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INTERSTELLAR SHOCK WAVES WITH MAGNETIC PRECURSORS

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ABSTRACT

The structure of steady, radiative, one-dimensional shock waves in partially ionized gas with a transverse magnetic field B_0 is investigated. Under a broad range of conditions applicable to the interstellar medium it is found that such shocks may be preceded by a "magnetic precursor" which heats and compresses the medium *ahead* of the front where the neutral gas undergoes a discontinuous change of state; indeed, if B_0 is sufficiently large, a shock can exist with *no* discontinuities in hydrodynamical variables. Within this "magnetic precursor" both ions and electrons stream through the neutral fluid with velocities which may be a significant fraction of the shock speed. The physical processes operative in such shocks are examined, including the effects of charged dust grains in dense molecular clouds. Numerical examples are shown for $v_s = 10$ km s^{-1} shocks propagating into diffuse H I or H_2. Shocks with magnetic precursors may have important consequences for the interstellar medium, some of which are briefly considered.

Subject headings: hydrodynamics — interstellar: magnetic fields — interstellar: matter — shock waves

I. INTRODUCTION

Shock waves appear to play a major role in the dynamics and evolution of the interstellar medium. These shock waves—resulting from the expansion of H II regions, cloud-cloud collisions, stellar winds, supernova explosions, etc.—compress, heat, and accelerate interstellar gas. Numerous investigations of the structure of such shocks, including detailed modeling of the hydrodynamics, heating, cooling, chemistry, etc., have been undertaken in recent years (cf. McKee and Hollenbach 1980, and references therein).

In nearly all studies of interstellar shock waves the magnetic field has been either assumed to be zero (this is usually considered—incorrectly—to be a valid approximation if $\rho v_s^2 \gg B_0^2/8\pi$) or else has been taken to be purely transverse and "frozen" into the gas (e.g., Field *et al.* 1968) and amplified as the gas is compressed. The magnetic field, however, is coupled only to charged particles, and the "frozen field" assumption is valid only if both (i) the magnetic field is frozen into the ion-electron fluid, and (ii) the ions and electrons are sufficiently well coupled (by collisions) to the neutral particles so that both charged particles and neutral particles have the same bulk velocities. While the first of these requirements is usually satisfied in interstellar shocks, the second is often not.

In an important but infrequently cited paper, Mullan (1971) demonstrated that the frozen field assumption breaks down for interstellar shock waves when the fractional ionization is low, as is often the case in H I regions and molecular clouds. Mullan showed that the structure of the shock wave could be significantly changed if the usual frozen field assumption is relaxed and replaced by a hydrodynamic description allowing the neutrals and the ion-electron fluid to have different flow velocities and temperatures. For magnetic field compression to take place, momentum must be transferred from the neutral gas to the ion-electron gas via ion-neutral collisions; if the ion density is low so that the rate/volume of ion-neutral collisions is small, the magnetic field will be compressed only gradually, over a length which can be several orders of magnitude longer than the neutral-neutral mean free path. Mullan demonstrated by model calculations that the extent of this region of ion-electron "slippage" could be quite appreciable.

Under certain conditions, the magnetic field can begin to be compressed and accelerated by the shock *before* the neutral gas-dynamic shock discontinuity (the "jump front") arrives—in this case the shock will be said to possess a "magnetic precursor." Magnetic precursors occur whenever the ion-electron fluid can propagate compressive (magnetosonic) waves at velocities larger than the shock speed v_s. When the fractional ionization is low, a shock will tend to have a magnetic precursor if the magnetic field is at all significant.

The intent of the present paper is to discuss the general properties of transverse magnetohydrodynamic (MHD) shock waves with magnetic precursors in media of low fractional ionization, including the effects of radiative cooling. In § II it is seen that MHD shock waves can be divided into three classes: (i) shocks without magnetic precursors (which includes the limit of $B_0 \rightarrow 0$), (ii) "*J*-type" shocks with magnetic precursors, and (iii) "*C*-type" shocks; in "*J*-type" shocks the neutral fluid undergoes a discontinuous compression "jump," while in "*C*-type" shocks all of the flow variables are continuous.

The remainder of this paper concentrates on shocks with magnetic precursors. The basic MHD equations which determine the variation of the hydrodynamic variables in a time-independent flow are presented in § III. In § IV order-of-magnitude formulae are given which allow a crude estimate to be made of the size of the magnetic precursor and the rate at which energy is dissipated in the precursor. In § V attention is given to the various processes which can effect momentum and energy transfer between the neutral gas and the ion-electron fluid. In diffuse clouds atomic processes (ion-neutral scattering, for example) are predominant, but in molecular clouds the fractional ionization can be so small that charged dust grains play an important role in transferring momentum from the neutral gas to the ion-electron fluid and the magnetic field; the effects of the dust grains are therefore examined in some detail.

The boundary conditions appropriate to the time-independent shock problem are obtained in § VI. In the case of J-type shocks, the exact downstream boundary condition is applicable only very far downstream; an approximate downstream boundary condition (which can be applied at any point downstream from the jump front) is therefore derived. Obtaining numerical solutions, especially for J-type shocks, is nontrivial; a successful numerical approach is described in § VII, and some numerical models are presented in § VIII. These models—for 10 km s^{-1} shocks in both diffuse H I and diffuse H_2—illustrate the effects of varying the magnetic field, and also reveal the extent to which the shock structure can be affected by radiative cooling, which can enhance the influence of the magnetic fluid.

A number of shock-related processes where magnetic precursor or ion-streaming effects could be important are identified in § IX, and it is suggested that some apparent discrepancies between theory and observation may be removed when magnetic precursor effects are taken into account. Finally, the principal results of the paper are summarized in § X.

II. REGIMES

The present study will be restricted to plane-parallel steady-state shock waves propagating into homogeneous media. It will be assumed that the magnetic field $\boldsymbol{B}$ is at all times perpendicular to the direction of propagation of the shock. The term "shock wave" will be used to refer to the *entire structure* which is advancing into the unperturbed preshock medium; the term "jump front" (or "J front") will be used to refer to the point within the shock structure where hydrodynamical variables (e.g., density and temperature) undergo a "discontinuous" change (on a length scale on the order of the mean free path). The distinction between shock and J front is necessary because, as will be seen below, a shock wave need not contain a J front; furthermore, a shock wave could, in principle, contain more than one J front.

The reader may object that standard usage reserves the term "shock" only for disturbances in which flow variables change discontinuously and that, in particular, waves in which the hydrodynamic variables are everywhere continuous (as in the C-type flows discussed below) should not be referred to as shock waves. This would, however, be too narrow an interpretation of what is meant by "shock wave." The following definition will be adopted here: *A steady state shock wave is any pressure-driven disturbance which is time-independent (in a comoving reference frame) and which effects an irreversible change in the state of the medium.* C-type shocks satisfy this definition; they differ from nonmagnetic collisional shock waves in that entropy is generated throughout a broad transition region by the dissipative effects of ion-neutral scattering, rather than being produced only in a thin transition layer (of thickness roughly one mean free path) by the dissipative effects of ordinary viscosity. The definition excludes ionization fronts (not pressure-driven) and rarefaction waves (not irreversible).

Consider a partially ionized plasma, and temporarily assume that charged particles never collide with neutral particles. Two "signal" speeds are then of interest: compressive waves can propagate through the neutral gas at the neutral gas sound speed $(5n_n kT/3\rho_n)^{1/2}$, and compressive waves can propagate through the ion-electron plasma in directions perpendicular to $\boldsymbol{B}$ with the ion magnetosonic speed (Spitzer 1962):

$$v_{\rm ims} = \left\{\frac{[B^2/4\pi + \frac{5}{3}(n_i + n_e)kT]/(\rho_i + \rho_e)}{1 + B^2/4\pi(\rho_i + \rho_e)c^2}\right\}^{1/2} . \tag{1}$$

Here n_n, n_i, n_e (ρ_n, ρ_i, ρ_e) are the number (mass) densities of neutral particles, ions, and electrons, respectively, and T is the temperature. When weak ion-neutral scattering is introduced, these waves are still allowed but become damped; very long wavelength compressive waves, however, can propagate (with little damping) at the effective magnetosonic speed

$$v_{\rm ms} = \left\{\frac{[B^2/4\pi + \frac{5}{3}(n_n + n_i + n_e)kT]/(\rho_n + \rho_i + \rho_e)}{1 + B^2/4\pi(\rho_n + \rho_i + \rho_e)c^2}\right\}^{1/2} . \tag{2}$$

Since the partially ionized preshock medium is able to communicate compressive disturbances over long distances at the speed $v_{\rm ms}$, it is clear that shock waves are possible only for shock speed $v_s > v_{\rm ms}$. It will be seen, however, that the structure of the shock depends crucially on whether $v_s > v_{\rm ims}$ or $v_s < v_{\rm ims}$.

Consider first the case where $v_s > v_{\rm ims}$; this will usually apply in the limit $B \to 0$.[1] Since the shock speed now exceeds all three of the signal speeds considered above, it is not possible for information to travel upstream in the unperturbed medium fast enough to stay ahead of the shock, and it follows that either the neutral fluid and/or the ion-electron fluid

[1] Except in the event that v_s is less than the ion-electron sound speed, which (since $v_s > v_{\rm ms}$ is required if there is to be a shock) is unlikely.

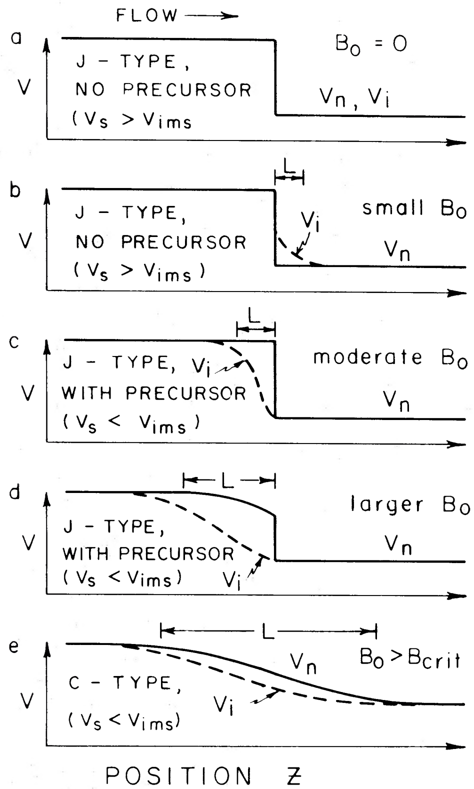

FIG. 1.—Schematic illustration of the effects of a transverse magnetic field B_0 on the structure of shock waves in a partially ionized gas: neutral velocity v_n and ion velocity v_i are shown in the frame of reference comoving with the shock. (*a*) With $B_0 = 0$ the shock is of the familiar "*J*-type" (i.e., with a "*J* front" where v_n undergoes a discontinuous "jump") with no magnetic precursor and with v_i closely following v_n. (*b*) With a small magnetic field B_0, again no magnetic precursor is present, and both v_n and v_i undergo discontinuous jumps; because of the magnetic field, however, the ions (which have a different compression ratio from the neutrals) maintain a "postjump" v_i different from v_n over a damping length *L* which can be much longer than the mean free path for collisions with neutral particles. If B_0 is increased until the ion-electron magnetosonic speed $v_{\rm ims}$ exceeds v_s, the shock acquires a "magnetic precursor" as shown in (*c*), where the magnetic field compression begins *ahead* of the *J* front and the ion velocity v_i becomes a continuous function of position, being submagnetosonic both upstream and downstream. As B_0 is increased further, as in (*d*), the extent *L* of the magnetic precursor increases, the amount of energy and momentum transferred to the neutral fluid *ahead* of the *J* front increases, and consequently the neutral gas compression ratio across the *J* front decreases. Finally, if B_0 becomes larger than some critical value $B_{\rm crit}$, the *J* front ceases to exist; the shock is now "*C*-type," and both v_n and v_i are continuous: the ion fluid is everywhere submagnetosonic, while the neutral fluid flow is everywhere supersonic.

must undergo a discontinuous change of state at a *J* front.[2] A guess at what the shock structure *might* look like is shown schematically in Figure 1*b*. The regime $v_s > v_{\rm ims}$ will not be considered further in this paper.

