```python
def main():
    Print_Function()
    (x, y, z) = xyz = symbols('x,y,z',real=True)
    (o3d, ex, ey, ez) = Ga.build('e_x e_y e_z', g=[1, 1, 1], coords=xyz)
    grad = o3d.grad
    (u, v) = uv = symbols('u,v',real=True)
    (g2d, eu, ev) = Ga.build('e_u e_v', coords=uv)
    grad_uv = g2d.grad
    v_xyz = o3d.mv('v','vector')
    A_xyz = o3d.mv('A','vector',f=True)
    A_uv = g2d.mv('A','vector',f=True)
    print '#3d orthogonal ($A$ is vector function)'
    print 'A =', A_xyz
    print '%A^{2} =', A_xyz * A_xyz
    print 'grad|A =', grad | A_xyz
    print 'grad*A =', grad * A_xyz
    print 'v|(grad*A) =',v_xyz|(grad*A_xyz)
    print '#2d general ($A$ is vector function)'
    print 'A =', A_uv
    print '%A^{2} =', A_uv * A_uv
    print 'grad|A =', grad_uv | A_uv
    print 'grad*A =', grad_uv * A_uv
    A = o3d.lt('A')
    print '#3d orthogonal ($A,\\;B$ are linear transformations)'
    print 'A =', A
    print '\\f{\\det}{A} =', A.det()
    print '\\overline{A} =', A.adj()
    print '\\f{\\Tr}{A} =', A.tr()
    print '\\f{A}{e_x^e_y} =', A(ex^ey)
    print '\\f{A}{e_x}^\\f{A}{e_y} =', A(ex)^A(ey)
    B = o3d.lt('B')
    print 'A + B =', A + B
    print 'AB =', A * B
    print 'A - B =', A - B
    print 'General Symmetric Linear Transformation'
    Asym = o3d.lt('A',mode='s')
    print 'A =', Asym
    print 'General Antisymmetric Linear Transformation'
    Aasym = o3d.lt('A',mode='a')
    print 'A =', Aasym

    print '#2d general ($A,\\;B$ are linear transformations)'
    A2d = g2d.lt('A')
    print 'A =', A2d
    print '\\f{\\det}{A} =', A2d.det()
    print '\\overline{A} =', A2d.adj()
    print '\\f{\\Tr}{A} =', A2d.tr()
    print '\\f{A}{e_u^e_v} =', A2d(eu^ev)
    print '\\f{A}{e_u}^\\f{A}{e_v} =', A2d(eu)^A2d(ev)
    B2d = g2d.lt('B')
    print 'B =', B2d
    print 'A + B =', A2d + B2d
    print 'AB =', A2d * B2d
    print 'A - B =', A2d - B2d
    a = g2d.mv('a','vector')
    b = g2d.mv('b','vector')
    print r'a|\f{\overline{A}}{b}-b|\f{\underline{A}}{a} =',((a|A2d.adj()(b))-(b|A2d(a))).simplify()
    m4d = Ga('e_t e_x e_y e_z', g=[1, -1, -1, -1],coords=symbols('t,x,y,z',real=True))
    T = m4d.lt('T')
```

```
    print 'g =', m4d.g
    print r'\underline{T} =',T
    print r'\overline{T} =',T.adj()
    print r'\f{\det}{\underline{T}} =',T.det()
    print r'\f{\mbox{tr}}{\underline{T}} =',T.tr()
    a = m4d.mv('a','vector')
    b = m4d.mv('b','vector')
    print r'a|\f{\overline{T}}{b}-b|\f{\underline{T}}{a} =',((a|T.adj()(b))-(b|T(a))).simplify()
    coords = (r, th, phi) = symbols('r,theta,phi', real=True)
    (sp3d, er, eth, ephi) = Ga.build('e_r e_th e_ph', g=[1, r**2, r**2*sin(th)**2], coords=coords)
    grad = sp3d.grad
    sm_coords = (u, v) = symbols('u,v', real=True)
    smap = [1, u, v]  # Coordinate map for sphere of r = 1
    sph2d = sp3d.sm(smap,sm_coords,norm=True)
    (eu, ev) = sph2d.mv()
    grad_uv = sph2d.grad
    F = sph2d.mv('F','vector',f=True)
    f = sph2d.mv('f','scalar',f=True)
    print 'f =',f
    print 'grad*f =',grad_uv * f
    print 'F =',F
    print 'grad*F =',grad_uv * F
    tp = (th,phi) = symbols('theta,phi',real=True)
    smap = [sin(th)*cos(phi),sin(th)*sin(phi),cos(th)]
    sph2dr = o3d.sm(smap,tp,norm=True)
    (eth, ephi) = sph2dr.mv()
    grad_tp = sph2dr.grad
    F = sph2dr.mv('F','vector',f=True)
    f = sph2dr.mv('f','scalar',f=True)
    print 'f =',f
    print 'grad*f =',grad_tp * f
    print 'F =',F
    print 'grad*F =',grad_tp * F
    return
```

