A ring of radius $a = r_o \sin\theta_o$ and uniform charge density λ, is centerd on the z axis at $z_o = r_o \cos\theta_o$, so that the face of the ring is parallel to the xy plane. The ring is inside a grounded sphere of radius R. The potential at point r, θ due to a ring with coordinates r_o, θ_o is

$$\varphi(r, \theta; r_o, \theta_o) = \sum_\ell \frac{\lambda a}{2R} P_\ell(x) P_\ell(x_o) \left[\left(\frac{r_<}{R}\right)^\ell \left(\left(\frac{R}{r_>}\right)^{\ell+1} - \left(\frac{r_>}{R}\right)^\ell \right) \right] \tag{1}$$

where $r_>$ is the greater of of r and r_o, and $r_<$ is the lesser of r and r_o, i.e. the potential has discontinuous derivative at $r = r_o$.

(a) Determine the charge per area, σ on the surface of the sphere.

(b) By integrating the charge per area, show that the induced charge is $-Q = -\lambda(2\pi a)$. Explain why this is the expected result.

(c) (Optional) Show that the charge per area as the ring approaches the surface of the sphere, approaches a physically intuitive result, *i.e.* a negative ringlike band on the surface of the sphere[1].

[1] Answer: $\sigma = -\frac{Q}{2\pi R^2} \delta(\cos\theta - \cos\theta_o)$