Three Phase Induction Motors

Introduction:

The three phase induction motor is the most widely used electrical motor. Almost 80% of the mechanical power used by industries is provided by **three phase induction motors** because of its simple and rugged construction, low cost, good operating characteristics, the absence of commutator, good speed regulation and easy to maintain.

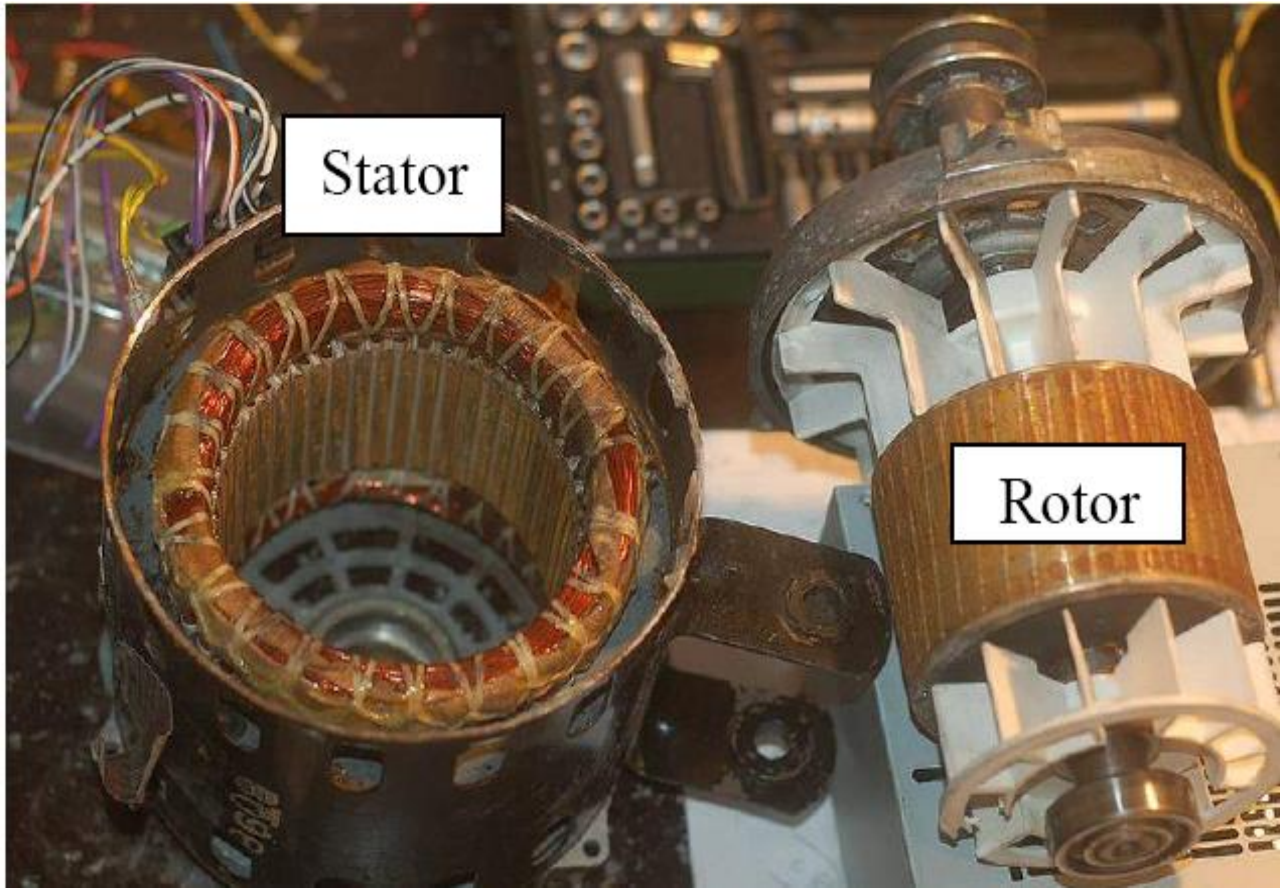
In three phase induction motor, the power is transferred from stator to rotor winding through **induction**. The induction motor is also called asynchronous motor as it runs at a speed other than the synchronous speed.

Constructional details of 3-Phase Induction Motor

Stator : (stationary part of induction motor)

- **It consists of stator frame, stator core, distributed winding, two end covers, bearings etc.**
- Slots are on the internal circumference of the stator.
- A three phase supply is placed in these slots as a stator winding of induction motor.
- **The two end convers are made of cast-iron.**

Air Gap- gap between Stator and Rotor
(gap length from 1.25 mm to 2.5 mm).

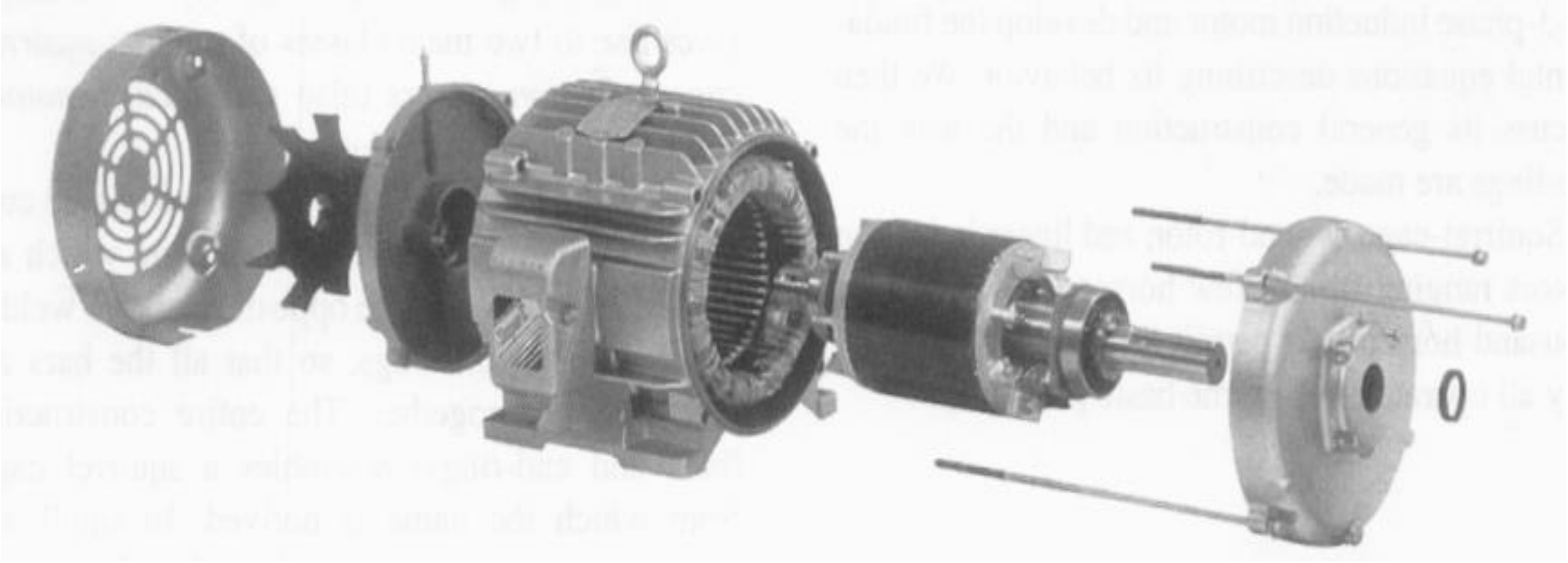


Rotor : (rotating part of induction motor)

- The rotor is connected to the mechanical load through the shaft.
- Also composed of punched laminations with rotor slots for rotor winding.

The construction of a stator for both of the kinds of three phase induction motors is identical, and is discussed briefly below. Please ensure you are using the appropriate electrical tools if you're going to be deconstructing a motor yourself.

induction motors



Stator of Three Phase Induction Motor

- The stator of the three-phase induction motor consists of three main parts :
- Stator frame,
- Stator core,
- Stator winding or field winding.

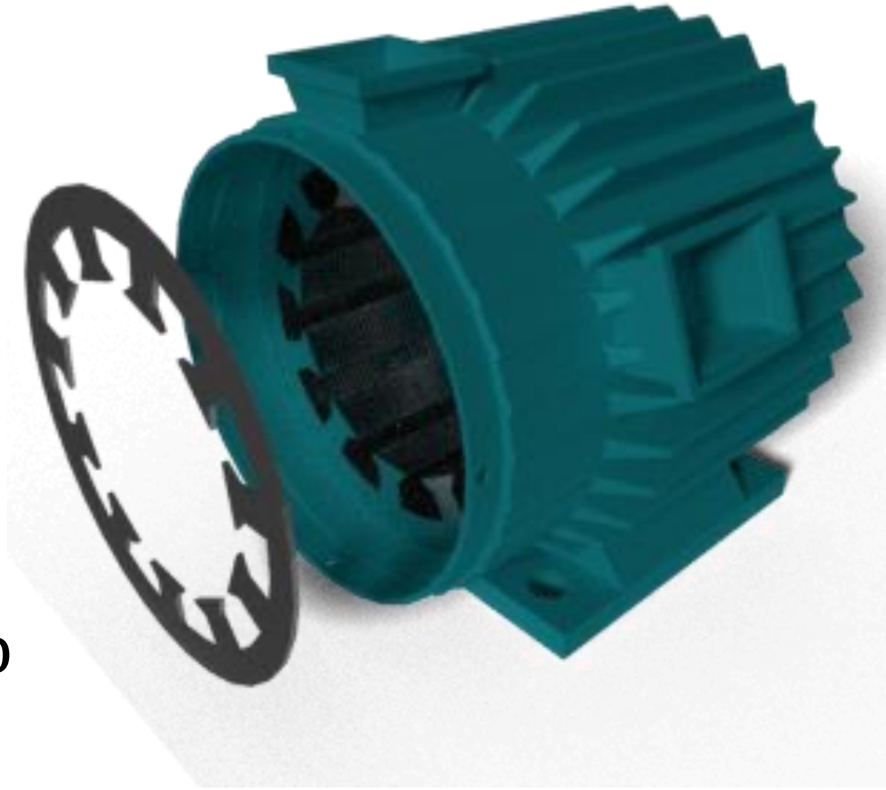
Stator Frame

It is the outer part of the **three phase induction motor**.

- support the stator core and the field winding.
- acts as a covering, and it provides protection and mechanical strength to all the inner parts of the induction motor.
- made up of die-cast or fabricated steel.
- strong and rigid

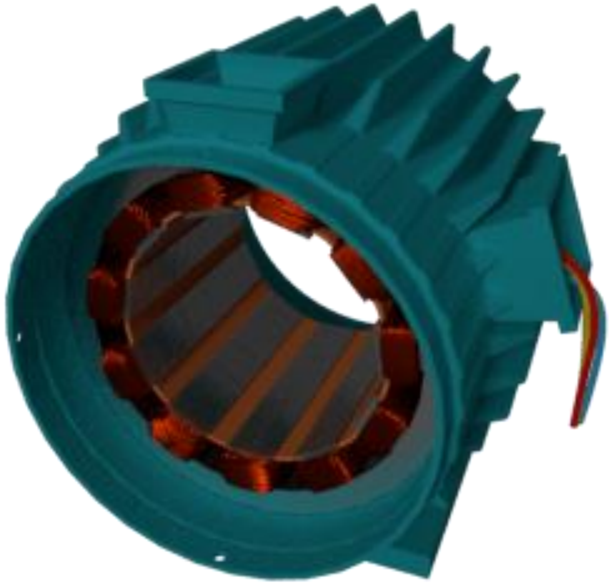
Stator Core

- The main function of the stator core is to carry the alternating flux.
- stator core is laminated- to reduce the eddy current loss.
- These laminated types of structure are made up of stamping which is about 0.4 to 0.5 mm thick.
- The stamping is made up of silicon steel, which helps to reduce the hysteresis loss occurring in the motor.



Stator Winding or Field Winding

The slots on the periphery of the stator core of the three-phase induction motor carry three phase windings. We apply three phase ac supply to this three-phase winding. The three phases of the winding are connected either in star or delta depending upon which type of starting method we use. We start the squirrel cage motor mostly with star-delta stator and hence the stator of squirrel cage motor is delta connected.



We start the slip ring three-phase induction motor by inserting resistances so, the stator winding of slip ring induction motor can be connected either in star or delta. The winding wound on the stator of three phase induction motor is also called field winding, and when this winding is excited by three phase ac supply, it produces a rotating magnetic field.

Types of Induction Motors:

Depending upon the type of rotor construction used the **three phase induction motor** are classified as:

1. Squirrel cage induction motor
2. Slip ring induction motor or wound rotor induction motor or phase wound induction motor.

Squirrel Cage Induction Motor (most common)

For squirrel cage rotor it consists of copper bars, slightly longer than the rotor, which are pushed into slots. The ends are welded to copper and rings so that all bars are short circuited.

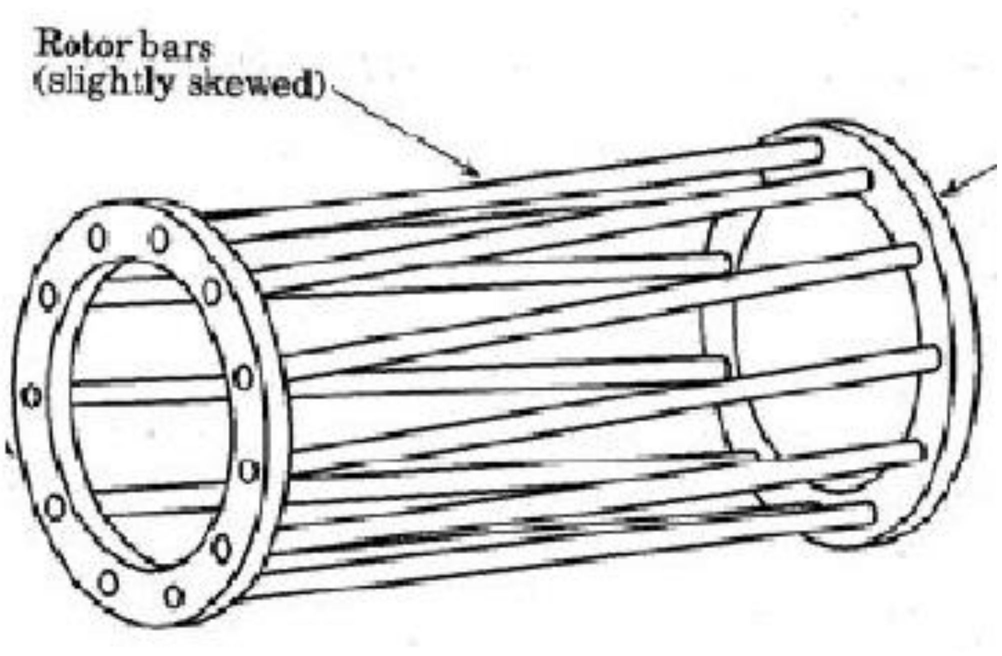
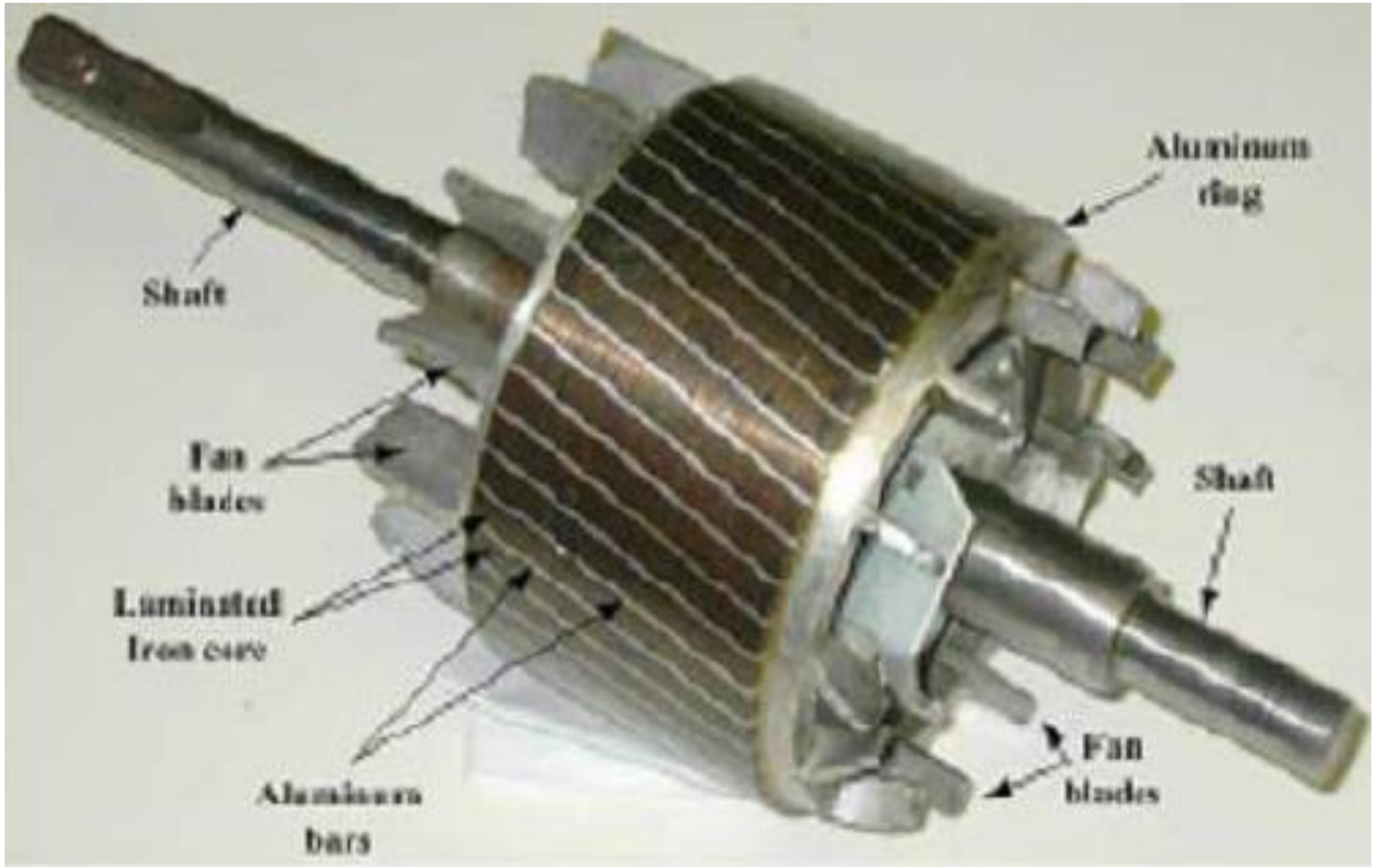
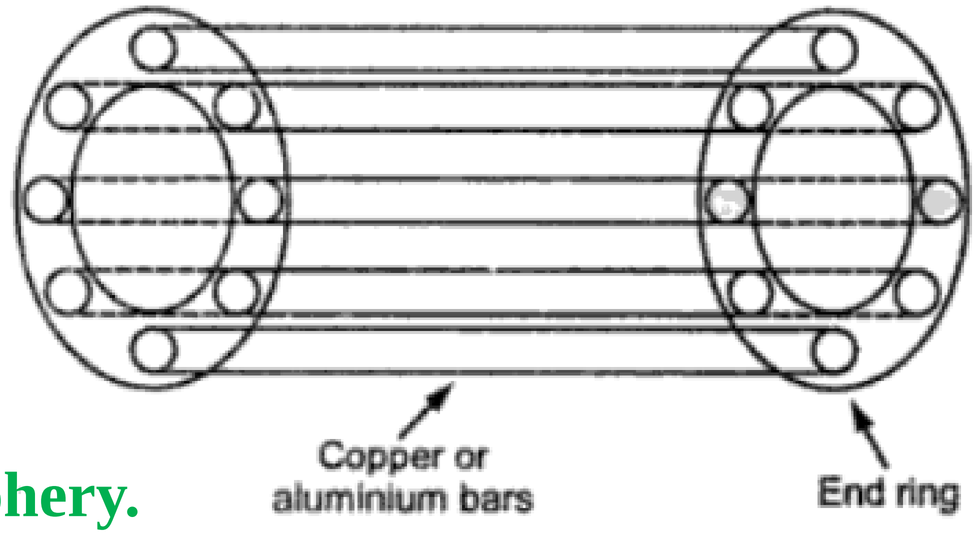


Figure 2: Squirrel Cage Rotor.





