Verifying & Improving Halide's Term Rewriting System with Program Synthesis

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Halide

```
Func blur_3x3(Func input) {
  Func blur_x, blur_y;
  Var x, y, xi, yi;

  // The algorithm - no storage or order
  blur_x(x, y) = (input(x-1, y) + input(x, y) + input(x+1, y))/3;
  blur_y(x, y) = (blur_x(x, y-1) + blur_x(x, y) + blur_x(x, y+1))/3;

  // The schedule - defines order, locality; implies storage
  blur_y.tile(x, y, xi, yi, 256, 32)
        .vectorize(xi, 8).parallel(y);
  blur_x.compute_at(blur_y, x).vectorize(x, 8);

  return blur_y;
}
```

A key component!

Running benchmark suite without this engine resulted in a 5x increase in compilation times and a 26x increase in runtimes

Uses of the reasoning engine

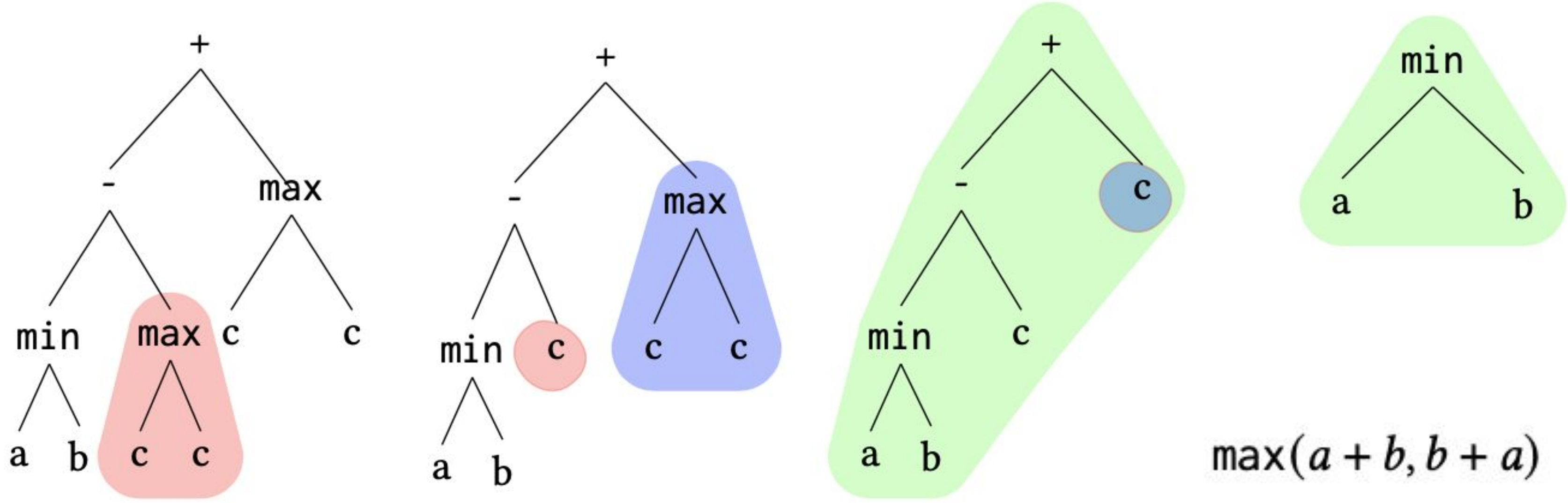
Halide code	**If we can show ….**	**Optimization**
Euclidean division	Denominator > 0	Machine insn division
Loop	Constant loop extent	Map loop to CUDA threads
`malloc`	Constant allocation size	Keep on stack

Talk roadmap

- Background
- Soundness
- Termination
- Greater reasoning power

Term rewriting systems

$$\mathsf{max}(x, x) \rightarrow_R x$$
$$(x - y) + y \rightarrow_R x$$



$$\mathsf{max}(a + b, b + a)$$

Limits to reasoning power

Reasoning power vs. performance

Nonlinear integer arithmetic is undecidable → the Halide TRS cannot be complete

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- **Soundness**
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Proofs of Correctness: Soundness

SMT Solver

Verified 88% of ruleset

Proof assistant

Verified 12% of ruleset

An example of an unsound rule

Wrong	$\frac{x*c_0}{c_1} \rightarrow_R \frac{x}{(c_1/c_0)}$ if $c_1 \ \% \ c_0 = 0 \wedge \boxed{c_1 > 0}$
Fixed	$\frac{x*c_0}{c_1} \rightarrow_R \frac{x}{(c_1/c_0)}$ if $c_1 \ \% \ c_0 = 0 \wedge \boxed{c_0 > 0 \wedge \frac{c_1}{c_0} \neq 0}$
Counterexample	$c_0 = -1, c_1 = 2, x = 1$

Reverification of new semantics

Division redefined:

- Previously x/0 was undefined behavior
- Now, x/0 == 0
- Some sample rules that became incorrect:

$$
\begin{aligned}
(x/y) * y + x\%y &\rightarrow_R x \\
-1/x &\rightarrow_R \texttt{select}(x < 0, 1, -1) \\
(x+y)/x &\rightarrow_R y/x + 1
\end{aligned}
$$

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Proofs of correctness: termination

$$x + y \rightarrow_R y + x$$

$3 + x \rightarrow x + 3 \rightarrow 3 + x \rightarrow$ …...

