Numerical Analysis Final Project Code

April 28, 2016

```
In [1]: import scipy
        import numpy as np
        import matplotlib.pyplot as plt
        from scipy import sparse
        from scipy.sparse import linalg
        from tabulate import tabulate
```

Problem 3

```
In [2]: def poissonmat(n):
            #Returns an nxn sparse tridiagonal matrix with 2 on the diagonal and -1 on both off-diagona
            return(sparse.diags([-1, 2, -1], [-1, 0, 1], shape=(n, n)))

In [3]: def poissonsolve(f):
            return(linalg.spsolve(poissonmat(len(f)), (np.power(1./(len(f)+1),2)*f)))

In [5]: table = []
        f = lambda t: np.sin(np.pi*t)
        f_r = lambda t: 1/(np.power(np.pi,2))*np.sin(np.pi*t)
        for i in [10,100,500,1000]:
            t = np.linspace(0,1,i)
            time = %time ans = poissonsolve(f(t))
            table.append([i, '1/{}'.format(len(t)+1), np.linalg.norm(ans - f_r(t)), np.linalg.norm(ans-
```

```
Wall time: 0 ns
Wall time: 0 ns
Wall time: 1 ms
Wall time: 1e+03 µs
```

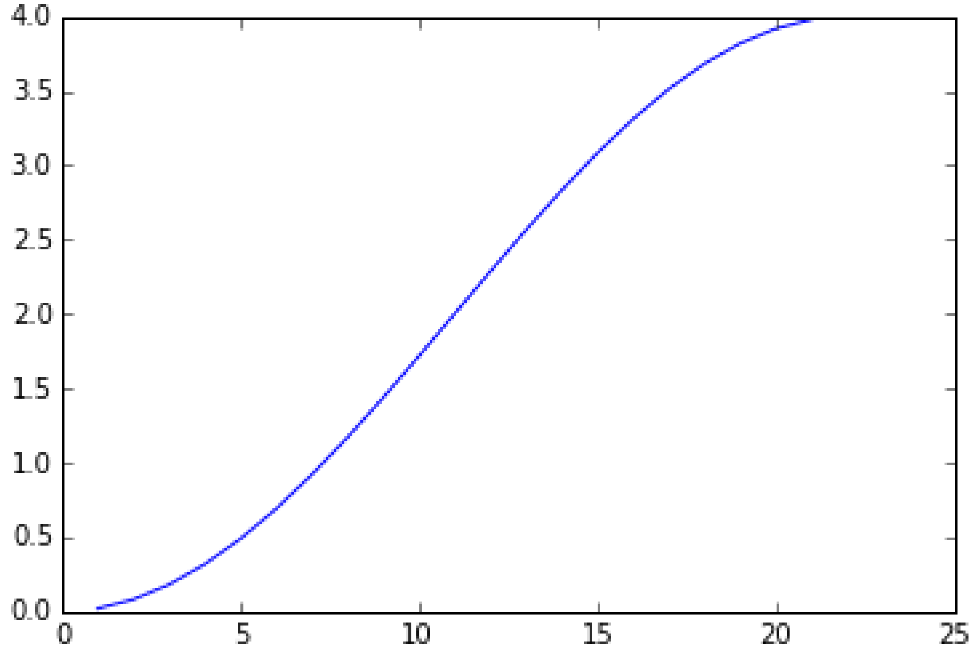
```
In [6]: table[0][4] = '0 ns'
        table[1][4] = '0 ns'
        table[2][4] = '1 ms'
        table[3][4] = '1e+3 micros'
        print(tabulate(table, headers = ['n', 'h', 'Absolute Error', 'Relative Error', 'CPU Time']))
```

n	h	Absolute Error	Relative Error	CPU Time
10	1/11	0.0401828	0.186954	0 ns
100	1/101	0.0137353	0.0192679	0 ns
500	1/501	0.00618504	0.00386462	1 ms
1000	1/1001	0.00437725	0.00193301	1e+3 micros

