

ELG7113/MCG5470
MACHINE LEARNING FOR ADAPTIVE AND INTELLIGENT CONTROL SYSTEMS
ASSIGNMENT 3

Prof. Davide Spinello

Due date posted in Virtual Campus

Policy

The solutions of the attached problems must be submitted to Virtual Campus as a single file by using the dedicated Assignment. The file must include your name and the assignment number, and ideally it should be a pdf, for example `LastName_FirstName_hw3.pdf`.

It is acceptable to upload handwritten solutions, but please be sure that your writing is readable. Whenever required, code and outputs have to be attached.

Problem 1

Consider a position servo described by $\ddot{y} = -a\dot{y} + bu$, where a and b are unknown parameters. The control law is $u = \theta_1(y - u_c) - \theta_2\dot{y}$ with two adjustable parameters θ_1 and θ_2 and reference signal u_c . Consider the desired output $y_m = G_m(p)u_c$, with transfer function $G_m(p) = \frac{\omega^2}{p^2 + 2\zeta\omega p + \omega^2}$.

1. (30pt) Use the MIT rule with loss function $J = e^2/2$ to obtain adaptive control laws that drive the output y to the desired output y_m . (*Hint: write the process as $y = G(p)u$*).
2. (20pt) Let $a = 3$, $b = 1$, $\omega = 1.5$, and $\zeta = 0.6$. Moreover, let

$$u_c(t) = \begin{cases} 1 & \sin \frac{2\pi t}{30} \geq 0 \\ 0 & \sin \frac{2\pi t}{30} < 0 \end{cases} \quad (1)$$

Simulate the system with the MIT rule and with the normalized MIT rule for the same value of the adaptation rate γ . For both cases, plot the error $y - y_m$ versus time and the input u .

Problem 2

A system with transfer function (integrator) $G(p) = \frac{b}{p}$, where b is an unknown parameter, is controlled by the two parameter control $u = \theta_1 u_c - \theta_2 y$, where y is the output and u_c is a reference signal. The desired response has first order dynamics with transfer function $G_m(s) = \frac{\beta}{s + \alpha}$ with β and α non-negative constants. Let the error be $e = y - y_m$.

1. **(25pt)** Show that

$$V(e) = \frac{1}{2} \left(e^2 + \frac{1}{b\gamma} (\alpha - b\theta_2)^2 + \frac{1}{b\gamma} (\beta - b\theta_1)^2 \right) \quad (2)$$

is a Lyapunov function for the system for some $\gamma > 0$. Simultaneously, derive adaptation laws for the control parameters that stabilize the error to zero.

2. **(20pt)** Simulate the system with the same reference signal as in Problem 1. Let $b = 2$, $\beta = \alpha = 1$. Plot the output y along the desired output y_m as well as the estimated control parameters. Simulate the system for two values of γ .

Problem 3

Consider the system in Problem 1.

1. **(15pt bonus)** Derive a Lyapunov function and adaptation laws for the control parameters that stabilize the errors $e_y = y - y_m$ and $e_v = v - v_m$ to zero, where $v = \dot{y}$ and $v_m = \dot{y}_m$. Assume that both y and v can be measured. It could be convenient to rewrite the system as

$$\dot{y} = v \quad (3)$$

$$\dot{v} = -av + bu \quad (4)$$

and

$$\dot{y}_m = v_m \quad (5)$$

$$\dot{v}_m = -2\zeta\omega v_m + \omega^2(u_c - y_m) \quad (6)$$

Alternatively, you can consider the second order dynamics for y and y_m along with control u defined in Problem 1.