Financial Time Series Final Project

Prediction of foreign exchange rate by GARCH(1,1) model – an empirical examination of trading strategy associated with UIP

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1. Motivation

As GARCH model is playing a very crucial role in the financial market today, we would like to apply it on a specific market and test the reliability of it. According to our literature review, foreign exchange market is the largest and most liquid financial market with daily turnover $5.0 billion in 2013. Therefore we decided to apply GARCH to 3 currencies (JPY, AUD, GBP) and then combining with the potential arbitrage opportunity after testing Uncovered Interest Rate Parity (UIP), we would like to examine the performance of such trading strategy by using GARCH to model volatility and predict exchange rate.

2. Background

2.1 GARCH

2.1.1 Conditional heteroscedastic models

By using a linear regression model,

$$y_t = \alpha + \beta x_t + e_t$$

We often see some characteristics of volatility in financial instruments: (1) there exists volatility clusters (i.e. volatility may be high for some certain time periods and low for other periods; (2) returns display fat tail behavior; (3) returns have little or weak autocorrelation and are not independent.

This can be explained by the terminology: *heteroscedasticity*, the presence of which will invalidate statistic tests of significance that assume that the error terms e_t are uncorrelated and normally distributed and that their variances do not vary with the effects being modeled.

2.1.2 ARCH, GARCH and IGARCH

Hence, some volatility models came up. According to Engle (1982), the first model provided a systematic framework, which is ARCH model (Autoregressive conditional heteroscedasticity). The basic idea of ARCH models is that, (1) the shock a_t of an asset return is serially uncorrelated but dependent; (2) the dependence of a_t can be described by a simple quadratic function of its lagged values.

$ARCH(m)$:

$$a_t = \sigma_t \epsilon_t, \quad \sigma^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2 \,,$$

where $\{\epsilon_t\}$ is a sequence of independent and identically distributed random variables with mean 0 and variance 1, $\alpha_0 > 0$, and $\alpha_i \geq 0$ for $i > 0$.

However, although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of time series data. So, Bollerslev (1986) proposed a useful extension model, which is the generalized ARCH (GARCH) model.

$GARCH(m, s)$:

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i \alpha_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2,$$

where again $\{\epsilon_t\}$ is a sequence of i.i.d. random variables with mean 0 and variance 1.0, $\alpha_0 > 0, \alpha_i \geq 0$, and $\sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_i) < 1$.

And GARCH(1,1) is most commonly used.

$GARCH(1,1)$:

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2,$$

where $0 < \alpha_1, \beta_1 < 1, (\alpha_1 + \beta_1) < 1$.

$IGARCH(1,1)$:

The GARCH(1,1) can be written as an ARMA(1,1) for a_t^2:

$$a_t^2 = \alpha_0 + (\alpha_1 + \beta_1) a_{t-1}^2 + \eta_t - \beta_1 \eta_{t-1}, \quad \eta_t = a^2 - \sigma^2$$

If the AR polynomial has a unit root, then we have the IGARCH(1,1). In this case, the impact of past squared shocks a_{t-1}^2 is persistent.

In practice, if we observe that $\alpha_1 + \beta_1$ almost equals to 1, we can apply IGARCH(1,1) to fit the data. And IGARCH(1,1) can be defined as

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) a_{t-1}^2$$

2.2 Uncovered Interest rate Parity (UIP)

Suppose that you have 1 $ to invest in the foreign exchange market for 1 month and that you can choose whether to invest it in the dollar denominated market or in the pound denominated security. How do you choose to invest?

We will use the following notations:
r_t---interest rate in the dollar denominated market at time t with maturity time t+1;
r_t^*---interest rate in the pound denominated market at time t with maturity time t+1;
S_t---spot exchange rate between a foreign currency and the U.S. dollar at time t, quoted as dollar price per foreign currency unit, e.g., USD/GBP;
$s_t = \ln(S_t)$;
F_t---1-month forward exchange rate at time t with maturity time t+1 between a foreign currency

and the U.S. dollar, quoted as dollar price per foreign currency unit, e.g. USD/GBP;
$f_t = \ln(F_t)$;

If we invest 1$ in U.S. market for 1 month, we will get $\$1 * (1 + r_t)$, which is for sure.
If we exchange 1$ to pound, invest in the pound denominated market for 1 month and sell 1-month forward contract to have a complete hedge, we will get $\$1/S_t * (1 + r_t^*) * F_t$, which is certain.
If we exchange 1$ to pound and then invest in the pound denominated market for 1 month and then get back dollar income $\$1/S_t * (1 + r_t^*) * S_{t+1}$ after one year, in which S_{t+1} (the spot exchange rate in 1 year) is uncertain.

So under fixed income condition:
If $\$1 * (1 + r_t) > \$1/S_t * (1 + r_t^*) * F_t$, we had better invest in the dollar denominated market;
If $\$1 * (1 + r_t) < \$1/S_t * (1 + r_t^*) * F_t$, we had better invest in the pound denominated market.

2.2.1 Borrowing and investing for an arbitrage profit

Now assume that we do not have money at pocket, but we are allowed to long and short currencies.
If $\$1 * (1 + r_t) > \$1/S_t * (1 + r_t^*) * F_t$, we can borrow in UK and use the proceeds to invest in US to get profit $= \$1 * (1 + r_t) - \$1/S_t * (1 + r_t^*) * F_t > 0$, and vice versa.

Arbitrage will continue until $\$1 * (1 + r_t) = \$1/S_t * (1 + r_t^*) * F_t$.
Taking log on both sides, we will get interest differential=forward premium, i.e.
$r_t - r_t^* = \ln(F_t) - \ln(S_t) = f_t - s_t$, which is also called Covered Interest Parity(CIP).

2.2.2 Uncovered Interest Parity (UIP)

The main assumption of this theorem is that investors like us are risk-neutral ,which means the forward exchange rate will be equal to the expected spot exchange rate of next period. $i.e. F_t = E_t(S_t)$ from the view of risk neutral investors.

Investing in US, we get sure return $\$1 * (1 + r_t)$.
Investing in UK, we will get expected return $\$1/S_t * (1 + r_t^*) * E_t(S_{t+1}) = \$1/S_t * (1 + r_t^*) * F_t$.
Thus, in order to avoid arbitrage opportunity, we must have

$$\$1 * (1 + r_t) = 1/S_t * (1 + r_t^*) * E_t(S_{t+1})$$

Using log approximation

$$r_t - r_t^* = E_t(\ln(S_{t+1})) - \ln(S_t) = E_t(s_{t+1}) - s_t = f_t - s_t$$

This equation is called Uncovered Interest Parity (UIP).

2.2.3 Arbitrage Opportunity with the violation of UIP

However, sometimes we will find that uncovered interest rate parity condition does not hold, i.e. $r_t - r_t^* = f_t - s_t \neq E_t(s_{t+1}) - s_{t.}$ There will exist arbitrage opportunities.

If $E_t(s_{t+1}) - s_t > f_t - s_t$, i.e. the forecasted exchange rate change next period is greater than the current forward premium. Then investing in the foreign currency will yield a higher return then investing in the dollar. In this case, we need to short dollar and long the dollar worth of the foreign currency. And the realized excess return is as follows: $er_{t+1} = s_{t+1} - s_t - (f_t - s_t) = s_{t+1} - f_t$.

In contrast, if $E_t(s_{t+1}) - s_t < f_t - s_t$, we should short the foreign currency and long the U.S. dollar. The realized excess return is as follows: $er_{t+1} = (f_t - s_t) - s_{t+1} - s_t = f_t - s_{t+1}$.

3. Data

In this project, we will use exchange rate to implement some arbitrage trading strategies based on deviations from the uncovered interest rate parity. The file"proj15_spot_forward_exchange_rate.csv" contains monthly data of spot and 1-month forward exchange rates for the following 3 currencies against the U.S. dollar: Australian dollar, Japanese yen and U.K. pound sterling and for the period from 1985.01 to 2014.07. The reasons why we choose these three currencies are that they are the major currencies in the world, so that the trading volumes of them rank the top three among all the currencies and they have more liquidity than other currencies.

4. Methodology

4.1 Ordinary Least Squares(OLS)

To test whether UIP holds or not, we can run the following regression:

$s_{t+1} - s_t = \alpha_t + \beta_t(f_t - s_t) + \varepsilon_{t+1}$. If the UIP condition holds, one should get α=0 and β=1, and there will be no arbitrage opportunity. However, our research is based on the situation of violation of UIP. So we do the followings:

Here we use OLS method to estimate the above parameters, assuming all the OLS assumptions are satisfied, which make OLS estimator BLUE. After we find that the UIP condition does not hold, we use these estimated parameters to predict future exchange rate change and compare it with the current forward premium. Based on the results, we will carry out our trading strategy: if the future exchange rate change is higher than the current forward premium, we will take a long position in foreign currency and short position in USD (here our exchange rate is quoted in dollar price per unit foreign currency). We will reverse our position for the opposite case. Finally we get the realized excess returns.

4.2 GARCH(1,1) Model

Since OLS requires rigorous assumptions, it may not be a good way to predict log return of exchange rate. Motivated by filtered historical simulation, we try to use GARCH(1,1) to predict log return of exchange rate. Also, based on EACF analysis, constant is chosen to be the mean equation.

$$
\begin{aligned}
r_t &= c + a_t \\
a_t &= \sigma_t \epsilon_t \\
\sigma_t^2 &= \omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2
\end{aligned}
$$

Fit data to the historical log returns of exchange and extract historical standardized residuals:

$$
\epsilon_t = \frac{a_t}{\sigma_t}
$$

In order to predict log returns in the future, we need something stable in the history. In the model, standardized residuals ϵ_t are assumed to be independently and identically distributed (IID). Therefore, past standardized residuals can be used as future standardized residuals. Since standardized residuals are stationary, we think the best choice for the next period's standardized residual is the last standardized residual. After using maximum likelihood estimation (MLE) to estimate parameters, next period's log return can be forecasted by the following formula:

$$\sigma_{T+1}^2 = \omega + \alpha a_T^2 + \beta \sigma_T^2$$
$$a_{T+1}^* = \sigma_{T+1}\epsilon_T$$
$$r_{T+1}^* = c + a_{T+1}^*$$

This forecasted log return of exchange rate can be used to check uncovered interest rate parity (UIP) condition. If there is deviation from UIP condition, we exploit the same arbitrage trading strategy mentioned above, and finally get the monthly realized excess return.

4.3 Out-of-Sample Test

For OLS method and GARCH(1,1) model, we use the data from 1985-01 to 2013-07 to estimate the models and data from 2013-08 to 2014-07 to do out-of-sample test. After obtaining the first realized return, we will re-estimate the model using data from 1985-01 to 2013-08 and do the arbitrage trading strategy. Hence, cumulative window is used to estimate parameters. Finally, we can get 12 realized excess return for the out-of-sample period (2013-08 to 2014-07).

