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ELG7113/MCG5470
Machine Learning for Adaptive and Intelligent Control Systems

Final Exam

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Due date posted in Virtual Campus

Policy

The solutions of the assigned problems must submitted to Virtual Campus as a single file by using the dedicated Assignment. The file must include your name and the assignment number, and ideally it should be a pdf, for example `LastName_FirstName_final.pdf`.

It is acceptable to upload handwritten solutions, but please be sure that your writing is readable. Whenever required, code and outputs have to be attached.

System description

Figure 1 schematizes a mechanical load (pendulum) driven by a DC motor through a gear system, for which non-ideal effects such as backlash can be neglected. The mass M of the pendulum is lumped at the tip, at a distance L from the axis of rotation z which is perpendicular to the plain of motion $\{x, y\}$, and it is parallel to z', the axis of rotation of the motor's rotor. The absolute rotation θ_L is measured with respect to x, and it related to the motor's shaft rotation θ by the gear relation . The input is the armature voltage E, inducing the armature current I.

The nonlinear governing equations of the system are

$$\dot{\theta} = \omega \tag{1a}$$

$$J\dot{\omega} + D\omega + MgL\frac{N_1}{N_2}\sin\left(\frac{N_1}{N_2}\theta\right) = K_t I \tag{1b}$$

$$L_a\dot{I} + R_a I + K_b\omega = E \tag{1c}$$

where L_a and R_a are the armature's inductance and resistance, K_b is the proportionality parameter between the back electromotive voltage and the angular velocity ω, K_t is the proportionality parameter between motor torque and motor angular velocity, and N_1/N_2 is the gear ratio. The pendulum bar is assumed to be of negligible mass with respect to M. The mechanical inertia and damping are given by $J = J_a + \left(\frac{N_1}{N_2}\right)^2 J_L$ and $D = D_a + \left(\frac{N_1}{N_2}\right)^2 D_L$ in terms of the armature parameters and of the load parameters $J_L = ML^2$ and D_L.

Let $(\bar{E}, \theta_d)$ be an equilibrium point corresponding at the constant angle θ_d at the constant armature voltage $\bar{E}$. By setting the equilibrium conditions in (1), the equilibrium point is

$$\theta = \theta_d, \quad \bar{I} = \frac{\bar{E}}{R_a}, \quad \bar{E} = \frac{R_a N_1}{K_t N_2}MgL\sin\left(\frac{N_1}{N_2}\theta_d\right) \tag{2}$$

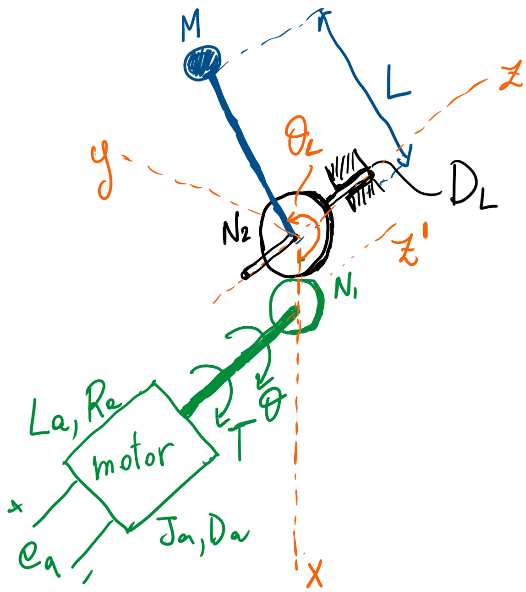

Fig. 1: Electromechanical system.

Define the errors $e_\theta = \theta - \theta_d$, $e_I = I - \frac{\bar{E}}{R_a}$, and $e_\omega = \omega$ (since θ_d is constant). Substitution into (1) give

$$\dot{e}_\theta = \omega \tag{3a}$$

$$J\dot{\omega} + D\omega + MgL\frac{N_1}{N_2}\left(\sin\left(\frac{N_1}{N_2}(e_\theta + \theta_d)\right) - \sin\left(\frac{N_1}{N_2}\theta_d\right)\right) = K_t e_I \tag{3b}$$

$$L_a\dot{e}_I + R_a e_I + K_b\omega = u \tag{3c}$$

where $u = E - \bar{E}$.