Squirrel cage rotor

- The rotor core is cylindrical and slotted on its periphery.
- The rotor consists of uninsulated copper or aluminium bars called rotor conductors.
- The bars are placed in the slots.
- These bars are permanently shorted at each end with the help of conducting copper ring called end ring.
- As rotor itself is short circuited, no external resistance can be introduced in the rotor circuit.
- Fan blades are generally provided at the ends of the rotor core.
- This circulates the air through the machine while operation , providing the necessary cooling .
- The air gap between stator and rotor is kept uniform and as small as possible.
- In this type of rotor the slots are not arranged parallel to the shaft axis but are skewed.

The below diagram shows a squirrel cage induction rotor having aluminum bars short circuit by aluminum end rings.

➢ **The advantages of skewing;**

1. A magnetic hum gets reduced.
2. Smooth motor operation.
3. Magnetic locking gets reduced.

Advantages of Squirrel Cage Induction Rotor

- Its construction is very simple and rugged.
- As there are no brushes and slip ring, these motors requires less maintenance.

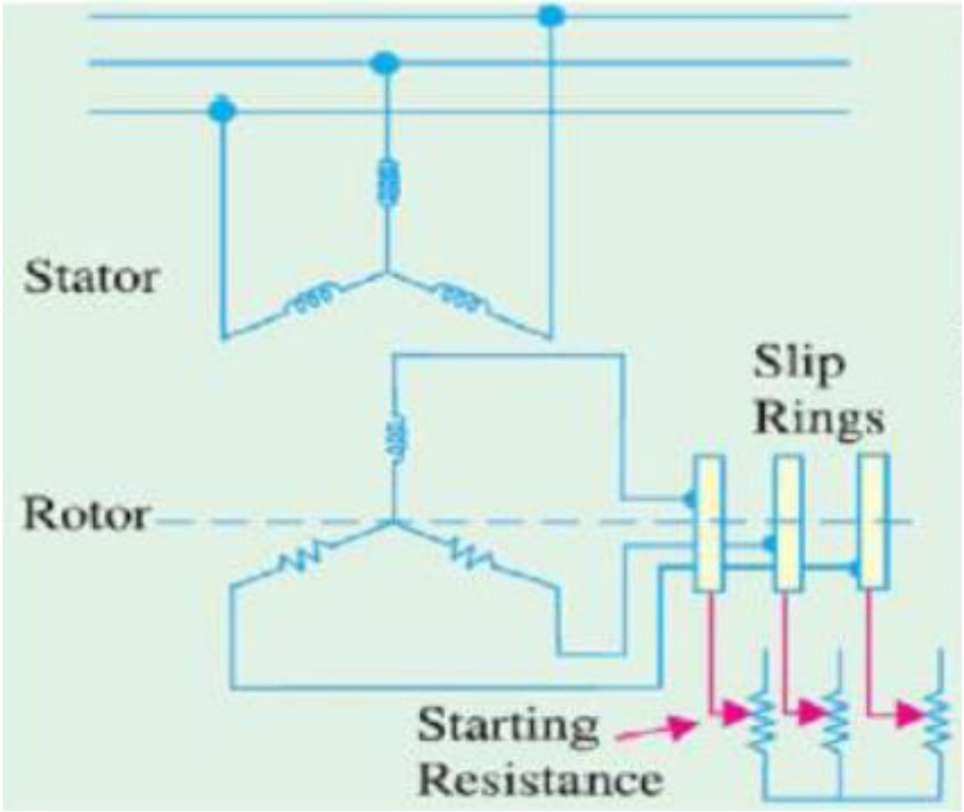
Applications of Squirrel Cage Induction Rotor

- We use the squirrel cage induction motors in lathes, drilling machine, fan, blower printing machines, etc.

3 phase wound-rotor induction motor

A wound rotor, has a 3-phase winding similar to the stator winding. The rotor terminals are connected to three slip rings which turn with the rotor. The slip (and brushes) allow external resistors to be connected in series with the winding.

The external resistors are mainly used during start up and under normal running conditions, the windings are short circuited externally.



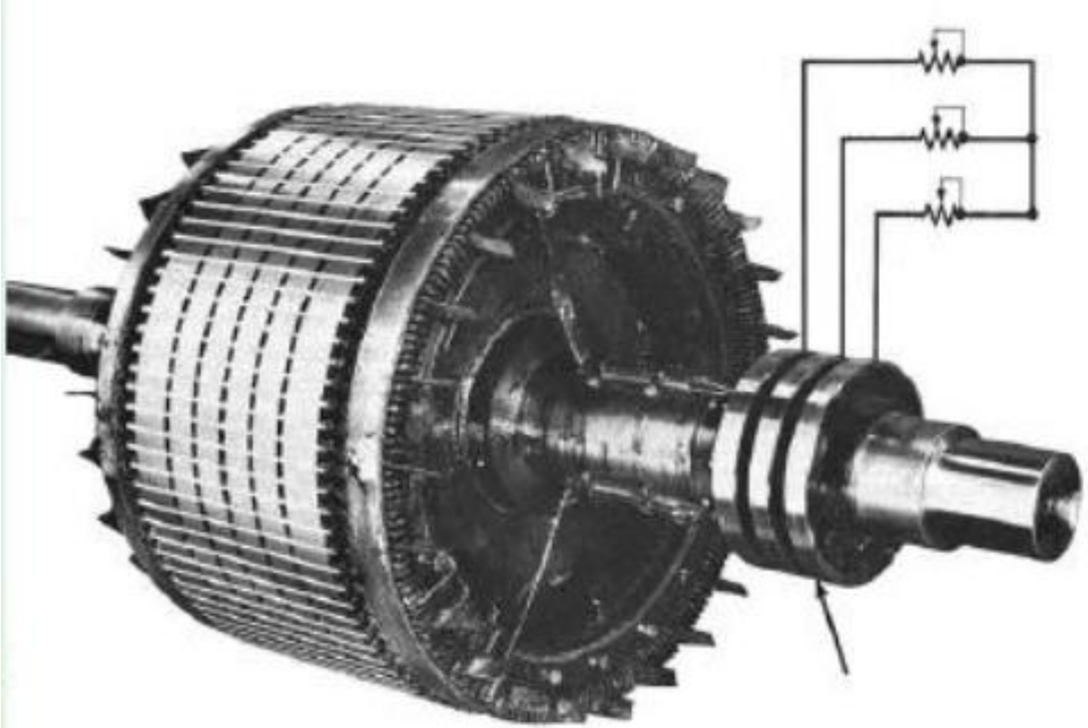
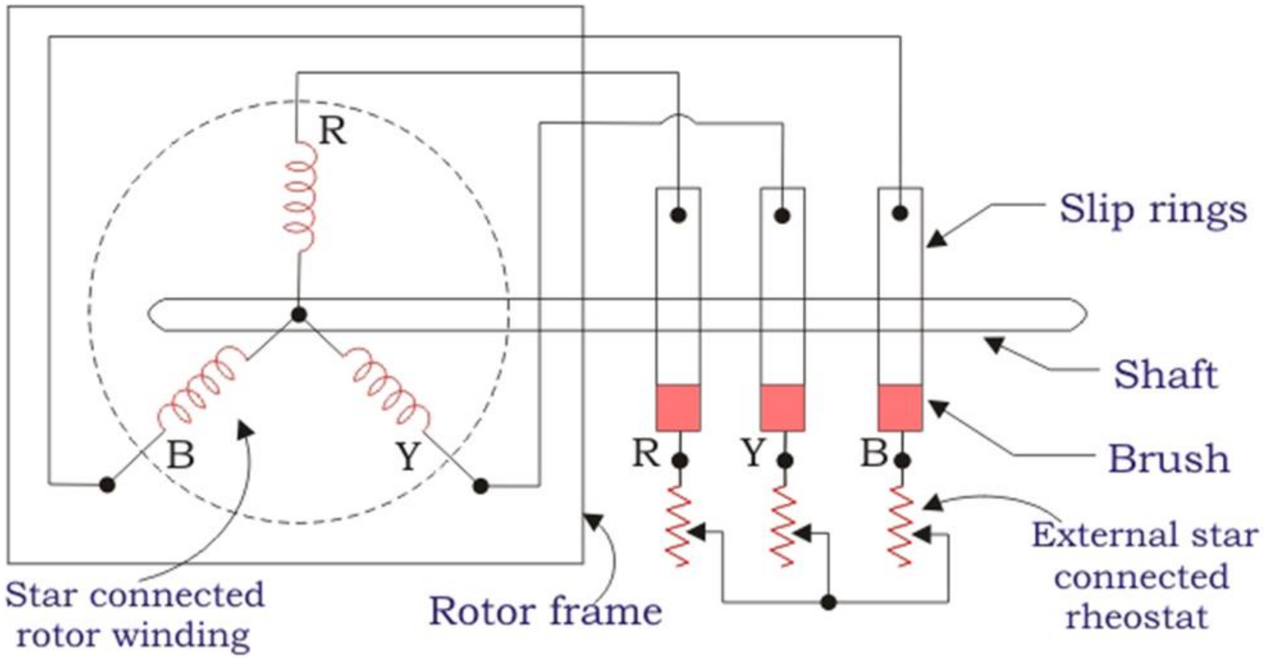


Figure: Wound Rotor.

Slip ring rotor or wound rotor

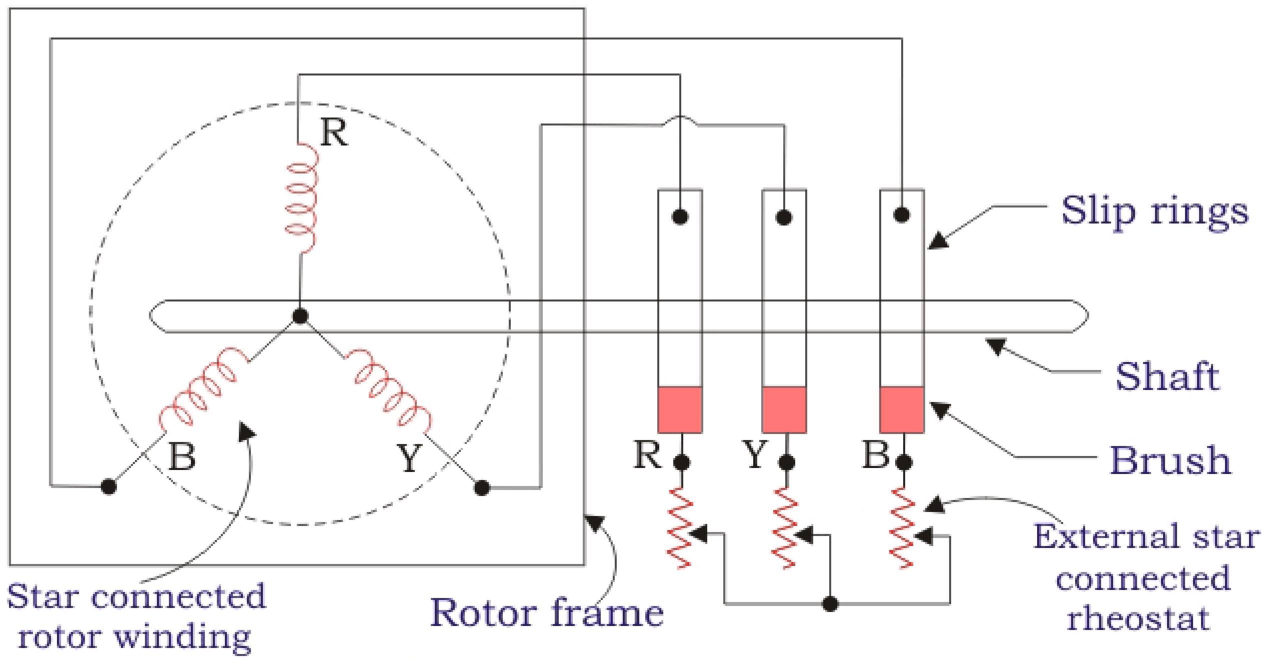
➢In this type of construction, rotor winding is similar to the stator.
➢The rotor construction is laminated and slotted. The slots contain the rotor winding.
➢The three ends of the three phase winding are permanently connected to the slip rings. The slip rings are mounted on the same shaft.
➢The external resistances can be added with the help of brushes and slip ring arrangement in series with each phase of the rotor winding.



Slip Ring Three Phase Induction Motor

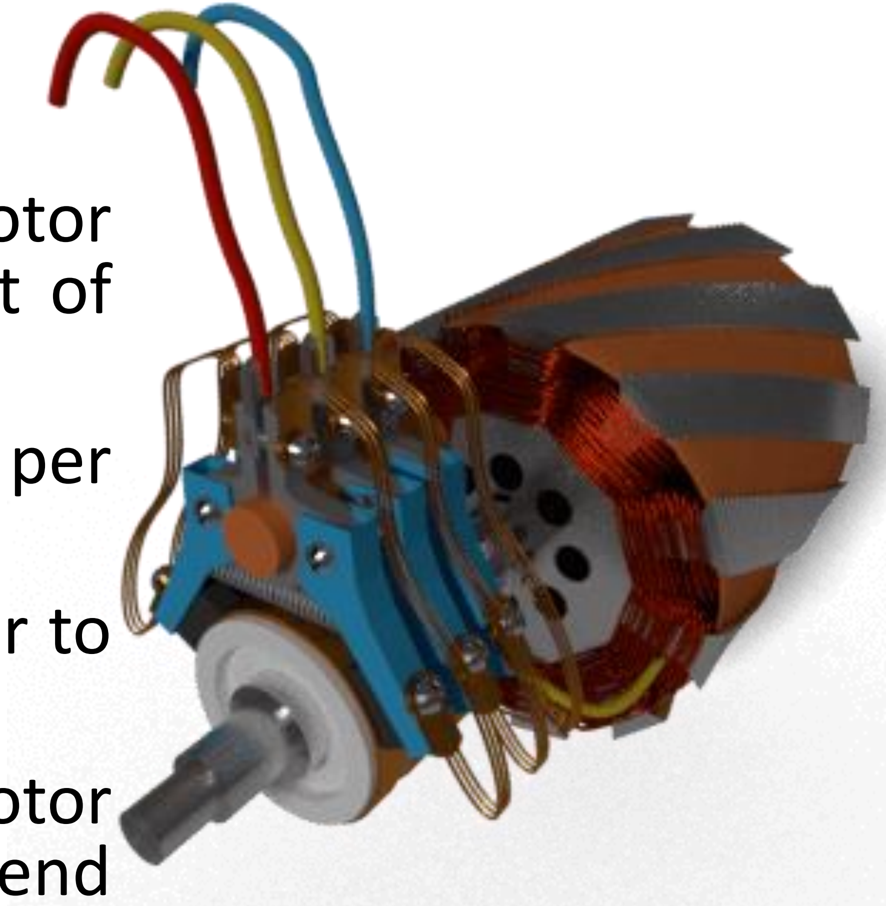
The three ends of three-phase windings are permanently connected to these slip rings. The external resistance can be easily connected through the brushes and slip rings and hence used for speed controlling and improving the starting torque of three phase induction motor. The brushes are used to carry current to and from the rotor winding. These brushes are further connected to three phase star connected resistances. An electrical diagram of a slip ring three phase induction motor is shown below.

At starting, the resistance is connected to the rotor circuit and is gradually cut out as the rotor pick up its speed. When the motor is running the slip ring are shorted by connecting a metal collar, which connects all slip ring together, and the brushes are also removed. This reduces the wear and tear of the brushes. Due to the presence of slip rings and brushes the rotor construction becomes somewhat complicated therefore it is less used as compare to squirrel cage induction motor.



Slip Ring Three Phase Induction Motor

Slip Ring or Wound Rotor Three Phase Induction Motor



In this type of three phase induction motor the rotor is wound for the same number of poles as that of the stator,

it has less number of slots and has fewer turns per phase of a heavier conductor.

The rotor also carries star or delta winding similar to that of the stator winding.

