lecture_07

February 7, 2017

```
In [13]: %plot --format svg
```

```
In [2]: setdefaults
```

1 Roots: Open methods

1.1 Newton-Raphson

First-order approximation for the location of the root (i.e. assume the slope at the given point is constant, what is the solution when f(x)=0)

$f'(x_i) = \frac{f(x_i)-0}{x_i-x_{i+1}}$

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

Use Newton-Raphson to find solution when $e^{-x} = x$

```
In [2]: f= @(x) exp(-x)-x;
        df= @(x) -exp(-x)-1;

        x_i= 0;
        x_r = x_i-f(x_i)/df(x_i)
        error_approx = abs((x_r-x_i)/x_r)
        x_i=x_r;
```

```
x_r =  0.50000
error_approx =  1
```

```
In [3]: x_r = x_i-f(x_i)/df(x_i)
        error_approx = abs((x_r-x_i)/x_r)
        x_i=x_r;
```

```
x_r =  0.56631
error_approx =  0.11709
```

```
In [4]: x_r = x_i-f(x_i)/df(x_i)
        error_approx = abs((x_r-x_i)/x_r)
        x_i=x_r;
```

```
x_r =  0.56714
error_approx =  0.0014673
```

```
In [5]: x_r = x_i-f(x_i)/df(x_i)
        error_approx = abs((x_r-x_i)/x_r)
        x_i=x_r;
```

```
x_r =  0.56714
error_approx =    2.2106e-07
```

In the bungee jumper example, we created a function f(m) that when f(m)=0, then the mass had been chosen such that at t=4 s, the velocity is 36 m/s.

$f(m) = \sqrt{\frac{gm}{c_d}}\tanh(\sqrt{\frac{gc_d}{m}}t) - v(t)$.

to use the Newton-Raphson method, we need the derivative $\frac{df}{dm}$

$\frac{df}{dm} = \frac{1}{2}\sqrt{\frac{g}{mc_d}}\tanh(\sqrt{\frac{gc_d}{m}}t) - \frac{g}{2m}\text{sech}^2(\sqrt{\frac{gc_d}{m}}t)$

```
In [6]: setdefaults
        g=9.81; % acceleration due to gravity
        m=linspace(50, 200,100); % possible values for mass 50 to 200 kg
        c_d=0.25; % drag coefficient
        t=4; % at time = 4 seconds
        v=36; % speed must be 36 m/s
        f_m = @(m) sqrt(g*m/c_d).*tanh(sqrt(g*c_d./m)*t)-v; % anonymous function f_m
        df_m = @(m) 1/2*sqrt(g./m/c_d).*tanh(sqrt(g*c_d./m)*t)-g/2./m*sech(sqrt(g*c_d./m)*t).^2;
```

```
In [8]: [root,ea,iter]=newtraph(f_m,df_m,140,0.00001)
```

```
root =  142.74
ea =    8.0930e-06
iter =  48
```

1.2 Secant Methods

Not always able to evaluate the derivative. Approximation of derivative:

$f'(x_i) = \frac{f(x_{i-1})-f(x_i)}{x_{i-1}-x_i}$

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$x_{i+1} = x_i - \frac{f(x_i)}{\frac{f(x_{i-1})-f(x_i)}{x_{i-1}-x_i}} = x_i - \frac{f(x_i)(x_{i-1}-x_i)}{f(x_{i-1})-f(x_i)}$

What values should x_i and x_{i-1} take?

To reduce arbitrary selection of variables, use the

1.3 Modified Secant method

Change the x evaluations to a perturbation δ.

