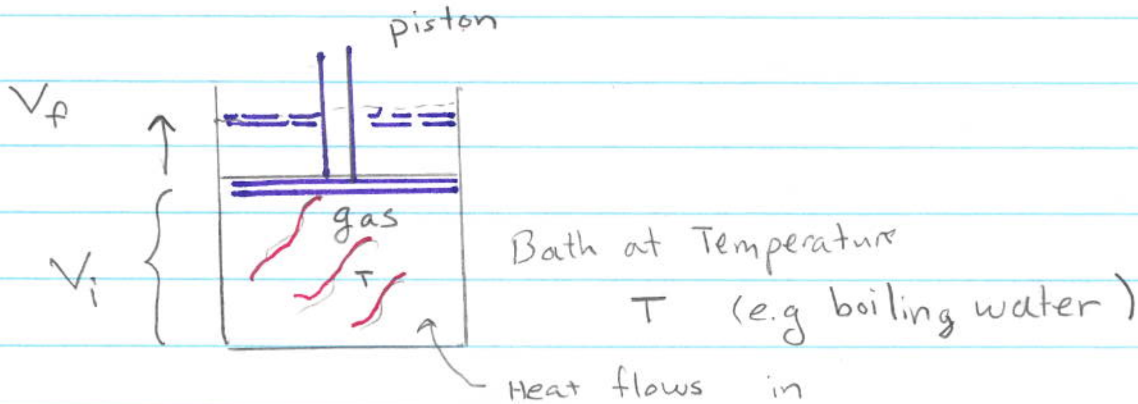
# Isothermal Expansion of Ideal Gas




- As the piston is raised the gas does work $p\,dV$ (we do negative work $\text{đ}W = -p\,dV$.) Heat flows in from bath to maintain a constant temperature. We do it <u>slowly</u> enough so that $T$ is constant at all times. Lets consider ideal gasses where $U = N f(T)$. Since $T$ is fixed $dU = 0$. What is the heat flowing in?

$$d\cancel{U}^{\,0} = \text{đ}Q + dW$$

$$\text{đ}Q = -\text{đ}W = p\,dV$$

So

$$\Delta Q = \int_i^f \text{đ}Q = \int_i^f p(T,V)\,dV = \int_{V_i}^{V_f} \frac{N k_B T}{V}\,dV$$

At all times we are in equilibrium so

$$p = \frac{N k_B T}{V}$$

So integrating

$$\boxed{\Delta Q = N k_B T \ln\left(\frac{V_f}{V_i}\right)}$$

heat inflow of ideal gas during isothermal expansion

We remind that $\Delta U = 0$ but $-W_{if} = Q_{if}$. The work done <u>by</u> the gas is $W_{if}^{gas} = Q_{if}$

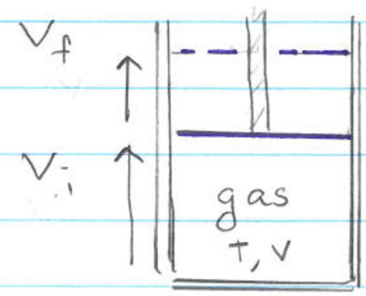
# Adiabatic Expansion of Ideal Gas @ Constant $C_V$

- We will consider a ideal gas with constant specific heat $C_V = \text{const}$, and, $C_p = C_V + Nk_B$, is also constant.

- In an adiabatic expansion we do not allow heat to flow into the cylinder, $đQ = 0$.

- Adiabatic expansions are much more common in practice since if the expansion is relatively quick, there isn't time for heat exchange.

Temperature drops as gas expands




← thermally insulated walls, or just do the expansion fast enough (but not too fast) that no heat can be exchanged

$$dQ = 0$$

$$dU = đW \qquad (1) \qquad \text{Use } đW = p\,dV$$

$$C_V\,dT = -p\,dV \qquad (2) \quad \text{use ideal gas relation: } dU = C_V\,dT + \overset{0}{\cancel{\left(\frac{\partial U}{\partial V}\right)_T}}\,dV$$

ideal gas

$$C_V\,dT = -\frac{Nk_BT}{V}\,dT \qquad (3) \quad \text{Use ideal gas EOS}$$

- So we can integrate this assuming constant specific heat

$$\frac{dT}{T} = -\frac{Nk_B}{C_V}\frac{dV}{V} \qquad C_p = C_V + Nk_B$$

$$\gamma \equiv \frac{C_p}{C_V} = 1 + \frac{Nk_B}{C_V}$$

$$\frac{dT}{T} = -(\gamma-1)\frac{dV}{V}$$

- Integrating both sides $\ln\frac{T_f}{T_i} = -(\gamma-1)\ln\frac{V_f}{V_i}$ or

$$\boxed{T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}} \quad \text{or} \quad TV^{\gamma-1} = \text{const}$$

So since $pV \propto T$ we find

$$\boxed{p_i V_i^{\gamma} = p_f V_f^{\gamma}} \quad \text{or} \quad pV^{\gamma} = \text{const}$$