The rotor consists of numbers of slots and rotor winding are placed inside these slots. The three end terminals are connected together to form a star connection. As its name indicates, three phase slip ring induction motor consists of slip rings connected on the same shaft as that of the rotor.

Advantages of Slip Ring Induction Motor

1.It has high starting torque and low starting current.

2.Possibility of adding additional resistance to control speed.

Application of Slip Ring Induction Motor

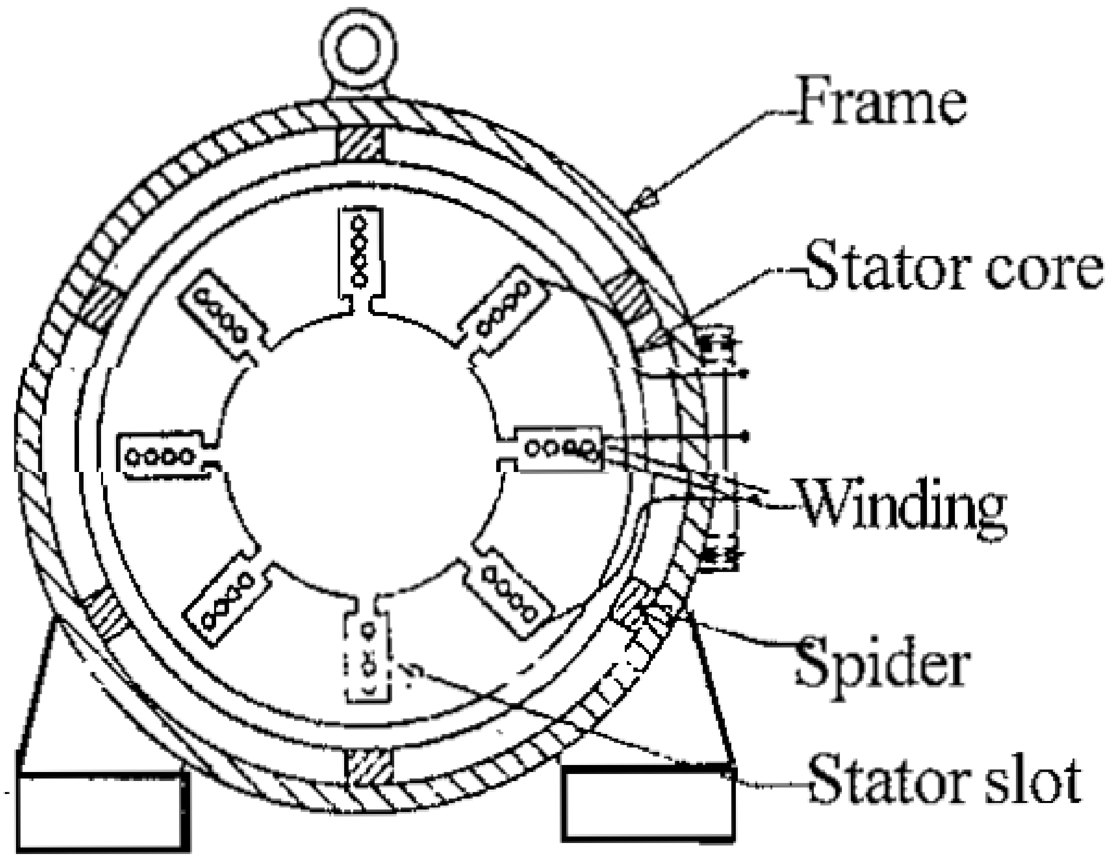
Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc.

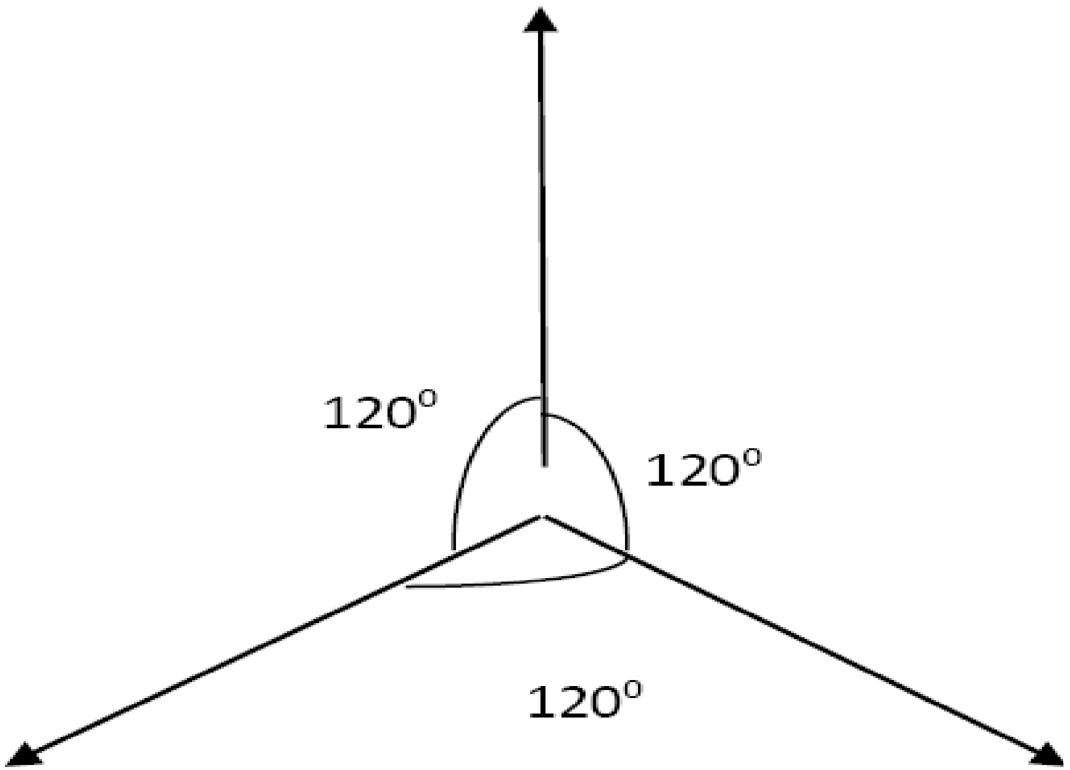
The other parts of a 3 phase induction motor are:

- Shaft for transmitting the torque to the load. This shaft is made up of steel.
- Bearings for supporting the rotating shaft.
- One of the problems with electrical motor is the production of heat during its rotation. To overcome this problem, we need a fan for cooling.
- For receiving external electrical connection Terminal box is needed.
- There is a small distance between rotor and stator which usually varies from 0.4 mm to 4 mm. Such a distance is called air gap.

Difference between Slip Ring and Squirrel Cage Induction Motor

Slip ring or phase wound Induction motor	Squirrel cage induction motor
Construction is complicated due to presence of slip ring and brushes	Construction is very simple
The rotor winding is similar to the stator winding	The rotor consists of rotor bars which are permanently shorted with the help of end rings
We can easily add rotor resistance by using slip ring and brushes	Since the rotor bars are permanently shorted, its not possible to add external resistance
Due to presence of external resistance high starting torque can be obtained	Staring torque is low and cannot be improved
Slip ring and brushes are present	Slip ring and brushes are absent
Frequent maintenance is required due to presence of brushes	Less maintenance is required
The construction is complicated and the presence of brushes and slip ring makes the motor more costly	The construction is simple and robust and it is cheap as compared to slip ring induction motor
This motor is rarely used only 10% industry uses slip ring induction motor	Due to its simple construction and low cost. The squirrel cage induction motor is widely used
Rotor copper losses are high and hence less efficiency	Less rotor copper losses and hence high efficiency
Speed control by rotor resistance method is possible	Speed control by rotor resistance method is not possible
Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc	Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc


Frame
Stator core
Winding
Spider
Stator slot

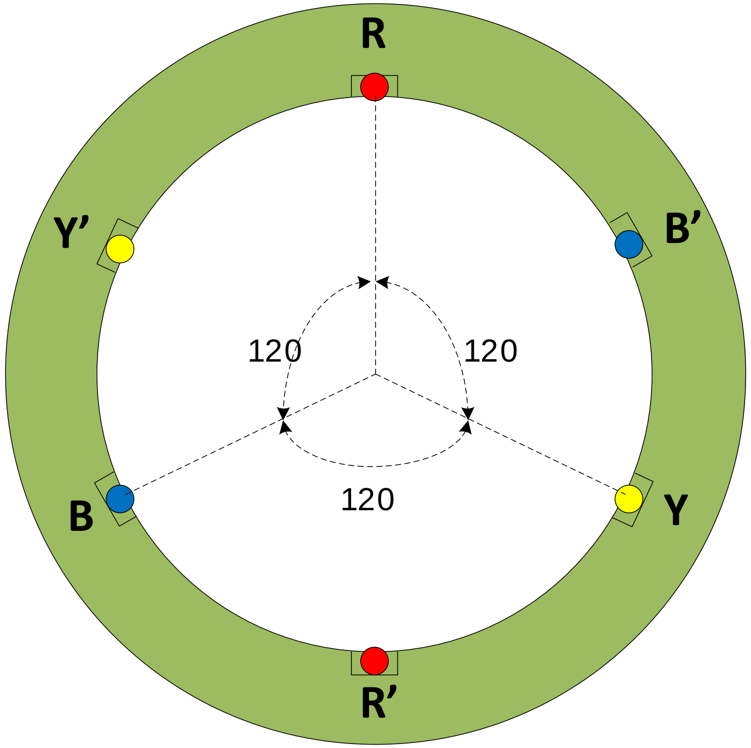

120°
120°
120°

Rotating Magnetic Field

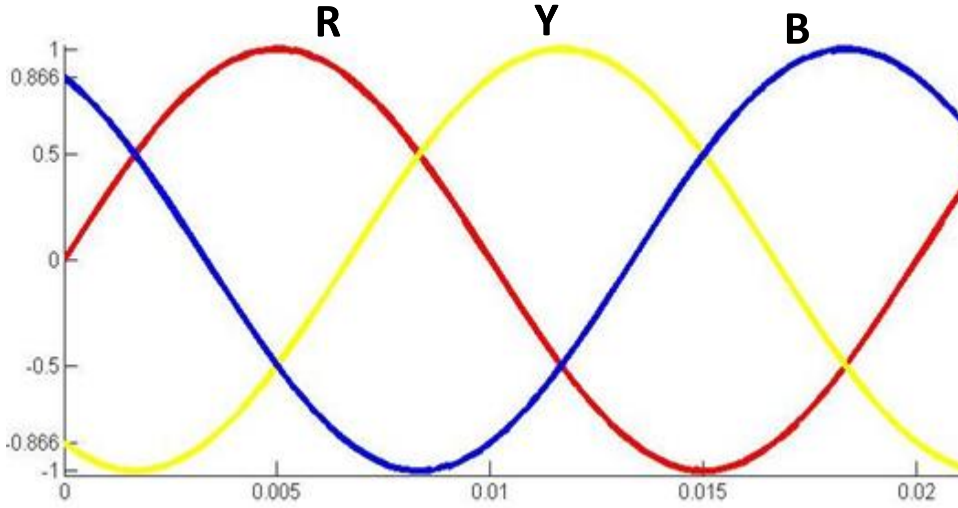
of

Three-phase Induction Motor

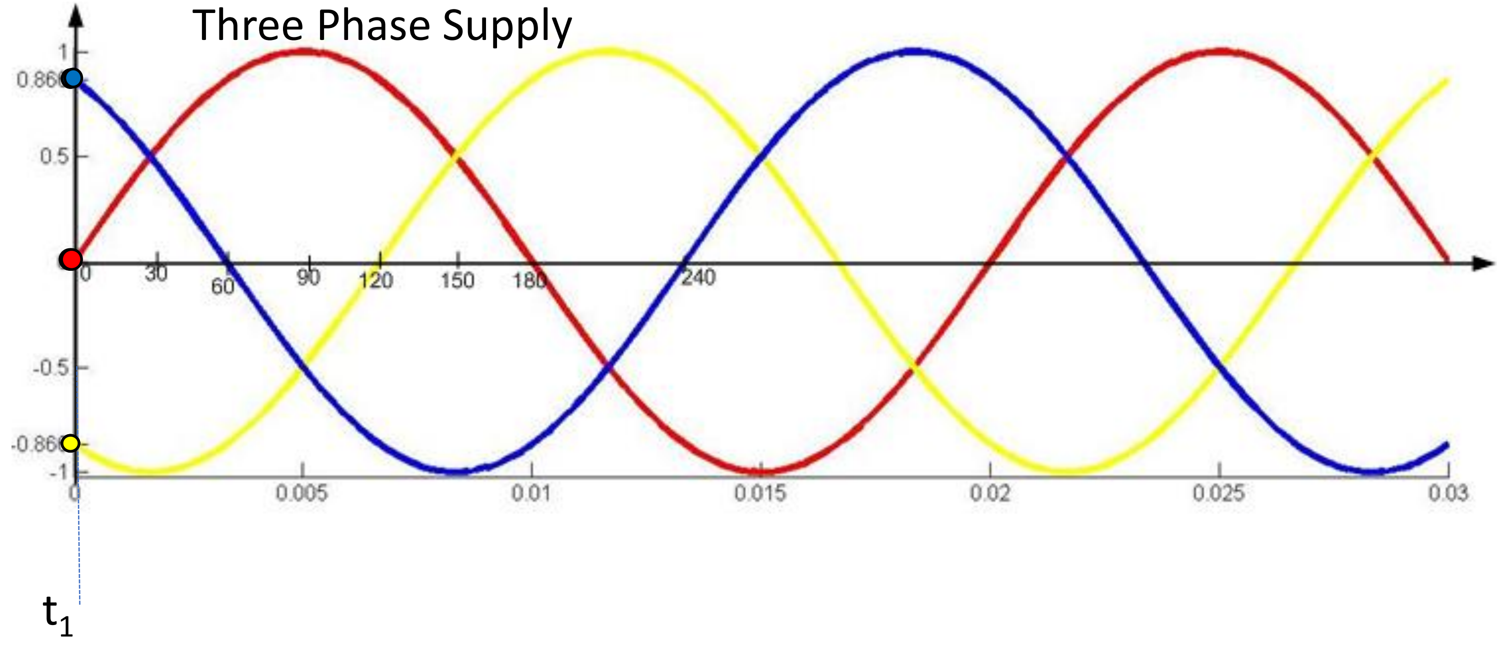
Three- phase Induction Motor



- Three winding are place around the circular stator
- Windings are physically apart by 120° (electrical degrees) in space
- three phase supply (120° time phase) voltage is applied to closed stator winding.
- Three phase current produces three phase magnetic field
- These magnetic field creates Rotating Magnetic Field (RMF)



Generation of RMF



At t_1 time instance ($t_1 = \omega t = 0$)

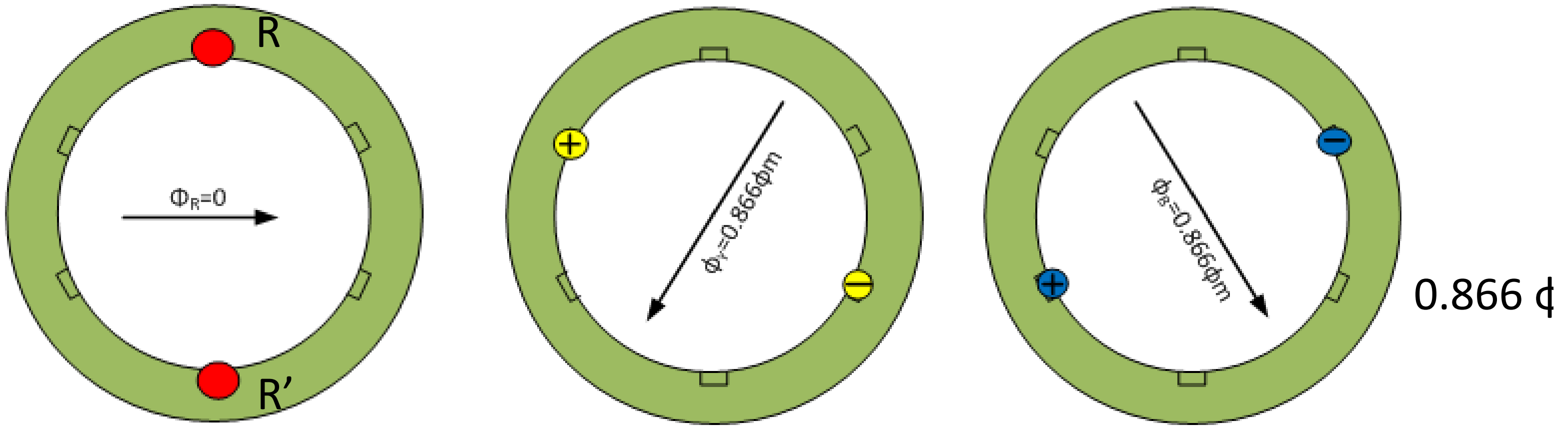
$I_R = 0.0\ I_m$ $\qquad \varphi_R = 0.0\varphi_m$

$I_Y = -0.866\ I_m$ $\qquad \varphi_Y = -0.866\varphi_m$

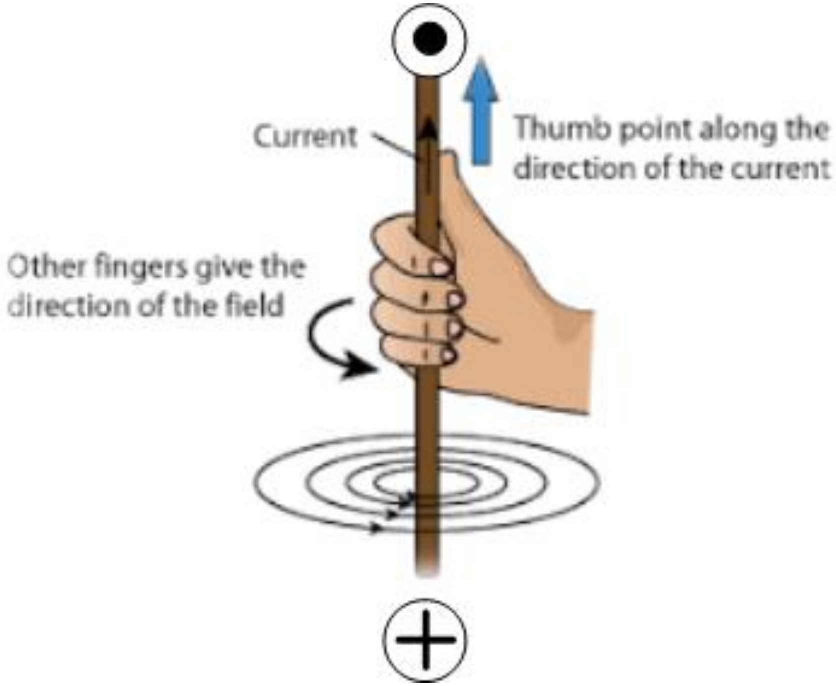
$I_B = 0.866\ I_m$ $\qquad \varphi_B = 0.866\varphi_m$

$$\boldsymbol{I_R = I_m \sin(\omega t) \quad I_Y = I_m \sin(\omega t - 2\pi/3) \qquad I_B = I_m \sin(\omega t + 2\pi/3)}$$

