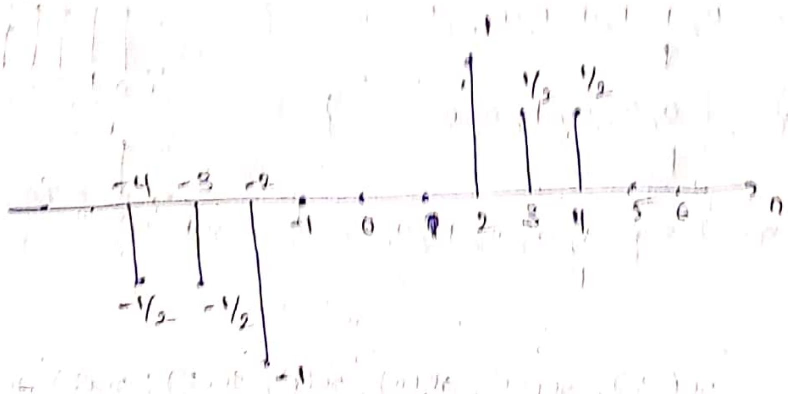




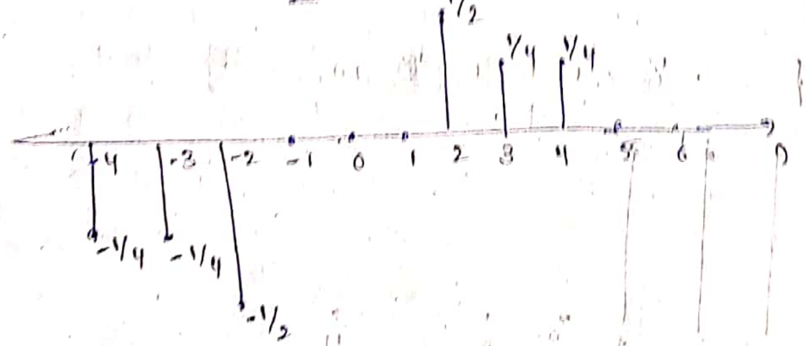
h) odd part of $x(n) \Rightarrow x_o(n) = \frac{x(n) - x(-n)}{2}$

sol:

$x(n) - x(-n)$



$x_o(n) = \frac{x(n) - x(-n)}{2}$



(3) Show that

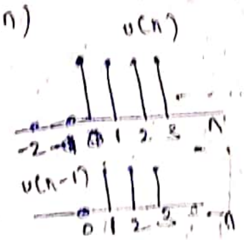
a) $\delta(n) = u(n) - u(n-1)$

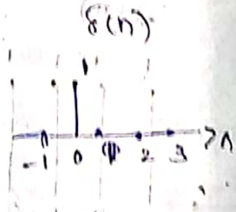
sol: $u(n) - u(n-1) = \{\ldots 0, 1, 0, \ldots\}$

$= \delta(n)$

$u(n) = \{\ldots 0, \underset{\uparrow}{1}, 1, \ldots\}$

$u(n-1) = \{\ldots 0, \underset{\uparrow}{0}, 1, 1, \ldots\}$





$u(n) - u(n-1) = \{\ldots 0, \underset{\uparrow}{1}, 0, \ldots\}$

b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

$\sum_{k=-\infty}^{n} \delta(k) = \delta(-k) + \ldots + \delta(n)$

(1) $\rightarrow$ $u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$

$\sum_{k=0}^{\infty} \delta(n-k) \Rightarrow \delta(n) + \delta(n-1) + \delta(n-2) + \ldots + \delta(n-\infty)$

(2) $\rightarrow$ $u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$

Reason :- 0, ∞ => all positive numbers

if $n-k=0$ => $\delta(0) = 1$;

hence 'n' must be a +ve integer.

hence from ① & ② ;

Hence

$$u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$$

4) Show that any signal can be decomposed in to an even and an odd component. Is the decomposition unique ? Illustrate your arguments using the signal

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

∵ $x_e(-n) = x_e(n)$

$x_o(-n) = -x_o(n)$

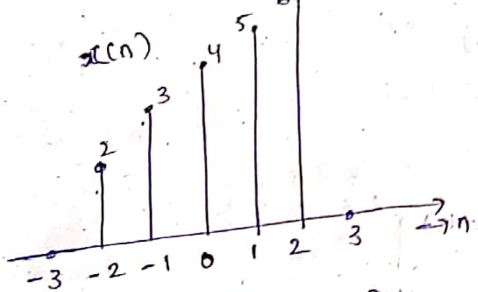
$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

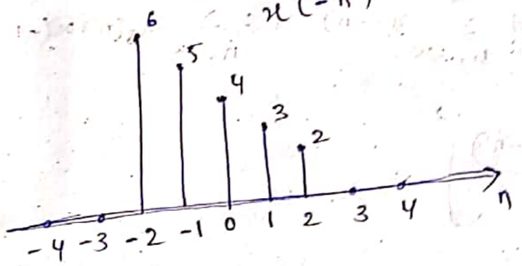
$x(n) = \underbrace{x_e(n) + x_o(n)}$

when decomposition is unique.

x(n)

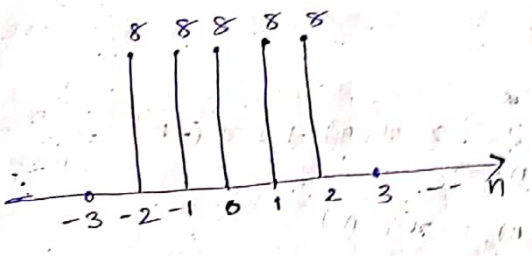


x(-n)



$x(-n) = \{\ldots 0, 6, 5, \underset{\uparrow}{4}, 3, 2, 0, \ldots\}$

x(n) + x(-n)



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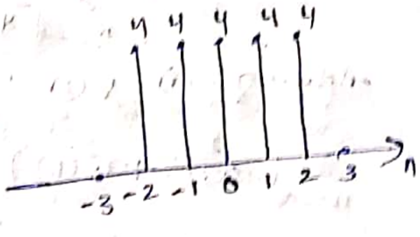
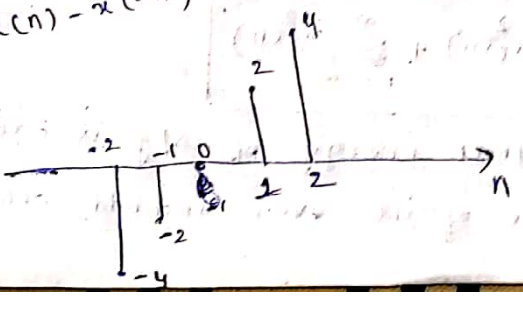
$x_e(n)$



x(n) - x(-n)



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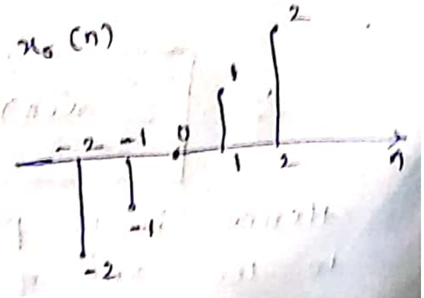
$x_o(n)$



$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$, $x(-n) = \{6, 5, 4, 3, 2\}$

$$x_e(n) = \frac{x(n) + x(-n)}{2} = \frac{\{8, 8, 8, 8, 8\}}{2}$$

$$= \{4, 4, \underset{\uparrow}{4}, 4, 4\}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$x_o(n) = \frac{\{-4, -2, 0, 2, 4\}}{2} = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$$

$$x(n) = x_e(n) + x_o(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

Hence the decomposition is always unique.

5) Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energies (powers) of its even and odd components.

method 1 :

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$\sum_{n=-\infty}^{\infty} x_e^2(n) = \sum_{n=-\infty}^{\infty} \left[\frac{x(n) + x(-n)}{2}\right]^2$$

$$= \frac{1}{4}\left[\sum_{n=-\infty}^{\infty} x^2(n) + \sum_{n=-\infty}^{\infty} x(-n)^2 + 2\sum_{n=-\infty}^{\infty} x_e(n)x(-n)\right] \rightarrow ①$$

Similarly,

$$\sum_{n=-\infty}^{\infty} x_o^2(n) = \sum_{n=-\infty}^{\infty} \left[\frac{x(n) - x(-n)}{2}\right]^2$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{4}\left[x^2(n) + x^2(-n) - 2x(n)x(-n)\right] \rightarrow ②$$

Adding ① & ②

$$\sum_{n=-\infty}^{\infty} \left(x_e^2(n) + x_o^2(n)\right) = \frac{1}{4}\sum_{n=-\infty}^{\infty} 2x^2(n) + 2x^2(-n) = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$\because x^2(n) = x^2(-n)$$

$$\boxed{\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n)}$$

Hence it is proved that energy of s/g is equal to the sum of energies of its even and odd components.

method 2 :- first we prove that

$$\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n) = 0$$

$$\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n) = \sum_{m=-\infty}^{\infty} x_e(-m)\, x_o(-m)$$

$$= -\sum_{m=-\infty}^{\infty} x_e(m)\, x_o(m)$$

$$= -\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n) = 0$$

then

$$\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \sum_{n=-\infty}^{\infty} 2x_e(n)\, x_o(n)$$

$$= E_e + E_o \text{ (sum of energies)}$$

6) Consider the system

$$y(n) = T[x[n]] = x(n^2)$$

a) Determine if the system is time invariant :- No

Given, $y(n) = T[x(n)] = x(n^2)$

$$x(n-k) \rightarrow y_1(n) = x[(n-k)^2]$$

$$= x(n^2 + k^2 - 2nk]$$

$$x(n-k) \neq y(n-k)$$

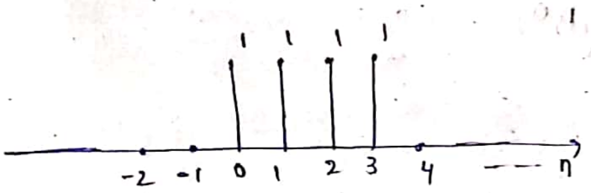
so the given system is time variant.

b) To clarify the result in part (a) assume ~~that the~~ signal

$$x(n) = \begin{cases} 1; & 0 \le n \le 3 \\ 0; & \text{elsewhere} \end{cases}$$ is applied in to the system.

1) sketch the signal $x(n)$.

$$x(n) = \{ \ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, 0 \ldots \}$$



2) Determine and sketch the signal $y(n) = T[x(n)]$;

Given, $y(n) = T[x(n)] = x(n^2)$

$= \{x(0), x(1), x(2^2), x(3^2), x(4^2), \ldots\}$

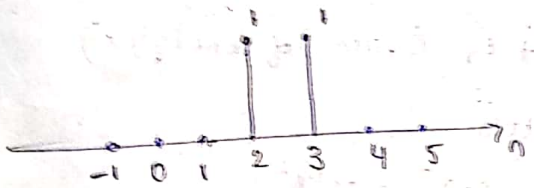
$\Rightarrow \{x(0), x(1), x(4), x(9), x(16), \ldots\}$

$y(n) = x(n^2) = \{\ldots 0, \underset{\uparrow}{1}, 1, 0, 0, 0, \ldots\}$

0 1 2 3 4

3) Sketch the signal $y_2'(n) = y(n-2)$

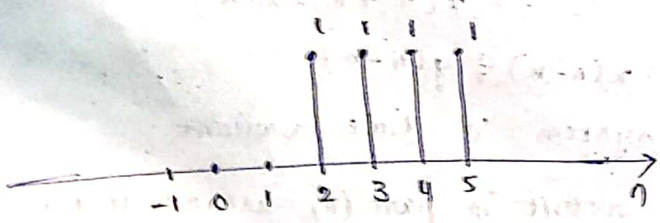
$y(n-2) = \{\ldots\; \underset{\uparrow}{0}, 0, 1, 1, 0, \ldots\}$



4). Determine and sketch the signal $x_2(n) = x(n-2)$

$x(n) = \{\ldots\; 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\}$

$x(n-2) = \{\ldots\; \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0, \ldots\} = x_2(n)$



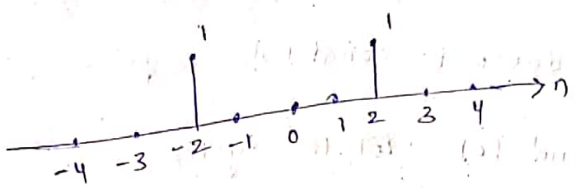
5) Determine and sketch the signal $y_2(n) = T[x_2(n)]$

$y_2(n) = x_2[n^2] = \{\ldots\; y_2(-2), y_2(-1), y_2(0) \ldots\}$

$= \{\ldots\; x((-3)^2), x(16), x(9), x(4), x(1), x(0), x(1), x(4),$

$= \{\ldots\; 0, 0, \underset{\uparrow}{1}, 0, 0, \ldots\; 0, 0,$

$y_2(n) = T[x_2(n-2)] = \{ \ldots 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0, \ldots \}$



$= \{ x(0), x(1), x(2), x(3), x(4), x(5), \ldots \}$

6) Compare the signals $y_2(n)$ and $y(n-2)$. what is your conclusion?

$y_2(n) \neq y(n-2) \Rightarrow$ S/m is time variant.

c) Repeat part (b) for the system

$$y(n) = x(n) - x(n-1)$$

can you use this result to make any statement about the time invariance of this system? why?

sol: $x(n) = \{ \ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots \}$

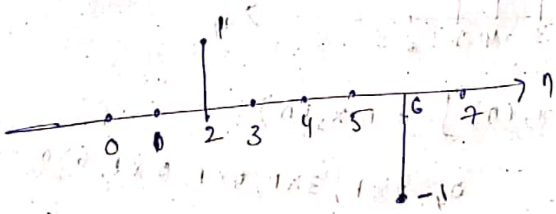
$x(n-1) = \{ \ldots \underset{\uparrow}{0}, 1, 1, 1, 1, 0, \ldots \}$

$y(n) = x(n) - x(n-1)$

$= \{ \ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots \} - \{ \ldots \underset{\uparrow}{0}, 1, 1, 1, 1, 0 \ldots \}$

$= \{ \ldots 0, \underset{\uparrow}{1}, 0, 0, 0, -1, \ldots \}$

$y(n-2) = \{ \ldots \underset{\uparrow}{0}, 0, 1, 0, 0, 0, -1, 0, \ldots \}$



let $x(n-2) = \{ \ldots \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0, \ldots \} = x_2(n)$

$y_2(n) = x_2(n) - x_2(n-1)$

$= \{ \ldots \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0 \ldots \} - \{ \ldots \underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1, 0, \ldots \}$

$= \{ \ldots \underset{\uparrow}{0}, 0, 1, 0, 0, -1, 0, \ldots \}$

$y_2(n) = y(n-2)$. Hence the given system is time-invariant.

By proof that if

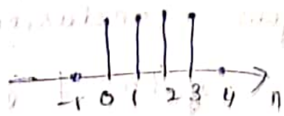
$y(n,k) = y(n-k)$; then it is time invariant.

but this ex alone doesn't constitute a proof.

d) Repeat parts (b) and (c) for the system.

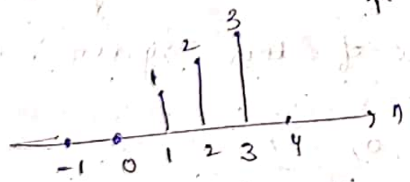
$$y(n) = T[x(n)] = n\,x(n).$$

1) $x(n) = \{ \cdots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \cdots \}$

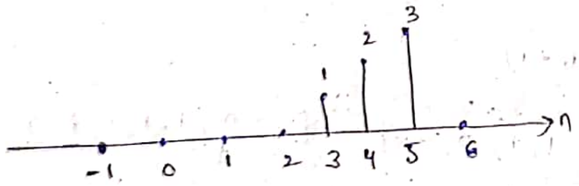


2) $y(n) = n\,x(n) = \{ \cdots 0\times1, 1\times1, 2\times1, 3\times1, 0\times4 \cdots \}$

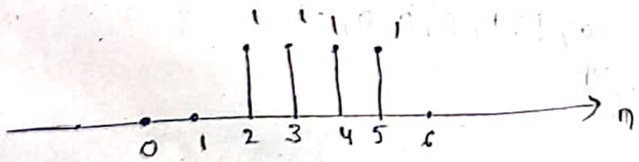
$y(n) = \{ \cdots \underset{\uparrow}{0}, 1, 2, 3, 0, \cdots \}$



3) $y_2'(n) = y(n-2) = \{ \cdots \underset{\uparrow}{0}, 0, 0, 1, 2, 3, 0, \cdots \}$



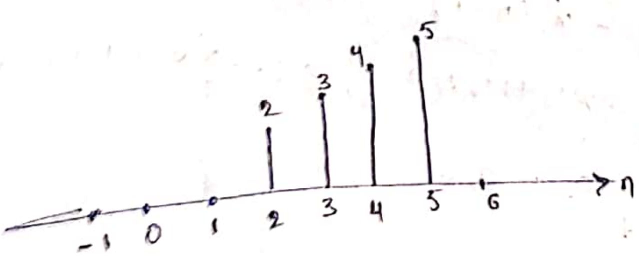
4) $x_2(n) = x(n-2) = \{ \cdots \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0, \cdots \}$



5) $y_2(n) = T[x_2(n)] = n\,x_2(n)$

$= \{ \cdots 0, 2\times1, 3\times1, 4\times1, 5\times1, 6\times0, \cdots \}$

$= \{ \cdots \underset{\uparrow}{0}, 0, 2, 3, 4, 5, 0, \cdots \}$



6] $y_2(n) \neq y(n-2)$.

~~Hence~~ ~~the~~ given system is time variant.

7). A discrete-time system can be

(1) Static & dynamic

(2) Linear & non linear

(3) Time invariant & time varying.

(4) Causal & non causal

(5) stable & unstable.

Examine the following systems with respect to the properties above.

(a) $y(n) = \cos[x(n)]$

1) Static as $y[1] = \cos[x(1)]$

$y[n] = f(x(n))$ only static.

2) $y_1(n) = \cos(x_1(n))$

$y_2(n) = \cos(x_2(n))$

$y_1(n) + y_2(n) = \cos(x_1(n)) + \cos[x_2(n)] = y(n)$

$y'(n) = \cos[x_1(n) + x_2(n)]$

$y(n) \neq y'(n)$

Hence it Non-Linear.

3) Time invariant

① $\leftarrow y(n) = \cos[x(n)] \Rightarrow y(n,k) = \cos[x(n-k)]$

② $\leftarrow y(n-k) = \cos[x(n-k)]$

from ① & ② system is time-invariant.