Problem 4

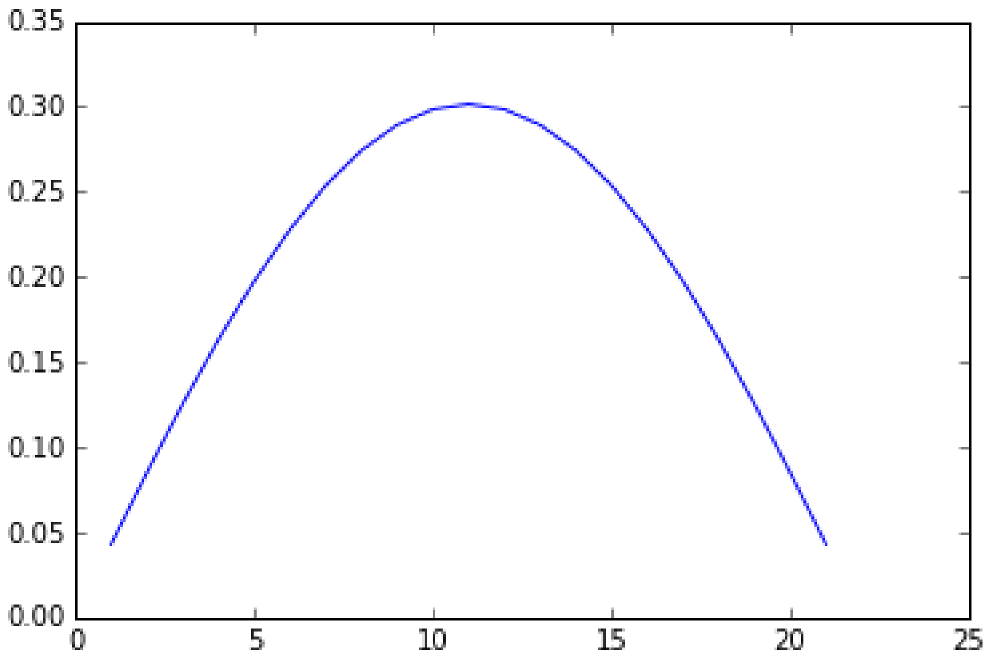
b)

In [7]:
```
N = 21
f_v = lambda x: 2*(1-np.cos((np.pi*x)/(N+1)))
x = np.arange(1, N+1, 1)
plt1 = plt.plot(x, f_v(x))
```

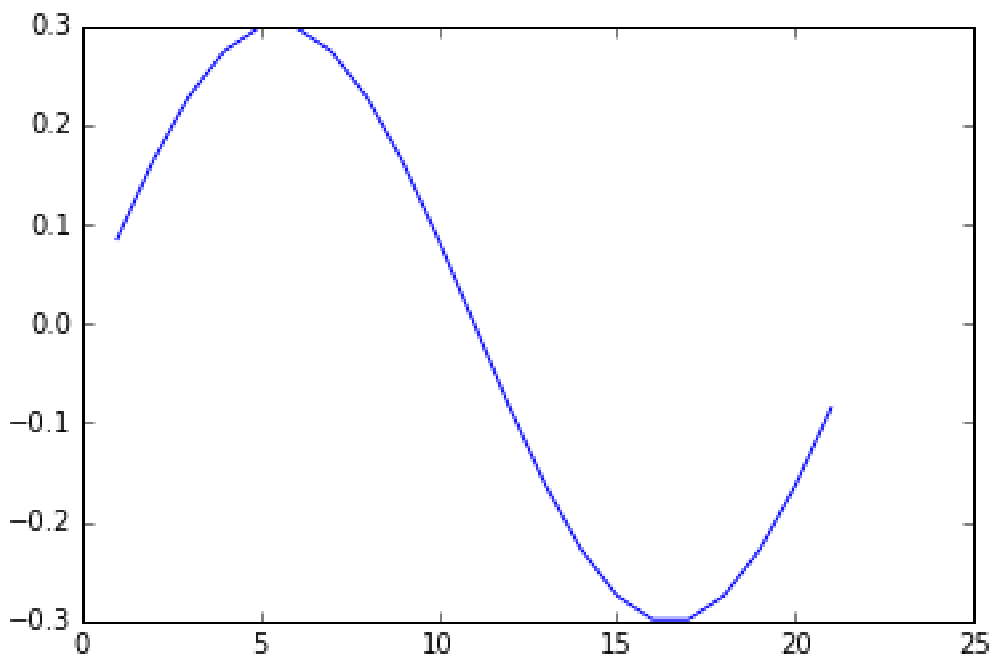


c)