Code Output: 3d orthogonal (A is vector function)

$$A = A^{x}\boldsymbol{e}_{x} + A^{y}\boldsymbol{e}_{y} + A^{z}\boldsymbol{e}_{z}$$

$$A^{2} = \left(A^{x}\right)^{2} + \left(A^{y}\right)^{2} + \left(A^{z}\right)^{2}$$

$$\boldsymbol{\nabla} \cdot A = \partial_{x} A^{x} + \partial_{y} A^{y} + \partial_{z} A^{z}$$

$$\boldsymbol{\nabla} A = \left(\partial_{x} A^{x} + \partial_{y} A^{y} + \partial_{z} A^{z}\right) + \left(-\partial_{y} A^{x} + \partial_{x} A^{y}\right) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{y} + \left(-\partial_{z} A^{x} + \partial_{x} A^{z}\right) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{z} + \left(-\partial_{z} A^{y} + \partial_{y} A^{z}\right) \boldsymbol{e}_{y}\wedge \boldsymbol{e}_{z}$$

$$v\cdot (\boldsymbol{\nabla} A) = \left(v^{y} \partial_{y} A^{x} - v^{y} \partial_{x} A^{y} + v^{z} \partial_{z} A^{x} - v^{z} \partial_{x} A^{z}\right) \boldsymbol{e}_{x} + \left(-v^{x} \partial_{y} A^{x} + v^{x} \partial_{x} A^{y} + v^{z} \partial_{z} A^{y} - v^{z} \partial_{y} A^{z}\right) \boldsymbol{e}_{y} + \left(-v^{x} \partial_{z} A^{x} + v^{x} \partial_{x} A^{z} - v^{y} \partial_{z} A^{y} + v^{y} \partial_{y} A^{z}\right) \boldsymbol{e}_{z}$$

2d general (A is vector function)

$$A = A^{u}\boldsymbol{e}_{u} + A^{v}\boldsymbol{e}_{v}$$

$$A^{2} = \left(e_{u}\cdot e_{u}\right) \left(A^{u}\right)^{2} + 2 \left(e_{u}\cdot e_{v}\right) A^{u} A^{v} + \left(e_{v}\cdot e_{v}\right) \left(A^{v}\right)^{2}$$

$$\boldsymbol{\nabla} \cdot A = \partial_{u} A^{u} + \partial_{v} A^{v}$$

$$\boldsymbol{\nabla} A = \left(\partial_{u} A^{u} + \partial_{v} A^{v}\right) + \frac{- \left(e_{u}\cdot e_{u}\right) \partial_{v} A^{u} + \left(e_{u}\cdot e_{v}\right) \partial_{u} A^{u} - \left(e_{u}\cdot e_{v}\right) \partial_{v} A^{v} + \left(e_{v}\cdot e_{v}\right) \partial_{u} A^{v}}{\left(e_{u}\cdot e_{u}\right) \left(e_{v}\cdot e_{v}\right) - \left(e_{u}\cdot e_{v}\right)^{2}} \boldsymbol{e}_{u}\wedge \boldsymbol{e}_{v}$$