4) Causal → as static it is causal

$f = \{present\}$

5) Stable as $|x(n)| < \infty$;

$y(n) = \cos[|x(n)|]$ {as cos is always -1 to 1 bounded}

∴ s/m is stable.

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Given,

$y(n) = \sum_{k=-\infty}^{n+1} x(k) = x(-\infty) + \dots + x(n) + x(n+1)$

i.e $y(1) = y(-\infty) + \dots + y(1) + x(2)$

1) dynamic as $y(1)$ depends on $x(2)$

2) Linear , $y_1(n) = \Sigma x_1(n)$;

$y_2(n) = \Sigma x_2(n)$

$y(n) = y_1(n) + y_2(n) = \Sigma (x_1(n) + x_2(n))$

similarly scaling also.

3) Time invariant.

$$y(n) = \sum_{k=-\infty}^{n+1} x(k) \; ; \; y(n,k) = \sum_{k=-\infty}^{n+1-k} x(k)$$

$$\text{lly} \quad y(n-k) = \sum_{k=-\infty}^{n+1-k} x(k)$$

Hence Time Invariant.

4) s/m is Non-causal

$y(1) = y(-\infty) + \dots + y(1), + x(2)$

s/m depends on future values.

5). s/m is unstable because

if $x(k) = u(k)$; the o/p becomes:.

$$y(n) = \sum_{k=-\infty}^{n+1} u(k) = \begin{cases} 0; & n<0 \\ n+2; & n \geq 0 \end{cases}$$

c) $y(n) = x(n) \cos(\omega_0 n)$

1) $y(1) = x(1) \cos(\omega_0)$

$y(2) = x(2) \cos(2\omega_0)$

static.

2) Linear $y_1(n) = x_1(n) \cos(\omega_0 n)$.

$y_2(n) = x_2(n) \cos(\omega_0 n)$

$y(n) = y_1(n) + y_2(n) = [x_1(n) + x_2(n)] \cos(\omega_0 n)$

3) Time variant :

$y(n) = x(n) \cos(\omega_0 n)$

$y(n,k) = x(n-k) \cos(\omega_0 n) \rightarrow (1)$

$y(n-k) = x(n-k)\cos(\omega_0(n-k))$ → ②

$1 \neq 2$; Hence time invariant.

4) static or

sol: Causal as it is static.

5) stable as $|x(n)| < \infty = N$,

$|\cos(\omega_0 n)| = \pm 1$

then $y(n)$ is also stable

d) $y(n) = x(-n+2)$

1) Dynamic, $y(0) = x(2)$; hence dynamic

ii) Linear $y_1(n) = x_1(-n+2)$, $y_2(n) = x_2(-n+2)$

$y(n) = y_1(n) + y_2(n)$

3) Time variant

$y[n] = x(-n+2)$, $y(n,k) = x(-n-k+2)$

$y(n-k) = x(-(n-k)+2) = x(-n+k+2)$

4) Non causal [depends on future i/ps]

$y[0] = x[2]$

5) stable ; as $|x(x)| < \infty$,

e) $y(n) = \text{Trun}[x(n)]$, where $\text{Trun}[x(n)]$ denotes the integer part of $x(n)$; obtained after truncation.

sol:- 1) Static , $y[0] = \text{Trunc}[x(0)]$

ii) Non-linear $y_1(n) = \text{Trunc}[x_1(n)]$

$x_1(n)$ $y_2(n) = \text{Trunc}[x_2(n)]$

$y(n) \neq y_1(n) + y_2(n)$

Eg : 1.6 ; 1.5

o/p 1 1 ; 1+1 = 2

1.6 + 1.5 = 3.1 → o/p = 3.

Hence Non-linear.

3) Time invariant

$y(n,k) = y(n-k)$

$[\text{Trunc}[x(n-k)]] = y(n,k)$

$\text{Trunc}[x(n-k)] = y(n-k)$

Hence time invariant.

4) Causal as it is static

5) stable as bounded i/p gives bounded o/p

f) $y(n) = \text{Round}[x(n)]$, where $\text{Round}[x(n)]$ denotes the integer part of $x(n)$ obtained by rounding.

Sol: similar to above system is static, Non-linear, Time invariant, causal and stable.

Non-linearity let i/p ⇒ 1.6, 1.3

2 + 2 = 4

i/p $(1.6+1.3) = 2.9 \rightarrow 3$

Hence Non-linear.

Remark: The systems in parts (e) and (f) are quantizers that perform truncation and rounding, respectively.

g). $y(n) = |x(n)|$

1) static; $y(0) = |x(0)|$

2) Time invariant; $y(n,k) = y(n-k)$

causal; as it is static

stable; As bounded i/p gives bounded o/p

$x[n] = 1$; $y[n] = 1$;

Non-linear $x_1[n] = 1 \rightarrow y_1[n] = 1$

$x_2[n] = 1 \rightarrow y_2[n] = 1$

$y[n] = |x_1[n] + x_2[n]| \neq y_1[n] + y_2[n]$

S/m is Non-linear.

h) $y(n) = x(n)\,u(n)$

static ; as $y[0] = x(0)\,u(0)$

Time variant

$y(n) = x(n)\,u(n) \rightarrow x(n-k)\,u(n)$

$y(n-k) = x(n-k)\,u(n-k)$

linear, $y_1[n] = x_1[n]\,u(n)$,

$y_2(n) = x_2(n)\,u(n)$

$y(n) = y_1(n) + y_2(n) = [x_1(n) + x_2(n)]\,u(n)$

causal as it is static.

stable

i) $y(n) = x(n) + n\,x(n+1)$

Dynamic - as $y[0] = x(0) + 0$

$y[1] = x(1) + x(2)$

Hence it is dynamic.

Time variant as

$y[n] = x(n) + n\,x(n+1)$

$y(n,k) = x(n-k) + n\,x(n+1-k) \rightarrow (1)$

$y(n-k) = x(n-k) + n-k\;x(n+1-k) \rightarrow (2)$

As (1) $\neq$ (2), hence time variant.

linear ;

$y_1(n) = x_1(n) + n\,x_1(n+1)$

$y_2(n) = x_2(n) + n\,x_2(n+1)$

$y(n) = y_1(n) + y_2(n) = [x_1(n) + x_2(n)] + n\,[x_1(n+1) + x_2(n+1)]$

Non-causal ; as it depends on future i/p values.

stable ; Bounded i/p produces ~~the~~ unbounded o/p.

j) $y(n) = x(2n)$

sol :- * Dynamic ; as $y[2] = x(4)$

* linear ; as $y_1[n] \rightarrow x_1[2n]$

$y_2[n] \rightarrow x_2[2n]$

$y(n) = y_1[n] + y_2[n] = x_1(2n) + x_2(2n)$

⇒ Time variant as

$y(n,k) = x(2n-k)$;

$y(n-k) = x(2n-2k)$

$y(n,k) \neq y(n-k)$.

⇒ Non - causal [depends on future values]

$y(2) = x(4)$.

⇒ Stable ;

$|x(2n)| < N < \infty$;

$|y(n)| < 2$,

bounded.

k) $y(n) = \begin{cases} x(n), & \text{if } x(n) \geq 0 \\ 0, & \text{if } x(n) < 0 \end{cases}$

→ static ; $y(0) = x(0)$, as it depends on present i/p only

→ Non-linear ; let $x_1[n] = 0.3 \Rightarrow y_1[n] = 0.3$

$x_2[n] = -0.7 \Rightarrow y_2[n] = 0$

$y[n] = x_1[n] + x_2[n] = -0.4$

$y[n] \neq y_1[n] + y_2[n]$

→ It is time invariant

$y(n,k) = y(n-k)$

→ Causal as it is static

→ Stable → as bounded i/p produces the bounded o/p

l) $y(n) = x(-n)$

→ Dynamic as $y[-2] = x(2)$

→ linear → $y_1[n] = x_1[-n]$; $y_2[n] = x_2[-n]$

$y[n] = y_1[n] + y_2[n] = x_1[-n] + x_2[-n]$

→ Time variant.

→ Non causal [depends on future values]

→ Stable → as bounded input produces the bounded output.

(m) $y(n) = sign[x(n)]$

1. Static : $y[0] = sign[x(0)]$

$y[1] = sign[x(1)]$

2. Non-linear ; $x_1[n] = 0.3 \rightarrow 1$

$x_2[n] = -0.7 \rightarrow 2$

$x = y_1[n] + y_2[n] = sign[x_1(n)] + sign[x_2(n)]$

$y[n] = sign[x_1(n) + x_2(n)]$ ∴ $y[n] \neq x$

Hence it is Non-linear.

3. Time Invariant

$y(n) = sign[x(n)]$

$y(n, k) = sign[x(n-k)]$

$y(n-k) = sign[x(n-k)]$

4. stable s/m

(n) The ideal sampling system with input $x_a(t)$ and output $x(n) = x_a(nT)$, $-\infty < n < \infty$.

sol: static → $x(0) = x_a(0)$

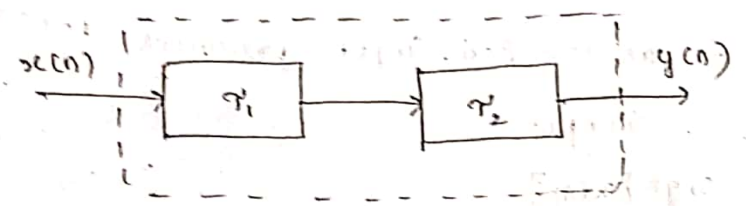
linear → $x_{1a}(nT) + x_{2a}(nT) = y(n)$

Time Invariant

causal as, it is static

stable → bounded input gives the bounded output.

8) Two discrete-time systems $\mathcal{T}_1$ and $\mathcal{T}_2$ are connected in cascade to form a new system $\mathcal{T}$ as shown in fig. Prove or disprove the following statements.



$$\mathcal{T} = \mathcal{T}_1 + \mathcal{T}_2$$

a) If $\mathcal{T}_1$ and $\mathcal{T}_2$ are linear, then $\mathcal{T}$ is ~~time~~ ~~invariant~~ linear (i.e the cascade connection of two linear systems is linear).

sol: True if

$v_1(n) = T_1[x_1(n)]$ and $v_2(n) = T_1[x_2(n)]$

then $\alpha_1 x_1(n) + \alpha_2 x_2(n)$

yields $\alpha_1 v_1(n) + \alpha_2 v_2(n)$

by linearity property of T_1.

lly if $y_1(n) = T_2[v_1(n)]$

$y_2(n) = T_2[v_2(n)]$

then $\beta_1 v_1(n) + \beta_2 v_2(n) \rightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$

by linearity property of $\mathcal{T}_2$.

$v_1(n) = \mathcal{T}_1[x_1(n)]$

$v_2(n) = T_2[x_2(n)]$; if it follows that

$A_1 x_1(n) + A_2 x_2(n)$ yields the output

$A_1 T[x_1(n)] + A_2 T[x_2(n)]$

where $T = \mathcal{T}_1 \mathcal{T}_2$. Hence $\mathcal{T}$ is linear.

b) If $\mathcal{T}_1$ and $\mathcal{T}_2$ are time invariant, then $\mathcal{T}$ is time invariant.

sol: True.

for $\mathcal{T}_1$, if $x(n) \rightarrow v(n)$ &

$x(n-k) \rightarrow v(n-k)$

for T_2, if

$$x(n) \rightarrow y(n) \text{ and}$$

$$x(n-k) \rightarrow y(n-k)$$

$\therefore$ $T = T_1 T_2$ is time invariant.

c) If T_1 and T_2 are causal, the T is causal.

sol: True. T_1 is causal $\Rightarrow$ $v(n)$ depends only on $x(k)$ for $k \leq n$

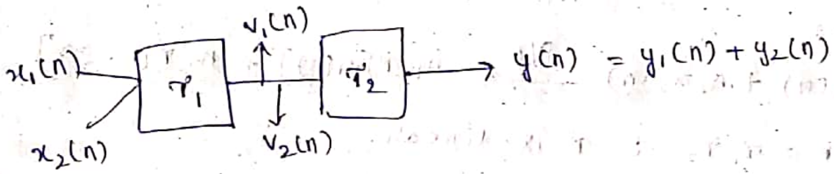
T_2 is causal $\Rightarrow$ $y(n)$ depends only on $v(k)$ for $k \leq n$.

Therefore, $y(n)$ depends only on $x(k)$ for $k \leq n$.

Hence, T is causal.

d) If T_1 and T_2 are linear and time invariant, the same holds for T.

sol:



$$\alpha_1 x_1(n) \xrightarrow{T_1} \alpha_1 v_1(n)$$

$$\alpha_2 x_2(n) \xrightarrow{T_1} \alpha_2 v_2(n)$$

$$\alpha_1 x_1(n) + \alpha_2 x_2(n) \xrightarrow[\text{linear}]{T_1} \alpha_1 v_1(n) + \alpha_2 v_2(n)$$

$$\alpha_1 x_1(n-k) + \alpha_2 x_2(n-k) \xrightarrow[T_1]{LTI} \alpha_1 v_1(n-k) + \alpha_2 v_2(n-k)$$

Actual

$$\alpha_1 v_1(n-k) + \alpha_2 v_2(n-k) \xrightarrow[\text{as LTI}]{T_2} \alpha_1 y_1(n-k) + \alpha_2 y_2(n-k)$$

for T_2 $\quad v_1(n) \xrightarrow{T_2} y_1(n)$

$$v_2(n) \xrightarrow{T_2} y_2(n)$$

$$\beta_1 v_1(n) + \beta_2 v_2(n) \xrightarrow{T_2} \beta_1 y_1(n) + \beta_2 y_2(n)$$

$$\beta_1 v_1(n-k) + \beta_2 v_2(n-k) \xrightarrow{T_2} \beta_1 y_1(n-k) + \beta_2 y_2(n-k)$$

Hence for $T_1 T_2$ if $x(n) \rightarrow y(n)$ and,

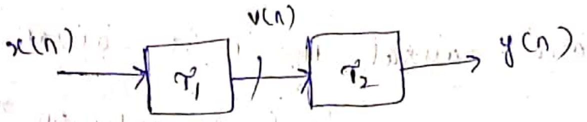
$$x(n-k) \rightarrow y(n-k).$$

$\therefore$ $T = T_1 T_2$ is time invariant

$\rightarrow$ linear.

e) If T_1 and T_2 are linear and time invariant, then interchanging their order does not change the system T.

Sol: True. This follows from $h_1(n) * h_2(n) = h_2(n) * h_1(n)$



i) 1) $v(n) \rightarrow T_1[x(n)]$

$\alpha_1 x_1(n) + \alpha_2 x_2(n) \xrightarrow{L} \alpha_1 v_1(n) + \alpha_2 v_2(n)$

$\alpha_1 x_1(n-k) + \alpha_2 x_2(n-k) \xrightarrow{LTI} \alpha_1 v_1(n-k) + \alpha_2 v_2(n-k)$

2) $y(n) \rightarrow T_2[v(n)]$

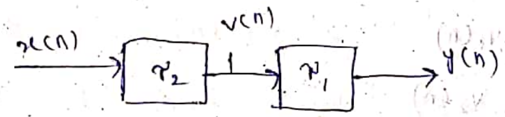
$\beta_1 v_1(n) + \beta_2 v_2(n) \xrightarrow{L} \beta_1 y_1(n) + \beta_2 y_2(n)$

$\beta_1 v_1(n-k) + \beta_2 v_2(n-k) \xrightarrow{LTI} \beta_1 y_1(n-k) + \beta_2 y_2(n-k)$

$A_1 x_1(n) + A_2 x_2(n) \xrightarrow{o/p} A_1 T[x_1(n)] + A_2 T[x_2(n)]$

$T = T_1 T_2$; T is linear.

ii)



$v(n) \rightarrow T_2[x(n)]$

$\alpha_1 x_1(n) + \alpha_2 x_2(n) \rightarrow \alpha_1 v_1(n) + \alpha_2 v_2(n)$

$y(n) \rightarrow T_1[v(n)] = T_1[T_2[x(n)]]$

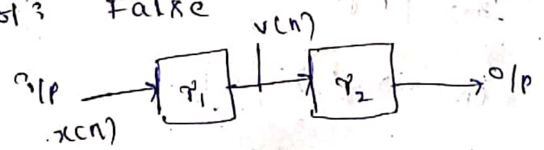
$= T_1[\alpha_1 v_1(n) + \alpha_2 v_2(n)]$

$= y_1(n) + y_2(n) = T_1 T_2[\alpha_1 x_1(n) + \alpha_2 x_2(n)]$

yields same output.

f) As in part (e) except that T_1, T_2 are now time varying. (Hint: use an example)

Sol: False



$v(n) \rightarrow T_1[x(n)]$

$\alpha_1 x_1(n) + \alpha_2 x_2(n) \xrightarrow{L} \alpha_1 v_1(n) + \alpha_2 v_2(n)$

$\alpha_1 x_1(n-k) + \alpha_2 x_2(n-k) \longrightarrow \alpha_1 v_1(n-g) + \alpha_2 v_2(n-g)$.

$k \neq g$.

$y(n) \longrightarrow \tau_2[v(n)]$

$\alpha_1 v_1(n-g) + \alpha_2 v_2(n-g) \longrightarrow \alpha_1 y_1(n-h) + \alpha_2 y_2(n-h)$

$g \neq h$.

Similarly if we reverse the blocks, then overall output changes accordingly.

Ex: consider

$\tau_1 : y(n) = n\,x(n)$

$\tau_2 : y(n) = n\,x(n+1)$.

$\tau_2[\tau_1[\delta(n)]] = \tau_2(0) = 0$

$\tau_1[\tau_2[\delta(n)]] = \tau_1[\delta(n+1)]$

$= -\delta(n+1)$

$\neq 0$.

g) If τ_1 and τ_2 are nonlinear, then τ is nonlinear.

Sol: False.

Ex: $\tau_1 : y(n) = x(n) + b$

$\tau_2 : y(n) = x(n) - b$ where $b \neq 0$

$\tau[x(n)] = \tau_2[\tau_1[x(n)]]$

$= \tau_2[x(n) + b]$

$= x(n) + \cancel{b} - \cancel{b}$

$= x(n)$; linear

h) If τ_1 and τ_2 are stable, then τ is stable.

Sol: True,

τ_1 is stable $\Rightarrow v(n)$ is bounded if $x(n)$ is bounded

τ_2 is stable $\Rightarrow y(n)$ is bounded if $v(n)$ is bounded.

Hence, $y(n)$ is bounded if $x(n)$ is bounded $\Rightarrow \tau = \tau_1 \tau_2$ is stable.

i) Show by an example that the inverse of parts (c) and (h) do not hold in general.