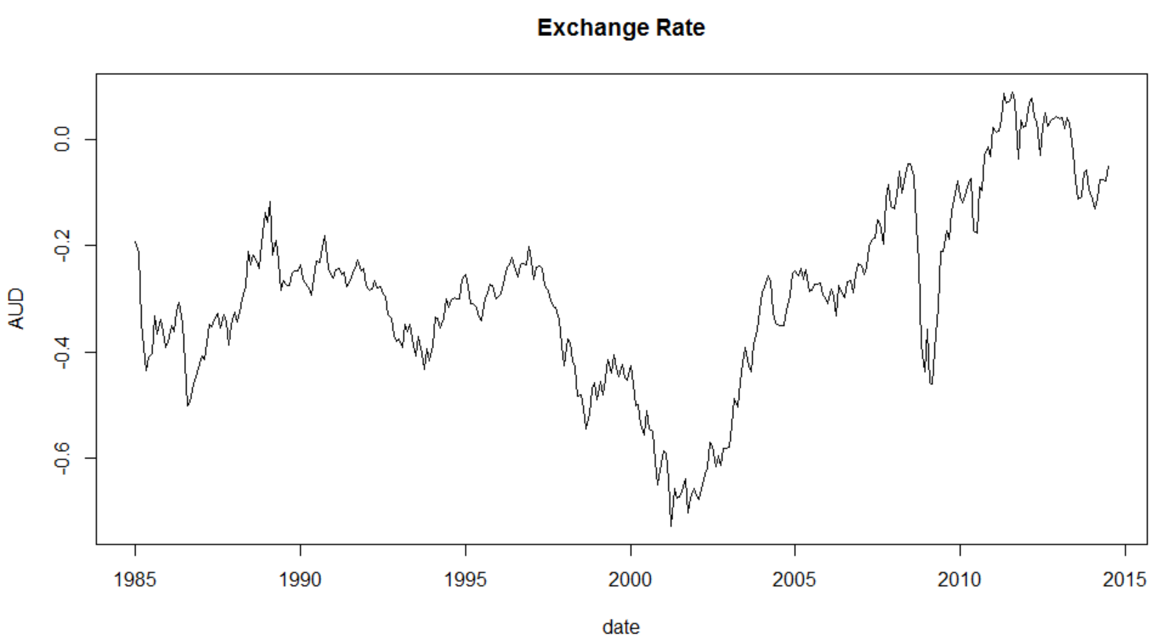
4.4 Assumptions

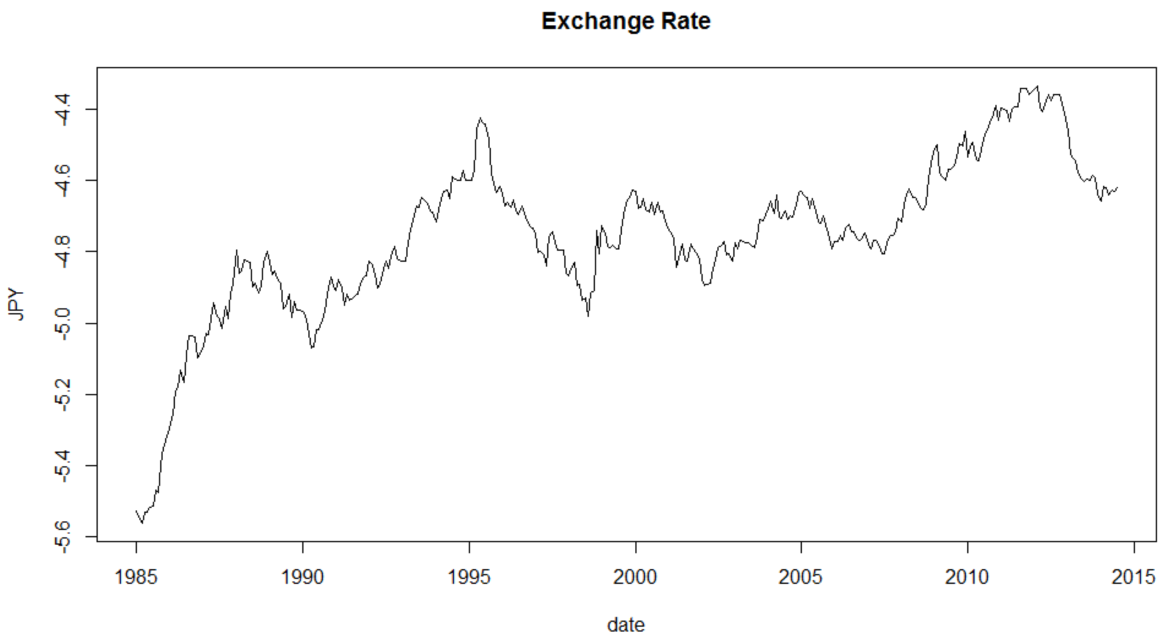
- Trading frequency is monthly.
- No transaction cost, including interest cost, borrowing cost, etc.
- Trading on margin is allowed.
- No constraints on short sale.

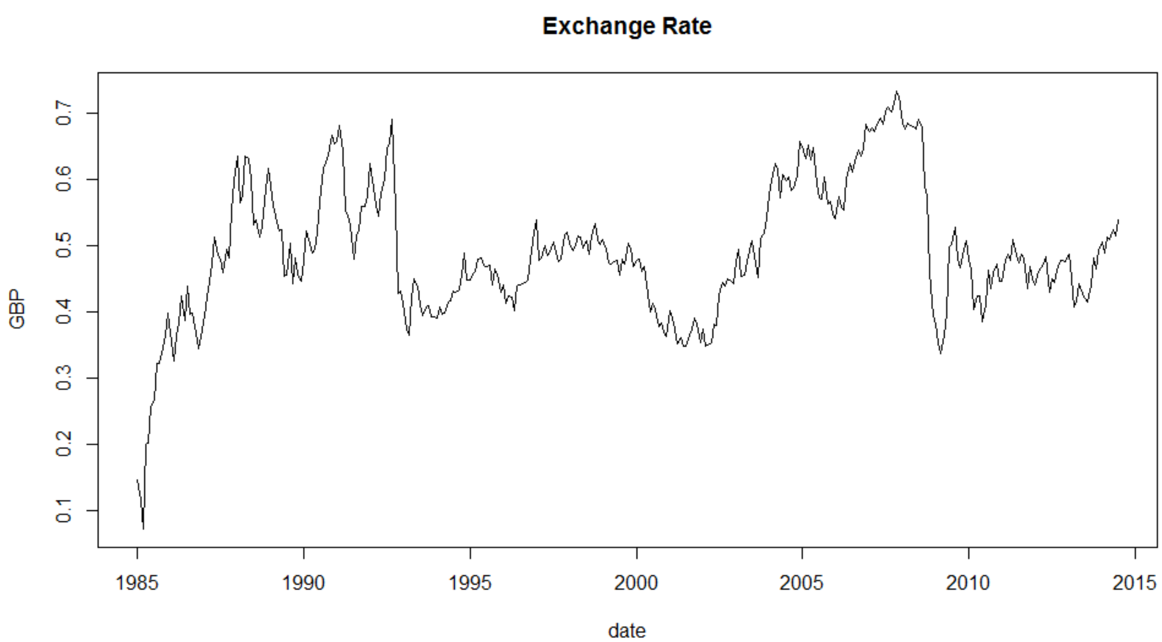
5. Results

5.1 Feel the data

- We plot data in different ways to get a general idea.

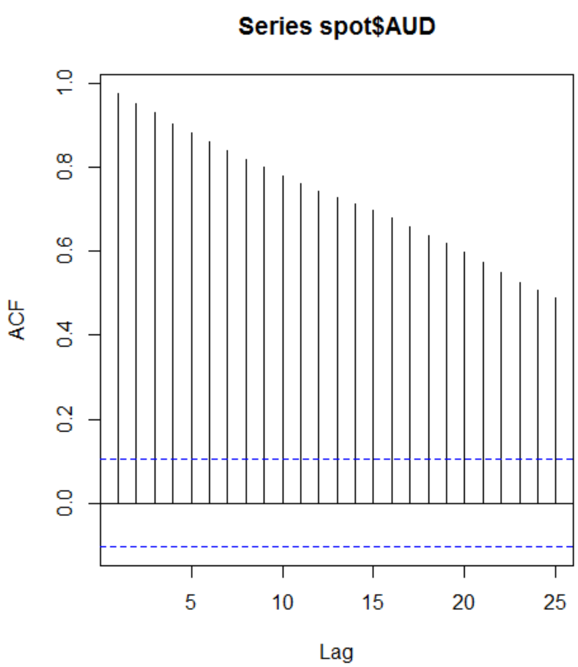


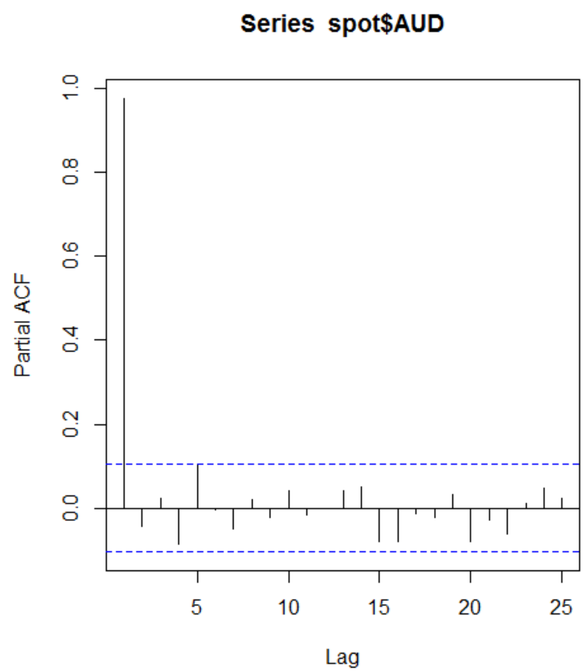

Exchange Rate
JPY
-4.4
-4.6
-4.8
-5.0
-5.2
-5.4
-5.6
1985
1990
1995
2000
2005
2010
2015
date

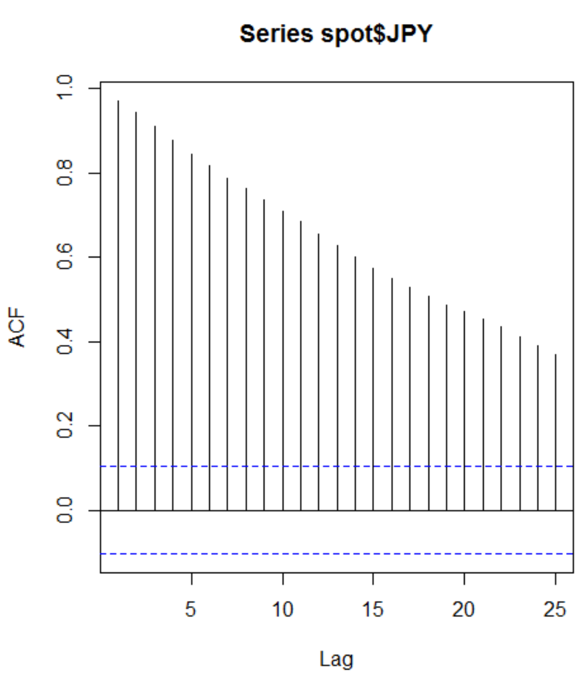

Exchange Rate
GBP
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1985
1990
1995
2000
2005
2010
2015
date

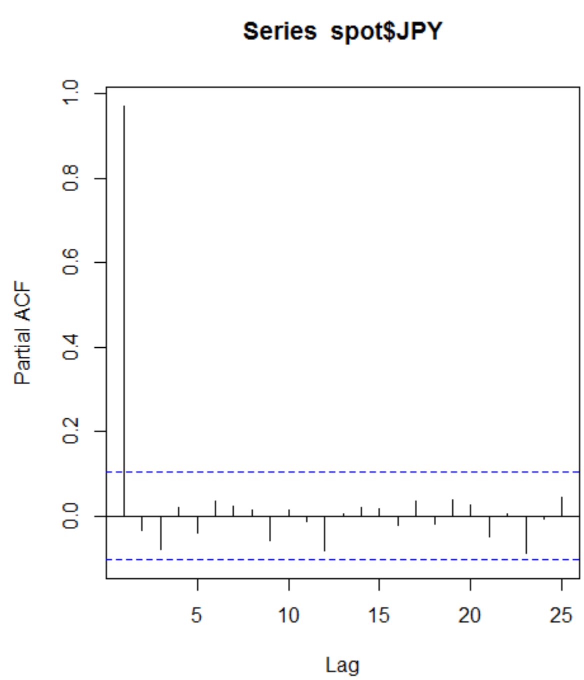
From the above plots, in y-axis, we have log of exchange rate quoted in dollar price per unit foreign currency and we can see that JPY and GBP has very similar shape while we cannot visualize any correlation between AUD and them.