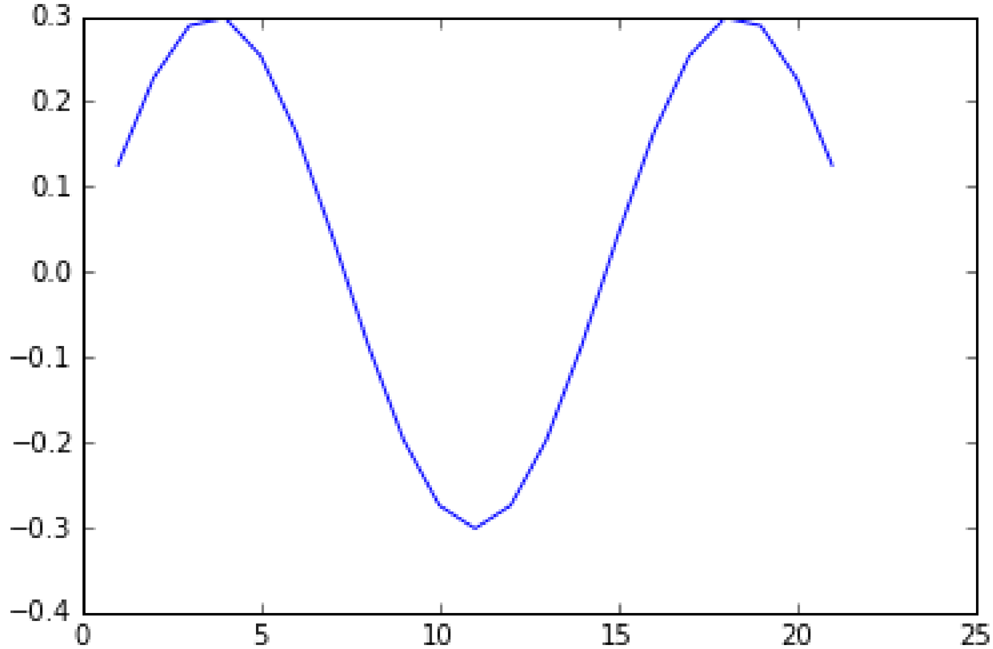
In [8]:
```
f_u = lambda j, k: np.sqrt(2./(N+1))*np.sin((j*k*np.pi)/(N+1))
plt2 = plt.plot(x, f_u(1,x))
```



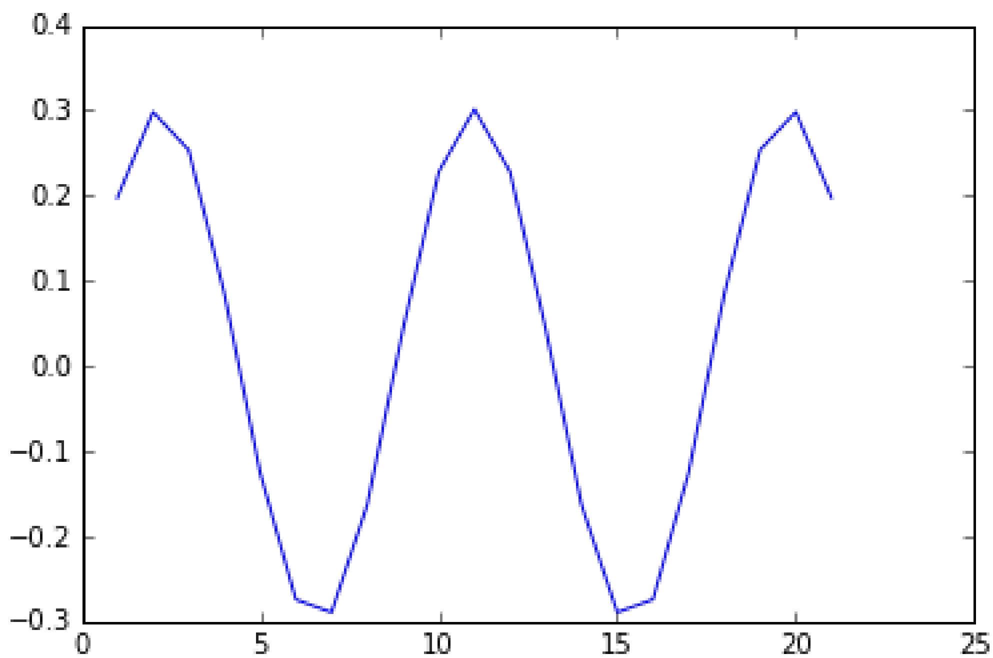

```
In [9]: plt3 = plt.plot(x, f_u(2,x))
```



