SPH4U-C

Lesson 4

Circular Motion

Revised August 31, 2016

Introduction

Transportation has undoubtedly advanced. Not only have vehicles changed, but the means by which people can get themselves to and from different locations has changed as well. Maps and compasses now take a backseat to the Global Positioning System (GPS). Access to the GPS is now very widespread; 24 satellites above our atmosphere are used to send and receive information in order to accurately determine locations of objects to within 15 m of their actual position.

If you have used a GPS, or even a mobile phone or satellite TV or radio, you have used some of the nearly 300 artificial satellites that exist above our atmosphere to send and receive information. Satellites are projectiles that, through our knowledge of force and energy, have been launched into orbit. Because the earth curves downward by approximately 5 metres over 8000 metres along the horizon, satellites launched with a horizontal speed of 8000 m/s can take orbit. At this speed, thanks to earth's curved surface and a satellite's constant horizontal velocity, the satellites, which are in free fall once projected, keep falling toward the earth, but always miss it. Thus, under the influence of gravity, satellites maintain motion in a circular pattern at a uniform speed.

Most satellites are launched using rockets that fall to the ocean when their fuel is spent. Sometimes a satellite may require minor adjustments to its orbit. This is accomplished through the use of built-in rockets, called thrusters. Once placed in the proper orbit, a satellite can stay there for a long time, with its operation and location monitored by computers and human operators at a control centre on earth. Solar panels provide a source of power for some satellites. A satellite keeps its solar panels facing the sun and its antennae ready to receive information. A satellite remains in orbit until its speed decreases and gravity pulls it down into the atmosphere, where it slows down, due to its collisions with air molecules in the atmosphere. As the satellite falls further down into the denser atmosphere, it compresses the air in front of it, causing the air to become so hot that the satellite burns up. In this lesson, you will learn more about the forces that contribute to the motion of a satellite and to other circular motions.

Planning Your Study

You may find this time grid helpful in planning when and how you will work through this lesson.

Suggested Timing for This Lesson (hours)	
Circular Motion	1
Investigation into Circular Motion	1
Problem Solving with Circular Motion	1
Frames of Reference	1
Key Questions	½

What You Will Learn

After completing this lesson, you will be able to

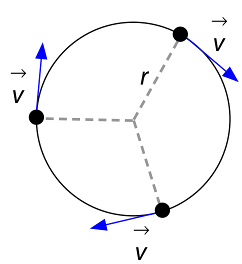
- describe the relationships between variables affecting uniform circular motion
- identify and analyze the forces contributing to uniform circular motion
- distinguish between inertial and non-inertial frames of reference, and real and fictitious (apparent) forces

Circular Motion

Before you begin this lesson, find an object that you can safely tie to a string and swing around in a circle, for example, a yo-yo, or even a belt with a heavy buckle. A heavier object at the end of the rope will give you a better "feel" for the ideas in this lesson, but safety should be your primary concern. Make sure that you have lots of space in which to swing the object and put on some safety glasses or sports goggles, if you have them. Try to swing your object so that it stays on a constant horizontal circular path (as if you were getting ready to lasso an animal). After this, try to make a vertical circular motion. Notice the differences. Can you feel how the tug on your hand changes at different times, in the vertical cycle? That's what this lesson is all about: the forces involved in creating uniform circular motion.

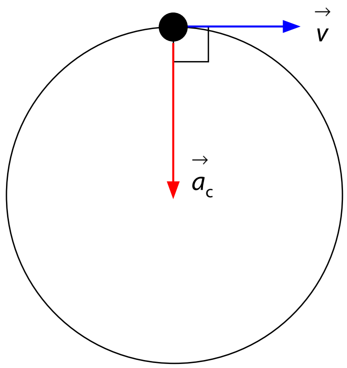
Uniform Circular Motion

Uniform circular motion is motion in which an object travels in a circle, maintaining a constant radius and constant speed. The following diagram shows the path of an object in uniform circular motion and indicates three specific points in its path. The vectors indicate the object's instantaneous velocity. Notice that the length of each vector is the same, indicating that the magnitude of velocity at all locations is the same. The direction of the vectors indicates the direction of the instantaneous velocity at each location, that is, the direction in which the object would travel, if the string to which it is connected suddenly snapped. Therefore, even though the magnitude of the velocity of the ball is not changing, the direction of the velocity is constantly changing, indicating that there is uniform acceleration, despite the constant speed.