Assume the true values of the system's parameters to be $M = 5\,\text{kg}$, $L = 1\,\text{m}$, $D_L = 1\,\text{N}\,\text{m}\,\text{s}\,\text{rad}^{-1}$, $N_1 = 15$ and $N_2 = 45$, $J_a = 2\,\text{kg}\,\text{m}^2\,\text{rad}^{-1}$ and $D_a = 0.1\,\text{N}\,\text{m}\,\text{s}\,\text{rad}^{-1}$. The armature resistance is $R_a = 10\,\Omega$ and the inductance is $L_a = 0.1\,\text{H}$. Moreover, the coupling parameters are $K_t = 11$ and $K_b = 6.36$. The coupling parameters are obtained from the motor characteristic torque/angular velocity curve $T + 7\omega = 110$. Let $\theta_d = \frac{\pi}{6}$.

Problem 1

1. (**2pt**) For a constant sampling time step $\Delta = 0.01\,\text{s}$, use the Euler's forward difference scheme to write the error dynamics (3) in discrete time form

$$x_{k+1} = f(x_k) + g(x_k)u_k \tag{4}$$

   where k is an integer labelling the discrete time sample. Explicitly identify drift and input dynamics $f(\cdot)$ and $g(\cdot)$.

2. (**20pt**) Consider a quadratic performance index with stage reward $r = x_k^T Q x_k + R u_k^2$, where $Q > 0$ and $R > 0$ are state and control energy weights. Consider a temporal difference learning formulation of the optimal control problem and implement a **value iteration** algorithm to obtain

a control sequence $u_1, u_2 \ldots$ with critic function approximation

$$V(x) = \sum_{i=1}^{N_L} \phi_i(x) w_i = \boldsymbol{\phi}^T(x)\boldsymbol{W} \tag{5}$$

Consider a quadratic homogeneous basis functions set, which determines the dimension of the hidden layer/number of neurons. Simulate the system with initial condition $\theta_0 = 0$, $\omega_0 = 0$ and $I_0 = 0$ (be sure to translate it in terms of errors). Plot the critic gains, the input $u(t) + \bar{E}$ and the trajectory $\theta(t)$.

3. (**20pt**) Repeat the problem at point 2 with the actor function approximation

$$u(x) = \sum_{i=1}^{N_L} \phi_i(x) U_i = \boldsymbol{\phi}^T(x)\boldsymbol{U} \tag{6}$$

Use the same basis function as the critic approximation. Plot the actor and critic gains, the input u and the trajectory θ.

4. (**20pt**) Let $Q(x_k, u_k) = r(x_k, u_k) + \gamma V(x_{k+1}, u_{k+1})$ be the Q function (not to be confused with the state energy weight in the stage reward). Consider a Q function approximation with homogeneous quadratic basis $\boldsymbol{\psi}(x, u)$ and weights $\boldsymbol{W}$

$$Q(x, u) = \sum_{i=1}^{N_L} \psi_i(x, u) w_i = \boldsymbol{\psi}^T(x, u)\boldsymbol{W} \tag{7}$$

Simulate the system with the same initial conditions and parameters. Plot the Q function gains, the input $u(t)$ and the trajectory $\theta(t)$.

Problem 2

1. (**15pt bonus**) Linearize the discrete time error dynamics around the origin to obtain the linear representation

$$x_{k+1} = Ax_k + Bu_k \tag{8}$$

Clearly identify state and input matrices A and B. Use Q-learning with value function iterations to simulate the system with initial condition $x_0 = (10°, 0, 0)$, and compare the trajectory $\theta(t)$ with the one obtained by solving offline the Riccati equation that gives the sequence of Kalman gains K_k. Pose the offline optimal control as a LQR problem with kernel at the final time S_N to be the identity matrix.