3d orthogonal (A, B are linear transformations)

$$A = \left\{ \begin{array}{ll} L \left (\boldsymbol{e}_{x} \right) = & A_{xx}\boldsymbol{e}_{x} + A_{yx}\boldsymbol{e}_{y} + A_{zx}\boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{y} \right) = & A_{xy}\boldsymbol{e}_{x} + A_{yy}\boldsymbol{e}_{y} + A_{zy}\boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{z} \right) = & A_{xz}\boldsymbol{e}_{x} + A_{yz}\boldsymbol{e}_{y} + A_{zz}\boldsymbol{e}_{z} \end{array} \right\}$$

$$\det\left(A\right) = A_{xz} \left(A_{yx} A_{zy} - A_{yy} A_{zx}\right) - A_{yz} \left(A_{xx} A_{zy} - A_{xy} A_{zx}\right) + A_{zz} \left(A_{xx} A_{yy} - A_{xy} A_{yx}\right)$$

$$\overline{A} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{x}\right) = & A_{xx}\boldsymbol{e}_{x} + A_{xy}\boldsymbol{e}_{y} + A_{xz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & A_{yx}\boldsymbol{e}_{x} + A_{yy}\boldsymbol{e}_{y} + A_{yz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & A_{zx}\boldsymbol{e}_{x} + A_{zy}\boldsymbol{e}_{y} + A_{zz}\boldsymbol{e}_{z} \end{array} \right\}$$

$$\mathrm{Tr}\left(A\right) = A_{xx} + A_{yy} + A_{zz}$$

$$A\left(e_{x} \wedge e_{y}\right) = \left(A_{xx}A_{yy} - A_{xy}A_{yx}\right)\boldsymbol{e}_{x} \wedge \boldsymbol{e}_{y} + \left(A_{xx}A_{zy} - A_{xy}A_{zx}\right)\boldsymbol{e}_{x} \wedge \boldsymbol{e}_{z} + \left(A_{yx}A_{zy} - A_{yy}A_{zx}\right)\boldsymbol{e}_{y} \wedge \boldsymbol{e}_{z}$$

$$A\left(e_{x}\right) \wedge A\left(e_{y}\right) = \left(A_{xx}A_{yy} - A_{xy}A_{yx}\right)\boldsymbol{e}_{x} \wedge \boldsymbol{e}_{y} + \left(A_{xx}A_{zy} - A_{xy}A_{zx}\right)\boldsymbol{e}_{x} \wedge \boldsymbol{e}_{z} + \left(A_{yx}A_{zy} - A_{yy}A_{zx}\right)\boldsymbol{e}_{y} \wedge \boldsymbol{e}_{z}$$

$$A + B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{x}\right) = & \left(A_{xx} + B_{xx}\right)\boldsymbol{e}_{x} + \left(A_{yx} + B_{yx}\right)\boldsymbol{e}_{y} + \left(A_{zx} + B_{zx}\right)\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & \left(A_{xy} + B_{xy}\right)\boldsymbol{e}_{x} + \left(A_{yy} + B_{yy}\right)\boldsymbol{e}_{y} + \left(A_{zy} + B_{zy}\right)\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & \left(A_{xz} + B_{xz}\right)\boldsymbol{e}_{x} + \left(A_{yz} + B_{yz}\right)\boldsymbol{e}_{y} + \left(A_{zz} + B_{zz}\right)\boldsymbol{e}_{z} \end{array} \right\}$$

$$AB = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{x}\right) = & \left(A_{xx}B_{xx} + A_{xy}B_{yx} + A_{xz}B_{zx}\right)\boldsymbol{e}_{x} + \left(A_{yx}B_{xx} + A_{yy}B_{yx} + A_{yz}B_{zx}\right)\boldsymbol{e}_{y} + \left(A_{zx}B_{xx} + A_{zy}B_{yx} + A_{zz}B_{zx}\right)\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & \left(A_{xx}B_{xy} + A_{xy}B_{yy} + A_{xz}B_{zy}\right)\boldsymbol{e}_{x} + \left(A_{yx}B_{xy} + A_{yy}B_{yy} + A_{yz}B_{zy}\right)\boldsymbol{e}_{y} + \left(A_{zx}B_{xy} + A_{zy}B_{yy} + A_{zz}B_{zy}\right)\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & \left(A_{xx}B_{xz} + A_{xy}B_{yz} + A_{xz}B_{zz}\right)\boldsymbol{e}_{x} + \left(A_{yx}B_{xz} + A_{yy}B_{yz} + A_{yz}B_{zz}\right)\boldsymbol{e}_{y} + \left(A_{zx}B_{xz} + A_{zy}B_{yz} + A_{zz}B_{zz}\right)\boldsymbol{e}_{z} \end{array} \right\}$$