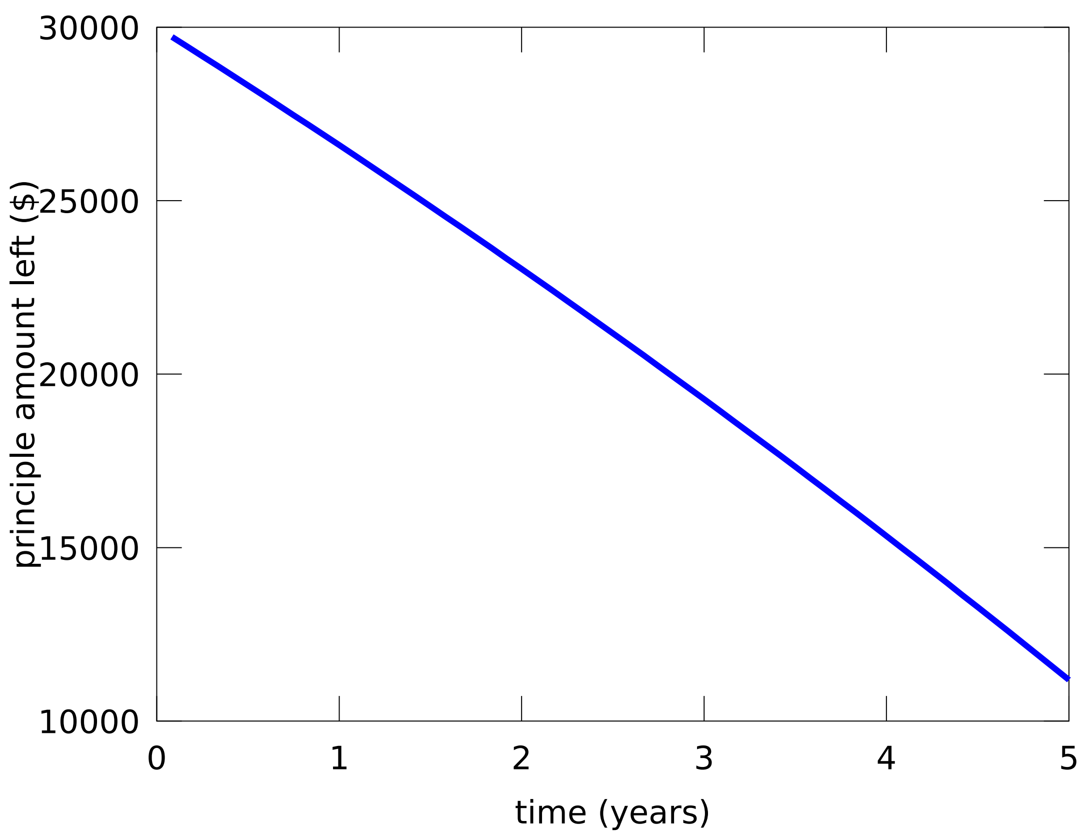
$$x_{i+1} = x_i - \frac{f(x_i)(\delta x_i)}{f(x_i+\delta x_i)-f(x_i)}$$

```
In [11]: [root,ea,iter]=mod_secant(f_m,1,50,0.00001)
```

```
root =  142.74
ea =    3.0615e-07
iter =  7
```

