In [1]:

```python
import matplotlib.pyplot as plt
import control as co
import pandas as pd
import numpy as np
import sympy as sp
import time

from sympy.abc import a, b, q, s, z, omega, zeta
from numpy.linalg import inv
```

In [2]:

```python
u, uc, vs, y, ys, Gs, Hz, Hq = sp.symbols('u(k),u_c{(k)},v(s),y(k),y(s),G(s),H(z),H(q)')
Ts = sp.symbols('T_s')

am1, am2, bm0, bm1 = sp.symbols('a_{m_1},a_{m_2},b_{m_0},b_{m_1}')
ao, a1, a2, b0, b1 = sp.symbols('a_o,a_1,a_2,b_0,b_1')
r0, s0, s1, t0, t1 = sp.symbols('r_0,s_0,s_1,t_0,t_1')

Gs_eq = sp.Eq(Gs, b/(s*(s + b)))
```

The model is given by $G(s) = \frac{b}{s(b+s)}$. To discretize,

$$G(z) = (1 - z^{-1})Z\left\{\frac{b}{s^2(s+b)}\right\}$$

$$G(z) = (1 - z^{-1})Z\left\{\frac{1}{s^2} - \frac{1}{bs} + \frac{1}{b(b+s)}\right\}$$

$$G(z) = (1 - z^{-1})\left(\frac{T_s z}{(z-1)^2} - \frac{1}{b}\frac{z}{z-1} + \frac{1}{b}\frac{z}{z-e^{-bT_s}}\right)$$

In [3]:

```python
Gz_eq_Ts_b = sp.simplify(sp.expand((1-1/z)*(Ts*z/(z-1)**2 - z/(b*(z-1)) + z/(b*(z-sp.exp(-b*Ts))))))

num, den = sp.fraction(Gz_eq_Ts_b)
den_poly = sp.Poly(den, z)
mono_div = den_poly.coeffs()[0]
sp.Eq(Hz, sp.collect(sp.simplify(Gz_eq_Ts_b), z))
```

Out[3]:

$$H(z) = \frac{-T_s b + z\left(T_s b e^{T_s b} - e^{T_s b} + 1\right) + e^{T_s b} - 1}{b\left(z^2 e^{T_s b} + z\left(-e^{T_s b} - 1\right) + 1\right)}$$

Pulse Function

In [4]:

```python
radius = 38/2000  # 38 mm diameter to radius in m
mass = 4/7000  # grams per ball
area_ball = np.pi * radius**2
volume_ball = 4/3*np.pi*radius**3
density = 1.2  # kg/m^3
veq = 2.8 # m/s

Ts_val = 0.2
b_nom = 2*9.81*(mass-density*volume_ball)/(mass*veq)
```

In [5]:

```python
## Example 3.1: since it has the same structure TF I used it to validate my approach, it's consistent
# sub_vals = [(Ts, Ts_val), (b, 1)]

## This is for our project
sub_vals = [(Ts, Ts_val), (b, b_nom)]

num_mono_z = sp.collect(num/mono_div, z).subs(sub_vals)
den_mono_z = sp.collect(den/mono_div, z).subs(sub_vals)

Hq_eq = sp.Eq(Hq, num_mono_z/den_mono_z)
Hq_eq
# np.roots(sp.Poly(num_mono_z).coeffs())
```

Out[5]:

$$H(q) = \frac{0.0888233086497573z + 0.0575823297146081}{1.0z^2 - 1.26797180817817z + 0.267971808178173}$$

In [6]:

```python
n_coeffs_nom, d_coeffs_nom = sp.fraction(Hq_eq.rhs)
pulse_coeffs = sp.Poly(d_coeffs_nom).coeffs()[1:]
for bi in sp.Poly(n_coeffs_nom).coeffs():
    pulse_coeffs.append(bi)

zeros = np.roots(sp.Poly(n_coeffs_nom).coeffs())
poles = np.roots(sp.Poly(d_coeffs_nom).coeffs())
zeros[0]
```

Out[6]:

```
-0.6482794954381095
```

This is a sanity check with the nominal values to check the stability of the zeros. Here, the zero is unstable for a nominal value of $b =$ -0.6482794954381095 and the poles are 1.0 and 0.26797180817817307.

Control Parameter Derivation

Given that the zeros are unstable, the parameters are derived without zero cancelation.

Here we know from the compatability conditions that,

$$\mathrm{degA}_m = \mathrm{degA} = 2$$

$$\mathrm{degB}_m = \mathrm{degB} = 1$$

Since the zeros in B are stable then $B^+ = q + \frac{b_1}{b_0}$ and $B^- = b_0$

Then,

$$\mathrm{degA}_o = \mathrm{degA} - \mathrm{degB}^+ - 1 = 2 - 1 - 1 = 0$$

Using the Diophantine equation we get,

$$AR' + b_0 S = A_o A_m$$

Let $A_o = 1$

Since the process is second order, then,

$$\mathrm{degR} = \mathrm{degS} = \mathrm{degT} = 1,$$

with R being monic.