$$A - B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{x}\right) = & \left(A_{xx} - B_{xx}\right)\boldsymbol{e}_{x} + \left(A_{yx} - B_{yx}\right)\boldsymbol{e}_{y} + \left(A_{zx} - B_{zx}\right)\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & \left(A_{xy} - B_{xy}\right)\boldsymbol{e}_{x} + \left(A_{yy} - B_{yy}\right)\boldsymbol{e}_{y} + \left(A_{zy} - B_{zy}\right)\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & \left(A_{xz} - B_{xz}\right)\boldsymbol{e}_{x} + \left(A_{yz} - B_{yz}\right)\boldsymbol{e}_{y} + \left(A_{zz} - B_{zz}\right)\boldsymbol{e}_{z} \end{array} \right\}$$

$$General Symmetric Linear Transformation$$

$$A = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{x}\right) = & A_{xx}\boldsymbol{e}_{x} + A_{xy}\boldsymbol{e}_{y} + A_{xz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & A_{xy}\boldsymbol{e}_{x} + A_{yy}\boldsymbol{e}_{y} + A_{yz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & A_{xz}\boldsymbol{e}_{x} + A_{yz}\boldsymbol{e}_{y} + A_{zz}\boldsymbol{e}_{z} \end{array} \right\}$$

$$General Antisymmetric Linear Transformation$$

$$A = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{x}\right) = & A_{yx}\boldsymbol{e}_{y} + A_{zx}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & -A_{yx}\boldsymbol{e}_{x} + A_{zy}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & -A_{zx}\boldsymbol{e}_{x} - A_{zy}\boldsymbol{e}_{y} \end{array} \right\}$$

2d general (A, B are linear transformations)

$$A = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{u}\right) = & A_{uu}\boldsymbol{e}_{u} + A_{vu}\boldsymbol{e}_{v} \\ L\left(\boldsymbol{e}_{v}\right) = & A_{uv}\boldsymbol{e}_{u} + A_{vv}\boldsymbol{e}_{v} \end{array} \right\}$$

$$\det\left(A\right) = \frac{A_{uu}A_{vv}\boldsymbol{e}_{u} \wedge \boldsymbol{e}_{v} - A_{uv}A_{vu}\boldsymbol{e}_{u} \wedge \boldsymbol{e}_{v}}{\sqrt{\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right) - \left(e_{u} \cdot e_{v}\right)^{2}}}$$

$$\overline{A} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{u}\right) = & \frac{1}{\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right) - \left(e_{u} \cdot e_{v}\right) \wedge 2}\left(-\left(e_{u} \cdot e_{u}\right)\left(e_{u} \cdot e_{v}\right)A_{uv} + \left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right)A_{uu} - \left(e_{u} \cdot e_{v}\right) \wedge 2A_{vv} + \left(e_{u} \cdot e_{v}\right)\left(e_{v} \cdot e_{v}\right)A_{vu}\right)\boldsymbol{e}_{u} + \frac{1}{\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right) - \left(e_{u} \cdot e_{v}\right) \wedge 2}\left(\left(e_{u} \cdot e_{u}\right) \wedge 2A_{uv} - \left(e_{u} \cdot e_{u}\right)\left(e_{u} \cdot e_{v}\right)A_{uu} + \left(e_{u} \cdot e_{u}\right)\left(e_{u} \cdot e_{v}\right)A_{vv} - \left(\right.\right. \\ L\left(\boldsymbol{e}_{v}\right) = & \frac{1}{\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right) - \left(e_{u} \cdot e_{v}\right)^{2}}\left(-\left(e_{u} \cdot e_{v}\right)^{2}A_{uv} + \left(e_{u} \cdot e_{v}\right)\left(e_{v} \cdot e_{v}\right)A_{uu} - \left(e_{u} \cdot e_{v}\right)\left(e_{v} \cdot e_{v}\right)A_{vv} + \left(e_{v} \cdot e_{v}\right)^{2}A_{vu}\right)\boldsymbol{e}_{u} + \frac{1}{\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right) - \left(e_{u} \cdot e_{v}\right)^{2}}\left(\left(e_{u} \cdot e_{u}\right)\left(e_{u} \cdot e_{v}\right)A_{uv} + \left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right)A_{vv} - \left(e_{u} \cdot e_{v}\right)^{2}A_{uu} - \left(e_{u} \cdot e_{v}\right)\left(e_{v} \cdot e\right.\right. \end{array} \right.$$