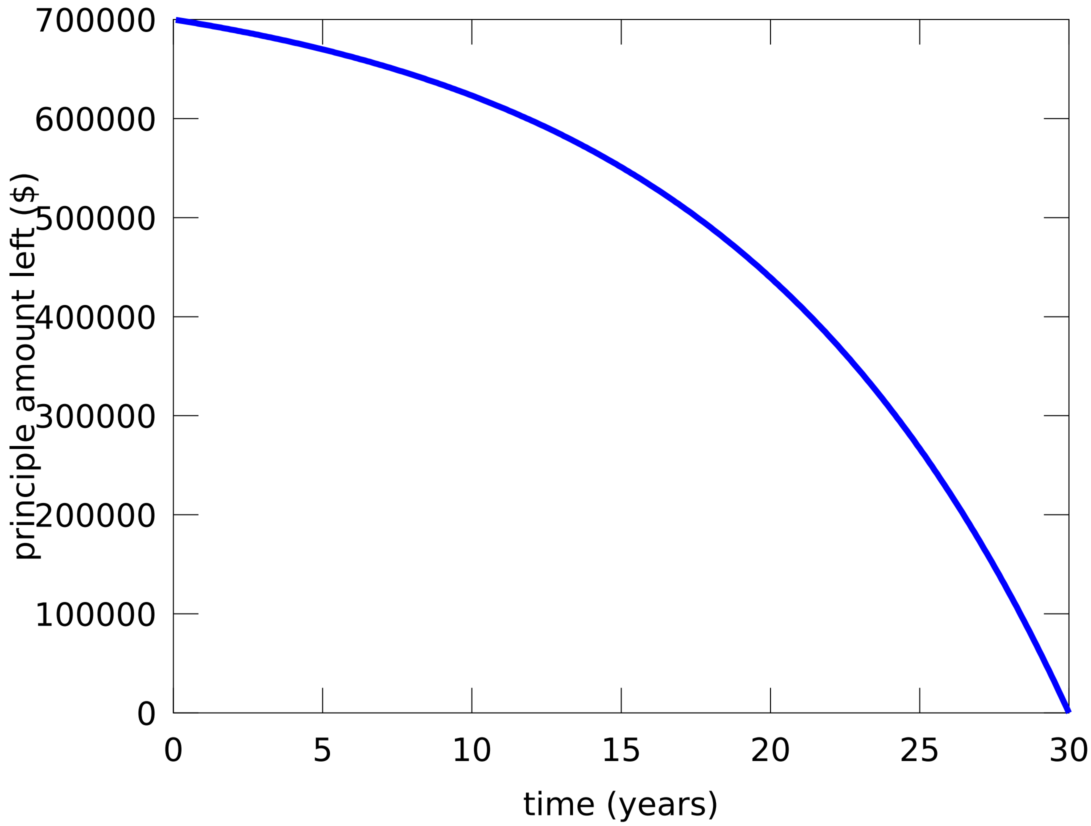
```
In [15]: car_payments(400,30000,0.05,5,1)
```

```
ans =    1.1185e+04
```




```
In [17]: Amt_numerical=mod_secant(@(A) car_payments(A,700000,0.0875,30,0),1e-6,50,0.001)
         car_payments(Amt_numerical,700000,0.0875,30,1)
```

```
Amt_numerical =  5467.0
ans =    3.9755e-04
```




```
In [18]: Amt_numerical*12*30

ans =    1.9681e+06
```

Amortization calculation makes the same calculation for the monthly payment amount, A, paying off the principle amount, P, over n pay periods with monthly interest rate, r.

```
In [19]: % Amortization calculation
         A  = @(P,r,n) P*(r*(1+r)^n)./((1+r)^n-1);
         Amt=A(30000,0.05/12,5*12)

Amt =  566.14
```

1.4 Matlab's function

Matlab and Octave combine bracketing and open methods in the `fzero` function.

```
In [13]: help fzero
```

```
'fzero' is a function from the file /usr/share/octave/4.0.0/m/optimization/fzero.m

 -- Function File: fzero (FUN, X0)
 -- Function File: fzero (FUN, X0, OPTIONS)
 -- Function File: [X, FVAL, INFO, OUTPUT] = fzero (...)
     Find a zero of a univariate function.

     FUN is a function handle, inline function, or string containing the
     name of the function to evaluate.

     X0 should be a two-element vector specifying two points which
     bracket a zero.  In other words, there must be a change in sign of
     the function between X0(1) and X0(2).  More mathematically, the
     following must hold

          sign (FUN(X0(1))) * sign (FUN(X0(2))) <= 0

     If X0 is a single scalar then several nearby and distant values are
     probed in an attempt to obtain a valid bracketing.  If this is not
     successful, the function fails.

     OPTIONS is a structure specifying additional options.  Currently,
     'fzero' recognizes these options: "FunValCheck", "OutputFcn",
     "TolX", "MaxIter", "MaxFunEvals".  For a description of these
     options, see *note optimset: XREFoptimset.

     On exit, the function returns X, the approximate zero point and
     FVAL, the function value thereof.

     INFO is an exit flag that can have these values:

        * 1 The algorithm converged to a solution.

        * 0 Maximum number of iterations or function evaluations has
          been reached.

        * -1 The algorithm has been terminated from user output
          function.

        * -5 The algorithm may have converged to a singular point.

     OUTPUT is a structure containing runtime information about the
     'fzero' algorithm.  Fields in the structure are:

        * iterations Number of iterations through loop.

        * nfev Number of function evaluations.
```

```
    * bracketx A two-element vector with the final bracketing of the
      zero along the x-axis.

    * brackety A two-element vector with the final bracketing of the
      zero along the y-axis.

 See also: optimset, fsolve.

Additional help for built-in functions and operators is
available in the online version of the manual.  Use the command
'doc <topic>' to search the manual index.

Help and information about Octave is also available on the WWW
at http://www.octave.org and via the help@octave.org
mailing list.
```

```
In [20]: fzero(@(A) car_payments(A,30000,0.05,5,0),500)
```

```
ans =  563.79
```