At t_1 instance for each phase flux behavior



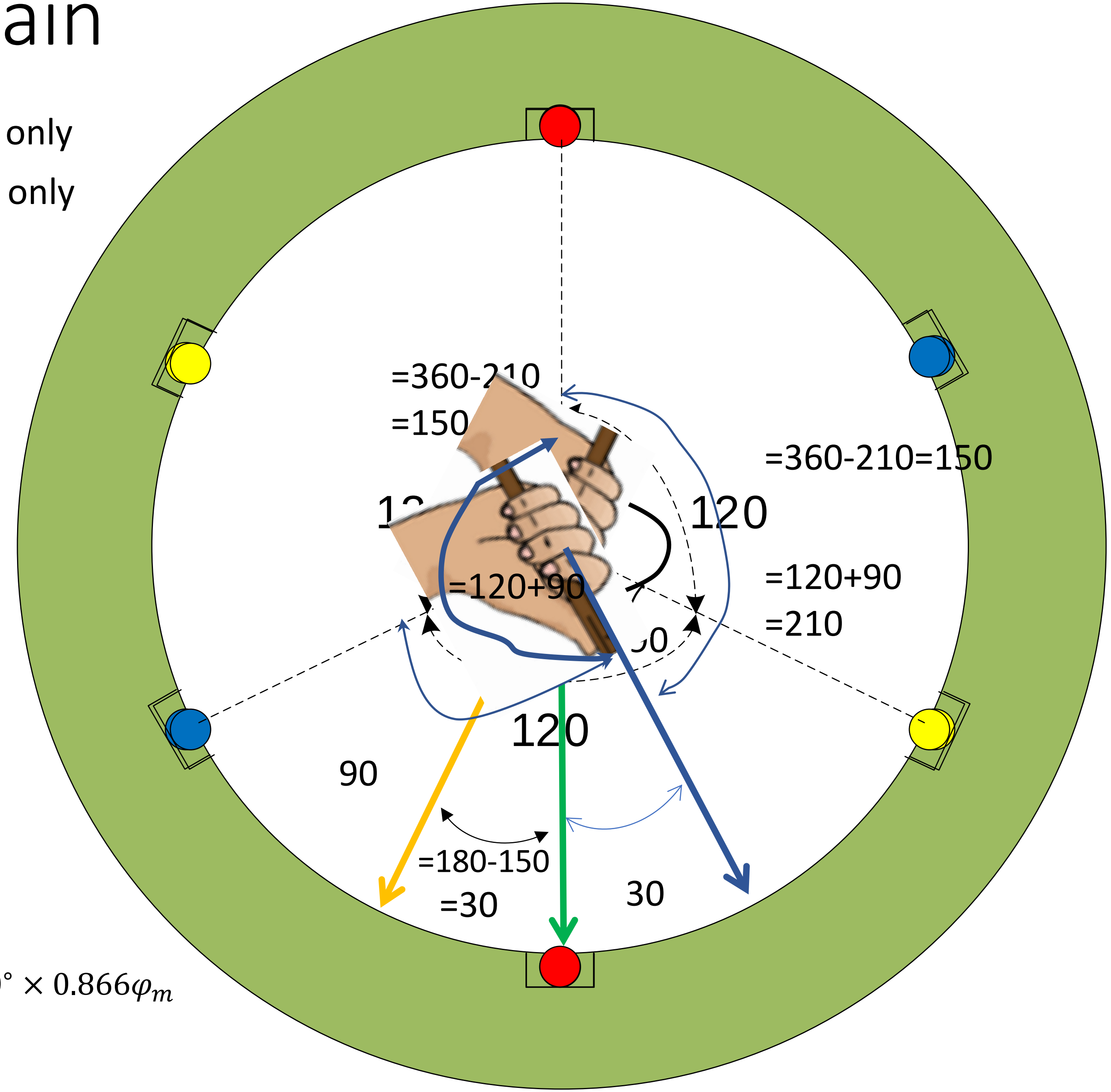
R phase Y phase B phase



Think again

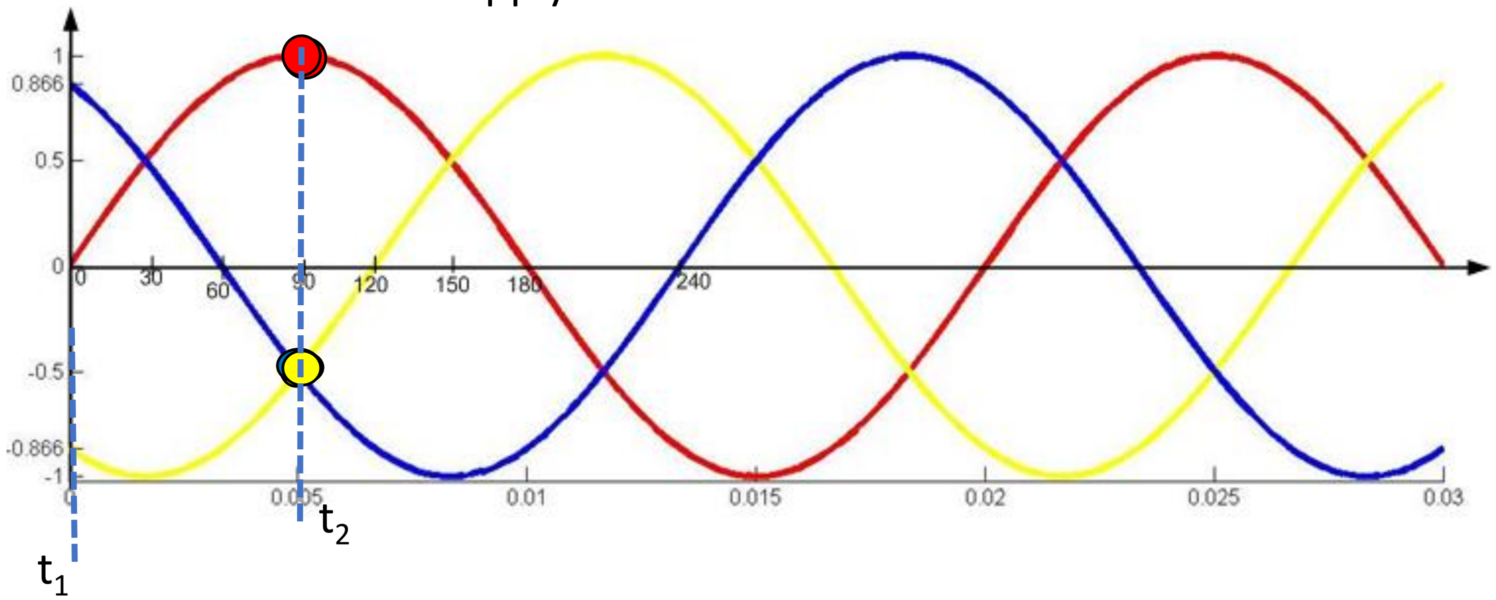
Now think of Y phase only

Now think of B phase only



$$= \cos 3\,0^\circ \times 0.866\varphi_m + \cos 3\,0^\circ \times 0.866\varphi_m$$

Three Phase Supply



At t_2 time instance ($t_2 = \omega t = 90^\circ$)

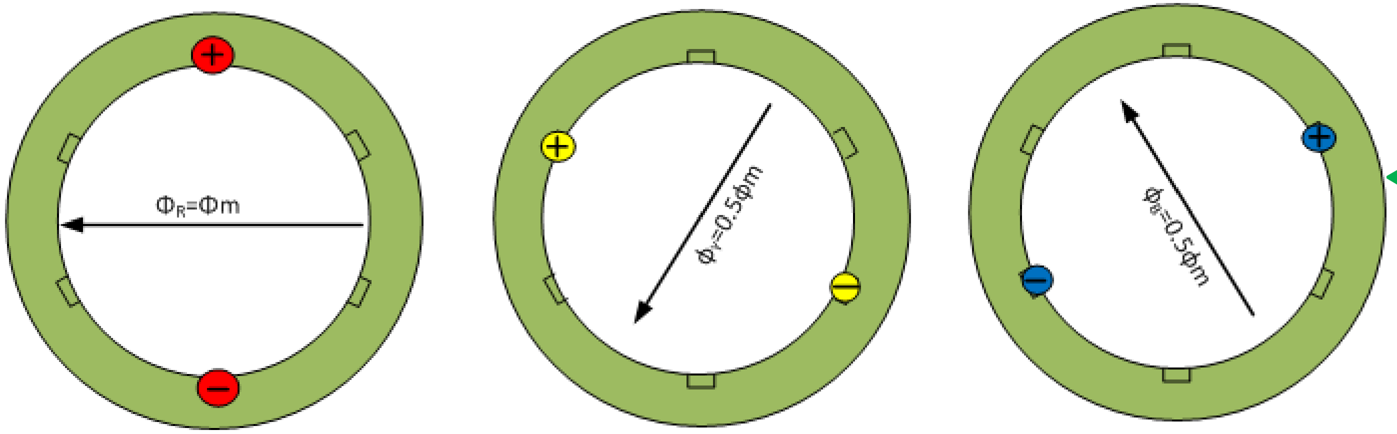
$I_R = I_m$ $\qquad \varphi_R = \varphi_m$

$I_Y = -0.5\, I_m$ $\qquad \varphi_Y = -0.5\varphi_m$

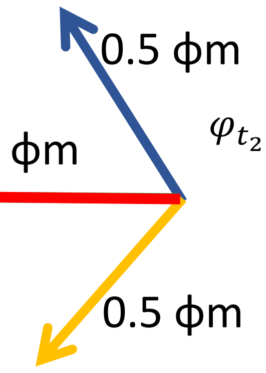
$I_B = -0.5\, I_m$ $\qquad \varphi_B = -0.5\varphi_m$

$$I_R = I_m \sin(\omega t) \quad I_Y = I_m \sin(\omega t - 2\pi/3) \quad I_B = I_m \sin(\omega t + 2\pi/3)$$

At t_2 instance for each phase flux behavior



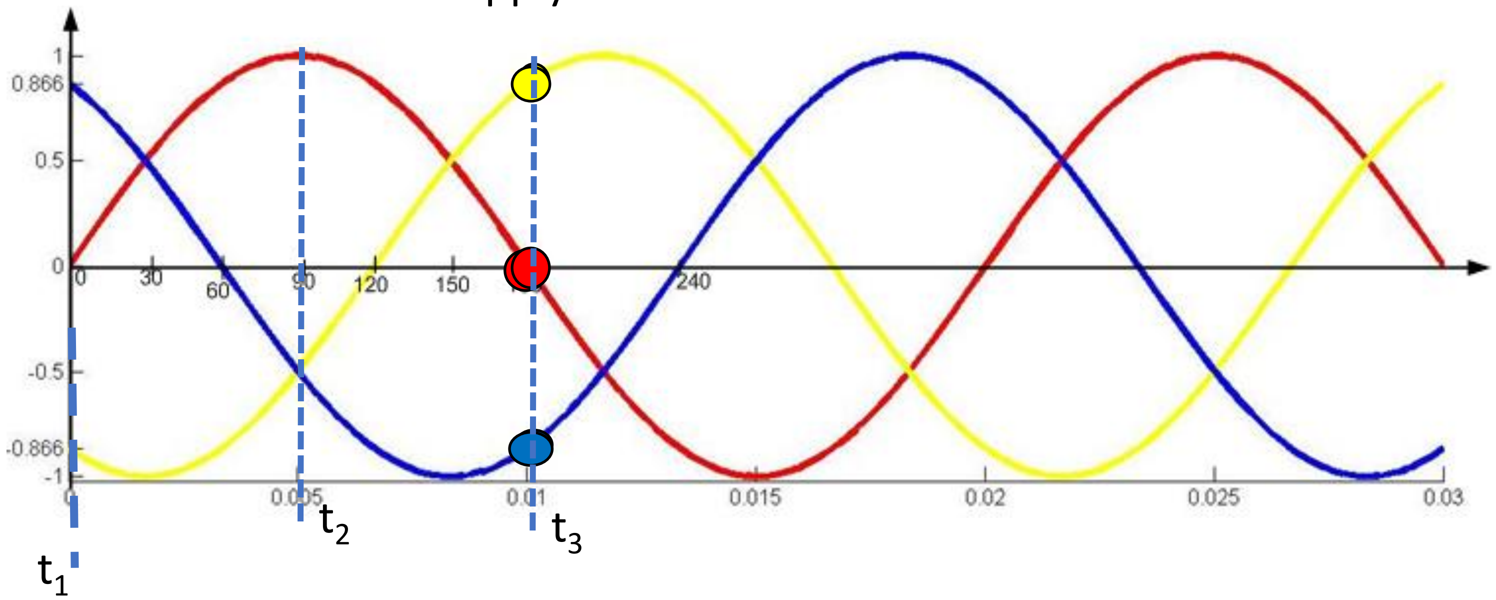
R phase Y phase B phase



$$\varphi_{t_2} = \varphi_R + \varphi_Y + \varphi_B \quad 1.5\varphi_m$$

$$= \varphi_m + \cos 60^\circ \times 0.5\varphi_m + \cos 60^\circ \times 0.5\varphi_m$$

Three Phase Supply



At t_3 time instance ($t_3 = \omega t = 180^\circ$)

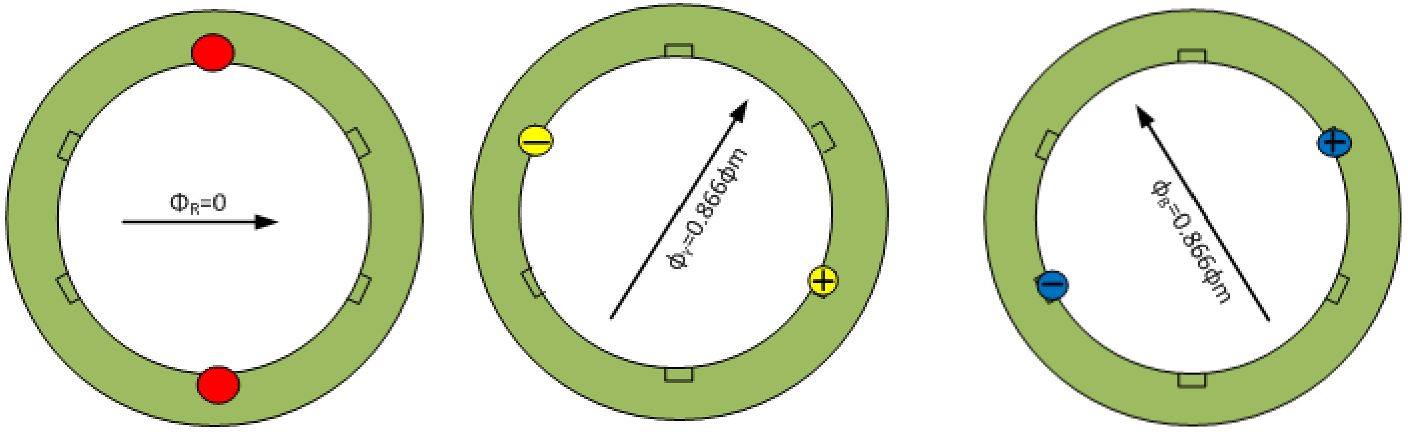
$I_R = 0.0 I_m$ $\quad \varphi_R = 0.0\varphi_m$

$I_Y = 0.866\ I_m$ $\quad \varphi_Y = 0.866\varphi_m$

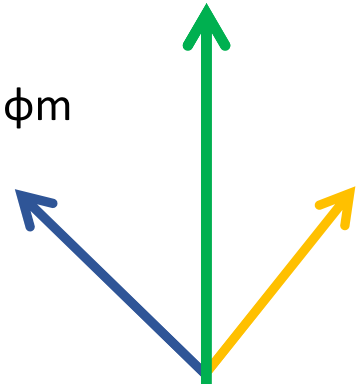
$I_B = -0.866\ I_m$ $\quad \varphi_B = -0.866\varphi_m$

$I_R = I_m \sin(\omega t)$ $\quad I_Y = I_m \sin(\omega t - 2\pi/3)$ $\quad I_B = I_m \sin(\omega t + 2\pi/3)$

