lecture_07

February 6, 2017

```
In [25]: %plot --format svg
```

```
In [30]: setdefaults
```

1 Roots: Open methods

1.1 Newton-Raphson

First-order approximation for the location of the root (i.e. assume the slope at the given point is constant, what is the solution when f(x)=0)

$f'(x_i) = \frac{f(x_i)-0}{x_i-x_{i+1}}$

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

Use Newton-Raphson to find solution when $e^x = x$

```
In [1]: f= @(x) exp(-x)-x;
        df= @(x) -exp(-x)-1;

        x_i= 0;
        c
        error_approx = abs((x_r-x_i)/x_r)
        x_i=x_r;
```

```
error: 'c' undefined near line 1 column 1
error: 'x_r' undefined near line 1 column 21
error: evaluating argument list element number 1
error: 'x_r' undefined near line 1 column 5
```

```
In [2]: x_r = x_i-f(x_i)/df(x_i)
        error_approx = abs((x_r-x_i)/x_r)
        x_i=x_r;
```

```
x_r =  0.50000
error_approx =  1
```

```
In [3]: x_r = x_i-f(x_i)/df(x_i)
        error_approx = abs((x_r-x_i)/x_r)
        x_i=x_r;
```

```
x_r =  0.56631
error_approx =  0.11709
```

```
In [4]: x_r = x_i-f(x_i)/df(x_i)
        error_approx = abs((x_r-x_i)/x_r)
        x_i=x_r;
```

```
x_r =  0.56714
error_approx =  0.0014673
```

In the bungee jumper example, we created a function f(m) that when f(m)=0, then the mass had been chosen such that at t=4 s, the velocity is 36 m/s.

$f(m) = \sqrt{\frac{gm}{c_d}}\tanh(\sqrt{\frac{gc_d}{m}}t) - v(t)$.

to use the Newton-Raphson method, we need the derivative $\frac{df}{dm}$

$\frac{df}{dm} = \frac{1}{2}\sqrt{\frac{g}{mc_d}}\tanh(\sqrt{\frac{gc_d}{m}}t) - \frac{g}{2m}\text{sech}^2(\sqrt{\frac{gc_d}{m}}t)$

```
In [6]: setdefaults
        g=9.81; % acceleration due to gravity
        m=linspace(50, 200,100); % possible values for mass 50 to 200 kg
        c_d=0.25; % drag coefficient
        t=4; % at time = 4 seconds
        v=36; % speed must be 36 m/s
        f_m = @(m) sqrt(g*m/c_d).*tanh(sqrt(g*c_d./m)*t)-v; % anonymous function f_m
        df_m = @(m) 1/2*sqrt(g./m/c_d).*tanh(sqrt(g*c_d./m)*t)-g/2./m*sech(sqrt(g*c_d./m)*t).^2;
```

```
In [7]: newtraph(f_m,df_m,140,0.00001)
```

```
ans =  142.74
```

1.2 Secant Methods

Not always able to evaluate the derivative. Approximation of derivative:

$f'(x_i) = \frac{f(x_{i-1})-f(x_i)}{x_{i-1}-x_i}$

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$x_{i+1} = x_i - \frac{f(x_i)}{\frac{f(x_{i-1})-f(x_i)}{x_{i-1}-x_i}} = x_i - \frac{f(x_i)(x_{i-1}-x_i)}{f(x_{i-1})-f(x_i)}$

What values should x_i and x_{i-1} take?

To reduce arbitrary selection of variables, use the

1.3 Modified Secant method

Change the x evaluations to a perturbation δ.

