Thermo and Differentials

- Take an ideal gas with constant specific heat

$$dU = đQ + đW$$

$$đQ = dU + p\,dV$$

$$đQ = C_V\,dT + \frac{Nk_BT}{V}\,dV = A(T,V)\,dT + B(T,V)\,dV$$

$đQ$ is not exact:

$$\frac{\partial A}{\partial V} \neq \frac{\partial B}{\partial T}$$

- But notice:

$$\frac{đQ}{T} = \underbrace{\frac{C_V}{T}}_{=A'}\,dT + \underbrace{\frac{Nk_B}{V}}_{=B'}\,dV = A'\,dT + B'\,dV$$

is exact

$$\frac{\partial A'}{\partial V} = \frac{\partial B'}{\partial T} \checkmark$$

So there is a function $S(T,V)$ which is a property of the equilibrium state

$$dS = \frac{đQ}{T}$$

We will begin to interpret $S(T,V)$, as the entropy

From a thermodynamic perspective (and for ideal gas)

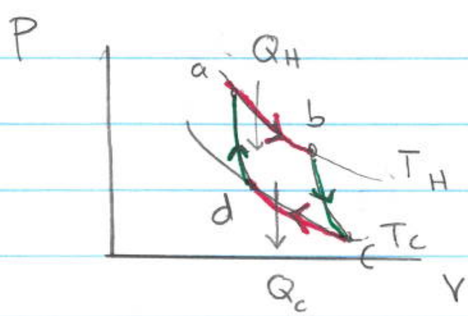
$$\boxed{dS = \frac{C_V}{T}\,dT + \frac{Nk_B}{V}\,dV}$$ (ideal gas)

and (ideal gas only)

$$\boxed{\Delta S = S_f - S_i = C_V \ln\left(\frac{T_f}{T_i}\right) + Nk_B \ln\left(\frac{V_f}{V_i}\right)}$$

entropy of ideal gas. + constant C_V

- At least from a mathematical perspective, the relation from the carnot cycle is clear



$$\Delta S = \oint \frac{đQ}{T} = 0$$

It is a closed loop:

$$\Delta S = \underbrace{\frac{Q_H}{T_H}}_{\Delta S_{ab}} + \underbrace{0}_{\Delta S_{bc}} + \underbrace{\frac{Q_c}{T_c}}_{\Delta S_{cd}} + \underbrace{0}_{\Delta S_{da}} = 0$$