One Population t Test

Test Statistic:

$$t = \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}} \sim \mathcal{N}(0,1)$$

1. $H_0 : \mu \leq \mu_0 \quad vs. \quad H_I : \mu_0 > \mu_0$
   Reject H_0 if $t > t_{\alpha,n-1}$
2. $H_0 : \mu \geq \mu_0 \quad vs. \quad H_I : \mu_0 < \mu_0$
   Reject H_0 if $t <_{\alpha,n-1}$
3. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 \neq \mu_0$
   Reject H_0 if $|t| > t_{\frac{\alpha}{2},n-1}$

Two Population t Test

Test Statistic:

$$t = \frac{\bar{x} - \bar{y}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

1. $H_0 : \mu \leq \mu_0 \quad vs. \quad H_I : \mu_0 > \mu_0$
   Reject H_0 if $t > t_{\alpha,n_1+n_2-2}$
2. $H_0 : \mu \geq \mu_0 \quad vs. \quad H_I : \mu_0 < \mu_0$
   Reject H_0 if $t <_{\alpha,n_1+n_2-2}$
3. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 \neq \mu_0$
   Reject H_0 if $|t| > t_{\frac{\alpha}{2},n_1+n_2-2}$

NonParametric One Population Test

Test Statistic:
Let y = # of observations that excede η_0 among the population.

$$Z = \frac{y - \frac{n}{2}}{\sqrt{\frac{n}{4}}}$$

1. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 > \mu_0$
   Reject H_0 if $Z > Z_\alpha$
2. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 < \mu_0$
   Reject H_0 if $Z < -Z_\alpha$
3. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 \neq \mu_0$
   Reject H_0 if $|Z| > Z_\alpha$

NonParametric Two Population Test

Method: Form n differences $d_i = x_i - y_i$ for i = 1, 2, ..., n and rank the absolute differences $|d_i|$ in increasing order. Write the ranks of $|d_i|$ as 1 to n, then attack the sign of d_i to its rank. The ranks with the appropriate signs attached are called the signed ranks, R_i. If a tie occurs, replace tied ranks by their average rank. Compute the sum of the signed ranks, $T = \sum_{i=1}^{n} R_i$. If $d_i = 0$ remove it from the calculation and reduce n accordingly.

Test Statistic:

$$Z = \frac{T}{\sqrt{\frac{n(n+1)(2n+1)}{6}}}$$

1. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 > \mu_0$
   Reject H_0 if $Z > Z_\alpha$
2. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 < \mu_0$
   Reject H_0 if $Z < -Z_\alpha$
3. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 \neq \mu_0$
   Reject H_0 if $|Z| > Z_\alpha$

Exact Test:
T_+ = sum of the positive ranks
T_- = sum of the negative ranks
$T_{min} = Min(T_+, T_-)$

1. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 > \mu_0$
   Reject H_0 if $T_- < T_{\alpha(1),n}$
2. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 < \mu_0$
   Reject H_0 if $T_+ < T_{\alpha(1),n}$
3. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 \neq \mu_0$
   Reject H_0 if $T_{min} > T_{\alpha(2),n}$

Mann-Whitney Test

Used to determine if there is a difference between two independent populations. Let $x_1, x_2, \ldots, x_{n_2}$ be a random sample of size n_2 from the first population and $y_1, y_2, \ldots, y_{n_2}$ be a random sample of size n_2 from the second population.
Method:
Rank the combined sample observations from lowest to highest, with tied values assigned as the average of the rankings. compute T_1 - the sum of the ranks of the first sample.
Test Statistic:
Let $\mu_{T_1} = \frac{1}{2}n_1(n_1 + n_2 + 1)$ and
$\sigma_{T_1} = \sqrt{\frac{1}{12}n_1 n_2(n_1 + n_2 + 1)}$

$$Z = \frac{T_1 - \mu_{T_1}}{\sigma_{T_1}}$$

1. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 > \mu_0$
   Reject H_0 if $Z > Z_\alpha$
2. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 < \mu_0$
   Reject H_0 if $Z < -Z_\alpha$
3. $H_0 : \mu = \mu_0 \quad vs. \quad H_I : \mu_0 \neq \mu_0$
   Reject H_0 if $|Z| > Z_\alpha$

One Way ANOVA

Total number of observations:

$$N = n_1 + n_2 + \ldots + n_k$$

Grand total:

$$G = T_1 + T_2 + \ldots + T_k$$

Correction Term:

$$CT = \frac{G^2}{N}$$

Treatment Sum of Squares:

$$SS_{TR} = \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \ldots + \frac{T_k^2}{n_k} \right) - CT$$

Total Sum of Squares:

$$SS_T = \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 - CT$$

Error Sum of Squares:

$$SS_E = SS_T - SS_{TR}$$

ANOVA Table:

Source	dof	SS	MS	F
Treatment	$k-1$	SS_{TR}	$MS_{TR} = \frac{SS_{TR}}{k-1}$	$F = \frac{MS_{TR}}{MS_E}$
Error	$N-k$	SS_E	MS_E	
Total	$N-1$	SS_T		

$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$
Reject H_0 if $F > F_{\alpha,k-1,N-k}$

1 Hartley's Test

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots \sigma_k^2$ vs. H_I: At least one different
Test Statistic:
For S_i^2 = sample variance for ith population.

$$F_{max} = \frac{max(S_i^2)}{min(S_i^2)}$$

Reject H_0 if $F_{max} > F_{max(\alpha),k,\nu}$
for k = number of groups
and $\nu = max(n_1 - 1, n_2 - 1, \ldots, n_i - 1)$

2 Brown-Forsythe-Levene (BFL) Test