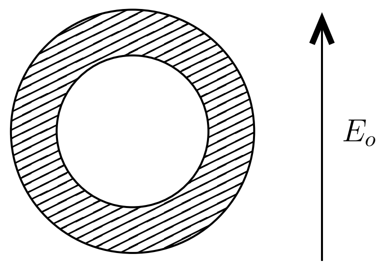
Problem 1. Electric field in a dielectric shell.



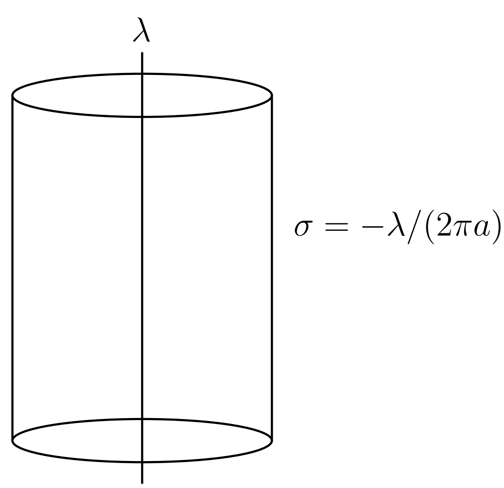
A spherical dielectric shell, with a hollow interior, has inner radius a and outer radius b and dielectric constant ε. The dielectric shell sits in an external electric field of magnitude E_o pointing in the z direction.

(a) Find the system equations which determines the electric field inside the sphere, but (for lack of time) do not try to solve this system.

(b) Taking $a \to 0$ (so that the sphere is solid) determine the electric field within the sphere for $r < b$.

(c) It was not part of the exam, but solve for the electric field everywhere without using mathematica. Use two-by-two matrices to relate the solution in one region to another.

Problem 2. Induced rotation.



An infinitely long dielectric wire runs along the z axis with charge density λ, and is surrounded by a thin dielectric cylindrical shell with radius a, carrying charge density $\sigma = -\lambda/(2\pi a)$. The suspended cylinder can rotate freely about the z-axis, but is initially at rest in a constant magnetic field in the z-direction, $\boldsymbol{B}_{\text{ext}} = B_o\hat{\boldsymbol{z}}$

(a) Determine the electric field for $t < 0$.

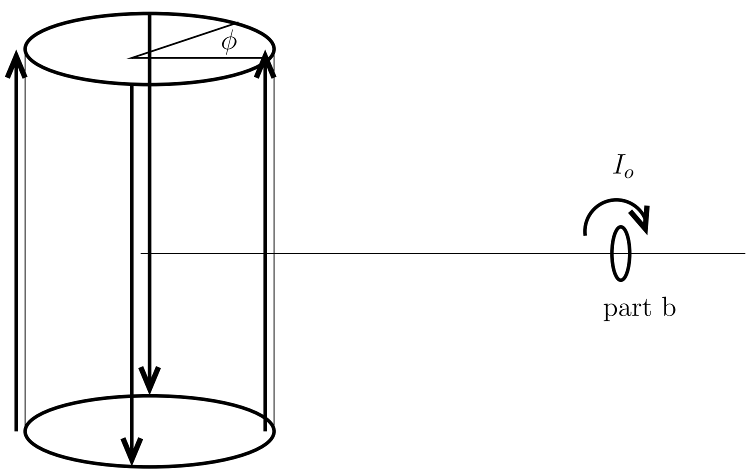
(b) At $t = 0$ we slowly reduce the magnetic field to zero over a time $T \gg a/c$. What happens and why? Draw a sketch of the resulting motion indicating the way that the cylinder rotates.

(c) Find the angular velocity of the cylinder as a function of time, taking its moment of inertia per unit length to be I.

(d) How did the condition $T \gg a/c$ help you in part (c) to find an approximate solution to the Maxwell equations. Point to a specific term in the Maxwell equations which was neglected/dropped/approximated using this condition. Give an estimate for the magnitude of the corrections to your result.

(e) Calculate the angular momentum per unit length for $t > T$ and show that it is conserved, *i.e.* that the final angular momentum equals the angular momentum for $t < 0$.

Problem 3. Currents in a cylindrical shell.



An infinite cylindrical shell of radius, a, carries a surface current in the z direction which is a function of angle, $\boldsymbol{K}(\phi) = K_o \cos(2\phi)\hat{\boldsymbol{z}}$.

(a) Determine the magnetic field outside and inside the shell produced by the surface current.

(b) A second *small* circular loop lies carries current I_o and has radius r_o and sits on the x-axis at distance ρ_o from the center. The current is oriented as shown. Determine the magnitude of the force on the current loop as a function of distance ρ from the center of the cylinder. What is the direction of the force.

Problem 4.   Reflection and transmission from glass

Consider a plane wave of light in vacuum normally incident (*i.e.* head on) on a semi-infinite slab of glass filling the space $z > 0$. The glass has index of refraction $n > 1$ and magnetic permeability $\mu \simeq 1$. The frequency of the light is ω. The incoming wave, is polarized in the $-\hat{\boldsymbol{x}}$ direction and has amplitude E_I, thus taking the form

$$E_I e^{-i\omega t + ikz} \, \widehat{-\boldsymbol{x}} \tag{1}$$

(a) Starting from the Maxwell equations, determine the electric field amplitudes of the reflected and transmitted waves, in terms of the incident amplitude.

(b) Determine the reflection and transmission coefficients, *i.e.* the ratio of the reflected and transmitted power to the incident power.

(c) Determine the time averaged electromagnetic stress tensor both just in front, and just behind the vacuum-glass interface. Use this result to determine the force per unit area on the front face of the glass.