Problem 2

a) $$Z = \sum_n e^{-E_n/kT}$$ (8 points)

$$Z = 1 + e^{-\Delta/kT} + e^{-5\Delta/kT}$$ 4+4 points

$$\langle E \rangle = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = \frac{\Delta e^{-\Delta/kT} + 5\Delta e^{-5\Delta/kT}}{(1+e^{-\Delta/kT}+e^{-5\Delta/kT})}$$

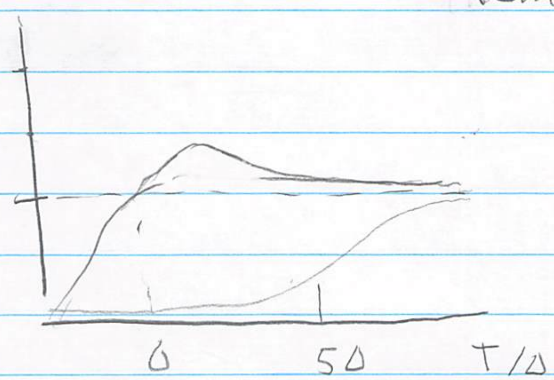
b) $$P_1 = \frac{e^{-\Delta/kT}}{(1+e^{-\Delta/kT}+e^{-5\Delta/kT})}$$ (10 points)

4 points

$$P_2 = \frac{e^{-5\Delta/kT}}{(1+e^{-\Delta/kT}+e^{-5\Delta/kT})}$$

i) So at low temperature P_1 + P_2 are zero, while at high temperature $P_0 = P_1 = P_2 = \frac{1}{3}$, all states are equally likely. The P_1 curve reaches its high temperature limit sooner

6 points



- We can estimate what high temperature means. We want

$$5\beta\Delta \longrightarrow 0$$

So

$$P_2 \simeq \frac{1}{(1+1+1)} \simeq \frac{1}{3}$$

So the high temperature limit is when

$$5\beta\Delta \ll 1 \quad \text{or} \quad \frac{5\Delta}{k_B T} \ll 1 \quad \text{or} \quad T \gg \frac{5\Delta}{k_B}$$

Substituting

$$T \gg \frac{5 \cdot (0.1\,eV)}{\left(\frac{0.025\,eV}{300^\circ K}\right)} \quad \text{or} \quad \boxed{T \gg 6000^\circ K}$$