Control Parameters

In [7]:

```python
# Process Values
_A = q**2 + a1*q + a2
_B = b0*q + b1
_Bplus = q + b1/b0
_Bminus = b0

# Model Values
_Am = q**2 + am1*q + am2
_Bm = _Am.subs(q, 1)*q
_Ao = ao

# Control Values
_R = q + r0
_S = s0*q + s1
_T = t0*q
```

Diophantine equation:

In [8]:

```python
diophantine = sp.Eq((_A*1 + b0*_S), (_Ao*_Am))
dio_LHS_coeffs = sp.Poly(diophantine.lhs, q).coeffs()
dio_RHS_coeffs = sp.Poly(diophantine.rhs, q).coeffs()
```

In [9]:

```python
# for i in range(len(dio_LHS_coeffs)):
#     display([dio_LHS_coeffs[i], dio_RHS_coeffs[i]])
```

Finding s_0

In [10]:

```python
_s0 = sp.solve(dio_LHS_coeffs[1] - dio_RHS_coeffs[1], s0)[0]
sp.Eq(s0, _s0)
```

Out[10]:

$$s_0 = \frac{-a_1 + a_o a_{m_1}}{b_0}$$

Finding s_1

In [11]:

```python
_s1 = sp.solve(dio_LHS_coeffs[2] - dio_RHS_coeffs[2], s1)[0]
sp.Eq(s1, _s1)
```

Out[11]:

$$s_1 = \frac{-a_2 + a_o a_{m_2}}{b_0}$$

Finding T

In [12]:

```python
_t0 = _Ao*_Bm/b0
_t0
```

Out[12]:

$$\frac{a_o q\left(a_{m_1} + a_{m_2} + 1\right)}{b_0}$$

In [13]:

```python
R = _Bplus
S = _s0*q + _s1
T = _Ao*_Bm/_Bminus

display(R)
display(S)
display(T)
```

$$q + \frac{b_1}{b_0}$$

$$\frac{q\left(-a_1 + a_o a_{m_1}\right)}{b_0} + \frac{-a_2 + a_o a_{m_2}}{b_0}$$

$$\frac{a_o q\left(a_{m_1} + a_{m_2} + 1\right)}{b_0}$$

Control Action

In [14]:

```python
uk1,  uck1, yk1 = sp.symbols('u(k-1),u_c{(k-1)},y(k-1)')
control_action = (sp.expand(sp.Eq((_R*u)/q, (_T*uc - _S*y)/q)))
control_action
control_subs = [(1/q*u, uk1), (1/q*uc, uck1), (1/q*y, yk1)]
control_action_sol = sp.Eq(u, sp.solve(control_action.subs(control_subs), u)[0])
control_action_sol.subs([(r0, b1/b0),
                          (s0, _s0),
                          (s1, _s1),
                          (t0, _t0),
                          (ao, 1)])
```

Out[14]:

$$u(k) = -\frac{b_1 u(k-1)}{b_0} + \frac{q u_c(k)\left(a_{m_1} + a_{m_2} + 1\right)}{b_0} - \frac{y(k)\left(-a_1 + a_{m_1}\right)}{b_0} - \frac{y(k-1)\left(-a_2 + a_{m_2}\right)}{b_0}$$

Simulation

Process Deivation

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_0 u(k-1) + b_1 u(k-2) = \phi(t-1)^T\theta$$

where,

$$\phi(t-1) = \left[-y(k-1)\ \ -y(k-2)\ u(k-1)\ u(k-2)\right]^T$$

and

$$\theta = \left[a_1\ a_2\ b_0\ b_1\right]^T$$

In [15]:

```python
final_time = 90
t = np.arange(0, final_time + Ts_val, Ts_val)
def reference_signal(end_time=final_time, Ts_func=Ts_val, lower_set=0.2, upper_set=0.1, period=30):
    uc_func = []
    time = np.arange(0, end_time + Ts_func, Ts_func)
    for _t in time:
        rat = 2*np.pi/period
        if np.sin(rat*_t) >= 0:
            uc_func.append(upper_set)
        else:
            uc_func.append(lower_set)
    return np.array(uc_func, float)
uc_val = reference_signal()
# plt.plot(np.arange(0, 60 + Ts_val, Ts_val), uc)
# plt.show()
```

