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REVIVAL OF A STALLED SUPERNOVA SHOCK BY NEUTRINO HEATING

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ABSTRACT

We analyze the mechanism for revival of a stalled supernova shock found by one of us (J. R. W.) in a computation. Neutrinos from the hot, inner core of the supernova are absorbed in the outer layers, and although only about 0.1% of their energy is so absorbed, this is enough to eject the outer part of the star and leave only enough mass to form a neutron star. The neutrino absorption is independent of the density of material. After the shock recedes to some extent, neutrino heating establishes a sufficient pressure gradient to push the material beyond about 150 km outward, while the material further in falls rapidly toward the core. This makes the density near 150 km decrease spectacularly, creating a quasi-vacuum in which the pressure is mainly carried by radiation. This is a perfect condition to make the internal energy of the matter sufficient to escape from the gravitational attraction of the star. The net energy of the outgoing shock is about 4×10^{50} ergs.

Subject headings: neutrinos — shock waves — stars: supernovae

I. INTRODUCTION

One of us (J. R. W.) has recently shown (Wilson 1982, hereafter Paper I) that neutrino capture in the mantle of a high-mass star may give new life to an outgoing shock which had previously been stalled. It is the purpose of this paper to elucidate the physical conditions required for this phenomenon.

Essentially all computer calculations and analytical theories agree that after the gravitational collapse of the core of a highly evolved, massive star, a shock wave will be formed which propagates outward. However, in the great majority of computations, the shock gets out to only a moderate distance, roughly 100–300 km. It then stalls, and the material which falls through the shock subsequently accretes onto the dense core already formed. Clearly, this sequence of events does not lead to a supernova; the material finally accreted will exceed the stability limit of a neutron star (about 2 $M_\odot$), and a black hole results.

An exception is the calculation by Cooperstein, Bethe, and Brown (1984), in which they assumed that the inner core of the star has initially a very low entropy, about $0.5k_B$ per nucleon. In this case, Cooperstein *et al.* found an outgoing shock which appears not to stall. But it remains to be seen whether such a low initial entropy is compatible with the presupernova evolution of the star; the best present presupernova calculations (Weaver, Woosley, and Fuller 1985) yield an entropy of about 0.8 units.

We shall here assume, as was found in Paper I, that the shock stalls. The core (including all the material accreted by it) will then emit copious neutrinos. For the present, we shall just take the result of the computer calculation, viz., that the energy flux in each of the electron neutrinos and antineutrinos is about

$$L_{\nu e} \approx L_{\bar{\nu} e} \approx 4 \times 10^{52} \text{ ergs s}^{-1} . \tag{1}$$

The flux of ν_μ, $\bar{\nu}_\mu$, ν_τ, and $\bar{\nu}_\tau$ is only slightly smaller. However, these neutrinos cannot be captured by nucleons, and hence give a much smaller contribution to the energy which can be transferred to the outer layers of the star (see below).

II. HEATING BY NEUTRINOS

The net rate at which energy is absorbed by a gram of matter at large distance R_m from the center is

$$\dot{E} = k(T_\nu)\left[\frac{L_\nu}{4\pi R_m^2} - \left(\frac{T_m}{T_\nu}\right)^2 acT_m^4\right] \text{ ergs g}^{-1}\text{ s}^{-1} . \tag{2}$$

Here the subscript m refers to the matter element and ν to the "neutrino sphere," i.e., the sphere at which the neutrinos are emitted from the core. Its position may be defined by the condition that the collision mean free path of an average neutrino is equal to the radius R_ν. Generally, R denotes the radius (distance from the star center) and T the temperature in MeV. $K(T_\nu)$ is the neutrino absorption coefficient in cm^2 g^{-1} for the temperature T_ν of the neutrino sphere. From the computer output, $T_\nu \approx 5$ MeV, and $R_\nu \approx 30$ km. L_ν is the energy flux in electron neutrinos alone.

The first term in equation (2), the energy gain, assumes that the neutrinos move radially outward at R_m, which is a good assumption if $R_m \gg R_\nu$. The second term is the energy loss due to electron and positron capture at the matter temperature T_m. Here aT_m^4 is the energy per unit volume of a "blackbody" neutrino gas,

$$a = \tfrac{7}{16} \times 1.37 \times 10^{26} \text{ ergs cm}^{-3}\text{ MeV}^{-4} , \tag{3}$$

and it has been assumed that the absorption coefficient for neutrinos is proportional to T^2.

The expression for the negative term in equation (2) is correct only if the electrons and positrons also form a blackbody gas, i.e., if the electron chemical potential $\mu_e = 0$. Because in fact $\mu_e > 0$, the energy loss by neutrino emission is greater, as is discussed in the Appendix; for an actual case, it is about 50% larger than in equation (2). But we are mainly interested in the case when the energy loss by neutrino emission is small compared to the energy gain by neutrino absorption, and then the numerical value of the loss is not important.

The absorption processes are

$$n + \nu_e \rightarrow p + e^- \ , \quad p + \bar{\nu}_e \rightarrow n + e^+ \ . \qquad (4)$$

The absorption therefore is actually proportional to

$$K(T_\nu)(L_\nu Y_n + L_{\bar{\nu}} Y_p) \ , \qquad (5)$$

where Y_n and Y_p are, as usual, the mass fractions of neutrons and protons. Since, according to equation (1), $L_\nu \approx L_{\bar{\nu}}$, we get

$$L_\nu(Y_n + Y_p) \equiv L_\nu Y_N \ , \qquad (5a)$$

where Y_N is the total mass fraction of nucleons (as distinct from heavier nuclei). If the matter is fully dissociated into nucleons, $Y_N = 1$. But even in complex nuclei (except for He), processes (4) can occur with somewhat reduced cross section, just as for electron capture on the infall (Bethe *et al.* 1979). We therefore replace Y_N by $\tilde{Y}_N$, where $\tilde{Y}_N$ lies between Y_N and 1. It is reasonable to simply multiply equation (2) by $\tilde{Y}_N$.

