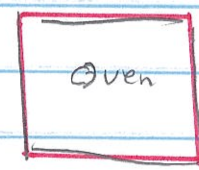
Last Time

$$N = \sum_{\text{modes}} \frac{1}{e^{\beta(\varepsilon-\mu)} \mp 1}$$



$$U = \sum_{\text{modes}} \frac{\varepsilon}{e^{(\varepsilon-\mu)} \mp 1}$$

Number of photons or particles in oven

$-$ for bosons, $+$ for fermions

★ We derived this from

$$\mathcal{Z}_p = \frac{1}{1 - e^{-\beta(\varepsilon_p-\mu)}} \qquad \text{bosons}$$

$$\mathcal{Z}_p = 1 + e^{\beta(\varepsilon_p-\mu)} \qquad \text{fermions}$$

So for one mode

$$\Phi_G = \mp \ln(1 \mp e^{-\beta(\varepsilon-\mu)})$$

And in total

$$\boxed{\Phi_G = \sum_{\text{modes}} \mp \ln(1 \mp e^{-\beta(\varepsilon-\mu)})}$$

Photons

polarization is factor of 2

$$\sum_{\text{modes}} \longrightarrow \int \frac{2V\, d^3p}{(2\pi\hbar)^3}$$

So

$$N = 2V \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{e^{\beta cp} - 1} \Rightarrow \frac{N}{V} = \left(\frac{k_B T}{\hbar c}\right)^3 0.244$$

$$U = 2V \int \frac{d^3p}{(2\pi\hbar)^3} \frac{cp}{e^{\beta cp} - 1} \Rightarrow \frac{U}{V} = \left(\frac{k_B T}{\hbar c}\right)^3 k_B T \frac{\pi^2}{15}$$

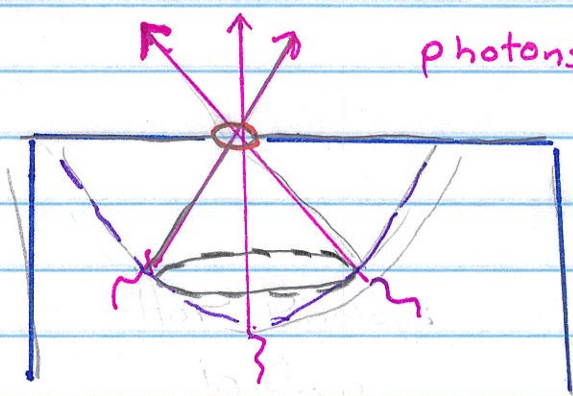
$$\Phi_G = 2V \int \frac{d^3p}{(2\pi\hbar)^3} - \ln(1 - e^{\beta cp})$$

Also interesting to find the energy per frequency interval $d\omega$

$$\boxed{\frac{du}{d\omega} = \frac{1}{\pi^2} \frac{1}{c^3} \frac{\omega^3}{e^{\hbar\omega/kT} - 1}}$$

Flux of Photons

- Now Lets consider a physical question and The number and energy that escapes a hole. It is similar to problems we studied at the beginning of the semester



photons escaping the hole ; coming through the hole at Various angles.

- In a given volume V the density of photons with momentum in $[p_x + dp_x, p_y + dp_y, p_z + dp_z]$ is

$$\boxed{dn_\gamma = 2 \frac{1}{e^{\beta \varepsilon(p)} - 1} \frac{d^3 p}{(2\pi\hbar)^3}} \qquad n_\gamma = \frac{N}{V}$$

So

$$n_\gamma = \int \frac{2}{e^{\beta \varepsilon(p)} - 1} \frac{d^3 p}{(2\pi\hbar)^3}$$

- Now how do I find the number of photons in a given angular range

$$[\theta, \theta + d\theta] \quad \text{and} \quad [\phi, \phi + d\phi]$$

- Then $d\Omega = \sin\theta \, d\theta \, d\phi$

$$d^3p = p^2 \, dp \, \sin\theta \, d\theta \, d\phi$$

$$d^3p = 4\pi p^2 \, dp \, \frac{\sin\theta \, d\theta \, d\phi}{4\pi} \leftarrow \frac{d\Omega}{4\pi}$$

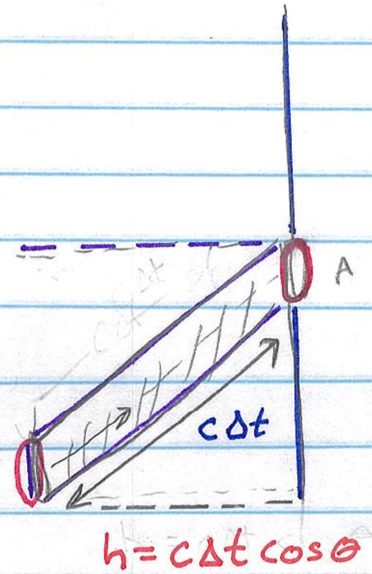
- So summing up all momenta

$$dn_\gamma = \left[\int_0^\infty \frac{2}{e^{\beta \varepsilon(p)} - 1} \frac{4\pi p^2 \, dp}{(2\pi\hbar)^3} \right] \frac{\sin\theta \, d\theta \, d\phi}{4\pi}$$

$$\boxed{dn_\gamma = n_\gamma \frac{d\Omega}{4\pi}}$$

this is the number of photons per volume, "flying" in angular range $[\theta, d\theta]$ and $[\phi; d\phi]$. The distribution is uniform in solid angl

