Université d'Ottawa
Faculté de génie

Département de
génie mécanique

uOttawa

University of Ottawa
Faculty of Engineering

Department of
Mechanical Engineering

ELG7113/MCG5470
Machine Learning for Adaptive and Intelligent Control Systems

Final Exam

Prof. Davide Spinello

Due date posted in Virtual Campus

Policy

The solutions of the assigned problems must submitted to Virtual Campus as a single file by using the dedicated Assignment. The file must include your name and the assignment number, and ideally it should be a pdf, for example `LastName_FirstName_final.pdf`.

It is acceptable to upload handwritten solutions, but please be sure that your writing is readable. Whenever required, code and outputs have to be attached.

System description

Figure 1 schematizes a mechanical load (pendulum) driven by a DC motor through a gear system, for which non-ideal effects such as backlash can be neglected. The mass M of the pendulum is lumped at the tip, at a distance L from the axis of rotation z which is perpendicular to the plain of motion $\{x, y\}$, and it is parallel to z', the axis of rotation of the motor's rotor. The absolute rotation θ_L is measured with respect to x, and it related to the motor's shaft rotation θ by the gear relation . The input is the armature voltage E, inducing the armature current I.

The nonlinear governing equations of the system are

$$\dot{\theta} = \omega \tag{1a}$$

$$J\dot{\omega} + D\omega + MgL\frac{N_1}{N_2}\sin\left(\frac{N_1}{N_2}\theta\right) = K_t I \tag{1b}$$

$$L_a\dot{I} + R_a I + K_b\omega = E \tag{1c}$$

where L_a and R_a are the armature's inductance and resistance, K_b is the proportionality parameter between the back electromotive voltage and the angular velocity ω, K_t is the proportionality parameter between motor torque and motor angular velocity, and N_1/N_2 is the gear ratio. The pendulum bar is assumed to be of negligible mass with respect to M. The mechanical inertia and damping are given by $J = J_a + \left(\frac{N_1}{N_2}\right)^2 J_L$ and $D = D_a + \left(\frac{N_1}{N_2}\right)^2 D_L$ in terms of the armature parameters and of the load parameters $J_L = ML^2$ and D_L.

Let $(\bar{E}, \theta_d)$ be an equilibrium point corresponding at the constant angle θ_d at the constant armature voltage $\bar{E}$. By setting the equilibrium conditions in (1), the equilibrium point is

$$\theta = \theta_d, \quad \bar{I} = \frac{\bar{E}}{R_a}, \quad \bar{E} = \frac{R_a g N_1}{K_t L N_2}\sin\left(\frac{N_1}{N_2}\theta_d\right) \tag{2}$$

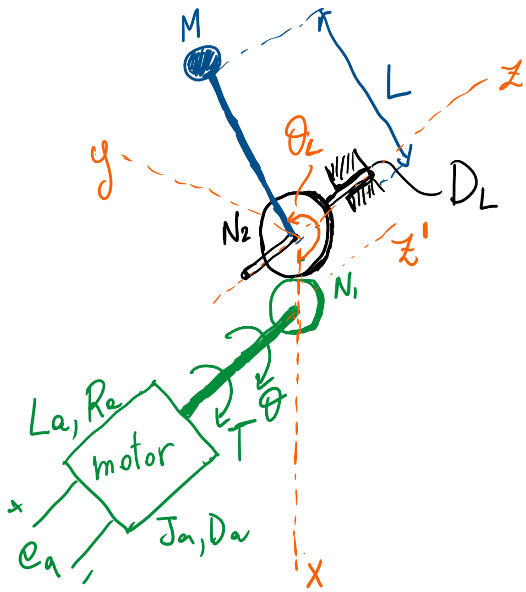

Fig. 1: Electromechanical system.