The absorption coefficient is

$$K = \mathscr{A}\sigma \ , \qquad (5b)$$

where $\mathscr{A}$ is Avogadro's number (number of nucleons per gram) and σ the capture cross section,

$$\sigma = 2 \times 4.5 \times 10^{-44} \langle \epsilon_\nu^2 \rangle \ . \qquad (6)$$

Without the factor 2, this is the capture cross section for electrons, i.e., $p + e^- \rightarrow n + \nu$. The factor 2 arises because neutrinos have only one spin direction. The neutrino energy is measured in MeV; we take

$$\langle \epsilon_\nu^2 \rangle = 7T_\nu^2 \ . \qquad (6a)$$

If the neutrinos had a true Boltzmann spectrum, the factor would be 21, but the spectrum is actually poor in high-energy neutrinos (because of their short mean free path), and Bethe, Applegate, and Brown (1980) have given arguments for a factor of about 7; this point must be further investigated. Then

$$K(T_\nu) = 3.8 \times 10^{-19} T_\nu^2 \ \mathrm{cm}^2 \ \mathrm{g}^{-1} \ , \qquad (7)$$

and the first term in equation (2) becomes

$$\dot{E}_+ = 3.0 \times 10^{18} \times L_{52}(T_\nu^2/R_{m7}^2)\tilde{Y}_N \ \mathrm{ergs\ g^{-1}\ s^{-1}} \ , \qquad (7a)$$

where L_{52} is the neutrino luminosity in units of 10^{52} ergs s^{-1}, and R_{m7} the distance from the center in units of 10^7 cm. Note that

$$10^{18} \ \mathrm{ergs\ g^{-1}} = 1.04 \ \mathrm{MeV\ per\ nucleon} \ . \qquad (7b)$$

Neutrinos are not only captured but also scattered. Scattering by nuclei transfers almost no energy because of the heavy mass of nuclei. Scattering by electrons is discussed in the Appendix. The approximate result is given by equation (A22), which is

$$\frac{\dot{E}_{\mathrm{sc}}}{\dot{E}} \approx \frac{T_m}{T_\nu} \ . \qquad (8)$$

Typically, $T_m \approx \frac{1}{5}T_\nu$, so scattering adds about 20% to the heating by capture. Scattering becomes relatively more important when the nucleons are in complex nuclei rather than free, because Y_e does not contain the extra factor $\tilde{Y}_N$ of equation (7a).

Our final formula, replacing equation (2), is then

$$\dot{E} = 3.8 \times 10^{-19}\left(\tilde{Y}_N + \frac{T_m}{T_\nu}\right)\left(T_\nu^2 \frac{L_\nu}{4\pi R_m^2} - T_m^2 acT_m^4\right) \times \mathrm{ergs\ g^{-1}\ s^{-1}} \ . \qquad (9)$$

We realize that it is not justified to put the factor $F = \tilde{Y}_N + T_m/T_\nu$ also with the second, negative term in equation (9). But in the most important cases that second term is only a correction. For most of our further discussion, we shall replace the first factor in equation (9) by unity, i.e., we go back to equation (2).

It is useful to write L_ν in terms of the temperature of the neutrino sphere, viz.,

$$L_\nu = 4\pi R^2 \tfrac{1}{4} acT_\nu^4 \ . \qquad (10)$$

T_ν is usually about 5 MeV. Then equation (2) becomes

$$\dot{E} = K(T_\nu)acT_\nu^4\left[\left(\frac{R_\nu}{2R_m}\right)^2 - \left(\frac{T_m}{T_\nu}\right)^6\right] . \qquad (11)$$

To have positive heating, we must have

$$\frac{T_m}{T_\nu} < \left(\frac{R_\nu}{2R_m}\right)^{1/3} . \qquad (11a)$$

III. HEATING AND MOTION

A most important feature of the heating formulae (2) and (7) is that they are independent of the density of the matter; dilute matter is heated as much (per unit mass) as dense matter. The energy first goes into dissociating nuclei: this is very important because it makes it unnecessary for the shock to supply the dissociation energy—it has been shown that the need for supplying the dissociation energy is chiefly responsible for the stalling of the original shock. At a temperature of slightly over 1 MeV, dissociation into nucleons is accomplished, thereby incidentally making the factor $\tilde{Y}_N$ in equation (9) equal to one. Further neutrino capture then gives additional (pressure producing) energy to the nucleon gas.

We shall now discuss detailed results from the calculation for a star of mass 25 $M_\odot$ in Paper I. Table 1 shows, as a function of the time t from collapse, the internal energy of the material behind the shock, ϵ, the fraction of free nucleons Y_N,

TABLE 1

INTERNAL ENERGY, p/ρ, AND NUCLEON FRACTION

t (s)	M ($M_\odot$)	ϵ_{int} (10^{18} ergs g^{-1})	p/ρ	Y_N	rp/ρ (10^{25} $\mathrm{cm}^3\ \mathrm{s}^{-2}$)
0.417...	1.659	3.1	0.72	0.010	3.4
0.433...	1.659	3.5	0.76	0.016	3.1
	1.665[a]	3.5	0.80	0.011	3.5
0.464...	1.659	5.7	1.22	0.18	2.8
	1.665	5.8	1.24	0.20	3.2
	1.669[a]	4.3	0.94	0.064	2.7
0.478...	1.659	9.4	1.76	0.55	2.6
	1.665	9.5	1.82	0.57	2.9
	1.678[a]	5.5	1.19	0.17	2.3
0.493...	1.659	13.7	2.74	1.00	3.7
	1.665	13.2	2.48	1.00	3.7
	1.678[a]	6.9	1.43	0.30	2.7
0.504...	1.641	34	10.8	1.00	4.6
	1.650	19	4.8	1.00	4.6
	1.659	16.1	3.7	1.00	4.7
	1.665	15.5	3.4	1.00	4.9
	1.673	7.3	1.64	0.50	2.8
	1.684[a]	3.3	1.76	0.38	4.4

[a] These mass points are directly behind the shock.

and the ratio p/ρ. For matter far out in the star, all three quantities increase with time, both for a given mass element,[1] e.g., $M = 1.659\ M_\odot$, and for the material directly behind the shock. At any given t, the quantities increase as M decreases, because the lower-M material has been heated for a longer time. The neutrino heating increases the internal energy, this leads to dissociation of nuclei, and this again to increased p/ρ. Both ϵ and p/ρ are given in units of 10^{18} ergs g^{-1}, i.e., essentially in MeV per nucleon, equation (7b); the two quantities are roughly proportional. Writing

$$p/\rho = (\Gamma - 1)\epsilon\ , \tag{12}$$

$$\Gamma - 1 = 0.19\text{–}0.25 \tag{12a}$$

for $M \geq 1.659$ over the time interval investigated. If the "energy-Γ," as defined by (12), is independent of ρ, then it is also equal to the γ in the equation of an adiabat,

$$p \approx \rho^\gamma (\text{at constant entropy } S)\ . \tag{13}$$

As we know, an energy of 8.5×10^{18} is required to dissociate the nuclei into nucleons, and in addition about 2×10^{18} to give the nucleons the required thermal energy. At ϵ_{18} of[2] ~ 3, dissociation goes only to α-particles, and Y_N is very small. At $\epsilon_{18} = 5\text{–}10$, the nucleon fraction gradually increases to about 0.5. At $\epsilon_{18} > 13$, full dissociation is achieved. Beyond this ϵ, Γ increases with increasing ϵ because the the excess of the energy over dissociation energy can all produce pressure.

