

Linear Algebra Review

(Gauss Elimination) Suggested problems

No due date

1. Determine the lower (L) and upper (U) triangular matrices with LU-decomposition for the following matrices:

a. $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

c. $A = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$

d. $A = \begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & 3 & 6 \end{bmatrix}$

e. $A = \begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & 3 & 6 \end{bmatrix}$

f. $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$

g. $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 2 \\ 1 & -1 & 2 \end{bmatrix}$

2. Calculate the determinant of A from 1a-g.
3. Determine the Cholesky factorization, C, of the following matrices, where

$$C_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} C_{ki}^2}$$

$$C_{ij} = \frac{a_{ij} - \sum_{k=1}^{i-1} C_{ki} C_{kj}}{C_{ii}}.$$

a. $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix}$

c. $A = \begin{bmatrix} 10 & -10 & 20 \\ -10 & 20 & 10 \\ 20 & 10 & 30 \end{bmatrix}$

d. $A = \begin{bmatrix} 21 & -1 & 0 & 0 \\ -1 & 21 & -1 & 0 \\ 0 & -1 & 21 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

4. Verify that $C^T C = A$ for 3a-d