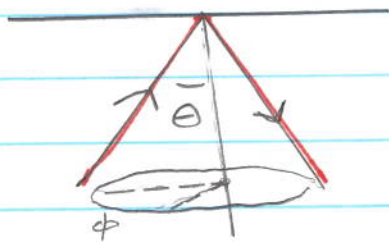


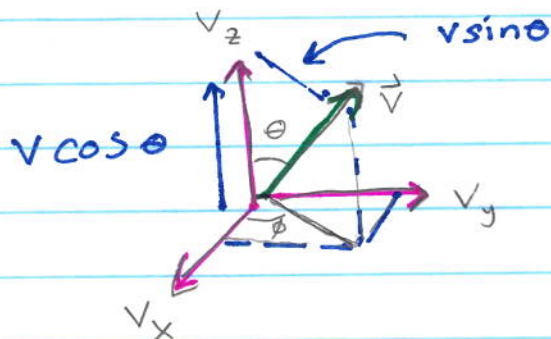
Pressure and Effusion



- Particles bounce off the walls with a variety of angles θ, ϕ a variety of speeds V each second. This gives a net force per area. We need the distribution of angles and speeds
- Then the velocity distribution is

$$d\rho = \left(\frac{m}{2\pi k_B T} \right)^{3/2} e^{-\frac{1}{2}mv^2/kT} dv_x dv_y dv_z$$

- So we can change variables to speeds and angles
 $v = |\vec{v}|$, $v_z = v \cos \theta$ $v_x = v \sin \theta \cos \phi$ etc like before



$$\sin \theta d\theta d\phi$$

$$d\rho = \left(\frac{m}{2\pi k_B T} \right)^{3/2} e^{-\frac{1}{2}mv^2/kT} v^2 dv d\Omega$$

- So we multiply and divide by 4π and write

$$d\mathcal{P} = P(v) dv \frac{d\Omega}{4\pi} \quad \text{where}$$

$$P(v) = \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} 4\pi v^2 \quad \text{is the distribution of speeds discussed before.}$$

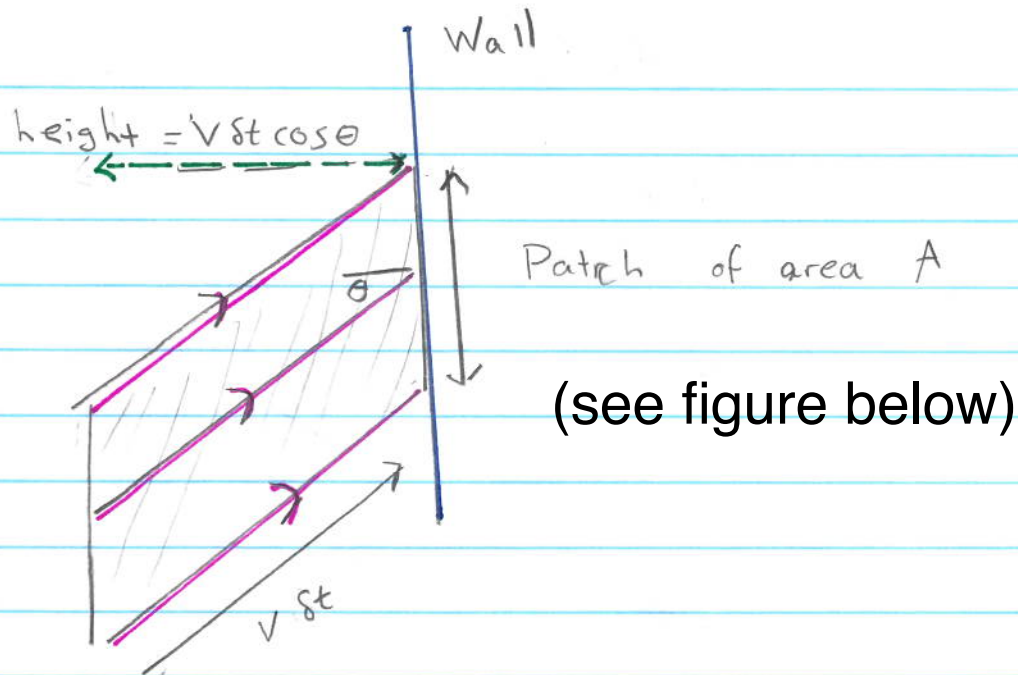
This gives the distribution of speeds and angles. The angular part is uniformly distributed over the sphere $d\Omega = \sin\theta d\theta d\phi / 4\pi$, as we could have hoped.