In [16]:

```python
omega_n = 1
zeta = 1
std = 0
# std = 0.001
Bmz_tf, Amz_tf = co.tfdata(co.sample_system(co.tf([1], [1, 2*zeta*omega_n, omega_n**2]), method='zoh', Ts=Ts_va
AM1 = Amz_tf[0][0][1]
AM2 = Amz_tf[0][0][2]
# AM1 = 2*zeta*omega_n
# AM2 = omega_n**2
AM_SUM = 1 + AM1 + AM2
A0 = np.array([1])
lam = 0.98
initial_P_weights = [10000]*4
# initial_P_weights = [100, 100, 10, 10]
theta = np.array(pulse_coeffs, float).reshape(4, -1)
```

In [17]:

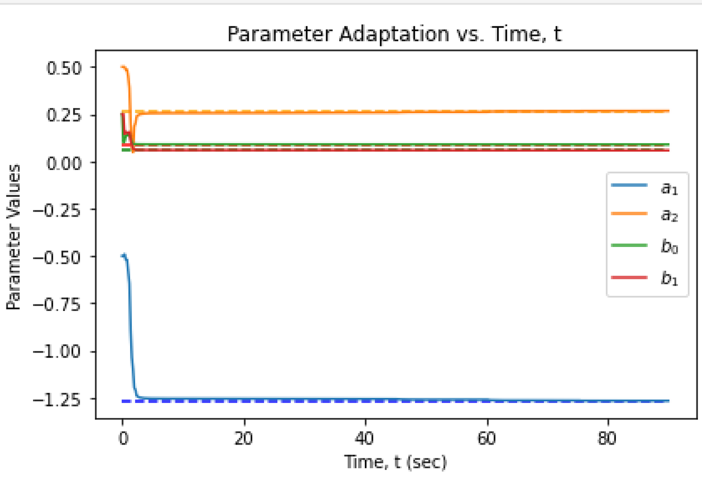
```python
    # Estimates k = 0
time_ns = time.time_ns()
theta_hat = np.array([-0.5, 0.5, 0.25, 0.25], float).reshape(4, -1) # 0: a1 1: a2 2: b0 3: b1
# theta_hat = np.array([-0.5, 0.5, 0.5, 0.5], float).reshape(4, -1)
# theta_hat = np.ones((4,1), float)
theta_arr = theta_hat
P = np.diag(initial_P_weights)
phi = np.zeros((4,1))

y_measure = (phi.T@theta + np.random.normal(0, std)).reshape(-1,)

r0_est = theta_hat[3]
s0_est = A0*AM1 - theta_hat[0]
s1_est = A0*AM2 - theta_hat[1]
t0_est = A0*AM_SUM

M = np.array([r0_est, s0_est, s1_est, t0_est], float).reshape(-1, 1)
N = np.array([0, -y_measure[0], 0, uc_val[0]], float).reshape(M.shape)
u_val = 1/theta[2]*(N.T@M).reshape(-1,)

    # Estimates k = 1
phi = np.array([-y_measure[0], 0, u_val[0], 0], float).reshape(-1,1) # phi of 0
K = P@phi@inv(lam + phi.T@P@phi)
y_measure = np.concatenate((y_measure,
                            (phi.T@theta + np.random.normal(0, std)).reshape(-1,)))
theta_hat = theta_hat + K@(y_measure[-1] - phi.T@theta_hat)
theta_arr = np.concatenate((theta_arr,
                            theta_hat.reshape(-1, 1)), axis=1)
P = (np.eye(len(phi)) - K@phi.T) @P/lam

r0_est = theta_hat[3]
s0_est = A0*AM1 - theta_hat[0]
s1_est = A0*AM2 - theta_hat[1]
t0_est = A0*AM_SUM

M = np.array([r0_est, s0_est, s1_est, t0_est], float).reshape(-1, 1)
N = np.array([-u_val[0], -y_measure[1], -y_measure[0], uc_val[1]],float).reshape(M.shape)
u_val = np.concatenate((u_val,
                        1/theta[2]*(N.T@M).reshape(-1,)))

for k in range(2, len(t)):
        phi = np.array([-y_measure[k-1], -y_measure[k-2], u_val[k-1], u_val[k-2]], float).reshape(-1,1)
        K = P@phi@inv(lam + phi.T@P@phi)
        y_measure = np.concatenate((y_measure,
                                    (phi.T@theta + np.random.normal(0, std)).reshape(-1,)))
        theta_hat = theta_hat + K@(y_measure[-1] - phi.T@theta_hat)
        theta_arr = np.concatenate((theta_arr,
                                    theta_hat.reshape(-1, 1)), axis=1)
        P = (np.eye(len(phi)) - K@phi.T)@P/lam

        r0_est = theta_hat[3]
        s0_est = A0*AM1 - theta_hat[0]
        s1_est = A0*AM2 - theta_hat[1]
        t0_est = A0*AM_SUM

        M = np.array([r0_est, s0_est, s1_est, t0_est], float).reshape(-1, 1)
        N = np.array([-u_val[k-1], -y_measure[k], -y_measure[k-1], uc_val[k]]).reshape(M.shape)
        u_val = np.concatenate((u_val,
                                1/theta[2]*(N.T@M).reshape(-1,)))
(time.time_ns() - time_ns)*1e-9/len(t)*1e6
```

