

FREQUENTLY ASKED Questions

Q: How can you use the graph of a linear or quadratic function to graph its reciprocal function?

A: If you have the graph of a linear or quadratic function, you can draw the graph of its reciprocal function as follows:

1. Draw a vertical asymptote for the reciprocal function at each zero of the original function. The x -axis is a horizontal asymptote for the reciprocal function, unless the original function is a constant function.
2. The reciprocal function has the same positive/negative intervals that the original function has, so you can shade the regions where there will be no graph.
3. Mark any points where the y -value of the original function is 1 or -1 . The reciprocal function also goes through these points.
4. The y -intercept of the reciprocal function is the reciprocal of the y -intercept of the original function. Determine and mark the y -intercept of the reciprocal function.
5. If the original function is quadratic, the reciprocal function has a local maximum/minimum at the same x -value as the vertex of the quadratic function. The y -value of this local maximum/minimum is the reciprocal of the y -value of the vertex. Determine and mark the local maximum/minimum point.
6. Draw the pieces of the graph of the reciprocal function through the marked points, approaching the asymptotes.

Q: What are rational functions, and what are the characteristics of their graphs?

A: Rational functions are quotients of polynomial functions. Their equations have the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$. Graphs of rational functions may have vertical, horizontal, or oblique asymptotes. Some rational functions have holes in their graphs.

Study Aid

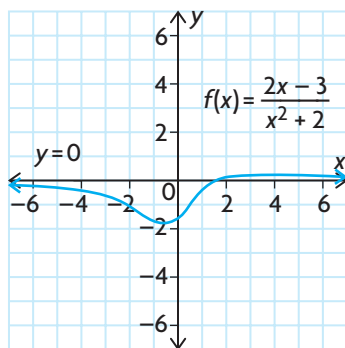
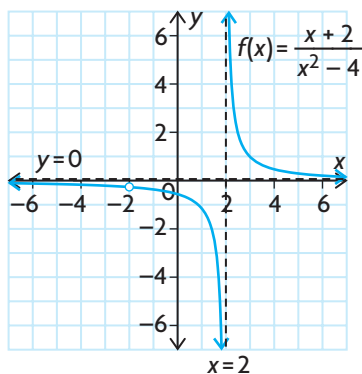
- See Lesson 5.1, Examples 1 and 2.
- Try Mid-Chapter Review Questions 1 and 2.

Study Aid

- See Lesson 5.2.
- Try Mid-Chapter Review Question 3.

Study Aid

- See Lesson 5.2.
- Try Mid-Chapter Review Question 4.

**Study Aid**

- See Lesson 5.3, Example 2.
- Try Mid-Chapter Review Questions 5 to 8.

Q: Most rational functions have one or more discontinuities. Where and why do these discontinuities occur? When is a rational function continuous?

A: If the polynomial function in the denominator of a rational function has one or more zeros, the rational function will be discontinuous at these points. If a value of x can be zero in both the numerator and the denominator of a rational function (that is, if the numerator and denominator have a common linear factor), the result is a hole. This type of discontinuity is called a point discontinuity. If a zero in the denominator does not correspond to a zero in the numerator, there will be a vertical asymptote at the x -value. This is called an infinite discontinuity.

For example, $f(x) = \frac{x+2}{x^2-4} = \frac{x+2}{(x-2)(x+2)}$ has a point

discontinuity where $x = -2$ because -2 is a zero of both the denominator, $q(x) = x^2 - 4$, and the numerator, $p(x) = x + 2$. The graph of $f(x)$ has an infinite discontinuity where $x = 2$ because 2 is a zero of $q(x)$ but not of $p(x)$. The graph also has a hole at $x = -2$ and a vertical asymptote at $x = 2$. Note that $\frac{p(-2)}{q(-2)} = \frac{0}{0}$, but $\frac{p(2)}{q(2)} = \frac{4}{0}$.

If the polynomial function in the denominator of a rational function does not have any zeros, the rational function is continuous. Its graph is a smooth curve, with no breaks.

For example, $f(x) = \frac{2x-3}{x^2+2}$ is a continuous rational function with a horizontal asymptote at $y = 0$.

Q: How do you determine the equations of the vertical and horizontal asymptotes of a rational function of the form

$$f(x) = \frac{b}{cx+d} \text{ and } f(x) = \frac{ax+b}{cx+d}?$$

A: You can determine the equations of the vertical and horizontal asymptotes directly from the equation of a rational function of the form $f(x) = \frac{b}{cx+d}$ or $f(x) = \frac{ax+b}{cx+d}$. The vertical asymptote occurs at the zero of the function in the denominator. The equation of the vertical asymptote is $x = -\frac{d}{c}$. The horizontal asymptote describes the end behaviour of the function when $x \rightarrow \pm\infty$.

All rational functions of the form $f(x) = \frac{ax+b}{cx+d}$ have $y = \frac{a}{c}$ as a horizontal asymptote.

All rational functions of the form $f(x) = \frac{b}{cx+d}$ have $y = 0$ (the x -axis) as a horizontal asymptote.

PRACTICE Questions

Lesson 5.1

- State the reciprocal of each function, and determine the locations of any vertical asymptotes.
 - $f(x) = x - 3$
 - $f(q) = -4q + 6$
 - $f(z) = z^2 + 4z - 5$
 - $f(d) = 6d^2 + 7d - 3$
- For each function, determine the domain and range, intercepts, positive/negative intervals, and intervals of increase/decrease. Use this information to sketch the graphs of the function and its reciprocal.
 - $f(x) = 4x + 6$
 - $f(x) = x^2 - 4$
 - $f(x) = x^2 + 6$
 - $f(x) = -2x - 4$

Lesson 5.2

- Different characteristics of the graph of a rational function are created by different characteristics of the function. List at least four characteristics of a graph and the characteristic of the function that causes each one. Make sure that you include a characteristic of a continuous rational function in your list.
- For each function, determine the equations of any vertical asymptotes, the locations of any holes, and the existence of any horizontal asymptotes (other than the x -axis) or oblique asymptotes.
 - $y = \frac{x}{x - 2}$
 - $y = \frac{x - 1}{3x - 3}$
 - $y = \frac{-7x}{4x + 2}$
 - $y = \frac{x^2 + 2}{x - 6}$
 - $y = \frac{1}{x^2 + 2x - 15}$

Lesson 5.3

- List the functions that had a horizontal asymptote in question 4, and give the equation of the horizontal asymptote.
- For each function, determine the domain, intercepts, asymptotes, and positive/negative intervals. Use these characteristics to sketch the graph of the function. Then describe where the function is increasing or decreasing.
 - $f(x) = \frac{5}{x - 6}$
 - $f(x) = \frac{3x}{x + 4}$
 - $f(x) = \frac{5x + 10}{x + 2}$
 - $f(x) = \frac{x - 2}{2x - 1}$
- Kevin is trying to develop a reciprocal function to model some data that he has. He wants the horizontal asymptote to be $y = 7$. He also wants the graph to decrease and approach $y = 7$ as x approaches infinity, so he chooses the equation $y = \frac{7x + 6}{x}$. Then he decides that he needs the vertical asymptote to be $x = -1$, so he changes the equation to $y = \frac{7x + 6}{x + 1}$. What happened to the graph of Kevin's function? Did it give him the result he wanted? Explain why or why not.
- For the function $f(x) = \frac{7x - m}{2 - nx}$, find the values of m and n such that the vertical asymptote is at $x = 6$ and the x -intercept is 5.
- Create a rational function that has a domain of $\{x \in \mathbf{R} | x \neq -2\}$ and no vertical asymptote. Describe the graph of this function.