When $v_{\rm ims} > v_s$, a new phenomenon occurs: since ion-electron magnetosonic waves can propagate faster than the shock speed, it is possible for the medium *upstream* from the neutral gas *J* front to be disturbed by magnetosonic waves which travel ahead of the *J* front and, as they are damped, compress the preshock magnetic field throughout a region comparable to the damping length *L*. Thus the *J* front will be preceded by a magnetic *precursor* wherein the magnetic field ahead of the shock is gradually compressed as the shock approaches, forcing ions and electrons upstream from the *J* front to stream through the neutral gas, thereby heating and compressing the neutral gas prior to the arrival of the *J* front. In fact, it is not even necessary that there be a *J* front at all: if the magnetic field is large enough (or the shock speed small enough) the magnetic precursor will "quench" the *J* front, and the neutral flow variables will be everywhere continuous.

[2] Indeed, it is conjectured that *both* fluids must undergo jump discontinuities (though the two *J* fronts need not necessarily be spatially coincident), based on the following argument: suppose that one fluid undergoes a jump discontinuity, but all flow variables for the other fluid are continuous. It will be seen below that the derivative dv/dz of the velocity (in the frame of reference of the shock) of the continuous fluid should everywhere be given by either eq. (49) or eq. (50), depending on whether the fluid in question is the neutral fluid or the ion-electron fluid. By hypothesis the initial flow is supersonic, so that the denominator in the expression for dv/dz is initially negative. On the other hand, the final flow far downstream should be subsonic, so that there must be a (sonic) point where the denominator vanishes. By hypothesis, however, dv/dz is finite, so that the continuous flow hypothesis requires that the numerator in the expression (either [49] or [50]) for dv/dz vanish at the sonic point. While the equations have not been investigated in detail, there does not seem to be any guarantee that a solution exists with this numerator vanishing at the sonic point, and it is therefore conjectured that both fluids will contain jump discontinuities when $v_s > v_{\rm ims}$. The two *J* fronts need not be spatially coincident and will in general have different compression ratios. The thickness of the *J* front in the neutral fluid is of order one collisional mean free path. Since "collisionless" plasma shocks are possible (Tidman and Krall 1971), the thickness of the ion-electron *J* front could be less than the ion-ion collisional mean free path.

The physics involved is perhaps clearest if one considers a gas with no radiative cooling and, at constant shock speed v_s, examines the effect of increasing the preshock magnetic field. Figure 1*a* shows schematically the variation in gas velocity (relative to the stationary shock) as a function of position, for the case $B_0 = 0$. Here the upstream flow is unperturbed until it reaches the *J* front, at which point the velocity decreases by a factor ~ 4 (for a strong shock in a monatomic gas). Since ions and electrons experience no Lorentz forces, they basically flow together with the neutrals, so that v_i and v_e are indistinguishable. Figure 1*b*, discussed above, shows the effect of introducing a weak magnetic field into the preshock medium, where the field is assumed to be weak enough so that $v_{\text{ims}} < v_s$.

Now suppose that the magnetic field is increased to the point where $v_{\text{ims}} > v_s$: compressional waves in the ion-electron fluid can now propagate ahead of the *J* front, although these waves are damped in a length *L* due to ion-neutral collisions. The shock now possesses a magnetic precursor, and the velocity structure is qualitatively shown in Figure 1*c*. If the magnetic field is increased further, the damping length *L* becomes longer and more ion-neutral collisions occur in the magnetic precursor, leading to the exchange of more momentum and an appreciable change in the neutral flow velocity v_n *prior* to the arrival of the *J* front, as illustrated in Figure 1*d*. The neutral gas, in addition to having v_n reduced, is also heated ahead of the *J* front, and the density enhancement across the *J* front is thereby reduced.

Finally, suppose the magnetic field B_0 to be increased beyond some critical value B_{crit} where the effects of the magnetic precursor on the neutral flow are so pronounced that the compression ratio across the *J* front drops to unity—at this point the *J* front ceases to exist, all hydrodynamic variables become continuous, and the shock will be referred to as of the "continuous type," or "*C*-type." It will be seen in § VIII that interstellar magnetic fields are large enough to produce *C*-type shock fronts for a range of interesting conditions.

Following the above discussion, it is clear that MHD shocks in partially ionized media fall into three broad classes: (1) J-type (i.e., containing at least one *J* front) with no magnetic precursor ($v_s > v_{\text{ims}}$); (2) *J*-type with a magnetic precursor ($v_s < v_{\text{ims}}$, $B < B_{\text{crit}}$); (3) *C*-type ($v_s < v_{\text{ims}}$, $B > B_{\text{crit}}$).

The critical magnetic field B_{crit} dividing *J*-type shocks from *C*-type shocks (which Mullan [1971] referred to as "strong" and "weak" shocks, respectively) can be found analytically (cf. Mullan 1971, eq. [41]) for adiabatic shocks where radiative cooling does not occur by requiring that the downstream neutral flow be supersonic for $B > B_{\text{crit}}$, since the upstream neutral flow is supersonic and a continuous solution is possible only if no sonic point exists. In the more realistic case where radiative cooling is present, B_{crit} is found to depend upon the details of the radiative cooling, which depends, in turn, upon the detailed temperature profile of the shock. There appear to be two reasons for this. The first is that cooling of the neutral gas within the shock structure makes it more likely that the neutral flow will remain everywhere supersonic. The second is that cooling of the postshock gas alters the downstream values of the ion velocity and magnetic field strength, which, in turn, affect the magnitude of the ion-neutral velocity difference as well as the extent of the region throughout which magnetic slip occurs. Radiative cooling, therefore, tends to *reduce* the value of B_{crit}. In practice, the only way to determine B_{crit} is by obtaining numerical solutions for the shock structure for various values of B_0, starting with a small value and increasing B_0 until the *J* front compression ratio drops to unity.

III. BASIC EQUATIONS

Consider a steady, one dimensional flow, and let z be the distance measured from some arbitrary point in the shock structure. The flow will be assumed to have three components: neutrals, ions, and electrons, denoted by subscript n, i, and e, respectively. Each component will be assumed to have a velocity $v(z)$, density $\rho(z)$, and temperature $T(z)$, with $\partial v/\partial t = \partial \rho/\partial t = \partial T/\partial t = 0$. It will further be assumed that $v_i = v_e$; this approximation is highly accurate, since any tendency for electrons and ions to flow with different velocities will produce a large electric field opposing this velocity difference. Finally, it will be assumed that the electric field vanishes in the frame of reference of the ions and electrons; this approximation will be valid provided

$$L \gg c^2(4\pi v_s \sigma_{\text{cond}})^{-1} , \tag{3}$$

where σ_{cond} is the electrical conductivity and L is a characteristic length scale for variation of the ion velocity. The inequality (3) is satisfied for all conditions of interest here. If the electric field vanishes in the frame of reference of the electrons and ions, then it follows that the magnetic flux is conserved, and the magnetic field B is just

$$B = B_0(v_s/v_i) . \tag{4}$$

Finally, let μ_n and $\frac{1}{2}\mu_i$ be the mean mass per neutral and the mean mass per charged particle, respectively, and specialize to the case of singly charged positive ions (μ_i is then the mean ion mass plus the electron mass). It is convenient to define the "sum" and "difference" temperatures as

$$T_s \equiv T_i + T_e , \tag{5}$$

$$T_d \equiv T_i - T_e . \tag{6}$$

The (nonrelativistic) equations expressing conservation of mass, momentum, and energy then become, in the absence of both viscosity and thermal conduction,

$$\frac{d}{dz}[\rho_n v_n] = S_n , \tag{7}$$

$$\frac{d}{dz}[\rho_i v_i] = -S_n , \tag{8}$$

$$\frac{d}{dz}\left[\rho_n v_n^2 + \rho_n \frac{kT_n}{\mu_n}\right] = F_n , \tag{9}$$

$$\frac{d}{dz}\left[\rho_i v_i^2 + \rho_i \frac{kT_s}{\mu_i} + \left(\frac{v_s}{v_i}\right)^2\left(\frac{B_0^2}{8\pi}\right)\right] = -F_n , \tag{10}$$

$$\frac{d}{dz}\left[\tfrac{1}{2}\rho_n v_n^3 + \frac{\rho_n v_n}{\mu_n}(\tfrac{5}{2}kT_n + u_n)\right] = G_n + F_n v_n - \tfrac{1}{2}S_n v_n^2 , \tag{11}$$

$$\frac{d}{dz}\left[\tfrac{1}{2}\rho_i v_i^3 + \tfrac{5}{2}\rho_i v_i \frac{kT_s}{\mu_i} + v_i\left(\frac{v_s}{v_i}\right)^2\left(\frac{B_0^2}{4\pi}\right)\right] = G_i + G_e - F_n v_i + \tfrac{1}{2}S_n v_i^2 , \tag{12}$$

$$\frac{d}{dz}\left[\tfrac{5}{2}\rho_i v_i \frac{kT_d}{\mu_i}\right] = G_i - G_e . \tag{13}$$

The source terms S_n, F_n, G_n, G_i, and G_e have the following definitions: S_n is the rate, per volume, at which electron-ion mass is converted to neutral mass; F_n is the rate, per volume, at which momentum is transferred from the ion-electron fluid to the neutral fluid; G_n, G_i, and G_e are the rates, per volume, at which *thermal* energy is added to the neutral, ion, and electron fluids, respectively, as a result of interactions between the different fluids or coupling to the external radiation field. The ions and electrons are assumed to be structureless; u_n is the internal thermal energy (due to rotation or vibration) of the neutrals per particle. Equations (11) and (12) neglect the effects of thermal conduction. In the neutral fluid thermal conduction is important only when viscosity is important, i.e., only within J fronts, which the present study is not attempting to resolve. Thermal conduction by the electrons, on the other hand, will be suppressed by the transverse magnetic field which is assumed in the present work.

One also has equations prescribing the variation of μ_n and μ_i:

$$\frac{d}{dz}\left[\frac{\rho_n v_n}{\mu_n}\right] = N_n , \tag{14}$$

$$\frac{d}{dz}\left[\frac{\rho_i v_i}{\mu_i}\right] = N_i , \tag{15}$$

where N_n and N_i are the rates, per volume, at which the *number* of neutrals and ions, respectively, changes as a result of dissociation, ionization, or recombination.

One can generalize to the case where the neutral fluid has a finite viscosity by adding dD/dx to the left-hand side of (9) and $(d/dx)[v_n D]$ to the left-hand side of (11), where $D = (4\eta/3 + \xi)(dv_n/dx)$ is the viscous stress, and η and ξ are the shear and bulk viscosities (Landau and Lifshitz 1959). In this form, equations (7)–(13) admit continuous solutions which, for some initial conditions, contain a "J front" in which ρ_n, v_n, and T_n change significantly in a length of order $(4\eta/3 + \xi)/\rho_n v_n$. When the viscous stress terms are included, however, (9)–(12) become second order in v_n. This presents no numerical difficulty for a single-component fluid (i.e., eqs. [7], [9], and [11] with $S_n = F_n = 0$); however, when the full set of equations (7)–(13) is considered, the problem becomes numerically intractable as an initial value problem: the equations have spurious solutions which grow more rapidly than the physically meaningful solution. While these spurious solutions are, strictly speaking, excluded by the upstream boundary condition, they are introduced whenever a finite-accuracy numerical integration is performed; once introduced, they quickly blow up.

