Radiance Conversion From W/(cm^2 sr cm^{-1}) to W/(cm^2 sr μm)

Under certain circumstances, MODTRAN outputs at-sensor radiances in units of W/(cm^2 sr cm^{-1}). The use of cm^{-1} traces back to the fields of chemistry and spectroscopy. Generally, the remote sensing community writes radiance in units of W/(cm^2 sr μm). The conversion is not simply multiplying by 10^4 to go from cm to μm. What is really meant by the units on the radiance is

$$L(\nu) = \frac{\text{Watts}}{(\Delta\text{Area})(\Delta\text{Angle})(\Delta\text{wavenumber bin})}$$

or

$$L(\nu) = \frac{W}{(\Delta A)(\Delta\phi)(\Delta\nu)}.$$

When converting to $L(\lambda)$ in W/(cm^2 sr μm) we wish to get

$$L(\lambda) = \frac{\text{Watts}}{(\Delta\text{Area})(\Delta\text{Angle})(\Delta\text{wavelength bin})}$$

or

$$L(\lambda) = \frac{W}{(\Delta A)(\Delta\phi)(\Delta\lambda)}.$$

So what is required is the relationship between $\Delta\nu$ and $\Delta\lambda$.

The first step is to convert the wavenumber to μm^{-1} by $10^4\,\frac{\mu\text{m}}{\text{cm}}$:

$$L(\nu)\left[\frac{\text{W}}{(\text{cm}^2\text{sr}\,\mu\text{m}^{-1})}\right] = L(\nu)\left[\frac{\text{W}}{(\text{cm}^2\text{sr cm}^{-1})} \times 10^4\right].$$

Now, the definition of the wavenumber, ν, is

$$\nu[\,\mu\text{m}^{-1}] = \frac{1}{\lambda[\,\mu\text{m}]}.$$

This provides us with the relationship between $\Delta\nu$ and $\Delta\lambda$. Take the derivative of ν with respect to λ:

$$\frac{d\nu}{d\lambda} = \frac{-1}{\lambda^2}.$$

In this case, the negative sign in the derivative indicates that increasing wavenumbers correspond to decreasing wavelengths. It can be ignored for the rest of this exercise. Similarly, we can write

$$\frac{d\lambda}{d\nu} = \frac{1}{\nu^2}.$$

Note that the wavenumber ν above must now be in μm^{-1}. Now we make the (good) approximation

$$\frac{d\lambda}{d\nu} = \frac{\Delta\lambda}{\Delta\nu}$$

to give us the relationship for the conversion:

$$\begin{aligned} L(\lambda) &= \frac{W}{(\Delta A)(\Delta\phi)(\Delta\nu) \times \frac{\Delta\lambda}{\Delta\nu}} \\ &= L(\nu) \times \frac{\Delta\lambda}{\Delta\nu}^{-1} \\ &= L(\nu) \times \nu^2. \end{aligned}$$

So, the radiance per unit wavelength is dependent on not only the radiance per unit wavenumber, but also *the wavenumber itself.*