The equation of motion of a mass element is

$$\ddot{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{GM(r)}{r^2}\ , \tag{14}$$

where $M(r)$ is the mass included in the sphere r; this is kept constant with time. We rewrite equation (14):

$$r\ddot{r} = -\frac{\partial \log p}{\partial \log r}\frac{p}{\rho} - \frac{GM}{r}\ . \tag{15}$$

Defining

$$k = -\frac{\partial \log p}{\partial \log r}\ , \tag{15a}$$

we have

$$r^2\ddot{r} = kr\frac{p}{\rho} - GM\ . \tag{16}$$

The range of masses of interest is rather narrow, so the last term in equation (16) is nearly constant:

$$GM \approx 22 \times 10^{25}\ \text{cm}^3\ \text{s}^{-2}\ . \tag{16a}$$

The product, rp/ρ, is tabulated in Table 1. It changes slowly with t and M and is of order $2.5\text{–}5 \times 10^{25}$. At the last time given, $t = 0.504$, we have included in Table 1 some masses $M < 1.659\ M_\odot$.

It is important to know when (if at all) the acceleration $\ddot{r}$ becomes positive. With $rp/\rho = 3$, this would require $k \geq 7$. For $t = 0.504$ and $M = 1.665$, however, $rp/\rho = 4.9$, and $k = 4.5$ is sufficient. This still may appear large, because a "normal" value of the related quantity

$$n = -\frac{\partial \log \rho}{\partial \log r} \tag{16b}$$

is $n = 3$. Then we would expect

$$k \approx \Gamma n \approx 4\ . \tag{16c}$$

However, we are now helped by a peculiar property in the dynamics of the infalling material. Before collapse of the center, the exponent n in equation (16b) is ~ 3. In our case, the density distribution is

$$\rho \approx 1.0 \times 10^{31} r^{-3}\ . \tag{17}$$

Well after collapse of the center, however, Cooperstein, Bethe, and Brown (1984) have shown that the density outside the shock wave is

$$\rho_1 = A_1 t^{-1} r^{-3/2}\ , \tag{18}$$

if the initial distribution was as shown in equation (17). Thus ρ, at given r, falls off with time, but for given t, it falls off more slowly with r than does equation (17). In our numerical results, equation (18) is very well fulfilled, from $t = 0.1\text{–}0.8$, with

$$A_1 = 1.2 \times 10^{18}\ \text{cgs}\ . \tag{18a}$$

Right after the shock has hit a given mass element M, the density is a certain multiple of the pre-shock density, i.e.,

$$\rho_2 = A_2 t_M^{-1} r_M^{-3/2}\ , \tag{18b}$$

where t_M and r_M are the time and radius at which M is hit by the shock. Once behind the shock, the density for given M increases again roughly as r^{-3}, as in equation (17), so when M has arrived at $r < r_M$, its density is

$$\rho(M, r) \approx \rho_2 (r_M/r)^3 = A_2 r^{-3} r_M^{3/2}/t_M\ . \tag{19}$$

Density proportional to r^{-3} is the appropriate polytrope for material of $\gamma = 4/3$ in hydrostatic equilibrium. (Near $r = 150$ km, ρ is somewhat higher than indicated by eq. [19], because there the material moves very slowly so that its density is increased, by continuity.)

Now it is characteristic of computations with a stalled shock that, for a period of time, the shock front actually recedes, i.e.,

$$dr_M/dM < 0\ . \tag{20}$$

In our computation, this is the case for $0.4 < t < 0.5$ s. The recession is not too surprising because the inner core, which after all supports the shock, shrinks due to loss of leptons and energy. Material velocities behind the shock, extending over about $0.05\text{–}0.1\ M_\odot$, are negative and several times 10^8. Velocities outside the shock are of order -2 to -3 times 10^9. The shock velocity, between $t = 0.42$ and 0.48, is about (see Fig. 1)

$$U_M = dr_M/dt_M \approx -5 \times 10^8\ . \tag{20a}$$

Of course,

$$dt_M/dM > 0\ . \tag{20b}$$

If we now consider a time after the recession of the shock e.g., $t = 0.478$, the pressure (and density) behind the shock fall very rapidly with r (Table 2). This is because, in equation (19), the mass points, which are now at smaller r, have smaller M and therefore were hit by the shock at a larger radius r_M and earlier time t_M (large r_M is more important). As is seen from Table 2,

$$k \equiv -\frac{d \log p}{d \log r} \approx 7\text{–}9\ . \tag{21}$$

(The density falls about as $r^{-6.5}$, on the average.) Table 2 also gives the product krp/ρ from Table 1. It is seen that krp/ρ is just

[1] All masses quoted here are masses of the particles, not gravitational masses.

[2] Expressions like ϵ_{18}, or R_6, mean the value of ϵ in units of 10^{18} cgs units, or R in units of 10^6 cm, etc.

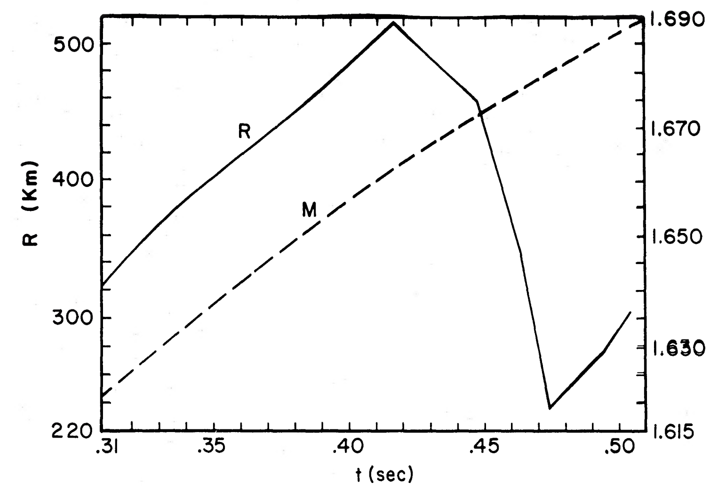

FIG. 1.—Position of the shock wave in the explosion of a star of 25 $M_\odot$. The solid line gives the radius of the shock (*left scale*). Note especially the sharp drop between $t = 0.45$ and 0.47 s. The dashed line gives the mass enclosed by the shock (*right scale*, in units of the solar mass). It is difficult to determine the shock position accurately from the computer output; therefore we did not draw the curves accurately through the computer points.

about 22, so that $\ddot{r} \approx 0$. The detailed computation gives $\ddot{r} > 0$. This has been achieved due to the very steep pressure profile, i.e., the large value of k.