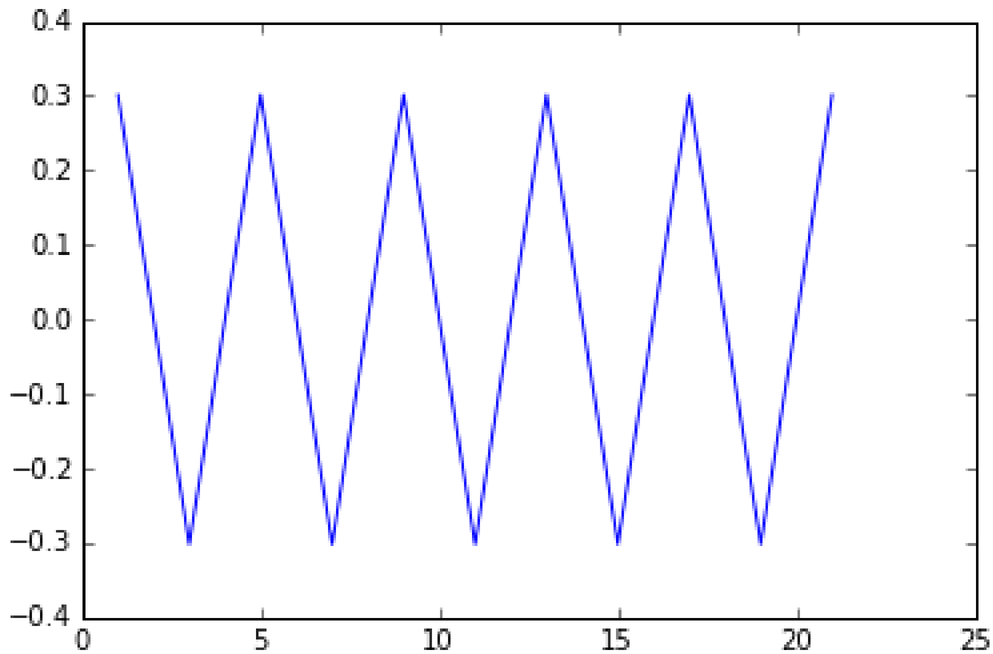

```
In [10]: plt4 = plt.plot(x, f_u(3,x))
```