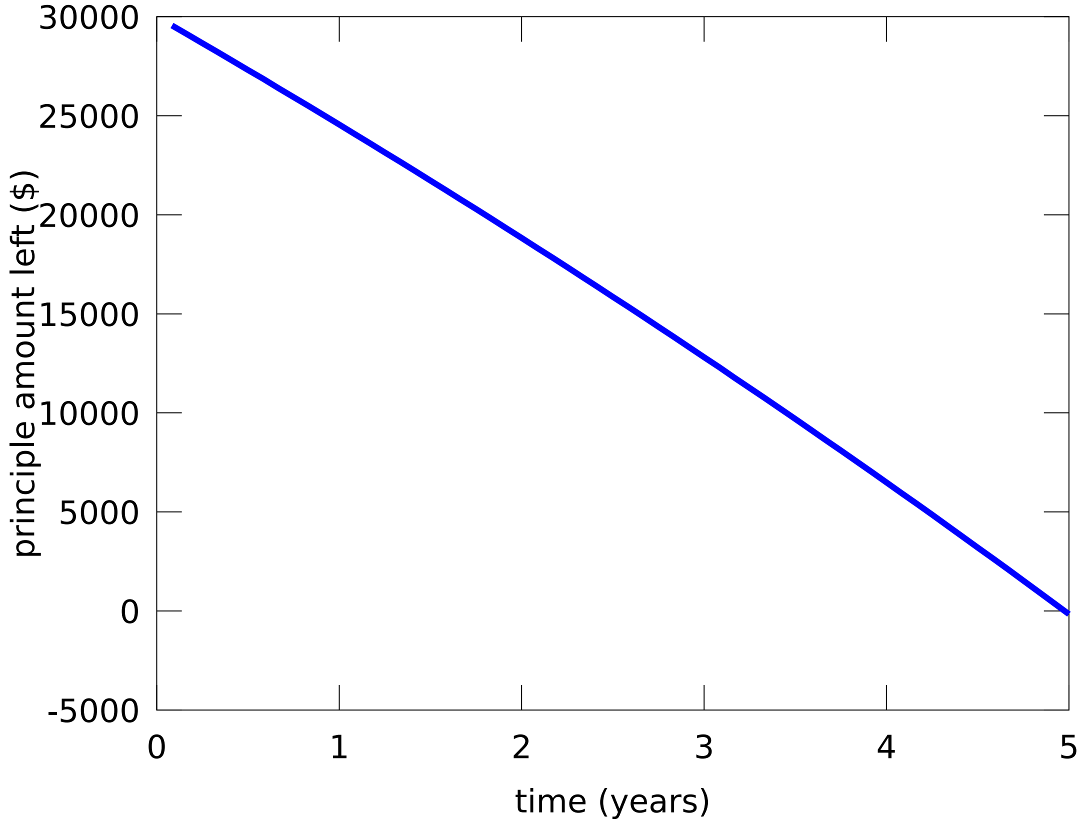
$x_{i+1} = x_i - \frac{f(x_i)(\delta x_i)}{f(x_i+\delta x_i)-f(x_i)}$

```
In [8]: mod_secant(f_m,1e-6,50,0.00001)
```

```
ans =  142.74
```

```
In [32]: Amt_numerical=mod_secant(@(A) car_payments(A,30000,0.05,5),1e-6,50,0.001)
         car_payments(Amt,30000,0.05,5)
```

```
Amt_numerical =  563.79
ans = -160.42
```




```
In [23]: Amt*12*5
```

```
ans =    3.3968e+04
```

Amortization calculation makes the same calculation for the monthly payment amount, A, paying off the principle amount, P, over n pay periods with monthly interest rate, r.

```
In [27]: % Amortization calculation
         A  = @(P,r,n) P*(r*(1+r)^n)./((1+r)^n-1);
         Amt=A(30000,0.05/12,5*12)
```

```
Amt =  566.14
```