Define the errors $e_\theta = \theta - \theta_d$, $e_I = I - \frac{\bar{E}}{R_a}$, and $e_\omega = \omega$ (since θ_d is constant). Substitution into (1) give

$$\dot{e}_\theta = \omega \tag{3a}$$

$$J\dot{\omega} + D\omega + MgL\frac{N_1}{N_2}\left(\sin\left(\frac{N_1}{N_2}(e_\theta + \theta_d)\right) - \sin\left(\frac{N_1}{N_2}\theta_d\right)\right) = K_t e_I \tag{3b}$$

$$L_a\dot{e}_I + R_a e_I + K_b\omega = u \tag{3c}$$

where $u = E - \bar{E}$.

Assume the true values of the system's parameters to be $M = 5\,\mathrm{kg}$, $L = 1\,\mathrm{m}$, $D_L = 1\,\mathrm{N\,m\,s\,rad^{-1}}$, $N_1 = 15$ and $N_2 = 45$, $J_a = 2\,\mathrm{kg\,m^2\,rad^{-1}}$ and $D_a = 0.1\,\mathrm{N\,m\,s\,rad^{-1}}$. The armature resistance is $R_a = 10\,\Omega$ and the inductance is $L_a = 0.1\,\mathrm{H}$. Moreover, the coupling parameters are $K_t = 11$ and $K_b = 6.36$. The coupling parameters are obtained from the motor characteristic torque/angular velocity curve $T + 7\omega = 110$. Let $\theta_d = \frac{\pi}{6}$.

Problem 1

1. (**2pt**) For a constant sampling time step $\Delta = 0.01\,\mathrm{s}$, use the Euler's forward difference scheme to write the error dynamics (3) in discrete time form

$$x_{k+1} = f(x_k) + g(x_k)u_k \tag{4}$$

   where k is an integer labelling the discrete time sample. Explicitly identify drift and input dynamics $f(\cdot)$ and $g(\cdot)$.

2. (**20pt**) Consider a quadratic performance index with stage reward $r = x_k^T Q x_k + R u_k^2$, where $Q > 0$ and $R > 0$ are state and control energy weights. Consider a temporal difference learning formulation of the optimal control problem and implement a **value iteration** algorithm to obtain

a control sequence $u_1, u_2 \dots$ with critic function approximation

$$V(x) = \sum_{i=1}^{N_L} \phi_i(x) w_i = \boldsymbol{\phi}^T(x)\boldsymbol{W} \tag{5}$$

Consider a quadratic homogeneous basis functions set, which determines the dimension of the hidden layer/number of neurons. Simulate the system with initial condition $\theta_0 = 0$, $\omega_0 = 0$ and $I_0 = 0$ (be sure to translate it in terms of errors). Plot the critic gains, the input $u(t) + \bar{E}$ and the trajectory $\theta(t)$.

3. (**20pt**) Repeat the problem at point 2 with the actor function approximation

$$u(x) = \sum_{i=1}^{N_L} \phi_i(x) U_i = \boldsymbol{\phi}^T(x)\boldsymbol{U} \tag{6}$$

Use the same basis function as the critic approximation. Plot the actor and critic gains, the input u and the trajectory θ.

4. (**20pt**) Let $Q(x_k, u_k) = r(x_k, u_k) + \gamma V(x_{k+1}, u_{k+1})$ be the Q function (not to be confused with the state energy weight in the stage reward). Consider a Q function approximation with homogeneous quadratic basis $\boldsymbol{\psi}(x, u)$ and weights $\boldsymbol{W}$

$$Q(x, u) = \sum_{i=1}^{N_L} \psi_i(x, u) w_i = \boldsymbol{\psi}^T(x, u)\boldsymbol{W} \tag{7}$$

Simulate the system with the same initial conditions and parameters. Plot the Q function gains, the input $u(t)$ and the trajectory $\theta(t)$.

Problem 2

1. (**15pt bonus**) Linearize the discrete time error dynamics around the origin to obtain the linear representation

$$x_{k+1} = Ax_k + Bu_k \tag{8}$$

Clearly identify state and input matrices A and B. Use Q-learning with value function iterations to simulate the system with initial condition $x_0 = (10°, 0, 0)$, and compare the trajectory $\theta(t)$ with the one obtained by solving offline the Riccati equation that gives the sequence of Kalman gains K_k. Pose the offline optimal control as a LQR problem with kernel at the final time S_N to be the identity matrix.