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Faculty of Engineering
Department of
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ELG7113/MCG5470: Machine Learning for Adaptive and Intelligent Control Systems
Assignment 1

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Due date posted in Virtual Campus

Policy

The solutions of the attached problems must submitted to Virtual Campus as a single file by using the dedicated Assignment. The file must include your name and the assignment number, and ideally it should be a pdf, for example `LastName_FirstName_hw1.pdf`.

It is acceptable to upload handwritten solutions, but please be sure that your writing is readable. Whenever required, code and outputs have to be attached.

Problem 1

The data file `dataHw1.dat` includes discrete time stamps and corresponding scalar data generated by the model $y(k) = \varphi^T(k)\theta + e(k)$, where $e(k)$ is a scalar white noise with standard deviation $\sigma = 11$, and θ is the set of true parameters of undisclosed size. Assume the set of regressors to be polynomial in the discrete time variable t, that is $\varphi = \begin{pmatrix} 1 & t & t^2 & \cdots \end{pmatrix}$.

1. (**20pt**) Obtain a least squares estimate $\hat{\theta}$ for different numbers of parameters from 1 to 5.

2. (**10pt**) Use the value function to estimate the right number of parameters used to generate the data, that is, the dimension of the "true set" θ. Plot the data long with the fitting curve.

3. (**10pt**) Calculate the unbiased estimate $\hat{\sigma}$ of the standard deviation σ for different dimensions of the regression vectors as above. If you had to use this parameter as convergence ctirion, would you revise your previous guess of the number of system's parameters?

Problem 2

Consider a discrete time system with transfer function model

$$y(t+2) + a_1 y(t+1) + a_2 y(t) = b_0 u(t+1) + b_1 u(t) + e(t+2) \tag{1}$$

where $e(t)$ is white noise. Notice that this is a transfer function model with $n = 2$, $m = 1$, and uncorrelated noise. The data is generated with the true set of parameters $a_1 = 1.3, a_2 = 0.75, b_0 = 1.1, b_1 = -0.35$, and the noise standard deviation is $\sigma = 0.65$.

1. (**20pt**) Obtain a recursive least squares estimate of the system for $u(t) = \delta(t - 100)$ and $u(t) = \mathfrak{u}(t - 100)$, where $\mathfrak{u}$ is the unit step function and δ is the unit pulse function. In both cases, plot the estimates $\hat{\theta}$ and the true parameters θ, and comment on the accuracy based on the excitation.
2. (**10pt**) Show that the order of persistent excitation of the signal excitation $u(t) = \sin \frac{2\pi t}{5} + \cos \frac{4\pi t}{5}$ is 4.
3. (**10pt**) Use the multi-frequency sinusoidal excitation above to estimate the system's parameters. Plot the estimates $\hat{\theta}$.
4. (**10pt**) From the simulation data, calculate the estimated covariances $\hat{\sigma}_{\hat{b}_1}(t = 3000)$ and $\hat{\sigma}_{\hat{b}_2}(t = 3000)$.
5. (**5pt bonus**) Analyze the results in view of Theorem 2.10 in the textbook.