2D fluid flow between parallel plates

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Introduction

The two dimensional equation of motion for systems of constant density and viscosity is solved using the finite volume method for flows between two parallel plates. For the x component of the equation of motion central differencing is applied to the diffusion terms and the Min-Mod scheme [1] is used to determine the convective flux at the node, i.e.: control volume, boundaries. For the y component of the equation of motion central differencing is applied to both the diffusion and convection terms. Time discretization is semi implicit. Details are given in the next section, "Model equations".

Model equations

The partially discrete equation of motion for systems of constant density and viscosity is:

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t}\rho = [-\nabla p - \nabla \cdot \rho \mathbf{u}\mathbf{u} - \nabla \cdot \boldsymbol{\tau}]^n - \nabla \cdot \boldsymbol{\tau}^{n+1} \tag{1}$$

Where $\mathbf{u}$ is the fluid velocity ρ the fluid density, Δt is the size of the timestep, μ the viscosity, p the pressure, $\mathbf{uu}$ the convective flux tensor, $\boldsymbol{\tau}$ the stress tensor , t time and n the timestep. The stress tensor is split into an implicit part and an explicit part, thereby decoupling the dependencies on other dependent variables:

$$\boldsymbol{\tau} = \boldsymbol{\tau}^{n+1} + \boldsymbol{\tau}^n = \begin{bmatrix} -2\mu\frac{\partial u_x^{n+1}}{\partial x} & -\mu(\frac{\partial u_y^{n+1}}{\partial x} + \frac{\partial u_x^n}{\partial y}) \\ -\mu(\frac{\partial u_x^{n+1}}{\partial y} + \frac{\partial u_y^n}{\partial x}) & -2\mu\frac{\partial u_y^{n+1}}{\partial y} \end{bmatrix}$$

The no slip condition applies to the boundary walls. At the outlet boundary the velocity gradients are equal to zero.

Discretization

The fully discrete form of equation (1) for the x velocity u_x is:

$$\begin{aligned}\rho\frac{\Delta u_x}{\Delta t} = & \frac{(\rho u_x u_x|_x - \rho u_x u_x|_{x+\Delta x})}{\Delta x} + \frac{(\rho u_y u_x|_y - \rho u_y u_x|_{y+\Delta y})}{\Delta y} \\ & + \frac{(\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x})}{\Delta x} + \frac{(\tau_{yx}|_y - \tau_{yx}|_{y+\Delta y})}{\Delta y} + \frac{(p|_x - p|_{x+\Delta x})}{\Delta x}\end{aligned} \tag{2}$$

The fully discrete form of equation (1) for the y velocity u_y is:

$$\begin{aligned}\rho\frac{\Delta u_y}{\Delta t} = & \frac{(\rho u_y u_y|_y - \rho u_y u_y|_{y+\Delta y})}{\Delta y} + \frac{(\rho u_x u_y|_x - \rho u_x u_y|_{x+\Delta x})}{\Delta x} \\ & + \frac{(\tau_{yy}|_y - \tau_{yy}|_{y+\Delta y})}{\Delta y} + \frac{(\tau_{xy}|_x - \tau_{xy}|_{x+\Delta x})}{\Delta x} + \frac{(p|_y - p|_{y+\Delta y})}{\Delta y}\end{aligned} \tag{3}$$

Verification

In order to verify that the computation of the flow field is being performed correctly results obtained from simulation are compared with the case of steady flow between parallel plates, for which an analytical solution can be obtained:

$$u_x = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - h^2) \tag{4}$$

Where h is half the width of the domain in the y direction. Comparison of results obtained with a grid resolution of 60 nodes in the x direction and 25 nodes in the y direction shows that the discrepancy between numerical results and analytical solution is in the order of 0.3 %.

References

[1] H.K. Versteeg, W. Malalasekara. An introduction to computational fluid dynamics the finite volume method. 2007