For this reason, only the inviscid hydrodynamic equations are integrated here; this requires that the Rankine-Hugoniot relations (Landau and Lifshitz 1959) be invoked to describe the J front, if one is present, since the inviscid equations are unable to continuously connect the upstream and downstream solutions. The J front is, in reality, a transition zone of finite thickness with a complex structure, containing nonthermal particle distribution functions and, in the case of "collisionless" plasma shocks, strong collective motions. In the present study no attempt is made to resolve the internal structure of the J front, which will be represented as a simple discontinuity. Since the present study will be restricted to shocks possessing magnetic precursors (i.e., $v_s < v_{\rm ims}$), only the neutral fluid will be assumed to contain a discontinuity in the flow variables; ρ_n, v_n, and T_n are then given by the Rankine-Hugoniot relations, where the subscripts 1 and 2 denote preshock and postshock values:

$$\rho_{n1} v_{n1} = \rho_{n2} v_{n2} , \tag{16}$$

$$\rho_{n1} v_{n1}^2 + \rho_{n1} \frac{kT_{n1}}{\mu_n} = \rho_{n2} v_{n2}^2 + \rho_{n2} \frac{kT_{n2}}{\mu_n} , \tag{17}$$

$$\tfrac{1}{2}\rho_{n1} v_{n1}^3 + \frac{\rho_{n1} v_{n1}}{\mu_n} (\tfrac{5}{2}kT_{n1} + u_{n1}) = \tfrac{1}{2}\rho_{n2} v_{n2}^3 + \frac{\rho_{n2} v_{n2}}{\mu_n} [\tfrac{5}{2}kT_{n2} + u_{n2}] . \tag{18}$$

These equations assume that no ionization, dissociation, or recombination occurs within the transition zone (hence μ_n and $\rho_n v_n$ are unchanged across the discontinuity), and that coupling between the neutrals and the charged particles may be ignored within the thin J front.

IV. ORDER-OF-MAGNITUDE ESTIMATES

It is useful to obtain order-of-magnitude estimates for the overall properties of the magnetic precursor. Consider a strong J-type shock with $\rho_{n0} v_s^2 \gg B_0^2/8\pi$, $\rho_{n0} \gg \rho_{i0}$, and $B_0^2/4\pi\rho_{i0} > v_s^2$. Then the neutrals will hardly have changed velocity when they pass through the J front, whereas the ion velocity will have changed significantly. Consider then a point (upstream from the J front) where $v_n \approx v_s$, $v_i \approx v_s/2$. Let the ion mass be μ_i, the neutral mass μ_n, and ignore dust. Let $L \equiv |d \ln v_i/dx|^{-1}$ be the characteristic length scale for the magnetic precursor. If the inertia and thermal pressure of the ions are neglected, and momentum exchange between ions and neutrals occurs via ion-neutral scattering with a rate coefficient $\langle \sigma v \rangle$ (cf. § V*a* below), then L is found from equation (10) to be

$$L \approx \frac{(\mu_n + \mu_i) B_0^2}{\pi \rho_{i0} \rho_{n0} \langle \sigma v \rangle v_s} . \tag{19}$$

If Γ is the energy dissipation rate per volume, then ΓL is the approximate energy dissipation rate per area of the shock, where, using equation (25) for Γ,

$$\Gamma L \approx \frac{B_0^2 v_s}{2\pi} . \tag{20}$$

One may estimate the peak temperature $T_{n,\max}$ to which the *preshock* neutral gas is heated by assuming that the temperature change is due to a heating Γ minus a cooling $\sim \Lambda(T_{n,\max})$ acting over a distance L (or time L/v_s). This provides an implicit equation for $T_{n,\max}$:

$$T_{n,\max} \approx T_{n0} + \frac{2\mu_n L}{3\rho_n v_s k} \left[\frac{B_0^2 v_s}{2\pi L} - \Lambda(T_{n,\max}) \right] . \tag{21}$$

This will be approximately the temperature of the neutral gas just prior to the arrival of the shock; passage through the J front will, of course, raise the neutral temperature to a considerably higher value.

V. SOURCE TERMS

a) *Atomic Processes*

The usual kinetic equations are used to obtain the source terms N_n, N_i, and S_n which describe changes in composition in the flow; for example, radiative recombination of protons with electrons makes a contribution $\alpha n_e n(H^+)$ to N_n, $-\alpha n_e n(H^+)$ to N_i, and $\alpha n_e n(H^+) m_H$ to S_n (α being the rate coefficient for radiative recombination, and m_H the mass of an H atom).

Momentum transfer between ions and neutrals results from both interconversion and scattering. Radiative recombination, for example, since it produces atoms with the center-of-mass velocity of the ions and electrons, makes a contribution $\alpha n_e n(H^+) m_H v_i$ to F_n. Elastic scattering of ionic species I by neutral species N contributes a term

$$(F_n)_{\substack{\text{ion-neutral}\\ \text{scattering}}} = \left(\frac{m_N m_I}{m_N + m_I} \right) n_N n_I \langle \sigma v \rangle_{NI} (v_i - v_n) \tag{22}$$

where m_N and m_I are the masses of species N and I, n_N and n_I their number densities, and $\langle \sigma v \rangle_{NI}$ the *momentum transfer* rate coefficient as defined by Osterbrock (1961). At the center-of-mass energies of interest ($\lesssim 1$ eV) Osterbrock estimates the ion-neutral momentum transfer rate coefficient to be

$$\langle \sigma v \rangle_{NI} = 2.41\pi \left(\frac{m_N + m_I}{m_N m_I} \right)^{1/2} e \alpha_N^{1/2} , \tag{23}$$

where α_N is the polarizability of the neutral partner. This rate coefficient is used here. It should be noted that equation (23), with $\sigma \propto v^{-1}$, will underestimate σ at high velocities. If the high-velocity cross section is just the geometric value $\sim 10^{-15}$ cm^2, then equation (23) ($\langle \sigma v \rangle_{NI} = 2.40 \times 10^{-9}$ cm^3 s^{-1} for $C^+ - H^0$) will be valid for $v \lesssim 24$ km s^{-1} where $v = (8kT_n/\pi m_N + 8kT_i/\pi m_i + (v_i - v_n)^2)^{1/2}$ is a good approximation to the mean relative velocity; for larger velocities, one should instead take

$$\langle \sigma v \rangle_{NI} \approx 10^{-15}\ \mathrm{cm}^2\ [8kT_n/\pi m_N + 8kT_i/\pi m_I + (v_i - v_n)^2]^{1/2}\ . \tag{24}$$

Ion-neutral elastic scattering, of course, can change the thermal energy of both neutrals and ions, with contributions which are, if $\langle \sigma v \rangle$ is velocity independent,

$$(G_n)_{\substack{\text{ion-neutral}\\ \text{elastic}}} = \frac{2m_N m_I}{(m_N + m_I)^2} n_N n_I \langle \sigma v \rangle_{NI} [\tfrac{3}{2}k(T_i - T_n) + \tfrac{1}{2}m_I(v_i - v_n)^2]\ ; \tag{25}$$

$$(G_i)_{\substack{\text{ion-neutral}\\ \text{elastic}}} = \frac{2m_N m_I}{(m_N + m_I)^2} n_N n_I \langle \sigma v \rangle_{NI} [\tfrac{3}{2}k(T_n - T_i) + \tfrac{1}{2}m_N(v_i - v_n)^2]\ . \tag{26}$$

Energy transfer between ions and electrons occurs at a rate (Spitzer 1962)

$$\begin{aligned}(G_i)_{\substack{\text{ion-electron}\\ \text{scattering}}} &= -(G_e)_{\substack{\text{ion-electron}\\ \text{scattering}}}\ , \\ &= \frac{4m_e}{m_I}\frac{\pi e^4}{2(kT_e)^2}\left(\frac{8kT_e}{\pi m_e}\right)^{1/2} \ln \Lambda\ n_I n_e k(T_e - T_i)\ ,\end{aligned} \tag{27}$$

$$\Lambda \equiv \frac{3}{2e^3}\left(\frac{k^3 T_e{}^3}{\pi n_e}\right)^{1/2}\ . \tag{28}$$

In addition to these elastic processes, one must consider various inelastic processes which either inject energy into the system (e.g., cosmic ray heating) or remove it in the form of radiation. These heating and cooling terms generally take the same form as in conventional hydrodynamic calculations (e.g., Field *et al.* 1968; Hollenbach and McKee 1979) except that in considering the excitation of ions by neutrals (or vice versa) one should recognize that, in the absence of significant streaming, the effective temperature is $T_{\rm eff} = (m_N T_i + m_I T_n)/(m_N + m_I)$; if $v_n \neq v_i$, then the velocity distribution is non-Maxwellian and the rate coefficient should be recomputed; however, it is sometimes adequate to use the thermal rate coefficient but with an effective temperature

$$T_{\rm eff} = \frac{m_N T_i + m_I T_n}{m_N + m_I} + \frac{1}{2}\frac{m_N m_I}{(m_N + m_I)k}(v_n - v_i)^2\ . \tag{29}$$

Streaming is generally unimportant for collisional processes involving electrons because their thermal velocities are normally much larger than the streaming speeds.

If H^+ ions constitute a significant fraction of the ionization, then H^+–H charge exchange may be an important process for momentum and energy transfer. Taking the momentum transfer coefficient to be the larger of the elastic scattering value (23) and the value obtained from the charge exchange cross section (Peek 1966), one obtains

$$\begin{aligned}\langle \sigma v \rangle_{\mathrm{H-H^+}} &\approx 3.26 \times 10^{-9}\ \mathrm{cm^3\ s^{-1}} \qquad \text{for} \quad v < 2\ \mathrm{km\ s^{-1}} \\ &\approx 2.0 \times 10^{-9}(v/\mathrm{km\ s^{-1}})^{0.73}\ \mathrm{cm^3\ s^{-1}} \qquad \text{for} \quad 2 < v < 1000\ \mathrm{km\ s^{-1}}\ .\end{aligned} \tag{30}$$

The momentum transfer cross sections found by Hunter and Kuriyan (1977) for 0.2–62 km s^{-1} are in good agreement with equation (30). The ratio of momentum transfer in H–H^+ collisions to the momentum transfer in collisions of H with, say, C^+ ions, is (from [22], [23], and [30])

$$\begin{aligned}\frac{(F_n)_{\mathrm{H-H^+}}}{(F_n)_{\mathrm{H-C^+}}} &= 0.74\frac{n(\mathrm{H^+})}{n(\mathrm{C^+})} \qquad \text{for} \quad v < 2\ \mathrm{km\ s^{-1}} \\ &= 0.45\frac{n(\mathrm{H^+})}{n(\mathrm{C^+})}\left(\frac{v}{\mathrm{km\ s^{-1}}}\right)^{0.73} \qquad \text{for} \quad 2 < v < 24\ \mathrm{km\ s^{-1}}\ .\end{aligned} \tag{31}$$

Thus for $v < 2$ km s^{-1}, H^+ is important only if it contributes appreciably to the ionization, whereas at $v \gtrsim 10$ km s^{-1}, H^+ can be important even if not the dominant ion species.

b) Dust Grains

i) *Grain Charge*

In regions of very low fractional ionization (e.g., dark molecular clouds) it is possible that charged dust grains may become important in the coupling of momentum from the magnetic field to the neutrals; this has been discussed by

Elmegreen (1979) for ambipolar diffusion in hydrostatic, magnetically supported dense molecular clouds. Here this question is reexamined to assess the role of grains in the hydromagnetic precursor to a shock in a dense cloud. It will be found that dust plays an important role in coupling neutrals to the magnetic field only when the fractional ionization $n_e/n_H \lesssim 10^{-7}$; readers interested only in diffuse clouds ($n_e/n_H \gtrsim 10^{-5}$) may therefore proceed directly to § VI.