Thus the very fact that the shock recedes in space for a while yields a steep pressure distribution, so that the pressure gradient overcomes gravitation and gives an outward acceleration. The gravitational acceleration is large: since $r \approx 1.5 \times 10^7$ in the most important region,

$$GM/r^2 \approx 10^{12} . \tag{22}$$

Thus even a slight imbalance between pressure and gravitation will give an acceleration of order 10^{11}. Thus a time interval

$$\Delta t \approx 0.01 \text{ s} \tag{22a}$$

changes the velocity by order 10^9. If $\ddot{r} > 0$, this will change an infall velocity, $V = -5 \times 10^8$, to an outgoing velocity, $V = +5 \times 10^8$. The pressure gradient thus causes outward motion, at least of some of the material.

At $t = 0.504$ and $M \geq 1.659$, the result is similar. The pressure profile is now less steep, but p/ρ is bigger due to the full dissociation of the nuclei, Table 1. The resulting krp/ρ is near the critical value 22, and again detailed computation gives $\ddot{r} > 0$. The situation is different for $M < 1.659$: Here the pressure profile is much flatter, $k = 2.1$ only, and krp/ρ is only 10, about half the gravitational attraction. This material therefore falls in rapidly, and is stopped only by hitting the rather solid, inner core at about $r = 30$ km. The rapid infall prevents any substantial neutrino heating beyond that already accomplished at $r = 150$ km, and in addition the temperature T_m gets high enough so that the cooling, the second term in (2), becomes comparable to the neutrino heating, i.e., the first term in (2). It appears that the material at about 150 km is in a precarious balance: slightly farther out, material is accelerated outward, while farther in it falls rapidly onto the core.

The outward moving material gets the shock going again, as we shall show in the next section. Aside from neutrino heating, the main cause of the outward acceleration is the fact that for some period the shock actually went inward. A secondary benefit of the shock recession is that while the shock is retreating more matter falls onto the core and raises the luminosity by about 50%. For this reason, which is already evident in Paper I, we have called (Cooperstein, Bethe, and Brown 1984) this phenomenon "the pause that refreshes."

TABLE 2

PRESSURE VS. RADIUS

t (s)	M ($M_\odot$)	r (km)	p (10^{27}dynes cm^{-2})	k	kpr/ρ (10^{25} cm^3 s^{-2})
0.478	1.659	150	7.8		
	1.669	169	3.3	7.1	20
	1.678	197	0.81	9.1	22
0.504	1.641	43	96		
	1.659	128	9.6	2.1	10
	1.669	163	4.0	3.7	17
	1.678	193	1.57	6.2	19
	1.687	289	0.22	4.6	17

IV. QUASI-VACUUM AND BIFURCATION

In the last section, we have shown that material inside a certain radius, of about 150 km, tends to move in toward the center of the star, while material at $r > 150$ km tends to move out (for $t > 0.50$). Thus a bifurcation takes place. We believe that essentially all the material which is at $r > 150$ km at $t = 0.50$ s will continue to move out and escape from the star (Fig. 2). We shall give arguments for this belief later on.

Around 150 km, density becomes very low and a quasi-vacuum is created. Pressure, of course, is maintained; in fact the pressure must be (and is) higher than in the outgoing material. The combination of fairly high pressure and low density means that most of the pressure cannot be in the material, but is in radiation. Electron pairs also contribute. The energy density of radiation is

$$w'_r = 1.37 \times 10^{26} T^4 \text{ ergs cm}^{-3} , \tag{23}$$

where the temperature T is MeV. Electron pairs contribute

$$w_e = \tfrac{7}{4} w'_r , \tag{23a}$$

if $T \gtrsim mc^2 = 0.51$ MeV, i.e., if the electrons are relativistic. For $T < 0.5$ MeV, the electron contribution is less, but at 0.5 MeV, the deviation is only about 1%. We write for the sum

$$w_r = w'_r + w_e = 3.77 \times 10^{26} T^4 \text{ ergs cm}^{-3} \equiv a' T^4 . \tag{23b}$$

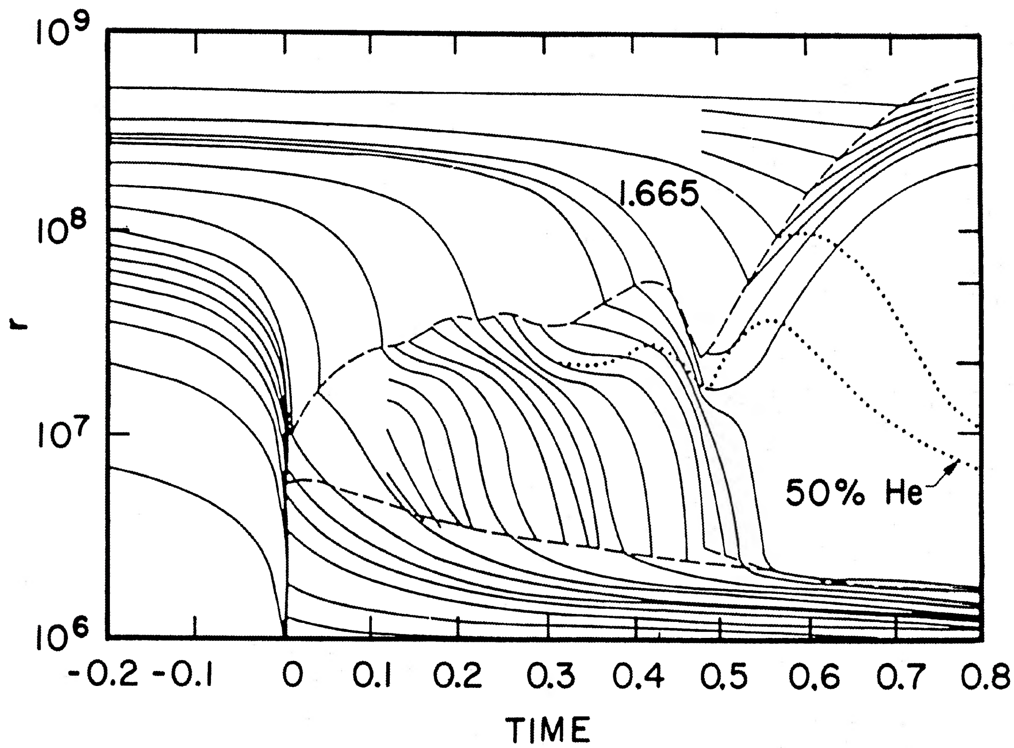

FIG. 2.—Trajectories of various mass points. Radius r is in cm, time from collapse in s. The lower dashed curve is the position of the neutrino sphere, the upper one is the shock. At about $t = 0.48$, two neighboring trajectories begin to diverge widely; the region between is the quasi-vacuum described in the text. Before this time, some trajectories are held in balance at about $r = 2 \times 10^7$.