1.5 Comparison of Solvers

It's helpful to compare to the convergence of different routines to see how quickly you find a solution.

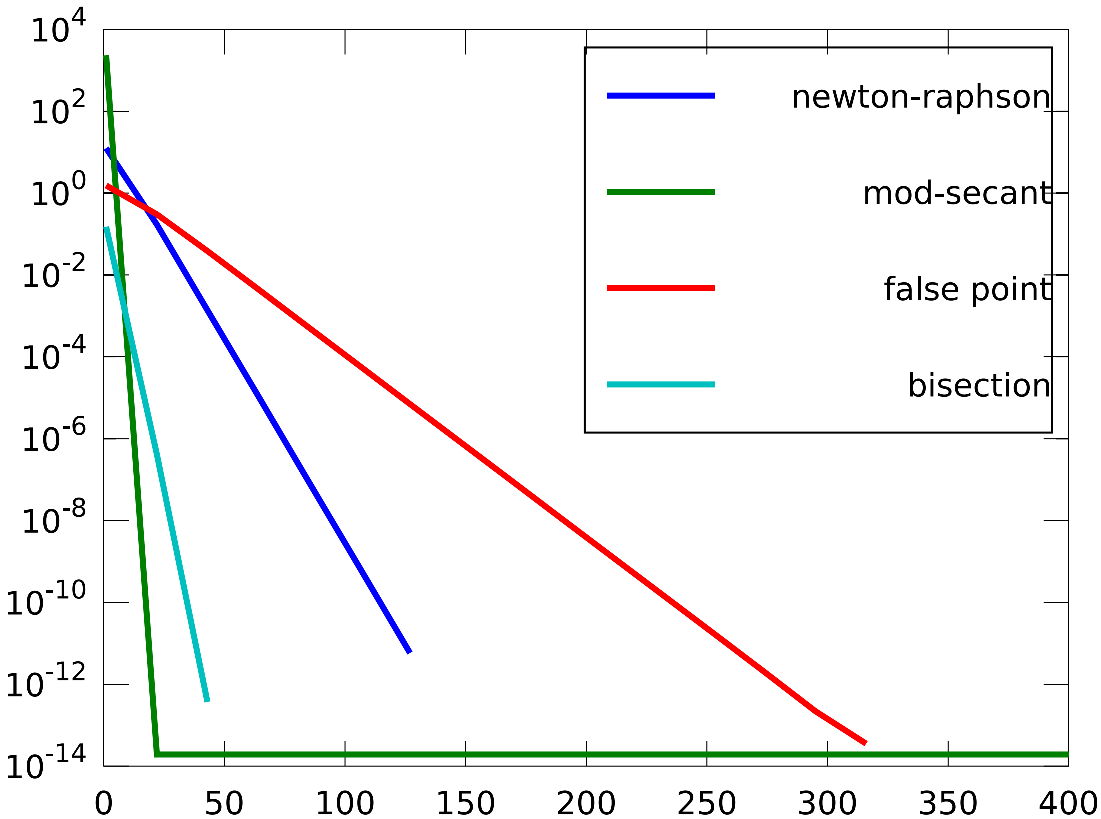
Comparing the freefall example

```
In [23]: N=20;
         iterations = linspace(1,400,N);
         ea_nr=zeros(1,N); % appr error Newton-Raphson
         ea_ms=zeros(1,N); % appr error Modified Secant
         ea_fp=zeros(1,N); % appr error false point method
         ea_bs=zeros(1,N); % appr error bisect method
         for i=1:length(iterations)
             [root_nr,ea_nr(i),iter_nr]=newtraph(f_m,df_m,300,0,iterations(i));
             [root_ms,ea_ms(i),iter_ms]=mod_secant(f_m,1e-6,300,0,iterations(i));
             [root_fp,ea_fp(i),iter_fp]=falsepos(f_m,1,300,0,iterations(i));
             [root_bs,ea_bs(i),iter_bs]=bisect(f_m,1,300,0,iterations(i));
         end

         setdefaults
         semilogy(iterations,abs(ea_nr),iterations,abs(ea_ms),iterations,abs(ea_fp),iterations,a
         legend('newton-raphson','mod-secant','false point','bisection')
```

```
warning: axis: omitting non-positive data in log plot
warning: called from
    __line__ at line 120 column 16
    line at line 56 column 8
```

```
    __plt__>__plt2vv__ at line 500 column 10
    __plt__>__plt2__ at line 246 column 14
    __plt__ at line 133 column 15
    semilogy at line 60 column 10
warning: axis: omitting non-positive data in log plot
warning: axis: omitting non-positive data in log plot
warning: axis: omitting non-positive data in log plot
```