At t_3 instance for each phase flux behavior

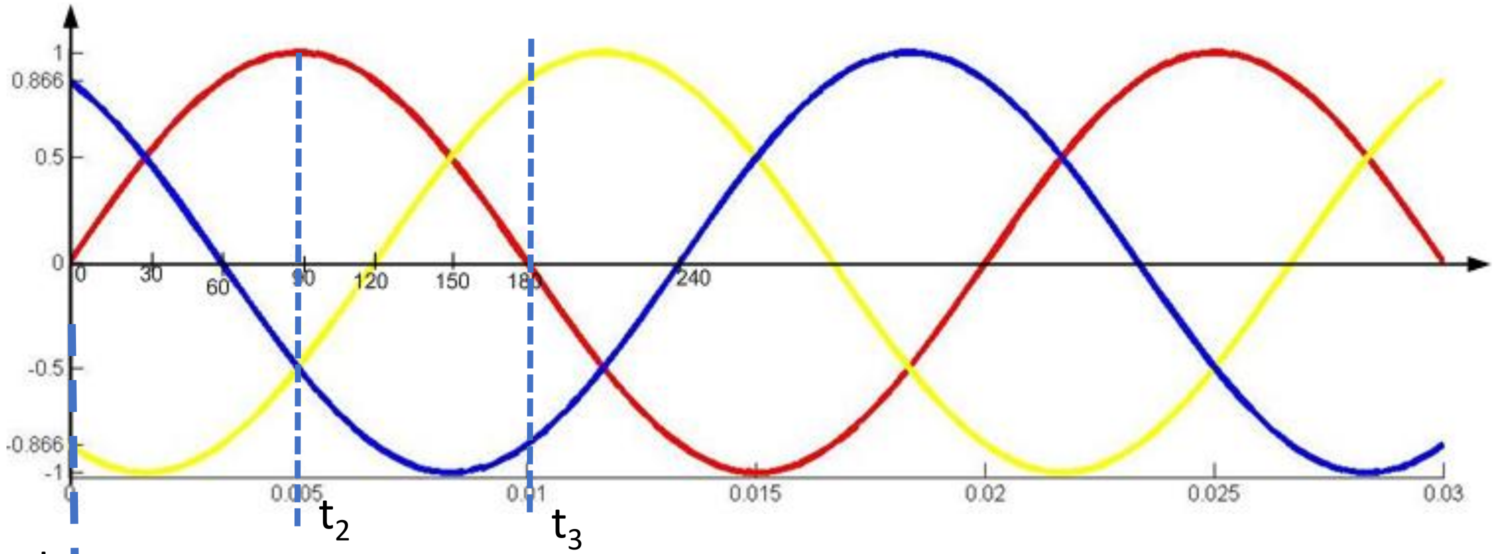


R phase Y phase B phase



$\varphi_{t_3} = \varphi_Y + \varphi_B$ $1.5\varphi_m$

$= \cos 30^\circ \times 0.866\varphi_m + \cos 30^\circ \times 0.866\varphi_m$

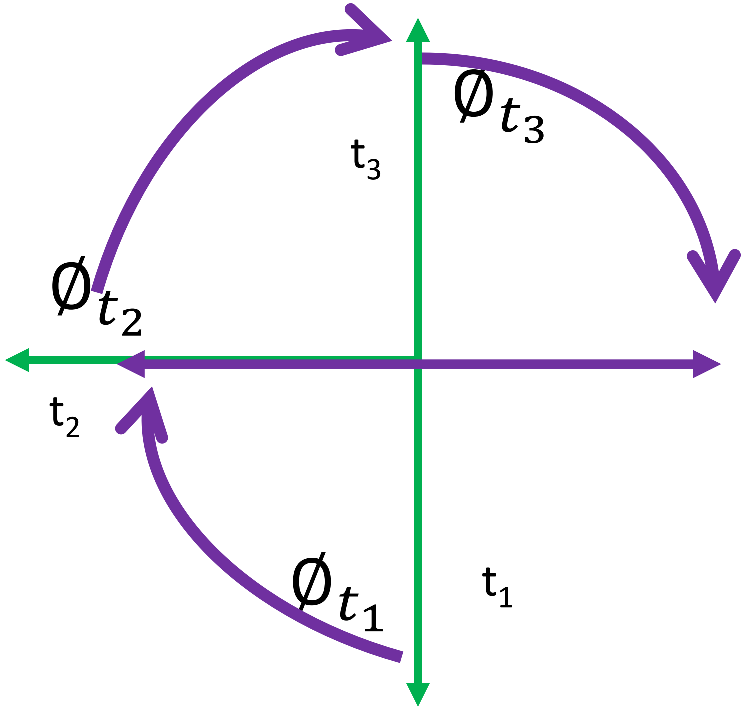


t1 $I_R = I_m \sin(\omega t)$ $I_Y = I_m \sin(\omega t - 2\pi/3)$ $I_B = I_m \sin(\omega t + 2\pi/3)$

At t_1 time instance (t_1 =ωt=0˚)

At t_2 time instance (t_2 =ωt=90˚)

At t_3 time instance (t_3 =ωt=180˚)



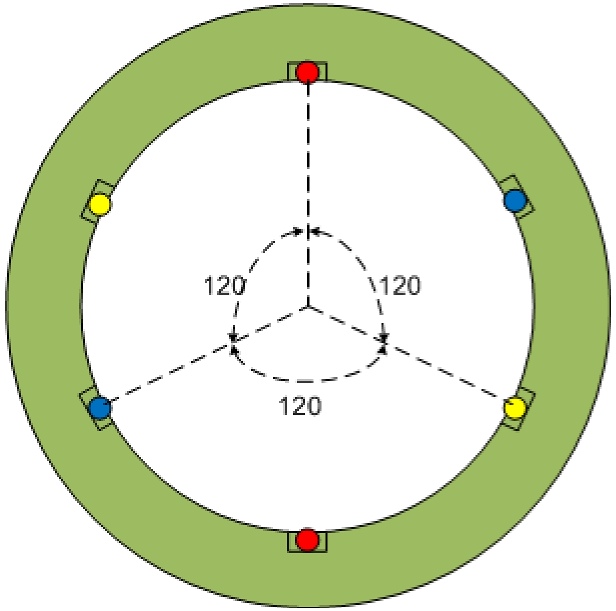
This is called RMF

This RMF rotate with the speed of $N_s = \frac{120f}{p}$ rpm

This RMF helps in the rotor winding to produce emf within itself by electromagnetic induction principal.

Three phase induction motor and Rotating Magnetic Field (RMF).

In three phase induction motor, three winding are placed in around the stator near the air gap.

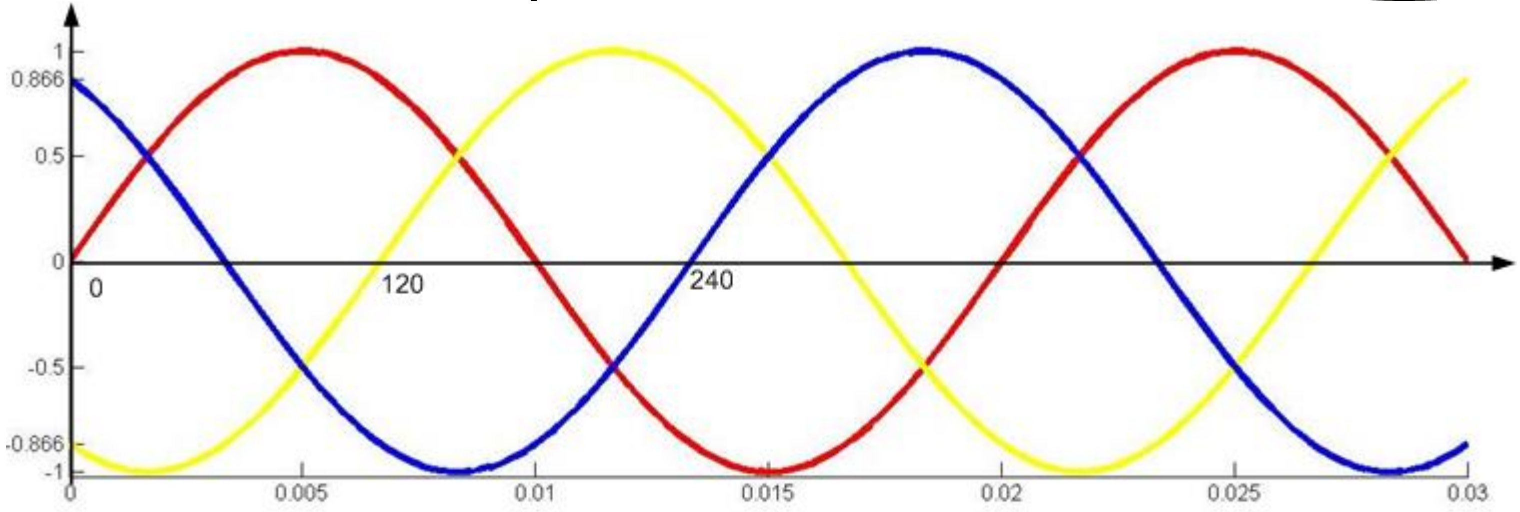


<u>These windings are</u>

a) Physically apart by 120°(electrical degree) in space.

b) 120° apart in time phase

c) conductors are placed in circular space of cylindrical stator

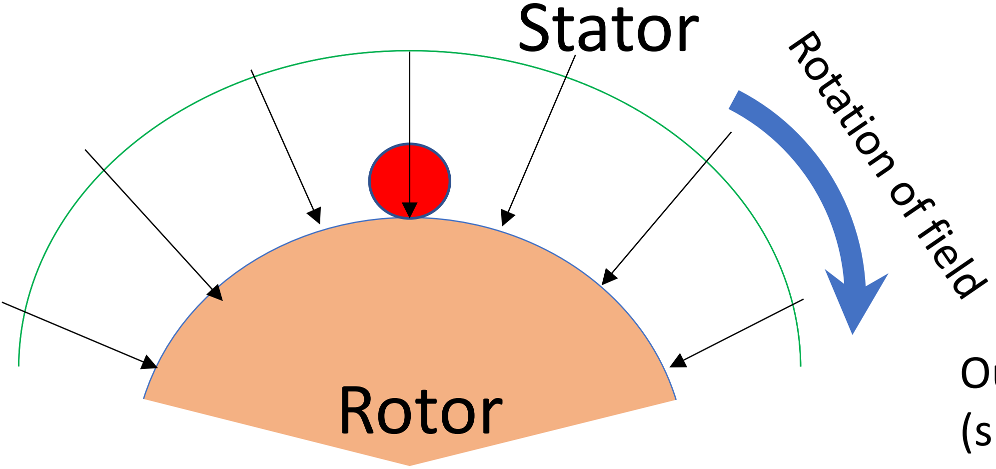


Working Principle of Induction Motor

We need to give double excitation to make a DC motor to rotate. In the DC motor, we give one supply to the stator and another to the rotor through brush arrangement.

But in induction motor, we give only one supply, so it is interesting to know how an induction motor works.

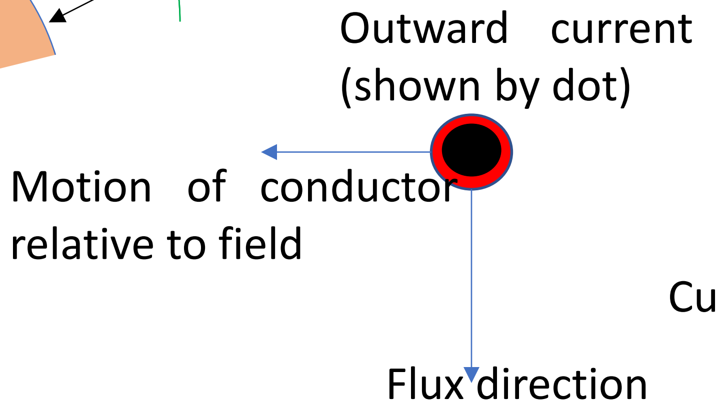
It is simple, from the name itself we can understand that here, the induction process is involved. When we give the supply to the stator winding, a magnetic flux **(RMF)** $\emptyset_s$ gets produced in the stator due to the flow of current in the coil.



Let the rotation of the magnetic field be clockwise.

A magnetic field moving clockwise has the same effect as a conductor moving anticlockwise in a stationary field.

By faraday's law of electromagnetic induction, a voltage will induce in the conductor.



Direction of the rotor conductor

BY flaming's right hand rule (generator operation)

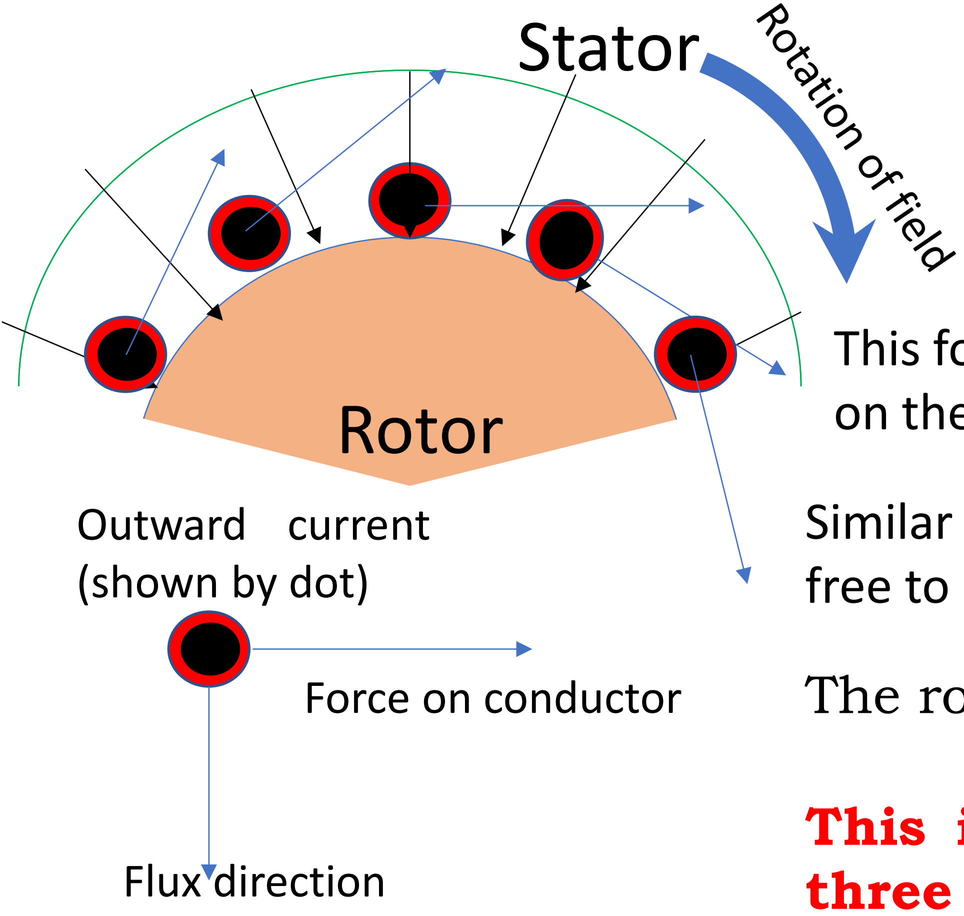
Current in rotor conductor produces its own magnetic field

The rotor winding is so arranged that each coil becomes short-circuited.

The flux from the stator cuts the short-circuited coil in the rotor. As the rotor coils are short-circuited, according to Faraday's law of electromagnetic induction, the current will start flowing through the coil of the rotor. When the current through the rotor coils flows, another flux gets generated in the rotor.

ϕ_r

Now there are two fluxes, one is stator flux, and another is rotor flux.



When a conductor carrying current is put in a magnetic field a force is produced on it (rotor conductor).

Direction of this force

BY flaming's left hand rule (motor operation)

This force acts in a tangential direction of rotor and develops a torque on the rotor conductor.

Similar torques are produced on all the rotor conductors. As the rotor is free to move, it start rotating in the same direction as the RMF.

The rotor flux will be lagging in respect of the stator flux.

This is the working principle of both single and three phase induction motors.

Induction motor speed

- At what speed will the IM run?
  - Can the IM run at the synchronous speed, why?
  - If rotor runs at the synchronous speed, which is the same speed of the rotating magnetic field, then the rotor will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor. So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed
  - When the speed falls, the rotating magnetic field will cut the rotor windings and a torque is produced

Induction motor speed

- So, the IM will always run at a speed lower than the synchronous speed
- The difference between the motor speed and the synchronous speed is called the *Slip*

$$\boxed{n_{slip} = n_{sync} - n_m}$$

Where n_{slip}= slip speed
n_{sync}= speed of the magnetic field
n_m = mechanical shaft speed of the motor

The Slip

$$\boxed{s = \frac{n_{sync} - n_m}{n_{sync}}}$$

Where *s* is the *slip*

Notice that : if the rotor runs at synchronous speed

$s = 0$

if the rotor is stationary

$s = 1$

Slip may be expressed as a percentage by multiplying the above eq. by 100, notice that the slip is a ratio and doesn't have units

Synchronous Speed and the Slip:

The synchronous speed ($N_{syc.}$) of the Stator magnetic field is given by:

$$N_{syc.} = \frac{120\,f}{p}$$

Where :

f is the frequency of the supply (stator).
p is the number of poles.

If N_r is the speed of the rotor (in *rpm*) then the difference between the synchronous speed and rotor speed is called the (slip) and is defined in per unit as:

$$S = \frac{N_{syc.} - N_r}{N_{syc.}}$$

unit less (per unit)

- For induction motors $N_{syc.} > N_r$ and the slip is always positive.
- At no-load the slip is nearly zero.

The Operation of 3-Phase IM:

As the magnetic flux rotates, it cuts the rotor conductors. Hence, a voltage is induced in the conductors. This voltage will circulate a current in the rotor conductors (shorted by two rings). This current interact with the air gap flux to produce torque.

The torque is maintained as long as the rotating flux and rotor current is present. The rotor will always rotate in the direction of the rotating field at the speed ***Nr*** (***Nr*** < ***Nsyc.***). The operation of an induction motor is divided into two parts, stand still and running periods. At stand still ***Nr*** = 0 and ***S*** = 1.

Due to relative speed between rotating flux and stationary conductors, an *e.m.f* is induced in the latter (Faraday's law). The frequency of this rotor *e.m.f* is the same as the stator frequency (supply).

While the magnitude of this *e.m.f* is proportional to the relative velocity between flux and rotor conductors, the direction is determined by *right hand rule* (*RHR*). This *e.m.f* will set up current in the rotor conductors , whose direction is determined by Lenz's law.

Modes of Operation of 3-Phase IM:

1- At starting $\boldsymbol{S} = 1$, $\boldsymbol{N_r} = 0$ $\qquad f_r = f_s$

2- $\boldsymbol{N_r} < \boldsymbol{N_{syc.}}$, $\boldsymbol{S}$ positive ($\boldsymbol{0 < S < 1}$) $\qquad f_r = S * f_s$

The machine is an induction motor

3- $\boldsymbol{N_r} = \boldsymbol{N_{syc.}}$, $\boldsymbol{S = 0}$, $\boldsymbol{E_r} = 0$, $\boldsymbol{I_r} = 0$ $\qquad f_r = S * f_s = 0$

Where: E_r : rotor induced *e.m.f* . ; I_r : rotor induced current .