The pressure due to radiation and electron pairs is

$$p_r = \tfrac{1}{3} w_r \; . \tag{23c}$$

Of course, some matter is still present. As was pointed out in § II, the neutrino absorption *per unit mass* is independent of the density. Thus the matter in the quasi-vacuum continues to absorb neutrinos. Using, in equation (7a), $L_{52} = 4$, $T_\nu = 5$, and $\tilde{Y}_N = 1$, we have

$$\dot{E} = 300 \times 10^{18}/R_{m7}^2 \; . \tag{24}$$

This continues for several tenths of a second, so

$$E \approx 100 \times 10^{18}/R_{m7}^2 \text{ ergs g}^{-1} \; , \tag{24a}$$

which agrees roughly with the computer output at $t = 0.8$ s. Assuming, as before, that most of the energy is in radiation, equation (23b) gives

$$T^4 \approx 2.7\rho_7/R_{m7}^2 \; . \tag{24b}$$

Typical densities at $t = 0.8$ are 6 to 2×10^5, and the quasi-vacuum extends from about 30 to 1000 km, so that

$$0.25 \lesssim T \lesssim 0.75 \; . \tag{25}$$

Since the temperature is low, and the energy so high, the entropy S per nucleon is very high; at $t = 0.8$,

$$50 \lesssim S \lesssim 400 \; . \tag{26}$$

Such entropies, for matter alone, would be absurd. But the entropy also resides mostly in radiation. Neglecting the fact that $w_e < (7/4)w_r'$ at low T, the radiation entropy is

$$S_r' = \tfrac{4}{3} a' T^3 \text{ ergs MeV}^{-1} \text{ cm}^{-3} \; , \tag{26a}$$

where a' is given in equation (23b). The entropy per nucleon is then

$$S_r = \frac{S_r'}{\mathscr{A}\rho} = \frac{4}{3} \frac{a'T^3}{\mathscr{A}\rho} \frac{\text{ergs}}{\text{MeV}^{-1}} \; , \tag{26b}$$

where $\mathscr{A}$ = Avogadro's number. Now

$$\mathscr{A} \text{ MeV erg}^{-1} = 0.96 \times 10^{18} \; , \tag{26c}$$

so

$$S_r = \frac{4}{3} \frac{a'}{0.96 \times 10^{18}} \frac{T^3}{\rho} \text{ per nucleon} \; . \tag{27}$$

The ratio of radiation to matter energy (in nucleons alone)

$$\frac{w_r}{w_m} = \frac{a'T^4}{(3/2)0.96 \times 10^{18}\rho T} \; , \tag{27a}$$

so

$$S_r = 2 \frac{w_r}{w_m} \; . \tag{28}$$

The entropy in matter in the shock is typically 8 ± 2 per nucleon.

The high radiation pressure in the quasi-vacuum is important for the motion of the outgoing material. Some astrophysicists have worried that if there is a bifurcation, the vacuum (rather, rarefied region) between ingoing and outgoing material will suck in the outgoing matter, either slowing all of it down, or removing material from its inner edge and making that fall back on the core. But there is no low-pressure region in the quasi-vacuum; therefore these worries are unnecessary.

Our description of the bifurcation applies only to the case when the original shock stalls and is then revived by neutrino heating after the "pause that refreshes." We consider it possible that the original shock succeeds, as discussed by Cooperstein, Bethe, and Brown (1984). In this case, bifurcation does not occur at a small radius like 150 km. Instead, the shock may proceed to several thousand km before any bifurcation becomes noticeable. Ultimately, some material will have a positive total energy

$$\epsilon_{\text{tot}} = \epsilon_{\text{int}} - GM/r > 0 \; ; \tag{29}$$

this will probably be true for $M(r)$ greater than some separation mass M_0. Material just inside M_0 will then still be moving out, but after some further time will come to rest and then fall back to the center. A neutron star of mass M_0 will ultimately be formed.

It is likely that radiation energy will play a part in this case also. Once the shock radius is several thousand km, radiation energy will dominate over material energy. Therefore the space in the bifurcation gap will be occupied by radiation which again will protect the outgoing material from being sucked in.

Returning to our case, the material on the inside of the quasi-vacuum forms a dense core, of radius about 20 km, at $t = 0.8$. Its entropy is about $S = 3$ at $t = 0.5$, falling to $S = 2$ at $t = 0.8$. The core has a strong negative energy, of order -10^{53} ergs because of the emission of so many neutrinos. Such a core necessarily has a finite radius, which can be calculated without any regard to the near-vacuum region because the pressure in the near-vacuum is negligibly small as far as the core is concerned. To compute the core radius, we only need to know the (negative) core energy, and the entropy versus mass inside it; the pressure at the core surface may be set to zero for this purpose.

Near the center of the core, the density is very high, up to 7×10^{14}, but near the surface, it falls off precipitously, Figure 3: it decreases from 4×10^{13} at $M = 1.602$, $R = 15.7$ km, to 3×10^6 at $M = 1.665$, $R = 20.1$ km. This is to be expected for matter of low entropy in a strong gravitational field. Outside $R = 20$ km, the density falls to 10^6; we are now in the quasi-vacuum: that medium may be considered a vapor surrounding the solid core.

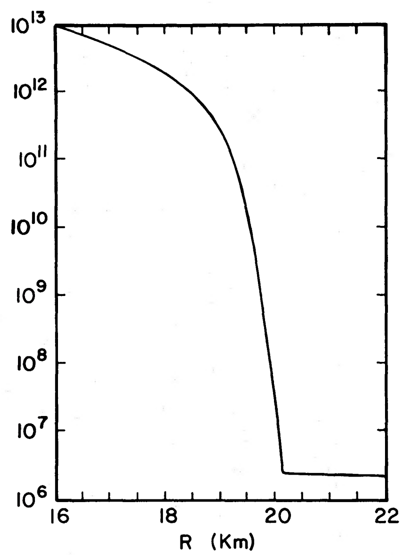

FIG. 3.—The cliff at the outer edge of the core, at $t = 0.8$ s. The density decreases by a factor 10^6 in about 2 km distance. Outside about $r = 20$ km, there is the "vapor" of density 2×10^6, slowly decreasing as r increases.

