Entropy of Ideal Gas (Mono-atomic)

- We have the first law of thermodynamics

$$dE = dQ - dW_{out}$$

$$dE = T\,dS - p\,dV$$

Or

$$dS = \frac{1}{T}\,dE + \frac{p}{T}\,dV \qquad (\star)$$

Now for an ideal gas we found:

$$E = \frac{3}{2}NkT \implies \frac{1}{T} = \frac{3}{2}\frac{Nk}{E} \qquad (\star\star)$$

$$pV = NkT \implies \frac{p}{T} = \frac{Nk}{V} \qquad (\star\star\star)$$

So

$$dS = \frac{3}{2}Nk\frac{dE}{E} + Nk\frac{dV}{V}$$

So integrating this equation we have using d ln E = dE/E, etc, that:

$$\boxed{S_{\substack{\text{ideal}\\\text{gas}}} = \frac{3}{2}Nk\ln E + Nk\ln V + \text{const}} \qquad (\star\star\star\star)$$

MAIG Only

$$= k\ln\left(E^{3N/2}V^{N}\right)$$

We can use this to find $\Omega(E,V)$

$$\boxed{\begin{array}{ll} \Omega(E,V) = e^{S/k} \quad \text{or} & S = k \ln \Omega \\ \Omega(E,V) = C\, E^{3N/2}\, V^N & (\text{MAIG}) \end{array}}$$

In the next section we will work in reverse. we will directly count the number of configurations (positions & momenta of the particles). This will determine the entropy (Eq. ~~****~~ on previous page). From the entropy, one can find the energy temperature relation

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V = \frac{3}{2}\frac{Nk}{E}$$

and the ideal gas law

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_E = \frac{Nk}{V}$$