

Orbital dynamics

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Introduction

The motion of a set of objects due to gravity, similar to those encountered in the solar system, is simulated.

Model equations

Newton's law of gravity applied to a set of objects interacting with one another gives:

$$\mathbf{F}_p = \sum_{k=1}^n G \frac{m_p \cdot m_k}{|\mathbf{r}|^2} \frac{\mathbf{r}}{|\mathbf{r}|} \quad (1)$$

Where \mathbf{F}_p is the force exerted on object p by all the other objects k , n is the number of objects, G is the gravitational constant, m_p is the mass of object p , m_k is the mass of object k and \mathbf{r} the relative position vector $\mathbf{r}_k - \mathbf{r}_p$ with \mathbf{r}_p the position vector of object p and \mathbf{r}_k the position vector of object k .

The force acting on a given object p is equal to the product of its mass and acceleration:

$$\mathbf{F}_p = m_p \frac{d\mathbf{v}_p}{dt} \quad (2)$$

Where \mathbf{v}_p is the velocity of object p and t time. Equating (1) and (2) gives:

$$m_p \frac{d\mathbf{v}_p}{dt} = \sum_{k=1}^n G \frac{m_p \cdot m_k}{|\mathbf{r}|^2} \frac{\mathbf{r}}{|\mathbf{r}|} \quad (3)$$

Equation (3) is used to update the velocities of the objects:

$$\mathbf{v}_{p,j+1} = \mathbf{v}_{p,j} + \delta\mathbf{v}_p = \mathbf{v}_{p,j} + \sum_{k=1}^n G \frac{m_k}{|\mathbf{r}|^2} \frac{\mathbf{r}}{|\mathbf{r}|} \delta t \quad (4)$$

Where j specifies the timestep. The positions of the objects are updated as follows:

$$\mathbf{r}_{p,j+1} = \mathbf{r}_{p,j} + \mathbf{v}_{p,j} \delta t \quad (5)$$