Consider a dust grain of radius a located in a gas of equal numbers of ions (charge $+e$, mass m_I) and electrons ($-e$, m_e). Assume the ions and electrons to have Maxwellian velocity distributions with temperatures T_i and T_e, respectively. Provided that neither charge quantization nor photoelectric emission is important, the grain (if at rest relative to the ion and electron gases) will acquire a charge $Q = \phi a k T_e/e$, where, if $\psi \equiv T_i/T_e$, ϕ is the solution to (cf. Spitzer 1978 for the case $\psi = 1$)

$$(\psi - \phi)e^{-\phi} = (\psi m_I/m_e)^{1/2} . \tag{32}$$

In a quiescent H_2 region, $\psi = 1$ to high accuracy. In the magnetic precursor to an interstellar shock, on the other hand, ψ can become significantly larger than 1 since the ions streaming through the neutral gas are heated much more than the electrons (due to the large mass mismatch between an electron and a neutral atom or molecule); model calculations (Draine, Roberge, and Dalgarno 1980) indicate that $\psi \gtrsim 10$ is not uncommon. In a region of low fractional ionization $n_e/n_H \lesssim 10^{-7}$, most of the positive ions are likely to have masses $\sim 30\, m_H$ (e.g., S^+ or HCO^+; de Jong, Dalgarno, and Bohland 1980). For $m_I = 30 m_H$, the solution to equation (32) varies from $\phi = -3.875$ at $\psi = 1$, has an extremum $\phi = -4.064$ at $\psi = 4.064$, and slowly rises to $\phi = -3.787$ at $\psi = 20$; $\phi = -4$ to within 5% over the entire range $1 < \psi < 20$.

Neglect of charge quantization (and the associated fluctuations in the grain charge Q around its mean) is valid provided $|Q/e| \gg 1$; this reduces to the condition $aT_e \gg (e^2/k)|\phi|^{-1} \approx 4.2 \times 10^{-4}$ cm K, assuming $\phi \approx -4$. This condition is only marginally satisfied for typical preshock conditions of interest ($T_e \approx 50$ K, $a \approx 2 \times 10^{-5}$ cm), but is fulfilled in those regions where the shock precursor has heated the electrons to $T_e \gtrsim 200$ K. In the discussion below, it will be assumed that the instantaneous grain charge is just $Q = \phi a k T_e/e$, where ϕ is the solution to equation (32), since the grain dynamics are greatly simplified if fluctuations in the grain charge may be neglected (see Elmegreen 1979 for discussion of the effects of charge fluctuations).

ii) *Grain Dynamics*

Consider the frame of reference in which the neutral gas (at the point of interest) is at rest. Let ions and electrons be streaming through the neutrals in the z direction. In the frame of reference of the neutral gas there will exist, in addition to the magnetic field $\boldsymbol{B} = B\boldsymbol{j}$ in the y direction, an electric field $\boldsymbol{E} = E\boldsymbol{i}$ in the x direction. There exists a second frame of reference, moving with a velocity $\boldsymbol{v}_B = (cE/B)\hat{\boldsymbol{k}}$ in which the electric field (at the point of interest) vanishes. In the approximation that the conductivity due to the electrons is infinite, this second reference frame coincides with the center-of-mass frame for the electrons; if the conductivity due to the ions is also large, they too are at rest in this reference frame. These approximations will be adopted here; thus $\boldsymbol{v}_B = (\boldsymbol{v}_i - \boldsymbol{v}_n)$ is the velocity with which electrons, ions, and magnetic field are streaming through the neutrals.

Now consider a dust grain, with mass M and charge Q, which is moving with velocity $\boldsymbol{u} = u_x\hat{\boldsymbol{i}} + u_z\hat{\boldsymbol{k}}$ relative to the neutral gas. Let the drag force due to the neutral gas be $\boldsymbol{F}_{\rm drag}$, and define the viscous damping time τ by $\tau \equiv Mu/F_{\rm drag}$. Letting $\omega \equiv QB/Mc$ be the gyrofrequency, the acceleration of the dust grain is

$$\frac{d}{dt}\boldsymbol{u} = \omega\boldsymbol{u} \times \hat{\boldsymbol{j}} + \frac{QE}{M}\hat{\boldsymbol{i}} - \frac{1}{\tau}\boldsymbol{u} . \tag{33}$$

An additional assumption will now be made: assume that the quantities v_B, B, Q, and τ are all nearly constant on a time scale τ. In this case, the grain motion will, to an excellent approximation, have $|(d/dt)\boldsymbol{u}| \ll u/\tau$. In the limit $d\boldsymbol{u}/dt = 0$, routine algebra yields the simple result for the steady state velocity $\boldsymbol{u}$:

$$\boldsymbol{u} = v_B\frac{(\omega\tau)}{1 + (\omega\tau)^2}\hat{\boldsymbol{i}} + v_B\frac{(\omega\tau)^2}{1 + (\omega\tau)^2}\hat{\boldsymbol{k}} ;$$

$$u = v_B\omega\tau/[1 + (\omega\tau)^2]^{1/2} . \tag{34}$$

From (34) it is clear that, for $\omega\tau \gg 1$, the grain is essentially carried along by the magnetic field; for $\omega\tau \ll 1$ it is viscously coupled to the neutral gas.

The damping time τ is independent of u only at low velocities. To excellent accuracy $\omega\tau$ can be written (cf. eq. [5B] of Draine and Salpeter 1979*a*):

$$\omega\tau \approx \frac{3}{8}\frac{B|\phi|T_e}{1.4 n_H m_H aec}\left(\frac{k\mu}{2\pi T_n}\right)^{1/2}\frac{1}{[1 + (9\pi/128)(\mu u^2/kT_n)]^{1/2}} \tag{35}$$

$$\approx 0.29\left(\frac{B}{\rm mG}\right)\left(\frac{10^6\ {\rm cm^{-3}}}{n_H}\right)\left(\frac{10^{-5}\ {\rm cm}}{a}\right)\left(\frac{T_e}{50\ {\rm K}}\right)\left(\frac{50\ {\rm K}}{T_n}\right)^{1/2}\left[1 + 1.25\frac{(u/{\rm km\ s^{-1}})^2}{(T_n/50\ {\rm K})}\right]^{-1/2} , \tag{36}$$

where $\phi = -4$ and $\mu = 7m_{\rm H}/3$ have been assumed in equation (36). It is evident from equation (36) that, for plausible parameters for dense molecular clouds (e.g., $B = 3$ milligauss [mG], $n_{\rm H} = 10^6\ {\rm cm}^{-3}$, $T_e = 50$ K), $\omega\tau$ can be of order unity for grains in the expected size range $a \approx 0.1$–$0.3\ \mu$m, especially if T_e (and therefore the grain charge) is increased as it would be in a hydromagnetic shock precursor. Using equations (34) and (35), $\omega\tau$ can be obtained analytically:

$$\omega\tau = \left\{\frac{\alpha - 1 + [(\alpha-1)^2 + 4\alpha\beta]^{1/2}}{2\beta}\right\}^{1/2}, \tag{37}$$

$$\alpha \equiv \left(\frac{3}{8}\frac{B\phi T_e}{1.4 n_{\rm H} m_{\rm H} a e c}\right)^2 \frac{k\mu}{2\pi T_n}, \tag{38}$$

$$\beta \equiv 1 + \frac{9\pi}{128}\frac{\mu v_B{}^2}{kT_n}. \tag{39}$$

iii) *Source Terms F_n and G_n*

The rate of momentum transfer (along the $\hat{z}$ direction) (see note added in proof) to the neutral gas due to dust is $(F_n)_{\rm dust} = n_{\rm gr} M u_z/\tau$, which becomes

$$(F_n)_{\rm dust} = \tfrac{8}{3} 1.4 n_H m_H n_{\rm gr} a^2 \left[\frac{1 + \beta(\omega\tau)^2}{1 + (\omega\tau)^2}\right]^{1/2} \left(1 + \frac{9\pi}{128}\frac{\mu u^2}{kT_n}\right)^{1/2} \frac{(\omega\tau)^2}{1 + (\omega\tau)^2}(v_i - v_n). \tag{40}$$

The importance of dust relative to ions in coupling the neutral gas to the magnetic field may now be evaluated by computing the ratio of the momentum transfer rates for the two processes, assuming $(\rho_{\rm gr}/\rho_n) = 0.01$, $a\rho = 5 \times 10^{-5}$ g cm^{-2}, and $\langle\sigma v\rangle_{NI} = 2 \times 10^{-9}$ cm^3 s^{-1}:

$$\frac{(F_n)_{\rm dust}}{(F_n)_{\rm ions}} \approx 1.57 \frac{(\omega\tau)^2}{1 + (\omega\tau)^2}\left(\frac{10^{-8}}{n_I/n_{\rm H}}\right)\left(\frac{T_n}{50\ {\rm K}}\right)^{1/2}\left[1 + 1.25\frac{(u/{\rm km\ s^{-1}})^2}{T_n/50\ {\rm K}}\right]^{1/2}. \tag{41}$$

Using equation (36) for $\omega\tau$, it can be seen from the above expression that grains are likely to be important only for fractional ionizations $n_I/n_{\rm H} \lesssim 10^{-8}$, *except* for high speed shocks where $u \gtrsim 30$ km s^{-1} and in which the electron temperature is sufficiently high that $\omega\tau \gtrsim 1$; for these cases, dust can be more important than the ions even for $n_I/n_{\rm H} \approx 10^{-7}$.

In addition to transferring momentum to the neutrals, grain-neutral collisions tend to (*a*) heat the neutrals, and (*b*) heat the dust grains (which may then either radiate this energy in the infrared or return it to the gas by further collisions). The rate at which the neutral gas is heated by collisions with the dust is just (neglecting the heat capacity of the dust)

$$(G_n)_{\rm dust} = (v_i - v_n)(F_n)_{\rm dust} - \Lambda_{\rm dust}, \tag{42}$$

where $\Lambda_{\rm dust}$ is the net rate at which energy is radiated by the dust. A good approximation to $\Lambda_{\rm dust}$ may be obtained by noting that the mean kinetic energy (relative to the grain) of an impacting gas molecule is $\sim 2kT(1 + s^2/2)$, the rate of impact is $\sim n\pi a^2(8kT/\pi\mu)^{1/2}(1 + \pi s^2/4)^{1/2}$, and the mean energy of the reflected gas atoms is $\sim\tilde{\alpha}2kT_{\rm gr} + (1 - \tilde{\alpha})2kT(1 + s^2/2)$, where $\tilde{\alpha}$ is an accommodation factor and $s \equiv (mv_0{}^2/2kT_n)^{1/2}$. Thus the rate at which the dust is heated by the gas is

$$\Lambda_{\rm dust} = n_{\rm gr} n\pi a^2 \left(\frac{8kT_n}{\pi\mu}\right)^{1/2}\left(1 + \frac{\pi s^2}{4}\right)^{1/2}[2\tilde{\alpha}kT_n(1 + s^2/2) - 2\tilde{\alpha}kT_{\rm gr}]. \tag{43}$$

Equation (43) assumes that impinging molecules do not stick to the dust grain; this is a good approximation in molecular clouds, but equation (43) should be modified to include the effects of H_2 formation on grain surfaces if appreciable atomic H is present. The dust temperature must, of course, also satisfy the condition

$$4\pi a^2 n_{\rm gr} Q_{\rm abs}(T_{\rm gr})\sigma T_{\rm gr}{}^4 = \Lambda_{\rm dust} + \Gamma_{\rm abs}, \tag{44}$$

where $\Gamma_{\rm abs}$ is the rate (per volume) at which dust grains absorb electromagnetic energy from the ambient radiation field, and $Q_{\rm abs}(T_{\rm gr})$ is the Planck-averaged absorption efficiency for the dust (Spitzer 1978).

VI. BOUNDARY CONDITIONS

a) Upstream Boundary Conditions

In order to constitute a well-posed problem, the differential equations (7)–(15) need to be supplemented by boundary conditions. Initially, $v_n = v_s$ and $v_i = v_s$. The undisturbed medium far upstream from the shock provides boundary conditions for the seven state variables: $\rho_{n0}, \rho_{i0}, \mu_{n0}, \mu_{i0}, T_{n0}, T_{i0}, T_{d0}$; these initial values must satisfy the seven equations

$$0 = N_n = N_i = S_n = S_i = G_n = G_i = G_e. \tag{45}$$

In the above form, these boundary conditions cannot be used as the starting point for a numerical integration, since they apply only infinitely far upstream from the shock. At any finite distance upstream from the shock, the physical properties of the fluids will be perturbed away from the undisturbed values. To obtain a starting point for a numerical integration, it is necessary to linearize the differential equations to obtain, in terms of an arbitrary (small) perturbation in one of the quantities, the corresponding perturbation in the remaining dependent variables, with all perturbations assumed to be growing exponentially as exp (ψz).

