Physics 306: Thermal Physics

Final Exam

Stony Brook University

Spring 2022

General Instructions:

You may use one page (front and back) of handwritten notes and a calculator. Graphing calculators are allowed. **No other materials may be used**.

Problem 1. 2D world with gravity

Consider a mono-atomic ideal gas in two dimensions, (x, y), at temperature T. The atoms are in the gravitational field of the 2d "earth", and the potential energy of the atoms is thus

$$U(y) = mgy\,. \tag{1}$$

Here the "earth's" surface is at $y = 0$, and the atoms are above the surface, i.e. in the region $y \in [0, \infty]$.

(a) Determine the root mean square velocity of the gas molecules from the equipartition theorem. Explain your reasoning.

(b) Determine the mean total energy (kinetic and potential) of the gas molecules.

(c) What is the probability that the velocity of an atom is less than the root mean square velocity?

Hint: express your result as a dimensionless integral, before attempting an evaluation of this integral.

Problem 2. Partition function of three level system

Consider an ensemble at temperture T of N independent "atoms", each of which have the three energy levels shown below.

$$E_2 = 5\Delta$$

$$E_1 = \Delta$$

$$E_0 = 0$$

(a) Determine the partition function and mean energy of an atom as a function of temperature.

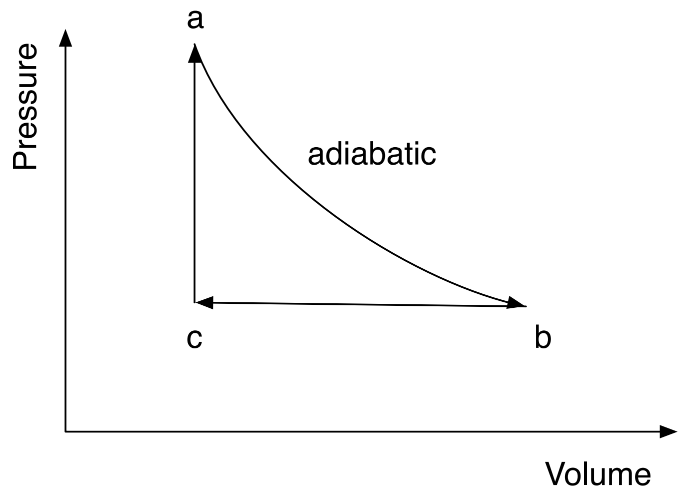
(b) Determine the probabilities to be in the first excited and second excited states, P_1 and P_2, and qualitatively sketch these probabilities versus temperature on the same graph, from very low to very high temperatures:

(i) Explain the qualitative features of your graph by pointing to specific terms in your equations.

(ii) Determine a Taylor series expansion for the probability P_2 at high temperature, including the leading and first subleading terms. Explain the value of the leading term physically. Taking $\Delta = 0.1\,\text{eV}$, estimate the temperature (in Kelvin) when the Taylor series becomes approximately valid.

Problem 3. An engine cycle

One mole of an ideal mono-atomic gas operates in an engine cycle shown below. Here the givens are the temperatures, $T_a = 400\,K$ and $T_c = 250\,K$, and the pressure, $P_c = 1\,\text{bar}$. These values are recorded in the table below.



Using the ideal gas law $PV = nRT$, it is straightforward to show that the volumes at a and c are $V_a = V_c = 20.8\,\text{L}$, and that the pressure at a is $P_a = 1.6\,\text{bar}$, as recorded in the table below.

(a) Find the volume and temperature at b, completing the table shown below.

state	pressure	volume	temperature
a	1.6 bar	20.8 L	400 K
b	1.0 bar	?	?
c	1.0 bar	20.8 L	250 K

(b) Find the work done by the gas per cycle.

(c) Find the efficiency of the engine.

(d) Find the change in entropy of the gas from c to a.

(e) Using part (d), find the change in entropy from a to b and from b to c