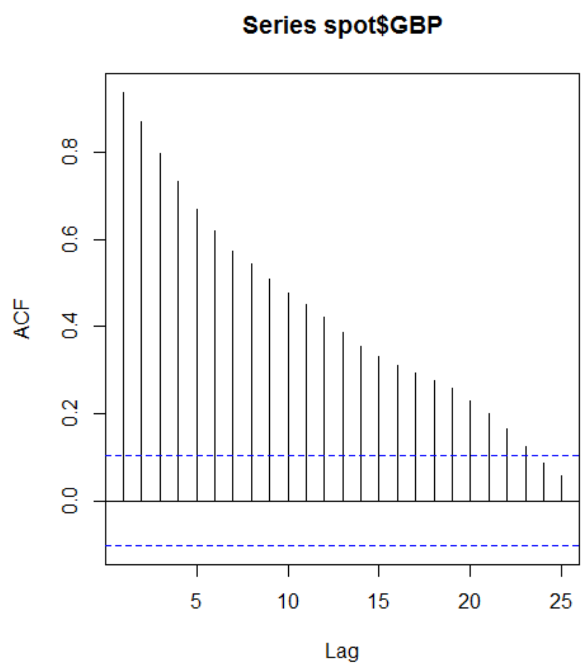
- We also plot ACF and PACF to visualize the degree of serial correlation.

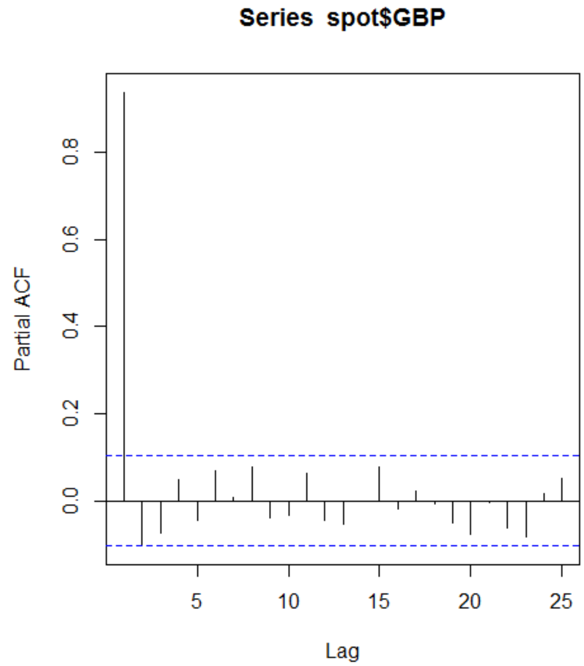












All ACF and PACF tell us the similar stories: we have AR(1) process and possibly unit root since ACF dies out slowly.

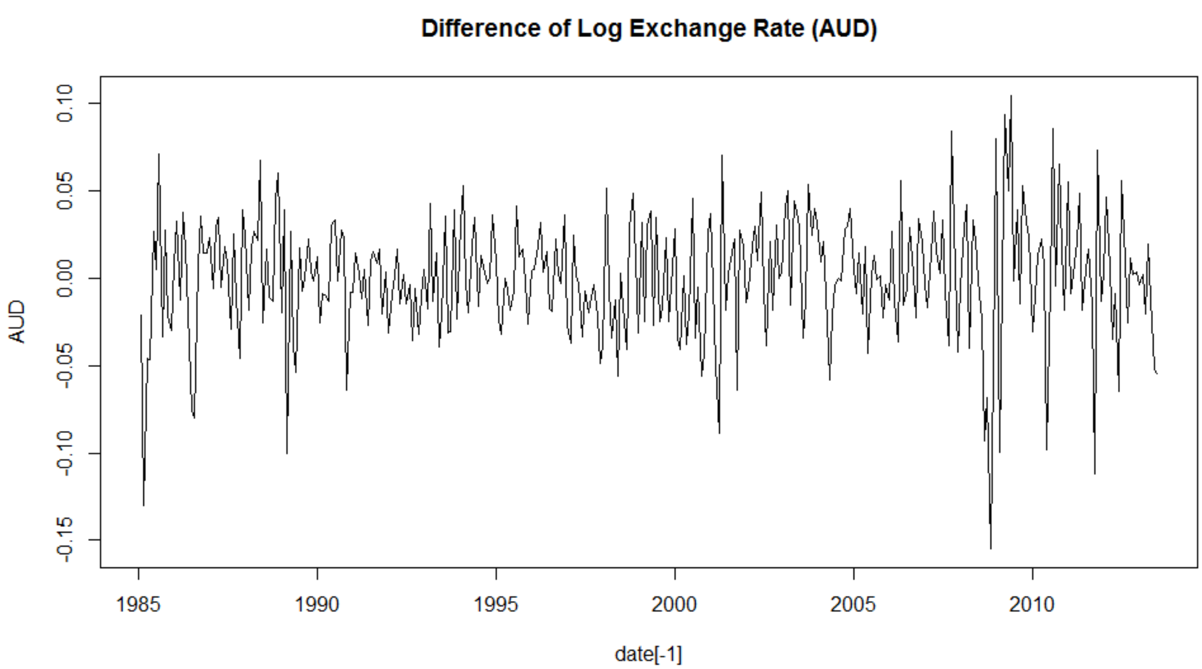
5.2 Unit Root test

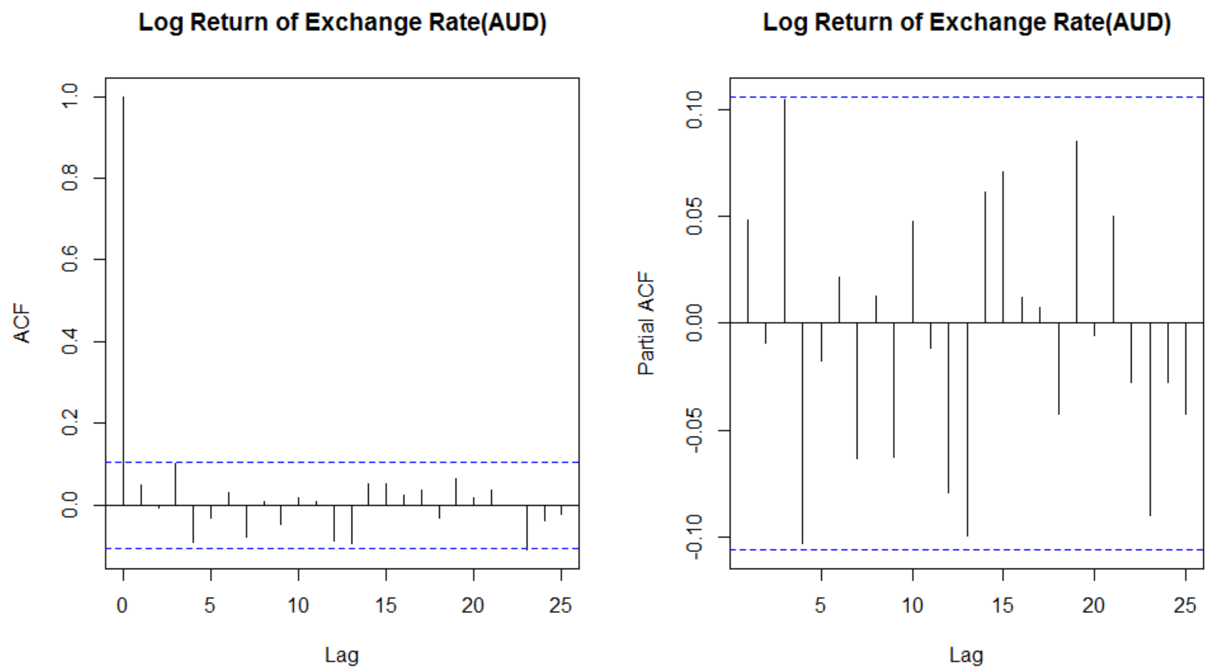
On top of everything, we need to make sure the time series we are interested are stationary, otherwise any analysis would be meaningless. Therefore, we first need to test unit root by ADF test.

```
> library(fUnitRoots)
> adfTest(spot$AUD,type=c("nc"));adfTest(spot$JPY,type=c("nc"));adfTest(spot
$GBP,type=c("nc"))

Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 1
  STATISTIC:
    Dickey-Fuller: -1.1033
  P VALUE:
    0.2647

Title:
 Augmented Dickey-Fuller Test

Test Results:
```

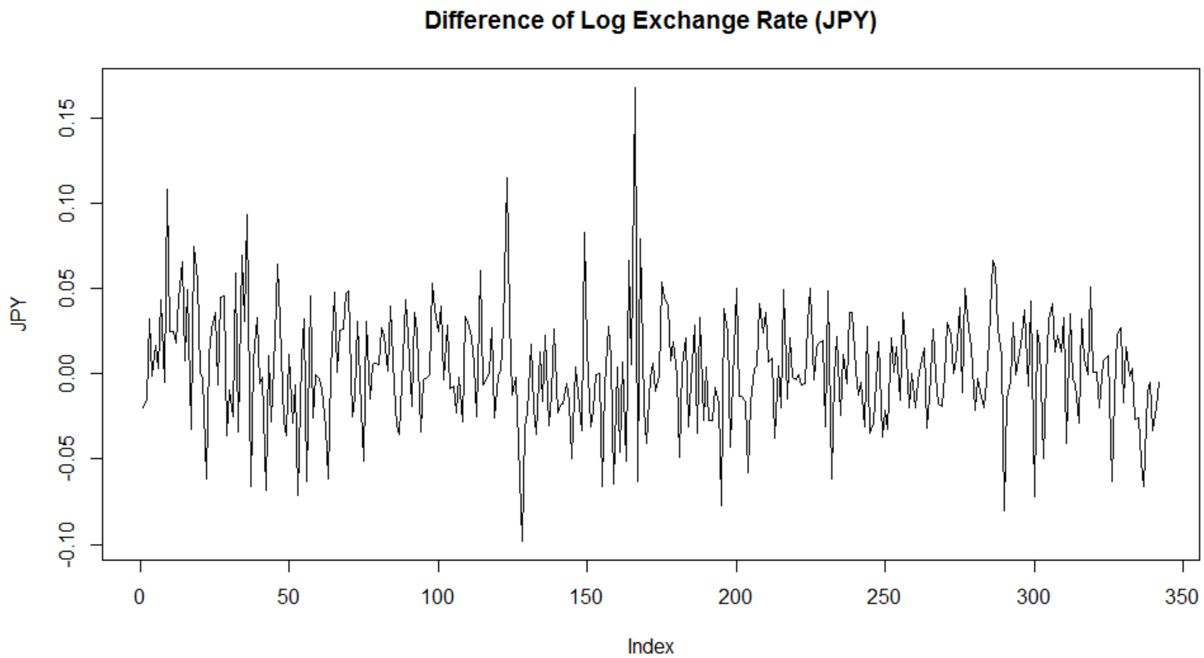
```
  PARAMETER:
    Lag Order: 1
  STATISTIC:
    Dickey-Fuller: -1.6369
  P VALUE:
    0.09744

Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 1
  STATISTIC:
    Dickey-Fuller: -0.1438
  P VALUE:
    0.5705
```

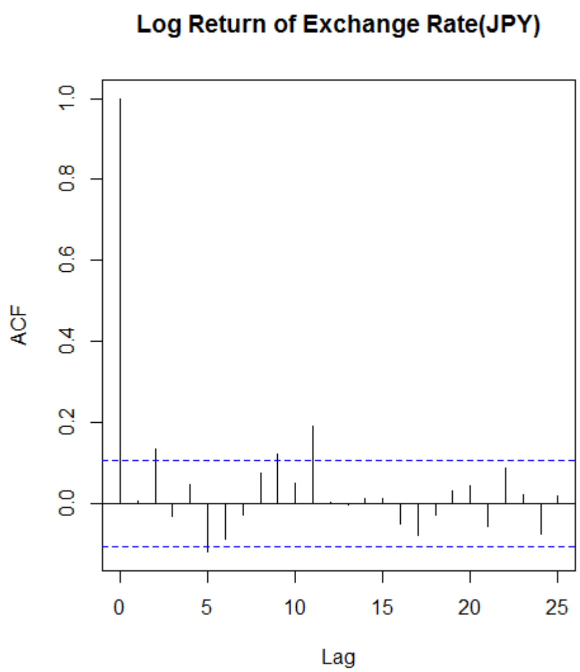
We find that all of the three currency pairs have unit root. Then we take difference and plot ACF and PACF and run ADF test again. From both ACF and PACF we observe after log difference they are stable and the processes no longer have unit root. ADF tests also show they no longer have unit root, because p-values are very small.