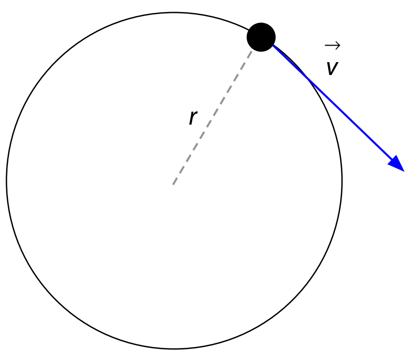


Centripetal Acceleration

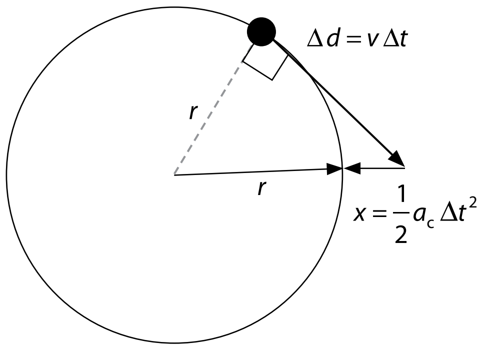
The type of acceleration that results in uniform circular motion is called centripetal acceleration ($\vec{a}_c$), also known as radial acceleration (because it is directed along the radius of the path of motion). Centripetal acceleration is always directed toward the centre of the circular path created by the motion; therefore, it is always perpendicular to the direction of the instantaneous velocity.



Consider an object going around a circle with a constant speed and a constant radius.



If this object, at this particular location in the circular path, had continued to travel in a straight line instead of being forced to maintain a circular path, then, rearranging the basic equation for determining velocity, the displacement would have been equal to $\Delta\vec{d} = \vec{v}\Delta t$. However, the displacement was not equal to $\vec{v}\Delta t$ because the object experienced acceleration in order to stay on the circle. As a result, the object's displacement is actually equal to $\vec{v}\Delta t$, plus the displacement that occurred due to the uniform acceleration: $\frac{1}{2}\vec{a}_c\Delta t^2$. This is based on the kinematics equation you know for finding the displacement of an object that is experiencing uniform acceleration: $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$. Notice that, in the diagram, the direction of the displacement due to acceleration is in the same direction as the centripetal acceleration: toward the centre of the circle. In the rest of this proof, we do not require vector symbols, as we are dealing with magnitudes (scalar values).



By drawing in the various displacement vectors for this object, a right triangle is formed. Notice that the displacement due to the acceleration is being set as equal to x.

Applying the Pythagorean theorem:

$r^2 + \Delta d^2 = (r + x)^2$

Given that $\Delta d = v\Delta t$, the equation can be rewritten as:

$r^2 + (v\Delta t)^2 = (r + x)^2$

Expanding and simplifying, you get:

$r^2 + v^2\Delta t^2 = r^2 + 2rx + x^2$

$v^2\Delta t^2 = 2rx + x^2$

Look at the diagram again. If the two object locations that had been chosen were much closer together, what would happen to x? Can you see that as the two points get closer, x will become smaller? In fact, as the time interval of the change in position approaches zero, x^2 will approach zero faster, becoming negligible. (This actually involves a little bit of calculus.)

Given that it becomes negligible, you can eliminate x^2 to get:

$v^2\Delta t^2 = 2rx$

Solving for x, you get:

$$v^2\Delta t^2 = 2rx$$

$$x = \frac{v^2\Delta t^2}{2r}$$

Recall that:

$$x = \frac{1}{2}a_c \Delta t^2$$

Substituting this into the equation and simplifying, you get an equation for centripetal acceleration:

$$\frac{1}{2}a_c \Delta t^2 = \frac{v^2 \Delta t^2}{2r}$$

$$a_c = \frac{v^2}{r}$$

Summary of Centripetal Acceleration

- The direction of centripetal acceleration is always toward the centre of the circle.
- The direction of the instantaneous velocity is always tangent to the circle.