$$\mathrm{Tr}\left(A\right) = -\frac{\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right)A_{uu}}{-\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right) + \left(e_{u} \cdot e_{v}\right)^{2}} - \frac{\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right)A_{vv}}{-\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right) + \left(e_{u} \cdot e_{v}\right)^{2}} + \frac{\left(e_{u} \cdot e_{v}\right)^{2}A_{uu}}{-\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right) + \left(e_{u} \cdot e_{v}\right)^{2}} + \frac{\left(e_{u} \cdot e_{v}\right)^{2}A_{vv}}{-\left(e_{u} \cdot e_{u}\right)\left(e_{v} \cdot e_{v}\right) + \left(e_{u} \cdot e_{v}\right)^{2}}$$

$$A\left(e_{u} \wedge e_{v}\right) = \left(A_{uu}A_{vv} - A_{uv}A_{vu}\right)\boldsymbol{e}_{u} \wedge \boldsymbol{e}_{v}$$

$$A\left(e_{u}\right) \wedge A\left(e_{v}\right) = \left(A_{uu}A_{vv} - A_{uv}A_{vu}\right)\boldsymbol{e}_{u} \wedge \boldsymbol{e}_{v}$$

$$B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{u}\right) = & B_{uu}\boldsymbol{e}_{u} + B_{vu}\boldsymbol{e}_{v} \\ L\left(\boldsymbol{e}_{v}\right) = & B_{uv}\boldsymbol{e}_{u} + B_{vv}\boldsymbol{e}_{v} \end{array} \right\}$$

$$A + B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{u}\right) = & \left(A_{uu} + B_{uu}\right)\boldsymbol{e}_{u} + \left(A_{vu} + B_{vu}\right)\boldsymbol{e}_{v} \\ L\left(\boldsymbol{e}_{v}\right) = & \left(A_{uv} + B_{uv}\right)\boldsymbol{e}_{u} + \left(A_{vv} + B_{vv}\right)\boldsymbol{e}_{v} \end{array} \right\}$$

$$AB = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{u}\right) = & \left(A_{uu}B_{uu} + A_{uv}B_{vu}\right)\boldsymbol{e}_{u} + \left(A_{vu}B_{uu} + A_{vv}B_{vu}\right)\boldsymbol{e}_{v} \\ L\left(\boldsymbol{e}_{v}\right) = & \left(A_{uu}B_{uv} + A_{uv}B_{vv}\right)\boldsymbol{e}_{u} + \left(A_{vu}B_{uv} + A_{vv}B_{vv}\right)\boldsymbol{e}_{v} \end{array} \right\}$$

$$A - B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{u}\right) = & \left(A_{uu} - B_{uu}\right)\boldsymbol{e}_{u} + \left(A_{vu} - B_{vu}\right)\boldsymbol{e}_{v} \\ L\left(\boldsymbol{e}_{v}\right) = & \left(A_{uv} - B_{uv}\right)\boldsymbol{e}_{u} + \left(A_{vv} - B_{vv}\right)\boldsymbol{e}_{v} \end{array} \right\}$$

$$a \cdot \overline{A}\left(b\right) - b \cdot \underline{A}\left(a\right) = 0$$

$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\underline{T} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{t}\right) = & T_{tt}\boldsymbol{e}_{t} + T_{xt}\boldsymbol{e}_{x} + T_{yt}\boldsymbol{e}_{y} + T_{zt}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{x}\right) = & T_{tx}\boldsymbol{e}_{t} + T_{xx}\boldsymbol{e}_{x} + T_{yx}\boldsymbol{e}_{y} + T_{zx}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & T_{ty}\boldsymbol{e}_{t} + T_{xy}\boldsymbol{e}_{x} + T_{yy}\boldsymbol{e}_{y} + T_{zy}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & T_{tz}\boldsymbol{e}_{t} + T_{xz}\boldsymbol{e}_{x} + T_{yz}\boldsymbol{e}_{y} + T_{zz}\boldsymbol{e}_{z} \end{array} \right\}$$

