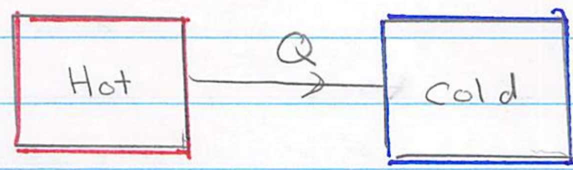
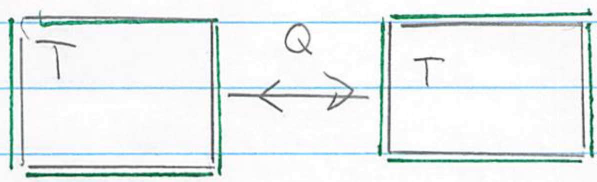
Reversible vs. Irreversible Exchange of Heat

- The carnot cycle is reversible since the transfer of heat occurs at the same temperature

- Example

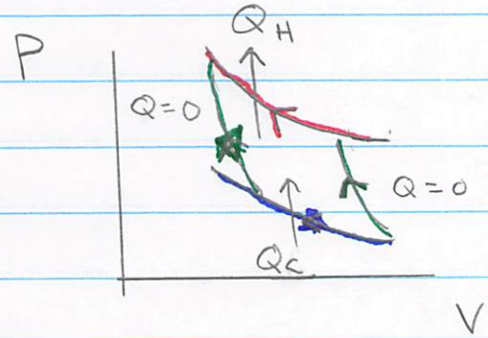


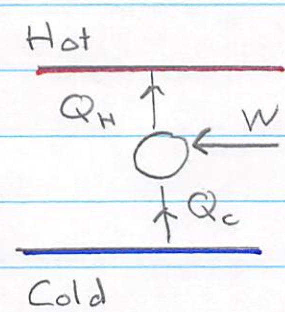
- Heat only spontaneonsly flows from the hot to the cold. The exchange is irrevesible



- If two objects have the same temperature heat can flow in both directions, the process is reversible.

In a Carnot Refrigerator we run the Carnot Cycle in reverse.





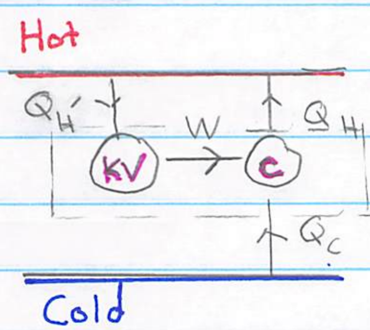
Now I do work, and force the transfer of heat from the cold reservoir to the hot resevoir. This is a refrigerator

Clausius + Kelvin Statements

① No process is possible whose sole result is the transfer from a cold to a hot body (Clausius)

② No engine can completely convert heat to work (Kelvin)

These are equivalent. The picture below shows why. Here we hooking up a Kelvin Violator Engine to a carnot refrigerator



Look at the energy flows in and out of box

- This is energy consv or first Law

$$W = Q_{H'} \quad \text{first law to KV}$$

$$W + Q_c = |Q_H| \quad \text{first low applied to Carnot}$$

So $\quad Q_c = |Q_H| - Q_H' > 0$

- This says we are transferring Q_c heat from the cold to the hot reservoirs without work, Violating the Clausius statement

- The reverse logic can also be proved, i.e. a Clausius violator produces a perfect engine

Kelvin / Carnot / Clausius Statements

① All reversible engines (not just based on ideal gasses) have the same efficiency

$$\eta = 1 - \frac{|Q_c|}{Q_H} = 1 - \frac{T_c}{T_H}$$

or by simple algebra

$$\boxed{\frac{Q_c}{T_c} + \frac{Q_H}{T_H} = 0} \qquad \text{(reversible)}$$

② An irreversible engine will necessarily have an efficiency less than Carnot Efficiency

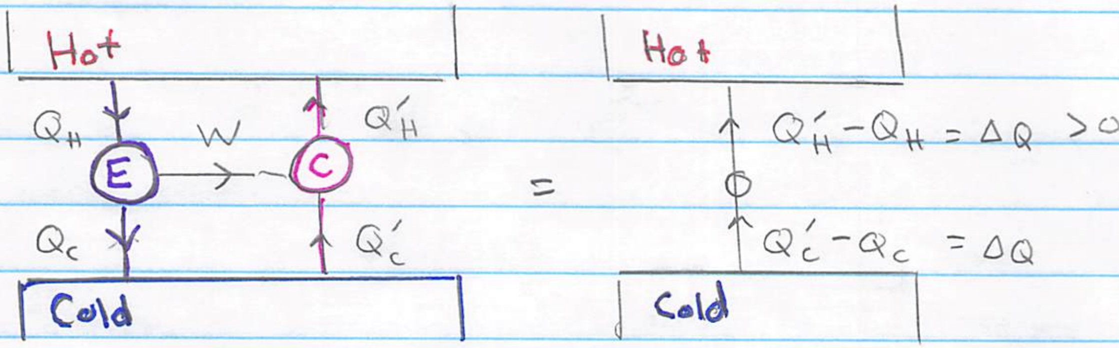
$$\eta_{irrev} = 1 - \frac{|Q_c|}{Q_H} < 1 - \frac{T_c}{T_H}$$

or by algebra

$$\boxed{\frac{Q_H}{T_H} + \frac{Q_c}{T_c} < 0} \qquad \text{(irreversible)}$$

- The arguement is based on logic. If I have a new reversible engine that is more efficient than the Carnot engine, I can use it to drive a carnot refrigerator, the reverse of a carnot engine. The "excess" efficiency ammounts to a spontaneous flow of heat in the wrong direction (see below). The picture is below:

O = new engine O = Carnot engine/refrig.