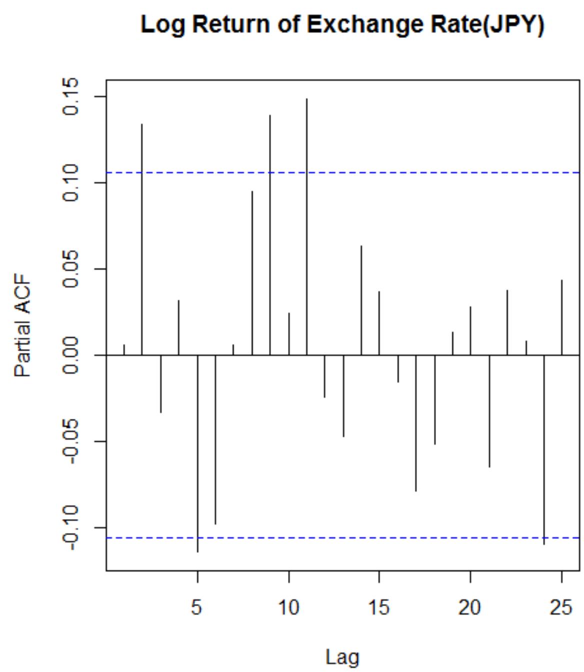





```
> adfTest(AUD_diff)

Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 1
  STATISTIC:
    Dickey-Fuller: -13.0374
  P VALUE:
    0.01

Warning message:
In adfTest(AUD_diff) : p-value smaller than printed p-value
```

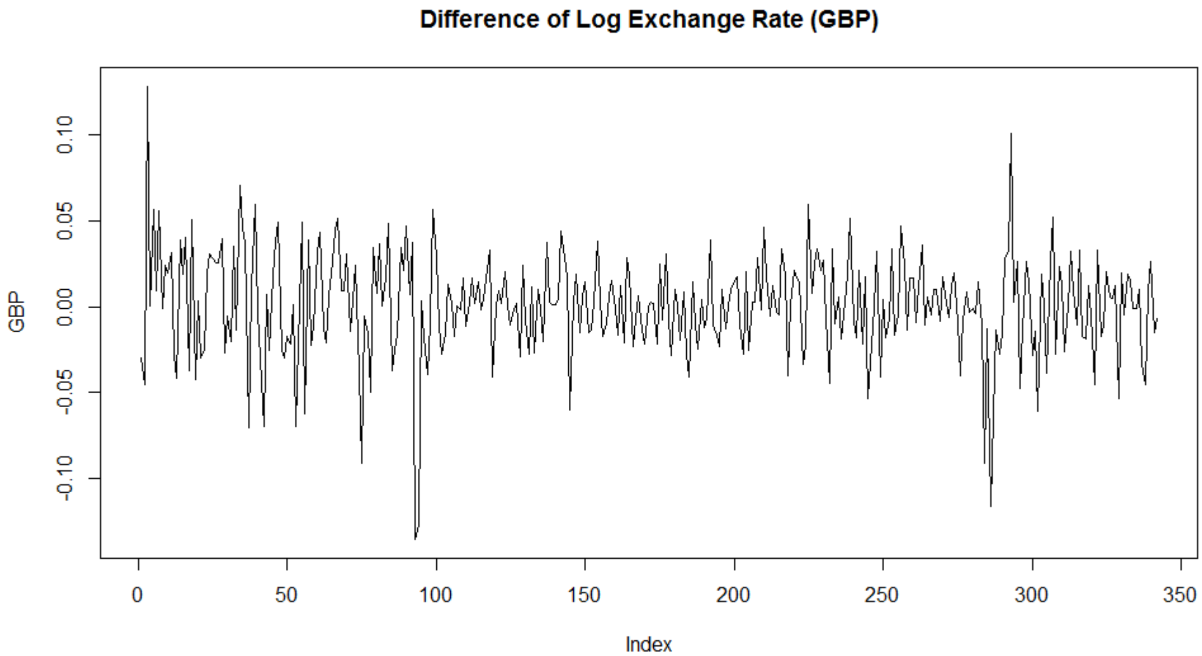


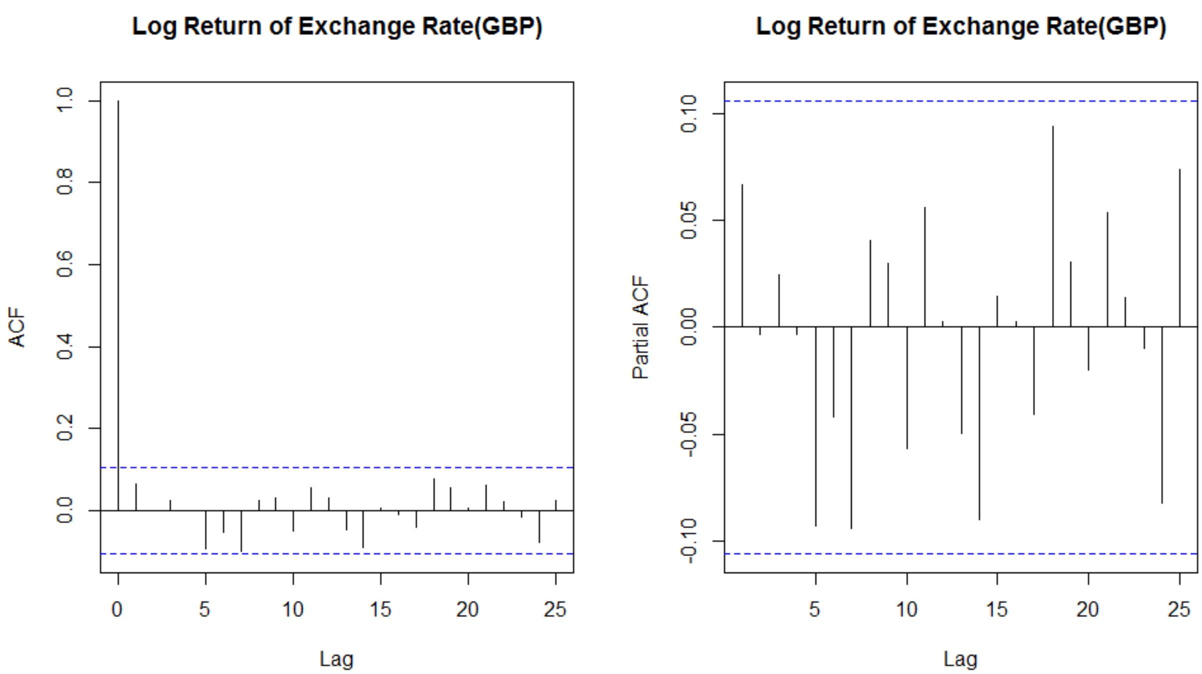





```
> adfTest(JPY_diff);adfTest(GBP_diff)

Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 1
```

```
  STATISTIC:
    Dickey-Fuller: -11.2363
  P VALUE:
    0.01

Warning message:
In adfTest(JPY_diff) : p-value smaller than printed p-value
```





```
> adfTest(GBP_diff)
Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 1
  STATISTIC:
    Dickey-Fuller: -12.6817
  P VALUE:
    0.01

Warning message:
In adfTest(GBP_diff) : p-value smaller than printed p-value
```

5.3 Identify the ARMA process

After obtaining the stationary processes, we would like to identify the order of ARMA(p,q) for these time series data. From our experience of homework, we learnt that it is very rare for ACF and PACF to give a direct solution for the choice of p and q. The alternative is to compute AIC or BIC values, but we try to use Extended Autocorrelation Function (EACF) to identify the order of ARMA(p,q). In the tables of EACF, we can see that, after taking difference, all of the three currency pairs are ARMA(0,0) process. The results are consistent with the ACF and PACF of log return of exchange rates of AUD and GBP, because, in ACF and PACF, they do not show any serial correlations.

```
> # Identify a good ARMA order using EACF
> eacf(AUD_diff, ar.max = 8, ma.max = 8)
AR/MA
  0 1 2 3 4 5 6 7 8
0 o o o o o o o o o
1 x o o x o o o o o
2 o x o o o o o o o
3 x x x o o o o o o
4 x x x x o o o o o
5 x o x o o o o o o
6 x x x o o x o o o
7 x x x o x o o o o
8 x x x o x o o o o
> eacf(JPY_diff, ar.max = 8, ma.max = 8)
AR/MA
  0 1 2 3 4 5 6 7 8
0 o x o o x o o o x
1 o x o o o o o o o
2 x x o o o o o o o
3 x o x o o o o o o
4 x x x x o o o o o
5 x x o x x o o o o
6 o x o x o o o o o
7 o x x o o x x o o
8 x x o o o x x x o
> eacf(GBP_diff, ar.max = 8, ma.max = 8)
AR/MA
  0 1 2 3 4 5 6 7 8
0 o o o o o o o o o
1 o o o o o o o o o
2 x x o o o o x o o
3 x x o o o o x o o
4 o x o x o o o o o
5 x x x x x o o o o
6 x x x x x x o o o
7 x x x x o o x o o
```

```
8 x x o x x o x x o
```