In previous math courses, you learned that the circumference of a circle is equal to $2\pi r$. Similarly, you may recall that the time required to complete one rotation is referred to as the period of revolution and is given the symbol T. Since the magnitude of velocity of an object is $v = \frac{\Delta d}{\Delta t}$, the magnitude of velocity in uniform circular motion can be calculated through the equation: $v = \frac{2\pi r}{T}$.

Substituting this expression into the centripetal acceleration equation you have derived gives you:

$$a_c = \frac{|v|^2}{r}$$

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

You may also recall that $T = \frac{1}{f}$ where f is the frequency of rotation (that is, how often the rotations happen, measured in hertz). Therefore, if you know the frequency (f) in a situation, you may use the equation:

$$a_c = 4\pi^2 r f^2$$

Equation Summary

You now have three equations for calculating centripetal acceleration:

1. $a_c = \dfrac{v^2}{r}$

2. $a_c = \dfrac{4\pi^2 r}{T^2}$

3. $a_c = 4\pi^2 r f^2$

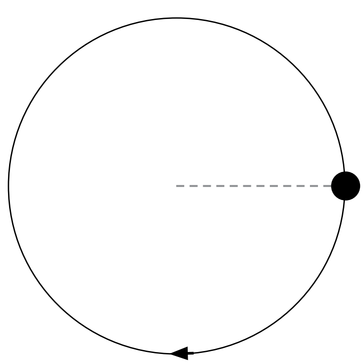
So, if an object is in uniform circular motion, you have the tools to analyze that motion. However, as you learned in the last lesson, where there is acceleration, there must be force. What causes uniform circular motion?

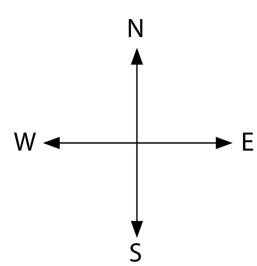
Centripetal Force

According to Newton's second law, whenever there is acceleration, there must be a net force causing that acceleration. The net force that keeps an object moving in uniform circular motion is referred to as centripetal force. Centripetal force is not a new "type" of force; it is simply the sum of the combination of forces that contribute to the circular motion. In other words, it is just another name for net force in a situation where there is uniform circular motion. Regardless of which forces contribute to the centripetal force causing the object to follow a circular path, the direction of this net force is always aimed toward the centre of the circle, in the same direction as the centripetal acceleration.

Example 1

In this diagram of a ball that is rotating clockwise in a horizontal plane, determine the directions of the velocity, centripetal acceleration, and centripetal force at the location indicated.





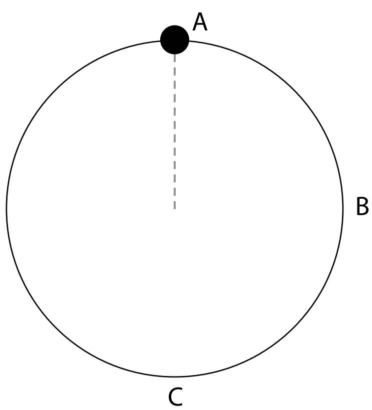
Solution

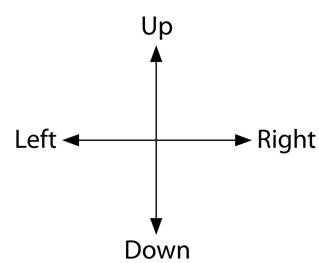
- The velocity is directed south (tangent to the circle and in the direction in which the ball would move if the string broke, at this instant).
- The centripetal acceleration is always directed toward the centre of the circle, which, at this instant, happens to be west.
- The centripetal force is always directed toward the centre of the circle, which, at this instant, happens to be west (the same as acceleration).