Since

$$\eta_E = \frac{|W|}{Q_H} > \frac{|W|}{Q'_H} = \eta' \quad \text{we have} \quad Q'_H > Q_H$$

carnot efficiency

- We also have from energy conservation in the first figure

$$W = |Q_H| - |Q_c| = |Q'_H| - |Q'_c|$$

So:

$$\Delta Q = |Q'_H| - |Q_H| = |Q'_c| - |Q_c| > 0$$

$\Delta Q > 0$ means heat is flowing from the cold to the hot resevoir spontaneously which cant happen. So $\eta \leq \eta'$.

- If the Carnot is more efficient than the new reversible one, we can interchange the roles of the engines and refrigerators, again leading to contradiction. So $\eta' \leq \eta$. The two together imply $\boxed{\eta' = \eta}$.

- Now if the new the engine is irreversible. The same argument just given applies, saying that $\eta_{irrev} > \eta'_{carnot}$ is impossible, So we must have

$$\eta_{irrev} < \eta_{carnot}$$

- But, unlike in the reversible case we cant interchange the role of the engines and refridgerators since the new engine is irreversible, and thus can't be used as a refrigerator.

So

$$\boxed{\eta_{irrev} < \eta_{carnot}}$$

strictly.

Generalization

- Now consider an arbitrary reversible cycle in the P, V plane. We may approximate it by a set of carnot cycles (see slide). Then for instance for the first cycle we have $\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$

The full loop is

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} + \frac{Q_4}{T_4} + \cdots = 0$$

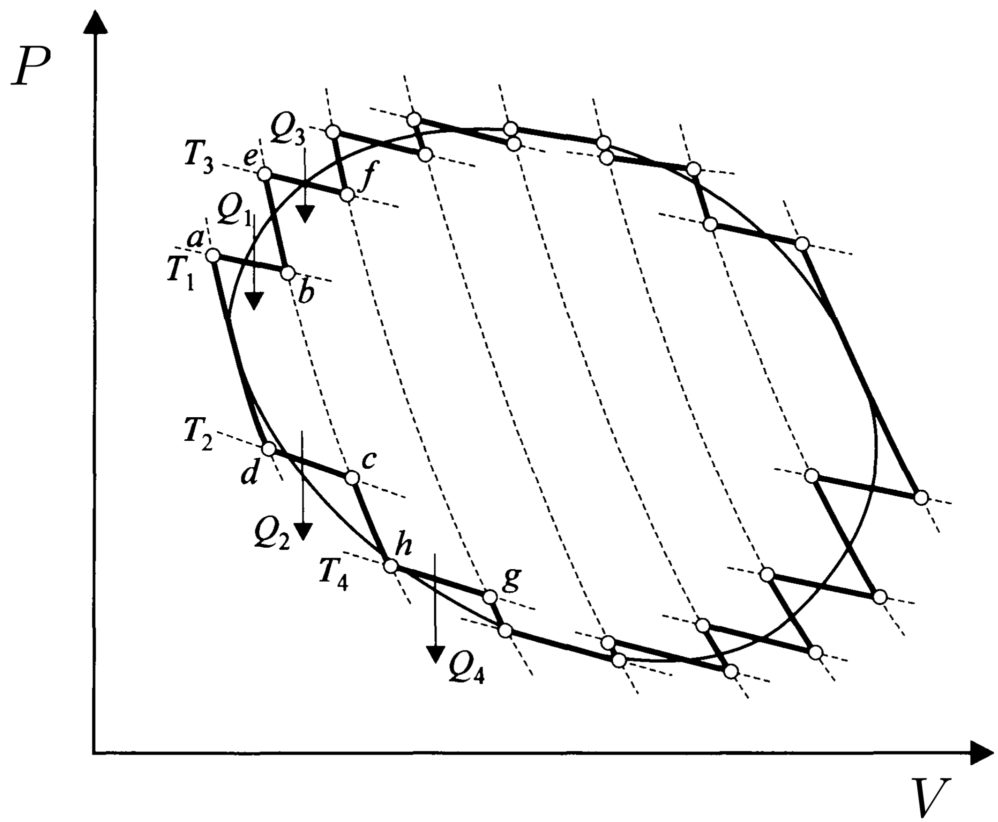
Becoming

$$\boxed{\oint \frac{dQ}{T} = 0 \qquad \text{(reversible)}}$$

in the limit.

More generally for an irreversible cycle we have, e.g. $\frac{Q_1}{T_1} + \frac{Q_2}{T_2} < 0$

Breaking up a general cycle into carnot cycles



leading to

$$\boxed{\oint \frac{\text{đ}Q}{T} < 0 \qquad \text{(irreversible)}}$$