5.4 GARCH and IGARCH

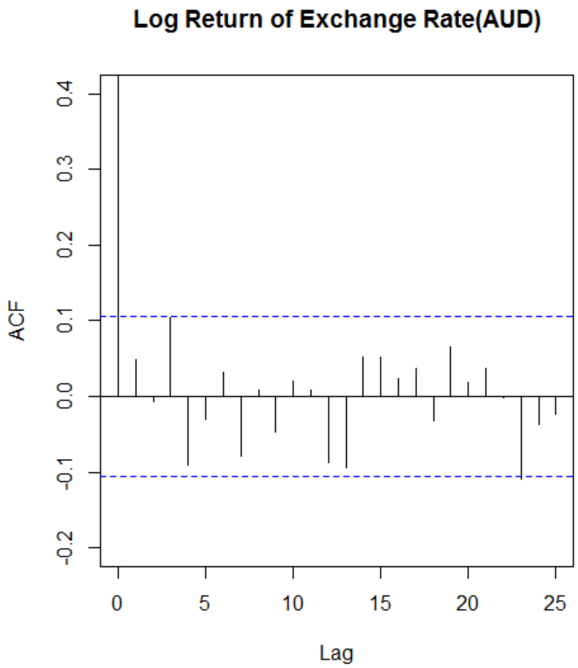
5.4.1 ARCH effect analysis

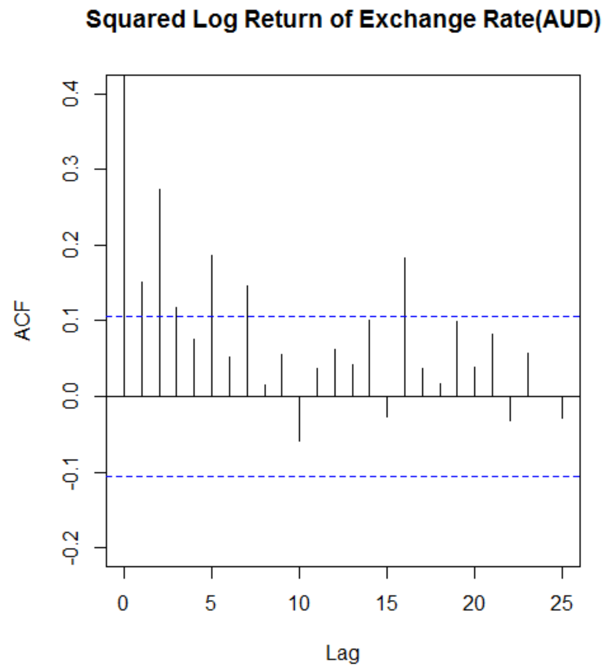
Because the mean of our time series data is not significantly different from zero, we can just use the original time series data to test ARCH effect.

```
> t.test(AUD_diff)

	One Sample t-test

data:  AUD_diff
t = 0.1758, df = 341, p-value = 0.8605
alternative hypothesis: true mean is not equal to 0

> t.test(JPY_diff)

	One Sample t-test

data:  JPY_diff
t = 1.4894, df = 341, p-value = 0.1373
alternative hypothesis: true mean is not equal to 0

> t.test(GBP_diff)

	One Sample t-test

data:  GBP_diff
t = 0.4839, df = 341, p-value = 0.6288
alternative hypothesis: true mean is not equal to 0
```

- Plot ACFs

From the plots of ACFs of log return of exchange rate and squared log return of exchange rate, we can see that AUD and GBP show no autocorrelation for log return of exchange rate, but squared log returns of exchange rate have serial correlations. This can explain they have ARCH effect. However, JPY does not show ARCH effect.





- Ljung-Box tests

The results of Ljung-Box tests are consistent with the analysis of ACFs. AUD and GBP show ARCH effect, but JPY does not.

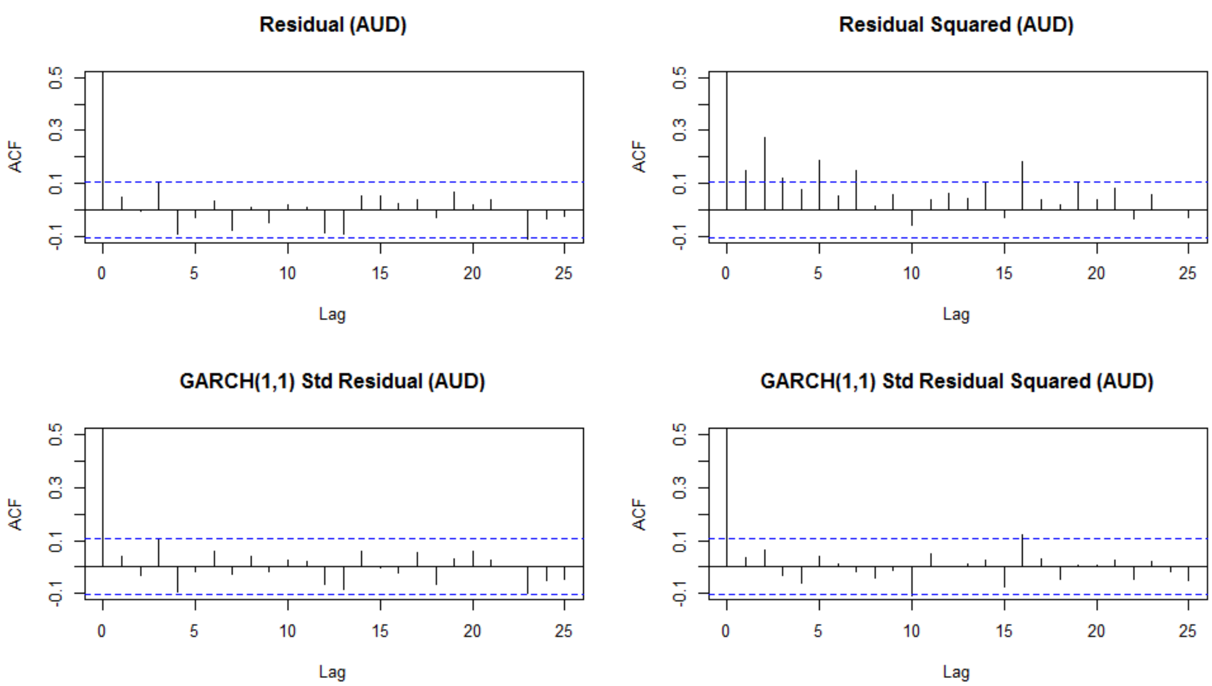
```
> Box.test(AUD_diff,lag=12,type=("Ljung-Box"))

        Box-Ljung test
data:  AUD_diff
X-squared = 13.9234, df = 12, p-value = 0.3056

> Box.test(AUD_diff^2,lag=12,type=("Ljung-Box"))

        Box-Ljung test

data:  AUD_diff^2
X-squared = 65.2146, df = 12, p-value = 2.488e-09

> Box.test(JPY_diff,lag=12,type=("Ljung-Box"))

        Box-Ljung test

data:  JPY_diff
X-squared = 36.4177, df = 12, p-value = 0.0002775

> Box.test(JPY_diff^2,lag=12,type=("Ljung-Box"))

        Box-Ljung test

data:  JPY_diff^2
X-squared = 15.0982, df = 12, p-value = 0.2361

> Box.test(GBP_diff,lag=12,type=("Ljung-Box"))

        Box-Ljung test
```

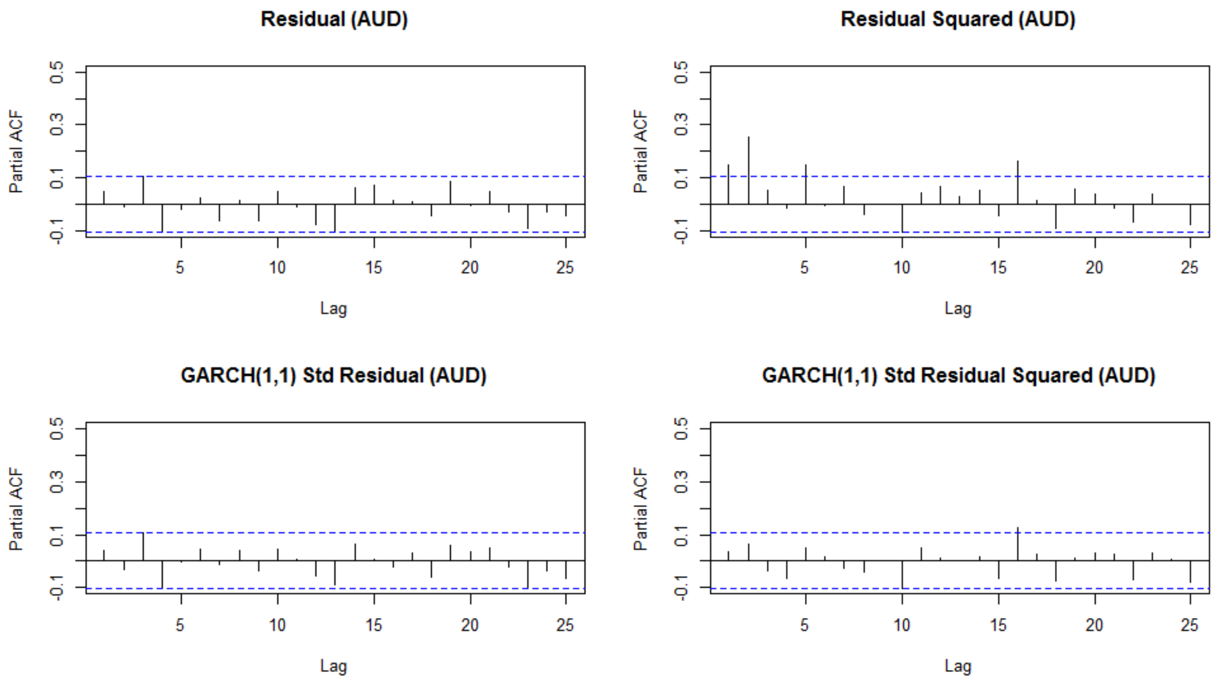
```
data:  GBP_diff
X-squared = 11.9718, df = 12, p-value = 0.4479

> Box.test(GBP_diff^2,lag=12,type=("Ljung-Box"))

        Box-Ljung test

data:  GBP_diff^2
X-squared = 26.1233, df = 12, p-value = 0.01031
```

5.4.1 Volatility Model GARCH(1,1)

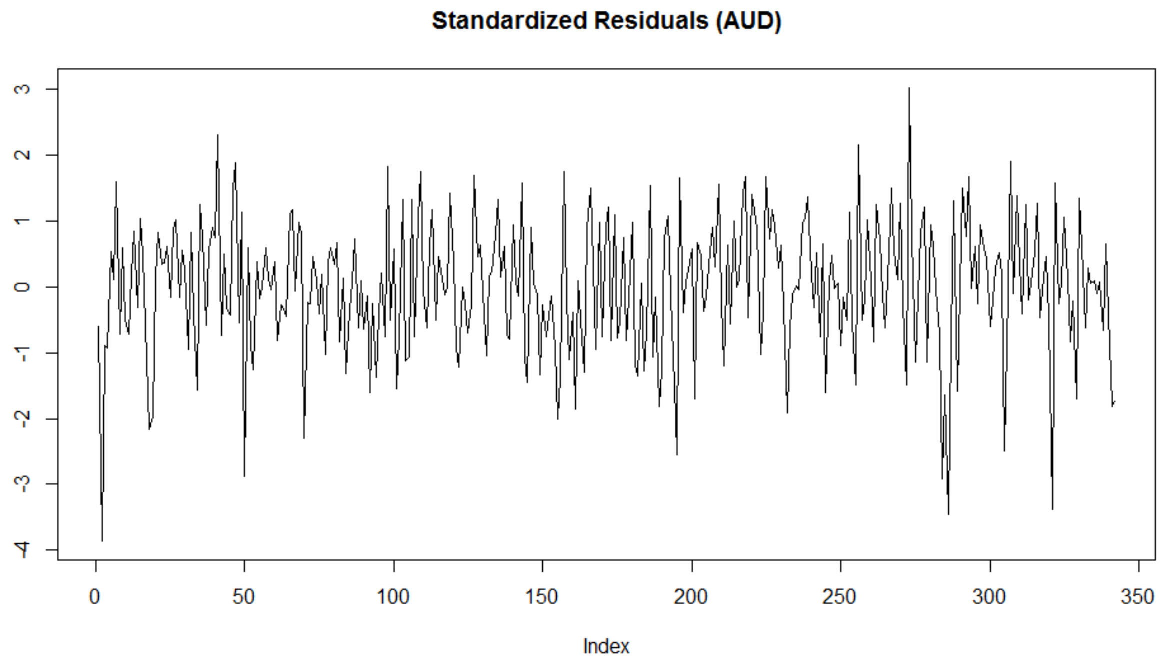
After testing the ARCH effect, we can specify a volatility model if ARCH effects are statistically significant. Although ARCH effect is not significant for JPY, we still apply a volatility model to it, in order for comparing with the other two currency pairs.