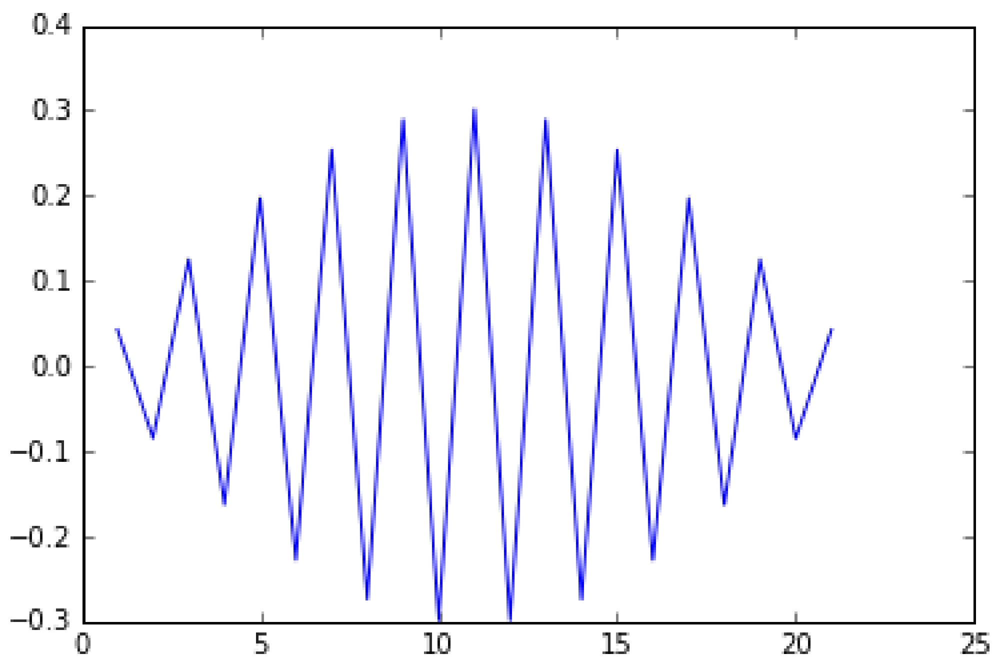

```
In [11]: plt5 = plt.plot(x, f_u(5,x))
```




```
In [12]: plt6 = plt.plot(x, f_u(11,x))
```




```
In [13]: plt7 = plt.plot(x, f_u(21,x))
```



d)

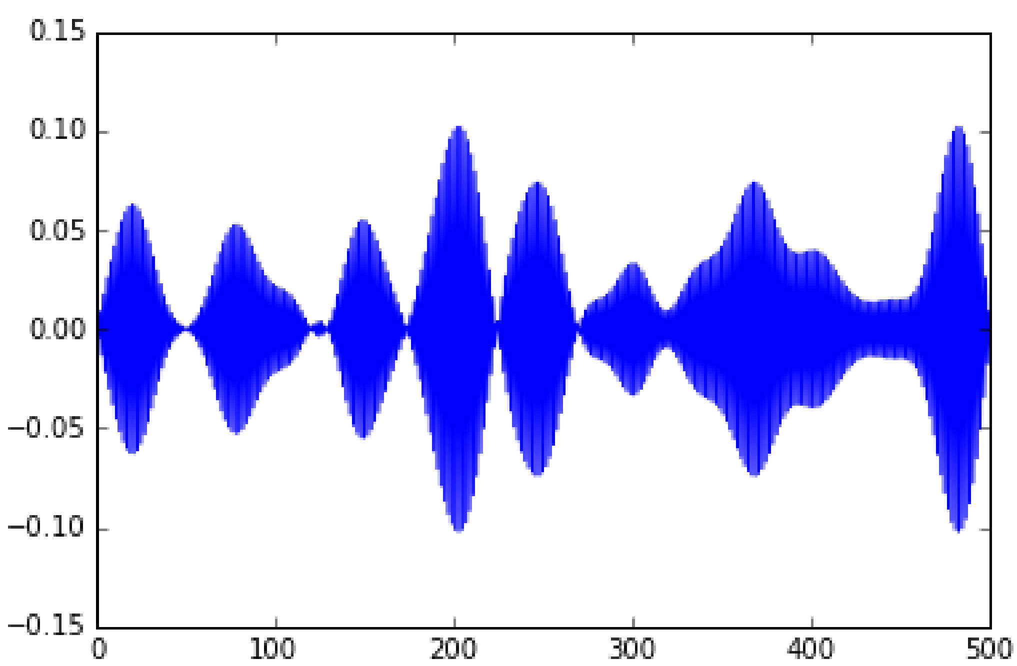
In [14]:
```
def power_method(A, guess, maxiter = 10000, tau = 10e-6):
    v = guess
    prev_norm = 0
    for i in range(maxiter):
        v = A*v
        curr_norm = np.linalg.norm(v)
        v = v/np.linalg.norm(v)
        if np.abs(prev_norm - curr_norm) < tau:
            print 'Found in {} iterations.'.format(i+1)
            return v
        prev_norm = curr_norm
    print 'Did not converge for specified number of iterations.({})'.format(np.abs(prev_norm -

def check(A, output, low = False):
    prod = A*output
    eigval = prod[0]/output[0]
    print 'Computed eigenvalue: {}'.format(eigval[0])
    eig, _ = linalg.eigsh(A, k = A.shape[0]-1)
    if low:
        print 'Smallest eigenvalue: {}'.format(np.sort(np.absolute(eig))[0])
    else:
        print 'Largest eigenvalue: {}'.format(np.sort(np.absolute(eig))[-1])
```