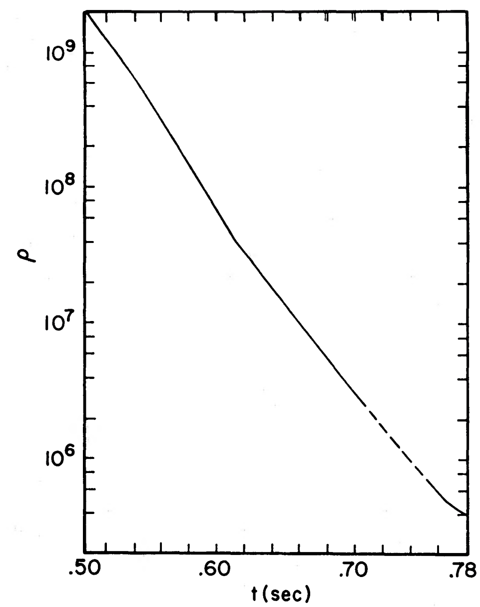

FIG. 4.—Decrease of density with time at $r = 140$ km. Before $t = 0.50$ and after $t = 0.78$, the density decrease is slow. Since accurate data are hard to obtain from the computer output, the curve passes by some of the points.

Finally, we discuss the manner in which the near-vacuum is formed from an initial, smooth density distribution. We use the continuity equation

$$\frac{\partial \ln \rho}{\partial t} = -u \frac{\partial \ln \rho}{\partial r} - \frac{2u}{r} - \frac{\partial u}{\partial r} \, . \tag{30}$$

The most important term is $\partial u/\partial r$: we have shown in § 3 that at $t = 0.5$, the material inside a certain radius flows inward, the material farther out flows outward. The radius for which $u = 0$ is about 140 km, and $\partial u/\partial r \approx 30 \text{ s}^{-1}$; similar results are obtained at later times. Thus ρ should decrease by a factor e in about 0.03 s. Following ρ at $r = 140$ km for about 0.25 s (Fig. 4), we see that this is indeed the case, the slope in $\ln \rho$ versus t is 33 s^{-1}. After $t = 0.77$, the drop of ρ stops because ρ has reached the level existing in the outflowing material, which in turn is determined by that in the infalling matter.

In this manner, ρ decreases by about a factor of 4000 in 0.25 s. This decrease is far stronger than that in the infalling material, equation (18). The very low resulting density will be important in the next section.

V. ENERGY AND ESCAPE

If a material element has a positive total energy per unit mass

$$\epsilon_{\text{tot}} = \epsilon_{\text{int}} - GM/r + \tfrac{1}{2}u^2 \, , \tag{31}$$

it can escape from the star provided it has no further interactions. The kinetic energy $\frac{1}{2}u^2$ is in general small compared to the two other terms. The internal energy continually increases due to neutrino heating, equation (2), so ultimately there is a good chance that equation (31) indeed becomes positive.

The computer output shows that the bifurcation region centers about $r = 150$ km and $M = 1.665 \, M_{\odot}$. In order to allow this material to escape from gravity, we must therefore have

$$\epsilon_{\rm int} > GM/r = 14.7 \times 10^{18} \text{ ergs g}^{-1} . \tag{31a}$$

On the other hand, equation (11a) gives an upper limit to the temperature of the material. Using $R_v = 30$ km,

$$T_m < T_v(R_v/2R_m)^{1/3} = 5 \times 10^{-1/3} = 2.3 \text{ MeV} . \tag{31b}$$

If all the energy were in the material, and the nuclei were fully dissociated, with $Y_e = \frac{1}{2}$, the energy per nucleon for $T_m = 2.3$ MeV would be

$$\begin{aligned} \epsilon_{\rm int} &= 8.8 + (\tfrac{3}{2} + \tfrac{1}{2} \times 3) \times 2.3 \\ &= 15.7 \text{ MeV per nucleon} , \\ &= 15.1 \times 10^{18} \text{ ergs g}^{-1} . \end{aligned} \tag{31c}$$

(The 3/2 refers to the thermal energy of the nucleons, the (1/2) × 3 to the electrons.) Thus equation (31a) would be only barely fulfilled. To have any leeway at all, it is necessary that much of the energy resides in radiation rather than in the material. In other words, in order to permit escape from gravity, it is *necessary* to have a quasi-vacuum region in which radiation energy dominates. (Of course, our choice of $R_m = 150$ km is arbitrary, and the material energy at T_m would be sufficient at larger R_m. But there the neutrino heating, according to eq. [2], would become very small.)

If equation (31a) is to be fulfilled by radiation energy, we must have

$$\frac{w_r}{\rho} > \frac{GM}{r} . \tag{32}$$

Using (23b) and (31a),

$$3.77 \times 10^{26} T^4/\rho > 14.7 \times 10^{18} . \tag{32a}$$

This gives an upper limit to the permitted density,

$$\rho_{\rm max} = 2.5 \times 10^7 T^4 . \tag{33}$$

At time 0.504 s, the beginning of Figure 4, this is not fulfilled: $T \approx 2$ MeV, so

$$\rho_{\rm max} = 4 \times 10^8 , \tag{33a}$$

while the actual density is

$$\rho = 1.8 \times 10^9 . \tag{33b}$$

So the rarefraction due to div u, as described at the end of § IV, is essential to have inequality (32) fulfilled. At the end of the rarefraction, $t = 0.80$. the inequality is well fulfilled at 150 km, where now

$$T = 0.41 , \quad \rho_{\rm max} = 7 \times 10^5 , \quad \rho = 3.3 \times 10^5 . \tag{34}$$

Incidentally, in these conditions, the internal energy is almost all in radiation,

$$w_r/w_m \approx 30 . \tag{34a}$$

More than half the material energy is in electrons, because the energy in nuclear particles is kept down by the small fraction of dissociation (7% α-particles, 4% nucleons).

Table 3 gives the internal and the gravitational energy in the outgoing material as functions of the included mass. Clearly, the internal energy is much greater than the gravitational one everywhere. The total energy is therefore positive everywhere. The rise for the outermost mass interval is probably due to the shock front. The last column of Table 3 gives the pressure, in units of 10^{23} dynes cm^{-2}.

As we mentioned earlier, positive total energy per unit mass does not yet insure that the mass element can escape the gravitational field, because there are still external forces on it. However, we are helped by the fact that radiation energy dominates all material energies. This means that the material can be kept in balance with only slight gradients of the radiation pressure over the entire volume inside the shock wave; this is indeed shown by the last column of Table 3. So the shock progresses, keeping the pressure inside essentially uniform at any time, which means the radiation field expands homologously. Thus the velocity of any material elment is very nearly linear in r. This also is confirmed by the computation, except that the innermost 1000 km (< 1% of the shock volume) remain nearly at rest, because here gravity acting on the material is relatively stronger.