- USD/AUD currency pair
  After applying GARCH(1,1),we can see improvement in ACF and PACF. Based on Ljung-Box tests, there are no serial correlations in standardized residuals and squared standardized residuals. Also, α_1 and β_1 are significant. Hence, GARCH(1,1) model is suitable for fitting the log return of exchange of USD/AUD.

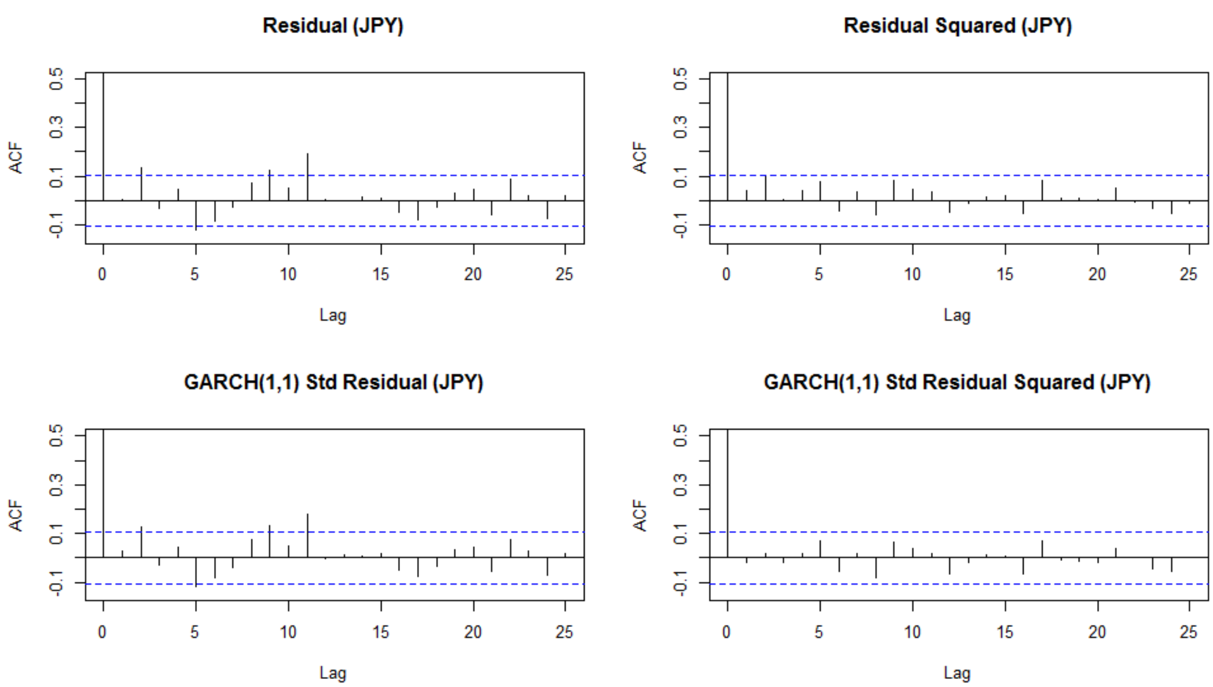


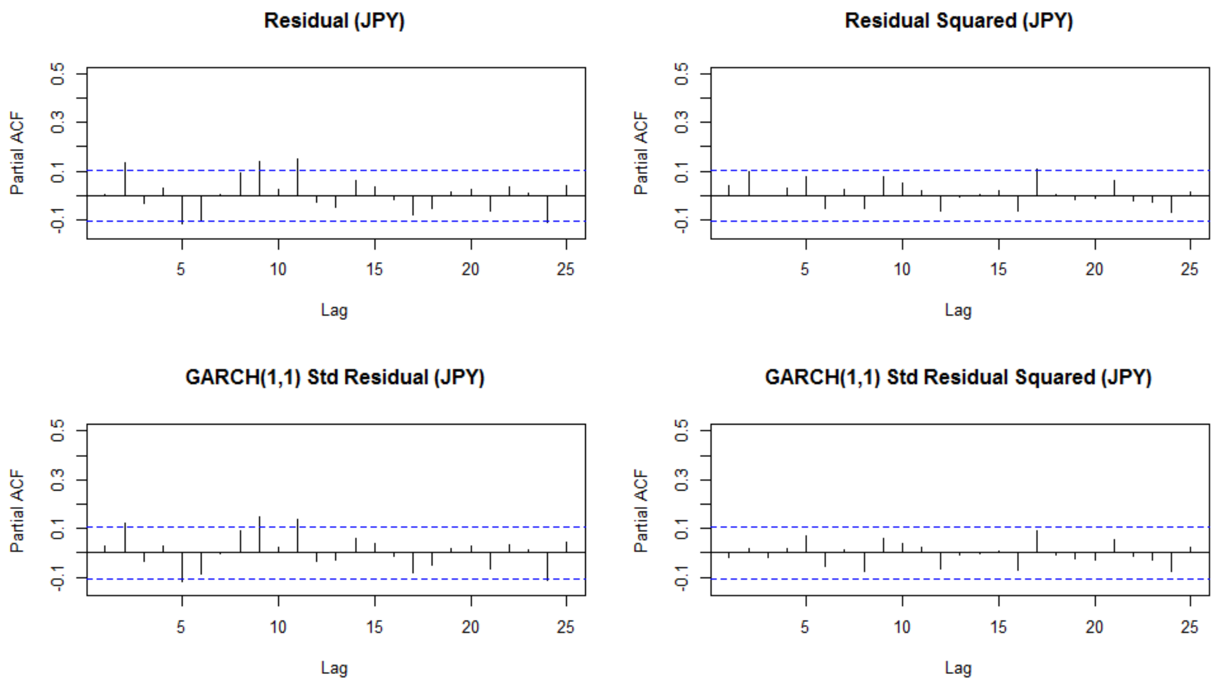



```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = AUD_diff, trace = F)

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -5.900e-05   1.648e-03   -0.036  0.97144
omega   4.197e-05   2.863e-05    1.466  0.14267
alpha1  9.439e-02   3.280e-02    2.878  0.00401 **
beta1   8.691e-01   4.382e-02   19.834  < 2e-16 ***

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  20.99683  2.758008e-05
 Shapiro-Wilk Test  R    W      0.985574  0.00171053
 Ljung-Box Test     R    Q(10)  10.19697  0.4233862
 Ljung-Box Test     R    Q(15)  15.67025  0.404299
 Ljung-Box Test     R    Q(20)  20.19033  0.4460818
 Ljung-Box Test     R^2  Q(10)  9.050965  0.5272738
 Ljung-Box Test     R^2  Q(15)  12.16475  0.6665183
 Ljung-Box Test     R^2  Q(20)  18.74349  0.5385511
 LM Arch Test       R    TR^2   12.13459  0.4349317
```



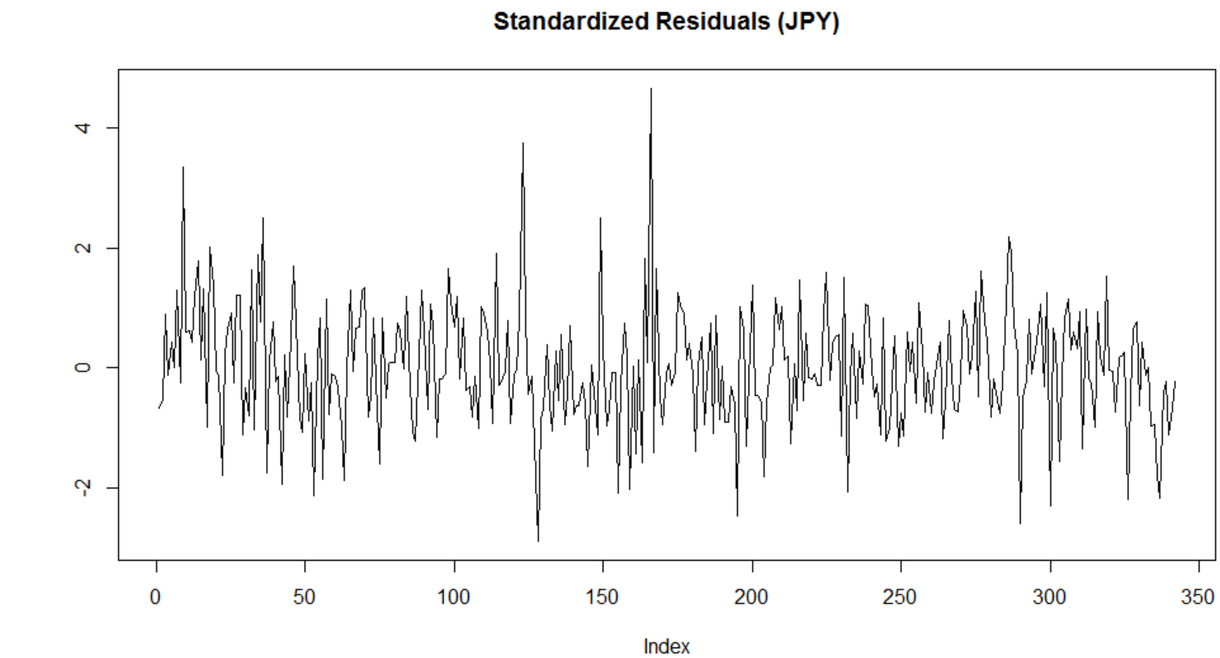
- USD/JPY currency pair
  Because USD/JPY does not have ARCH effect, after applying GARCH(1,1), we cannot see clear improvement in ACF and PACF. Also, standardized residuals still have serial correlations and parameter α_1 is not significant in the model. GARCH(1,1) may not suitable for these log return data.