1.4 Matlab's function

Matlab and Octave combine bracketing and open methods in the `fzero` function.

```
In [33]: help fzero
```

```
'fzero' is a function from the file /usr/share/octave/4.0.0/m/optimization/fzero.m

 -- Function File: fzero (FUN, X0)
 -- Function File: fzero (FUN, X0, OPTIONS)
 -- Function File: [X, FVAL, INFO, OUTPUT] = fzero (...)
     Find a zero of a univariate function.

     FUN is a function handle, inline function, or string containing the
     name of the function to evaluate.

     X0 should be a two-element vector specifying two points which
     bracket a zero.  In other words, there must be a change in sign of
     the function between X0(1) and X0(2).  More mathematically, the
     following must hold

          sign (FUN(X0(1))) * sign (FUN(X0(2))) <= 0

     If X0 is a single scalar then several nearby and distant values are
     probed in an attempt to obtain a valid bracketing.  If this is not
     successful, the function fails.

     OPTIONS is a structure specifying additional options.  Currently,
     'fzero' recognizes these options: "FunValCheck", "OutputFcn",
     "TolX", "MaxIter", "MaxFunEvals".  For a description of these
     options, see *note optimset: XREFoptimset.

     On exit, the function returns X, the approximate zero point and
     FVAL, the function value thereof.

     INFO is an exit flag that can have these values:

        * 1 The algorithm converged to a solution.

        * 0 Maximum number of iterations or function evaluations has
          been reached.

        * -1 The algorithm has been terminated from user output
          function.

        * -5 The algorithm may have converged to a singular point.

     OUTPUT is a structure containing runtime information about the
     'fzero' algorithm.  Fields in the structure are:
```

```
    * iterations Number of iterations through loop.

    * nfev Number of function evaluations.

    * bracketx A two-element vector with the final bracketing of the
      zero along the x-axis.

    * brackety A two-element vector with the final bracketing of the
      zero along the y-axis.

 See also: optimset, fsolve.

Additional help for built-in functions and operators is
available in the online version of the manual.  Use the command
'doc <topic>' to search the manual index.

Help and information about Octave is also available on the WWW
at http://www.octave.org and via the help@octave.org
mailing list.
```

In [40]:
```
fzero(@(A) car_payments(A,30000,0.05,5,0),500)
```

```
ans =  563.79
```

