Yufeng Li
cdf ID: liyufen1

1\. (a) By definition, $A\begin{bmatrix} a_{1x} & a_{2x} & a_{3x} \\ a_{1y} & a_{2y} & a_{3y} \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} b_{1x} & b_{2x} & b_{3x} \\ b_{1y} & b_{2y} & b_{3y} \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore A = \begin{bmatrix} b_{1x} & b_{2x} & b_{3x} \\ b_{1y} & b_{2y} & b_{3y} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_{1x} & a_{2x} & a_{3x} \\ a_{1y} & a_{2y} & a_{3y} \\ 1 & 1 & 1 \end{bmatrix}^{-1}$

(b) Suppose a homography matrix of 2D

$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$

Thus we can have 4 point mapping and produce 8 equations to get H

$\therefore$ for 2 point mapping, it produces 4 equations

(c) if we have $p' = Ap + b$ (by def of Affine transformation

the original centroid $= \frac{1}{3}(a_1 + a_2 + a_3)$, After $Ap+b$

$c' = \frac{1}{3}(Aa_1 + b + Aa_2 + b + Aa_3 + b) = \frac{1}{3}(A(a_1 + a_2 + a_3) + 3b)$

$= \frac{A}{3}(a_1 + a_2 + a_3) + b$

$\therefore c' = A\,\text{original\_centroid} + b$

$\therefore$ centroid is affine invariant

Counterexample for circumcentre isn't affine invariant:

$\cancel{A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$ $a_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $a_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $a_3 = \begin{bmatrix} 0 \\ \sqrt{3} \end{bmatrix}$ # right angle $\triangle$

circumcentre $= (1, 1)$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $Aa_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $Aa_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $Aa_3 = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ # equilateral ~~angle~~ $\triangle$

circumcentre $= (1, \frac{\sqrt{3}}{2})$

2. (a)

Since light travels in a straight line, the light from top of a scene travels through the pinhole and produce light at bottom of the image, same as other poisitions. So the picture is inverted.

(b) $w = (C-P)/\|C-P\|$

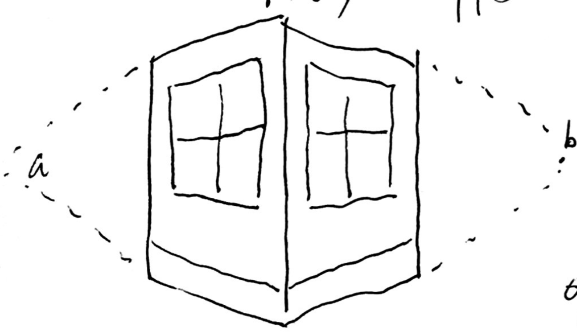
$u = u/\|u\|$

$v = w \times u$ # u is the camera facing up position

$$M_{camera} = \begin{pmatrix} v_x & v_y & v_z & -C_x \\ u_x & u_y & u_z & -C_y \\ w_x & w_y & w_z & -C_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(c) If $v/\|v\|$ is ~~either~~ $(V_{up} + u)$, suppose that a camera is facing $-w$ and camera's up vector is V_{up} and $u = w \times V_{up}$

(d) No, suppose we are looking from an alley corner



we can see that there are two points which different ~~parrell~~ parellel lines will converge and they are pairs of orthogonal parrellel lines.

3. (a) $\nabla f(x,y,z)$

$$= \langle x\left(2-\frac{2R}{\sqrt{x^2+y^2}}\right), y\left(2-\frac{2R}{\sqrt{x^2+y^2}}\right), 2z \rangle$$

(b) suppose our $P(x_0, y_0, z_0)$

$$T(x,y,z) = x\left(2-\frac{2R}{\sqrt{x^2+y^2}}\right)(x-x_0) +$$
$$y\left(2-\frac{2R}{\sqrt{x^2+y^2}}\right)(y-y_0) +$$
$$2z(z-z_0) = 0$$

(c) $f(q(\lambda)) = \left(R - \sqrt{(R\cos\lambda)^2+(R\sin\lambda)^2}\right)^2 + r^2 - r^2$

$$= 0^2 + 0 = 0$$

$\therefore$ $q(\lambda)$ lies on surface f

(d) $t(\lambda) = q'(\lambda) = (-R\sin\lambda, R\cos\lambda, 0)$

(e) $T(t(\lambda)) = \cancel{x(2-}$ $-R\sin\lambda\left(2-\frac{2R}{\sqrt{(R\sin\lambda)^2+(R\cos\lambda)^2}}\right)(-R\sin\lambda - x_0) +$

$$R\cos\lambda\left(2-\frac{2R}{\sqrt{(-R\sin\lambda)^2+(R\cos\lambda)^2}}\right)(R\cos\lambda - y_0) +$$
$$2\cdot 0(0-z_0)$$
$$= 0 + 0 + 0 = 0$$

$\therefore$ this tangent vector $t(\lambda)$ lies on tangent surface T

4. (a)

$$B_1(t) = P_1(1-t)^3 + P_2\,3(t-2t^2+t^3) + P_3\,3(t^2-t^3) + P_4t^3$$

$$B_2(t-1) = P_4(1-t)^3 + P_5\,3(t-2t^2+t^3) + P_6\,3(t^2-t^3) + P_7t^3$$

$$B_1'(t) = P_0(-3(1-t)^2) + P_1\,3(1-4t+3t^2) + P_2\,3(2t-3t^2) + P_3\,3t^2$$

$$B_2'(t-1) = P_4(-3(1-t)^2) + P_5\,3(1-4t+3t^2) + P_6\,3(2t-3t^2) + P_7\,3t^2$$

$$B_1'(1) = -3P_3 + 3P_4$$

$$B_2'(0) = -3P_4 + 3P_5$$

(b) $$B_1''(t) = P_0\,6(1-t) + P_1\,3(-4+6t) + P_2\,3(2-6t) + P_3\,6t$$

$$B_2''(t) = P_4\,6(1-t) + P_5\,3(-4+6t) + P_6\,3(2-6t) + P_7\,6t$$

$$B_1''(1) = 6P_2 - 12P_3 + 6P_4$$

$$B_2''(0) = 6P_4 - 12P_5 + 6P_6$$

(c) if C^2 continuous, then $B_1''(1) = B_2''(0)$, then

$$P_5 = P_4 + (P_4 - P_3)$$

$$P_6 = P_4 + (P_4 - P_2)$$ # thus P_4 will be seemingly inflection point.

P_7 can be free point (i.e. anywhere)

(d) ① the mathematical formula is simple

② it is easy to compute ~~and the~~

③ the output is 3D object which can be used directly

④ it can be composed into multiple shapes.