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Faculty of Engineering
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ELG7113/MCG5470
Machine Learning for Adaptive and Intelligent Control Systems

Assignment 4

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Due date posted in Virtual Campus

Policy

The solutions of the attached problems must submitted to Virtual Campus as a single file by using the dedicated Assignment. The file must include your name and the assignment number, and ideally it should be a pdf, for example `LastName_FirstName_hw#.pdf`.

It is acceptable to upload handwritten solutions, but please be sure that your writing is readable. Whenever required, code and outputs have to be attached.

Problem 1

Consider a system with scalar state x evolving in discrete time according the process

$$x_{k+1} = x_k u_k + \alpha \tag{1}$$

where α is a constant and u_k is the scalar input. Let the performance index be

$$J_0 = \frac{r}{2} \sum_{k=0}^{N-1} u_k^2 \tag{2}$$

where $r > 0$ is the control energy weight. Let the final time be $N = 2$, and the initial condition x_0 be given. The final time is constrained to $x_N = 0$.

1. (**10pt**) Write the Hamiltonian, and derive the state, costate and stationarity equations.
2. (**5pt**) Show that the state and costate equations after eliminating u_k are

$$x_{k+1} = \alpha - \frac{x_k^2}{r} \lambda_{k+1} \tag{3}$$

$$\lambda_k = -\frac{x_k}{r} \lambda_{k+1}^2 \tag{4}$$

where λ_k is the Lagrange multiplier.

3. (**20pt**) Obtain the state x_1 in terms of the final costate λ_2 by substituting for λ_1 in the state equation for x_1. Then substitute x_1 in the state equation for x_2 to obtain the characteristic equation for λ_2, that should be a polynomial equation for λ_2 with parameters x_0, α and r when you impose the boundary condition $x_2 = 0$.

4. (**20pt**) Obtain the optimal control sequence $u_0^\star, u_1^\star$ and the optimal trajectory $x_0, x_1^\star, x_2$. The controls should be expressed in terms of λ_2, x_0 and the parameters α and r.

5. (**10pt**) Simulate the system for $\alpha = 2$, $r = 1$, $x_0 = 1.5$. Obtain all the trajectories generated by values of λ_2 real and non-negative, obtained from the characteristic equation. Verify that $x_2 = 0$.

Problem 2

Consider the same system as in Problem 1 with $N = 2$ and modified performance index

$$J_N = \frac{1}{2}x_N^2 + \frac{r}{2}\sum_{k=0}^{N-1} u_k^2 \tag{5}$$

1. (**25pt**) Use dynamic programming to obtain an optimal feedback control law.

2. (**15pt**) Simulate the system for the same set of numerical parameters as in problem 1. Calculate a second solution for $r = 20$ and comment on eventual differences.