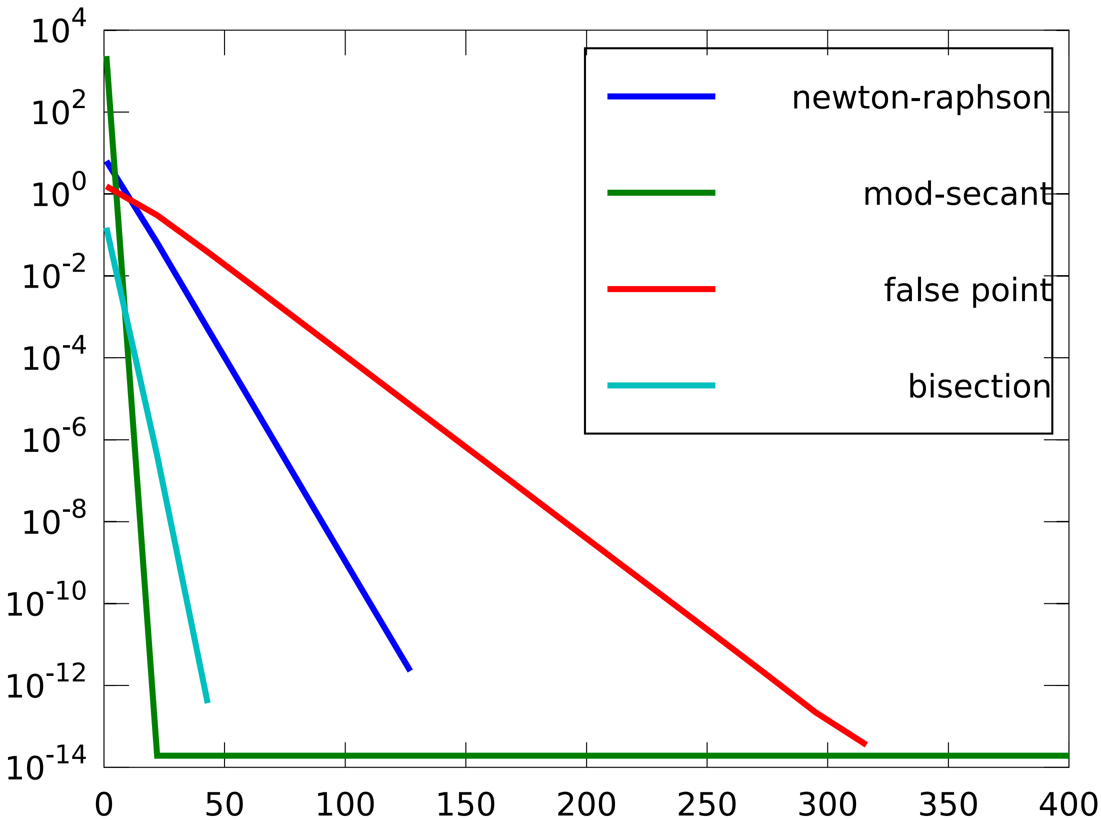
1.5 Comparison of Solvers

It's helpful to compare to the convergence of different routines to see how quickly you find a solution.

Comparing the freefall example

In [84]:
```
N=20;
iterations = linspace(1,400,N);
ea_nr=zeros(1,N); % appr error Newton-Raphson
ea_ms=zeros(1,N); % appr error Modified Secant
ea_fp=zeros(1,N); % appr error false point method
ea_bs=zeros(1,N); % appr error bisect method
for i=1:length(iterations)
    [root_nr,ea_nr(i),iter_nr]=newtraph(f_m,df_m,200,0,iterations(i));
    [root_ms,ea_ms(i),iter_ms]=mod_secant(f_m,1e-6,300,0,iterations(i));
    [root_fp,ea_fp(i),iter_fp]=falsepos(f_m,1,300,0,iterations(i));
    [root_bs,ea_bs(i),iter_bs]=bisect(f_m,1,300,0,iterations(i));
end

semilogy(iterations,abs(ea_nr),iterations,abs(ea_ms),iterations,abs(ea_fp),iterations,a
legend('newton-raphson','mod-secant','false point','bisection')
```

```
warning: axis: omitting non-positive data in log plot
warning: called from
    __line__ at line 120 column 16
    line at line 56 column 8
    __plt__>__plt2vv__ at line 500 column 10
    __plt__>__plt2__ at line 246 column 14
    __plt__ at line 133 column 15
    semilogy at line 60 column 10
warning: axis: omitting non-positive data in log plot
warning: axis: omitting non-positive data in log plot
warning: axis: omitting non-positive data in log plot
```