Support Questions

Be sure to try the Support Questions on your own before looking at the suggested answers provided. Click on each "Suggested answer" button to check your work.

28. In this diagram of a ball that is rotating clockwise within a vertical plane, determine the directions of the velocity, centripetal acceleration, and centripetal force, at the location indicated.





Suggested answer

Example 2

Draw FBDs for the following objects and state which forces contribute to the centripetal force:

a) A satellite in orbit around the earth

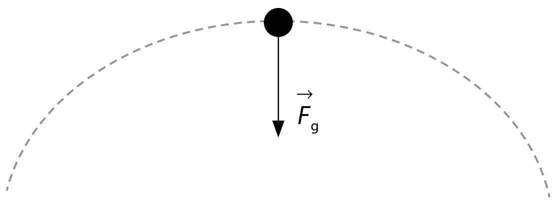
b) A car travelling around a (left-directed) turn on a horizontal road surface

c) A ball on a string being whirled in a circle in the vertical plane, at the bottom of the circle

d) A person on an amusement-park ride who is standing on a platform with her back against a wall and is being rotated in a horizontal circle

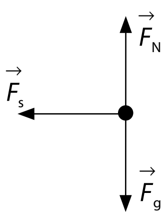
Solution

a)



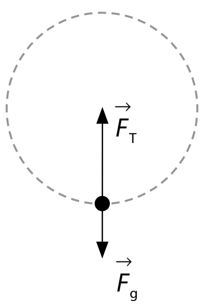
The force of gravity provides the centripetal force.

b) Back view:



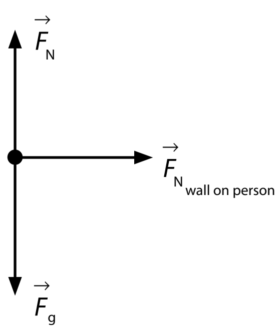
It is actually the force of static friction that enables a car to make a turn on a horizontal surface. Just imagine if the surface were icy—the car might just go straight, without this static friction to prevent it from slipping.

c)



It is the tension force in the string and the force of gravity on the ball that provide the centripetal force.

d)



Note that it is often a normal force that contributes to the centripetal force to keep an object in a "container" (vehicle, ride, and so on) during uniform circular motion.

Support Questions

29. Draw FBDs for the following objects and state which forces contribute to the centripetal force:

a) A ball on a string that is being whirled in a circle on a frictionless horizontal surface

b) A pilot in a jet who is making a loop in the vertical plane and is now at the top of the loop (the pilot is upside down at this location)

Suggested answer

Investigation into Circular Motion

Before moving on to look at problem solving in situations involving uniform circular motion, you are going to perform an online investigation, in order to take a closer look at the variables involved in circular motion and how they affect one another.

Open the following simulation called Uniform Circular Motion. Notice the following:

- At the end of the string, there is a mass that is hanging down. This mass provides the centripetal force for this motion.
- The radius of the circular path followed by the stopper remains constant.
- The speed of the stopper is constant, although its direction is constantly changing.
- The string is merely hanging; it is not "held" in place. It is only the uniform circular motion that is keeping it in place. The tension in the string is constant throughout, despite the bend. You could try to get a feel for this for yourself by hanging a string through a cylinder (for example, a paper towel roll), with some mass on either end. Try to rotate the top mass so that it attains and maintains uniform circular motion.

Support Questions

Hint: In the following questions, you will be using the abovementioned simulation to make observations while variables are being manipulated. One way of helping to verify your observations is through rearranging the appropriate equation to solve for the variable in question. For example, to determine how changing the radius affects frequency, rearrange the centripetal acceleration equation for frequency. If *r* appears in the numerator, it means that frequency is proportional to radius (as increasing radius causes frequency to increase). If *r* appears in the denominator, it means that frequency is inversely proportional to radius (as increasing radius causes frequency to decrease). Understanding such relationships helps with problem solving.

30. Click and drag to change the radius of the circular path. When the radius increases, what happens to the

a) period of rotation?

b) frequency of rotation?

c) speed of the stopper?

d) centripetal acceleration?

e) centripetal force?