Inverse of (c): τ_1 and τ_2 are non causal then τ is non-causal.

Ex: τ_1 : $y(n) = x(n+1)$

τ_2 : $y(n) = x(n-2)$

τ : $y(n) = x(n-1)$

which is causal. Hence, the inverse of (c) is false.

Inverse of (h) ∴ τ_1 and τ_2 are unstable, implies

τ is unstable.

Ex: τ_1 : $y(n) = e^{x(n)}$, stable

τ_2 : $y(n) = \ln[x(n)]$, which is unstable.

but τ : $y(n) = x(n)$, which is stable

Hence, the inverse of (h) is false.

9) Let τ be an LTI, relaxed, and BIBO stable system with input $x(n)$ and output $y(n)$.

Show that:

a) If $x(n)$ is periodic with period N [i.e., $x(n) = x(n+N)$ for all $n \geq 0$], the output $y(n)$ tends to a periodic signal with the same period.

Sol: $y(n) = \sum_{k=-\infty}^{n} h(k)x(n-k)$, $x(n) = 0$, $n<0$

$= \sum_{k=0}^{\infty} x(k)h[n-k]$ let $\sum_{m=0}^{\infty} x(m)h(n-m)$

$n-m = k$ $m = \infty$

at $m = 0$ $n - \infty = k$

$k = n$ $k = -\infty$ ($\because n = 0$)

$$y(n) = \sum_{k=-\infty}^{n} x(n-m)h(m)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k)x(n+N-k)$$

$$= \sum_{k=-\infty}^{n+N} h(k)x(n-k),$$ as $x(n) = x(n+N)$ periodic with period N.

$$= \sum_{k=-\infty}^{n} h(k)x(n-k) + \sum_{k=n+1}^{n+N} h(k)x(n-k)$$

$$= y(n) + \sum_{k=n+1}^{n+N} h(k)x(n-k).$$

for a BIBO system; $\lim_{n\to\infty} |h(n)| = 0$.

$$\therefore \lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k).x(n-k) = 0$$

$$\lim_{N\to\infty} \sum_{k=n+1}^{n+N} h(k)x(n-k) = 0 \text{ and}$$

$$\lim_{N\to\infty} y(n+N) = y(N)$$

$$\lim_{n\to\infty} y(n+N) = y(N)$$

b) If $x(n)$ is bounded and tends to a constant, the output will also tend to a constant.

Sol :- let $x(n) = x_0(n) + a\,u(n)$, where a is a constant

$x_0(n)$ is a bounded signal with $\lim_{n\to\infty} x_0(n) = 0$

$$y(n) = a\sum_{k=0}^{\infty} h(k)u(n-k) + \sum_{k=0}^{\infty} h(k)x_0(n-k)$$

$$= a\sum_{k=0}^{n} h(k) + y_0(n)$$

clearly, $\sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty$

Hence $\lim_{n\to\infty} |y_0(n)| = 0$.

thus $\lim_{n\to\infty} y(n) = a\sum_{k=0}^{n} h(k) = \text{constant}.$

c) If $x(n)$ is an energy signal, the output $y(n)$ will also be an energy signal.

Sol: $y(n) = \sum_k h(k)x(n-k)$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty}\left[\sum_k h(k)x(n-k)\right]^2$$

$$= \sum_k \sum_l h(k)h(l) \sum_n x(n-k)x(n-l)$$

But $\sum_n x(n-k)x(n-l) \le \sum_n x^2(n) = E_x$

$$\therefore \sum_n y^2(n) \le E_x \sum_k |h(k)| \sum_l |h(l)|.$$

for a BIBO stable system

$$\sum_k |h(k)| < M$$

$E_y \leq M^2 E_x$, so that

$E_y < 0$ if $E_x < 0$.

10) The following input-output pairs have been observed during the operation of a time-invariant system

$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{T} y_1(n) = \{0, \underset{\uparrow}{1}, 2\}$

$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} y_2(n) = \{0, \underset{\uparrow}{1}, 0, 2\}$

$x_3(n) = \{0, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$

Can you draw any conclusions ~~about the~~ regarding the linearity of the system. what is the impulse response of the system.

Sol:- The given system is non-linear.

Reason : Coming to $x_2(n)$ and $x_3(n)$

~~Sol~~:- The $x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xrightarrow{T} \{0, \underset{\uparrow}{1}, 0, 2\}$

$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xrightarrow{T} \{1, \underset{\uparrow}{2}, 1\}$

$x_3(n+1) = \{\underset{\uparrow}{0}, 0, 1\} \longrightarrow \{\underset{\uparrow}{1}, 2, \underset{\uparrow}{1}\}$

Now if the system is linear then

$3 * x_3(n+1) \longrightarrow \{\underset{\uparrow}{3}, 6, 3\}$

$\{\underset{\uparrow}{0}, 0, 3\}$

But $\{3, 6, 3\} \neq \{0, 1, 0, 2\}$

Hence the system is Non-linear.

11) The following input-output pairs have been observed during the operation of a linear system

$x_1(n) = \{-1, \underset{\uparrow}{2}, 1\} \xleftrightarrow{T} y_1(n) = \{1, \underset{\uparrow}{2}, -1, 0, 1\}$

$x_2(n) = \{1, \underset{\uparrow}{-1}, -1\} \xleftrightarrow{T} y_2(n) = \{-1, \underset{\uparrow}{1}, 0, 2\}$

$x_3(n) = \{0, \underset{\uparrow}{1}, 1\} \xleftrightarrow{T} y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$

Can you draw any conclusions about the time invariance of this system?

Sol: $x_1(n) + x_2(n) = \delta(n)$ and the system is linear

The impulse response of the system is given

by $y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$

If the system is further time-invariant; then it would

be an LTI system.

then $x_3(n) * h(n) = \{0, \underset{\uparrow}{1}, 1\} * \{0, \underset{\uparrow}{3}, -1, 2, 1\}$

$y(n) = \{3, 2, 1, 3, 1\}$

But the given o/p $y_3(n) = \{1, 2, 1\}$

Hence the system is time <u>variant</u>.

12) The only available information about a system

consists of N input-output pairs of signals

$y_i(n) = \mathcal{T}[x_i(n)]$, $i = 1, 2, \ldots, N$.

a) what is the class of input signals for which we can

determine the output, using the information above, if

the system is known to be ~~time invariant~~ linear

Sol:- Any weighted linear combination of the signals

$x_i(n) \Rightarrow i = 1, 2, \ldots, N$

b) The same as above, if the system is known to be

time invariant.

Sol: Any $x_i(n-k)$, where k is any integer and $i = 1, 2, \ldots, N$

13) show that the neccessary and sufficient condition for a

relaxed LTI system to be BIBO stable is.

$$\sum_{n=-\infty}^{\infty} |h(n)| \le M_h < \infty \quad \text{for some constant } M_h.$$

A system is BIBO stable if and only if a bounded

input produces a bounded output.

$y(n) = \sum_k h(k)\, x(n-k)$

$$|y(n)| \le \sum_k |h(k)||x(n-k)|$$

$$\le M_x \sum_k |h(k)|$$

where $|x(n-k)| \le M_x$.

$\therefore$ $|y(n)| < \infty$ for all n, if and only if

$$\sum_k |h(k)| < \infty.$$

14) Show that :

a) A relaxed linear system is causal if and only if for any input $x(n)$ such that

$$x(n) = 0 \text{ for } n < n_0 \Rightarrow y(n) = 0 \text{ for } n < n_0$$

Sol :- A system is causal $\Leftrightarrow$ the output becomes non zero after the input becomes non-zero. Hence,

$$x(n) = 0 \text{ for } n < n_0 \Rightarrow y(n) = 0 \text{ for } n < n_0.$$

b) A relaxed LTI system is causal if and only if

$$h(n) = 0 \quad \text{for } n < 0.$$

sol :- $y(n) = \sum_{-\infty}^{\infty} h(k)x(n-k)$, where $x(n) = 0$ for $n < 0$.

If $h(k) = 0$ for $k < 0$, then.

$y(n) = \sum_{0}^{n} h(k)x(n-k)$, and hence, $y(n) = 0$ for $n < 0$.

on the other hand, if $y(n) = 0$ for $n < 0$, then.

$$\sum_{-\infty}^{n} h(k)x(n-k) \Rightarrow h(k) = 0, \; k < 0.$$

15) a) Show that, for any real or complex constant a, and any finite integer numbers M and N, we have

$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M - a^{N+1}}{1-a} & \text{if } a \ne 1 \\ N-M+1 & \text{if } a = 1 \end{cases}$$

sol: for $a = 1$, $\sum_{n=M}^{N} a^n = N - M + 1$,

for $a \ne 1$, $\sum_{n=M}^{N} a^n = a^M + a^{M+1} + \cdots + a^N$

$$(1-a)\sum_{n=M}^{N} a^n = a^M + a^{M+1} - a^{M+1} + \cdots + a^N - a^N - a^{N+1}$$

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$= a^{M} - a^{N+1}$

b) Show that if $|a| < 1$, then

$$\sum_{n=0}^{\infty} a^{n} = \frac{1}{1-a}$$

Sol: for $M=0$, $|a| < 1$, $N \to \infty$

$$\sum_{n=0}^{\infty} a^{n} = \frac{1}{1-a}, \quad |a| < 1.$$

16) a) If $y(n) = x(n) * h(n)$, show that $\Sigma_y = E_x E_h$

where $E_x = \sum_{n=-\infty}^{\infty} x(n)$.

Sol: method 1

$$y(-\infty) + \dots + y(1) + y(2) + \dots y(\infty) = \sum_{k=-\infty}^{\infty} x(k) h(-\infty - k) + \dots + \sum_{k=-\infty}^{\infty} x(k) h(\infty - k)$$

$$= \sum_{k=-\infty}^{\infty} x(k) \left[h(-\infty - k) + \dots + h(\infty - k) \right]$$

$$\sum_n y(n) = \sum_{k=-\infty}^{\infty} x(k) \sum_{n=-\infty}^{\infty} h(k)$$

$$= \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)$$

method 2:-

$$y(n) = \sum_k h(k) x(n-k)$$

$$\sum_n y(n) = \sum_n \sum_k h(k) x(n-k) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k)$$

$$= \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)$$

b). Compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the corresponding correctness of the results by using the test in (a).

1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

Sol: $y(n) = h(n) * x(n) = \{1, 3, 7, 7, 7, 6, 4\}$

$h(n) \backslash x(n)$	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

$\sum_n y(n) = 35$,

$\sum_k h(k) \sum_n x(n) = 7 \times 5$

$\sum_k h(k) = 5$, $\sum_k x(k) = 7$

2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

Sol: $y(n) = \{1, 4, 2, -4, 1\}$

	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

$\sum y(n) = \sum x(n) \sum h(n) \Rightarrow 4 = 4$.

3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{1/2, 1/2, 1, 1/2\}$ by tabulation method

$y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, -2\}$

h(n) \ x(n)	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

$\sum_n y(n) = -5$

$\sum_n x(n) = -2$;

$\sum_n h(n) = 5/2$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$-5 = -2 \times \frac{5}{2}$

$-5 = -5$

4). $x(n) = \{1, 2, 3, 4, 5\}$; $h(n) = \{1\}$

$y(n) = x(n) * h(n)$

$y(n) = \{1, 2, 3, 4, 5\}$

h(n) \ x(n)	1	2	3	4	5
1	1	2	3	4	5

$\sum_n y(n) = 15$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$15 = 15(1)$

5) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$, $h(n) = \{0, 0, 1, 1, 1, 1\}$

$y(n) = x(n) * h(n)$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

h(n) \ x(n)	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$\sum_n y(n) = 8$

$\sum_n x(n) = 2$; $\sum_n h(n) = 4$.

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 8$.

6). $x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1\}$, $h(n) = \{1, -2, 3\}$

$y(n) = x(n) * h(n)$

$y(n) = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 1, 3\}$

h(n) \ x(n)	0	0	1	1	1
1	0	0	1	1	1
-2	0	0	-2	-2	-2
3	0	0	3	3	3

$\sum_n y(n) = 5$

7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{1, 0, -1, -1\}$

$y(n) = x(n) * h(n)$

$y(n) = \{\underset{\uparrow}{0}, 1, 4, -4, -5, -1, 3\}$

$\sum_n y(n) = -2$;

$\sum_n x(n) = -2$; $\sum_n h(n) = 1$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

h(n) \ x(n)	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$

$y(n) = x(n) * h(n)$

$= \{\underset{\uparrow}{1}, 2, 4, 3, 2\}$

$\sum_n y(n) = 12$; $\sum_n x(n) = 4$;

$\sum_n h(n) = 3$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$12 = 4 \times 3$

$12 = 12$.

h(n) \ x(n)	1	1	2
1	1	1	2
1	1	1	2
1	1	1	2

9) $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h(n) = \{1, -2, -3, 4\}$

$y(n) = x(n) * h(n)$

$y(n) = \{1, -1, -5, 2, 3, \underset{\uparrow}{-5}, 1, 4\}$

$\sum_n y(n) = 0$; $\sum_n h(n) = 0$, $\sum_n x(n) = 4$.

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$0 = 0$

h(n) \ x(n)	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	+4	4	0	4	4

10) $x(n) = \{1, 2, 0, 2, 1\}$, $h(n) = x(n)$

$y(n) = x(n) * h(n)$

$y(n) = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$

$\sum_n y(n) = 36$; $\sum_n x(n) = 6$

$\sum_n h(n) = 6$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$36 = 6 \times 6$

h(n) \ x(n)	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

11) $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$.

$$y(n) = \sum_{k=-\infty}^{\infty} (1/2)^k u(k) (1/4)^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} (1/2)^k (1/4)^{n-k} = \sum_{k=0}^{n} (1/2)^{2n-k}$$

$$y(n) = (1/2)^{2n} \sum_{k=0}^{n} 2^k$$

$$= (1/2)^{2n} \frac{1-2^{n+1}}{1-2}$$

$$= (1/2)^{2n} \left(2^{n+1}-1\right)$$

$$\sum_n h(n) \Rightarrow \frac{1}{1-1/4} = 4/3$$

$$\sum_n x(n) = 2.$$

$$\sum_n y(n) = 8/3$$

$$\Rightarrow 2^{-n+1} - 2^{-2n} \Rightarrow \left[2(1/2)^n - (1/4)^{4n}\right]^{-n} u(n).$$

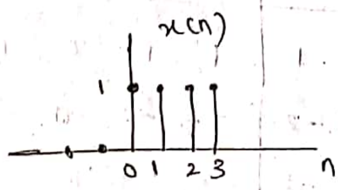
$2(2) -$

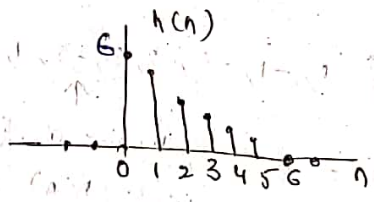
$$\Rightarrow 2 \frac{1}{1-1/2} - \frac{1}{1-1/4}$$

$$2(2) - 4/3 \Rightarrow 8/3$$

17) Compute and plot the convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signals shown in fig.

a)





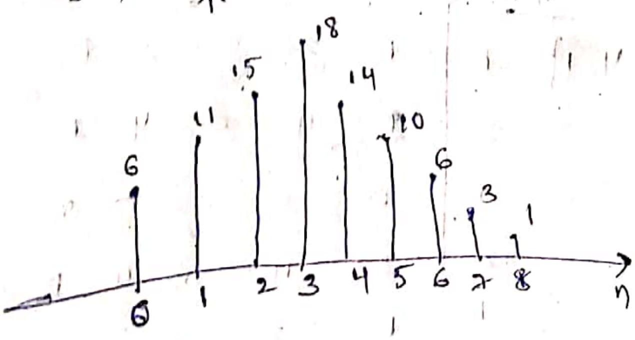
$x(n) = \{1, 1, 1, 1\}$
$\uparrow$

$h(n) = \{6, 5, 4, 3, 2, 1, 0\}$
$\uparrow$

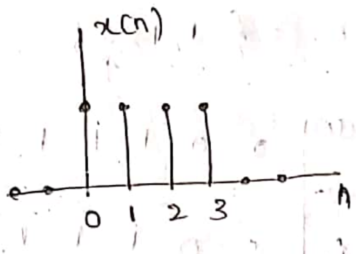
$y(n) = x(n) * h(n)$

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$
$\uparrow$

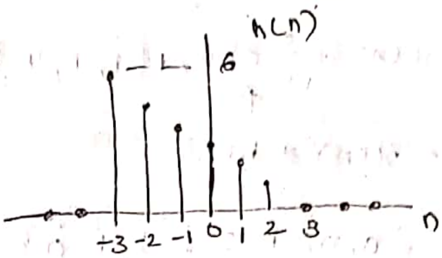
h(n) \ x(n)	1	1	1	1
6	6	6	6	6
5	5	5	5	5
4	4	4	4	4
3	3	3	3	3
2	2	2	2	2
1	1	1	1	1
0	0	0	0	0



b)



$*$

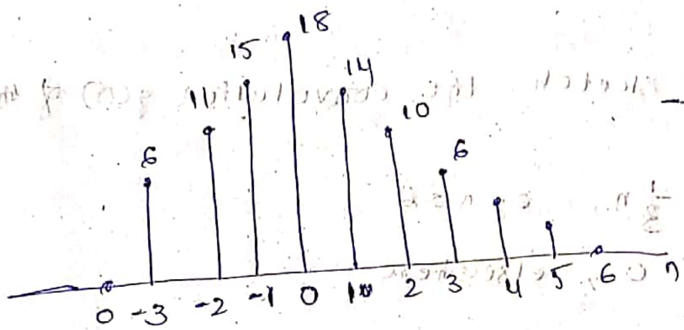


$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$ $\qquad$ $h(n) = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1, 0\}$

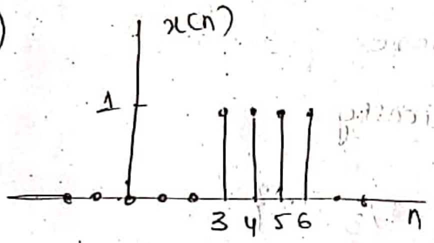
$y(n) = x(n) * h(n)$

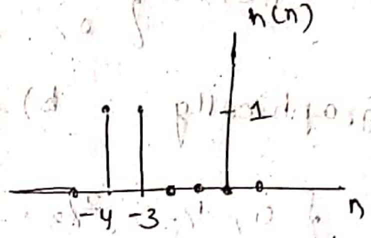
$y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$

h(n) \ x(n)	1	1	1	1
6	6	6	6	6
5	5	5	5	5
4	4	4	4	4
→ 3	3	3	3	3
2	2	2	2	2
1	1	1	1	1
0				



c)



