Problem 1. The electric stress tensor

Consider the electric force per volume, $\boldsymbol{f} = f^i \boldsymbol{e}_i$,

$$f_E^i = \rho E^i \tag{1}$$

Starting from Eq. (1), show that force can be written as

$$f_E^j = -\partial_i T_E^{ij} \tag{2}$$

where the electric stress tensor is

$$T_E^{ij} = -E^i E^j + \frac{1}{2} E^2 \delta^{ij} \tag{3}$$

Use the Maxwell equations, $\nabla \cdot \boldsymbol{E} = \rho$ and $\nabla \times \boldsymbol{E} = 0$

$$\partial_i E^i = \rho \qquad \partial_i E_j - \partial_j E_i = 0 \tag{4}$$