

Irreversible Processes and Entropy

- Irreversible processes are associated with an increase in entropy. We will illustrate this with examples
- First consider a completely ordered state of N harmonic oscillators, shown in (a). By a sequence of random hops transferring energy from site to site (see previous lectures) the system ends up in slide (b). It is equally to be in any of its states. It will never hop back to the completely ordered state although this is allowed energetically. The transition is irreversible and associated with an increase in the total # of possible states, $\Delta S_{\text{univ}} > 0$
(see also previous lectures)

- Next consider a gas initially on the left side (see slide) a container. The microstates are labelled by the positions and momenta of all particles $x_1, p_1, \dots, x_N, p_N$. When the plug is removed the available volume to each particle is increased. The gas will rush and fill the container, increasing the entropy of the system. For instance initially each particle was in the Left state. Afterwards it is either in Left or Right state. The number of states per particle has doubled and $\Omega_{\text{final}} / \Omega_{\text{int}} = 2^N$.
Left \equiv L
- (E.g. take two particles $N=2$. The initial state is LL. The possible final states are LL, LR, RL, RR: this is 4 times the number of initial states)

Ordered versus typical state:

[illegible]

Ordered state: one quantum of energy, $\hbar\omega_0$, per site

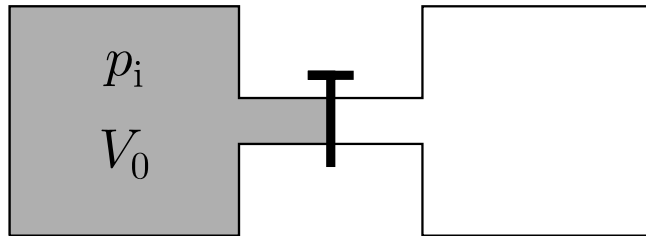
1	0	1	0	2	1	0	0	2	1	3	1	1	1	0	0	0	0	1	0
1	2	2	0	0	0	0	0	0	1	1	0	0	3	5	1	0	0	0	0
0	0	1	1	0	2	0	0	6	1	1	0	1	1	1	0	2	0	0	0
0	3	3	0	0	2	0	0	3	0	1	3	0	2	0	0	0	3	0	0
2	0	0	1	0	6	0	4	1	0	0	0	2	1	0	0	1	1	1	0
1	0	6	0	4	0	1	2	1	0	0	0	1	0	0	0	0	3	4	1
0	0	0	1	2	4	0	0	1	0	0	2	0	1	2	1	1	2	0	6
1	3	0	0	2	0	1	0	1	0	0	3	0	4	0	2	1	0	6	1
1	1	1	0	0	0	0	1	0	3	1	2	0	0	0	0	1	0	0	0
0	0	0	0	0	2	0	3	1	0	0	0	0	0	3	0	0	2	2	1
9	1	0	1	1	0	2	1	0	1	0	0	0	3	7	3	0	2	3	0
2	2	0	0	0	0	0	1	0	1	7	1	3	1	0	1	3	0	1	0
0	5	3	0	1	2	4	8	1	0	4	0	3	0	1	0	0	0	4	0
0	0	0	0	1	0	0	0	0	1	0	1	0	0	2	3	0	2	0	0
0	1	1	0	0	2	0	4	0	1	0	4	0	1	1	1	1	8	5	1
0	0	0	0	1	0	0	0	2	0	1	0	1	1	2	0	0	1	1	1
3	0	0	0	4	0	0	1	0	1	2	0	0	1	2	0	2	0	1	0
0	0	1	2	1	0	0	0	0	0	1	0	0	0	2	0	1	1	0	0
1	0	1	0	0	0	1	0	3	1	0	0	0	0	0	2	9	0	0	5
0	0	1	2	3	0	1	2	0	2	4	0	0	0	1	2	0	0	0	1

Typical state: on average one quantum per site, but the number in a given site can vary

Joule Expansion

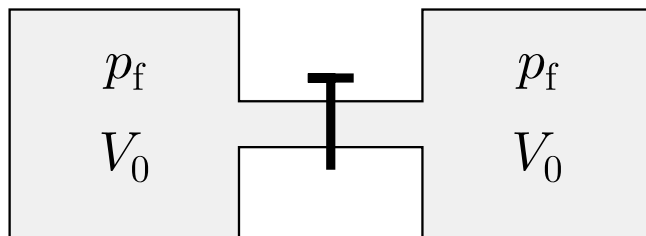
The gas is initially all on the left. When the valve is opened the gas rushes to fill the right hand side of the container.

(a)



The expansion is a highly non-equilibrium process.

(b)



During the expansion no heat enters the system. Thus the energy initial equals the final energy

So

$$\Delta S = k_B \ln \Omega_{\text{final}} - k_B \ln \Omega_{\text{init}} = N k_B \ln 2$$

We will derive this more formally later.

The particles will never go back to being on the left side (although it is not forbidden) since there is just way more configurations with the particles more equitably distributed.

- Summarizing all irreversible processes have

$$\Delta S_{\text{univ}} > 0$$

It is the weight of possibilities. There are of order 2^N more configurations with the gas completely filling the room, than with the gas on completely on the left. The gas explores these configurations for eternity, never returning to being only on the left.