Linearization of the inviscid hydrodynamic equations around the undisturbed upstream conditions is straightforward and is described in Appendix A. To simplify the analysis, it may be assumed that $S_n = S_i = N_n = N_i = 0$ so that $\rho_n v_n$, $\rho_i v_i$, μ_n, and μ_i may be treated as constants in the linearization of the equations (this approximation is valid provided the time scale for composition changes in the cool upstream solution is much longer than the dynamical time scale—composition changes being important only when the gas is shock-heated enough that thermal dissociation or ionization becomes rapid). One then obtains a quartic characteristic equation for the eigenvalue ψ. As expected (cf. § II), it is found that a single real root $\psi > 0$ exists if and only if $v_{\rm ms} < v_s < v_{\rm ims}$.[3] Since the disturbance far upstream is assumed to have originated in the shock, it must decay in the direction of decreasing z: thus the disturbance far upstream must be an eigenvector for the positive eigenvalue ψ.

b) *Downstream Boundary Conditions*

When the shock speed is such that a C-type shock occurs (i.e., $B_0 \geq B_{\rm crit}$) and a continuous hydrodynamic solution exists, the nine first order equations (7)–(15) may be integrated, starting upstream, using nine upstream boundary conditions (v_n, v_i, ρ_n, ρ_i, μ_n, μ_i, T_n, T_i, T_d) which are obtained by linearizing the equations around the unperturbed solution and assuming an arbitrary perturbation for one of the variables at the initial point $z = 0$ where the integration is begun. The equations are well behaved and, provided a small enough step size is used, one may integrate through the entire transition region and attain the steady-state regime far downstream. For C-type shocks, then, no downstream boundary conditions are needed.

For magnetic fields $B_0 < B_{\rm crit}$, continuous solutions to the inviscid hydrodynamic equations do not exist. One now has an eigenvalue problem since the location z_J of the J front becomes an additional degree of freedom in the problem, but only a single value of z_J will allow the postshock solution to smoothly evolve into the steady-state condition far downstream. With an extra variable z_J to be determined, one more boundary condition must be provided.

If one's computational resources allowed one to integrate far enough downstream, one could specify that $v_n = v_i$ at some point z_∞ far enough downstream so that one is certain that the steady-state downstream solution will have been attained (to the desired numerical accuracy). In using a relaxation technique, however, one is limited to a finite number of grid points (100, say) which, together with the requirement that the spacing between grid points not exceed the shortest characteristic length in the problem, will probably not bring one near to the downstream steady state, so this boundary condition, while formally correct, is not of practical use.

An *approximate* downstream boundary condition may be found as follows: suppose that T_n, T_i, and T_e evolve to a known final temperature T_∞ and a known composition $[(\rho_i/\rho_n)_\infty, \mu_{n\infty}, \mu_{i\infty}]$ in the steady-state region far downstream. By integrating the equations of momentum conservation (9) and (10) one can readily determine the final flow velocity $v_\infty \equiv v_s/\xi$ which both v_n and v_i approach asymptotically far downstream as the solution to a cubic equation:

$$\left[\frac{B_0{}^2}{8\pi(\rho_{n0} + \rho_{i0})v_s{}^2}\right](\xi^3 - \xi) + 1 - \xi + \frac{k}{v_s{}^2}\left[\left(\frac{\rho_n}{\rho_n + \rho_i}\right)\frac{T_n}{\mu_n} + \left(\frac{\rho_i}{\rho_n + \rho_i}\right)\frac{(T_i + T_e)}{\mu_i}\right]_\infty \xi^2 - \frac{k}{v_s{}^2}\left[\left(\frac{\rho_n}{\rho_n + \rho_i}\right)\frac{T_n}{\mu_n} + \left(\frac{\rho_i}{\rho_n + \rho_i}\right)\frac{(T_i + T_e)}{\mu_i}\right]_0 \xi = 0 \,. \tag{46}$$

Having obtained v_∞, one now makes the *Ansatz* that both v_n and v_i are exponentially approaching v_∞, i.e.,

$$\frac{dv_n}{dz} = \lambda(v_\infty - v_n)\,, \qquad \frac{dv_i}{dz} = \lambda(v_\infty - v_i)\,, \tag{47}$$

i.e.,

$$\frac{1}{(v_\infty - v_n)}\frac{dv_n}{dz} - \frac{1}{(v_\infty - v_i)}\frac{dv_i}{dz} = 0\,. \tag{48}$$

[3] In this case there is one root $\psi < 0$ and a doubly degenerate $\psi = 0$ root. For $v_s = v_{\rm ims}$ there is one $\psi < 0$ root and a triply degenerate $\psi = 0$ root.

From equations (7)–(13) one can obtain (exact) expressions for dv_n/dz and dv_i/dz:

$$\frac{dv_n}{dz} = \frac{\frac{2}{3}[G_n - N_n u_n - (\rho_n v_n/\mu_n)(du_n/dz)] - F_n v_n + S_n v_n{}^2}{\rho_n[\frac{5}{3}(kT_n/\mu_n) - v_n{}^2]}, \tag{49}$$

$$\frac{dv_i}{dz} = \frac{\frac{2}{3}(G_i + G_e) + F_n v_i - S_n v_i{}^2}{\rho_i[(B_0{}^2/4\pi)(v_s{}^2/\rho_i v_i{}^2) + \frac{5}{3}(kT_s/\mu_i) - v_i{}^2]}. \tag{50}$$

Insertion of these expressions into (48) gives the approximate downstream boundary condition:

$$\frac{\frac{2}{3}[G_n - N_n u_n - (\rho_n v_n/\mu_n)(du_n/dz)] - F_n v_n + S_n v_n{}^2}{\frac{2}{3}(G_i + G_e) + F_n v_i - S_n v_i{}^2} = \frac{\rho_n[\frac{5}{3}(kT_n/\mu_n) - v_n{}^2]}{\rho_i[(B_0{}^2/4\pi)(v_s{}^2/\rho_i v_i{}^2) + \frac{5}{3}(kT_s/\mu_i) - v_i{}^2]} \frac{(v_\infty - v_n)}{(v_\infty - v_i)}. \tag{51}$$

The advantage of the boundary condition (51) is that it is an explicit function of the hydrodynamic variables at the last mesh point only.

VII. NUMERICAL METHODS

Since the inviscid equations admit only one positive eigenvalue ψ, one could safely integrate the differential equations numerically (e.g., with a Runge-Kutta scheme) *provided* the differential equations are everywhere valid (i.e., no J front is present). For C-type shocks ($B_0 > B_{\rm crit}$), then, the integration may be continued through the entire shock and into the quiescent postshock region without numerical difficulties. In the case of a J-type shock ($B_0 < B_{\rm crit}$), however, this technique cannot be used because one does not know, *a priori*, where (or at what flow speed v_n) the discontinuity in (ρ_n, v_n, T_n) will occur. The procedure used by Mullan (1971) in this case was to also find the quiescent postshock solution, linearize it, and integrate in the direction of decreasing z until a point was found where the preshock solution and the postshock solution could be joined by a shock satisfying the Rankine-Hugoniot conditions, with v_i and T_i continuous. This approach is feasible for the idealized problem considered by Mullan, in which radiative cooling is unimportant, since the distance from the quiescent downstream solution to the shock is just the length scale for dynamical coupling of the ions to the neutrals. If radiative cooling is important, however, quiescent conditions may be attained only a very great distance downstream from the shock, and it is unfeasible to try to integrate over this length scale.

A one-dimensional relaxation code, generously provided by B. Flannery, was modified to handle the present problem. The code employs the Henyey relaxation technique (Kippenhahn, Weigert, and Hofmeister 1967; Eggleton 1971) to obtain, on a finite one-dimensional mesh, solutions to a set of coupled first order ordinary differential equations with boundary conditions imposed at either one or both ends of the mesh. For C-type shocks, where the solution is continuous, the relaxation code was used with a 100-point mesh with nonuniform mesh spacing (a smaller mesh spacing is desirable at the downstream end since the characteristic relaxation lengths are shorter in the compressed gas). The boundary conditions on the hydrodynamic variables were all imposed at the upstream end of the mesh; they were obtained by linearizing the equations and assuming a small perturbation in one of the variables (typically, $v_n = 0.99999v_s$).

For J-type shocks with magnetic precursors, the 100-point mesh was divided into prejump and postjump submeshes consisting of 40 and 60 mesh points, respectively. "Upstream" boundary conditions—one for each of the hydrodynamic variables—were again obtained by linearizing the equations. The last prejump and first postjump mesh point are assumed to be spatially coincident and located at a position z_J. The Rankine-Hugoniot jump conditions are used to relate the prejump and postjump hydrodynamic variables for the neutral fluid, while the hydrodynamic variables for the ion and electron fluids are assumed to be continuous. Since z_J—which the program treats as an eigenvalue—is now an additional unknown quantity, one more boundary condition is required; this is obtained by imposing equation (51) at the last postjump mesh point.

The relaxation code converges to a solution which satisfies the *difference* equations and boundary conditions to within the machine accuracy (i.e., mass, momentum, and energy fluxes are conserved to $\sim 10^{-13}$ using 64 bit arithmetic). Since the approximate boundary condition (51) might introduce spurious behavior in the vicinity of the position $z_{\rm down}$ where the downstream boundary condition is imposed, a J-type shock ($v_s = 10$ km s^{-1} into diffuse H I with $B_0 = 2.5\ \mu$G) was calculated using 40 preshock and 60 postshock mesh points, once with $z_{\rm down} - z_J = 3 \times 10^{15}$ cm, and again with $z_{\rm down} - z_J = 6 \times 10^{15}$ cm. Comparison revealed that z_J and the hydrodynamical variables for $z \le z_J + 3 \times 10^{15}$ cm differed by at most 3%, and usually less than 1%, indicating that the downstream boundary condition (51), though not exact, is a satisfactory approximation. The comparison also demonstrated that the results were insensitive to a doubling of the postjump mesh spacing.

One might have hoped that scaling laws would exist that would allow a family of solutions to be generated from a single numerical calculation. Unfortunately, as discussed in Appendix B, the radiative shocks which are of interest admit no scaling laws, so that every model must be computed individually.

VIII. SIX EXEMPLARY MODELS

In order to illustrate the effects of ion-neutral slip on the structure of interstellar shocks, models have been computed for $v_s = 10$ km s^{-1} shocks propagating into diffuse H I ($n_{\rm H} = 20$ cm^{-3}, $n_{\rm He} = 2$ cm^{-3}, $n_e = 6.6 \times 10^{-3}$ cm^{-3}) and diffuse

H_2($n_H = 20$ cm^{-3}, $n_{He} = 2$ cm^{-3}, $n(H^0) = 0.4$ cm^{-3}, $n(H_2) = 9.8$ cm^{-3}, $n_e = 6.6 \times 10^{-3}$ cm^{-3}). In both cases the electron density is assumed to be due primarily to photoionization of C, Mg, Si, etc., and the positive ions are assumed to have a mass $m_I = 20m_H$; H^+ ions are assumed to be $\lesssim 20\%$ of the total ionization, which will be true for the assumed steady-state preshock conditions ($T \approx 14$ K) provided the cosmic ray and X-ray primary ionization rate $\zeta \lesssim 10^{-17}$ s^{-1}.

Cooling of the H I is assumed to be dominated by C^+ and O^0 fine structure cooling, which is a good approximation since the peak temperatures attained in these models do not exceed ~ 5000 K. The radiative loss rate is calculated using the approximate formula (Elitzur and Watson 1980)

$$(G_n)_{\text{radiative}} = -[83 \exp(-92/T_n) + 35 \exp(-228/T_n) + 10 \exp(-326/T_n)] \times 10^{-28} n_H^2 \text{ ergs cm}^3 \text{ s}^{-1} . \quad (52)$$

Photoelectric heating by dust grains (Draine 1978) is ignored; the preshock temperature is thereby underestimated somewhat, but photoelectric heating is not of major importance in the shock-heated gas. The only external heat source included is cosmic ray heating for an assumed primary ionization rate $\zeta = 10^{-17}$ s^{-1}.

