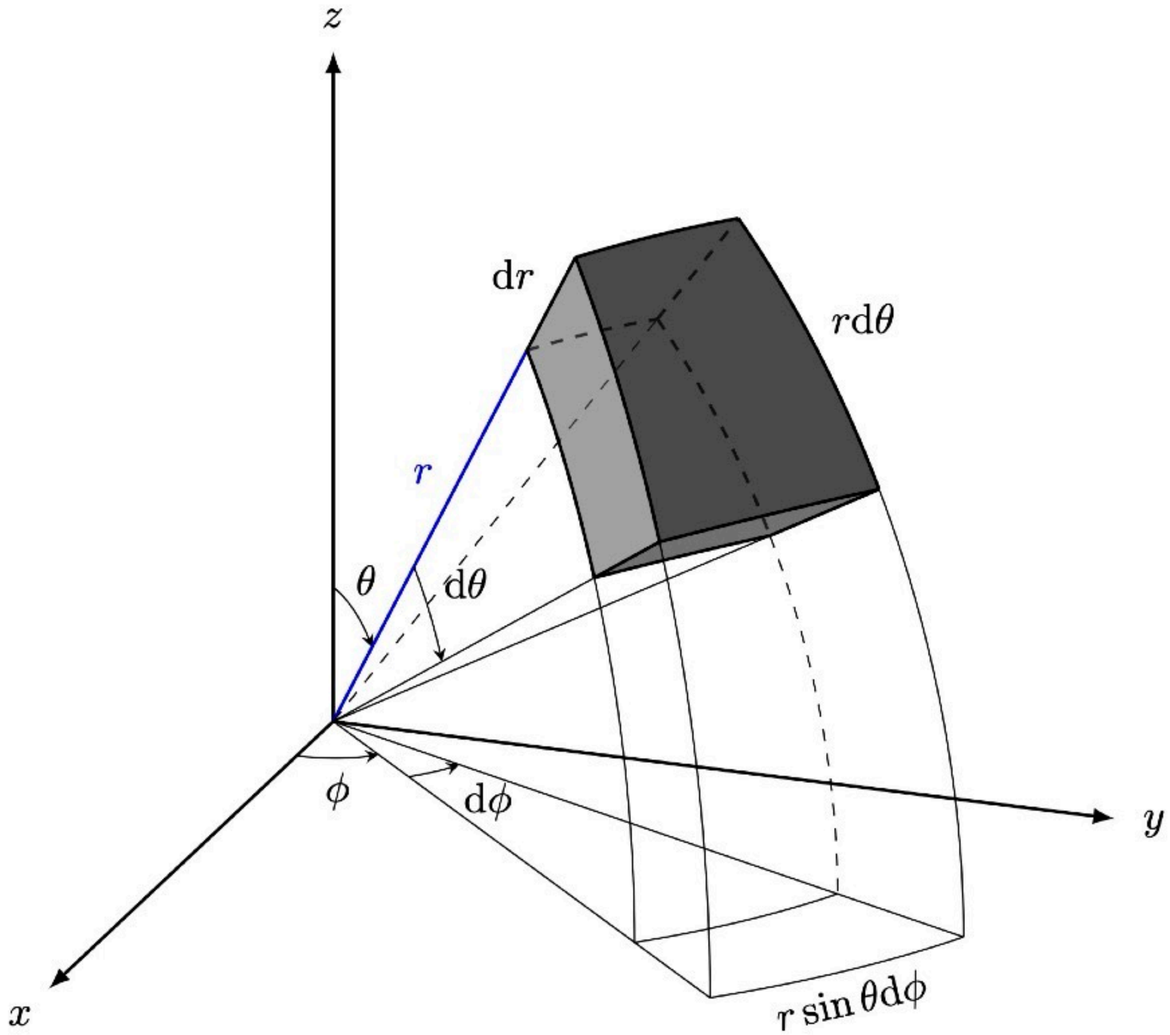
Spherical Coordinates

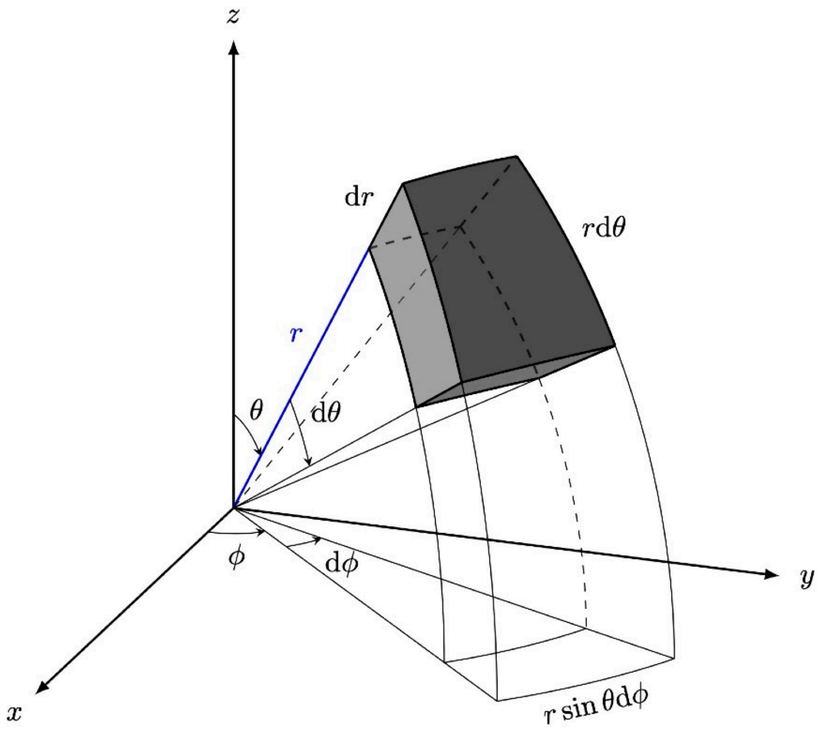


Volume and area elements

$$dV = \mathrm{d}A\,\mathrm{d}r = (r\mathrm{d}\theta)\,(r\sin\theta\mathrm{d}\phi)\,(\mathrm{d}r)$$
$$= r^2\sin\theta\,\mathrm{d}r\mathrm{d}\theta\mathrm{d}\phi$$

$$\mathrm{d}A = (r\mathrm{d}\theta)(r\sin\theta\mathrm{d}\phi)$$
$$= r^2\sin(\theta)\,\mathrm{d}\theta\mathrm{d}\phi$$

Jacobian Determinant



$$
\begin{aligned}
x &= r\sin\theta\cos\phi \\
y &= r\sin\theta\sin\phi \\
z &= r\cos\theta
\end{aligned}
$$

$$
\begin{vmatrix}
\frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\
\frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\
\frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi}
\end{vmatrix}
=
\begin{vmatrix}
\sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\
\sin\theta\cos\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\
\cos\theta & -r\sin\theta & 0
\end{vmatrix}
= r^2\sin\theta
$$