2D fluid flow between parallel plates

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Introduction

The two dimensional equation of motion for systems of constant density and viscosity is solved using the finite volume method for flows between two parallel plates. Central differencing is applied to the diffusion terms and the Min-Mod scheme is used to determine the convective flux at the node, i.e.: control volume, boundaries. Time discretization is semi implicit. Details are given in the next section, "**??**".

Model equations

The equation of motion for systems of constant density and viscosity:

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t}\rho = [-\nabla p - \nabla \cdot \mathbf{uu} - \nabla \cdot \boldsymbol{\tau}]^n - \nabla \cdot \boldsymbol{\tau}^{n+1} \tag{1}$$

Where $\mathbf{u}$ is the fluid velocity ρ the fluid density, Δt is the size of the timestep, μ the viscosity, p the pressure, $\mathbf{uu}$ the convective flux tensor, $\boldsymbol{\tau}$ the stress tensor , t time and n the timestep. The stress tensor is split into an implicit part and an explicit part, thereby decoupling the dependencies on other dependent variables:

$$\boldsymbol{\tau} = \boldsymbol{\tau}^{n+1} + \boldsymbol{\tau}^n = \begin{bmatrix} -2\mu\frac{\partial u_x^{n+1}}{\partial x} & -\mu(\frac{\partial u_y^{n+1}}{\partial x} + \frac{\partial u_x^n}{\partial y}) \\ -\mu(\frac{\partial u_x^{n+1}}{\partial y} + \frac{\partial u_y^n}{\partial x}) & -2\mu\frac{\partial u_y^{n+1}}{\partial y} \end{bmatrix}$$

The no slip condition applies to the boundary walls. At the outlet boundary the velocity gradients are equal to zero.

Discretization

The discrete form of equation (**??**) for the x velocity u_x is:

$$\begin{aligned} \rho\frac{\Delta u_x}{\Delta t} = & \frac{(\rho u_x u_x|_x - \rho u_x u_x|_{x+\Delta x})}{\Delta x} + \frac{(\rho u_y u_x|_y - \rho u_y u_x|_{y+\Delta y})}{\Delta y} \\ & + \frac{(\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x})}{\Delta x} + \frac{(\tau_{yx}|_y - \tau_{yx}|_{y+\Delta y})}{\Delta y} + \frac{(p|_x - p|_{x+\Delta x})}{\Delta x} \end{aligned} \tag{2}$$

The discrete form of equation (**??**) for the y velocity u_y is:

$$\begin{aligned} \rho\frac{\Delta u_y}{\Delta t} = & \frac{(\rho u_y u_y|_y - \rho u_y u_y|_{y+\Delta y})}{\Delta y} + \frac{(\rho u_x u_y|_x - \rho u_x u_y|_{x+\Delta x})}{\Delta x} \\ & + \frac{(\tau_{yy}|_y - \tau_{yy}|_{y+\Delta y})}{\Delta y} + \frac{(\tau_{xy}|_x - \tau_{xy}|_{x+\Delta x})}{\Delta x} + \frac{(p|_y - p|_{y+\Delta y})}{\Delta y} \end{aligned} \tag{3}$$

Verification

In order to verify that the computation of the flow field is being performed correctly results obtained from simulation are compared with the case of steady flow between parallel plates, for which an analytical solution can be obtained:

$$u_x = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - h^2) \tag{4}$$

In the equation above the y coordinate is relative to the center of the domain, whereas in numerical computations y is relative to the location of the bottom plate. Consequently results obtained from numerical computations must be shifted. Comparison of results obtained with a grid resolution of 60 nodes in the x direction and 25 nodes in the y direction shows that the discrepancy between numerical results and analytical solution is in the order of 0.3 %.