Ionization and recombination and H_2 formation/dissociation are assumed to be unimportant on the time scales of interest here ($\tau_{\text{hydro}} \lesssim 10^{11}$ s), so that the ionization and molecular fraction are taken to be "frozen"; this allows the molecular weight μ_n and μ_i to be treated as constants in the hydrodynamic equations, and permits S_n to be set to zero in equations (7)–(12). The assumption of constant fractional ionization is valid for the shocks considered here since (1) collisional ionization is negligible (T_n, $T_e < 4500$ K); and (2) the time scale for radiative recombination of metal ions is $\sim 10^{13}$ s $\gg \tau_{\text{hydro}}$. The assumption of constant molecular fraction in the H_2 shocks is valid since (1) collisional dissociation is negligible ($T_n < 2850$ K); (2) photoionization is negligible (by hypothesis, the H_2 must be protected by self-shielding and/or dust in order to exist in the preshock medium); and (3) the time scale for converting H to H_2 on dust grains is $\sim 5 \times 10^{16}$ cm^{-3} s/$n_H \sim 5 \times 10^{14}$s $\gg \tau_{\text{hydro}}$. Momentum transfer between ions and neutrals is assumed to be totally due to the ion-neutral elastic scattering of equation (22), where $\langle \sigma v \rangle_{NI} = 2 \times 10^{-9}$ cm^3 s^{-1} has been adopted.

Three models are shown in Figure 2, having preshock magnetic fields (transverse to the shock velocity) $B_0 = 3, 5$, and 8

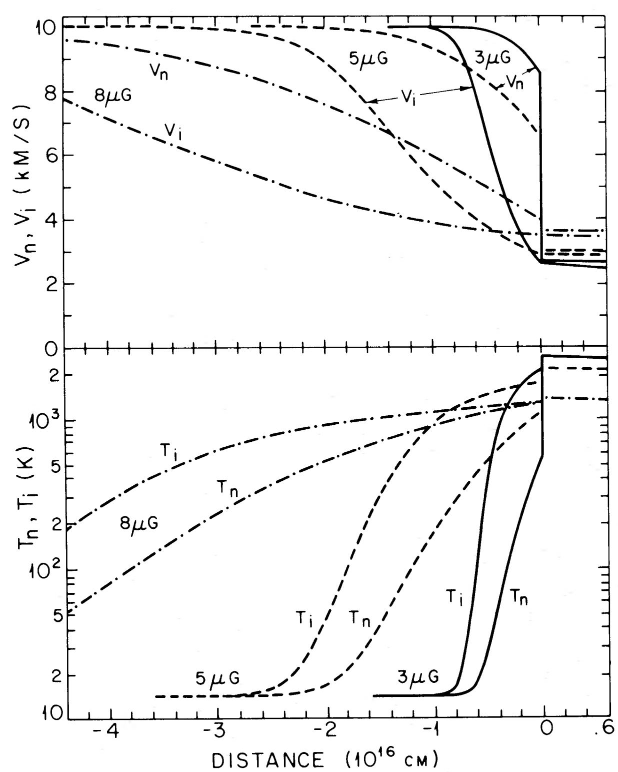

FIG. 2.—$v_s = 10$ km s^{-1} shocks in H I (preshock $n_H = 20$ cm^{-3}, $n_e = n(C^+) = 6.6 \times 10^{-3}$ cm^{-3}) for preshock transverse magnetic fields $B_0 = 3$ μG (*solid*), 5 μG (*broken*), and 8 μG (*dash-dot*). The upper plot shows v_i = ion velocity and v_n = neutral velocity in the frame of reference in which the shock is stationary. All three shocks are *J*-type with magnetic precursors; distances are measured from the *J* front. The spatial extent of the magnetic precursor can be seen to increase as $\sim B_0^2$, and the compression ratio across the *J* front is seen to decrease as B_0 increases; from the weakness of the 8 μG *J* front one may estimate $B_{\text{crit}} \approx 9$ μG.

The lower plot shows T_n = neutral temperature and T_i = ion temperature (virtually the same as the electron temperature T_e in these models). Considerable heating of the neutral gas takes place *ahead* of the *J* front; T_n just upstream of the *J* front increases as B_0 is increased from 3 μG to 8 μG, and, for $B_0 = 8$ μG, T_n only jumps slightly as the (very weak) *J* front passes. In these three models radiative cooling is of minor importance (observe that T_n decreases very slowly downstream from the *J* front).

TABLE 1

PROPERTIES OF 10 km/s MHD SHOCKS IN DIFFUSE H I AND H_2

		H I SHOCKS			H_2 SHOCKS		
VARIABLE	UNIT	3 μG[a]	5 μG[a]	8 μG[a]	2.5 μG[a]	4 μG[a]	5 μG[a]
1. Preshock $n_H = n(H) + 2n(H_2)$	cm^{-3}	20	20	20	20	20	20
2. Preshock $n(H_2)$	cm^{-3}	2×10^{-5}	2×10^{-5}	2×10^{-5}	9.8	9.8	9.8
3. Preshock n_e	cm^{-3}	6.6×10^{-3}	6.6×10^{-3}	6.6×10^{-3}	6.6×10^{-3}	6.6×10^{-3}	6.6×10^{-3}
4. Preshock T_n	K	14.1	14.1	14.1	14.1	14.1	14.1
5. T_n ahead of J front	K	583	1096	1288	2572	...	...
6. $T_{n,\max}$	K	2595	2147	1373	2850	1908	1569
7. $T_{e,\max}$	K	2707	2198	1369	4449	3362	2789
8. One fluid shock temperature T_s[b]	K	2598	2189	1480	4317	3808	3415
9. E_{rad}/k at $T_{n,\max}$	K	11.2	43.8	172	1021	1042	750
10. E_{rad}/k at $\lvert d \ln v_r/dz\rvert_{\max}$	K	11.2	43.8	172	1021	1304	2897
11. E_{rad}/k at ∞	K	5832	4404	2717	6224	5080	4398
12. v_n (rel. to shock) ahead of J front	$km\ s^{-1}$	8.54	6.59	3.99	4.15	...	...
13. $\lvert v_n - v_i\rvert_{\max}$	$km\ s^{-1}$	5.98	4.58	3.06	6.47	5.32	4.67
14. Compression across J front	...	3.17	2.19	1.10	1.06	...	...
15. One fluid compression ratio[b]	...	3.73	3.38	2.79	4.31	3.96	3.70

[a] Value of B_0.

[b] Calculated assuming the rotational levels of H_2 to instantaneously attain the steady-state populations at each density and temperature, using rotational excitation rates from Elitzur and Watson 1978*b*.

μG; quantitative information for the model shocks is given in Table 1. The value of 3 μG is considered to be a conservative estimate for the magnetic field in a diffuse cloud (Heiles 1976). It is evident that even this conservative value of B_0 implies a qualitative change in the shock structure compared to nonmagnetic shock models; the preshock gas is heated to $T_n = 583$ K and slowed from 10 to 8.54 km s^{-1} prior to the arrival of the J front (cf. Table 1). Throughout a region of thickness $\sim 5 \times 10^{15}$ cm the ions are streaming through the neutrals with slip velocities exceeding 3 km s^{-1}, or 30% of the shock speed (the maximum slip velocity is 5.98 km s^{-1}). Once the J front passes, however, the velocity difference between ions and neutrals drops to $\lesssim 0.2$ km s^{-1}—while slippage is occurring in the postjump gas, large slip velocities appear to be restricted to the region preceding the J front.

If the magnetic field is increased to $B_0 = 5$ μG, the magnetic effects become even more pronounced: the neutral gas is now heated to $T_n = 1096$ K and slowed to 6.59 km s^{-1} prior to the arrival of the J front and the ion-neutral slip velocity exceeds 3 km s^{-1} in a region of thickness $\sim 1.4 \times 10^{16}$ cm.

If, finally, the field is increased to $B_0 = 8$ μG, the transition from J-type to C-type is nearly attained: the neutral gas is heated to $T_n = 1288$ K ahead of the J front, and the compression ratio across the J front is only 1.10. From Figure 2 the characteristic size of the magnetic precursor is seen to be $\sim 4 \times 10^{16}$ cm; note that the order-of-magnitude formula (19) predicts characteristic lengths $L = 5 \times 10^{15}$ cm, 1.4×10^{16} cm, and 3.5×10^{16} cm for the 3, 5, and 8 μG models, in excellent agreement with the numerical results.

From the fact that the compression ratio is only 1.10 for $B_0 = 8$ μG, B_{crit} is estimated to be $B_{crit} \approx 9$ μG for the H I model; for $B_0 > B_{crit}$ a C-type shock would result. While models with $B_0 > 8$ μG were not considered here, it should be noted that such magnetic fields are by no means implausible: Heiles (1976) has estimated $B_0 \approx 20$ μG for diffuse H I clouds.

Radiative cooling is relatively unimportant for the H I shocks of Figure 2: the hydrodynamic length scales L are considerably shorter than the length scale required for C^+ and O^0 fine structure cooling to remove energy from the system. The efficacy of radiative cooling can be measured by

$$E_{rad}(z) \equiv \frac{1}{n_{H0} v_s} \int_{-\infty}^{z} (\Lambda_{rad} - \Gamma_{ext}) dz \,, \tag{53}$$

the net energy radiated per H nucleus up to a position z in the shock (Λ_{rad} is the radiative cooling rate per volume, Γ_{ext} the heating rate per volume due to cosmic rays, etc.). In Table 1, $E_{rad}(z)$ is tabulated for $z =$ position where T_n peaks, $z =$ position where $\lvert d \ln v_n/dz\rvert$ peaks (the location of the J front if one is present), and $z = \infty$ (infinitely far downstream after the gas has attained the postshock steady state). Since E_{rad}/k is only 172 K at the J front for even the 8 μG H I shock (with the most extensive magnetic precursor), it is no surprise that $T_{n,\max}$, the peak temperature attained by the neutral gas (line 6 of Table 1) is almost as large (for the H I shocks) as the "shock temperature" T_s (line 8 of Table 1) which is predicted by a one-fluid model for these MHD shocks (Field *et al.* 1968). However, the difference of 107 K between T_s and $T_{n,\max}$ would become more pronounced if the magnetic field strength were increased beyond $B_{crit} \approx 9$ μG.

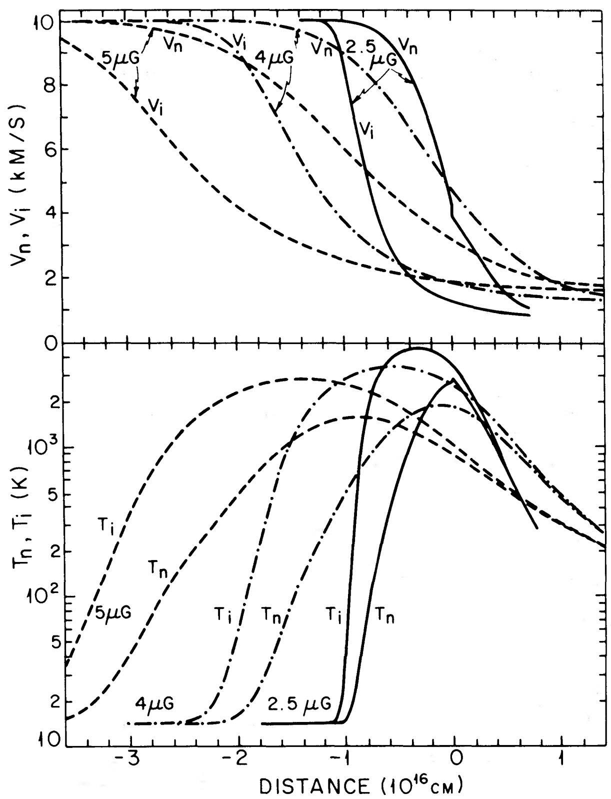

FIG. 3.—$v_s = 10\ \mathrm{km\ s^{-1}}$ shocks in diffuse H_2 [preshock $n_H = 20\ \mathrm{cm^{-3}}$, $n(H_2) = 9.8\ \mathrm{cm^{-3}}$, $n_e = n(C^+) = 6.6 \times 10^{-3}\ \mathrm{cm^{-3}}$] for preshock transverse magnetic fields $B_0 = 2.5\ \mu$G (*solid*), 4 μG (*dash-dot*), and 5 μG (*broken*). The $B_0 = 2.5\ \mu$G shock is *J*-type with a magnetic precursor, and the *J* front is located at distance = 0; the 4 μG and 5 μG shocks are *C*-type and have been positioned so that $|d \ln v_n/dx|$ is maximum at the distance origin. The upper curves show v_i = ion velocity and v_n = neutral velocity in the frame of reference comoving with the shock. The lower curves are T_n = neutral temperature and T_i = ion temperature; the rapid decrease of T_n and T_i after the peak temperature has been reached is evidence that radiative cooling (by H_2 rotational transitions) is effective on the length scales of interest. It is seen that cooling *decreases* the critical magnetic field strength B_{crit} separating *J*-type and *C*-type shocks, from $B_{\mathrm{crit}} \approx 9\ \mu$G for the H I shocks of Fig. 2 to $B_{\mathrm{crit}} \approx 3\ \mu$G for the case shown here.