Suggested answer

31. Press "reset" and restart the simulation. This time, add more mass to the hanging mass. As you add mass, you are actually increasing the tension in the string. As the hanging mass increases, what happens to the

a) period of rotation?

b) frequency of rotation?

c) speed of the stopper?

d) centripetal acceleration?

e) centripetal force?

Suggested answer

32. Press "reset" again, so that the two set-ups have the same motion. This time, increase the mass that is experiencing circular motion. As the mass of the object in motion increases, what happens to the

a) period of rotation?

b) frequency of rotation?

c) speed of the stopper?

d) centripetal acceleration?

e) centripetal force?

Suggested answer

Problem Solving with Circular Motion

Recall that you now know three equations for calculating centripetal acceleration. Because $\vec{F}_{\text{net}} = m\vec{a}$, all of the equations developed for centripetal acceleration are easily transformed into equations that can be used to calculate centripetal force by multiplying the centripetal acceleration by "mass." Therefore, the three equations for calculating centripetal force (F_C) are:

1. $F_C = \dfrac{mv^2}{r}$

2. $F_C = \dfrac{m4\pi^2 r}{T^2}$

3. $F_c = m4\pi^2 rf^2$

In uniform circular motion, it is a common convention to omit the arrows above the vector symbols, even though you are talking about vector quantities.

Example 1

A rope with a length of 2.00 m is used to swing a 0.250 kg object in a horizontal circular motion with a uniform speed of 20.0 m/s. What is the force of tension in the rope?

Solution

Given:

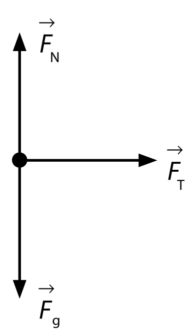
$r = 2.00$ m

$m = 0.250$ kg

$v = 20.0$ m/s

Required: F_T

Analysis and solution:



While you are not told the vertical forces, you are told that the object is maintaining horizontal motion. Therefore, the vertical forces must cancel each other out. The centripetal force is provided solely by the tension force in the rope. Let this direction be positive.

$F_{net} = F_T$

F_{net} is F_C and so $F_C = F_T$

$$F_C = \frac{mv^2}{r}$$

$$F_C = \frac{(0.250\text{ kg})(20.0\text{ m/s})^2}{2.00\text{ m}}$$

$$F_C = 50.0\text{ N}$$

Paraphrase: The force of tension in the rope is 50.0 N.

Example 2

Determine the period of rotation (that is, the time it takes for the ball to complete one entire circle) in the situation described in example 1 (the previous example).

Solution

Given:

$r = 2.00$ m

$m = 0.250$ kg

$v = 20.0$ m/s

We know F_{net} is F_C

and, from the last part of the problem, that $F_C = 50.0$ N.

Required: T

Analysis and solution:

We choose the appropriate form of F_C:

$$F_C = \frac{m4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{m4\pi^2 r}{F_C}}$$

$$T = \sqrt{\frac{(0.250\text{ kg})4\pi^2(2.00\text{ m})}{50.0\text{ N}}}$$

$$T = 0.628\text{ s}$$

Paraphrase: Therefore, the period of rotation is 0.628 s.

Example 3

The same ball (0.250 kg) is now swung with uniform circular motion using the same rope (2.00 m) so that it maintains the same speed (20.0 m/s) in a vertical plane. Determine the tension force in the rope at the highest point, and then again at the lowest point, in the circular path.