This picture was already proposed by Weaver, Zimmerman, and Woosley in 1978. It means that every material element outside the core of mass 1.665 $M_\odot$ is swept out with the shock. The matter is of course attached to the radiation (or vice versa), because the diffusion of radiation relative to matter is minimal (mean free path of order 10^{-5} cm). The external force acting on each matter element is chiefly the radiation, and this carries the matter out. This remains true also for the outer mantle and envelope of the star beyond $r = 5000$ km; in fact, here the gravitational energy becomes even smaller. A detailed computation of these late stages of shock development remains to be done.

TABLE 3

ENERGIES PER UNIT MASS

($t = 0.78$)

Mass Interval	R (10^8 cm)	Gravitational[a]	Internal[a]	Kinetic[a]	Total[a]	Pressure (10^{23} dynes cm^{-2})
1.666	1.25	1.78	5.28	0.05	3.55	5.5
1.667	1.54	1.44	4.39	0.18	3.13	4.6
1.668–69	1.86	1.20	3.83	0.38	3.01	4.1
1.670–76	2.31	0.97	3.00	0.48	2.51	3.9
1.676–86	2.89	0.78	2.12	0.71	2.05	2.6
1.686–95	3.30	0.68	1.40	0.90	1.62	2.3
1.695–709 ...	3.64	0.62	1.38	1.05	1.81	1.8
1.709–27	4.06	0.56	1.58	1.14	2.16	2.2
1.727–56	4.63	0.50	1.60	1.17	2.27	2.1

[a] Energies, in 10^{18} ergs g^{-1}.

VI. TOTAL ENERGY

The total energy can be computed from Table 3. We take the energy of all the material between the central core and the shock. This includes the material listed in Table 3, and also the quasi-vacuum between 30 and 1100 km which contains about 0.001 $M_\odot$. The latter contribution we simply calculate as the energy of the radiation in the sphere of 1100 km; it is 0.13×10^{50} ergs. The total energy, at time 0.78 s, is

$$E = 4.0 \times 10^{50} \text{ ergs} . \tag{35}$$

Subsequently, the shock has to absorb the material outside 5000 km which has a negative energy. On the other hand, the shocked material will still receive energy from neutrinos emitted by the core. We have not estimated these contributions quantitatively. There will not be much energy from nuclear reactions after 0.78 s, because the temperature of the shocked mantle is now too low, <0.25 MeV.

The energy of 4×10^{50} ergs equation (35), is rather low. Generally, supernova energies are reported to be 10^{51} or higher. It is possible, then, that our mechanism applies only to some supernovae. Possibly in others the initial shock moves straight out and the energy release is greater. Or perhaps the energetic supernovae are of low mass, 9 $M_\odot$, for which Hillebrandt (1982) has found large energy release. A lot more evidence is needed, both observational and theoretical.

VII. DISCUSSION

We have analyzed the various stages in the revival of the shock by neutrino heating. First, the shock recedes toward the center of the star. This increases the rate of neutrino heating for the material behind the shock, and it establishes a strong pressure gradient. That gradient is sufficient to overcome gravitation and give the material an outward acceleration and velocity. On the other hand, material close to the core moves inward with about one-half of free-fall acceleration. The combination of these motions leads to an enormous decrease in density in the region in between, around 150 km. The resulting low density (of order 10^6–10^7) makes it possible for the material to have simultaneously high specific energy (energy per unit mass) and low temperature.

The low temperature is required to keep the reradiation of neutrinos from the material to a small fraction of the neutrino energy absorbed. The high specific energy can overcome the negative gravitational potential and permit escape of the material from the star. The pressure in the low-density region is almost entirely due to radiation.

There are, however, many open questions. How does the mechanism depend on conditions? For instance, if the neutrino energy flux were only one-half of that obtained in our computation, would it still work, and how would the results be changed quantitatively? Are there other parameters to which the mechanism is sensitive?

Most important, how does the mechanism relate to another supernova mechanism? Cooperstein, Bethe, and Brown (1984) have shown that for certain initial conditions, viz., very low entropy in the Fe core, and small mass of that core, the initial shock wave goes right out without stalling. Where is the limit between these two mechanisms? If the initial shock were to stall very far out, let us say at 1000 km, could it then be revived by neutrino heating? Or is there perhaps an intermediate range of presupernova conditions where neither mechanism works?

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APPENDIX

NEUTRINO EMISSION AND SCATTERING

The main mechanism of neutrino emission is the inverse of the processes (4). The rate of energy loss per nucleon per second is

$$Q_1 = [w(e^+)\sigma_0\langle\epsilon_+^2\rangle Y_n + w(e^-)\sigma_0\langle\epsilon_-^2\rangle Y_p]c , \tag{A1}$$

$$\sigma_0 = 4.5 \times 10^{-44} , \tag{A2}$$

where $w(e^+)$ is the total energy of all the positrons in a unit volume of the gas, and $\langle\epsilon_+^2\rangle$ is the mean square energy per positron. The velocity of the electrons, + and −, has been assumed to be c. The cross section, $\sigma_0\langle\epsilon^2\rangle$, is essentially one-half of equation (6), because of the statistical factor.

In the computer output for the 25 $M_\odot$ star, in the relevant range of R and t, the electron chemical potential is in the range

$$1 < \eta_e \equiv \mu_e/T < 2 . \tag{A3}$$

The energy density of electrons, $w(e^-)$, is then substantially greater (factor 5–100) than that of the positrons, so we rewrite equation (A1):

$$Q_1 \approx [w(e^+)\langle\epsilon_+^2\rangle + w(e^-)\langle\epsilon_-^2\rangle]\sigma_0 c Y_p . \tag{A4}$$

Y_p is generally of order $\frac{1}{3}$. $w\langle\epsilon^2\rangle$ is given by the Fermi distribution (Bethe, Applegate, and Brown 1980).

$$w(e^-)\langle\epsilon_-^2\rangle = \frac{T^3}{\pi^2}\left(\frac{T}{\hbar c}\right)^3 F_5(\eta_e) , \tag{A5}$$

$$F_k(\eta) = \int \frac{t^k dt}{1 + \exp(t - \eta)} . \tag{A6}$$

For positrons, η is to be replaced by $-\eta$. It has been assumed that electrons and positrons are fully relativistic, which is a good approximation for $T > \frac{1}{2}$ MeV.