In contrast to the H I models of Figure 2, where radiative cooling is a minor effect, Figure 3 displays three models of MHD shocks in diffuse H_2. Here cooling by rotationally excited states of H_2,[4] computed using the collisional excitation rates of Elitzur and Watson (1978*b*), is very rapid, is able to cool the gas appreciably within one damping length *L*, and plays an important role in enhancing the importance of the magnetic effects. It is evident from Figure 3 that even $B_0 = 2.5\ \mu$G is close to B_{crit}: the compression ratio is only 1.06. For these H_2 models $B_{\mathrm{crit}} \approx 3\ \mu$G, in contrast to the H I models (with the same preshock mass density, shock speed, and fractional ionization) for which $B_{\mathrm{critt}} \approx 9\ \mu$G. This enhancement of magnetic effects will always occur when the cooling length is comparable to or less than the damping length *L* of equation (19).

The two H_2 shock models in Figure 3 with $B_0 = 4$ and 5 μG are examples of *C*-type shocks. It is clear that as the magnetic field continues to increase, the structure gets wider and the peak temperatures drop. For moderate magnetic fields the peak temperature $T_{n,\mathrm{max}}$ can be considerably less than the single-fluid shock temperature T_s for the same magnetic field, as is seen by comparing lines 6 and 8 of Table 1: for the 4 μG H_2 shock, $T_{n,\mathrm{max}}$ is only 50% of T_s.

IX. DISCUSSION

It is not the intention of this paper to explore the astrophysical implications of the above theory for magnetohydrodynamic shocks in media of low fractional ionization—the H I and H_2 models presented here are for a single density, a single fractional ionization, and a single shock speed, and are intended only to provide illustrations of the general character of the hydrodynamic solutions, and a demonstration of the importance of magnetic field-driven ion-neutral slip under conditions which are thought to be representative of some of the components of the interstellar medium. It is, however, clear that magnetic precursors to interstellar shock waves may entail a number of fairly dramatic astrophysical consequences, some of which are briefly considered below.

[4] The H_2 is assumed for simplicity, to be all para-H_2 (even *J*). Elitzur and Watson (1978*b*) have shown that both para-H_2 and ortho-H_2 have nearly the same radiative cooling rate per molecule, so that the hydrodynamical calculations are insensitive to the ortho/para ratio.

One of the more striking features of shock waves with magnetic precursors is that an appreciable fraction of the available "free energy" in an MHD shock wave is dissipated throughout an extended region whose temperature rises only gradually from the unperturbed preshock value. This means, for example, that high-velocity shocks in H_2 can have an appreciable fraction of their total energy radiated away by H_2, even if the shock speed exceeds the ~ 24 km s^{-1} threshold above which nonmagnetic shocks dissociate H_2 before it can radiate (Kwan 1977; London, McCray, and Chu 1977). (Indeed, if the magnetic field is large enough that the shock is C-type rather than J-type, the H_2 cooling may be so effective that the temperature never rises to the level at which dissociation begins.)

With this in mind, models of high-velocity shocks in dense molecular clouds have been constructed (Draine, Roberge, and Dalgarno 1980). For reasonable preshock conditions, it has been found that the observed intense H_2 $v = 1 \rightarrow 0$ line emission from OMC-1 may be produced by a 50 km s^{-1} MHD shock, with virtually all of the H_2 line emission originating in the magnetic precursor ahead of a J front. In general, the emission spectra of shock waves will be modified if a magnetic precursor is present; whenever the ambient magnetic field is large enough (and the preshock ionization low enough) for a magnetic precursor to exist, attempts to infer shock speeds and gas densities from observed optical emission by means of nonmagnetic shock models (e.g., Dopita 1977) should be done with caution. It will be of interest to perform model calculations to assess the importance of magnetic effects in altering the optical spectra.

The destruction of dust grains in interstellar shock waves (e.g., Shull 1977, 1978; Cowie 1978; Barlow 1978; Draine and Salpeter 1979*b*) depends considerably on the relative velocity between dust grains and gas, especially when the "betatron" acceleration process (Spitzer 1976) is operative. When no magnetic precursor is present, large relative velocities arise at the moment the J front suddenly changes the gas velocity, since the grain velocity does not partake of this sudden change. In shock waves with magnetic precursors, however, it is evident that the grain dynamics—and hence the destruction rates—may be significantly modified. It remains to be seen by numerical calculation what quantitative effect this will have on the rate at which grains are destroyed in interstellar shocks, but (by reducing grain destruction in shocks) it may perhaps help to resolve the discrepancy between theoretical estimates of grain destruction rates and observed depletions of interstellar metals (Draine and Salpeter 1979*b*; Dwek and Scalo 1980).

Finally, chemistry in interstellar shocks (e.g., Elitzur and Watson 1978*a*, 1980; Elitzur and de Jong 1978; Iglesias and Silk 1978; Hartquist, Oppenheimer, and Dalgarno 1980) can be expected to be modified. It is clear that magnetic precursors will be a new and interesting place where interstellar chemistry can occur. Indeed, reactions between ions and neutrals will proceed at nonthermal rates since the ions are streaming through the neutrals with a velocity which can greatly exceed the sound speed. In particular, this streaming may help to overcome the energy barrier in endothermic ion-neutral reactions (such as $C^+ + H_2 \rightarrow CH^+ + H$). Work to investigate the chemistry of MHD shocks is now in progress.

X. SUMMARY

The principal conclusions of this paper are:

1. MHD shocks in media of low fractional ionization (i.e., H I regions or molecular clouds) will have "magnetic precursors" if $B_0^2/4\pi\rho_{i0} > v_s^2$, where B_0 is the preshock transverse magnetic field strength, ρ_{i0} the preshock ion density, and v_s the shock speed. This condition is satisfied for a broad range of shock parameters of astrophysical interest.

2. MHD shocks with magnetic precursors fall into two classes. (i) If the magnetic field is weak, one has a "J-type" shock, in which the neutral gas undergoes a "discontinuous" change of state (or "jump") at a J front; the jump, which is mediated by viscosity in the neutral gas, is, of course, not a true discontinuity, but the transition occurs in a distance of order one molecular mean free path. (ii) If the magnetic field is strong enough, the shock will be a "C-type" shock, in which all of the hydrodynamic variables are continuous; the dissipative effects of ion-neutral slip generate enough entropy so that the fluid system can evolve to a compressed, higher entropy state without recourse to a gas-dynamic viscous jump.

3. MHD shocks with magnetic precursors possess an extended region in which the ions, driven by the magnetic field, stream through the neutrals with an appreciable fraction of the shock speed. For J-type shocks, this region always extends *ahead* of the J front (hence the term "magnetic precursor").

4. A substantial fraction of the energy dissipation (i.e., conversion of bulk kinetic energy to thermal energy) in the shock can occur in this extended magnetic precursor. For C-type shocks, sudden heating of the gas at a J front does not occur, so that, if cooling is effective, the gas temperature may never approach the value $\sim 3\mu v_s^2/16k$ which it would attain for a strong nonmagnetic monatomic shock. For J-type shocks, if cooling is effective, a substantial fraction of the available energy may be radiated by the gas (at a low temperature) prior to sudden heating in the J front. Because of these effects, shock spectra can differ drastically from those for nonmagnetic shocks (or magnetic shocks in which ion-neutral slip is neglected).

5. Radiative cooling of the gas can enhance the importance of the magnetic precursor. In particular, the critical shock speed which (for constant preshock density, ionization, and magnetic field) divides C-type shocks from J-type shocks is increased when cooling is taken into account (or, for constant preshock density, ionization, and shock speed, the "critical" magnetic field is reduced). For reasonable parameters, $v_s \lesssim 10$ km s^{-1} shocks in diffuse molecular clouds should often be C-type.

6. The structure of MHD shocks with magnetic precursors is qualitatively different from that of conventional (nonmagnetic or significantly ionized) shock models; these differences can be expected to have important consequences for a number of processes occurring in interstellar shock waves, for example: (i) emission from collisionally excited ions, atoms, or molecules (both intensity and line shape may be affected); (ii) collisional excitation and dissociation of H_2 in shocks; (iii) grain dynamics (especially "betatron" acceleration) and grain destruction; (iv) chemistry (especially that involving ion-neutral reactions). Research on all of these problems is currently in progress.

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APPENDIX A

LINEARIZATION AROUND THE UPSTREAM SOLUTION

The unperturbed upstream solution is determined by equations (45). Assume an exponentially growing perturbation:

$$v_n = v_s(1 + ue^{\psi z}) ,$$
$$v_i = v_s(1 + \phi e^{\psi z}) ,$$
$$T_n = T_{n0}(1 + te^{\psi z}) ,$$
$$T_s = T_{s0}(1 + \theta e^{\psi z}) ,$$
$$T_d = T_{d0} + T_{s0}\chi e^{\psi z} .$$

The time scale $(\psi v_s)^{-1}$ on which the hydrodynamic perturbation grows will normally be short compared to the time scales for ionization, dissociation, and recombination. For purposes of obtaining the linearized equations one may therefore set $N_n = N_i = S_n = S_i = 0$, in which case $\rho_n = \rho_{n0} v_s/v_n$, $\rho_i = \rho_{i0} v_s/v_i$, $\mu_n = \mu_{n0}$, and $\mu_i = \mu_{i0}$ in the five equations (9)–(13). Expanding these five equations to first order in the dimensionless perturbations u, ϕ, t, θ, and χ, one obtains the matrix equation $\mathbf{M}A = 0$, where A is the column vector whose components are $(u, \phi, t, \theta, \chi)$, and

$$\mathbf{M} = \begin{vmatrix} (1-\beta_n)\psi - f_{nu} & f_{n\phi} & \beta_n\psi & 0 & 0 \\ f_{nu} & [\alpha(1-\beta_s) - \gamma]\psi - f_{n\phi} & 0 & \alpha\beta_s\psi & 0 \\ \psi - f_{nu} - g_{nu} & -(f_{n\phi} + g_{n\phi}) & 5\beta_n\psi/2 - g_{nt} & -g_{n\theta} & -g_{n\chi} \\ f_{nu} - g_{su} & (\alpha - \gamma)\psi + f_{n\phi} - g_{s\phi} & -g_{st} & 5\alpha\beta_s\psi/2 - g_{s\theta} & -g_{s\chi} \\ -g_{du} & -g_{d\phi} & -g_{dt} & -g_{d\theta} & 5\alpha\beta_s\psi/2 - g_{d\chi} \end{vmatrix} ,$$

where $f_n \equiv F_n/(\rho_{n0} v_s^2)$, $g_n \equiv G_n/(\rho_{n0} v_s^3)$, $g_s \equiv (G_i + G_e)/(\rho_{n0} v_s^3)$, and $g_d \equiv (G_i - G_e)/(\rho_{n0} v_s^3)$. The dimensionless coefficients are

$$\alpha \equiv \rho_{i0}/\rho_{n0} ,$$
$$\beta_n \equiv kT_{n0}/\mu_{n0} v_s^2 ,$$
$$\beta_s \equiv kT_{s0}/\mu_{i0} v_s^2 ,$$
$$\gamma \equiv B_0^2/4\pi\rho_{n0} v_s^2 ,$$

and the partial derivatives are defined by

$$f_{nu} \equiv \partial f_n/\partial \ln v_n ,$$
$$f_{n\phi} \equiv \partial f_n/\partial \ln v_i = -f_{nu} ,$$
$$g_{nu} \equiv \partial g_n/\partial \ln v_n , \quad g_{su} \equiv \partial g_s/\partial \ln v_n , \quad g_{du} \equiv \partial g_d/\partial \ln v_n ,$$
$$g_{n\phi} \equiv \partial g_n/\partial \ln v_i , \quad g_{s\phi} \equiv \partial g_s/\partial \ln v_i , \quad g_{d\phi} \equiv \partial g_d/\partial \ln v_i ,$$
$$g_{nt} \equiv \partial g_n/\partial \ln T_n , \quad g_{st} \equiv \partial g_s/\partial \ln T_n , \quad g_{dt} \equiv \partial g_d/\partial \ln T_n ,$$
$$g_{n\theta} \equiv \partial g_n/\partial \ln T_s , \quad g_{s\theta} \equiv \partial g_s/\partial \ln T_s , \quad g_{d\theta} \equiv \partial g_d/\partial \ln T_s ,$$
$$g_{n\chi} \equiv T_{s0}\partial g_n/\partial T_d , \quad g_{s\chi} \equiv T_{s0}\partial g_s/\partial T_d , \quad g_{d\chi} \equiv T_{s0}\partial g_d/\partial T_d .$$