Solution

Given:

$r = 2.00$ m

$m = 0.250$ kg

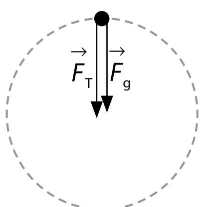
$v = 20.0$ m/s

Required: F_T at the top of the circle, F_T at the bottom of the circle

Analysis and solution:

At the top:

Let [down] be positive.



$$F_{net} = F_g + F_T$$

$$\frac{mv^2}{r} = mg + F_T$$

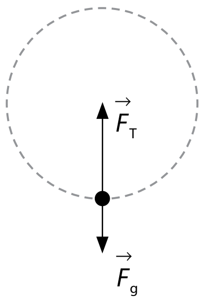
$$F_T = \frac{mv^2}{r} - mg$$

$$F_T = \frac{(0.250\text{ kg})(20.0\text{ m/s})^2}{2.00\text{ m}} - (0.250\text{ kg})(9.8\text{ m/s}^2)$$

$F_T = 47.6$ N, which rounds to 48 N.

At the bottom:

Let [up] be positive.



$F_{net} = F_T - F_g$

$\dfrac{mv^2}{r} = F_T - mg$

$F_T = \dfrac{mv^2}{r} + mg$

$F_T = \dfrac{(0.250\text{ kg})(20.0\text{ m/s})^2}{2.00\text{ m}} + (0.250\text{ kg})(9.8\text{ m/s}^2)$

$F_T = 52.5$ N, which rounds to 53 N.

Paraphrase: Therefore, the force of tension in the rope is 48 N when the ball is at the top of the circle, and 53 N when the ball is at the bottom of the circle.

Note: For both calculations, the direction of acceleration was made the positive direction, as this makes calculations easier. Also note that the tension in the circular loop changes. It goes from a maximum at the bottom to a minimum at the top. At the top, where the force of gravity is helping the object turn, there is a moment when only the force of gravity is turning the object.

Here, $F_C = F_g$, thus $\dfrac{mv^2}{r} = mg$.

This is the minimum speed needed to make the turn. Solving for v, we find $v = (rg)^{\frac{1}{2}}$.

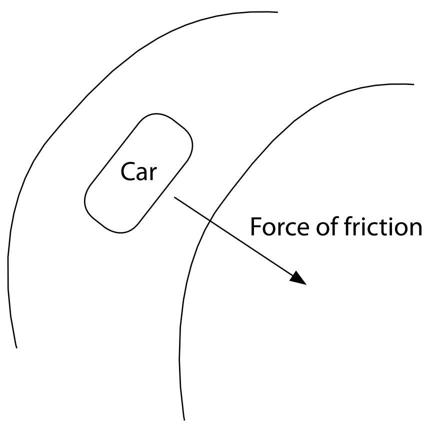
Support Questions

33. An airplane is flying in a loop, following a circular path with a radius of 2.50 km in the vertical plane. What is the minimum speed that the airplane can maintain without being forced out of the circular path?

Suggested answer

Turning Corners

When a car turns a corner on a flat road, only the friction of the tires on the road enables it to stay on the road. If the car were to hit a wet or icy patch, the tires would lose their grip and the car would move in a straight line. It is the component of friction, pointing toward the centre of the curve, that provides the unbalanced force creating centripetal acceleration, as shown in the top view diagram below.



Example 4

A 2000 kg car goes around a curve with a radius of 60 m. What is the maximum speed at which the car can safely make the turn, if the coefficient of static friction between the tires and the dry pavement is 1.0? Assume that the road is flat.

Solution

Given:

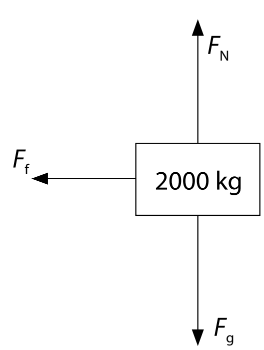
$m = 2000$ kg

$r = 60$ m

$\mu_s = 1.0$

Required: maximum v

Analysis and solution:



First we solve for F_{net} in the F_N direction.