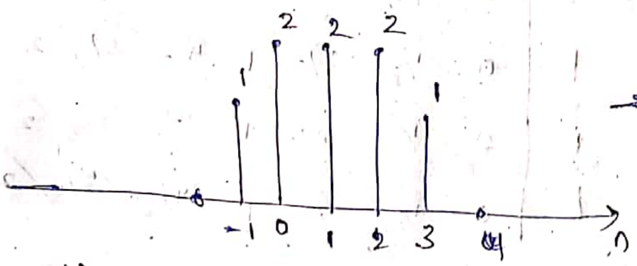


$x(n) = \{\underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1, 0 \ldots\}$ $\qquad$ $h(n) = \{\ldots 0, 1, 1, 0, 0, \underset{\uparrow}{0}, \ldots\}$

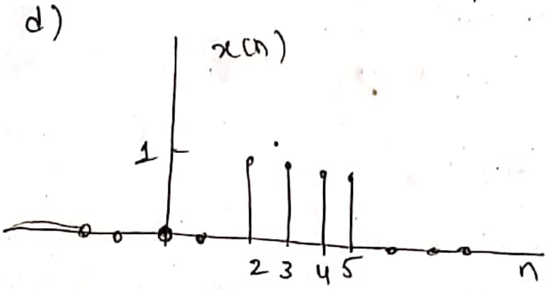
$y(n) = x(n) * h(n)$

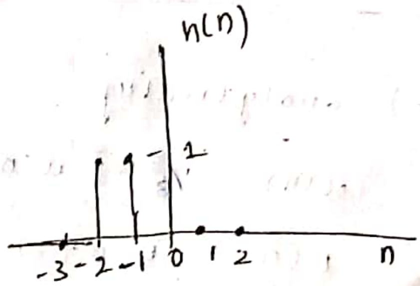
$y(n) = \{0, 0, 0, 1, \underset{\uparrow}{2}, 2, 2, 1, 0, 0, 0, \ldots\}$

h(n) \ x(n)	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
→ 0	0	0	0	0	0	0	0



d)



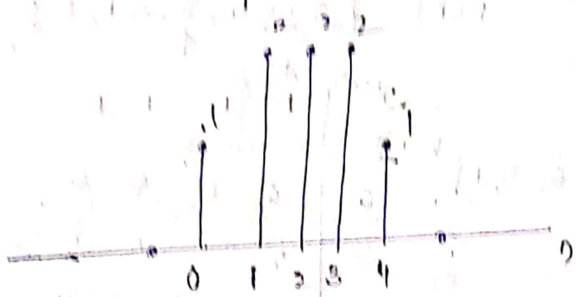


$x(n) = \{0, 0, 1, 1, 1, 1\}$ $\quad h(n) = \{1, 1, 0\}$

$y(n) = x(n) * h(n)$

$y(n) = \{0, 0, 1, 2, 2, 2, 1, 0\}$

$h(n)$ \ $x(n)$	0	0	1	1	1	1
1	0	0	1	1	1	1
1	0	0	1	1	1	1
0	0	0	0	0	0	0



18) Determine and sketch the convolution $y(n)$ of the signals.

$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

a) Graphically b) Analytically.

Sol:- $x(n) = \{0, 1/3, 2/3, 1, 4/3, 5/3, 2\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, 1/3, 1, 2, 10/3, 5, 20/3, 6, 5, 11/3, 2\}$

$h(n)$ \ $x(n)$	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2

b) analytically

$x(n) = \frac{1}{3} n [u(n) - u(n-7)]$

$h(n) = u(n+2) - u(n-3)$

$$y(n) = \frac{1}{3} n [u(n) - u(n-7)] * u(n+2) - u(n-3)$$

$$y(n) = \frac{1}{3} n\, u(n) * u(n+2) - \frac{1}{3} n\, u(n) * u(n-3)$$

$$- \frac{1}{3} n\, u(n-7) * u(n+2) + \frac{1}{3} n\, u(n-7) * u(n-3).$$

$y(n) =$

19) Compute the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \alpha^n, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

sol:-

$$x(n) = \{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5\}$$

$$h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$$

$$y(n) = \sum_{k=0}^{4} h(k)x(n-k)$$

$$y(n) = \sum_{k=0}^{4} x(n-k), \quad -3 \le n \le 9$$

$\therefore$ $y(-3) = \alpha^{-3}$

$y(-2) = x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}$

$y(-1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1}$

$y(0) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1$

$y(1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha$

$y(2) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^2$

$y(3) = \cancel{\alpha^{-3}} + \cancel{\alpha^{-2}} + \alpha^{-1} + 1 + \alpha + \alpha^2, \alpha^3$

$y(4) = \cancel{\alpha^{-3}} + \cancel{\alpha^{-2}} + \cancel{\alpha^{-1}} + 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4$

$y(5) = \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

$y(6) = \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

$y(7) = \alpha^3 + \alpha^4 + \alpha^5$ $\quad y(8) = \alpha^4 + \alpha^5$, $\quad y(9) = \alpha^5$

20) Consider the following three operations.

a) Multiply the integer numbers 131 and 122.

Sol: $131 \times 122 = 15982$

b) Compute the convolution of signals $\{1,3,1\} * \{1,2,2\}$

$y(n) = \{1, 5, 9, 8, 2\}$

	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

c) Multiply the polynomials:

$1 + 3z + z^2$ and $1 + 2z + 2z^2$

$(z^2 + 3z + 1) \times (2z^2 + 2z + 1)$

$\Rightarrow \quad 2z^4 + 6z^3 + 2z^2 + 2z^3 + 6z^2 + 2z + z^2 + 3z + 1$

$\Rightarrow \quad 2z^4 + 8z^3 + 9z^2 + 5z + 1$

d) Repeat part (a) for the numbers 1.31 and 12.2.

$1.31 \times 12.2 = 15.982$

e) Comment on your result.

These are different ways to perform convolution.

21) Compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

a) $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ when $a \neq b$ and when $a = b$

$y(n) = x(n) * h(n)$

$= a^n u(n) * b^n u(n)$

$= [a^n * b^n] u(n)$

$y(n) = \sum_{k=0}^{n} a^k u(k)\, b^{n-k} u(n-k)$

$= b^n \sum_{k=0}^{n} a^k u(k) b^{-k} = b^n \sum_{k=0}^{n} (a/b)^k$

$= b^n \sum_{k=0}^{n} (ab^{-1})^k \qquad b^n \left[\frac{1 - (a/b)^{n+1}}{1 - a/b} \right]$

if $a \neq b$, then $y(n) = \frac{b^{n+1} - a^{n+1}}{b - a} u(n) = \frac{b^{n+1}\left[1 - (a/b)^{n+1}\right]}{b - a}$

if $a = b \Rightarrow b^n (n+1) u(n)$

b) $x(n) = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0, & \text{elsewhere} \end{cases}$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

$x(n) = \{1, 2, \underset{\uparrow}{1}, 1\}$

$h(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 1, 1\}$

x(n) \ h(n)	1	-1	0	0	1	1
1	1	-1	0	0	1	1
2	2	-2	0	0	2	2
1	1	-1	0	0	1	1
1	1	-1	0	0	1	1

$y(n) = x(n) * h(n)$

$= \{1, 1, \underset{\uparrow}{-1}, 0, 0, 3, 3, 2, 1\}$

c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$

$h(n) = [u(n+2) - u(n-3)].(3 - |n|)$

sol :- $x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, -1\}$

	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

d) $x(n) = u(n) - u(n-5)$; $h(n) = u(n-2) - u(n-8) + u(n-11) - u(n-17)$

sol : $x(n) = \{0, \underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h'(n) = \{0, 0, 1, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$,

$y(n) = y'(n) + y'(n-9)$, where

$y'(n) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

22) let $x(n)$ be the input signal to a discrete-time filter with impulses response $h_i(n)$ and let $y_i(n)$ be the corresponding output.

a) Compute and sketch $x(n)$ and $y_i(n)$ in the following cases, using the same scale in all figures.

$x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1(n) = \{1, 1\}$

$h_2(n) = \{1, 2, 1\}$

$h_3(n) = \{1/2, 1/2\}$

$h_4(n) = \{1/4, 1/2, 1/4\}$

$h_5(n) = \{1/4, -1/2, 1/4\}$

Sketch $x(n), y_1(n), y_2(n)$ on one graph and $x(n), y_3(n), y_4(n), y_5(n)$ on another graph.

$y_1(n) = x(n) * h_1(n)$

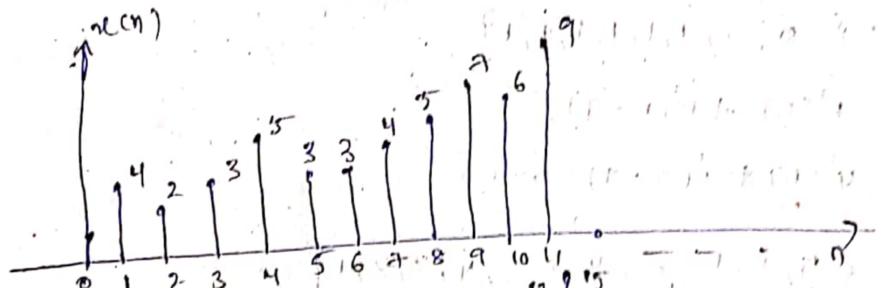
$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7
1	1	4	2	3	5	3	3	4	5	7
1	1	4	2	3	5	3	3	4	5	7

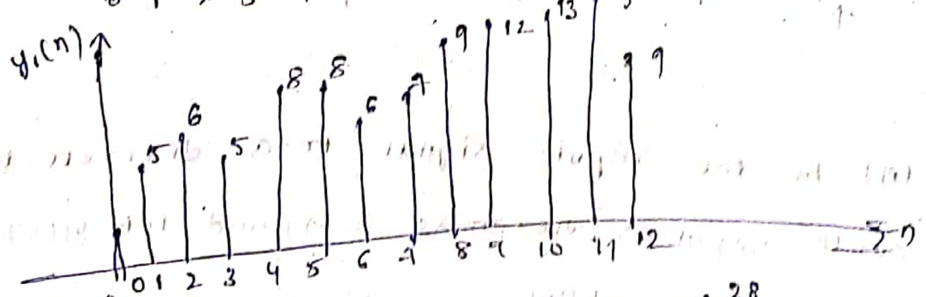
$y_1(n) = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 13, 15, 9\}$

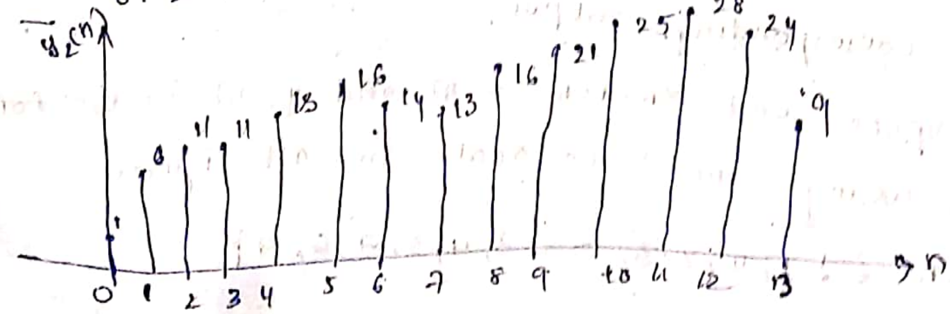
$y_2(n) = x(n) * h_2(n)$

\ $x(n)$	1	4	2	3	5	3	3	4	5	7	6
1	1	4	2	3	5	3	3	4	5	7	6
2	2	8	4	6	10	6	6	8	10	14	12
1	1	4	2	3	5	3	3	4	5	7	6

$y_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$



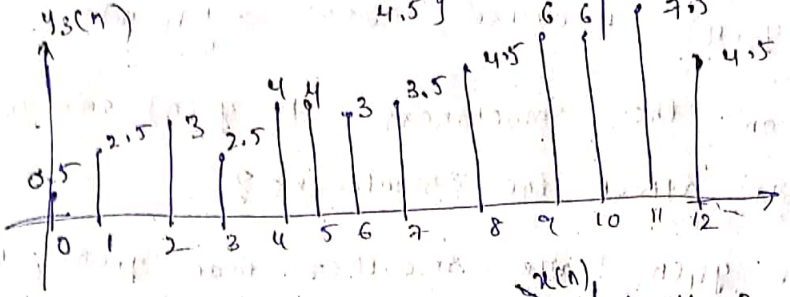




$y_3(n) = x(n) * h_3(n)$

	1	4	2	3	5	3	3	4	5	7	6	9
1/2	1/2	2	1	3/2	5/2	3/2	3/2	2	5/2	7/2	3	9/2
1/2	1/2	2	1	3/2	5/2	3/2	3/2	2	5/2	7/2	3	9/2

$y_3(n) = \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\}$

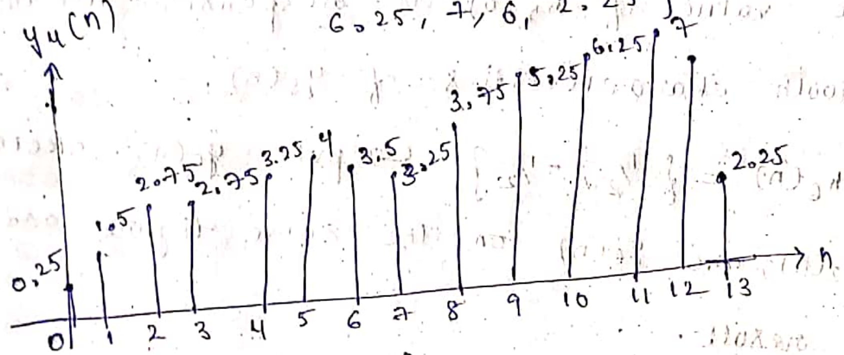


$y_4(n) = x(n) * h_4(n)$

$h_4(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
1/4	1/4	1	1/2	3/4	5/4	3/4	3/4	1	5/4	7/4	6/4	9/4
1/2	1/2	2	1	3/2	5/2	3/2	3/2	2	5/2	7/2	6/2	9/2
1/4	1/4	1	1/2	3/4	5/4	3/4	3/4	1	5/4	7/4	6/4	9/4

$y_4(n) = \frac{1}{4} y_2(n)$.

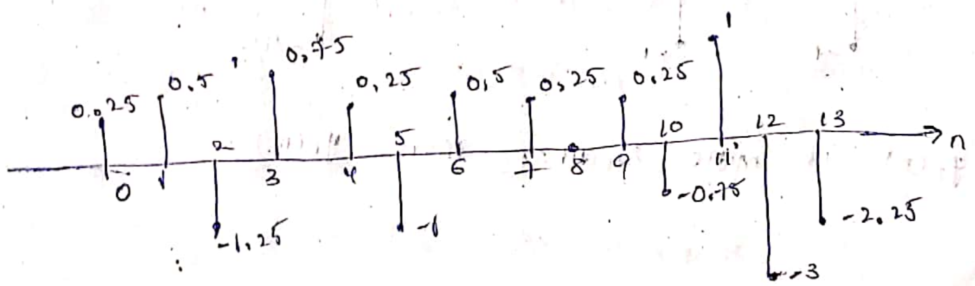
$= \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 3.75, 5.25, 6.25, 7, 6, 2.25\}$



$y_5(n) = x(n) * h_5(n)$

	1	4	2	3	5	3	3	4	5	7	6	9
1/4	1/4	1	1/2	3/4	5/4	3/4	3/4	1	5/4	7/4	6/4	9/4
-1/2	-1/2	-2	-1	-3/2	-5/2	-3/2	-3/2	-2	-5/2	-7/2	-6/2	-9/2
1/4	1/4	1	1/2	3/4	5/4	3/4	3/4	1	5/4	7/4	6/4	9/4

$y_5(n) = \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}$.



b) what is the difference between $y_1(n)$ and $y_2(n)$, and between $y_3(n)$ and $y_4(n)$?

sol: $y_3(n) = \frac{1}{2} y_1(n)$ because; $h_3(n) = \frac{1}{2} h_1(n)$.

$y_4(n) = \frac{1}{4} y_2(H)$ because; $h_4(n) = \frac{1}{4} h_2(m)$.

c) comment on the smoothness of $y_2(n)$ and $y_4(n)$, which factors affect the smoothness?

sol: $y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$, but $y_4(n)$ will appear even smoother because of the smaller scale factor.

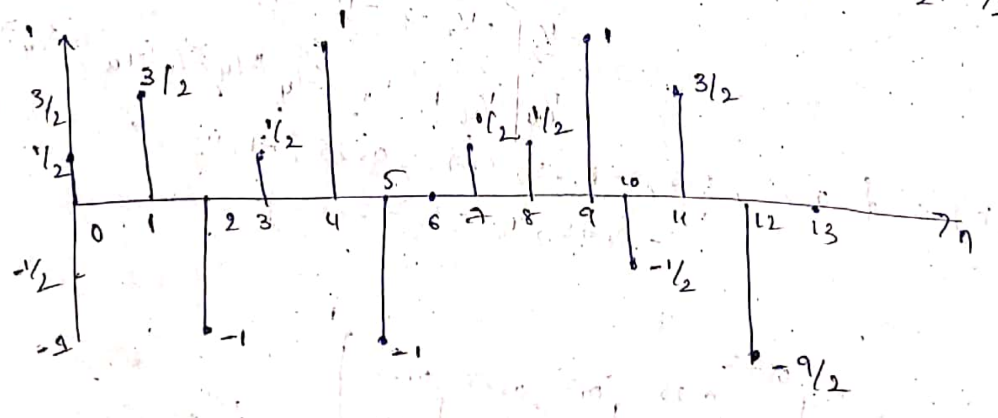
d) compare $y_4(n)$ with $y_5(n)$. what is the difference can you explain it?

sol:- system 4 results in a smoother output, The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$.

e) let $h_6(n) = \{ 1/2, -1/2 \}$ compute $y_6(n)$. sketch $x(n)$, $y_2(n)$, and $y_6(n)$ on the same figure and comment on the result.

sol:- $y_6(n) = x(n) * h_6(n)$

$y_6(n) = \{ 0/2, 3/2, -1, 1/2, 1, -1, 0, 1/2, 1/2, 1, -1/2, 3/2, -9/2 \}$



$y_2(n)$ is more smoother than $y_6(n)$

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23) Express the output $y(n)$ of a linear time-invariant system with impulse response $h(n)$ in terms of its step response $s(n) = h(n) * u(n)$ and the input $x(n)$.

we can express $\delta(n) = u(n) - u(n-1)$

$$h(n) = h(n) * \delta(n)$$
$$= h(n) * [u(n) - u(n-1)]$$
$$= h(n) * u(n) - h(n) * u(n-1)$$
$$= s(n) - s(n-1)$$

then, $y(n) = h(n) * x(n)$
$$= [\delta(n) - \delta(n-1)] * x(n)$$
$$= \delta(n) * x(n) - \delta(n-1) * x(n)$$

24) The discrete-time system

$$y(n) = n\,y(n-1) + x(n), \quad n \geq 0$$

is at rest [i.e., $y(-1) = 0$]. check if the system is linear time invariant and BIBO stable.

sol: $y_1(n) = n\,y_1(n-1) + x_1(n)$

$y_2(n) = n\,y_2(n-1) + x_2(n)$

$x(n) = a\,x_1(n) + b\,x_2(n)$

$y(n) = n\,y_1(n-1) + x_1(n) + n\,y_2(n-1) + x_2(n)$

$y(n) = a\,y_1(n) + b\,y_2(n)$

Hence the system is linear.