To evaluate equation (A4), we have to calculate

$$F_5(\eta_e) + F_5(-\eta_e) \,. \tag{A7}$$

This can be done by the method of Bludman and Van Riper (1978), with the result

$$F_5(\eta) + F_5(-\eta) = \tfrac{31}{126}\pi^6 + \tfrac{7}{6}\pi^4\eta^2 + \tfrac{5}{6}\pi^2\eta^4 + \tfrac{1}{6}\eta^6 \,. \tag{A8}$$

The first term here corresponds to a pure blackbody gas. Using this in Q_1, and putting $Y_p = \frac{1}{2}$, gives the negative term in equation (2). But a typical value of η_e is about $\pi/2$, and

$$\frac{F_5(\pi/2) + F_5(-\pi/2)}{2F_5(0)} = 2.408 \,. \tag{A9}$$

Multiplying now by $Y_p = \frac{1}{3}$ rather than $\frac{1}{2}$,

$$\frac{Q_1(\eta = \pi/2)}{Q_1(\eta = 0)} = 1.605 \,, \tag{A10}$$

a substantial increase of neutrino emission. This increase is almost entirely due to the energy density of electrons plus positrons being greater for $\eta_e = \pi/2$ than for the pure vacuum at the same temperature; the average square energy is almost the same, viz.,

$$\langle \epsilon_e^2 \rangle = 20.813 \text{ at } \eta = 0 \,, \qquad 22.724 \text{ at } \eta = \pi/2 \,. \tag{A11}$$

There is a small correction to equation (A10) because some of the neutrino states are occupied by the neutrinos coming from the core. At the neutrino sphere, the neutrino states of energy $\epsilon < T_\nu$ are nearly fully occupied, and since $T_m \ll T_\nu$, this includes most of the states into which ν and $\bar{\nu}$ will be emitted by capture of e^- and e^+. But, purely by geometry, the fraction of states occupied at R_m is only $(R_\nu/R_m)^2$, which is typically 1/25, a small correction to equation (A10). Similarly, in the positive term in equation (2), blocking of neutrino capture by occupation of the final electron state is very small, again because $T_m \ll T_\nu$.

Evidently, in this mechanism, only electron neutrinos and antineutrinos can be produced, not μ- or τ-neutrinos. To produce these, we would have to start from μ- or τ-mesons, whose concentration is extremely small anywhere in the star.

However, μ- (and τ-) neutrinos *can* be produced in pairs, from electron pairs

$$e^- + e^+ \rightarrow \nu_\mu + \bar{\nu}_\mu \,. \tag{A12}$$

A similar mechanism also produces pairs of electron neutrinos. The latter can be produced both by the charged and the neutral weak current, while ν_μ pairs can only be produced by the neutral current. The total rate of energy going into neutrino pairs is (Bethe, Applegate, and Brown 1980)

$$Q_2 \approx 1.4 \times 10^{25} T^9 \text{ ergs cm}^{-3} \text{ s}^{-1} \,. \tag{A13}$$

Of the factor 1.4, approximately 1.0 is for ν_e, and 0.2 for ν_μ and ν_τ each. The rate Q_2 is per unit volume, independent of the density of matter. It may be compared with the rate of electron capture. If $\mu_e = 0$, this is given by the negative term in (2),

$$Q_1 = acT^4 K(T_m) \,. \tag{A14}$$

Using a from equation (3) and K from equation (7),

$$Q_1 = 6.8 \times 10^{17} T^6 \text{ ergs g}^{-1} \text{ s}^{-1} \,. \tag{A15}$$

The same result can be obtained from equation (A4), with $\mu_e = 0$ and $Y_p = \frac{1}{2}$. Note that equation (A15) is per unit mass. The ratio of pair and single production of neutrinos is

$$\frac{Q_2}{Q_1 \rho} = 2.1 \times 10^7 \frac{T^3}{\rho} \,. \tag{A16}$$

In equations (27) and (23b) we showed that the entropy in radiation, per nucleon, is

$$S_r = 5.2 \times 10^8 T^3/\rho \,, \tag{A17}$$

so that

$$\frac{Q_2}{Q_1 \rho} \approx \frac{S_r}{25} \,. \tag{A18}$$

During most of the heating by neutrinos, $S_r \ll 25$, so that the emission of single neutrinos dominates. Only when the quasi-vacuum is already well developed does the radiation entropy become comparable with 25, but then the temperature is very low, equation (25), and the energy loss by neutrino emission, singly or in pairs, is very small compared with the energy gain given by the first term in equation (2).

Neutrino pair emission *is* important in the core, where temperatures are of the order of 20 MeV, and therefore the rate (A13) is very high. The neutrinos of all types which are produced there get into thermal equilibrium (Bethe, Applegate, and Brown 1980) and then have to diffuse out to the neutrino sphere from where they are emitted into the low-density regions of the star.

Finally, we discuss the scattering of neutrinos by electrons. Due to the universal Fermi interaction, the scattering

$$e^- + \nu_e \rightarrow \nu_e + e^- \tag{A19}$$

goes essentially like the absorption process (4), i.e., like the first term in equation (2). However, in equation (6) ϵ_ν^2 must be replaced by the square of the center-of-mass energy of electron plus neutrino, which is (neglecting the rest masses)

$$\epsilon_{\mathrm{cm}}^2 = 2\epsilon_\nu \epsilon_e - 2\boldsymbol{p}_\nu \cdot \boldsymbol{p}_e\ . \tag{A20}$$

Averaging over directions, the second term gives zero. The average energies $\langle \epsilon_\nu \rangle$ and $\langle \epsilon_e \rangle$, are respectively proportional to T_ν and T_m, so roughly

$$\frac{\langle \epsilon_{\mathrm{cm}}^2 \rangle}{\langle \epsilon_\nu^2 \rangle} \approx 2\,\frac{T_m}{T_\nu}\ . \tag{A21}$$

The scattering by electrons is of course proportional to Y_e, just as the capture of $\bar{\nu}_e$ is proportional to Y_p; we have $Y_e = Y_p$ if all nuclei are dissociated into nucleons. Scattering can occur for μ- and τ-neutrinos as well as for ν_e, but scattering of ν_μ and ν_τ can only occur by the neutral weak currents, while ν_e can be scattered by both charged and neutral currents. Thus scattering of ν_μ and ν_τ is about a factor of $\frac{1}{2}$ to $\frac{1}{4}$ less effective than ν_e. The total number of neutrinos $\nu_e + \nu_\mu + \nu_\tau$ is about 2.5 times that of ν_e. Collecting all factors, we find for the heating by scattering

$$\frac{\dot{E}_{\mathrm{sc}}}{\dot{E}} \approx \frac{T_m}{T_\nu}\ . \tag{A22}$$

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