lecture_15

March 23, 2017

```
In [1]: %plot --format svg
```

```
In [2]: setdefaults
```

0.1 UConn Hackathon Mar 24-25

https://www.hackuconn.org/

1 Final Project

1. Will be a team project (select team of 2-3 students)
2. You will create a repository and each of you will contribute code and documentation
3. If you have an idea feel free to suggest it, otherwise I will come up with a project, possible topics include:

   a. Conduction of heat through simple geometry
   b. Plate or beam mechanics (1-D and 2-D geometries)
   c.

2 Eigenvalues

Eigenvalues and eigen vectors are the solution to the set of equations where
$Ax = \lambda x$
or
$A - I\lambda = 0$
Where A is the description of the system and I is the identity matrix with the same dimensions as A and λ is an eigenvalue of A.
These problems are seen in a number of engineering practices:

1. Determining vibrational modes in structural devices
2. Material science - vibrational modes of crystal lattices (phonons)
3. Google searches - http://www.rose-hulman.edu/~bryan/googleFinalVersionFixed.pdf
4. Quantum mechanics - many solutions are eigenvalue problems

5. Solid mechanics, principle stresses and principle stress directions are eigenvalues and eigenvectors

One way of determining the eigenvalues is taking the determinant:
$|A-\lambda I|=0$
This will result in an n^{th}-order polynomial where A is $n\times n$.
Take, A
$A=\left[\begin{array}{ccc}2&1&0\\1&3&1\\0&1&4\end{array}\right]$
$|A-\lambda I|=\left|\begin{array}{ccc}2-\lambda&1&0\\1&3-\lambda&1\\0&1&4-\lambda\end{array}\right|=0$
$(2-\lambda)(3-\lambda)(4-\lambda)-(4-\lambda)-(2-\lambda)=0$
$-\lambda^3+9\lambda^2-24\lambda+18=0$
$\lambda=3,\ \sqrt{3}+3,-\sqrt{3}+3$
in Matlab/Octave:

```
A=[2,1,0;1,3,1;0,1,4];
pA=poly(A);
lambda = roots(pA)
```

```
In [3]: A=[2,1,0;1,3,1;0,1,4];
        pA=poly(A); % characteristic polynomial of A, e.g. l^3-9*l^2+24*l-18=0
        lambda = roots(pA)
```

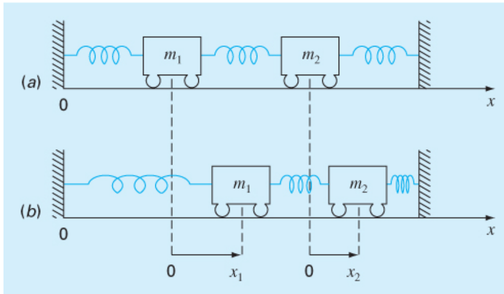
```
lambda =

   4.7321
   3.0000
   1.2679
```

2.1 Applications of Eigenvalue analysis

Consider the 2-mass, 3-spring system shown below
It might not be immediately obvious, but there are two resonant frequencies for these masses connected in series.
Take the two FBD solutions:
$m_1\frac{d^2x_1}{dt^2}=-kx_1+k(x_2-x_1)$
$m_2\frac{d^2x_2}{dt^2}=-k(x_2-x_1)-kx_2$
we know that $x_i(t)\propto sin(\omega t)$ so we can substitute
$x_i=X_isin(\omega t)$
$-m_1X_1\omega^2sin(\omega t)=-kX_1sin(\omega t)+k(X_2-X_1)sin(\omega t)$
$-m_2X_2\omega^2sin(\omega t)=-kX_2sin(\omega t)-k(X_2-X_1)sin(\omega t)$
where X_1 and X_2 are the amplitude of oscillations and ω is the frequency of oscillations.
now,

FIGURE 13.3
A two mass–three spring system with frictionless rollers vibrating between two fixed walls. The position of the masses can be referenced to local coordinates with origins at their respective equilibrium positions (*a*). As in (*b*), positioning the masses away from equilibrium creates forces in the springs that on release lead to oscillations of the masses.



masses and springs in series

$$\left(\frac{2k}{m_1} - \omega^2\right) X_1 - \frac{k}{m_1} X_2 = 0$$
$$-\frac{k}{m_2} X_1 + \left(\frac{2k}{m_2} - \omega^2\right) X_2 = 0$$

or

$$|K - \lambda I| = \begin{vmatrix} \left(\frac{2k}{m_1} - \omega^2\right) & -\frac{k}{m_1} \\ -\frac{k}{m_2} & \left(\frac{2k}{m_2} - \omega^2\right) \end{vmatrix} = 0$$

where $\lambda = \omega^2$

```
In [4]: help eig
```

```
'eig' is a built-in function from the file libinterp/corefcn/eig.cc

 -- Built-in Function: LAMBDA = eig (A)
 -- Built-in Function: LAMBDA = eig (A, B)
 -- Built-in Function: [V, LAMBDA] = eig (A)
 -- Built-in Function: [V, LAMBDA] = eig (A, B)
     Compute the eigenvalues (and optionally the eigenvectors) of a
     matrix or a pair of matrices

     The algorithm used depends on whether there are one or two input
     matrices, if they are real or complex, and if they are symmetric
     (Hermitian if complex) or non-symmetric.

     The eigenvalues returned by 'eig' are not ordered.

     See also: eigs, svd.

Additional help for built-in functions and operators is
available in the online version of the manual.  Use the command
'doc <topic>' to search the manual index.

Help and information about Octave is also available on the WWW
at http://www.octave.org and via the help@octave.org
```