$y(n-1) = (n-1)\,y(n-2) + x(n-1)$. But

delayed $\Rightarrow y(n-1) = n\,y(n-2) + x(n-1)$.

so the system is time variant.

$\Rightarrow$ If $x(n) = u(n)$, then $|x(n)| \leq 1$, for this bounded input, and output is $y(0) = 0$, $y(1) = 2$, $y(2) = 5$ --- unbounded so system is unstable.

2.5) Consider the signal $\gamma(n) = a^n u(n)$, $0 < a < 1$

a) Show that any sequence $x(n)$ can be decomposed as

$$x(n) = \sum_{n=-\infty}^{\infty} c_k \gamma(n-k)$$ and

express c_k in terms of $x(n)$.

$$\delta(n) = \gamma(n) - a\gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,[\delta(n-k) - a\delta(n-k-1)]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - a\,x(k-1)]\,\gamma(n-k)$$

Thus $c_k = x(k) - a\,x(k-1)$

b) Use the properties of linearity and time invariance to express the output $y(n) = T[x(n)]$ in terms of the input $x(n)$ and the signal $g(n) = T[\gamma(n)]$, where $T[\cdot]$ is an LTI system.

Sol:-
$$y(n) = T[x(n)] = T\left[\sum_{k=-\infty}^{\infty} c_k \gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k\, T[\gamma(n-k)]$$

$$= \sum_{k=-\infty}^{\infty} c_k\, g(n-k)$$

c) Express the impulse response $h(n) = T[\delta(n)]$ in terms of $g(n)$.

$$h(n) = T[\delta(n)]$$

$$= T[\gamma(n) - a\gamma(n-1)]$$

$$= g(n) - a\,g(n-1).$$

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26) Determine the zero-input response of the system described by the second-order difference equation.

$x(n) - 3y(n-1) - 4y(n-2) = 0$.

sol :- with $x(n) = 0$

$$-3y(n-1) - 4y(n-2) = 0 \quad (\div(-3))$$

$$y(n-1) + \frac{4}{3} y(n-2) = 0$$

at $n=0$

$$y(-1) = -\frac{4}{3} y(-2)$$

at $n=1$ $\quad y(0) = -\frac{4}{3} y(-1) = \left(-\frac{4}{3}\right)^2 y(-2)$

at $n=2$

$$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$$

$$\vdots$$

$$y(k) = \left(\frac{-4}{3}\right)^{k+1} y(-2)$$

$\rightarrow$ zero-input response.

27) Determine the particular solution of the difference equation

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n).$$

when the forcing function is $x(n) = 2^n u(n)$.

sol:- Given

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$$

$$\Rightarrow x(n) = y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2)$$

characteristic equation is

$$\lambda^2 - \frac{5}{6}\lambda + 1/6 = 0 \; ; \quad \lambda = 1/2, 1/3.$$

Hence, $\quad y_h(n) = c_1 (1/2)^n + c_2 (1/3)^n$.

The particular solution is $y_p(n) = K(2^n) u(n)$

$x(n) = 2^n u(n)$ is

substitute this solution in to the difference eq'n.

$$K(2^n)u(n) - K\left(\frac{5}{6}\right)(2^{n-1})u(n-1) + K(1/6)(2^{n-2})u(n-2) = 2^n u(n)$$

~~for~~ $n=2$.

$$K(2^2)u(n) - K\left(\frac{5}{6}\right)\times 2 \times u(n-1) + K(1/6)u(n-2) = 4u(n)$$

for $n=2$ $\quad 4K - \frac{5}{3}K + \frac{K}{6} = 4 \Rightarrow \frac{24K - 10K + K}{6} = 4$

$$\Rightarrow \frac{\cancel{15}^{5}K}{\cancel{6}_{2}} = 4 \qquad \Rightarrow K = \frac{8}{5}.$$

$\therefore$ Total sol is

$$y(n) = y_p(n) + y_h(n)$$

$$= \frac{8}{5}(2^n)u(n) + c_1\left(\frac{1}{2}\right)^n u(n) + c_2\left(\frac{1}{3}\right)^n u(n).$$

To determine c_1 & c_2; assume that

$$y(-1) = y(-2) = 0$$

$$\Rightarrow y(0) = 1; \quad y(1) = 5/6 + 2 = \frac{17}{6}$$

Thus,

put $n=0$: $\quad \frac{8}{5} + c_1 + c_2 = 1$

$$c_1 + c_2 = -\frac{3}{5} \rightarrow (1)$$

put $n=1$

$$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6}$$

$$3c_1 + 2c_2 = -\frac{11}{5} \rightarrow (2)$$

solving (1) & (2)

$$c_1 = -1; \quad c_2 = 2/5$$

Total solution is

$$y(n) = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n).$$

28) In the given equation

$$y(n) = (-a_1)^{n+1}\, y(-1) + \frac{(1-(-a_1)^{n+1})}{1+a_1} \quad \text{for } n \ge 0.$$

separate the output sequence $y(n)$ into the transient response and the steady-state response plot these two responses for $a_1 = -0.9$.

at $y(-1) = 1$

the given eq'n $y(n) = (-a_1)^{n+1} + \frac{(1-(-a_1)^{n+1})}{1+a_1}$

$y(n) = y_{zi}(n) + y_{zs}(n)$

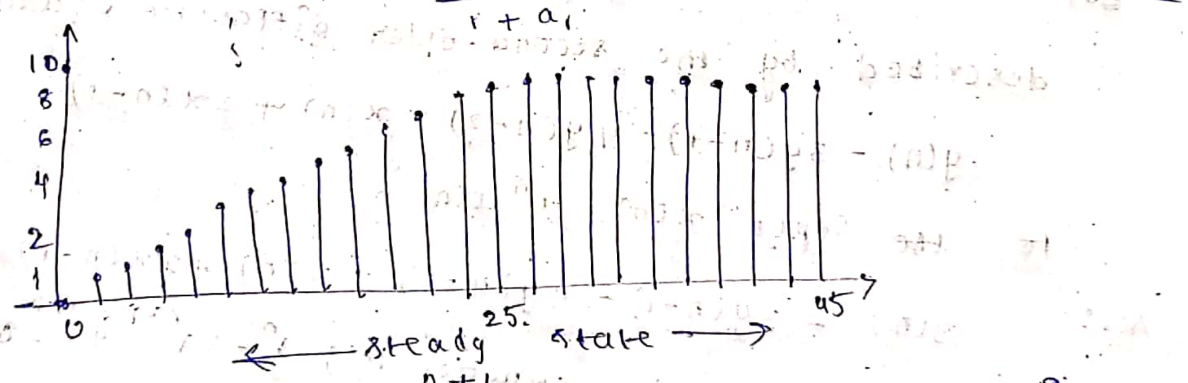
= Transient + steady state

as $y(\infty) = 0 + \frac{1}{1+a_1} \rightarrow$ steady state response

Hence steady state response.

at $n=0 \Rightarrow \frac{1+a_1}{1+a_1} = 1$

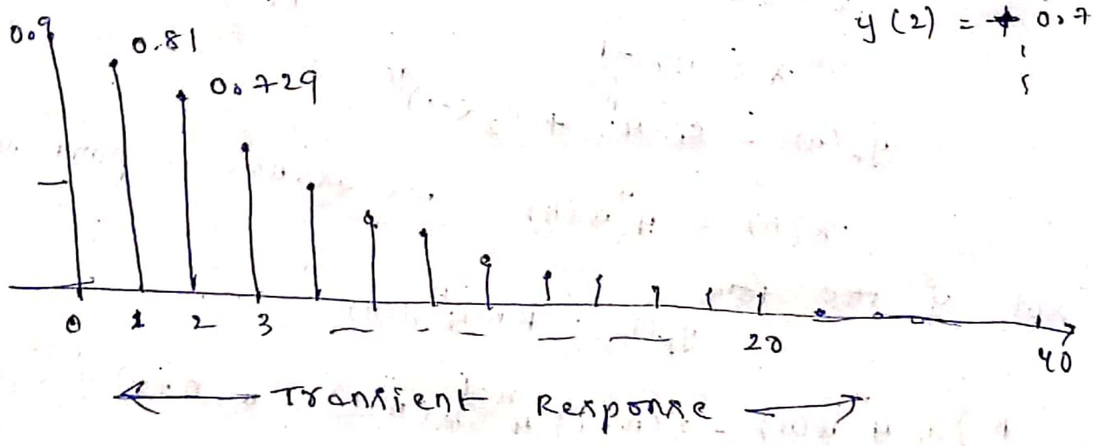
$n=1 \Rightarrow \frac{1-a_1^2}{1+a_1} = 1 - a_1 = 1 + 0.9 = 1.9 \simeq 2$



T.R $= (-a_1)^{n+1} \Rightarrow n=0 \Rightarrow y(0) = +0.9$

$n=1 \Rightarrow y(1) = 0.81$

$y(2) = +0.729$



29) Determine the impulse response for the cascade of two linear time-invariant systems having impulse responses,

$h_1(n) = a^n [u(n) - u(n-N)]$ and $h_2(n) = [u(n) - u(n-M)]$

sol:-

$h(n) = h_1(n) * h_2(n)$

$h(n) = \sum_{k=-\infty}^{\infty} a^k [u(k) - u(k-N)] [u(n-k) - u(n-k-M)]$

$= \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k-m)$

$- \sum_{k=-\infty}^{\infty} a^k u(k-N) u(n-k) + \sum_{k=\infty}^{\infty} a^k u(k-N) u(n-k-m)$

$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-m} a^k \right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-m} a^k \right)$

$h(n) = 0.$

30) Determine the response $y(n)$, $n \geq 0$, of the system described by the second-order difference equation.

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

to the input $x(n) = 4^n u(n)$

sol:- $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

characteristic equation is ; $\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$

$\lambda^2 - 3\lambda - 4 = 0$ let i/p to the sim be λ^n

$\lambda = 4, -1$

$y_h(n) = c_1 \cdot 4^n + c_2 (-1)^n$

$x(n) = 4^n u(n)$; we assume a particular sol of the form

$y_p(n) = K n 4^n u(n).$

$\Rightarrow K [n 4^n u(n) - 3(n-1) 4^{n-1} u(n-1) - 4(n-2) 4^{n-2} u(n-2)]$

$= 4^n u(n) + 2 (4)^{n-1} u(n-1).$

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for $n \geq 2$ $\left(2^{n-2}\right)$

$$K\left[n\,4^2 u(n) - 3(n-1)\times 4\, u(n-1) - 4(n-2)u(n-2)\right]$$
$$= 4^2 u(n) + 2\times 4\, u(n-1)$$

for $n = 2$

$$K\left[32 - 12\right] = 16 + 8$$

$$\Rightarrow K = \frac{24}{20} = 6/5$$

The total solution is

$$y(n) = y_p(n) + y_h(n)$$

$$= \left[6/5\; n\,4^n + c_1 4^n + c_2(-1)^n\right] u(n)$$

To solve for c_1 & c_2, let us assume that

$y(-1) = y(-2) = 0$; then

$y(0) = 1$ and $c_1 + c_2 = 1 \rightarrow (1)$

$y(1) = 3y(0) + 4 + 2$

$y(1) = 9 \Rightarrow \frac{6}{5}\times 4 + 4c_1 - c_2 = 9 \rightarrow (2)$

$\Rightarrow c_1 = \frac{26}{25}$ and $c_2 = \frac{-1}{25}$.

Hence the total solution is

$$\therefore y(n) = \left[6/5\; n.4^n + \frac{26}{25}4^n - \frac{1}{25}(-1)^n\right] u(n).$$

31) Determine the impulse response of the following causal system:

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

sol :- when the input is impulse, particular solution does not exist; only homogeneous solution exists

Hence the characteristic values are $\lambda = 4, -1$.

$\therefore \lambda^2 - 3\lambda - 4 = 0.$

$$y_h(n) = c_1 4^n + c_2(-1)^n$$

$$x(n) = \delta(n)$$

we find that

$y(0) = 1$; $y(1) - 3y(0) = 2$

$\Rightarrow y(1) = 5$

so $c_1 + c_2 = 1$

$4c_1 - c_2 = 5$

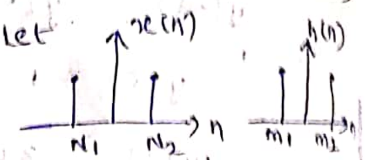
$5c_1 = 6$

$c_1 = 6/5$; $c_2 = -1/5$

Hence $h(n) = \left[\frac{6}{5} 4^n - \frac{1}{5} (-1)^n \right] u(n)$.

32) let $x(n)$, $N_1 \le n \le N_2$ and $h(n)$, $M_1 \le n \le M_2$ be two finite-duration signals.

a) Determine the range $L_1 \le n \le L_2$ of their convolution, in terms of N_1, N_2, M_1 and M_2. let



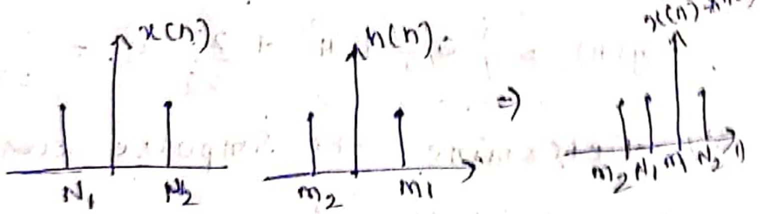
sol:- $L_1 = N_1 + M_1$ and $L_2 = N_2 + M_2$

b) Determine the limits of the cases of partial overlap from the left, full overlap, and partial overlap from the right. for convenience, assume that $h(n)$ has shorter duration than $x(n)$.

sol:-

partial overlap from left

$\rightarrow x(n) * h(n)$

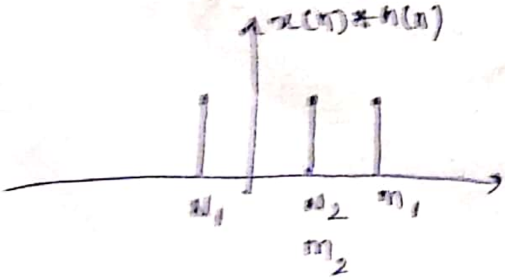


low $N_1 + M_1$ & high $M_2 + N_1 - 1$

If fully overlap then $N_1 + M_2$ (low)

$N_2 + M_1$ (high)

Partial overlap from right.



low $\Rightarrow N_2 + M_1 + 1$

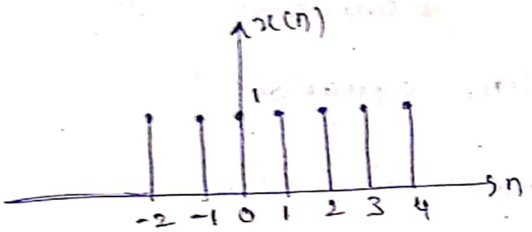
high $\Rightarrow N_2 + M_2$.

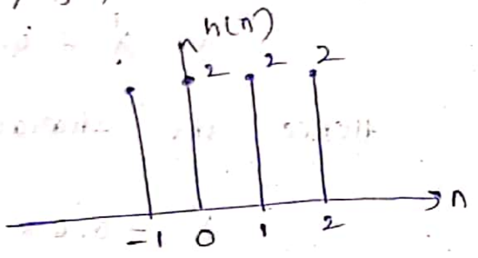
c) Illustrate the validity of your results by computing the convolution of the signals.

$$x(n) = \begin{cases} 1, & -2 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2; & -1 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

sol:- $x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$, $h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$





$N_1 = -2$ $N_2 = 4$ $M_1 = -1$ $M_2 = 2$

$y(n) = x(n) * h(n)$

$y(n) = \{2, \underset{\uparrow}{4}, 6, 8, 8, 8, 8, 6, 4, 2\}$

h(n) \ x(n)	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2

partial overlap from left

low $N_1 + M_1 = -3$

high $M_2 + N_1 - 1 = 2 - 2 - 1 = -1$

full overlap $n = 0$, $n = 3$

low, $N_1 + M_2 = -2 + 2 = 0$

high, $N_2 + M_1 = 4 - 1 = 3$.