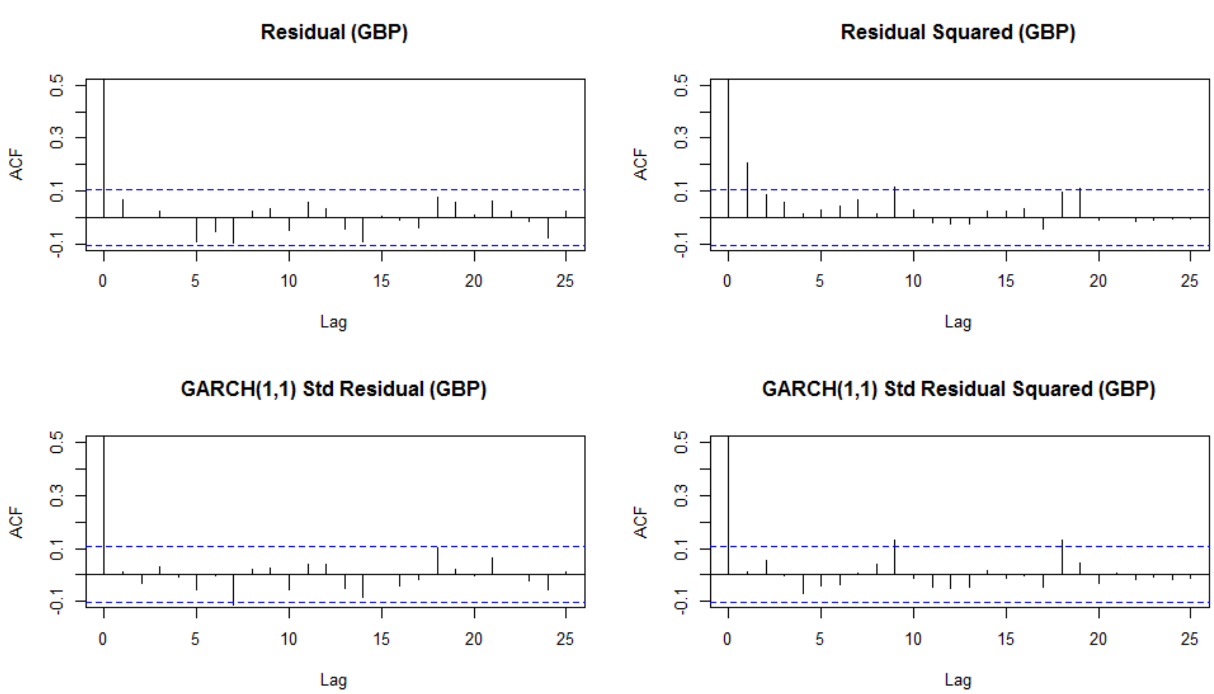


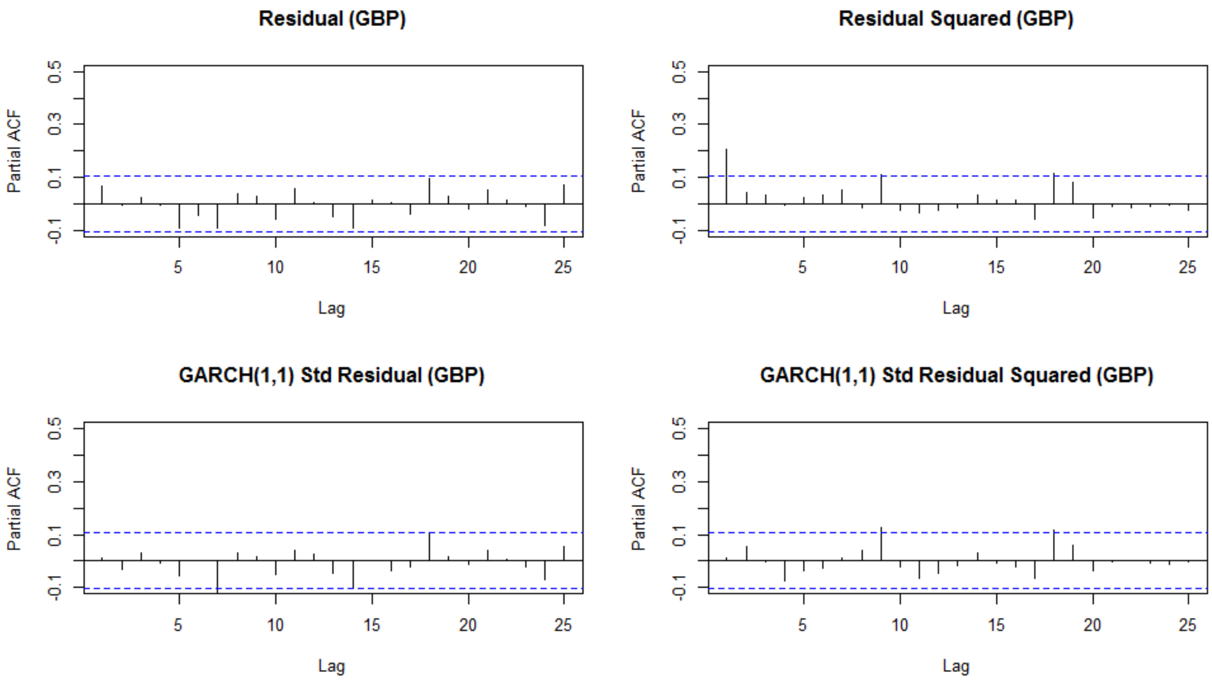

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = JPY_diff, trace = F)

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      2.670e-03   1.791e-03    1.491    0.136
omega   6.256e-05   5.920e-05    1.057    0.291
alpha1  3.475e-02   2.341e-02    1.485    0.138
beta1   9.090e-01   6.525e-02   13.930   <2e-16 ***

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  44.5247   2.145772e-10
 Shapiro-Wilk Test  R    W      0.981749  0.0002482357
 Ljung-Box Test     R    Q(10)  23.25912  0.009829498
 Ljung-Box Test     R    Q(15)  34.82927  0.002600428
 Ljung-Box Test     R    Q(20)  39.19927  0.006297052
 Ljung-Box Test     R^2  Q(10)  7.60955   0.6669154
 Ljung-Box Test     R^2  Q(15)  9.393031  0.8560872
 Ljung-Box Test     R^2  Q(20)  12.9386   0.880003
```

```
LM Arch Test        R     TR^2    9.217877   0.6842168
```

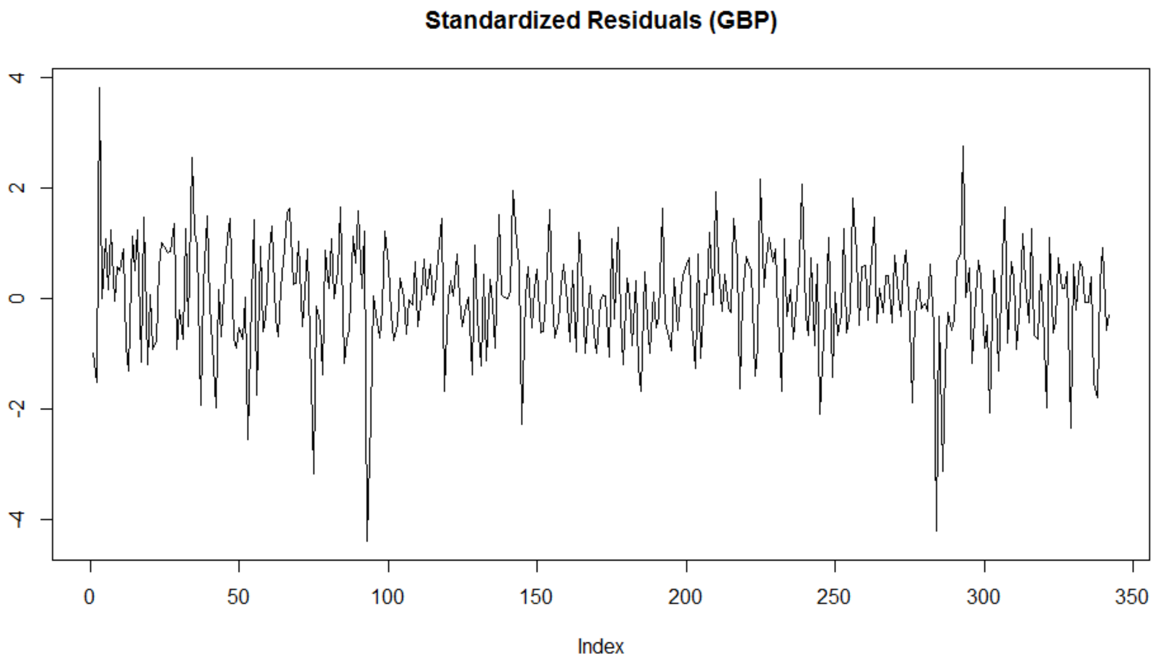


- GBP/USD currency pair
  After applying GARCH(1,1), ACF and PACF are improved. Based on Ljung-Box tests, there are no serial correlations in standardized residuals and squared standardized residuals. In addition, α_1 and β_1 are significant. Hence, GARCH(1,1) model is suitable for fitting the log return of exchange of GBP/AUD.






```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = GBP_diff, trace = F)

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     7.347e-04   1.426e-03    0.515   0.6064
omega  1.002e-04   5.523e-05    1.814   0.0697 .
alpha1 1.518e-01   6.404e-02    2.371   0.0178 *
beta1  7.421e-01   1.028e-01    7.219 5.24e-13 ***

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  72.70638  1.110223e-16
 Shapiro-Wilk Test  R    W      0.9754566 1.442523e-05
 Ljung-Box Test     R    Q(10)  7.847916  0.6436894
 Ljung-Box Test     R    Q(15)  12.58117  0.6346129
 Ljung-Box Test     R    Q(20)  17.1425   0.6437023
 Ljung-Box Test     R^2  Q(10)  10.75955  0.3765559
 Ljung-Box Test     R^2  Q(15)  13.26591  0.5817675
 Ljung-Box Test     R^2  Q(20)  21.3769   0.3752548
 LM Arch Test       R    TR^2   15.49248  0.2156022
```



After checking the suitability of GARCH(1,1), we will use this model predict volatility and also log return of exchange rate, and examining the deviation from UIP to implement our arbitrage trading strategy.

5.4.2 Volatility Model IGARCH(1,1)

We also notice that, for USD/AUD, $\alpha_1 + \beta_1$ of GARCH(1,1) model is very close to 1, hence we might have unit root for the AR polynomial. Therefore we would also like to introduce IGARCH as an alternative way to model volatility. In our trading strategy, we will exploit volatility and standardized residual from both GARCH and IGARCH to see their accuracy in predicting future log return of exchange rate.

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : iGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000023    0.001624 -0.014404 0.988508
omega   0.000014    0.000008  1.672150 0.094495
alpha1  0.114392    0.030873  3.705207 0.000211
beta1   0.885608          NA        NA       NA
```

GARCH(1,1) for USD/AUD:

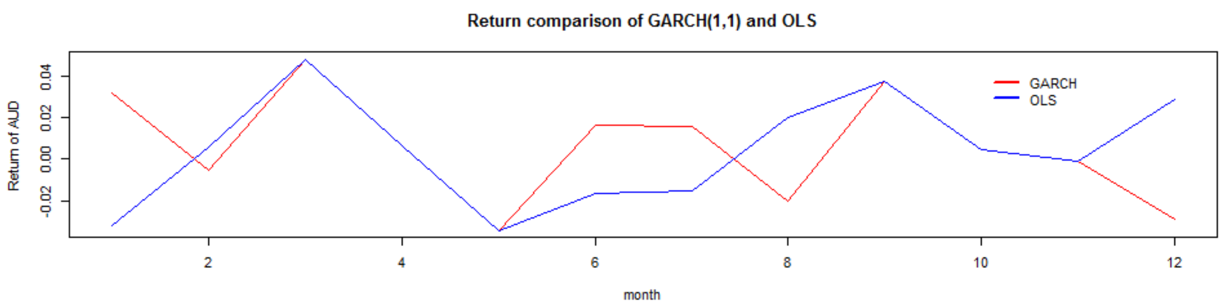
$$r_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = 0.094a_{t-1}^2 + 0.891\sigma_{t-1}^2$$

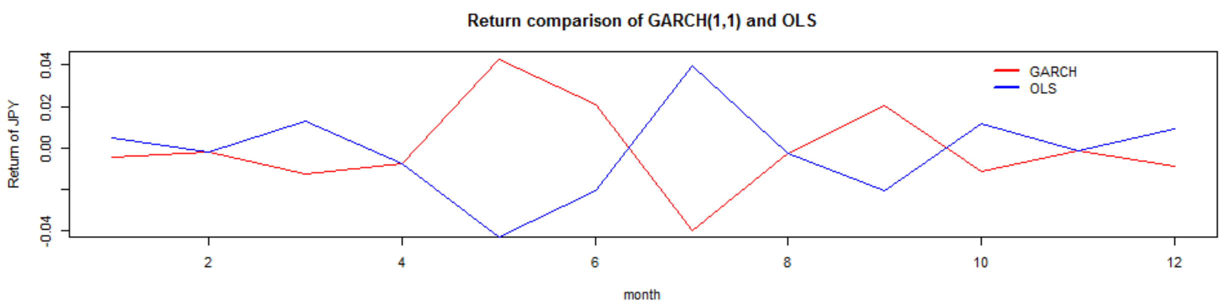
IGARCH(1,1) for USD/AUD:

