Physics 306: Thermal Physics

Final Exam

Stony Brook University

Fall 2023

General Instructions:

You may use one page (front and back) of handwritten notes and a calculator. Graphing calculators are allowed. **No other materials may be used**.

Problem 1. Distribution de Broglie wavelengths in two dimensions

Consider a two-dimensional ideal classical gas at temperature T consisting of N mono-atomic atoms confined in a box of area A.

(a) (4 points) Write down the probability for finding a particle with speed between v and $v + \mathrm{d}v$.

(b) (4 points answer + 4 points estimate) Determine the root mean square velocity and evaluate it numerically by estimating typical values for the temperature and other parameters of the air molecules in this room.

(c) (6 points answer + 4 points estimate) Determine the most probable speed v_* and the corresponding de Broglie wavelength λ_* for that speed. Evaluate λ_* numerically using same parameters estimated in part (b).

(d) (8 points) Determine the probability for finding a particle with de Broglie wavelength between λ and $\lambda + \mathrm{d}\lambda$. Sketch the probability distribution $P(\lambda)$.

Problem 2. A three state paramagnet

Consider a paramagnet at temperature T consisting of an Avogadro's number of atoms N_A in a constant magnetic field B pointing in the z direction. The atoms in the paramagnet have a magnetic moment μ and can be in one of three spin states: spin up ($\uparrow$), spin down ($\downarrow$), and neutral (0) as shown below.

$$\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\ 0\,0\,0\,0\ \uparrow\ 0\,0\ \downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\ 0\ \uparrow\ 0\ \uparrow\uparrow\uparrow\uparrow\ 0\ \uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\ 0\ \uparrow\ 0\ \uparrow\uparrow\uparrow$$

The energy of these three states is given by

$$E_\uparrow = -B\mu, \quad E_0 = 0, \quad E_\downarrow = B\mu\,, \tag{1}$$

as shown below. *Note:* The spin-down states ($\downarrow$) have higher energy than the spin-up states.

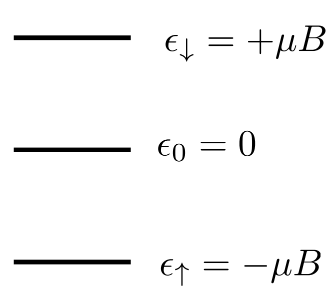


Figure 1: Energy level diagram for the three state paramagnet.

(a) (4 + 6) Determine the partition function and mean energy $\langle U \rangle$ of the system. Express your result using hyperbolic functions as appropriate:

$$\cosh(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right) \qquad \frac{\mathrm{d}\cosh(x)}{\mathrm{d}x} = \sinh(x) \tag{2}$$

$$\sinh(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right) \qquad \frac{\mathrm{d}\sinh(x)}{\mathrm{d}x} = \cosh(x) \tag{3}$$

(b) (6 points) Find the leading (non-zero) term in a Taylor series expansion for $\langle U \rangle$ when the magnetic field is very weak, $\mu B \ll kT$. Use the result to find the specific heat C_V when the magnetic field is weak.

These questions are independent of (a) and (b) :

(c) (8 points) Determine a specific temperature T_* when the number of atoms in the spin-down state is one quarter of those in the spin-up state? At this temperature, what fraction of the atoms are in the up, neutral, and down states, respectively?

(d) (8 points) At the temperature T_*, what is the entropy of the system?

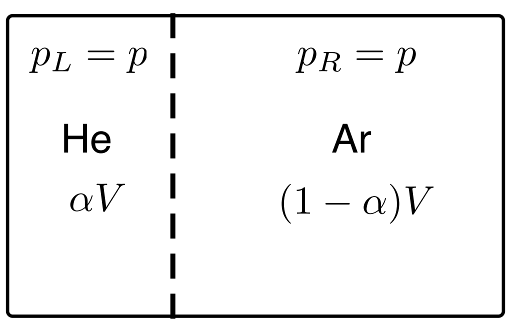
(e) (4 points) If the probability (or fraction) of each atomic state is held fixed to their values in part (c), how does the mean energy depend on the magnetic field? In other words, determine:

$$\left(\frac{\partial U}{\partial B}\right)_{\text{fixed-prob}} \tag{4}$$

This does *not* correspond to a fixed temperature!

Problem 3. Entropy changes

Two ideal mono-atomic gasses He and Ar are separated by a wall and have the same temperature T and pressure $p = p_L = p_R$. The Helium occupies a fraction α of the volume V and the Argon occupies a fraction $(1-\alpha)$. The wall is removed and the gasses intermingle with each other, ultimately reaching equilibrium.



(a) (2 points) Determine the number of Helium and Argon atoms in the container.

(b) (8 points) Determine the change in entropy of the system from the process.

(c) (4 points) How would your result change if the pressures of the two gasses were initially unequal with $p_L = p$ and $p_R = 2p$?