Numerical solution of a linear first order complex differential equation

Derek W. Harrison

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Introduction

A linear first order complex differential equation is solved numerically using the finite volume method. Time discretization is fully implicit.

Model equations

The following complex differential equation is solved:

$$ih\frac{d\psi}{dt} = \psi \tag{1}$$

Where i is the imaginary number, h a constant, ψ the dependent variable and t time.

Discretization

Since the dependent variable ψ depends only on time t a single node can be used to represent the domain of ψ. The discrete form of equation (1) for this single node is:

$$\psi_p = \frac{\psi_{p-1}}{\Delta t}\frac{(\frac{h^2}{\Delta t} - ih)}{1 + \frac{h^2}{\Delta t^2}} \tag{2}$$

Where the subscript p denotes the timestep.

Verification

In order to verify that the computation is proceeding correctly the numerical solution to equation (1) is compared with the analytical solution to equation (1):

$$\psi = \psi_0(\cos(t/h) - i\sin(t/h)) \tag{3}$$

Where the subscript 0 of ψ_0 denotes the initial value of ψ at $t = 0$. A comparison between numerical and analytical solutions is given in table [1]. Analysis shows that the discrepancy between numerical and analytical results is small.

Table 1: Numerical and analytical data.

t	analytical solution (real)	numerical solution (real)
0.1	1.09484	1.09484
0.2	1.17874	1.17874
0.3	1.25086	1.25086
0.5	1.35701	1.35701
1.0	1.38177	1.38176
2.0	0.49315	0.49311
4.0	-1.41045	-1.41033