Partial overlap from the right

low, $N_2 + M_1 + 1 = 4 - 1 + 1 = +4$

high, $N_2 + M_2 = 4 + 2 = 6$

33) Determine the impulse response and the unit step response of the systems described by the difference equation.

a) $y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$

sol: Impulse response $x(n) = \delta(n)$

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(a) the impulse response ; only the homogeneous solution exists ; but not particular solution.

given let $\lambda^n = y(n)$

$$y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$$

$$\Rightarrow \quad x(n) = y(n) - 0.6\,y(n-1) + 0.08\,y(n-2).$$

$$\Rightarrow \lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2}$$

Hence the characteristic equation is

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$\lambda = 1/5 ,\ 2/5$$

$$y_h(n) = c_1 (1/5)^n + c_2 (2/5)^n$$

Impulse response $x(n) = \delta(n)$, with $y(0) = 1$

$$y(1) - 0.6\,y(0) = 0$$

$$\Rightarrow y(1) = 0.6$$

$n=0$ $\quad c_1 + c_2 = 1 \rightarrow (1)$

$n=1$ $\quad 1/5\,c_1 + 2/5\,c_2 = 0.6 \rightarrow (2)$

from (1) & (2) $\quad c_1 = -1,\ c_2 = 2.$

$$\therefore \quad h(n) = \left[-(1/5)^n + 2(2/5)^n \right] u(n)$$

step response $x(n) = u(n)$

$$s(n) = \sum_{k=0}^{n} h(n-k) \quad , n \geq 0$$

$$= \sum_{k=0}^{n} \left[2(2/5)^{n-k} - (1/5)^{n-k} \right]$$

$$= 2\left(\frac{2}{5}\right)^n \sum_{k=0}^{n} (5/2)^k - (1/5)^n \sum_{k=0}^{n} 5^k$$

$$= 2\left(\frac{2}{5}\right)^n \left(\frac{1-(5/2)^{n+1}}{1-5/2} \right) - (1/5)^n \left(\frac{1-5^{n+1}}{1-5} \right)$$

$$= 2\left[\left(\frac{2}{5}\right)^n \left((5/2)^{n+1} - 1 \right)\right] - \left[(1/5)^n \left(5^{n+1} - 1\right) u(n)\right]$$

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b) $y(n) = 0.7\,y(n-1) - 0.1\,y(n-2) + 2x(n) - x(n-2)$.

sol: only homogeneous solution exists

Hence the characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0,$$

$$\Rightarrow \lambda_1 = 1/2 \;;\; \lambda_2 = 1/5$$

$$y_h(n) = c_1(1/2)^n + c_2(1/5)^n$$

Impulse response $x(n) = \delta(n)$, $y(0) = 2$.

$$y(1) = 0.7(2) = 1.4.$$

$$c_1 + c_2 = 2 \rightarrow (1) \;;\; \tfrac{1}{2}c_1 + \tfrac{1}{5}c_2 = 1.4$$

$$c_1 + \frac{2}{5}c_2 = \frac{14}{5} \rightarrow (2)$$

solving (1) & (2)

$$c_1 = 10/3 \;,\; c_2 = -4/3.$$

$$h(n) = \left[\frac{10}{3}(1/2)^n - 4/3\,(1/5)^n\right]u(n).$$

step Response $s(n) = \sum_{k=0}^{n} h(n-k)$

$$= \frac{10}{3}\sum_{k=0}^{n}(1/2)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}(1/5)^{n-k}.$$

$$= \frac{10}{3}(1/2)^n\sum_{k=0}^{n}2^k - \frac{4}{3}(1/5)^n\sum_{k=0}^{n}5^k.$$

$$= \frac{10}{3}\left[\frac{1}{2}^n(2^{n+1}-1)\,u(n)\right] - \frac{4}{3}\left[\frac{1}{5}^n(5^{n+1}-1)\,u(n)\right]$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n\left(\frac{1-2^{n+1}}{1-2}\right) - \frac{4}{3}(1/5)^n\left[\frac{1-5^{n+1}}{1-5}\right]$$

34) Consider a system with impulse response

$$h(n) = \begin{cases} (1/2)^n, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the input $x(n)$ for $0 \le n \le 8$ that will generate the output sequence.

$$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0, \ldots\}$$

sol: $h(n) = \{\underset{\uparrow}{1}, 1/2, 1/4, 1/8, 1/16, 0, \ldots\}$

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0, \dots\}$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
1/2	$x_0/2$	$x_1/2$	$x_2/2$	$x_3/2$	$x_4/2$	$x_5/2$	$x_6/2$	$x_7/2$
1/4	$\frac{x_0}{4}$	$\frac{x_1}{4}$	$\frac{x_2}{4}$	$\frac{x_3}{4}$	$\frac{x_4}{4}$	$\frac{x_5}{4}$	$\frac{x_6}{4}$	$\frac{x_7}{4}$
1/8	$\frac{x_0}{8}$	$\frac{x_1}{8}$	$\frac{x_2}{8}$	$\frac{x_3}{8}$	$\frac{x_4}{8}$	$\frac{x_5}{8}$	$\frac{x_6}{8}$	$\frac{x_7}{8}$
1/16	$\frac{x_0}{16}$	$\frac{x_1}{16}$	$\frac{x_2}{16}$	$\frac{x_3}{16}$	$\frac{x_4}{16}$	$\frac{x_5}{16}$	$\frac{x_6}{16}$	$\frac{x_7}{16}$

$x_0 = 1$; $\quad x_1 + \frac{x_0}{2} = 2$

$x_1 + \frac{1}{2} = 2 \Rightarrow x_1 = 2 - \frac{1}{2}$

$= 3/2$

$y_2 = x_2 + \frac{x_1}{2} + \frac{x_0}{4}$

$= x_2 + \frac{3}{4} + \frac{1}{4}$

$\cancel{\;= x_2 + \;} \Rightarrow \cancel{x_2} = \cancel{\tfrac{1}{4}} - 1$

$= \cancel{3}/4$

$2.5 = x_2 + \frac{3}{4} + \frac{1}{4}$

$\Rightarrow x_2 = 1.5 = 3/2$

$y_3 \Rightarrow 3 = x_3 + \frac{x_2}{2} + \frac{x_1}{4} + \frac{x_0}{8}$

$3 = x_3 + \frac{3}{4} + \frac{3}{8} + \frac{1}{8}$

$3 = x_3 + \frac{5}{4}$

$\Rightarrow x_3 = 3 - \frac{5}{4} = \frac{7}{4}$

$y_4 \Rightarrow x_4 + \frac{x_3}{2} + \frac{x_2}{4} + \frac{x_1}{8} + \frac{x_0}{16} = 3$ $\qquad -\frac{10}{8} - \frac{1}{4}$, $-\frac{5}{4} - \frac{1}{4}$

$x_4 = 3 - \frac{7}{8} - \frac{3}{8} - \frac{3}{16} - \frac{1}{16}$ $\qquad 3 - \frac{6}{4}$, $\frac{6}{4}$

$\Rightarrow x_4 = 3/2$

$y_5 \Rightarrow x_5 + \frac{x_4}{2} + \frac{x_3}{4} + \frac{x_2}{8} + \frac{x_1}{16} = 3.$

$x_5 = 3 - \frac{3}{4} - \frac{7}{16} - \frac{3}{16} - \frac{3}{16} = \frac{49}{32}$

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$y_6 \Rightarrow x_6 + \frac{x_5}{2} + \frac{x_4}{4} + \frac{x_3}{8} + \frac{x_2}{16} = 2$

$\Rightarrow x_6 = 2 - \frac{49}{64} - \frac{3}{8} - \frac{7}{32} - \frac{3}{64} = \frac{19}{32}$

$y_7 \Rightarrow x_7 + \frac{x_6}{2} + \frac{x_5}{4} + \frac{x_4}{8} + \frac{x_3}{16} = 1$

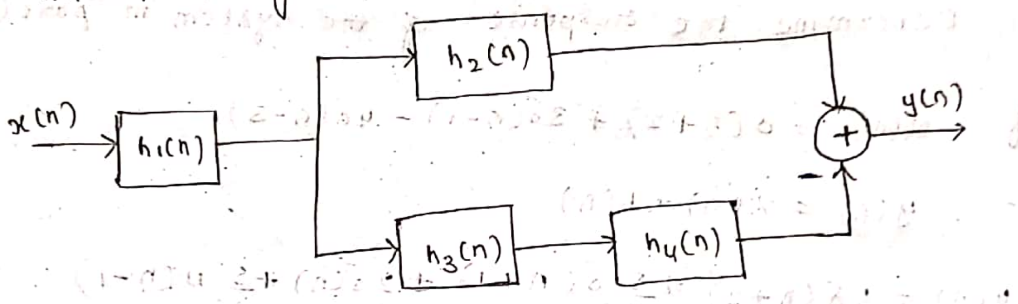
$\Rightarrow x_7 = 1 - \frac{7}{32} - \frac{3}{16} - \frac{49}{32\times 4} - \frac{19}{32\times 2} = \frac{-11}{128}$

$x_8 \Rightarrow x_8 + \frac{x_7}{2} + \frac{x_6}{4} + \frac{x_5}{8} + \frac{x_4}{16} = 0$

$x_8 = -\frac{3}{2\times 16} - \frac{49}{32\times 8} - \frac{19}{32\times 4} + \frac{11}{128\times 2}$

$x_8 = \frac{-25}{64}$

35) Consider the interconnection of LTI systems as shown in fig.



a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$, and $h_4(n)$.

Sol :- $h(n) = h_1(n) * [h_2(n) - \{h_3(n) * h_4(n)\}]$

b). Determine $h(n)$ when

$h_1(n) = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\}$

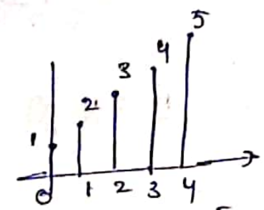
$h_2(n) = h_3(n) = (n+1)\,u(n)$

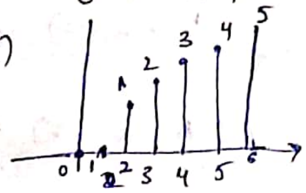
$h_4(n) = \delta(n-2)$.

n=2
$n-2<0$
$n-2>0$
$n>2$.

Sol :- $h_3(n) * h_4(n) = (n+1)\,u(n) * \delta(n-2)$

$= (n-1)\,u(n-2) = x(n)$





$h_2(n) - x(n)$

$\Rightarrow (n+1)\,u(n) - (n-1)\,u(n-2) = 2u(n) - \delta(n)$

$1\delta(n) + 2\delta(n-1) + 2\delta(n-2) + 2\delta(n-3) + \cdots$

$= -\delta(n) + 2\delta(n) + 2\delta(n-1$

$$h_1(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right]$$

36

$$h(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * \left[2u(n) - \delta(n)\right]$$

$$= u(n) + \frac{1}{2}\delta(n) + \frac{1}{2}u(n-1) - \frac{1}{4}\delta(n-1)$$
$$+ u(n-2) - \frac{1}{2}\delta(n-2)$$

$$= \left[\delta(n) - \frac{1}{2}\delta(n)\right] + \left[\delta(n-1) + \frac{1}{2}\delta(n-1) - \frac{1}{4}\delta(n-1)\right]$$
$$+ u(n-2) + \frac{1}{2}u(n-2) + u(n-2)$$

$$h(n) = \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$$

c) Determine the response of the system in part(b)

if $x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$

Sol :- $y(n) = x(n) * h(n)$

$$y(n) = \frac{1}{2}\delta(n+2) + \frac{5}{4}\delta(n+1) + 2\delta(n) + \frac{5}{2}u(n-1)$$
$$+ \frac{3}{2}\delta(n-1) + \frac{15}{4}\delta(n-2) + 6\delta(n-3) + \frac{15}{2}u(n-4)$$
$$- 2\delta(n-3) - 5\delta(n-4) - 8\delta(n-5) - 10u(n-6)$$

$n=-2 \Rightarrow 1/2$; $n=-1 \Rightarrow 5/4$; $n=0 \Rightarrow 2$

$n=1 \Rightarrow 5/2 + 3/2 = 4$; $n=3 \Rightarrow 6-2+5/2$

$n=2 \Rightarrow \frac{15}{4} + \frac{5}{2} = \frac{25}{4}$ $\qquad 4+5/2 = \frac{13}{2}$

$n=4 \Rightarrow 5/2 + \frac{15}{2} - 5 = \frac{15}{2} - \frac{5}{2} = 5$

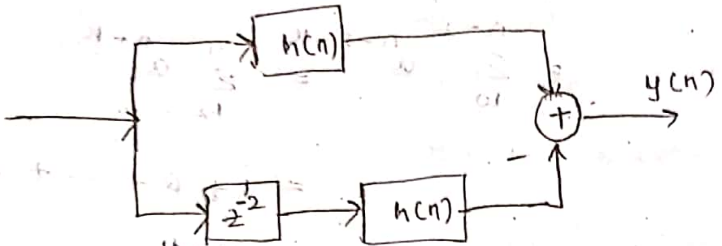
$n=5 \Rightarrow 5/2 + 15/2 - 8 = 2$

$n=6 \Rightarrow 5/2 + 15/2 - 10 = 0 \Rightarrow n=7 = 0 - \ldots$

$$y(n) = \left\{\frac{1}{2}, \frac{5}{4}, \underset{\uparrow}{2}, 4, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, \ldots\right\}$$

36) Consider the system in fig with $h(n) = a^n u(n)$, $-1 < a < 1$, Determine the response $y(n)$ of the system to the excitation.

$$x(n) = u(n+5) - u(n-10)$$



sol:-

$$h'(n) = h(n) - h(n-2)$$

$$= a^n u(n) - a^{n-2} u(n-2)$$

$$y(n) = x(n) * h'(n)$$

$$= a^n u(n) * u(n+5) - a^{n-2} u(n-2) * u(n+5)$$

$$- a^n u(n) * u(n-10) + a^{n-2} u(n-2) * u(n-10)$$

① $$= \sum_{k=-\infty}^{\infty} u(k+5) . a^{n-k} u(n-k)$$

$$= \sum_{-5}^{n} a^{n-k} = a^n \left[\sum_{k=-5}^{n} a^{-k} \right]$$

$$= a^n \left[a^5 + a^4 + \cdots + a^{-n} \right]$$

$$= 1 + a + \cdots + a^{n+5}$$

$$= \frac{1 - a^{n+6}}{1 - a} = \frac{a^{n+6} - 1}{a - 1} u(n+5).$$

$$- a^{n-2} u(n-2) * u(n+5)$$

$$= \sum_{k=-\infty}^{\infty} u(k+5) a^{n-2-k} u(n-2-k)$$

$$= \sum_{k=-5}^{n-2} a^{n-2-k} = \sum_{-3}^{n} a^{n-2-k+2} = \sum_{k=-3}^{n} a^{n-k}$$

$$= 1 + a + \cdots + a^{n-2+5}$$

$$= 1 + a + \cdots + a^{n+3} = \frac{a^{n+4} - 1}{a - 1} u(n+3)$$

$$+ a^n u(n) * u(n-10) \Rightarrow \sum_{-\infty}^{\infty} u(k-10) a^{n-k} u(n-k)$$

$$= \sum_{10}^{n} a^{n-k}$$

$$= 1 + \dots + a^{n-10}$$

$$= \frac{a^{n-9} - 1}{a - 1} u(n-10)$$

$$- a^{n-2} u(n-2) * u(n-10) = \sum_{k=-\infty}^{\infty} u(k-10) \cdot a^{n-2-k} u(n-2-k)$$

$$= \sum_{10}^{n-2} a^{n-2-k} = \sum_{12}^{n} a^{n-k}$$

$$= 1 + a + \dots + a^{n-12}$$

$$= \frac{a^{n-11} - 1}{a - 1} u(n-12)$$

Hence

$$y(n) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$$

$$- \frac{a^{n+4} - 1}{a-1} u(n+3) + a^{n-11}$$

$$+ \frac{a^{n-11} - 1}{a-1} u(n-12).$$

37) Compute and sketch the step response of the system.

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k).$$

sol:- $h(n) = \frac{1}{M} [u(n) - u(n-M)]$

$$s(n) = u(n) * \frac{1}{M} [u(n) - u(n-M)]$$

$$= \frac{1}{M} \left[\sum_{k=0}^{n} 1 - \sum_{M}^{n} 1 \right]$$

if $n \geq M = \frac{1}{M} \left[n+1 - [n-M+1] \right] = 1$

if $n < M \Rightarrow s(n) = \frac{1}{M} \left[\sum_{k=0}^{n} 1 - 0 \right]$

$$= \frac{1}{M}(n+1) = \frac{n+1}{M}$$

Hence $s(n) = \begin{cases} \frac{n+1}{M} ; & n < M \\ 1 ; & n \geq M. \end{cases}$

38) Determine the range of values of the parameter a for which the linear time-invariant s/m with impulse response

$$h(n) = \begin{cases} a^n, & n \geq 0,\ n \text{ even} \\ 0, & \text{otherwise.} \end{cases}$$ is stable.

sol :-
$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{\substack{n=0 \\ n \text{ is even}}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2}$$

stable if $|a| < 1$.

39) Determine the response of the system with impulse response $h(n) = a^n u(n)$ to the input signal

$$x(n) = u(n) - u(n-10)$$

(Hint : The solution can be obtained easily and quickly by applying the linearity and time-invariance properties to the result in Ex 2.3.5).

sol :- $h(n) = a^n u(n)$.

$$y_1(n) = \sum_{k=0}^{n} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k} = [1 + a + \dots + a^n]$$

$$= \frac{1-a^{n+1}}{1-a} u(n)$$

$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-a}\left[(1-a^{n+1})\, u(n) - (1-a^{n-9})\, u(n-10)\right]$$

40) Determine the response of the (relaxed) system characterized by the impulse response

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

to the input signal $x(n) = \begin{cases} 1, & 0 \le n \le 10 \\ 0, & \text{otherwise} \end{cases}$

Sol:- similar to above problem ;

with $a = \frac{1}{2} = 0.5$

$$y(n) = \frac{1}{1-0.5}\left[\left(1-\left(\tfrac{1}{2}\right)^{n+1}\right)u(n) - \left(1-\left(\tfrac{1}{2}\right)^{n-9}\right)u(n-10)\right]$$

$$= 2\left[\left(1-\left(\tfrac{1}{2}\right)^{n+1}\right)u(n) - \left(1-\left(\tfrac{1}{2}\right)^{n-9}\right)u(n-10)\right]$$

41) Determine the response of the (relaxed) syst characterized by the impulse response

$h(n) = \left(\frac{1}{2}\right)^n u(n)$, to the input signals

a) $x(n) = 2^n u(n)$ b) $x(n) = u(-n)$.

Sol:- a)

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k).\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^k u(k)\, 2^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^{2k} = 2^n \sum_{k=0}^{n} \left(\tfrac{1}{4}\right)^k$$

$$= 2^n \frac{1-(1/4)^{n+1}}{1-1/4}$$

$$= 2^n \left[1-(1/4)^{n+1}\right] \times \frac{4}{3}.$$

$$= \frac{2}{3}\left[2^{n+1} - 2^{-2n-2+n+1}\right]$$

$$= \frac{2}{3}\left[2^{n+1} - (1/2)^{n+1}\right] u(n)$$

b) $$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, x(n-k),$$

$$= \sum_{k=-\infty}^{\infty} u(-k)\left(\tfrac{1}{2}\right)^{n-k} u(n-k)$$

$$= \sum_{k=0}^{-\infty} \left(\tfrac{1}{2}\right)^{n-k}$$

* case i) $\Rightarrow n<0 \Rightarrow \sum_{k=0}^{\infty} (1/2)^{k} = 2 \rightarrow n=0$

case ii) $\Rightarrow n \geq 0 \Rightarrow \sum_{k=0}^{-\infty} (1/2)^{n-k}$

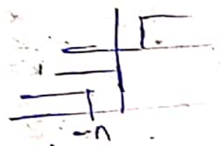
$$= (1/2)^{n} + (1/2)^{n+1} + \cdots$$

$$= (1/2)^{n}\left[1 + 1/2 + 1/2^{2} + \cdots\right]$$

$$= (1/2)^{n}[2] = 2(1/2)^{n} \; ; \; n \geq 0.$$

Reason :- $u(n-k) \Rightarrow$ shift by $u(k) \rightarrow u(n+k)$

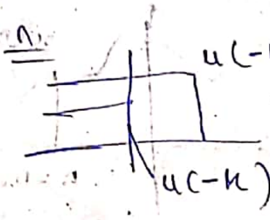
case 1)



$n=1$

$u(-k+n)$

limits $\rightarrow n$



case 2) for $n=-1$

* limits $-\infty$ to 0 $[n=0]$

42) Three systems with impulse responses $h_1(n) = \delta(n) - \delta(n-1)$, $h_2(n) = h(n)$, and $h_3(n) = u(n)$, are connected in cascade.

a) what is the impulse response, $h_c(n)$, of the overall system?

sol :- $h_c(n) = h_1(n) * h_2(n) * h_3(n)$

$$= [h_1(n) * h_3(n)] * h_2(n) \quad \text{(commutative property)}$$

$$= [\delta(n) - \delta(n-1)] * u(n) * h_2(n)$$

$$= [u(n) - u(n-1)] * h_2(n)$$

$$= \underbrace{[u(n) - u(n-1)]} * h(n)$$

$$= \delta(n) * h(n)$$

$$= h(n)$$

b) Does the order of interconnection affect the overall system?

