Orbital dynamics

Derek W. Harrison

October 7, 2020

Introduction

Orbital dynamics simulation.

Model equations

Newton's law of gravity applied to a set of objects interacting with one another gives:

$$\mathbf{F}_p = \sum_{k=1}^{n} G \frac{m_p \cdot m_k}{|\mathbf{r}|^2} \frac{\mathbf{r}}{|\mathbf{r}|} \tag{1}$$

Where $\mathbf{F}_p$ is the force exerted on object p by all the other objects k, G is the gravitational constant, m_p is the mass of object p, m_k is the mass of object k and $\mathbf{r}$ the relative position vector $\mathbf{r}_k - \mathbf{r}_p$ with $\mathbf{r}_p$ the position vector of particle p and $\mathbf{r}_k$ the position vector of particle k.

The force acting on a given object p is equal to the product of its mass and acceleration:

$$\mathbf{F}_p = m_p \frac{d\mathbf{v}_p}{dt} \tag{2}$$

Where $\mathbf{v}_p$ is the velocity of particle p and t time. Equating (1) and (2) gives:

$$m_p \frac{d\mathbf{v}_p}{dt} = \sum_{k=1}^{n} G \frac{m_p \cdot m_k}{|\mathbf{r}|^2} \frac{\mathbf{r}}{|\mathbf{r}|} \tag{3}$$

Equation (3) is used to update the velocities and positions of the objects:

$$\mathbf{v}_{p,j+1} = \mathbf{v}_{p,j} + \delta\mathbf{v}_p = \mathbf{v}_{p,j} + \sum_{k=1}^{n} G \frac{m_k}{|\mathbf{r}|^2} \frac{\mathbf{r}}{|\mathbf{r}|} \delta t \tag{4}$$

Where j specifies the timestep. The positions of the objects are updated as follows:

$$\mathbf{r}_{p,n+1} = \mathbf{r}_{p,n} + \mathbf{v}_{p,n} \delta t \tag{5}$$