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ELG7113/MCG5470
Machine Learning for Adaptive and Intelligent Control Systems

Assignment 3

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Due date posted in Virtual Campus

Policy

The solutions of the attached problems must submitted to Virtual Campus as a single file by using the dedicated Assignment. The file must include your name and the assignment number, and ideally it should be a pdf, for example `LastName_FirstName_hw3.pdf`.

It is acceptable to upload handwritten solutions, but please be sure that your writing is readable. Whenever required, code and outputs have to be attached.

Problem 1

Consider a position servo described by $\ddot{y} = -a\dot{y} + bu$, where a and b are unknown parameters. The control law is $u = \theta_1(y - u_c) - \theta_2\dot{y}$ with two adjustable parameters θ_1 and θ_2 and reference signal u_c. Consider the desired output $y_m = G_m(p)u_c$, with transfer function $G_m(p) = \frac{\omega^2}{p^2+2\zeta\omega p+\omega^2}$.

1. (**30pt**) Use the MIT rule with loss function $J = e^2/2$ to obtain adaptive control laws that drive the output y to the desired output y_m. (*Hint: write the process as* $y = G(p)u$).

2. (**15pt**) Let the reference signal u_c be bounded for all t, and let $u_c^0 = \max_t |u_c(t)|$. Estimate the adaptation rate γ such that the adaptation laws are stable.

3. (**25pt**) Let $a = 3$, $b = 1$, $\omega = 1.5$, and $\zeta = 0.6$. Moreover, let

$$u_c(t) = \begin{cases} 1 & \sin\frac{2\pi t}{30} \geq 0 \\ 0 & \sin\frac{2\pi t}{30} < 0 \end{cases} \tag{1}$$

   Solve the system for two values of the adaptation rate γ corresponding respectively to stable and unstable output. For both cases, plot the error $y - y_m$ versus time, showing that in the unstable case the error grows unbounded. Also plot the estimated parameters along with the true values.

4. (**15pt**) Simulate the unstable case above with the normalized MIT rule, showing that in this case the error decays.

Problem 2

A system with transfer function (integrator) $G(p) = \frac{b}{p}$, where b is an unknown parameter, is controlled by the two parameter control $u = \theta_1 u_c - \theta_2 y$, where y is the output and u_c is a reference signal. The desired response has first order dynamics with transfer function $G_m(s) = \frac{\beta}{s+\alpha}$ with β and α non-negative constants. Let the error be $e = y - y_m$.

1. (**25pt**) Show that

$$V(e) = \frac{1}{2}\left(e^2 + \frac{1}{\beta\gamma}(\alpha - b\theta_2)^2 + \frac{1}{\beta\gamma}(\beta - b\theta_1)^2\right) \tag{2}$$

   is a Lyapunov function for the system for some $\gamma > 0$. Simultaneously, derive adaptation laws for the control parameters that stabilize the error to zero.

2. (**20pt**) Simulate the system with the same reference signal as in Problem 1. Let $b = 2$, $\beta = \alpha = 1$. Plot the output y along the desired output y_m as well as the estimated control parameters. Simulate the system for two values of γ.

Problem 3

Consider the system in Problem 1.

1. (**15pt bonus**) Derive a Lyapunov function and adaptation laws for the control parameters that stabilize the errors $e_y = y - y_m$ and $e_v = v - v_m$ to zero, where $v = \dot{y}$ and $v_m = \dot{y}_m$. Assume that both y and v can be measured. It could be convenient to rewrite the system as

$$\dot{y} = v \tag{3}$$
$$\dot{v} = -av + bu \tag{4}$$

   and

$$\dot{y}_m = v_m \tag{5}$$
$$\dot{v}_m = -2\zeta\omega v_m + \omega^2(u_c - y_m) \tag{6}$$

   Alternatively, you can consider the second order dynamics for y and y_m along with control u defined in Problem 1.