1 Estimates

We came across a number of physical constants that I expect you to know. Outside of this list, the constants will be given.

(a) Avagadros number $N_A = 6 \times 10^{23}$.

(b) The speed of light $c = 3 \times 10^8\,\mathrm{m/s}$.

(c) The speed of sound in air is approximately $330\,\mathrm{m/s}$.

(d) A useful unit of volume is liters. One liter is $(10\,\mathrm{cm})^3 = 1000\,\mathrm{cm}^3$. One mole of an ideal gas at STP has a volume of $22\,\mathrm{L}$.

(e) A useful unit of pressure is a bar. One atmosphere is approximately 1 bar. 1 bar is $10^5\,\mathrm{N/m}^2$.

(f) The ideal gas constant is $R = 8.32\,J/^\circ\mathrm{K}$:

(g) The Boltzmann constant k_B you can remember in two ways:

- The macroscopic way: one avagadros number times k_B is R:

$$N_A k_B = R \tag{1}$$

- The microscopic way: $k_B T$ is one fourtieth of an eV at $T = 300\,^\circ\mathrm{K}$.

$$k_B = \frac{\frac{1}{40}\,\mathrm{eV}}{300\,^\circ\mathrm{K}} = \frac{0.025\,\mathrm{eV}}{300\,^\circ\mathrm{K}} \tag{2}$$

(h) You should remember the proton (and neutron mass) in two ways:

- The microscopic way: the rest energy in mega electron volts is

$$m_p c^2 \simeq 938\,\mathrm{MeV} \simeq 1000\,\mathrm{MeV} \tag{3}$$

- The macroscopic way: an avagadros number of protons weighs a gramm. This is the molar mass of the proton:

$$\mathcal{M}_{\mathrm{ml}} = m_p N_A = 1\,\mathrm{g} \tag{4}$$

Protons and neutrons and weigh nearly the same thus the mass of one avagadros number of diatomic oxygen weighs $32\,g$, since there are eight protons and neutrons in one oxygen nucleus, and two such nuclei. The electrons are light (see below) for the mass budget.

You might want to use both of these methods to evaluate v_{rms} in atomic hydrogen gas

$$v_{\mathrm{rms}} = \sqrt{\frac{k_B T}{m_p}} = c\sqrt{\frac{k_B T}{m_p c^2}} = \sqrt{\frac{3RT}{1\,\mathrm{g}}} \tag{5}$$

(i) You should remember the electron mass in two ways:

- The microscopic way: half a MeV

$$m_e \simeq 0.5\,\text{MeV} \tag{6}$$

- Comparison to the proton mass:

$$\frac{m_e}{m_p} \simeq \frac{1}{2000} \tag{7}$$

(j) Plancks constant is needed to convert wavelength to energy

$$\hbar c = 197\,\text{eV nm} \tag{8}$$

or using $h = 2\pi\hbar$

$$hc = 1240\,\text{eV nm} \tag{9}$$

Thus the energy of yellow light with $\lambda = 550\,\text{nm}$ (emitted by sodium) is

$$E = \frac{1240\text{eV nm}}{550\,\text{nm}} \tag{10}$$

Planks constant is also useful for measuring typical de Broglie wavelength at constant temperature

$$\frac{h}{\sqrt{m_p k_B T}} = \frac{hc}{\sqrt{(m_p c^2)(k_B T)}} \tag{11}$$

(k) A useful unit of distance in atomic physics is angstroms, $1\,\text{Å} = 0.1\,\text{nm}$. The Bohr radius

$$a_0 = 0.53\,\text{Å} \tag{12}$$

is about half an Angstrom. A typical bond length is normally between 1-5 Bohr Radii. (For N_2 the distance between the two nuclei is 1.09 Å)

(l) An electron volt is a good unit of microscopic energy. Avagadros number times 1 eV is a good unit of macroscopic chemical energy and is 100 kilo Joules.

$$N_A\,\text{eV} \simeq 100\,\text{kJ} \tag{13}$$

This is sometimes called the Faraday constant[1]. An explosion involves roughly an avagadros number of atomic transitions, with each atomic transition releasing about an electron volt of energy, for approximately 100 kJ of energy per mole. Burning gasoline gives roughly this amount of energy. In the Otto cycle we took the input heat to be $Q_{\text{in}} = 22\,\text{kJ}$ per mole, which is roughly the Faraday constant.

[1]We can quickly find the charge in Coulombs from this relation,

$$1e \simeq \frac{100\,\text{kJ}}{N_A \cdot \text{Volt}} \simeq \frac{100000}{6\times 10^{23}}\left(\frac{\text{J}}{\text{Volt}}\right) = 1.6\times 10^{-19}\,\text{C}. \tag{14}$$

(m) We have not used this one yet, and it will not be on the exam. But, it is useful to know that the binding energy of an electron to a proton in atomic hydrogen is

$$\frac{\hbar^2}{2m_e a_0^2} \simeq 13.6\,\mathrm{eV} \tag{15}$$

where m_e is the *electron* mass. This comes from the Bohr model.

This gives another way to estimate the thermal debroglie wavelength of a *proton*, by inserting a_0 and the electron mass

$$\lambda_{\mathrm{th}} \sim \frac{\hbar}{\sqrt{2m_p k_B T}} = a_0 \sqrt{\frac{\hbar^2}{2m_e a_0^2}\left(\frac{1}{k_B T}\right)} \sqrt{\frac{m_e}{m_p}} = 0.5\text{Å} \sqrt{\frac{13.6\,\mathrm{eV}}{k_B T}} \sqrt{\frac{1}{2000}} \tag{16}$$