Heat and Entropy as inexact and exact differentials

- Consider an ideal gas with constant specific heat $E = C_0 T$

$$đQ = dE + đW_{out}$$

$$= C_0\, dT + p\, dV$$

$$đQ = C_0\, dT + \frac{NkT}{V}\, dV \equiv F_T\, dT + F_V\, dV$$

This is not exact. The curl is non-zero:

$$\left(\frac{\partial F_T}{\partial V} - \frac{\partial F_V}{\partial T}\right) = \cancelto{0}{\frac{\partial C_0}{\partial V}} - \frac{\partial}{\partial T}\left(\frac{NkT}{V}\right) = -\frac{Nk}{V} \neq 0$$

- Now consider dS:

different F_T and F_V

$$dS = \frac{đQ}{T} = \frac{C_0}{T}\, dT + \frac{Nk}{V}\, dV \equiv F_T\, dT + F_V\, dV$$

So now the curl is zero

$$\frac{\partial}{\partial V}\left(\frac{C_0}{T}\right) - \frac{\partial}{\partial T}\left(\frac{Nk}{V}\right) = 0$$

So we see that heat is an inexact differential, while entropy is exact.