The condition det $\mathbf{M} = 0$ provides a fourth order equation for ψ with, if a magnetic precursor can exist, a single positive root.[5] For each of the nondegenerate eigenvalues, the corresponding eigenvector can be found (to within an arbitrary multiplicative constant) by solving the four simultaneous equations

$$\sum_{j=1}^{4} M_{ij}{}^* A_j{}^* = -B_i\,, \qquad i = 1, \ldots, 4\,,$$

where $M_{ij}{}^*$ is the 4×4 matrix obtained from $\mathbf{M}$ by discarding the first row and first column, $A_j{}^* = (\phi/u, t/u, \theta/u, \chi/u)$, and $B_i = M_{i+1,1}$.

APPENDIX B

SCALING LAWS

Scaling laws for the solutions to the hydrodynamic equations (7)–(15) may be obtained provided the scaling rules for the source terms are specified. Assume that the source functions can be written in the following forms:

$$N_n = c_1 \rho_n + \rho_n{}^2 f_1(T_n) + \rho_n \rho_i f_2(T_n, T_i) + \rho_i{}^2 f_3(T_i)\,,$$
$$N_i = c_2 \rho_n + \rho_n{}^2 f_4(T_n) + \rho_n \rho_i f_5(T_n, T_i) + \rho_i{}^2 f_6(T_i)\,,$$
$$S_n = c_3 \rho_n + \rho_n{}^2 f_7(T_n) + \rho_n \rho_i f_8(T_n, T_i) + \rho_i{}^2 f_9(T_i)\,,$$
$$F_n = c_4 \rho_n \rho_i (v_n - v_i)\,,$$
$$G_n = c_5 \rho_n \rho_i [(T_i - T_n) + c_6 (v_i - v_n)^2] + \rho_n f_{10}(T_n) + \rho_n{}^2 f_{11}(T_n)\,,$$
$$G_i = c_7 \rho_n \rho_i [(T_n - T_i) + c_8 (v_i - v_n)^2] + \rho_i{}^2 f_{12}(T_i, T_e)\,,$$
$$G_e = -\rho_i{}^2 f_{12}(T_i, T_e)\,,$$

where $\{c_1, \ldots, c_8\}$ are constants, and $\{f_1, \ldots, f_{12}\}$ are functions. Assume that $\{\rho_n{}^0(z), v_n{}^0(z), T_n{}^0(z), \rho_i{}^0(z), v_i{}^0(z), T_i{}^0(z), T_e{}^0(z), \mu_n{}^0(z), \mu_i{}^0(z)\}$ is a valid solution to equations (7)–(15) having initial magnetic field $B_0{}^0$.

Consider now the new family of functions $\{\rho_n(z), \ldots, \mu_i(z)\}$ defined by

$$\rho_j(z) = \alpha \rho_j{}^0(\beta z/\alpha)\,, \qquad j = n, i\,;$$
$$v_j(z) = \beta v_j{}^0(\beta z/\alpha)\,, \qquad j = n, i\,;$$
$$T_j(z) = \beta^2 T_j{}^0(\beta z/\alpha)\,, \qquad j = n, i, e\,;$$
$$\mu_j(z) = \mu_j{}^0(\beta z/\alpha)\,, \qquad j = n, i\,;$$
$$B_0 = \alpha^{1/2} B_0{}^0\,.$$

These functions will satisfy the differential equations (7)–(15) provided one of the following conditions is satisfied:

1. If (i) $f_{10} = 0$ (unsaturated cooling, no heating by cosmic rays or ambient radiation), (ii) $c_1 = c_2 = c_3 = 0$ (no dissociation or ionization by cosmic rays or ambient radiation), and (iii) the heating-cooling functions f_{11} and f_{12} have the property $f_{11}(\xi x) = f_{11}(x), f_{12}(\xi x, \xi y) = f_{12}(x, y)$ (including $f_{11} = 0$ or $f_{12} = 0$, i.e., adiabatic shocks, as special cases), then α and β are both arbitrary.
2. If (i) $f_{10}(\xi x) = \xi^\nu f_{10}(x)$, (ii) $c_1 = c_2 = c_3 = 0$, and (iii) $f_{11}(\xi x) = \xi f_{11}(x)$, $f_{12}(\xi x, \xi y) = \xi f_{12}(x, y)$, then α is arbitrary but $\beta = \alpha^{1/(2\nu-1)}$.
3. If (i) $f_{10} = 0$, (ii) $c_1 = c_2 = c_3 = 0$, but (iii) f_{11} and f_{12} are arbitrary, then α is arbitrary but $\beta = 1$.
4. If at least one of $\{c_1, c_2, c_3\}$ is nonzero, or if $f_{10} \neq 0$ and is not a power law, then $\alpha = \beta = 1$; i.e., no scaling is possible.

Unfortunately, condition (4) usually prevails, so that no exact scaling is possible.

[5] As mentioned in § III, viscous stresses can be added to the conservation equations (9) and (11) for the neutral fluid momentum and energy, converting these to second-order differential equations which are now valid through the J front itself as well as in the slowly varying preshock and postshock flows. When the above linearization procedure is carried through, the condition det $\mathbf{M} = 0$ now results in a *fifth* order equation for ψ, having two positive roots. In the limit of infinitesimal viscosity η, the new root $\psi \propto \eta^{-1}$. Thus even a small amount of viscosity (corresponding, in a dilute gas, to a *short* molecular mean free path) introduces an eigenvector which has an extremely rapid growth rate (indeed, the smaller the viscosity, the greater the growth rate). In the only slightly perturbed preshock region far upstream from the shock, the perturbation should be *orthogonal* to this most rapidly growing eigenvector (since, if one starts at the shock and integrates backwards upstream, this eigenvector corresponds to the most rapidly *decaying* part of the disturbance and, given the large value of ψ, would have decayed to virtually zero). In any numerical calculation, however, it is difficult to prevent a little bit of this eigenvector from being introduced into the calculation due to round-off error; once present, it will rapidly "blow up." It is on this fact that any attempt to directly integrate (as an initial value problem) the equations of viscous hydrodynamics for a multicomponent fluid will come to grief.

REFERENCES

Barlow, M. J. 1978, *M.N.R.A.S.*, **183**, 367.
Cowie, L. L. 1978, *Ap. J.*, **225**, 887.
de Jong, T., Dalgarno, A., and Boland, W. 1980, *Astr. Ap.*, in press.
Dopita, M. A. 1977, *Ap. J. Suppl.*, **33**, 437.
Draine, B. T. 1978, *Ap. J. Suppl.*, **36**, 595.
Draine, B. T., Roberge, W. G., and Dalgarno, A. 1980, in preparation.
Draine, B. T., and Salpeter, E. E. 1979*a*, *Ap. J.*, **231**, 77.
———. 1979*b*, *Ap. J.*, **231**, 438.
Dwek, E., and Scalo, J. M. 1980, *Ap. J.*, **239**, 193.
Eggleton, P. P. 1971, *M.N.R.A.S.*, **151**, 351.
Elitzur, M., and de Jong, T. 1978, *Astr. Ap.*, **67**, 323.
Elitzur, M., and Watson, W. D. 1978*a*, *Ap. J.* (*Letters*), **222**, L141.
———. 1978*b*, *Astr. Ap.* **70**, 443.
———. 1980, *Ap. J.*, **236**, 172.
Elmegreen, B. G. 1979, *Ap. J.*, **232**, 729.
Field, G. B., Rather, J. D. G., Aannestad, P. A., and Orszag, S. A. 1968, *Ap. J.*, **151**, 953.
Hartquist, T. W., Oppenheimer, M., and Dalgarno, A. 1980, *Ap. J.*, **236**, 182.
Heiles, C. 1976, *Ann. Rev. Astr. Ap.*, **14**, 1.
Hollenbach, D., and McKee, C. F. 1979, *Ap. J. Suppl.*, **41**, 555.
Hunter, G., and Kuriyan, M. 1977, *Proc. Roy. Soc. London*, **A353**, 575.
Iglesias, E. R., and Silk, J. I. 1978, *Ap. J.*, **226**, 851.
Kippenhahn, R., Weigert, A., and Hofmeister, E. 1967, *Methods Comput. Phys.*, **7**, 129.
Kwan, J. 1977, *Ap. J.*, **216**, 713.
Landau, L. D., and Lifshitz, E. M. 1959, *Fluid Mechanics* (Oxford: Pergamon).
London, R., McCray, R., and Chu, S. I. 1977, *Ap. J.*, **217**, 442.
McKee, C. F., and Hollenbach, D. J. 1980, *Ann. Rev. Astr. Ap.*, **18**, in press.
Mullan, D. J. 1969, Ph.D. thesis, University of Maryland (University Microfilms No. 70-16026).
———. 1971, *M.N.R.A.S.*, **153**, 145.
Osterbrock, D. E. 1961, *Ap. J.*, **134**, 270.
Peek, J. M. 1966, *Phys. Rev.*, **143**, 33.
Shull, J. M. 1977, *Ap. J.*, **215**, 805.
———. 1978, *Ap. J.*, **226**, 858.
Spitzer, L. 1962, *Physics of Fully Ionized Gases* (2d ed.; New York: Interscience).
———. 1976, *Comments Ap.*, **6**, 177.
———. 1978, *Physical Processes in the Interstellar Medium* (New York: Wiley).
Tidman, D. A., and Krall, N. A. 1971, *Shock Waves in Collisionless Plasmas* (New York: Wiley).

Note added in proof (see § V*b* [iii]).—The reader may observe that, since the dust velocity (34) has a component in the $+x$ direction [for $Q(v_i - v_n) > 0$], the neutral and ion fluids will be accelerated in the $+x$ and $-x$ directions, respectively, and the assumption of fluid flow only in the z direction, upon which the conservation laws (7)–(13) are based, is violated. The conservation laws can easily be reformulated, at the cost of introducing two new hydrodynamic variables (x components of $\boldsymbol{v}_n$ and $\boldsymbol{v}_i$). In the interests of simplicity, however, the present analysis will assume both $\boldsymbol{v}_n$ and $\boldsymbol{v}_i$ to remain in the z direction. In order to retain energy conservation, the heating term in eq. (42) will be written $(v_i - v_n)(F_n)_{\text{dust}}$, rather than $\boldsymbol{u} \cdot (\boldsymbol{F}_n)_{\text{dust}}$.

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