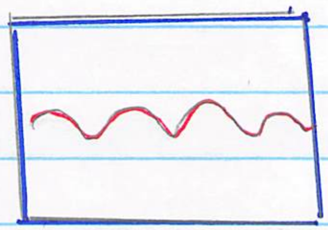
Last Time



- consider quantum particles in a box
- The waves are broken up into modes

$$\vec{p} = \hbar \vec{k} = \hbar \left(\frac{\pi n_x}{L}, \frac{\pi n_y}{L}, \frac{\pi n_z}{L} \right)$$

- Each mode is an independent subsystem sharing the available energy and particles. Temperature is a parameter describing how the energy is shared. μ is a parameter describing how the particles are shared

- The mean number of particles in a mode of single-particle energy $\varepsilon(p)$ is

$$\bar{n}_{BE}^{(p)} = \frac{1}{e^{\beta(\varepsilon(p)-\mu)} - 1} \leftarrow \text{if particles are bosons}$$

$$\bar{n}_{FD}^{(p)} = \frac{1}{e^{\beta(\varepsilon(p)-\mu)} + 1} \leftarrow \text{if particles are fermions}$$

- The energy in a mode is $E_p = n\,\varepsilon(p)$ and the mean energy of a mode is

$$\bar{E}_p = \bar{n}\,\varepsilon(p)$$

Black Body Radiation and the Photon Gas:

$$\mathcal{E}(p) = \frac{p^2}{2m}$$ non-relativist particles

$$\mathcal{E}(p) = cp$$ light

Then the total number of quantum particles in the box is

$$N = \sum_{\text{modes}} \frac{1}{e^{\beta(\mathcal{E}(p)-\mu)} \mp 1}$$

Photons

- Photons are bosons.
- $\mathcal{E}(p) = cp$, since they are relativistic & massless.
- $\mu = 0$ since photons are easily created and destroyed
- The sum over modes / states was:

$$\sum_{n_x} \sum_{n_y} \sum_{n_z} \cdots \longrightarrow \int \frac{V\, d^3p}{(2\pi\hbar)^3} \cdots = \int \frac{d^3r\, d^3p}{h^3} \cdots$$

- The two polarizations of photons gives "extra" an overall factor of 2:

$$\sum_{\text{modes}} \cdots \longrightarrow 2 \int \frac{V\, d^3p}{(2\pi\hbar)^3} \cdots$$

- Putting Together The ingrediants we found:

$$N = 2V \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{e^{cp/kT} - 1}$$

- Looking at this integral, there is a characteristic energy scale for the photons

$$\boxed{E_0 \sim kT}$$

Then, $p_0 = \frac{E_0}{c} \sim \frac{kT}{c}$, and the typical vavelength is:

$$\boxed{\lambda\!\!\!{}^{-}_0 \equiv \frac{\hbar}{p_0} \equiv \frac{\hbar c}{kT}} \quad \text{with} \quad \lambda\!\!\!{}^{-} \equiv \frac{\lambda}{2\pi}$$

- Then doing integral we found:

numerical integration

$$N = \underbrace{V \left(\frac{p_0}{\hbar}\right)^3}_{\text{units}} \# = V \left(\frac{kT}{\hbar c}\right)^3 0.244 \propto T^3$$

So the density of photons is $n_\gamma \equiv N/V$

$$n_\gamma \equiv \left(\frac{N}{V}\right) = 0.244 \left(\frac{kT}{\hbar c}\right)^3 = \frac{0.244}{\lambda\!\!\!{}^{-3}_0}$$