Reduction order

A reduction order is a total order defined over terms.

$$x + x + y >_{+} (x * 2) + y$$

The Halide simplification order

$s > t$ if

1. There are more vector operations in s than in t. If they have the same number of vector operations …
2. There are more division, multiplication, and modulo operations (sum) in s than t. If they are the same …
3. There are more occurrences of variables and constants in s than in t …
4. ...
5. ...
6. ...

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Strengthening the ruleset via synthesis

But: set of potential rules is infinite

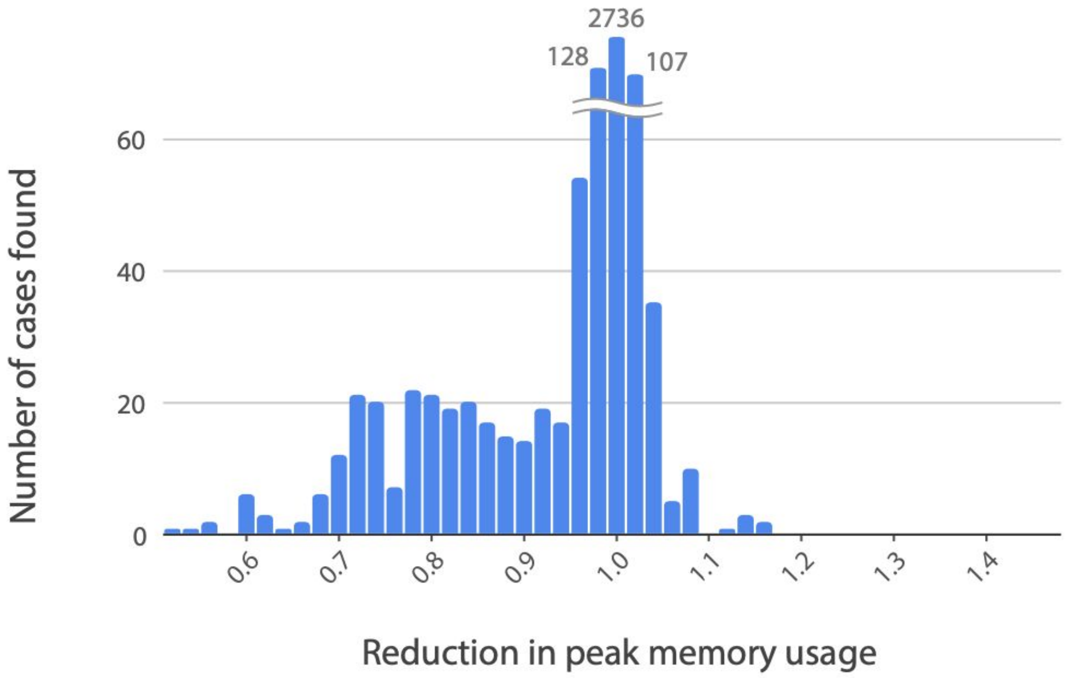
So we are guided by the bias in expressions seen by the compiler

Our synthesis pipeline (briefly)

Input expression	$(y+2) < \texttt{select}(u < z, -3, 4) + (y+2)$
Generated LHS patterns	$\dots$ $(x_1+2) < x_2 + (x_1+2)$ $x_1 < \texttt{select}(x_2, -3, 4) + x_1$ $\texttt{select}(x_1 < x_2, -3, 4)$ $\dots$
Superoptimized with CEGIS	$x_1 < \texttt{select}(x_2, -3, 4) + x_1 \rightarrow \boxed{\neg x_2}$
With symbolic constants	$x_1 < \texttt{select}(x_2, c_0, c_1) + x_1 \ \rightarrow_R \ \neg x_2$
Rule with predicate	$x_1 < \texttt{select}(x_2, c_0, c_1) + x_1 \ \rightarrow_R \ \neg x_2 \text{ if } 0 < c_1 \wedge c_0 \leq 0$

Case studies & anecdotes

- Synthesized bug fixes as good or better than the human-authored fixes
- Used corpus of ~100k expressions to synthesize ~4000 rules, resulting in reductions of peak memory usages and no appreciable increase in compilation times.
- Human developers have used the synthesizer as an assistant



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