Sol: No.

43) a) Prove and explain graphically the difference between the results

$x(n)\delta(n-n_0) = x(n_0)\delta(n-n_0)$ and $x(n) * \delta(n-n_0) = x(n-n_0)$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

$$x(n)\delta(n-n_0) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \cdot \delta(n-n_0)$$

$$\Rightarrow [x(-\infty)\delta(n+\infty) + \dots + x(n_0)\delta(n-n_0) + \dots x(\infty)\delta(n-\infty)]\,\delta(n-n_0)$$

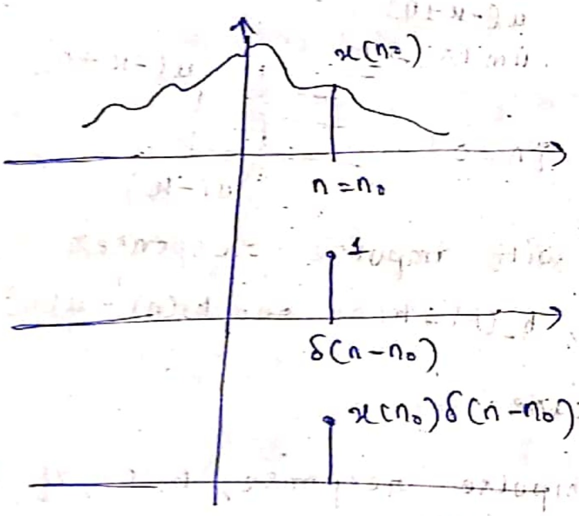
at $n=n_0$ $= x(n_0)\delta(n-n_0)$

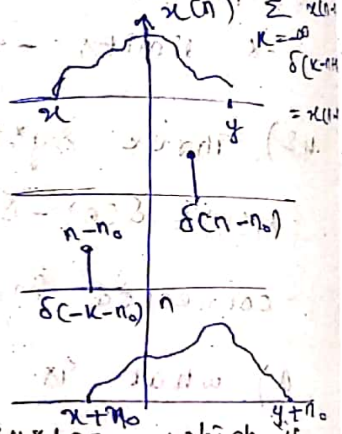
$$x(n) * \delta(n-n_0) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k-n_0)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\delta(n-n_0-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\delta(n-n_0-k)$$

$n-n_0$





$\sum_{k=-\infty}^{\infty} x(n-k)\,\delta(k-n_0) = x(n-n_0)$

b) Show that a discrete-time system, which is described by a convolution summation, is LTI and relaxed.

Sol:- $$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = h(n) * x(n)$$

linearity : $x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$

$$x(n) = \alpha x_1(n) + \beta x_2(n) \rightarrow y(n) = h(n) * x(n)$$

$$y(n) = h(n) * [\alpha x_1(n) + \beta x_2(n)]$$

$$= \alpha y_1(n) + \beta y_2(n)$$

Hence the given system is linear.

Time invariance :

$x(n) \rightarrow y_1(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$= \sum_k h(k)\, x(n-n_0-k)$

Relaxed = ? $= y(n-n_0)$

c) what is the impulse response of the system described by $y(n) = x(n-n_0)$?

sol: $h(n) = \delta(n-n_0)$ is the impulse response of the system.

44) Two signals $s(n)$ and $v(n)$ are related through the following difference equations.

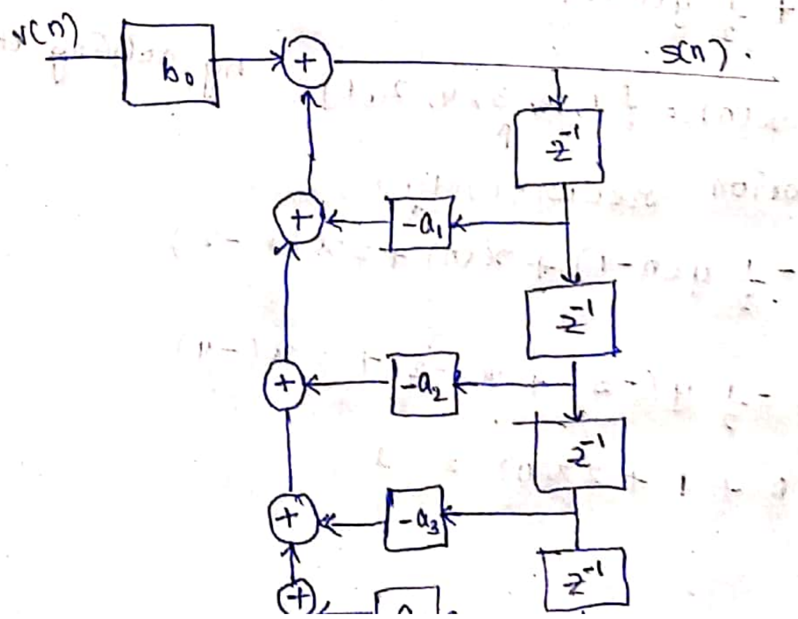
$$s(n) + a_1 s(n-1) + \dots + a_N s(n-N) = b_0 v(n)$$

Design the block diagram realization of :

a) The system that generates $s(n)$ when excited by $v(n)$

b) The system that generates $v(n)$ when excited by $s(n)$.

a) sol:- $s(n) = -a_1 s(n-1) - a_2 s(n-2) \dots - a_N s(n-N) + b_0 v(n)$



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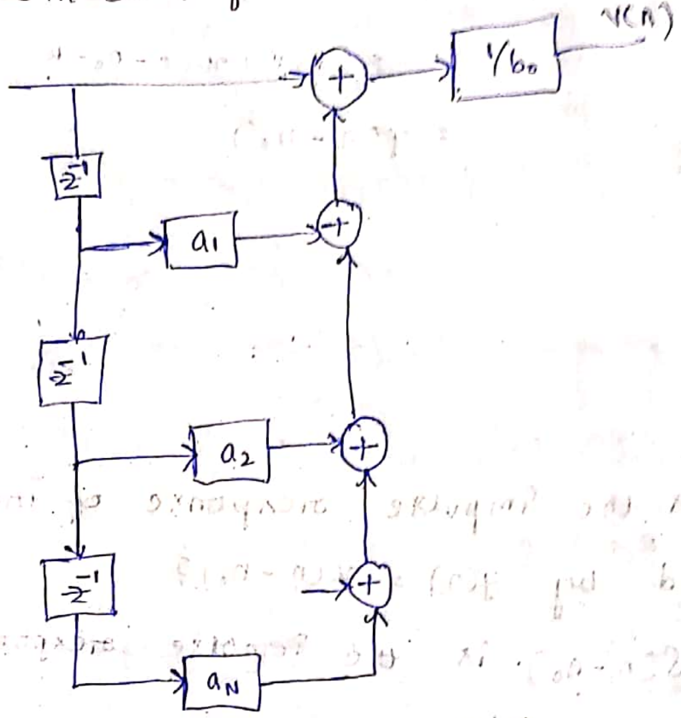
$$v(n) = \frac{1}{b_0}\left[s(n) + a_1' s(n-1) + a_2 s(n-2) + \cdots + a_N s(n-N) \right]$$

c) what is the impulse response of the cascade interconnection of system in parts a and b?

b) sol:



c)

sol:- $\lambda^n + a_1 \lambda^{n-1} + \cdots + a_N \lambda^{n-N} = 0$

$\lambda^N + a_1 \lambda^{N-1} + \cdots + a_N = 0 \rightarrow$ characteristic eq'n.

let roots be $\lambda_0, \lambda_1, \cdots \lambda_{N-1}$

sol is $c_1 \lambda_0^n + c_2 \lambda_1^n + \cdots + c_N \lambda_{N-1}^n$

45) Compute the zero - state response of the system described by the difference equation.

$$y(n) + \frac{1}{2} y(n-1) = x(n) + 2x(n-2)$$

to the input $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$, by solving the difference equation recursively.

sol:- $y(n) = -\frac{1}{2} y(n-1) + x(n) + 2x(n-2)$

$y(-2) = -\frac{1}{2} y(-3) + x(-2) + 2x(-4)$

$= 0 + 1 + 2x(0) = 1.$

$y(-1) = -\frac{1}{2} y(-2) + x(-1) + 2x(-3)$

$= -\frac{1}{2}(1) + 2 + 2 \times 0 = 3/2$

$y(0) = -\frac{1}{2} y(-1) + x(0) + 2x(-2)$

$= -1/2 (3/2) + 3 + 2(1)$

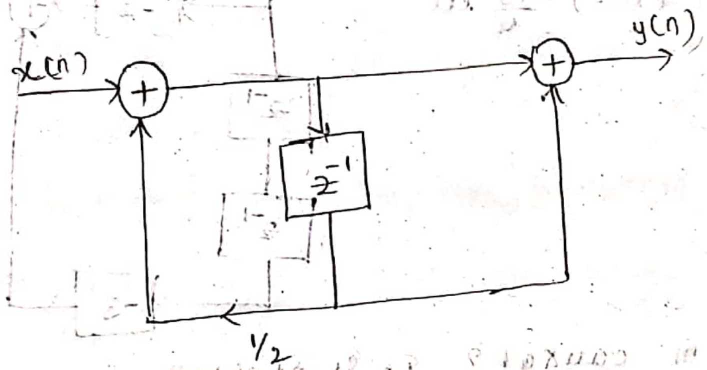
$= 5 - 3/4 = 17/4$

$y(1) = -\frac{1}{2} y(0) + x(1) + 2x(-1)$

$= -\frac{1}{2}(17/4) + 4 + 2(2) = 8 - \frac{17}{8} = 47$

Note :- we have consider from $y(-2) \Rightarrow$ as $y(-2)$ depends on $x(-2)$ where x starts $\Rightarrow y(-3)$ depends on $x(-3)$ which is zero; hence we start from $y(-2)$

47) Consider the discrete-time system shown in fig.



a) Compute the 10 first samples of its impulse response

sol :- $x(n) = \{ \underset{\uparrow}{1}, 0, 0, --- \}$

$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = 3/2$

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) = 3/4$, Thus, we obtain

$y(n) = \{ 1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, --- \}$

b) Find the input - output relation.

sol :- $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

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c) Apply the input $x(n) = \{\underset{\uparrow}{1}, 1, 1, \dots\}$ and compute the first 10 samples of the output.

c) as in part (a) we obtain

$$y(n) = \{1, 5/2, \frac{13}{4}, \frac{29}{8}, \frac{61}{16}, \dots\}$$

d) Compute the first 10 samples of the output for the input given in part (c) by using convolution

$y(n) = u(n) * h(n)$

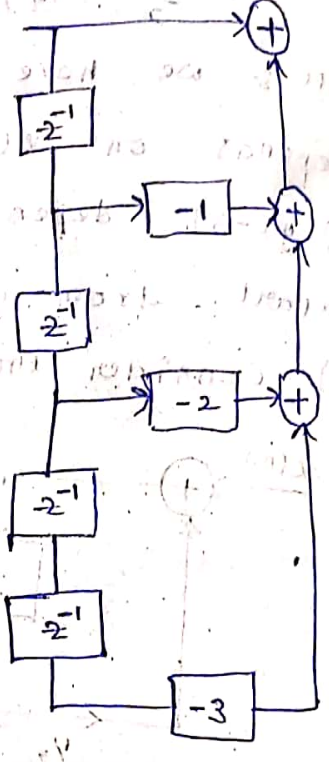
$= \sum_{k} u(k) h(n-k)$

$= \sum_{k=0}^{n} h(n-k)$

$y(0) = h(0) = 1$

$y(1) = h(0) + h(1) = 5/2$

$y(2) = h(0) + h(1) + h(2) = \frac{13}{4}$ etc.



e) Is the system causal? Is it stable?

from part (a), $h(n) = 0$ for $n<0$ ⇒ The system is causal

$$\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}(1 + 1/2 + 1/4 + \dots) = 4 ⇒ \text{sm is stable.}$$

48) Consider the system described by the difference equation $y(n) = a\,y(n-1) + b\,x(n)$.

a) Determine b in terms of a so that

$$\sum_{n=-\infty}^{\infty} h(n) = 1.$$

sol:- Homogeneous solution [Impulse response]

$\lambda^n - a\lambda^{n-1} = 0 \quad ⇒ \lambda = a$

$h(n) = c_1 a^n u(n)$

$y(0) = b \Rightarrow c_1 \Rightarrow h(n) = b a^n u(n)$

$$\sum_{n=-\infty}^{\infty} h(n) = \frac{b}{1-a} = 1 \Rightarrow \boxed{b = 1-a}$$

b) Compute the zero-state step response $s(n)$ of the system and choose b so that $s(\infty) = 1$;

sol;-

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$$

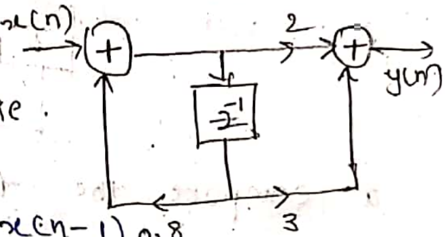
$$s(\infty) = \frac{b}{1-a} = 1$$

$$b = 1-a.$$

c) Compare the values of b obtained in parts (a) and (b). what did you notice ?

sol: $b = 1-a$ in both the cases.

49) A discrete-time system is realized by the structure shown in fig.



a) Determine the impulse response.

sol;-

$$y(n) = 0.8\, y(n-1) + 2x(n) + 3x(n-1)$$

$$y(n) - 0.8\, y(n-1) = 2x(n) + 3x(n-1)$$

The characteristic equation is

$$\lambda - 0.8 = 0$$

$$\lambda = 0.8$$

$$y_h(n) = c(0.8)^n$$

let us first consider the response of the system $y(n) - 0.8\, y(n-1) = x(n)$; to $x(n) = \delta(n)$.

since $y(0) = 1$; it follows that $c = 1$ then the impulse response of the original system is

$$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$$

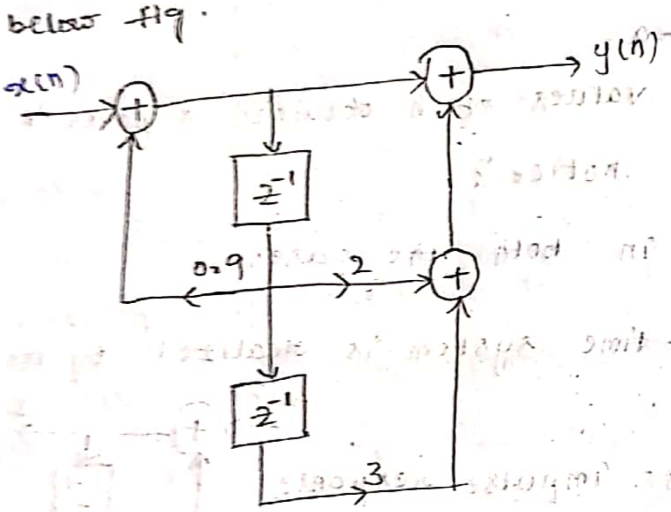
$$= 2\delta(n) + 4.6(0.8)^{n-1} u(n-1)$$

b) Determine a realization for its inverse system, that is, the system which produces $x(n)$ as an output when $y(n)$ is used as an input.

Sol: The inverse system is characterized by the difference equation.

$$x(n) = -1.5x(n-1) + \tfrac{1}{2}y(n) - 0.4\,y(n-1)$$

5) Consider the discrete-time system shown in below fig.



a) compute the first six values of the impulse response of the system.

Sol:- $y(n) = 0.9\,y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

for $x(n) = \delta(n)$

$y(0) = 0.9(1) + 2 = 2.9$

$y(2) = 0.9(2.9) + 3 = 5.61$

$y(3) = 5.61 \times 0.9 = 5.049$

$y(4) = 5.049 \times 0.9 = 4.544$

$y(5) = 4.544 \times 0.9 = 4.090$ ---

b) Compute the ~~first~~ first six values of the zero-state step response of the system.

Sol: step response is nothing but an accumulator

Hence $s(0) = y(0) = 1$.

$s(1) = y(0) + y(1) = 3.91$

$s(2) = y(0) + y(1) + y(2) = 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3) = 14.56$

$s(4) = \sum_{0}^{4} y(n) = 19.10$

$s(5) = \sum_{0}^{5} y(n) = 23.19$.