- To find the density of particles with speed $[v, dv]$ and angles $[\theta, \theta+d\theta]$, $[\phi, \phi+d\phi]$ we multiply by the ^{total} density of the system $n = \frac{N}{V}$

$$dn = n d\mathcal{P} = n P(v) dv \frac{d\Omega}{4\pi}$$

- Now consider a small patch of area A on the wall. We want to work out the flux $\bar{\Phi}$ of particles striking the surface per $dv, d\theta, d\phi$

$$\bar{\Phi} \equiv \frac{\# \text{ of particles}}{\text{Area} \cdot \text{time}} \equiv \text{Flux}$$



- The tube shows a swarm of particles in space flying with speed v , angles θ, ϕ . In a given δt all particles in the tube will strike the surface. The volume of the tube is

$$\text{Volume} = A \overbrace{v \delta t \cos \theta}^{\text{height}}$$

So the number per area per time in $dv, d\theta, d\phi$ is

$$\frac{dN}{A \delta t} = \frac{\text{Volume} \cdot dn}{A \delta t} = \frac{n P(v) v \cos \theta}{4\pi} dv d\Omega$$

So

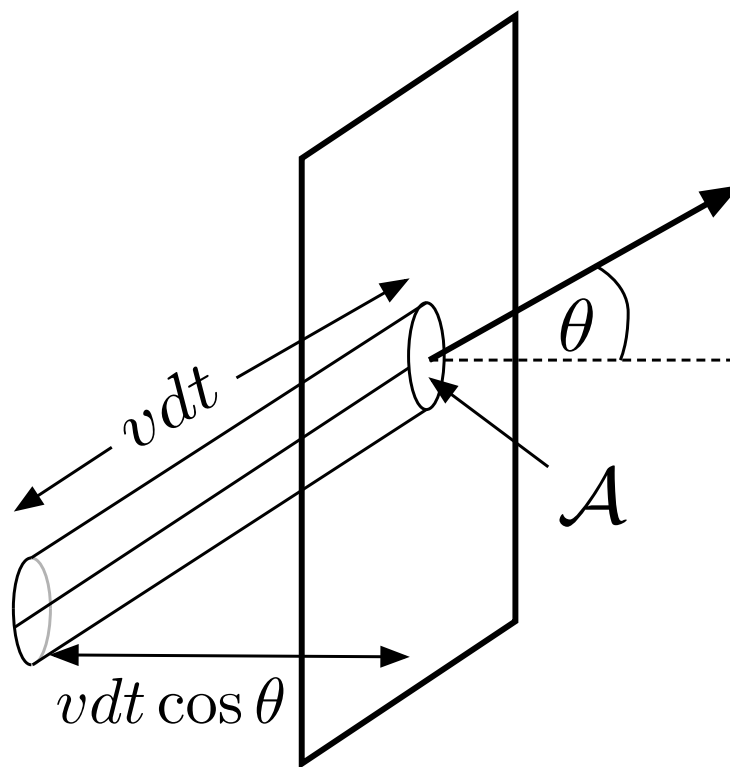
$$d\Phi = n P(v) v \cos \theta \frac{dv d\Omega}{4\pi}$$

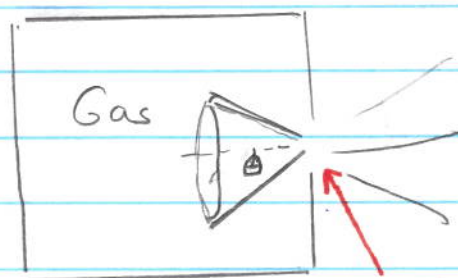
$d\Omega = \sin \theta d\theta d\phi$

↑

number of particles striking wall per area per time with $[v, v+dv]$ and angles $[\theta, \theta+d\theta]$ and $[\phi, \phi+d\phi]$

Flux through the hole



Effusion

- For example a slow bicycle leak. This is gas escaping through A

Hole of area A

- Find the total number escaping per area per time

$$\Phi = \int d\Phi = \int_0^\infty dv \int_0^{\pi/2} d\theta \int_0^{2\pi} d\phi \, n P(v) v \cos\theta \frac{\sin\theta}{4\pi}$$

↑ from $\frac{d\Omega}{4\pi}$

- You will do the angular integrals in homework

$$\Phi = \int_0^\infty dv \, v P(v) \underbrace{\int_0^{\pi/2} \cos\theta \frac{1}{2} \sin\theta d\theta \int_0^{2\pi} \frac{d\phi}{2\pi}}_{1/4 \text{ from homework}}$$

$$\Phi = \frac{1}{4} n \langle v \rangle$$

$$\Phi = n \left(\frac{k_B T}{m} \right)^{1/2} \frac{1}{\sqrt{2\pi}}$$

↑ velocity

$$\langle v \rangle = \left(\frac{8 k_B T}{\pi m} \right)^{1/2}$$

- units: $\frac{\text{Number}}{\text{m}^3} \times \frac{\text{m}}{\text{s}} = \frac{\text{number}}{\text{Area time}}$ ✓