# Orbital dynamics

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## Introduction

The motion of a set of objects due to gravity, similar to those encountered in the solar system, is simulated.

## Model equations

Newton's law of gravity applied to a set of objects interacting with one another gives:

$$\mathbf{F}_p = \sum_{k=1}^{n} G \frac{m_p \cdot m_k}{|\mathbf{r}|^2} \frac{\mathbf{r}}{|\mathbf{r}|} \tag{1}$$

Where $\mathbf{F}_p$ is the force exerted on object $p$ by all the other objects $k$, $n$ is the number of objects, $G$ is the gravitational constant, $m_p$ is the mass of object $p$, $m_k$ is the mass of object $k$ and $\mathbf{r}$ the relative position vector $\mathbf{r}_k - \mathbf{r}_p$ with $\mathbf{r}_p$ the position vector of particle $p$ and $\mathbf{r}_k$ the position vector of particle $k$.

The force acting on a given object $p$ is equal to the product of its mass and acceleration:

$$\mathbf{F}_p = m_p \frac{d\mathbf{v}_p}{dt} \tag{2}$$

Where $\mathbf{v}_p$ is the velocity of particle $p$ and $t$ time. Equating (1) and (2) gives:

$$m_p \frac{d\mathbf{v}_p}{dt} = \sum_{k=1}^{n} G \frac{m_p \cdot m_k}{|\mathbf{r}|^2} \frac{\mathbf{r}}{|\mathbf{r}|} \tag{3}$$

Equation (3) is used to update the velocities of the objects:

$$\mathbf{v}_{p,j+1} = \mathbf{v}_{p,j} + \delta \mathbf{v}_p = \mathbf{v}_{p,j} + \sum_{k=1}^{n} G \frac{m_k}{|\mathbf{r}|^2} \frac{\mathbf{r}}{|\mathbf{r}|} \delta t \tag{4}$$

Where $j$ specifies the timestep. The positions of the objects are updated as follows:

$$\mathbf{r}_{p,j+1} = \mathbf{r}_{p,j} + \mathbf{v}_{p,j} \delta t \tag{5}$$