~~c) Determine and sketch the impulse response of the following~~ x

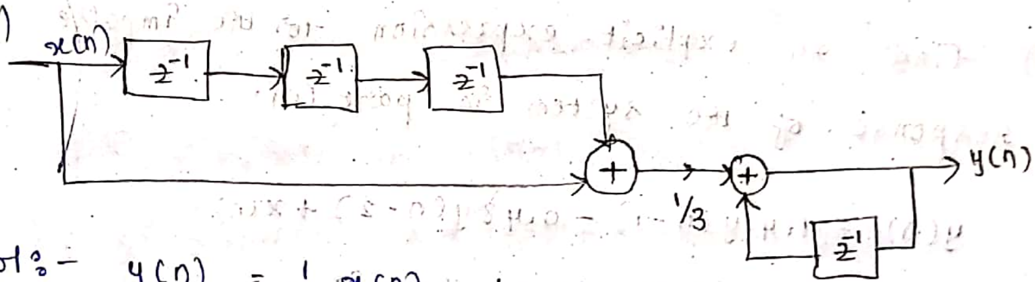
c) Determine an analytical expression for the impulse response of the sim.

sol:- $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$= \delta(n) + 2.9\,\delta(n-1) + 5.61\,(0.9)^{n-2} u(n-2)$

51) Determine and sketch impulse response of the following systems for $n = 0, 1, \ldots, 9$.

a)

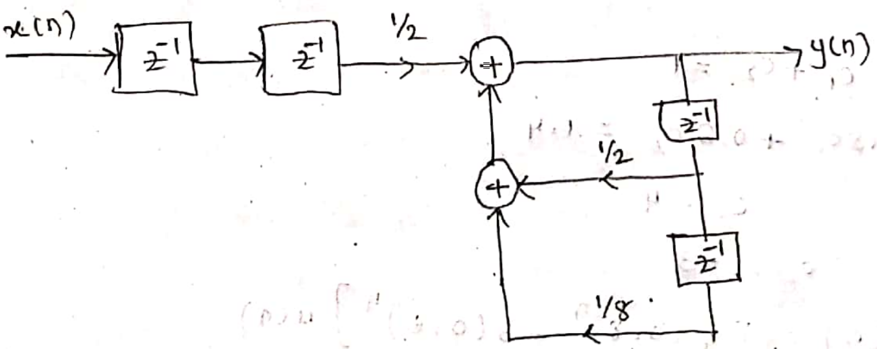


sol:- $y(n) = \frac{1}{3} x(n) + \frac{1}{3} x(n-3) + y(n-1)$

for $x(n) = \delta(n)$, we have

$h(n) = \{1/3, 1/3, 1/3, 2/3, 2/3, 2/3, 2/3, \ldots\}$

b)



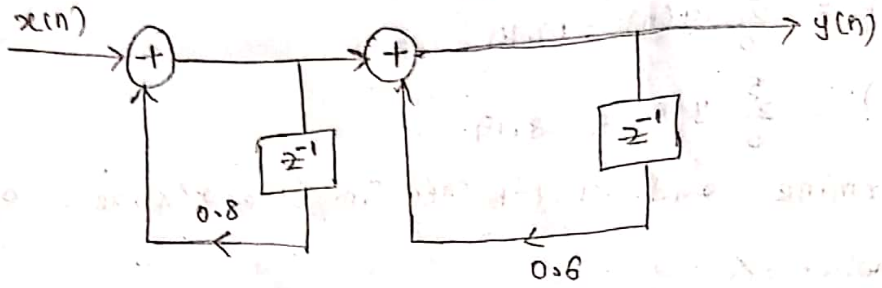
sol: $y(n) = \frac{1}{2} y(n-1) + \frac{1}{8} y(n-2) + \frac{1}{2} x(n-2)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$h(n) = \{0, 0, 1/2, 1/4, 3/16, 1/8, 11/128, \frac{18}{256}, \frac{41}{1024}, \ldots\}$

(c)



Sol :- $y(n) = 1.4\, y(n-1) - 0.48\, y(n-2) + x(n)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086, \ldots\}$

d) Classify the systems above as FIR or IIR.

sol :- All three systems are IIR.

e) Find an explicit expression for the impulse response of the system in part (c).

$y(n) = 1.4\, y(n-1) - 0.48\, y(n-2) + x(n)$

The characteristic equation is

$\lambda^2 - 1.4\lambda + 0.48 = 0$

$\lambda = 0.8, 0.6$

$y_h(n) = c_1 (0.8)^n + c_2 (0.6)^n$ for $x(n) = \delta(n)$ we have,

$c_1 + c_2 = 1$

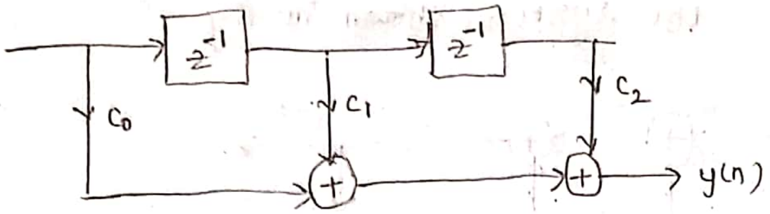
$0.8 c_1 + 0.6 c_2 = 1.4$

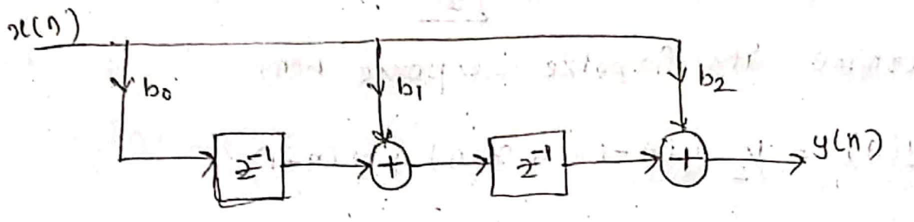
$c_1 = 4$

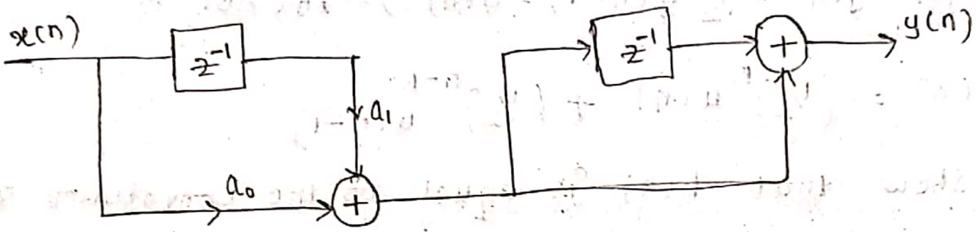
$c_2 = -3$

$h(n) = [4(0.8)^n - 3(0.6)^n]\, u(n)$.

52) Consider the systems shown in fig.







a) Determine and sketch their impulse responses $h_1(n)$, $h_2(n)$ and $h_3(n)$.

sol:- $y_1(n) = c_0 x(n) + c_1 x(n-1) + c_2 x(n-2)$

$h_1(n) = c_0 \delta(n) + c_1 \delta(n-1) + c_2 \delta(n-2)$

$y_2(n) = b_2 x(n) + b_1 x(n-1) + b_0 x(n-2)$

$h_2(n) = b_2 \delta(n) + b_1 \delta(n-1) + b_0 \delta(n-2)$

$y_3(n) = a_0 x(n) + (a_1 + a_0 a_2) x(n-1) + a_1 a_2 x(n-2)$

$h_3(n) = a_0 \delta(n) + (a_1 + a_0 a_2) \delta(n-1) + a_1 a_2 \delta(n-2)$.

b) Is it possible to choose the coefficients of these systems in such a way that $h_1(n) = h_2(n) = h_3(n)$

sol: The only question is whether

$h_3(n) = h_2(n) = h_1(n)$

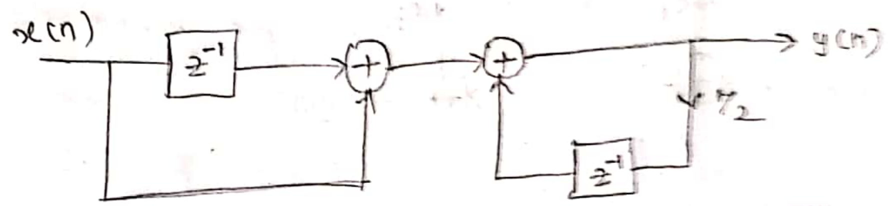
let $a_0 = c_0$; $a_1 + a_2 c_0 = c_1$,

$a_2 a_1 = c_2$ $\quad \Rightarrow \frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$

$\Rightarrow \frac{c_2}{a_2} = a_1$ $\quad \Rightarrow c_0 a_2^2 - c_1 a_2 + c_2 = 0$;

for $c_0 \neq 0$, the quadratic has a real solution if & only if $c_1^2 - 4c_0c_2 \geq 0$:

53) Consider the system shown in fig.



a) Determine its impulse response $h(n)$

sol:- $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

for $y(n) - \frac{1}{2} y(n-1) = \delta(n)$; the sol is

$$h(n) = \left(\tfrac{1}{2}\right)^n u(n) + \left(\tfrac{1}{2}\right)^{n-1} u(n-1)$$

b) Show that $h(n)$ is equal to the convolution of the following signals:

$h_1(n) = \delta(n) + \delta(n-1)$

$h_2(n) = \left(\frac{1}{2}\right)^n u(n)$.

sol:- $h_1(n) * h_2(n) = h_2(n) * [\delta(n) + \delta(n-1)]$

$$= \left(\tfrac{1}{2}\right)^n u(n) + \left(\tfrac{1}{2}\right)^{n-1} u(n-1)$$

54) Compute and sketch the convolution $y_i(n)$ and correlation $r_i(n)$ sequences for the following pair of signals and comment on the results obtained.

a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$ $h_1(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$y_1(n) = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

correlation $= \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

	1	1	1	1	1
4	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

b) $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$ $\quad h_2(n) = \{1/2, 1, \underset{\uparrow}{2}, 1, 1/2\}$.

$y_2(n) = x_2(n) * h_2(n)$

$y_2(n) = \{1/2, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$

Correlation $\gamma_2(n) = \{1/2, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$

Note $y_2(n) = \gamma_2(n)$, $\because h_2(-n) = h_2(n)$

c) $x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $\quad h_3(n) = \{\underset{\uparrow}{4}, 3, 2, 1\}$

Convolution : $y_3(n) = x_3(n) * h_3(n)$

	1	2	3	4
4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4

$y_3(n) = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

Correlation

$\gamma_3(n) = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

d) $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$;

$h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$y_4(n) = \{1, 4, 10, 20, 25, 24, 16\}$

$\gamma_4(n) = \{4, 11, 20, 30, 20, 11, 4\}$

Note that $h_3(-n) = h_4(n+3)$

hence $\gamma_3(n) = y_4(n+3)$

$h_4(-n) = h_3(n+3)$

$\gamma_4(n) = y_3(n+3)$.

55) The zero-state response of a causal LTI s/m to the input $x(n) = \{\underset{\uparrow}{1}, 3, 3, 1\}$ is $y(n) = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$. Determine its impulse response. $x(n) * y(n) = h(n)$

Sol :- length of $h(n) = 2$

$h(n) = \{h_0, h_1\}$

	1	3	3	1
h_0	h_0	$3h_0$	$3h_0$	h_0
h_1	h_1	$3h_1$	$3h_1$	h_1

$h_0 = 1$

$3h_0 + h_1 = 4$

$\Rightarrow h_0 = 1, \quad h_1 = 1$

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$4 + x - 1 = 5$

$x + 3 = 5$

$x = 2$ = length of $h(n)$.

56) Prove by direct substitution the equivalence of eq'ns (2.5.9) and (2.5.10), which describe the direct form II structure, to the relation (2.5.6), which describes the direct form I structure.

$$(2.5.6) \quad y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

$$(2.5.9), \quad w(n) = -\sum_{k=1}^{N} a_k\, w(n-k) + x(n)$$

$$(2.5.10) \quad y(n) = \sum_{k=0}^{M} b_k\, w(n-k)$$

from 2.5.9 we obtain

$$x(n) = w(n) + \sum_{k=1}^{N} a_k\, w(n-k). \longrightarrow (A).$$

By substituting (2.5.10) for $y(n)$ and (A) into (2.5.6)

$$\sum_{k=0}^{M} b_k w(n-k)$$

L.H.S = R.H.S

57) Determine the response $y(n)$, $n \geq 0$ of the system described by the second-order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1),$$

when the input is $x(n) = (-1)^n u(n)$ and the initial conditions are $y(-1) = y(-2) = 0$.

Sol:- $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0;$$

$\Rightarrow \lambda = 2, 2$. Hence

$$y_h(n) = c_1 2^n + c_2 n 2^n$$

The particular solution is $y_p(n) = K(-1)^n u(n)$

substituting this solution in to the difference eq'n ;

we obtain

$$K(-1)^n u(n) - 4K(-1)^{n-1}u(n-1) + 4K(-1)^{n-2} u(n-2)$$
$$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

$$[K(-1)^2 - 4K(-1) + 4K] = 1+1$$

for $n=2$, $K(1+4+4) = 2$

$$\Rightarrow K = 2/9.$$

Hence the total solution is

$$y(n) = [c_1 2^n + c_2 n 2^n + 2/9(-1)^n] u(n).$$

from the initial conditions ; we obtain

$y(0) = 1$, $y(1) = 2$ $\Rightarrow y(1) - 4 = -1 - 1$
$y(1) = 2$

$c_1 + 2/9 = 1$ $\quad\quad 2c_1 + 2c_2 - 2/9 = 2$
$\Rightarrow c_1 = 7/9$ $\quad\quad \Rightarrow c_2 = 1/3$

$$y(n) = [7/9(2)^n + 1/3 n 2^n + 2/9(-1)^n] u(n)$$

58) Determine the impulse response $h(n)$ for the system described by the second-order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1).$$

sol: from the above problem (57)

$$h(n) = [c_1 2^n + c_2 n 2^n] u(n)$$

with $y(0) = 1$; $y(1) = 3$, we have

$c_1 = 1$, $2c_1 + 2c_2 = 3 \Rightarrow c_2 = 1/2$.

Thus $h(n) = [2^n + 1/2\, n 2^n] u(n)$

59) Show that any discrete-time signal $x(n)$ can be expressed as $x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$

where $u(n-k)$ is a unit step delayed by k units

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in time, that is,

$$u(n-k) = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise.} \end{cases}$$

Sol:-

$$x(n) = x(n) * \delta(n)$$

$$= x(n) * [u(n) - u(n-1)]$$

$$= [x(n) - x(n-1)] * u(n)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$$

method-2

$$x(n) = x(n) * \delta(n)$$

$$= x(n) * \frac{d}{dn} u(n)$$

$$= \frac{d}{dn} x(n) * u(n)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$$

60) Show that the output of an LTI system can be expressed in terms of its unit step response $s(n)$ as follows.

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] s(n-k).$$

Sol:- let $h(n)$ be the impulse response of the s/m

$$s(k) = \sum_{m=-\infty}^{k} h(m)$$

$$h(k) = s(k) - s(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] s(n-k)$$

method 2 :-

$$s(n) = h(n) * u(n)$$

$$s(n-1) = h(n) * u(n-1)$$

$$\Rightarrow s(n) - s(n-1) = h(n) * u(n) = h(n) \rightarrow (1)$$

(or) $s(n) = h(n) * u(n)$; $s(n-1) = h(n-1) * u(n)$

$$s(n) - s(n-1) = [h(n) - h(n-1)] * u(n).$$

$$y(n) = x(n) * h(n) = x(n) * [s(n) - s(n-1)]$$

$$\Rightarrow y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, s(n-k)$$

61) compute the correlation sequence $r_{xx}(l)$ and $r_{xy}(l)$ for the following signal sequences.

$$x(n) = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

$$r_{xx}(l) = \sum_{k=-\infty}^{\infty} x(n)\, x(n-l)$$

The range of non-zero values of $r_{xx}(l)$ is determined by

$$n_0 - N \le n \le n_0 + N$$

$$n_0 - N \le n - l \le n_0 + N.$$

which implies

$$-2N \le l \le 2N.$$

for a given shift l, the no of terms in the summation for which both $x(n)$ and $x(n-l)$ are non-zero is $2N+1-|l|$ and the value of each term is 1. Hence,

$$r_{xx}(l) = \begin{cases} 2N+1-|l|; & -2N \le l \le 2N \\ 0 ; & \text{otherwise.} \end{cases}$$

for $r_{xy}(l)$ we have

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0| \,; & n_0 - 2N \le l \le n_0 + 2N \\ 0 \,, & \text{otherwise.} \end{cases}$$

62) Determine the autocorrelation sequences of the following signals.

a) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$

b) $y(n) = \{\underset{\uparrow}{1}, 1, 2, 1\}$

What is your conclusion?

Sol:-

$r_{xx}(l) = x(n) * x(-n)$

$x(-n) = \{1, 1, 2, \underset{\uparrow}{1}\}$

	1	2	1	1
1	1	2	1	1
1	1	2	1	1
2	2	4	2	2
1	1	2	1	1

$r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$

* $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$

$r_{xx}(-3) = x(0)x(3) = 1.$

b) $y(n) = \{1, 1, 2, 1\}$

$r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)y(n-l)$

	1	1	2	1
1	1	1	2	1
2	2	2	4	2
1	1	1	2	1
1	1	1	2	1

$r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

we obtain $y(n) = x(-n+3)$

which is equivalent to reversing the sequence

This has not changed the autocorrelation sequence

63) What is the normalized autocorrelation sequence of the signal $x(n)$ given by.

$$x(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise.} \end{cases}$$

Sol:- $\sum_{-N}^{n+N} = 2N+1+n$

$\sum_{n-N}^{N} 1.1 = 2N+1-n$

$$r_{xx}(n) = \begin{cases} 2N+1+n & -2N \le n < 0 \\ 2N+1-n, & 0 \le n \le 2N \end{cases}$$

Hence $$r_{xx}(l) = \begin{cases} 2N+1-|l|\ ; & -2N \le l \le 2N \\ 0\ ; & \text{otherwise} \end{cases}$$

$r_{xx}(0) = 2N+1$

∴ the normalized autocorrelation is

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|)\ ; & -2N \le l \le 2N \\ 0\ ; & \text{otherwise.} \end{cases}$$

64) An audio signal $s(t)$ generated by a loudspeaker is reflected at two different walls with reflection coefficients r_1 and r_2. The signal $x(t)$ recorded by a microphone close to the loudspeaker, after sampling is

$$x(n) = s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)$$

where k_1 and k_2 are the delays of the two echoes.

a) Determine the autocorrelation $r_{xx}(l)$ of the signal $x(n)$.

Sol:- $$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} [s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)] *$$

$$[s(n-l) + r_1 s(n-l-k_1) + r_2 s(n-l-k_2)]$$

$$= (1 + r_1^2 + r_2^2) \cdot r_{ss}(l) + r_1 [r_{ss}(l+k_1) + r_{ss}(l-k_1)]$$

$$+ r_2 [r_{ss}(l+k_2) + r_{ss}(l-k_2)] +$$

$$r_1 r_2 [r_{ss}(l+k_1-k_2) + r_{ss}(l+k_2-k_1)]$$

b) Can we obtain r_1, r_2, k_1, and k_2 by observing $r_{xx}(l)$?

Sol:- $r_{xx}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1 + k_2)$. suppose that $k_1 < k_2$, Then, we can determine r_1 and k_1. The problem is to determine r_2 and k_2 from the other peaks.

c) what happens if $r_2 = 0$?

If $r_2 = 0$; the peaks occur at $l = 0$ and $l = \pm k_1$, then, it is easily to obtain r_1 and k_1