Problem 1. Diffusion of a oscillating magnetic field

In class we showed that a magnetic field driven with $H_0 e^{-i\omega t}\hat{\boldsymbol{z}}$ on the exterior of semi-infinite metal slab diffuses into the metal. The semi-infinite metal slab fills the region $x > 0$. The magnetic field in the metal is

$$H^z(x,t) = H_o e^{-x/\delta}\cos(x/\delta - \omega t) \tag{1}$$

where $\delta = \sqrt{2c^2/\mu\omega\sigma}$ is the skin depth.

(a) Compute the current in the metal. You should find:

$$\frac{j^y}{c} = \frac{\sqrt{2}}{\delta} H_o e^{-x/\delta}\cos(x/\delta - \omega t - \pi/4) \tag{2}$$

(b) Estimate the size of the induced electric field relative to the applied magnetic field

$$\frac{E^{\text{ind}}}{H_o} \tag{3}$$

What is the condition on the frequency so that $E^{ind} \ll H_o$, which was our starting point?

(c) Compute the integral of the current j^y over x. You should find

$$\frac{K^y}{c} \equiv \int_0^\infty dx\, j^y(x,t)/c = H_o\cos(\omega t) \tag{4}$$

Interpret this simple result using the boundary conditions

$$\boldsymbol{n} \times (\boldsymbol{H}_{\text{out}} - \boldsymbol{H}_{\text{in}}) = \frac{\boldsymbol{K}}{c} \tag{5}$$