Entropy of Ideal Gas (Mono-atomic)

- We have

$$dE = dQ - dW_{out}$$

$$dE = T\,dS - p\,dV$$

Or

$$dS = \frac{1}{T}\,dE + \frac{p}{T}\,dV \qquad (\star)$$

Now for an ideal gas we found:

$$E = \frac{3}{2}NkT \Rightarrow \frac{1}{T} = \frac{3}{2}\frac{Nk}{E} \qquad (\star\star)$$

$$pV = NkT \Rightarrow \frac{p}{T} = \frac{Nk}{V} \qquad (\star\star\star)$$

So

$$dS = \frac{3}{2}Nk\frac{dE}{E} + Nk\frac{dV}{V}$$

So

$$\boxed{S_{\substack{ideal\\gas}} = \frac{3}{2}Nk\ln E + Nk\ln V + const} \qquad (\star\star\star\star)$$

MAIG Only

$$= k\ln\left(E^{3N/2}V^N\right)$$

We can use this to find $\Omega(E, V)$

$$\boxed{\begin{array}{ll} \Omega(E,V) = e^{S/k} \quad \text{or} & S = k \ln \Omega \\ \Omega(E,V) = C\, E^{3N/2}\, V^{N} & (\text{MAIG}) \end{array}}$$

In the next section we will work in reverse. we will directly count the number of configurations (positions & momenta of the particles). This will determine the entropy (Eq. ~~XXXX~~ on previous page). From the entropy, one can find the energy temperature relation

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V = \frac{3}{2}\frac{Nk}{E}$$

and the ideal gas law

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_E = \frac{Nk}{V}$$