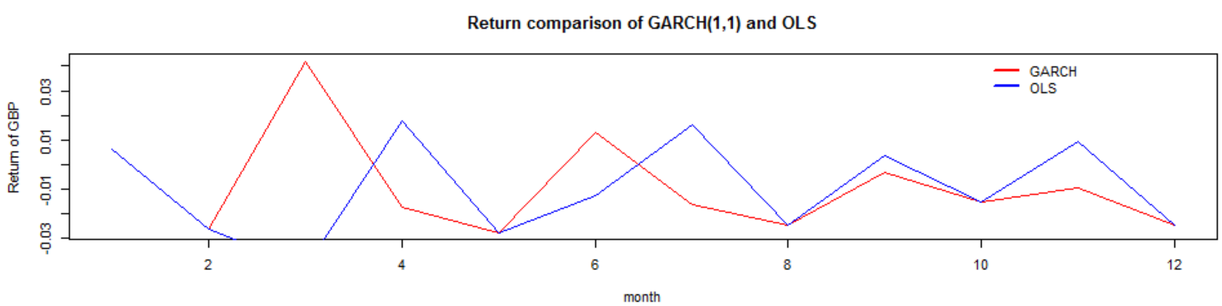
$$r_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = 0.114 a_{t-1}^2 + 0.886 \sigma_{t-1}^2$$

5.5 Implementing OLS, GARCH(1,1) and IGARCH(1,1) on arbitrage trading strategy

- Plots of realized returns of GARCH(1,1) and OLS for three currency pairs







- Mean of realized excess return by GARCH(1,1):

```
> colMeans(realized_return, na.rm = FALSE, dims = 1)
         AUD           JPY           GBP 
 0.0057904777 -0.0005422027 -0.0087054236 
```

- Mean of realized excess return by OLS:

```
> colMeans(realized_return_ols, na.rm = FALSE, dims = 1)
         AUD          JPY          GBP 
 0.004307174 -0.001676173 -0.010098155 
```

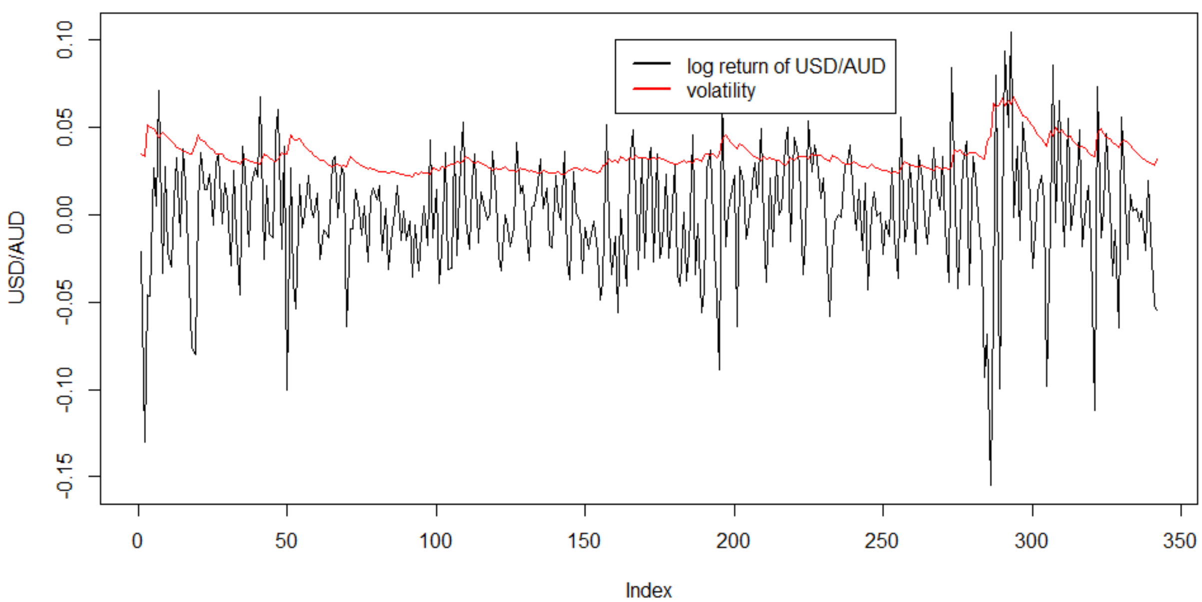
** The result of IGARCH(1,1) is the same with GARCH(1,1), because their model parameters are very close.

Form the plots of realized returns, it is not obvious which one is better. However, based on the mean of realized excess returns, the performance of GARCH(1,1) on our trading strategy is better than that of OLS. Therefore, we think GARCH(1,1) perform better in forecasting future exchange rate changes.

5.6 Using asymmetric volatility model

5.6.1 EGARCH(1,1) on USD/AUD monthly log return of exchange rate

We try to use asymmetric volatility model to fit our log return of exchange rate of USD/AUD. First, observe the relationship between volatility and log return:



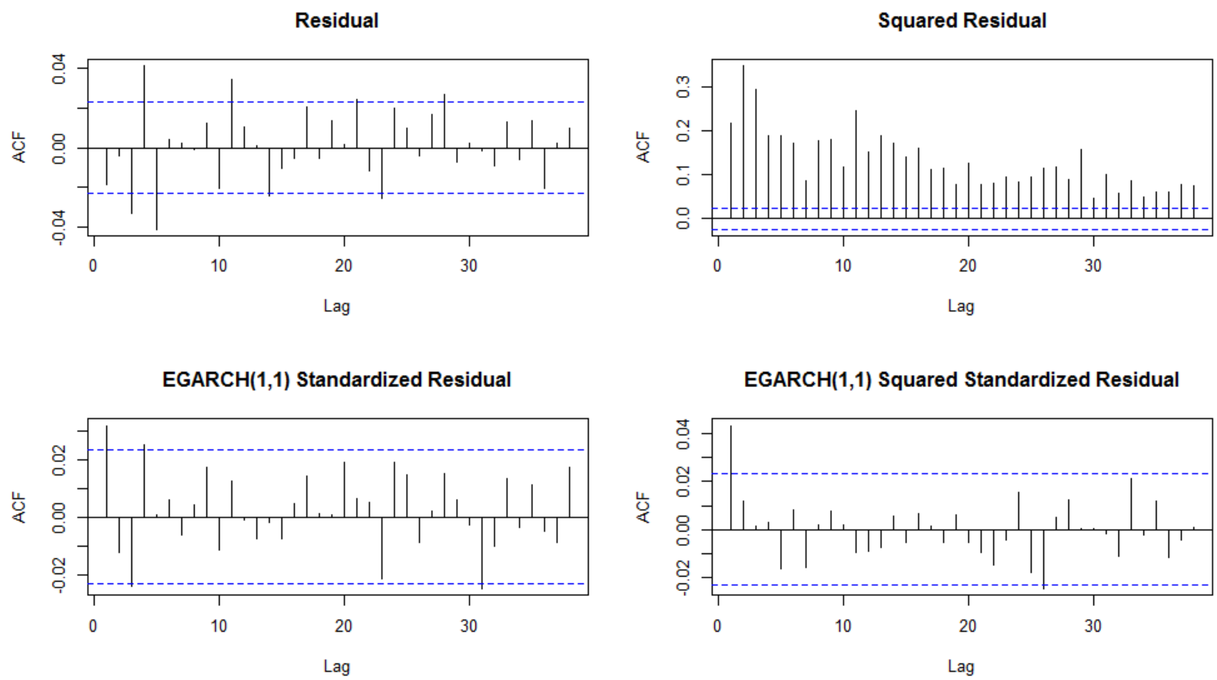
Although we cannot see clear asymmetric impact of log return on volatility, we still attempt to apply EGARCH(1,1) to fit our data. The result is corresponding to our expectation. The parameter α_1 is not significant, so EGARCH(1,1) is not effective here.

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000016    0.002527 -0.006462 0.994844
omega  -0.302021    0.302413 -0.998702 0.317939
```

```
alpha1 -0.041544    0.037033 -1.121802 0.261947
beta1   0.955747    0.044073 21.685336 0.000000
gamma1  0.207036    0.058726  3.525472 0.000423
```

5.6.2 EGARCH(1,1) on USD/AUD daily log return of exchange rate

According to the paper we read, we find that EGARCH(1,1) can fit log return of exchange rate well, but limited to daily data. So, we also attempt to apply EGARCH(1,1) to daily data of exchange rate.




```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.000069    0.000070    0.99606  0.31922
omega  -0.181806    0.007273  -24.99900  0.00000
alpha1 -0.037944    0.006286   -6.03594  0.00000
beta1   0.981082    0.000701 1398.97988  0.00000
gamma1  0.157977    0.010031   15.74899  0.00000
```

Based on the results, ACFs are improved dramatically, and the parameters are significant, so we think EGARCH(1,1) can fit daily data very well. The reason why EGARCH(1,1) can fit daily data but not monthly data may be due to asymmetric impact of log return of exchange rates on volatility only existing in daily data.

5.7 Performance analysis

- Sharpe ratios for excess return of three currency pairs and market Sharpe ratio

```
USD/AUD 0.2229827 USD/JPY -0.02707028 USD/GBP -0.4244997 MARKET 0.6815299
```

Although excess return of USD/AUD is below market Sharpe ratio, our trading strategy is zero net investment and we do not need initial capital to implement this strategy.

- Regarding to USD/AUD, we have 7 out of 12 months in which we have positive returns.

6. Conclusion

- After comparing the excess returns by using GARCH(1,1) model to predict log return of exchange rate with that by using OLS regression, we can find that performance of GARCH(1,1) is better than OLS.
- Based on the analysis of standardized residuals, GARCH(1,1) can solve the problem of volatility clustering in residuals of log returns of exchange rate of USD/AUD and USD/GBP.
- We have also tested the performance of our strategy by implementing GARCH(1,1) to model volatility. We made 0.58% monthly return on AUD but made losses on JPY and GBP and our trading strategy provides an arbitrage opportunity for USD/AUD.
- In terms of performance, Sharpe ratio of excess returns of USD/AUD is not better than market Sharpe ratio, but our strategy is long-short arbitrage trading strategy, we don't need any capital to invest. In addition, our trading strategy on USD/AUD gives us 7 winning months, which is a good result for an arbitrage strategy.

7. Evaluation

- **EGARCH(1,1) on daily data**
  Because EGARCH(1,1) can fit daily data well, maybe we could apply our trading strategy on daily data by using EGARCH(1,1) volatility model.
- **Further analysis of our performance**
  In terms of the performance of our trading strategy, we make substantial profit on USD/AUD, but no profit on USD/GBP and USD/JPY. Unlike regular financial products like stocks or bonds, currency pairs are more complicated and exchange rate are determined by a lot of factors, such as monetary policies and the degree of integration of two economies that are beyond our scope of discussion. We need further investigation to explain this.
- **Choice of currency**
  We could improve our projects by choosing more currencies, instead of just three main currencies in the world. In particular, we would love to choose EURUSD because it is the biggest currency market nowadays. But for the fact that EUR has only been issued for 13 years (since 2002), we cannot be sure whether we have enough historical data for our prediction. If we have further evidence that 13 years data is enough, we can confidently choose EURUSD.
- **Strong assumption**

We assume no trading friction in our trading strategy. In practical, there exists bid/ask spread and transaction cost in the trading which may lead to less profitable results. We could make our trading strategy more realistic by adding these costs into our model.

8. Reference

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D. Ng Cheong. Vee, P. Nunkoo Gonpot, N. Sookia. (2011), 'Forecasting Volatility of USD/MUR

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[5] John Dukich, Kyung Yong Kim, Huan-Hsun Lin. (2010), 'Modeling Exchange Rates using the GARCH Model'