$$\overline{T} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{t}\right) = & T_{tt}\boldsymbol{e}_{t} - T_{tx}\boldsymbol{e}_{x} - T_{ty}\boldsymbol{e}_{y} - T_{tz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{x}\right) = & -T_{xt}\boldsymbol{e}_{t} + T_{xx}\boldsymbol{e}_{x} + T_{xy}\boldsymbol{e}_{y} + T_{xz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & -T_{yt}\boldsymbol{e}_{t} + T_{yx}\boldsymbol{e}_{x} + T_{yy}\boldsymbol{e}_{y} + T_{yz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & -T_{zt}\boldsymbol{e}_{t} + T_{zx}\boldsymbol{e}_{x} + T_{zy}\boldsymbol{e}_{y} + T_{zz}\boldsymbol{e}_{z} \end{array} \right\}$$

$$\det\left(\underline{T}\right) = -T_{tz}\left(T_{xt}T_{yx}T_{zy} - T_{xt}T_{yy}T_{zx} - T_{xx}T_{yt}T_{zy} + T_{xx}T_{yy}T_{zt} + T_{xy}T_{yt}T_{zx} - T_{xy}T_{yx}T_{zt}\right) + T_{xz}\left(T_{tt}T_{yx}T_{zy} - T_{tt}T_{yy}T_{zx} - T_{tx}T_{yt}T_{zy} + T_{tx}T_{yy}T_{zt} + T_{ty}T_{yt}T_{zx} - T_{ty}T_{yx}T_{zt}\right) - T_{yz}\left(T_{tt}T_{xx}T_{zy} - T_{tt}T_{xy}T_{zx} - T_{tx}T_{xt}T_{zy} + T_{tx}T_{xy}T_{z}\right.$$

$$\operatorname{tr}\left(\underline{T}\right) = T_{tt} + T_{xx} + T_{yy} + T_{zz}$$

$$a \cdot \overline{T}\left(b\right) - b \cdot \underline{T}\left(a\right) = 0$$

$$f = f$$

$$\boldsymbol{\nabla} f = \partial_{u} f \boldsymbol{e}_{u} + \frac{\partial_{v} f}{\sin\left(u\right)} \boldsymbol{e}_{v}$$

$$F = F^{u}\boldsymbol{e}_{u} + F^{v}\boldsymbol{e}_{v}$$

$$\boldsymbol{\nabla} F = \left(\frac{F^{u}}{\tan\left(u\right)} + \partial_{u} F^{u} + \frac{\partial_{v} F^{v}}{\sin\left(u\right)}\right) + \left(\frac{F^{v}}{\tan\left(u\right)} + \partial_{u} F^{v} - \frac{\partial_{v} F^{u}}{\sin\left(u\right)}\right) \boldsymbol{e}_{u} \wedge \boldsymbol{e}_{v}$$

$$f = f$$

$$\boldsymbol{\nabla} f = \partial_{\theta} f \boldsymbol{e}_{\theta} + \frac{\partial_{\phi} f}{\sin\left(\theta\right)} \boldsymbol{e}_{\phi}$$

$$F = F^{\theta}\boldsymbol{e}_{\theta} + F^{\phi}\boldsymbol{e}_{\phi}$$

$$\boldsymbol{\nabla} F = \left(\frac{F^{\theta}}{\tan\left(\theta\right)} + \partial_{\theta} F^{\theta} + \frac{\partial_{\phi} F^{\phi}}{\sin\left(\theta\right)}\right) + \left(\frac{F^{\phi}}{\tan\left(\theta\right)} + \partial_{\theta} F^{\phi} - \frac{\partial_{\phi} F^{\theta}}{\sin\left(\theta\right)}\right) \boldsymbol{e}_{\theta} \wedge \boldsymbol{e}_{\phi}$$