4- $\boldsymbol{N_r} > \boldsymbol{N_{syc.}}$, $\boldsymbol{S}$ negative ($\boldsymbol{S < 0}$) $\qquad f_r = S * f_s$

The machine is an induction generator

5- $\boldsymbol{N_r}$ negative (opposite direction)

$$S = \frac{N_{syc.} - (-N_r)}{N_{syc.}} = \frac{N_{syc.} + N_r}{N_{syc.}}$$

The machine is in breaking mode (frequency multipliers).

Developed Torque

$$T \infty \frac{S E_{20}^2 R_2}{R_2^2 + S^2 X_{20}^2}$$

$$T \infty \frac{S \varphi^2 R_2}{R_2^2 + S^2 X_{20}^2}$$

Rotor induced emf ∞ air-gap flux ø

For stable input voltage, ø constant

$$T \infty \frac{S R_2}{R_2^2 + S^2 X_{20}^2}$$

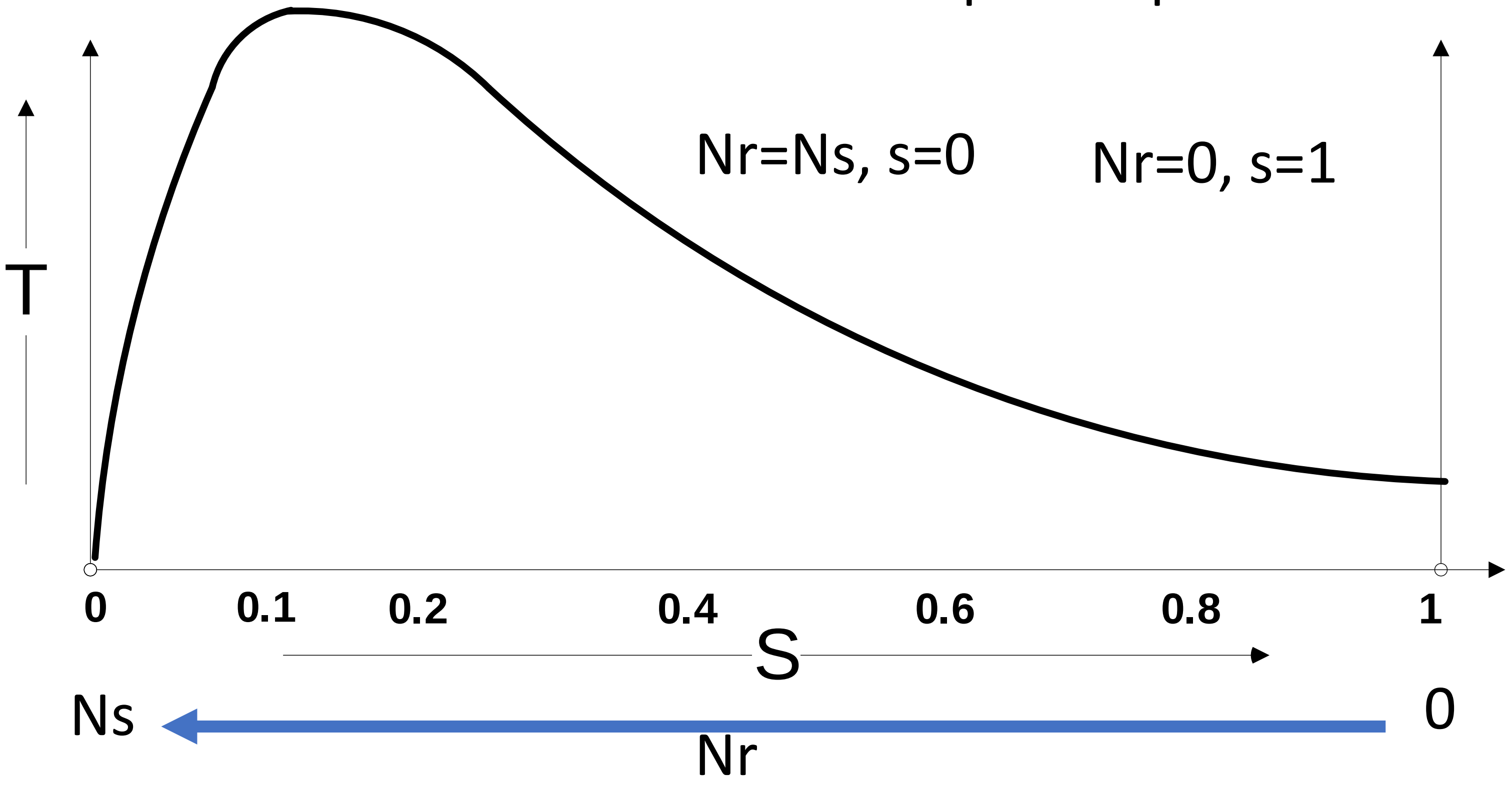
$$T = \frac{K S R_2}{R_2^2 + S^2 X_{20}^2}$$

Normally, $R_2 \ll X_{20}$

S, slip varies from 0 to 1

Torque vs slip

Torque vs speed



For s= (0 to 0.1)

as s^2 is very small $R_2^2 \gg S^2 X_{20}^2$

$$T \infty \frac{S}{R_2} \qquad T \infty S$$

For 0.1 -1

as s^2 is not very small $R_2^2 \ll S^2 X_{20}^2$

$$T \infty \frac{S R_2}{S^2 X_{20}^2} \qquad T \infty \frac{R_2}{S X_{20}^2} \qquad T \infty \frac{1}{S}$$

A 208-V, 10hp, four pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent

1. What is the synchronous speed of this motor?
2. What is the rotor speed of this motor at rated load?
3. What is the rotor frequency of this motor at rated load?
4. What is the shaft torque of this motor at rated load?

Solution

1. $$n_{sync} = \frac{120\, f_e}{P} = \frac{120(60)}{4} = 1800\ rpm$$

2. $$n_m = (1-s)n_s$$
$$= (1-0.05)\times 1800 = 1710\ rpm$$

3. $$f_r = sf_e = 0.05\times 60 = 3Hz$$

4. $$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{P_{out}}{2\pi \frac{n_m}{60}}$$
$$= \frac{10\, hp \times 746\, watt / hp}{1710\times 2\pi \times (1/60)} = 41.7\ N.m$$

Problem

A 220-V, three-phase, two-pole, 50-Hz induction motor is running at a slip of 5 percent. Find:

(a) The speed of the magnetic fields in revolutions per minute

(b) The speed of the rotor in revolutions per minute

(c) The slip speed of the rotor

(d) The rotor frequency in hertz

SOLUTION

(a) The speed of the magnetic fields is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50 \text{ Hz})}{2} = 3000 \text{ r/min}$$

(b) The speed of the rotor is

$$n_m = (1-s)\, n_{\text{sync}} = (1-0.05)(3000 \text{ r/min}) = 2850 \text{ r/min}$$

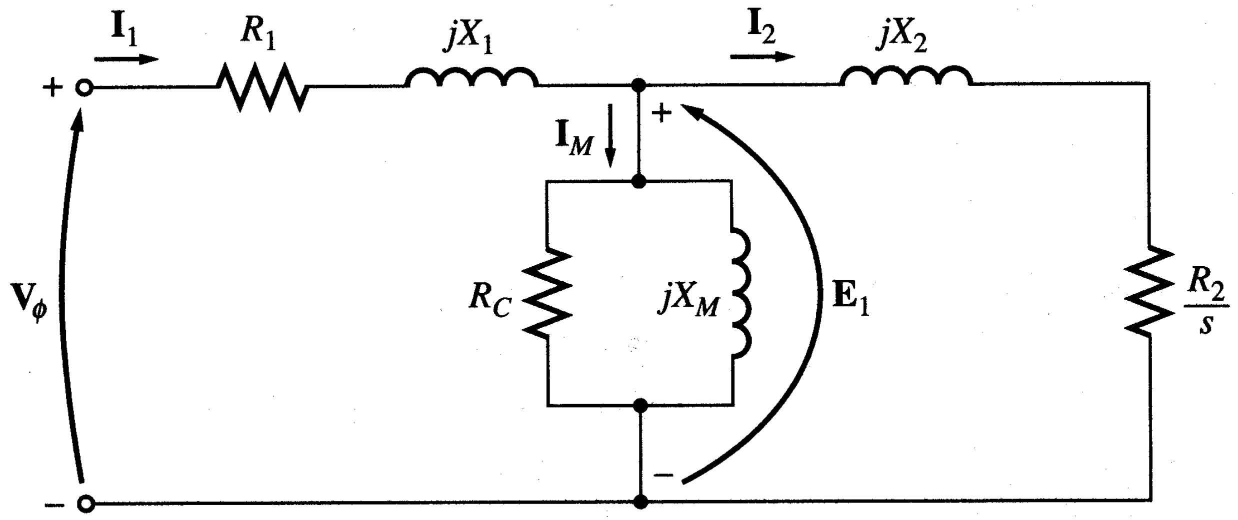
(c) The slip speed of the rotor is

$$n_{\text{slip}} = s n_{\text{sync}} = (0.05)(3000 \text{ r/min}) = 150 \text{ r/min}$$

(d) The rotor frequency is

$$f_r = \frac{n_{\text{slip}} P}{120} = \frac{(150 \text{ r/min})(2)}{120} = 2.5 \text{ Hz}$$

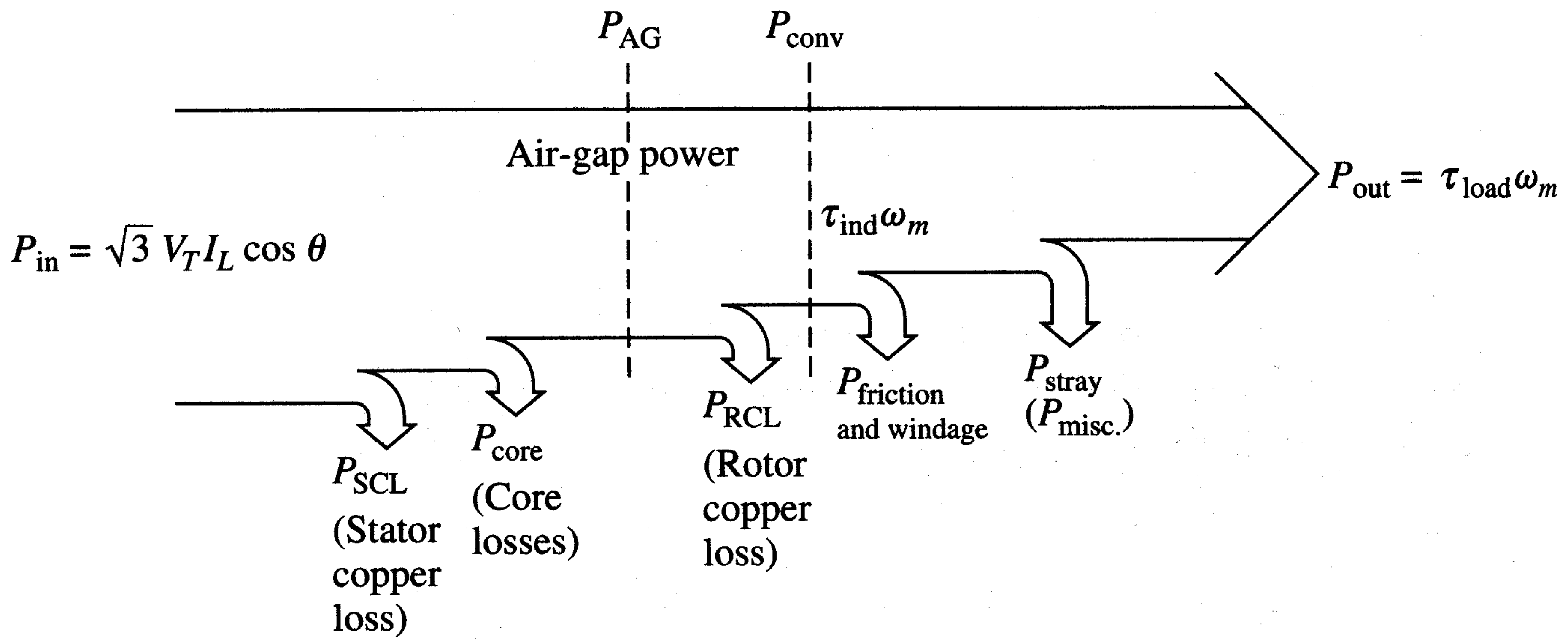
Equivalent Circuit



Power losses in Induction machines

- Copper losses
  - Copper loss in the stator (P_{SCL}) = $I_1^2R_1$
  - Copper loss in the rotor (P_{RCL}) = $I_2^2R_2$
- Core loss (P_{core})
- Mechanical power loss due to friction and windage
- How this power flow in the motor?

Power flow in induction motor



Power relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core})$$

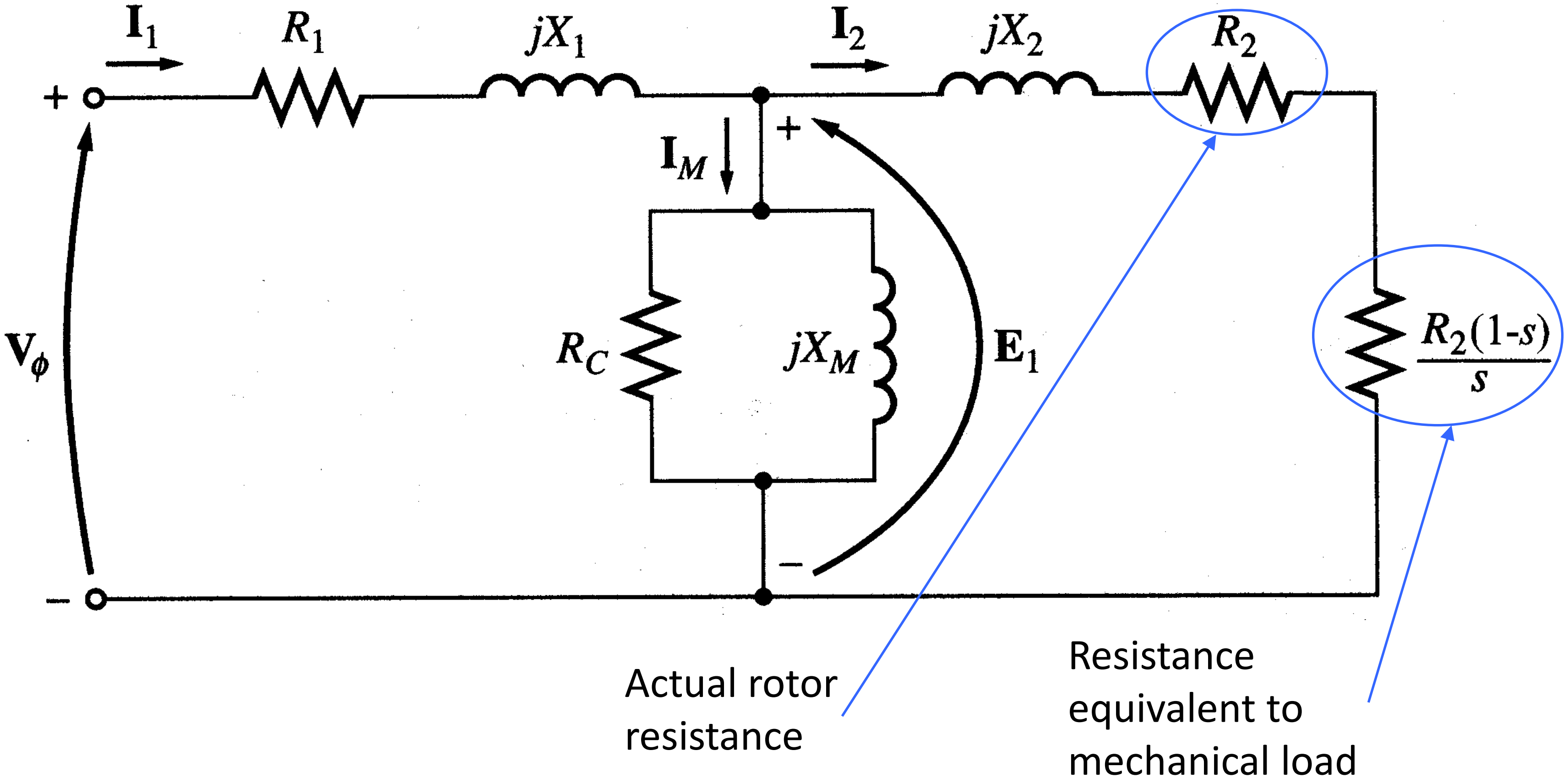
$$P_{RCL} = 3 I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray})$$

Equivalent Circuit

- We can rearrange the equivalent circuit as follows



Power relations

$$P_{in} = \sqrt{3}\,V_L I_L \cos\theta = 3V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\,I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core}) \quad = P_{conv} + P_{RCL} \quad = 3I_2^2 \frac{R_2}{s} \quad = \frac{P_{RCL}}{s}$$

