Physics 306: Thermal Physics

Second Midterm

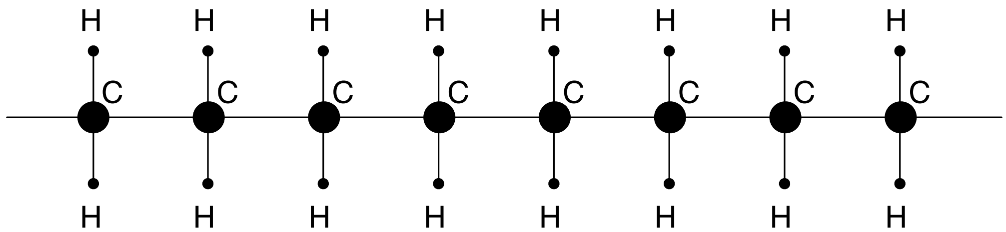
Stony Brook University

Spring 2023

General Instructions:

You may use one page (front and back) of handwritten notes and a calculator. Graphing calculators are allowed. **No other materials may be used**.

Problem 1. A Chain of Hydrogen Atoms



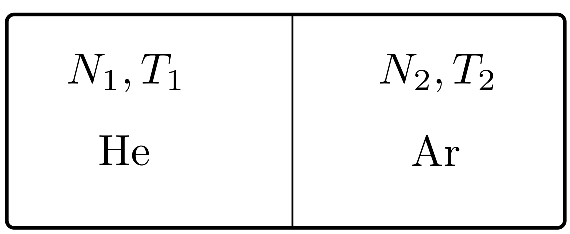
Consider a long chain of hydrogen atoms of mass m connected by a chain of N Carbon atoms. The Carbon atoms can be considered to be fixed in space[1]. Each hydrogen atom is harmonically bound to the Carbon with a spring constant k_0, and they are independent of each other[2]

(a) Derive the partition function of single hydrogen atom. Be explicit and explain your steps.

(b) Determine the mean vibrational energy and the variance in the vibrational energy of the chain of hydrogen atoms. Be explicit about your algebraic steps.

(c) Determine a Taylor series for the energy of part (b) in the high temperature limit, including both the leading term and the first correction. Use your series to answer the following: in the high temperature limit, if hydrogen is replaced by deuterium (which consists of a proton, a neutron, and an electron) what is the approximate difference in vibrational energies of the two system, $\Delta E = E_D - E_H$, at high temperatures.

Problem 2. Entropy change in the mixing of hot and cold gasses



Consider two mono-atomic ideal gasses, Helium and Argon, separated by a divider which partitions a container of volume V into two equal parts. There are N_1 Helium atoms on the left of the divider, and N_2 Argon atoms on the right of the divider. The Helium atoms are initially at a temperature of T_1, while the Argon atoms are initially at a temperature of T_2. After the dividing wall is removed, the two gasses mix and ultimately equilibrate.

(a) Determine the final temperature of the system.

(b) Determine the change in entropy of the system resulting from the mixing process.

[1] Carbon is twelve times heavier than hydrogen justifying this approximation.

[2] Recall that the resonant frequency is $\sqrt{k_0/m}$.

Problem 3. Ideal gas in 1D

Consider consider a classical gas of N atoms of mass M at temperature T in one spatial dimension. Each particle is free to move in the x direction, but is confined to a box of size L.

(a) Determine the partition function and free energy of the system.

(b) Starting from the first law of thermodynamics $dU = dQ + dW$, derive an expression for dF where F is the free energy.

(c) Use your result for the free energy and dF to derive an expression for the entropy of the gas.

Now consider the same 1D gas, consisting of N molecules. Each molecule is of mass M (as before), but now each molecule has internal energy states ϵ_s, so that the energy of one molecule is

$$E_1 = \frac{p^2}{2M} + \epsilon_s \,. \tag{1}$$

The internal energy levels can take on two possible values: the ground state has energy $\epsilon_0 = 0$ and is not degenerate, while the excited energy level has energy $\epsilon_1 = \Delta$ and degeneracy g.

(d) Determine the entropy of the gas of molecules.

(e) Determine the entropy of the gas in the limits where kT is low compared to Δ, and high compared to Δ. How do your limiting expressions compare to part (c)? In both limits, explain the similarities or differences with part (c) physically.