$$g_{ij} = \left[\begin{array}{ccc} (e_x \cdot e_x) & (e_x \cdot e_y) & (e_x \cdot e_z) \\ (e_x \cdot e_y) & (e_y \cdot e_y) & (e_y \cdot e_z) \\ (e_x \cdot e_z) & (e_y \cdot e_z) & (e_z \cdot e_z) \end{array} \right]$$

$$A = A + A^x \boldsymbol{e}_x + A^y \boldsymbol{e}_y + A^z \boldsymbol{e}_z + A^{xy} \boldsymbol{e}_x \wedge \boldsymbol{e}_y + A^{xz} \boldsymbol{e}_x \wedge \boldsymbol{e}_z + A^{yz} \boldsymbol{e}_y \wedge \boldsymbol{e}_z + A^{xyz} \boldsymbol{e}_x \wedge \boldsymbol{e}_y \wedge \boldsymbol{e}_z$$

$$\begin{aligned} A = & A \\ & + A^x \boldsymbol{e}_x + A^y \boldsymbol{e}_y + A^z \boldsymbol{e}_z \\ & + A^{xy} \boldsymbol{e}_x \wedge \boldsymbol{e}_y + A^{xz} \boldsymbol{e}_x \wedge \boldsymbol{e}_z + A^{yz} \boldsymbol{e}_y \wedge \boldsymbol{e}_z \\ & + A^{xyz} \boldsymbol{e}_x \wedge \boldsymbol{e}_y \wedge \boldsymbol{e}_z \end{aligned}$$

$$\begin{aligned} A = & A \\ & + A^x \boldsymbol{e}_x \\ & + A^y \boldsymbol{e}_y \\ & + A^z \boldsymbol{e}_z \\ & + A^{xy} \boldsymbol{e}_x \wedge \boldsymbol{e}_y \\ & + A^{xz} \boldsymbol{e}_x \wedge \boldsymbol{e}_z \\ & + A^{yz} \boldsymbol{e}_y \wedge \boldsymbol{e}_z \\ & + A^{xyz} \boldsymbol{e}_x \wedge \boldsymbol{e}_y \wedge \boldsymbol{e}_z \end{aligned}$$

$$A_+ = A + A^{xy} \boldsymbol{e}_x \wedge \boldsymbol{e}_y + A^{xz} \boldsymbol{e}_x \wedge \boldsymbol{e}_z + A^{yz} \boldsymbol{e}_y \wedge \boldsymbol{e}_z$$

$$A_- = A^x \boldsymbol{e}_x + A^y \boldsymbol{e}_y + A^z \boldsymbol{e}_z + A^{xyz} \boldsymbol{e}_x \wedge \boldsymbol{e}_y \wedge \boldsymbol{e}_z$$

$$X = X^x \boldsymbol{e}_x + X^y \boldsymbol{e}_y + X^z \boldsymbol{e}_z$$

$$Y = Y^x \boldsymbol{e}_x + Y^y \boldsymbol{e}_y + Y^z \boldsymbol{e}_z$$

$$\begin{aligned} XY = & \left((e_x \cdot e_x) X^x Y^x + (e_x \cdot e_y) X^x Y^y + (e_x \cdot e_y) X^y Y^x + (e_x \cdot e_z) X^x Y^z + (e_x \cdot e_z) X^z Y^x + (e_y \cdot e_y) X^y Y^y + (e_y \cdot e_z) X^y Y^z + (e_y \cdot e_z) X^z Y^y + (e_z \cdot e_z) X^z Y^z \right) \\ & + \left(X^x Y^y - X^y Y^x \right) \boldsymbol{e}_x \wedge \boldsymbol{e}_y + \left(X^x Y^z - X^z Y^x \right) \boldsymbol{e}_x \wedge \boldsymbol{e}_z + \left(X^y Y^z - X^z Y^y \right) \boldsymbol{e}_y \wedge \boldsymbol{e}_z \end{aligned}$$

$$X \wedge Y = \left(X^x Y^y - X^y Y^x \right) \boldsymbol{e}_x \wedge \boldsymbol{e}_y + \left(X^x Y^z - X^z Y^x \right) \boldsymbol{e}_x \wedge \boldsymbol{e}_z + \left(X^y Y^z - X^z Y^y \right) \boldsymbol{e}_y \wedge \boldsymbol{e}_z$$

$$X \cdot Y = (e_x \cdot e_x) X^x Y^x + (e_x \cdot e_y) X^x Y^y + (e_x \cdot e_y) X^y Y^x + (e_x \cdot e_z) X^x Y^z + (e_x \cdot e_z) X^z Y^x + (e_y \cdot e_y) X^y Y^y + (e_y \cdot e_z) X^y Y^z + (e_y \cdot e_z) X^z Y^y + (e_z \cdot e_z) X^z Y^z$$

$$g_{ij} = \left[\begin{array}{cc} (e_x \cdot e_x) & (e_x \cdot e_y) \\ (e_x \cdot e_y) & (e_y \cdot e_y) \end{array} \right]$$

$$X = X^x \boldsymbol{e}_x + X^y \boldsymbol{e}_y$$

$$A = A + A^{xy} \boldsymbol{e}_x \wedge \boldsymbol{e}_y$$

$$X \cdot A = -A^{xy} \left((e_x \cdot e_y) X^x + (e_y \cdot e_y) X^y \right) \boldsymbol{e}_x + A^{xy} \left((e_x \cdot e_x) X^x + (e_x \cdot e_y) X^y \right) \boldsymbol{e}_y$$

$$X \rfloor A = -A^{xy} \left((e_x \cdot e_y) X^x + (e_y \cdot e_y) X^y \right) \boldsymbol{e}_x + A^{xy} \left((e_x \cdot e_x) X^x + (e_x \cdot e_y) X^y \right) \boldsymbol{e}_y$$

$$A \lfloor X = A^{xy} \left((e_x \cdot e_y) X^x + (e_y \cdot e_y) X^y \right) \boldsymbol{e}_x - A^{xy} \left((e_x \cdot e_x) X^x + (e_x \cdot e_y) X^y \right) \boldsymbol{e}_y$$