$F_{net_y} = F_N - F_g$ But $F_{net} = 0$ so $F_N = mg$

This means that $F_f = \mu_s mg$ since $F_f = \mu F_N$

$F_f = (1.0)(2000\text{ kg})(9.8\text{ m/s}^2)$

$F_f = 19\,600\text{ N}$

Solving in the horizontal direction:

$F_{net_x} = F_f$ Since we need speed, we select the $\frac{mv^2}{r}$ form of F_{net}

$\frac{mv^2}{r} = 19\,600\text{ N}$

$v = \sqrt{(19\,600\text{ N})(60\text{ m})/(2000\text{ kg})}$

$v = 24.2\text{ m/s} = 24\text{ m/s}$

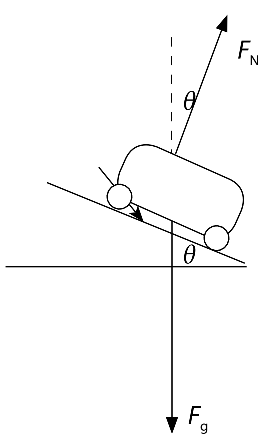
Note: By keeping the expression for friction instead of using a value, the mass would have cancelled out:

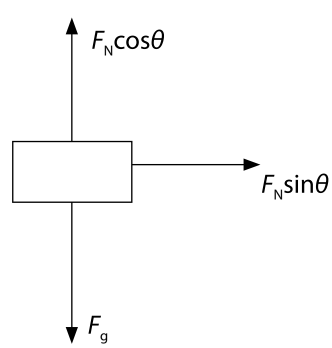
$\frac{mv^2}{r} = \mu_s mg$ producing $v = \sqrt{\mu_s gr}$

Banked Turns

When a car enters a highway, the ramp is banked (inclined) slightly. When a motorcycle or bike turns a corner, the driver leans; other sports such as bobsledding also rely on banked corners. In these cases, the road provides the normal force that, in turn, causes the object to turn.

Consider the following FBDs representing a car making a banked turn (the first diagram shows the turn in cross-section):





When we take components of F_N, there is a component pointing to the center of the turn. It creates the force required for centripetal acceleration.

If we solve the two F_{net} statements, we can derive an expression that is useful in situations involving banked turns.

In the y-direction:

$$F_{net_y} = F_N \cos\theta - mg \quad \text{so} \quad F_N = \frac{mg}{\cos\theta}$$

In the x-direction:

$$F_{net_x} = F_N \sin\theta - mg \quad \text{so} \quad \frac{mv^2}{r} = F_N \sin\theta$$

We now substitute in for F_N to obtain $\frac{mv^2}{r} = \frac{mg\sin\theta}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$ and the masses cancel to produce $\tan\theta = \frac{v^2}{gr}$

Example 5

For a banked turn, what is the bank angle required to enable a car to make a turn with a radius of 200 m at 80 km/h without using friction?

Solution

Given:

r = 200 m

v = 22 m/s

Required: θ

Analysis and solution:

$$\tan\theta = \frac{(22\text{ m/s})^2}{(9.8\text{ m/s}^2)(200\text{ m})}$$

$$\theta = \tan^{-1} 0.25 = 14°$$

Support Questions

34. A 1200.0 kg car goes around a curve with a radius of 45.0 m. What is the maximum speed at which the car can safely make the turn, if the coefficient of static friction between the tires and the dry pavement is 0.500? Assume that the road is flat. Suggested answer

35. For a banked turn, what is the bank angle required to enable a 600 kg sled to make a turn with a radius of 35 m at 50 km/h? Suggested answer

36. **a)** Find the optimal speed of a car on a banked turn with a bank angle of 10° and a radius of 30 m, so that that it completes the turn without using friction.

b) What coefficient of static friction would allow the car to turn a flat corner with a radius of 30 m at the speed you found in part a), above?

Suggested answer

Frames of Reference

Have you ever taken a ride on a roller coaster? If you have, you'll know that, as it goes around sharp turns, you feel pushed against the sides of the roller coaster car. You may also feel this way in a car that is going around a sharp bend. For a moment, you feel as if a force is pushing you in the opposite direction to the bend, away from the centre of the circle. There is, however, no such force directed away from the centre of the circular path; just because your frame of reference is accelerating, you imagine one!

Inertial versus Non-inertial Frames of Reference

An *inertial frame of reference* is a frame of reference that is either stationary or moving at constant velocity; that is, not accelerating. Motion in inertial frames of reference can be explained using Newton's laws of motion.