In [22]: ea_nr

```
ea_nr =

 Columns 1 through 8:

   6.36591    0.06436    0.00052    0.00000    0.00000    0.00000    0.00000    0.00000

 Columns 9 through 16:

   0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000
```

```
 Columns 17 through 20:

   0.00000   0.00000   0.00000   0.00000
```

```
In [26]: N=20;
         f= @(x) x^10-1;
         df=@(x) 10*x^9;
         iterations = linspace(1,50,N);
         ea_nr=zeros(1,N); % appr error Newton-Raphson
         ea_ms=zeros(1,N); % appr error Modified Secant
         ea_fp=zeros(1,N); % appr error false point method
         ea_bs=zeros(1,N); % appr error bisect method
         for i=1:length(iterations)
             [root_nr,ea_nr(i),iter_nr]=newtraph(f,df,0.5,0,iterations(i));
             [root_ms,ea_ms(i),iter_ms]=mod_secant(f,1e-6,0.5,0,iterations(i));
             [root_fp,ea_fp(i),iter_fp]=falsepos(f,0,5,0,iterations(i));
             [root_bs,ea_bs(i),iter_bs]=bisect(f,0,5,0,iterations(i));
         end

         semilogy(iterations,abs(ea_nr),iterations,abs(ea_ms),iterations,abs(ea_fp),iterations,a
         legend('newton-raphson','mod-secant','false point','bisection')
```

```
warning: axis: omitting non-positive data in log plot
warning: called from
    __line__ at line 120 column 16
    line at line 56 column 8
    __plt__>__plt2vv__ at line 500 column 10
    __plt__>__plt2__ at line 246 column 14
    __plt__ at line 133 column 15
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warning: axis: omitting non-positive data in log plot
```