Now take a time interval Δt and angular range $[\theta, d\theta]$ and $[\phi, \phi + d\phi]$. Then look at the picture below. All of the photons in the tube of length $c\Delta t$ will fly through the hole



- length of tube $= c\Delta t$
- height of tube $= h = c\,\Delta t \cos\theta$
- Base area A

- All of the photons in this tube will cross the face

$$dN_\gamma = dn_\gamma \text{ (tube volume)} \qquad \text{Volume} = h\,A$$

number crossing dN_γ

$$dN_\gamma = n_\gamma \frac{d\Omega}{4\pi} (c\Delta t \cos\theta)\, A$$

- Now we need to sum up the dN_γ to find ΔN_γ

$$\Delta N_\gamma = \frac{n_\gamma c \Delta t A}{4\pi} \int_0^{\pi/2} \sin\theta \, d\theta \cos\theta \int_0^{2\pi} d\phi$$

$$\Delta N_\gamma = \frac{1}{4} n_\gamma c \Delta t A$$

So

$$\boxed{\frac{1}{A}\frac{\Delta N}{\Delta t} = \frac{1}{4} n_\gamma c}$$

units number per area per time.

• Similarly

$$du_\gamma = \frac{2\, d^3p}{(2\pi\hbar)^3} \frac{cp}{e^{\beta cp} - 1}$$

energy per volume with momentum in range $[p_x, p_x, p_z]$, $[p_x + dp_x, p_y + dp_y, p_z + dp_z]$

$$d^3p = 4\pi p^2 dp \frac{d\Omega}{4\pi}$$

• So summing over momentum magnitude

$$du_\gamma = \left[\int_0^\infty \frac{2 \cdot 4\pi p^2 dp}{e^{\beta cp} - 1} cp \right] \frac{d\Omega}{4\pi}$$

$$du_\gamma = u_\gamma \frac{d\Omega}{4\pi}$$

energy density in angular range $[\theta, \phi] \leftrightarrow [\theta + d\theta, d\phi]$

• So the amount of energy dU_γ escaping with angles in range $[\theta ; d\theta, \phi ; d\phi]$ is

$$dU_\gamma = du_\gamma \times \text{tube volume}$$

Then as before

$$\boxed{\frac{1}{A} \frac{\Delta U_\gamma}{\Delta t} = \frac{1}{4} u_\gamma c}$$

So since

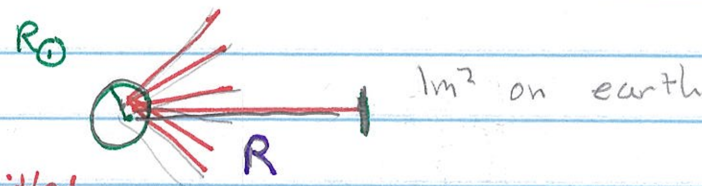
$$u_\gamma = \frac{\pi^2}{15}\left(\frac{kT}{\hbar c}\right)^3 kT$$

we get

$$\boxed{\begin{gathered}\text{Energy flux} = \frac{1}{A}\frac{\Delta u}{\Delta t} = \sigma T^4 \\ \sigma = \left(\frac{k_B}{\hbar c}\right)^3 k_B c \frac{\pi^2}{60} = 5.67\times10^{-8}\ \frac{W}{m^2\ {}^\circ K^4}\end{gathered}}$$

Ex,

- What is the Temperature of the Sun, given that $I = 1kW/m^2$ of energy per area per second comes from sun?



total energy emitted

$$\frac{dU}{dt} = \sigma T^4 \; 4\pi R_\odot^2$$

R ≡ distance from sun to earth

$R_\odot$ ≡ Radius of sun

$$\left.\frac{1}{A}\frac{dU}{dt}\right|_{earth} = \sigma T^4 \frac{4\pi R_\odot^2}{4\pi R^2} = \sigma T^4 \frac{R_\odot^2}{R^2}$$

$\Omega_{sun} = 6.8\times10^{-5}$

the energy per area absorbed on earth.

The dU/dt is spread out over area $4\pi R^2$

$$I = \sigma \frac{T^4}{\pi}\left(\frac{\pi R_\odot^2}{R^2}\right)$$

this is the Solid angle of sun as seen on earth

$$T = \left(\frac{I\pi}{\sigma\,\Omega_{sun}}\right)^{1/4}$$

$\Omega_{sun} = \dfrac{A}{r^2}$

you measure this with a protractor

$$\boxed{T = 5342\,°K}$$

pretty close!