

## Homework 2

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### Question 1:

Exercise 16.2.2: Give examples to show that:

- a) Projection cannot be pushed below set union.
- b) Projection cannot be pushed below set or bag difference.

#### Answer:

- a) The following example shows that projection cannot be pushed below set union.

Suppose relation A is,

| a1 | a2 |
|----|----|
| 1  | 2  |

Suppose relation B is,

| a1 | a2 |
|----|----|
| 1  | 3  |

Then,  $\pi_{a1}(A \cup B)$  is,

| a1 |
|----|
| 1  |
| 1  |

And  $\pi_{a1}(A) \cup \pi_{a1}(B)$  is,

| a1 |
|----|
| 1  |

The example shows that  $\pi_{a1}(A \cup B) \neq \pi_{a1}(A) \cup \pi_{a1}(B)$ , which means projection cannot be pushed below set union.

- b) The following example shows that projection cannot be pushed below set difference.

Suppose relation A is,

| a1 | a2 |
|----|----|
| 1  | 2  |
| 3  | 4  |

Suppose relation B is,

| a1 | a2 |
|----|----|
| 1  | 3  |

Then,  $\pi_{a1}(A - B)$  is

| a1 |
|----|
| 1  |
| 3  |

And  $\pi_{a1}(A) - \pi_{a1}(B)$  is

| a1 |
|----|
| 3  |

The example shows that  $\pi_{a1}(A - B) \neq \pi_{a1}(A) - \pi_{a1}(B)$ , which means projection cannot be pushed below set difference. Because a set is also a bag, then we proved that project cannot be pushed below bag difference either.

## Question 2:

*Exercise 16.2.4: Some laws that hold for sets hold for bags; others do not. For each of the laws below that true for sets, indicate whether or not it is true for bags by giving a proof that the law for bags is true, or by giving a counterexample.*

a)  $R \cup R = R$  (the idempotent law for union).

b)  $R \cap R = R$  (the idempotent law for intersection).

### Answer:

a) The following example shows that  $R \cup R = R$  does not hold for bags.

Suppose relation R is,

|    |
|----|
| a1 |
| 1  |

Then,  $R \cup R$  is,

|    |
|----|
| a1 |
| 1  |
| 1  |

The example shows that  $R \cup R \neq R$ , which means  $R \cup R = R$  does not hold for bags.

b) To prove  $R \cap R = R$  holds for bags, we suppose R have m ( $m \geq 0$ ) different tuples, denoted by  $t_1, t_2, \dots, t_m$ , the number of these tuples are  $n_1, n_2, \dots, n_m$ . Based on the definition of intersection for bags (see page 207 in the textbook),  $R \cap R$  will also have tuples  $t_1, t_2, \dots, t_m$ , and the number of these tuples will be  $\min\{n_1, n_1\}, \min\{n_2, n_2\}, \dots, \min\{n_m, n_m\}$ , which equals to  $n_1, n_2, \dots, n_m$ . So  $R \cap R$  is identical to R,  $R \cap R = R$  does hold for bags.

**Question 3:**

Write the following queries in both SQL and extended relational algebra (for bags), based on the Spy database schema

- Find the average salary of an agent.
- Find the average salary of agents in the USA.
- Find the average salary of agents on team # 12
- Find the number of agents who either speak German or are demolition experts.
- Find the average salary of an agent for each country.
- Find the average salary of an agent for each skill\_id.
- Find the agent id for agents who speak at least three languages.
- Find the maximum salary of agents who speak French.
- For each agent with clearance ID above 2, find the average salary
- Find the average salary of agents from the USA for each mission, where the mission has a clearance\_id = 5.

**Answer:**

a) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|                             |   |
|-----------------------------|---|
| SQL                         | SELECT AVG(salary)<br>FROM agent                  |
| extended relational algebra | $\forall \text{AVG}(\text{salary})(\text{agent})$ |

b) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|                             |  |
|-----------------------------|--|
| SQL                         | SELECT AVG(salary)<br>FROM agent<br>WHERE country='USA'                          |
| extended relational algebra | $\forall \text{AVG}(\text{salary})(\sigma_{\text{country}='USA'}(\text{agent}))$ |

c) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|                             |   |
|-----------------------------|---|
| SQL                         | SELECT AVG(salary)<br>FROM agent,teamrel<br>WHERE team_id=12<br>AND agent.agent_id=teamrel.agent_id   |
| extended relational algebra | $\forall \text{AVG}(\text{salary})(\sigma_{\text{team\_id}=12 \text{ AND agent.agent\_id}=\text{teamrel.agent\_id}}(\text{agent} \times \text{teamrel}))$ |

d) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|                             |   |
|-----------------------------|---|
| SQL                         | SELECT COUNT(agent_id)<br>FROM ((SELECT agent_id<br>FROM languagerel,language<br>WHERE languagerel.lang_id=language.lang_id<br>AND language.language='German')<br>UNION<br>(SELECT agent_id<br>FROM skillrel,skill<br>WHERE skillrel.skill_id=skill.skill_id<br>AND skill.skill='Demolition Expert')) res   |
| extended relational algebra | $\forall \text{COUNT}(\text{agent\_id})(\pi_{\text{agent\_id}}(\sigma_{\text{languagerel.lang\_id}=\text{language.lang\_id AND language.language}='German'}(\text{languagerel} \times \text{language}))) \cup (\pi_{\text{agent\_id}}(\sigma_{\text{skillrel.skill\_id}=\text{skill.skill\_id AND skill.skill}='Demolition Expert'}(\text{skillrel} \times \text{skill})))$ |

e) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|     |                            |
|-----|----------------------------|
| SQL | SELECT country,AVG(salary) |
|-----|----------------------------|

|                             |  |
|-----------------------------|--|
|                             | FROM agent<br>GROUP BY country                       |
| extended relational algebra | $\forall_{\text{country,AVG(salary)}}(\text{agent})$ |

f) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|                             |   |
|-----------------------------|---|
| SQL                         | SELECT skill_id,AVG(salary)<br>FROM agent,skillrel<br>WHERE agent.agent_id=skillrel.agent_id<br>GROUP BY skill_id                 |
| extended relational algebra | $\forall_{\text{skill\_id,AVG(salary)}}(\sigma_{\text{agent.agent\_id=skillrel.agent\_id}}(\text{agent} \times \text{skillrel}))$ |

g) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|                             |  |
|-----------------------------|--|
| SQL                         | SELECT agent_id<br>FROM languagerel<br>GROUP BY agent_id<br>HAVING COUNT(lang_id)>=3                     |
| extended relational algebra | $\sigma_{\text{COUNT(lang\_id) \geq 3}}(\forall_{\text{agent\_id,COUNT(lang\_id)}}(\text{languagerel}))$ |

h) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|                             |  |
|-----------------------------|--|
| SQL                         | SELECT MAX(salary)<br>FROM agent,languagerel,language<br>WHERE agent.agent_id=languagerel.agent_id AND<br>languagerel.lang_id=language.lang_id AND language.language='French'  |
| extended relational algebra | $\forall_{\text{MAX(salary)}}(\sigma_{\text{agent.agent\_id=languagerel.agent\_id AND languagerel.lang\_id=language.lang\_id AND language.language='French'}}(\text{agent} \times \text{languagerel} \times \text{language}))$ |

i) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|                             |   |
|-----------------------------|---|
| SQL                         | SELECT AVG(salary)<br>FROM agent<br>WHERE agent.clearance_id>2                        |
| extended relational algebra | $\forall_{\text{AVG(salary)}}(\sigma_{\text{agent.clearance\_id > 2}}(\text{agent}))$ |

j) The queries in both SQL and extended relational algebra (for bags) are listed bellow,

|                             |  |
|-----------------------------|--|
| SQL                         | SELECT mission_id,AVG(salary)<br>FROM agent,teamrel,mission<br>WHERE agent.country='USA' AND agent.agent_id=teamrel.agent_id AND<br>teamrel.team_id=mission.team_id AND mission.access_id=5<br>GROUP BY mission_id                             |
| extended relational algebra | $\forall_{\text{mission\_id,AVG(salary)}}(\sigma_{\text{agent.country='USA' AND agent.agent\_id=teamrel.agent\_id AND teamrel.team\_id=mission.team\_id AND mission.access\_id=5}}(\text{agent} \times \text{teamrel} \times \text{mission}))$ |

**Question 4:**

Suppose relations  $R$  and  $S$  have tuples  $m$  and  $n$  tuples, respectively. Give the minimum and maximum numbers of tuples that the results of the following expressions can have.

a)  $R \cap S$ .

b)  $R$  Full Outer Join  $S$ .

