1 Fermi and Bose Integrals

First thing – get the units right! Integrals with wrong units loose plenty of points. Integrals with right units (but wrong numbers) loose almost nothing. Ask yourself what the units of the integral are before you start, and check at the end that you get the right units. You will want to switch to some dimensionless variables, which separates the units part from doing the actual integral part. This will be necessary to put the integral into the form of this table.

The following integrals will be given[1]

$$\int_0^\infty \mathrm{d}x \, \frac{x}{e^x - 1} = \frac{\pi^2}{6} \tag{2}$$

$$\int_0^\infty \mathrm{d}x \, \frac{x^2}{e^x - 1} = 2\zeta(3) \simeq 2.404 \tag{3}$$

$$\int_0^\infty \mathrm{d}x \, \frac{x^3}{e^x - 1} = \frac{\pi^4}{15} \tag{4}$$

$$\int_0^\infty \mathrm{d}x \, \frac{x^4}{e^x - 1} = 24\,\zeta(5) \simeq 24.88 \tag{5}$$

$$\int_0^\infty \mathrm{d}x \, \frac{x^5}{e^x - 1} = \frac{8\pi^6}{63} \tag{6}$$

Fermionic integrals are similar

$$\int_0^\infty \mathrm{d}x \, \frac{x}{e^x + 1} = \frac{\pi^2}{12} \tag{7}$$

$$\int_0^\infty \mathrm{d}x \, \frac{x^2}{e^x + 1} = \frac{3}{2}\,\zeta(3) \simeq 1.80309 \tag{8}$$

$$\int_0^\infty \mathrm{d}x \, \frac{x^3}{e^x + 1} = \frac{7\pi^4}{120} \tag{9}$$

$$\int_0^\infty \mathrm{d}x \, \frac{x^4}{e^x + 1} = \frac{45}{2}\,\zeta(5) \simeq 23.33 \tag{10}$$

$$\int_0^\infty \mathrm{d}x \, \frac{x^5}{e^x + 1} = \frac{31\pi^6}{252} \tag{11}$$

Although these integrals are given, it will be your job to switch to put the integral into the form of this table. The advice below becomes very important.

[1] The integrals are done by expanding out the denominator,

$$\frac{1}{e^x - 1} = e^{-x} + e^{-2x} + e^{-3x} + \dots$$

and performing the integrals term by term. The resulting sum can be expressed in terms of the Riemann zeta function

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \tag{1}$$

which is the most studied function in human history.