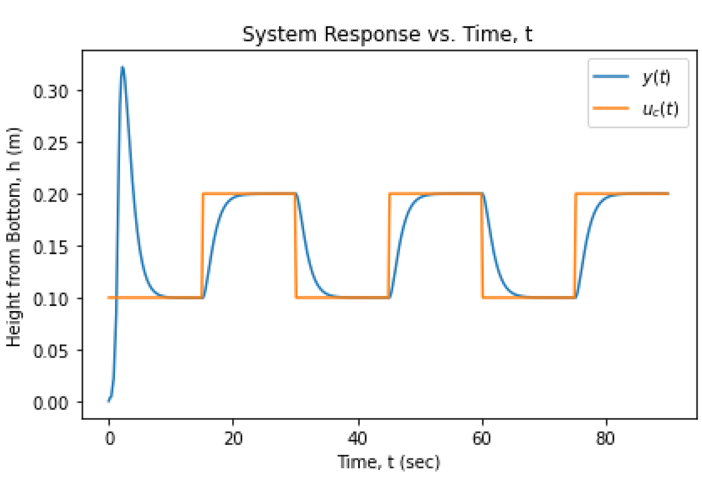
Out[17]:

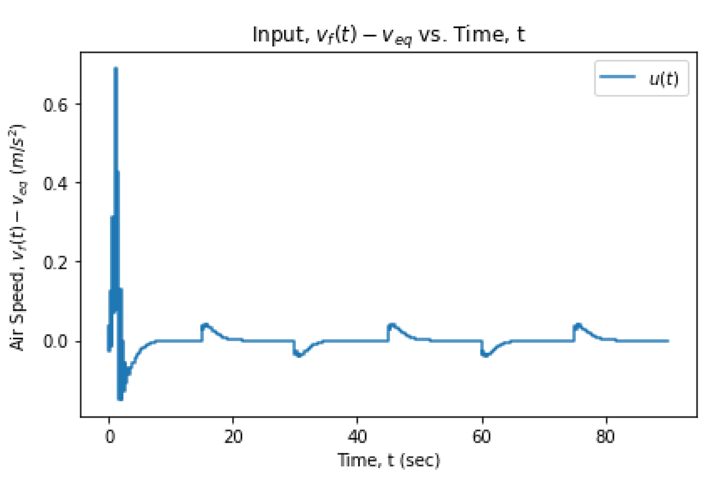
```
67.62372505543237
```

In [18]:

```python
filepath = 'C:/Users/dboas065/University of Ottawa/ELG7113/system_models/derek_model/with_pictures/'
colors = ['blue', 'orange', 'red', 'green']
for row in range(len(theta_arr)):
    plt.plot(t, theta_arr[row,:])
    plt.hlines(theta[row][0], 0, t[len(t) - 1], colors=colors[row], linestyles='--')
plt.title('Parameter Adaptation vs. Time, t')
plt.xlabel('Time, t (sec)')
plt.ylabel('Parameter Values')
plt.legend(['$a_1$', '$a_2$', '$b_0$','$b_1$'])
plt.tight_layout()
# plt.savefig(filepath+'adaptation.pdf', format='pdf')
plt.savefig(filepath+'adaptation_noise.pdf', format='pdf')
plt.show()

st = 0
et = len(t)

plt.plot(t[st:et], y_measure[st:et])
plt.plot(t[st:et], uc_val[st:et])
plt.title('System Response vs. Time, t')
plt.xlabel('Time, t (sec)')
plt.ylabel('Height from Bottom, h (m)')
plt.legend(['$y(t)$', '$u_c(t)$'])
plt.tight_layout()
# plt.savefig(filepath+'output.pdf', format='pdf')
plt.savefig(filepath+'output_noise.pdf', format='pdf')
plt.show()

plt.step(t[st:et], u_val[st:et])
plt.title('Input, $v_f(t) - v_{eq}$ vs. Time, t')
plt.xlabel('Time, t (sec)')
plt.ylabel('Air Speed, $v_f(t) - v_{eq}$ ($m/s^2$)')
plt.legend(['$u(t)$'])
plt.tight_layout()
# plt.savefig(filepath+'input.pdf', format='pdf')
plt.savefig(filepath+'input_noise.pdf', format='pdf')
plt.show()
```







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