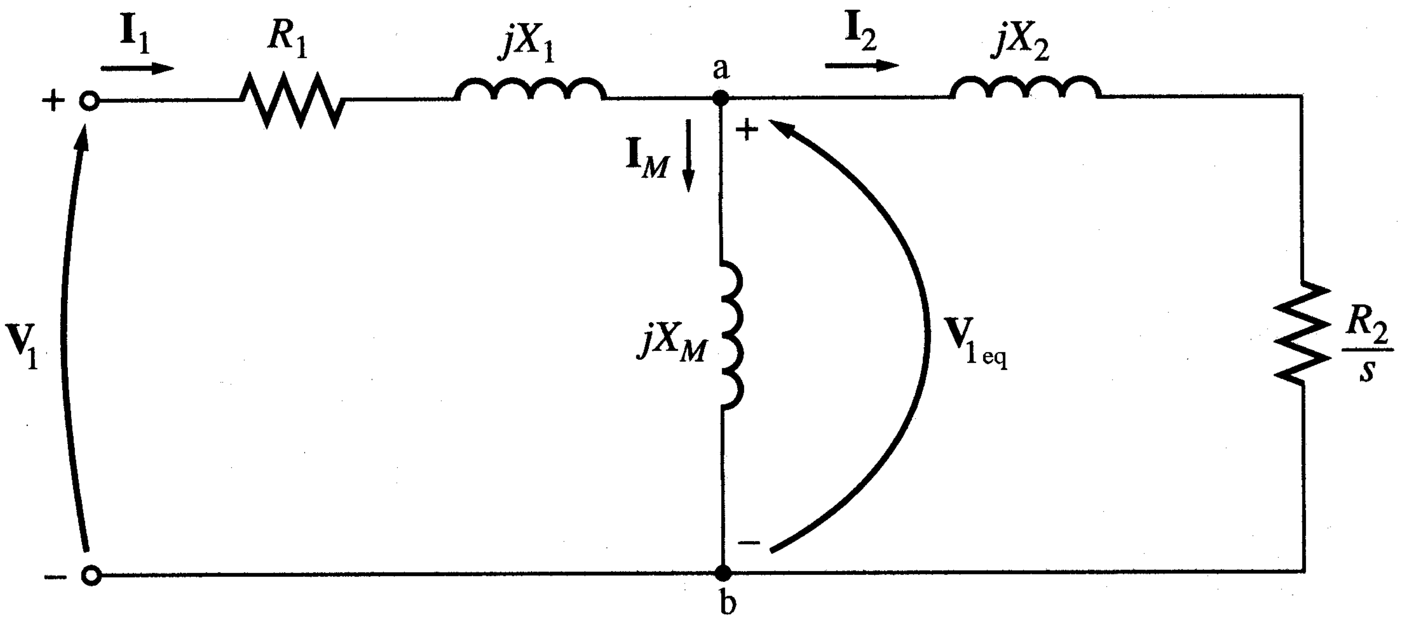
$$P_{RCL} = 3I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL} \quad = 3I_2^2 \frac{R_2(1-s)}{s} \quad = \frac{P_{RCL}(1-s)}{s}$$

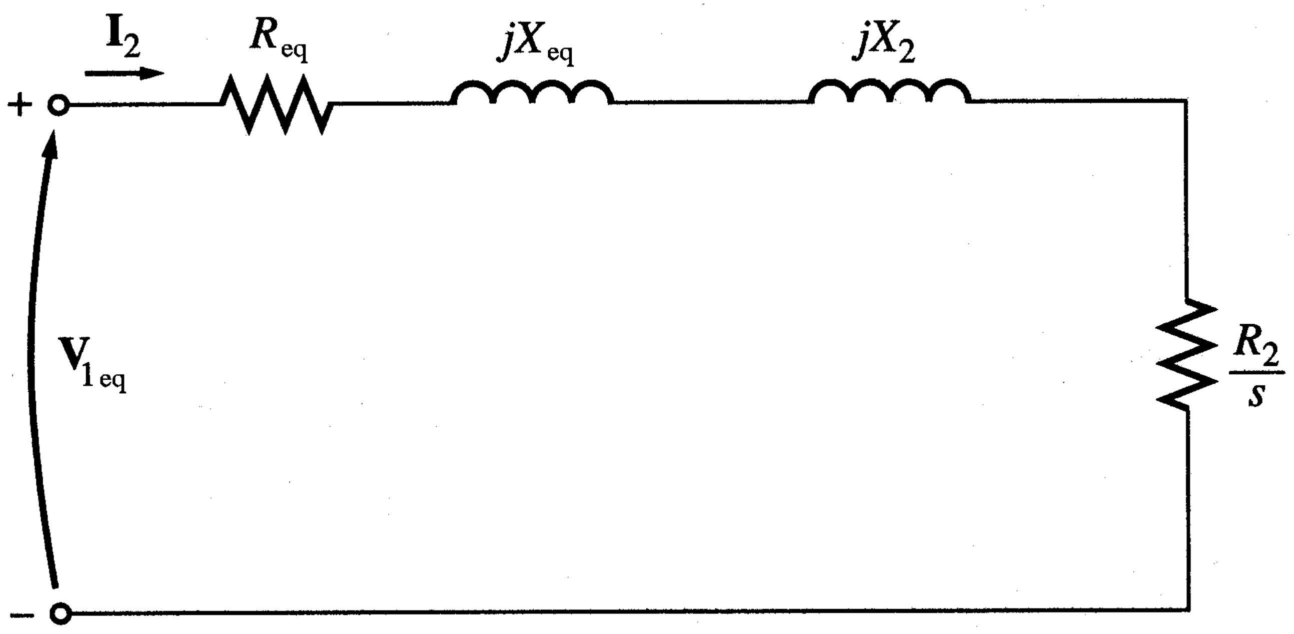
$$P_{out} = P_{conv} - (P_{f+w} + P_{stray})$$

Torque, power and Thevenin's Theorem

- Thevenin's theorem can be used to transform the network to the left of points 'a' and 'b' into an equivalent voltage source V_{1eq} in series with equivalent impedance $R_{eq}+jX_{eq}$



Torque, power and Thevenin's Theorem



$$V_{1eq} = V_1 \frac{jX_M}{R_1 + j(X_1 + X_M)}$$

$$R_{eq} + jX_{eq} = (R_1 + jX_1) // jX_M$$

Torque, power and Thevenin's Theorem

$$I_2 = \frac{V_{1eq}}{Z_T} = \frac{V_{1eq}}{\sqrt{\left(R_{eq} + \frac{R_2}{s}\right)^2 + (X_{eq} + X_2)^2}}$$

Then the power converted to mechanical (P_{conv})

$$P_{conv} = I_2^2 \frac{R_2(1-s)}{s}$$

And the internal mechanical torque (T_{conv})

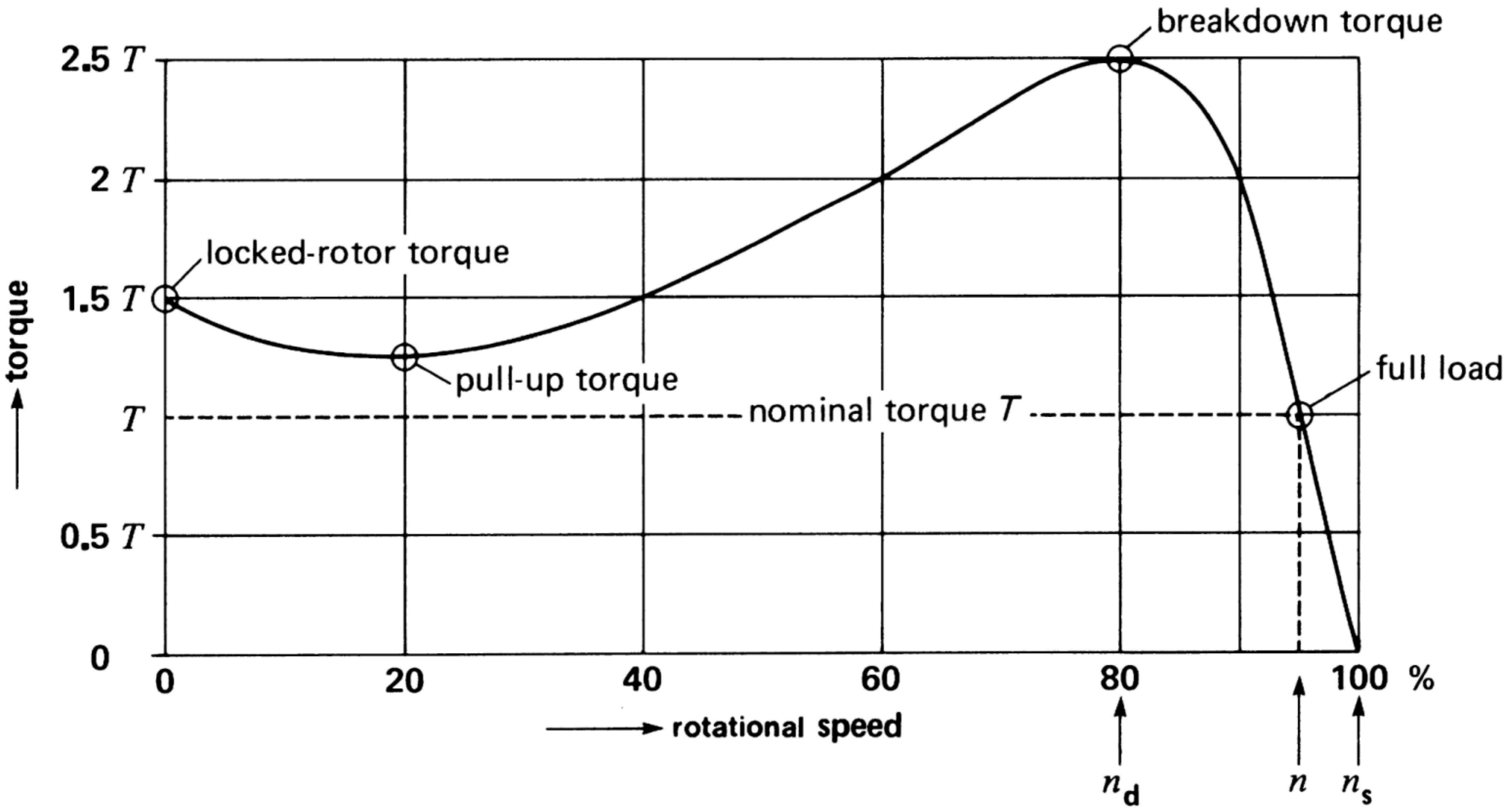
$$T_{conv} = \frac{P_{conv}}{\omega_m} = \frac{P_{conv}}{(1-s)\omega_s} = \frac{I_2^2 \frac{R_2}{s}}{\omega_s}$$

Torque, power and Thevenin's Theorem

$$T_{conv} = \frac{1}{\omega_s}\left(\frac{V_{1eq}}{\sqrt{\left(R_{eq}+\frac{R_2}{s}\right)^2+(X_{eq}+X_2)^2}}\right)^2\left(\frac{R_2}{s}\right)$$

$$\boxed{T_{conv} = \frac{1}{\omega_s}\frac{V_{1eq}^2\left(\frac{R_2}{s}\right)}{\left(R_{eq}+\frac{R_2}{s}\right)^2+(X_{eq}+X_2)^2}}$$

Torque-speed characteristics



Typical torque-speed characteristics of induction motor

Maximum torque

- Maximum torque occurs when the power transferred to R_2/s is maximum.
- This condition occurs when R_2/s equals the magnitude of the impedance $R_{eq} + j\,(X_{eq} + X_2)$

$$\frac{R_2}{s_{T_{\max}}} = \sqrt{R_{eq}^2 + (X_{eq} + X_2)^2}$$

$$\boxed{s_{T_{\max}} = \frac{R_2}{\sqrt{R_{eq}^2 + (X_{eq} + X_2)^2}}}$$

Maximum torque

- The corresponding maximum torque of an induction motor equals

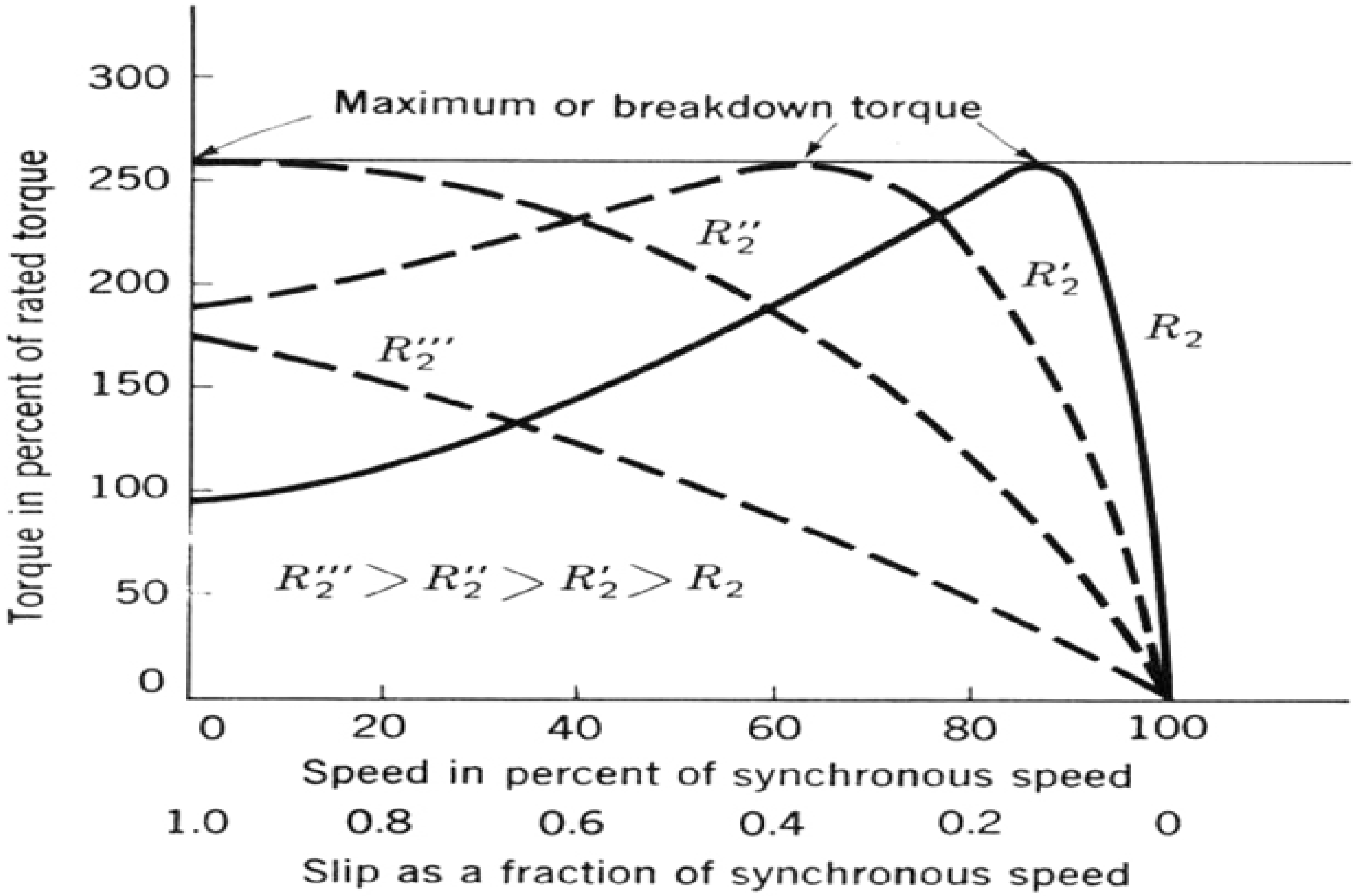
$$T_{\max} = \frac{1}{2\omega_s}\left(\frac{V_{eq}^2}{R_{eq} + \sqrt{R_{eq}^2 + (X_{eq} + X_2)^2}}\right)$$

- The slip at maximum torque is directly proportional to the rotor resistance R_2
- The maximum torque is independent of R_2

Maximum torque

- Rotor resistance can be increased by inserting external resistance in the rotor of a wound-rotor induction motor.
- The value of the maximum torque remains unaffected but the speed at which it occurs can be controlled.

Maximum torque



Effect of rotor resistance on torque-speed characteristic

Problem

A 50-kW, 440-V, 50-Hz, six-pole induction motor has a slip of 6 percent when operating at full-load conditions. At full-load conditions, the friction and windage losses are 300 W, and the core losses are 600 W. Find the following values for full-load conditions:

(a) The shaft speed n_m

(b) The output power in watts

(c) The load torque τ_{load} in newton-meters

(d) The induced torque τ_{ind} in newton-meters

(e) The rotor frequency in hertz

Solution to Problem

(a) The synchronous speed of this machine is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50 \text{ Hz})}{6} = 1000 \text{ r/min}$$

Therefore, the shaft speed is

$$n_m = (1 - s)\, n_{\text{sync}} = (1 - 0.06)(1000 \text{ r/min}) = 940 \text{ r/min}$$

(b) The output power in watts is 50 kW (stated in the problem).