In [15]:
```
N = 500
A = poissonmat(N)
guess = np.random.random((N,1))
v = power_method(A, guess)
check(A, v)
```

```
Found in 307 iterations.
Computed eigenvalue: 3.99768634436
Largest eigenvalue: 3.99996067915
```

In [16]:
```
plt8 = plt.plot(np.arange(1,501,1), v)
```



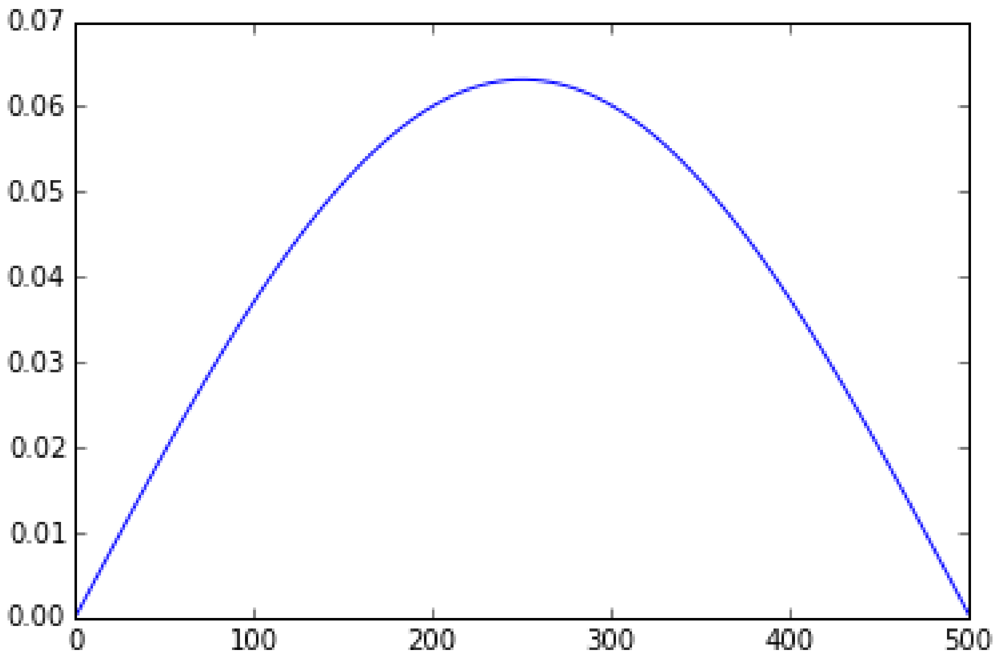
e)

```
In [17]: def inverse_power(A, guess, maxiter = 10000, tau = 10e-6):
             v = guess
             prev_norm = 0
             Ainv = linalg.splu(A)
             for i in range(maxiter):
                 v = Ainv.solve(v)
                 curr_norm = np.linalg.norm(v)
                 v = v/np.linalg.norm(v)
                 if np.abs(prev_norm - curr_norm) < tau:
                     print 'Found in {} iterations.'.format(i+1)
                     return v
                 prev_norm = curr_norm
             print 'Did not converge for specified number of iterations.({})'.format(np.abs(prev_norm -
```

```
In [19]: guess = np.random.random((N,1))
         v = inverse_power(A, guess)
         check(A, v, low = True)
```

```
Found in 7 iterations.
Computed eigenvalue: 3.9320978686e-05
Smallest eigenvalue: 0.00015728184415
```

```
In [20]: plt9 = plt.plot(np.arange(1,501,1), v)
```




```
In [ ]:
```