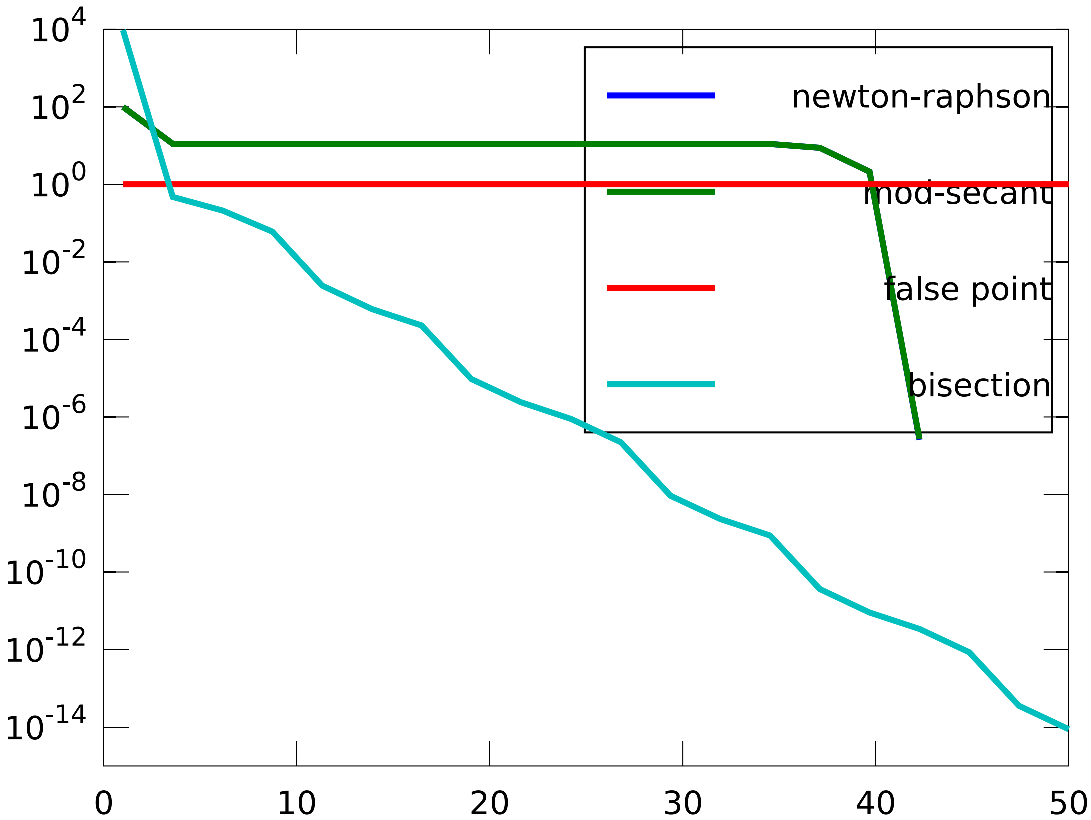


```
In [27]: ea_nr
         newtraph(f,df,0.5,0,12)

ea_nr =

 Columns 1 through 7:

   99.03195   11.11111   11.11111   11.11111   11.11111   11.11111   11.11111

 Columns 8 through 14:

   11.11111   11.11111   11.11111   11.11109   11.11052   11.10624   10.99684

 Columns 15 through 20:

    8.76956    2.12993    0.00000    0.00000    0.00000    0.00000

ans =  16.208


In [19]: df(300)
```

```
ans =    1.9683e+23
```

```
In [28]: % our class function
         f= @(x) tan(x)-(x-1).^2
         mod_secant(f,1e-3,1)
```

```
f =

@(x) tan (x) - (x - 1) .^ 2

ans =  0.37375
```

```
In [29]: f(ans)
```

```
ans =   -3.5577e-13
```

```
In [30]: tan(0.37375)
         (0.37375-1)^2
```

```
ans =  0.39218
ans =  0.39219
```

```
In [31]: f([0:10])
```

```
ans =

 Columns 1 through 8:

   -1.0000    1.5574   -3.1850   -4.1425   -7.8422  -19.3805  -25.2910  -35.1286

 Columns 9 through 11:

  -55.7997  -64.4523  -80.3516
```

```
In [ ]:
```