A *non-inertial frame of reference* is a frame of reference that is undergoing acceleration, that is, experiencing a net force. Motion with respect to non-inertial frames of reference cannot be explained using Newton's laws of motion. Therefore, people often use "fictitious" forces—forces that are not real—to try to explain the motion.

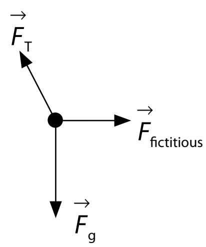
Now, go back to the example where you are riding in the turning roller coaster car. The fictitious "push" away from the centre of the circle is referred to as a centrifugal force. A centrifugal force is a fictitious force that is used to try to explain the push on objects to maintain circular motion. However, there is really no such force. If you were in a car that was rounding a bend, your body, according to Newton's first law, would tend to continue moving in a straight line. Because the car and seat belt prevent this, you, like the car, make the turn. The force you feel is the normal force of the vehicle on you. This is easy to explain from earth's frame of reference, but not so easy to explain from the frame of reference of the accelerating vehicle. If you had no idea of your motion with respect to some inertial frame (and here, you're assuming that the car is an inertial frame), you would have no way to explain the "push" that is being exerted on you.

Another example of a fictitious force that has been discussed already is "apparent weight." If you recall the elevator example you worked with, you "felt" as if you had more weight or less weight, depending on the acceleration of the elevator. Similarly, someone in free fall might say that they are "weightless," when really they weigh the same as they did when they were standing on solid ground! It is difficult to explain these force sensations, with respect to an accelerating frame of reference.

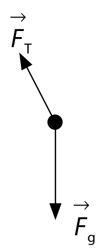
Example

You are riding in a car when, suddenly, the air freshener hanging above you swings forward. Is the car haunted? Explain your answer by constructing two FBDs—one in the car's frame of reference, and the other, in the earth's frame of reference.

In the car's frame of reference:



In the earth's frame of reference:



Because you are in the car with the air freshener, your frame of reference accelerates with the car, which makes it seem as if there is a force acting on the air freshener. An observer standing on the street would see the car's acceleration and would not need to include the fictitious force in the FBD.

Support Questions

37. Imagine that you are given a pail of water and are told that, somehow, you must make this pail turn upside down without the water falling out. No equipment is allowed other than you, the pail, and the water. Using the knowledge that you've gained in this lesson, how can you do this?

Do you need help? If so, watch this short video clip called [Water in a Bucket] for a demonstration.

Suggested answer

Review: Frames of Reference

Now watch this video called Frames of Reference, which demonstrates the ideas in this lesson and reviews relative velocity and circular motion.

Key Questions

Now work on your Key Questions in the online submission tool. You may continue to work at this task over several sessions, but be sure to save your work each time. When you have answered all the unit's Key Questions, submit your work to the ILC.

(24 marks)

12. Does the label "centripetal force" ever appear in an FBD? Explain. **(2 marks)**

13. Sometimes, road surfaces have banked curves. Use an FBD to explain how this helps cars to make turns more safely. **(3 marks)**

14. A bus passenger has her laptop sitting on the flat seat beside her as the bus, travelling at 10.0 m/s, goes around a turn with a radius of 25.0 m. What minimum coefficient of static friction is necessary to keep the laptop from sliding? **(5 marks)**

15. Keys with a combined mass of 0.100 kg are attached to a 0.25 m long string and swung in a circle in the vertical plane. **(9 marks)**

- **a)** What is the slowest speed that the keys can swing and still maintain a circular path?
- **b)** What is the tension in the string at the bottom of the circle?

16. Do research to find out what artificial gravity is and how it is related to centripetal motion. Explain how artificial gravity could be created in a weightless environment and give a reason why we would want to do this. Give at least one source that you used for your research. **(5 marks)**

This is the last lesson in Unit 1. When you have completed all the Key Questions, submit your work to the ILC. A teacher will mark it and you will receive your results online.