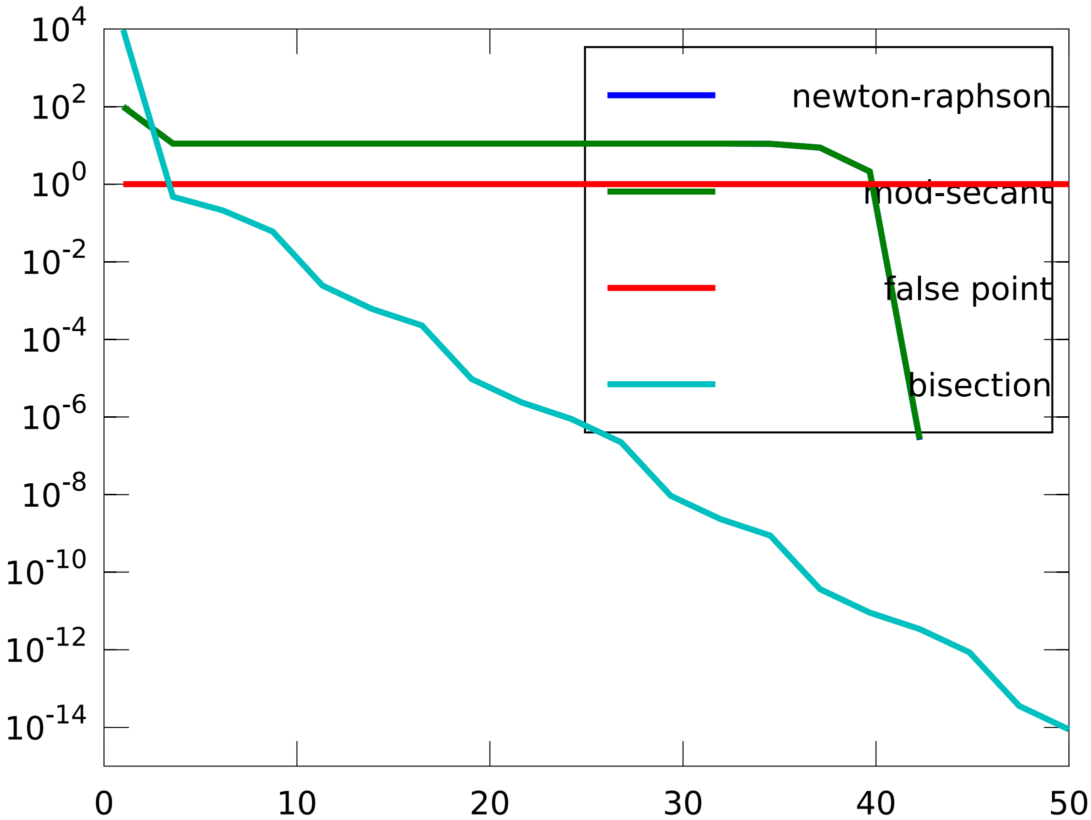



```
In [75]: ea_ms

ea_ms =

 Columns 1 through 7:

   43.43883     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000
```

```
 Columns 8 through 14:

    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000

 Columns 15 through 20:

    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000
```

```
In [101]: N=20;
          f= @(x) x^10-1;
          df=@(x) 10*x^9;
          iterations = linspace(1,50,N);
          ea_nr=zeros(1,N); % appr error Newton-Raphson
          ea_ms=zeros(1,N); % appr error Modified Secant
          ea_fp=zeros(1,N); % appr error false point method
          ea_bs=zeros(1,N); % appr error bisect method
          for i=1:length(iterations)
              [root_nr,ea_nr(i),iter_nr]=newtraph(f,df,0.5,0,iterations(i));
              [root_ms,ea_ms(i),iter_ms]=mod_secant(f,1e-6,0.5,0,iterations(i));
              [root_fp,ea_fp(i),iter_fp]=falsepos(f,0,5,0,iterations(i));
              [root_bs,ea_bs(i),iter_bs]=bisect(f,0,5,0,iterations(i));
          end

          semilogy(iterations,abs(ea_nr),iterations,abs(ea_ms),iterations,abs(ea_fp),iterations,
          legend('newton-raphson','mod-secant','false point','bisection')
```

```
warning: axis: omitting non-positive data in log plot
warning: called from
    __line__ at line 120 column 16
    line at line 56 column 8
    __plt__>__plt2vv__ at line 500 column 10
    __plt__>__plt2__ at line 246 column 14
    __plt__ at line 133 column 15
    semilogy at line 60 column 10
warning: axis: omitting non-positive data in log plot
warning: axis: omitting non-positive data in log plot
warning: axis: omitting non-positive data in log plot
```




```
In [102]: ea_bs
          newtraph(f,df,0.5,0,12)

ea_bs =

 Columns 1 through 6:

   9.5357e+03  -4.7554e-01  -2.1114e-01   6.0163e-02  -2.4387e-03   6.1052e-04

 Columns 7 through 12:

   2.2891e-04  -9.5367e-06   2.3842e-06   8.9407e-07  -2.2352e-07   9.3132e-09

 Columns 13 through 18:

  -2.3283e-09  -8.7311e-10   3.6380e-11  -9.0949e-12  -3.4106e-12   8.5265e-13

 Columns 19 and 20:

  -3.5527e-14   8.8818e-15
```

```
ans =  16.208
```

```
In [93]: df(300)
```

```
ans =    1.9683e+23
```

```
In [ ]:
```