(c) The load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{50 \text{ kW}}{(940 \text{ r/min}) \left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 508 \text{ N} \cdot \text{m}$$

(d) The induced torque can be found as follows:

$$P_{\text{conv}} = P_{\text{OUT}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{misc}} = 50 \text{ kW} + 300 \text{ W} + 600 \text{ W} + 0 \text{ W} = 50.9 \text{ kW}$$

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{50.9 \text{ kW}}{(940 \text{ r/min}) \left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 517 \text{ N} \cdot \text{m}$$

(e) The rotor frequency is

$$f_r = s f_e = (0.06)(50 \text{ Hz}) = 3.00 \text{ Hz}$$

Problem

A 208-V, two-pole, 60-Hz Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are

$R_1 = 0.200\ \Omega$	$R_2 = 0.120\ \Omega$	$X_M = 15.0\ \Omega$
$X_1 = 0.410\ \Omega$	$X_2 = 0.410\ \Omega$	
$P_{mech} = 250\ \text{W}$	$P_{misc} \approx 0$	$P_{core} = 180\ \text{W}$

For a slip of 0.05, find

(a) The line current I_L

(b) The stator copper losses P_{SCL}

(c) The air-gap power P_{AG}

(d) The power converted from electrical to mechanical form P_{conv}

(e) The induced torque τ_{ind}

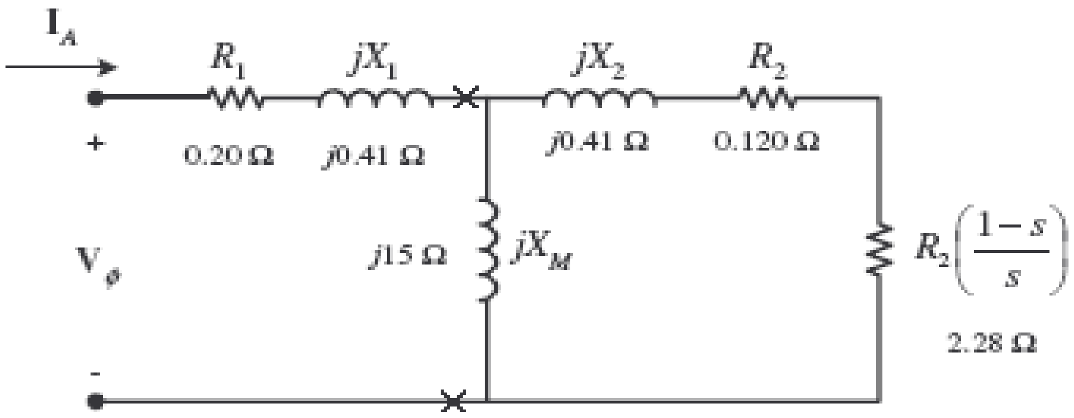
(f) The load torque τ_{load}

(g) The overall machine efficiency

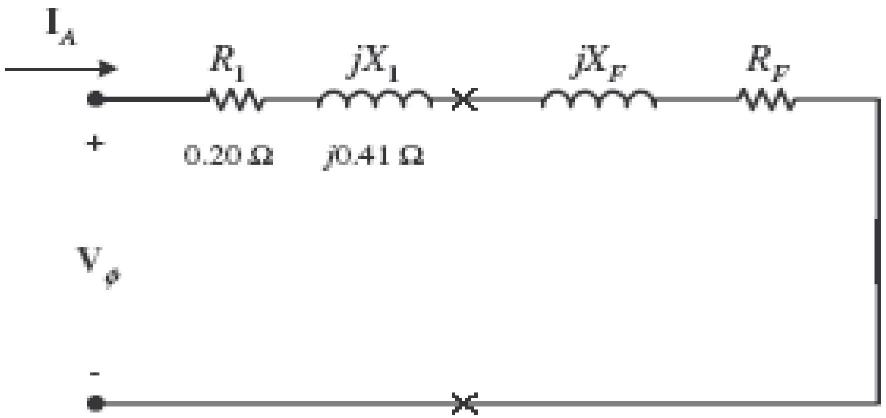
(h) The motor speed in revolutions per minute and radians per second

Solution

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j15\ \Omega} + \frac{1}{2.40 + j0.41}} = 2.220 + j0.745 = 2.34\angle 18.5°\ \Omega$$

The phase voltage is $208/\sqrt{3} = 120$ V, so line current I_L is

Cont'd

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0^\circ \text{ V}}{0.20\ \Omega + j0.41\ \Omega + 2.22\ \Omega + j0.745\ \Omega}$$

$$I_L = I_A = 44.8\angle -25.5^\circ \text{ A}$$

(b) The stator copper losses are

$$P_{\text{SCL}} = 3I_A^{\,2}R_1 = 3(44.8 \text{ A})^2(0.20\ \Omega) = 1205 \text{ W}$$

(c) The air gap power is $P_{\text{AG}} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F$

(Note that $3I_A^{\,2}R_F$ is equal to $3I_2^{\,2}\frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{\text{AG}} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F = 3(44.8 \text{ A})^2(2.220\ \Omega) = 13.4 \text{ kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = (1-0.05)(13.4 \text{ kW}) = 12.73 \text{ kW}$$

(e) The induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{13.4 \text{ kW}}{(3600 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 35.5 \text{ N}\cdot\text{m}$$

Cont'd

(f) The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 12.73 \text{ kW} - 250 \text{ W} - 180 \text{ W} - 0 \text{ W} = 12.3 \text{ kW}$$

The output speed is

$$n_m = (1-s)\, n_{\text{sync}} = (1-0.05)(3600 \text{ r/min}) = 3420 \text{ r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{12.3 \text{ kW}}{(3420 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 34.3 \text{ N}\cdot\text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{12.3 \text{ kW}}{3(120 \text{ V})(44.8 \text{ A})\cos 25.5^\circ} \times 100\% = 84.5\%$$

(h) The motor speed in revolutions per minute is 3420 r/min. The motor speed in radians per second is

$$\omega_m = (3420 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 358 \text{ rad/s}$$

Problem

A 460-V, four-pole, 50-hp, 60-Hz, Y-connected three-phase induction motor develops its full-load induced torque at 3.8 percent slip when operating at 60 Hz and 460 V. The per-phase circuit model impedances of the motor are

$$R_1 = 0.33\ \Omega \qquad X_M = 30\ \Omega$$

$$X_1 = 0.42\ \Omega \qquad X_2 = 0.42\ \Omega$$

Mechanical, core, and stray losses may be neglected in this problem.

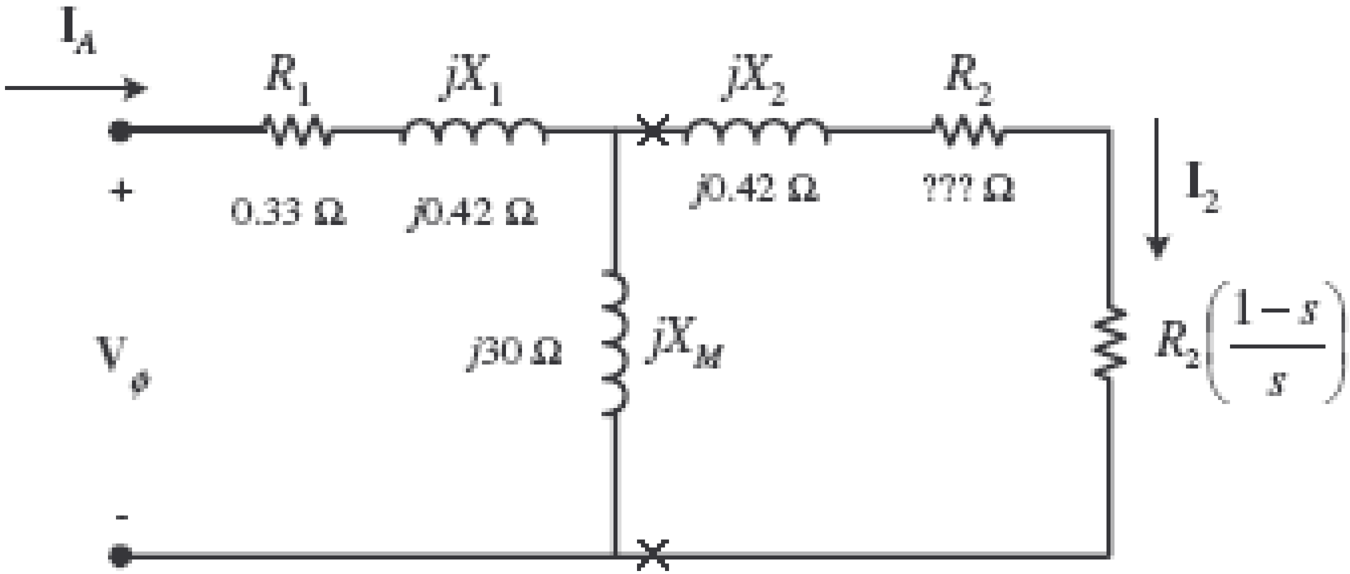
(a) Find the value of the rotor resistance R_2.

(b) Find τ_{max}, s_{max}, and the rotor speed at maximum torque for this motor.

(c) Find the starting torque of this motor.

Solution

SOLUTION The equivalent circuit for this motor is



The Thevenin equivalent of the input circuit is:

$$Z_{TH} = \frac{jX_M\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_M\right)} = \frac{(j30\ \Omega)(0.33\ \Omega\ +\ j0.42\ \Omega)}{0.33\ \Omega\ +\ j(0.42\ \Omega + 30\ \Omega)} = 0.321 + j0.418\ \Omega = 0.527\angle 52.5^\circ\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j\left(X_1 + X_M\right)}\mathbf{V}_\phi = \frac{(j30\ \Omega)}{0.33\ \Omega\ +\ j(0.42\ \Omega + 30\ \Omega)}\left(265.6\angle 0^\circ\ \text{V}\right) = 262\angle 0.6^\circ\ \text{V}$$

Cont'd

(a) If losses are neglected, the induced torque in a motor is equal to its load torque. At full load, the output power of this motor is 50 hp and its slip is 3.8%, so the induced torque is

$$n_m = (1-0.038)(1800 \text{ r/min}) = 1732 \text{ r/min}$$

$$\tau_{\text{ind}} = \tau_{\text{load}} = \frac{(50 \text{ hp})(746 \text{ W/hp})}{(1732 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 205.7 \text{ N}\cdot\text{m}$$

The induced torque is given by the equation

$$\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2 / s}{\omega_{\text{sync}} \left[(R_{\text{TH}} + R_2/s)^2 + (X_{\text{TH}} + X_2)^2\right]}$$

Substituting known values and solving for R_2/s yields

$$205.7 \text{ N}\cdot\text{m} = \frac{3(262 \text{ V})^2 R_2/s}{(188.5 \text{ rad/s})\left[(0.321 + R_2/s)^2 + (0.418 + 0.42)^2\right]}$$

$$38{,}774 = \frac{205{,}932 \;\; R_2/s}{(0.321 + R_2/s)^2 + 0.702}$$

$$(0.321 + R_2/s)^2 + 0.702 = 5.311 \;\; R_2/s$$

$$0.103 + 0.642 R_2/s + (R_2/s)^2 + 0.702 = 5.311 \;\; R_2/s$$

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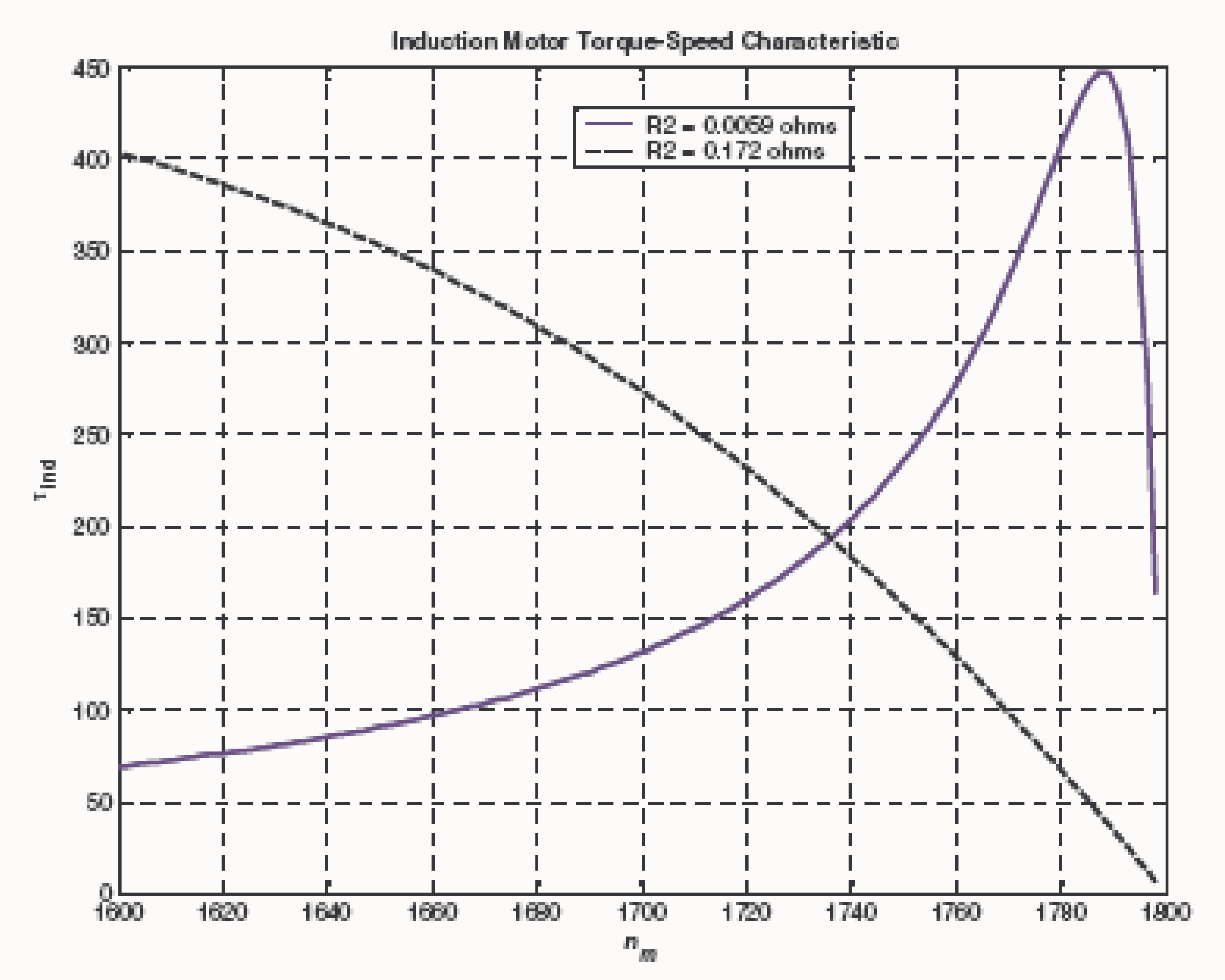


$$\left(\frac{R_2}{s}\right)^2 - 4.669\left(\frac{R_2}{s}\right) + 0.702 = 0$$

$$\frac{R_2}{s} = 0.156, \quad 4.513$$

$$R_2 = 0.0059\ \Omega, \quad 0.172\ \Omega$$

These two solutions represent two situations in which the torque-speed curve would go through this specific torque-speed point. The two curves are plotted below. As you can see, only the 0.172 Ω solution is realistic, since the 0.0059 Ω solution passes through this torque-speed point at an unstable location on the back side of the torque-speed curve.

Cont'd

(b) The slip at pullout torque can be found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model. The Thevenin equivalent of the input circuit was calculate in part *(a)*. The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}{}^2 + \left(X_{TH} + X_2\right)^2}}$$

$$s_{max} = \frac{0.172\ \Omega}{\sqrt{(0.321\ \Omega)^2 + (0.418\ \Omega\ + 0.420\ \Omega)^2}} = 0.192$$

The rotor speed a maximum torque is

$$n_{pullout} = (1 - s)\ n_{sync} = (1 - 0.192)(1800\ \text{r/min}) = 1454\ \text{r/min}$$

and the pullout torque of the motor is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\ \ R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}}$$

$$\tau_{max} = \frac{3(262\ \text{V})^2}{2(188.5\ \text{rad/s})\ \ 0.321\ \Omega + \sqrt{(0.321\ \Omega)^2 + (0.418\ \Omega\ + 0.420\ \Omega)^2}}$$

$$\tau_{max} = 448\ \text{N} \cdot \text{m}$$

(c) The starting torque of this motor is the torque at slip $s = 1$. It is

$$\tau_{ind} = \frac{3V_{TH}^2 R_2 / s}{\omega_{sync}\ \ \left(R_{TH} + R_2 / s\right)^2 + \left(X_{TH} + X_2\right)^2}$$

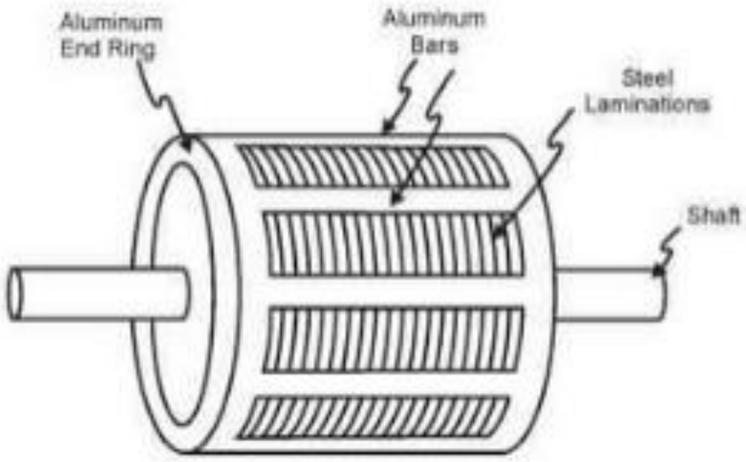
$$\tau_{ind} = \frac{3(262\ \text{V})^2\ (0.172\ \Omega)}{(188.5\ \text{rad/s})\ \ (0.321 + 0.172\ \Omega)^2 + (0.418 + 0.420)^2} = 199\ \text{N} \cdot \text{m}$$

what is a self-starting motor?

When the motor starts running automatically without any external force applied to the machine, then the motor is referred to as ‘self-starting’. For example, we see that when we put on the switch the fan starts to rotate automatically, so it is a self-starting machine. Point to be noted that fan used in home appliances is a single phase induction motor which is inherently not self-starting. How? Does a question arise as to how it works? We will discuss it now.

Induction Motor: Working Principle, Types, & Definition

- **What is an Induction Motor?**
- An **induction motor** (also known as an **asynchronous motor**) is a commonly used AC electric motor. In an induction motor, the electric current in the rotor needed to produce torque is obtained via electromagnetic induction from the rotating magnetic field of the stator winding. The rotor of an induction motor can be a squirrel cage rotor or wound type rotor.



- **Why is Three Phase Induction Motor Self Starting?**
- In a three phase system, there are three single phase lines with a 120° phase difference. So the rotating magnetic field has the same phase difference which will make the rotor to move. If we consider three phases a, b, and c when phase a gets magnetized, the rotor will move towards the phase a winding a, in the next moment phase b will get magnetized and it will attract the rotor and then phase c. So the rotor will continue to rotate.

Synchronous speed is the speed of rotation of the magnetic field in a rotary machine, and it depends upon the frequency and number poles of the machine. The induction motor always runs at speed less than its synchronous speed. The rotating magnetic field produced in the stator will create flux in the rotor, hence causing the rotor to rotate. Due to the lag between the flux current in the rotor and the flux current in the stator, the rotor will never reach its rotating magnetic field speed (i.e. the synchronous speed).

There are basically two **types of induction motor**. The **types of induction motor** depend upon the input supply. There are single phase induction motors and three phase induction motors. Single phase induction motors are not a self-starting motor, and three phase induction motor are a self-starting motor.

A permanent magnet moves at speed V, so that its magnetic field sweeps the conductors. The following sequence of events takes place:

1- A voltage ($E = B.L.V.$) is induced in each conductor while it's being cut by the flux (Faraday's law).
2- The induced voltage produces currents which circulate in a loop around the conductors (through the bars).
3- Since the current-carrying conductors lie in a magnetic field, they experience a mechanical force (Lorentz force).
4- The force always acts in a direction to drag the conductor along with the magnetic field.