Carnot Efficiency Revisited
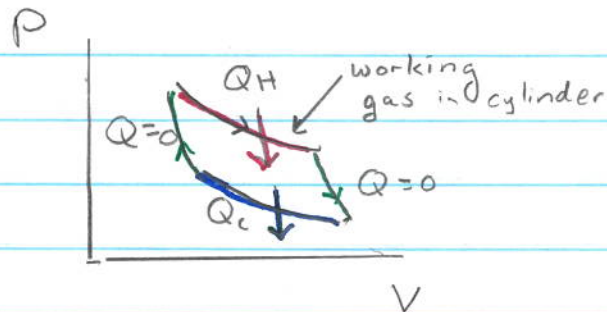
Lets calculate the change in entropy of the hot reservoir, the cold reservoir, and the working substance of the engine, i.e. the gas that is in the cylinder:
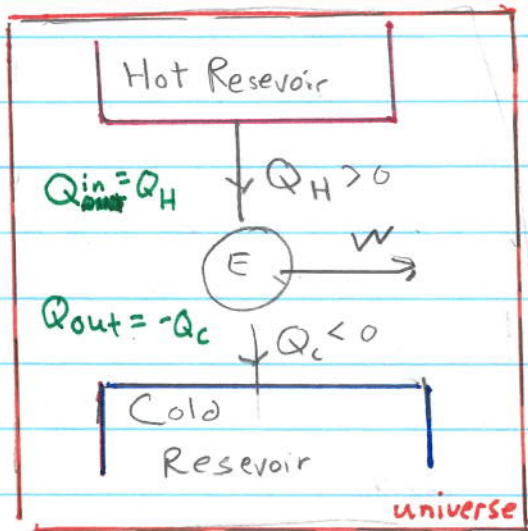
① $\Delta S_{Hot}^{reserve} = -\dfrac{Q_H}{T_H} \equiv -\dfrac{Q_{in}}{T_H}$

② $\Delta S_{Cold}^{res} = -\dfrac{Q_c}{T_c} \equiv \dfrac{Q_{out}}{T_c}$ $Q_c < 0$ $Q_{out} > 0$

③ $\Delta S_{working} = 0$

Substance of engine

This is a closed loop
The gas starts and stops at same place. So $\Delta S = 0$.

$\Delta S_{universe} = -\dfrac{Q_H}{T_A} + \dfrac{-Q_c}{T_c} \geq 0$

rearranging: $\quad -\dfrac{Q_c}{T_c} \geq \dfrac{Q_H}{T_H} \qquad$ i.e. $\boxed{\dfrac{|Q_c|}{T_c} \geq \dfrac{Q_H}{T_H}}$

Then   this implies that the efficiency is less than carnot efficiency:

$$\dfrac{|Q_c|}{Q_H} \geq \dfrac{T_c}{T_H} \qquad \text{or} \qquad \eta = 1 - \dfrac{|Q_c|}{Q_H} \leq 1 - \dfrac{T_c}{T_H}$$

$\underbrace{\qquad}_{\text{efficiency}}$

The condition:

$\dfrac{|Q_c|}{T_c} \geq \dfrac{Q_H}{T_H}$ is saying   equivalently $\boxed{S_{out} \equiv \dfrac{Q_{out}}{T_c} \geq S_{in} \equiv \dfrac{Q_{in}}{T_c}}$
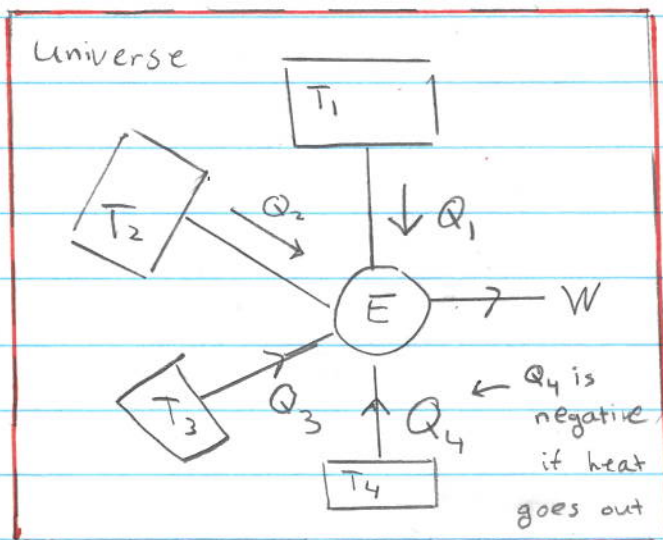
or entropy is produced!
by the engine cycle

Generalizing

(working substance)

- We can have an engine^ with many resevoirs at different temeperatures. At each resevoir the working substance exchanges heat.

Universe

This is reversible if each resevoir is in perfect equilibrium with the working substance

Q_4 is negative if heat goes out of engine

- The generalization is of Carnot to multicarnot is simple

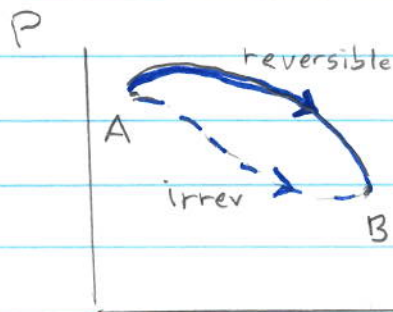$$\Delta S_{universe} = -\sum_i \frac{Q_i}{T_i} \geq 0 \quad \text{or} \quad \text{ultimately} \quad \boxed{-\oint \frac{dQ}{T} \geq 0}$$

Or dividing into in $(dQ>0)$ & out $(dQ<0)$

$$-\int_{in} \frac{dQ}{T} + \int_{out} \frac{|dQ|}{T} \geq 0$$

and thus, **engine produces entropy** $\longrightarrow$ $\boxed{S_{out} = \int_{out} \frac{|dQ|}{T} > \int_{in} \frac{dQ_{in}}{T} \equiv S_{in}}$

- For any process which is irreversible taking the system from A to B then we have

reversible

A

irrev

B

$$-\int_A^B \frac{dQ_{irrev}}{T} + \int_A^B \frac{dQ_{rev}}{T} \geq 0$$

↑ non equilibrium transition from A to B of working substance

equilibrium transition B to A

$$\int_B^A \frac{dQ_{rev}}{T} = -\int_A^B \frac{dQ_{rev}}{T}$$

First we go from A to B via a non-equilibrium process (irrev), and then back via an equilibrium process B to A (reversible)

Leading to

$$\int_A^B \frac{dQ_{rev}}{T} \geq \int_A^B \frac{dQ_{irrev}}{T}$$

in equilibrium we have $dS = dQ_{rev}/T$

or

$$\Delta S_{A \to B} \geq \int_A^B \frac{dQ_{irrev}}{T}$$

Thus we see that the change in entropy dS, is in general greater than dQ/T.

- If equilibrium between the reservoirs and working substance is maintained we have

$$\Delta S_{A \to B} = S(B) - S(A) = \int_A^B \frac{dQ}{T}$$