**Answer:**

a)  $R \cap S$ .

min: 0

max:  $\min\{m,n\}$

b)  $R$  Full Outer Join  $S$ .

min:  $\max\{m,n\}$

max:  $m+n$

**Question 5:**

The Circle and Sphere relations in a Geometry database give the center point of a circle or sphere of radius 5. Given the following data for these two tables:

Circle( $X,Y$ ):  $\{(1,2), (3,4), (1,2), (3,5), (4,5)\}$

Sphere( $X,Y,Z$ ):  $\{(1,2,3), (3,5,7), (3,6,2), (4,5,5), (1,3,5), (4,5,2)\}$

Compute the query answer for the following queries:

Note: these queries use the extended relational algebra operators that are defined for bags.

a.  $\pi_{X+Y, 2*X, 2*Y}(\text{Circle})$

b.  $\pi_{X+1, Y*2, Z-1}(\text{Sphere})$

c.  $\tau_{Y,X}(\text{Circle})$

d.  $\tau_{Y,Z}(\text{Sphere})$

e.  $\delta(\text{Circle})$

f.  $\delta(\text{Sphere})$

g.  $\gamma_{X, \text{SUM}(Y)}(\text{Circle})$

h.  $\gamma_{Y, \text{AVG}(Z), \text{AVG}(X)}(\text{Sphere})$

i.  $\gamma_X(\text{Circle})$

j.  $\gamma_{X,Y, \text{MAX}(Z)}(\text{Circle } C \bowtie_{C.X = S.X \text{ and } C.Y = S.Y} \text{Sphere } S)$

k. Circle Left Outer Join  $C.X = S.X \text{ and } C.Y = S.Y$  Sphere

l. Circle Right Outer Join  $C.X = S.X \text{ and } C.Y = S.Y$  Sphere

**Answer:**

a.  $\pi_{X+Y, 2*X, 2*Y}(\text{Circle}) = \{(3,2,4), (7,6,8), (3,2,4), (8,6,10), (9,8,10)\}$

b.  $\pi_{X+1, Y*2, Z-1}(\text{Sphere}) = \{(2,4,2), (4,10,6), (4,12,1), (5,10,4), (2,6,4), (5,10,1)\}$

c.  $\tau_{Y,X}(\text{Circle}) = \{(1,2), (1,2), (3,4), (3,5), (4,5)\}$

d.  $\tau_{Y,Z}(\text{Sphere}) = \{(1,2,3), (1,3,5), (4,5,2), (4,5,5), (3,5,7), (3,6,2)\}$

e.  $\delta(\text{Circle}) = \{(1,2), (3,4), (3,5), (4,5)\}$

f.  $\delta(\text{Sphere}) = \{(1,2,3), (3,5,7), (3,6,2), (4,5,5), (1,3,5), (4,5,2)\}$

g.  $\gamma_{X, \text{SUM}(Y)}(\text{Circle}) = \{(1,4), (3,9), (4,5)\}$

h.  $\gamma_{Y, \text{AVG}(Z), \text{AVG}(X)}(\text{Sphere}) = \{(2,3,1), (3,5,1), (5,4.66667, 3.66667), (6,2,3)\}$

i.  $\gamma_X(\text{Circle}) = \{(1),(3),(4)\}$

j.  $\gamma_{X,Y, \text{MAX}(Z)}(\text{Circle } C \bowtie_{C.X = S.X \text{ and } C.Y = S.Y} \text{Sphere } S) = \{(1,2,3), (3,5,7), (4,5,5)\}$

k. Circle Left Outer Join  $C.X = S.X \text{ and } C.Y = S.Y$  Sphere =  $\{(1,2,1,2,3), (3,4, \text{NULL}, \text{NULL}, \text{NULL}), (1,2,1,2,3), (3,5,3,5,7), (4,5,4,5,5), (4,5,4,5,2)\}$

l. Circle Right Outer Join  $C.X = S.X \text{ and } C.Y = S.Y$  Sphere =  $\{(1,2,1,2,3), (1,2,1,2,3), (3,5,3,5,7), (\text{NULL}, \text{NULL}, 3,6,2), (4,5,4,5,5), (\text{NULL}, \text{NULL}, 1,3,5), (4,5,4,5,2)\}$