Transmission Line Simulation Using the Finite Difference Time Domain Method

Derek Swanson, John Natzke, Ph.D.
George Fox University

Background

At high frequency, it becomes necessary to model a conductor as a transmission line because the voltage and current are highly dependent on position within the conductor. A transmission line is shown below:

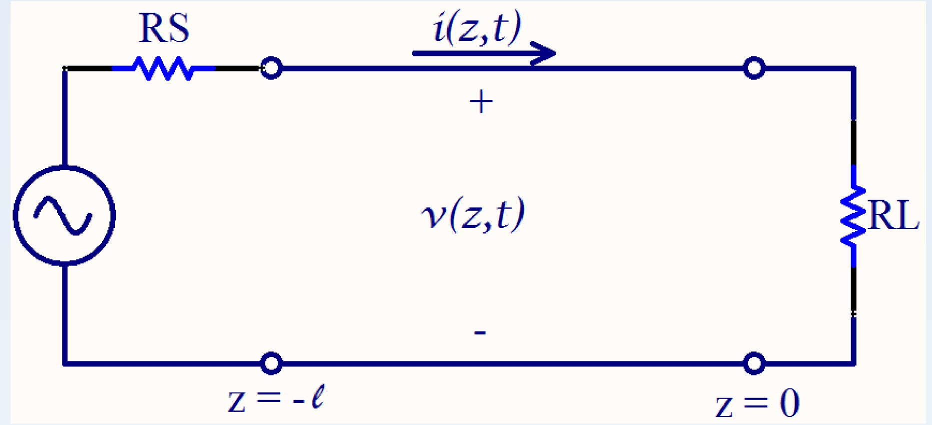

Figure 1: Transmission line connecting a voltage source to a resistive load. Note that the voltage and current are functions of the spatial dimension, z.

The voltage and current along the line are described by a pair of coupled partial differential equations:

$$-\frac{\partial i(z,t)}{\partial z} = v(z,t)G' + C'\frac{\partial v(z,t)}{\partial t}$$

$$-\frac{\partial v(z,t)}{\partial z} = i(z,t)R' + L'\frac{\partial i(z,t)}{\partial t}$$

The derivatives can be approximated with centered finite differences:

$$-\frac{i(z_{m+1}) - i(z_{m-1})}{\Delta z} = vG' + C'\frac{v(t_{n+1}) - v(t_{n-1})}{\Delta t}$$

$$-\frac{v(z_{m+1}) - v(z_{m-1})}{\Delta z} = iR' + L'\frac{i(t_{n+1}) - i(t_{n-1})}{\Delta t}$$

Which can easily be solved for the "next" values, $v(t_{n+1})$ and $i(t_{n+1})$, in terms of the "previous" values, $v(t_{n-1})$ and $i(t_{n-1})$. Given a set of initial conditions, the entire transmission line can be solved for one instant in time. Then those results are used to solve the transmission line for the next instant in time (i.e. one time step later). A computer program was developed based on this iterative method. The voltage and current were plotted vs. position after each time step, creating the effect of a moving wave.

Objectives

- Investigate numerical artifacts such as the "rounding" of square pulses
- Develop a more accurate set of boundary conditions at the source and load
- Investigate the effect on accuracy of higher-order derivative approximations.
- Investigate different solution configurations:
  - **<u>Collocated</u>** – discrete voltages and currents defined at same locations in space and time
  - **<u>Un-Collocated</u>** – discrete voltages and currents "staggered" in space and time
- Investigate non-uniform line parameters (i.e. where L', C', G', and R' vary with position)

Results

- Collocated techniques were unstable unless averaging was applied, which caused the "rounding" of square pulses.
- Un-collocated techniques were stable, did not have the "rounding" effect, and had 2nd order accuracy [2].
- Boundary conditions were developed such that the user can enter real resistances for the source and load. The reflected waveforms respond accordingly: if $R_L < Z_0$, then the reflected voltage waveform will be inverted (see Figure 2); if $R_L > Z_0$, then the reflected voltage wave will <u>not</u> be inverted; if $R_L = Z_0$, then the load is matched to the line and there will be no reflection.

Results Cont.

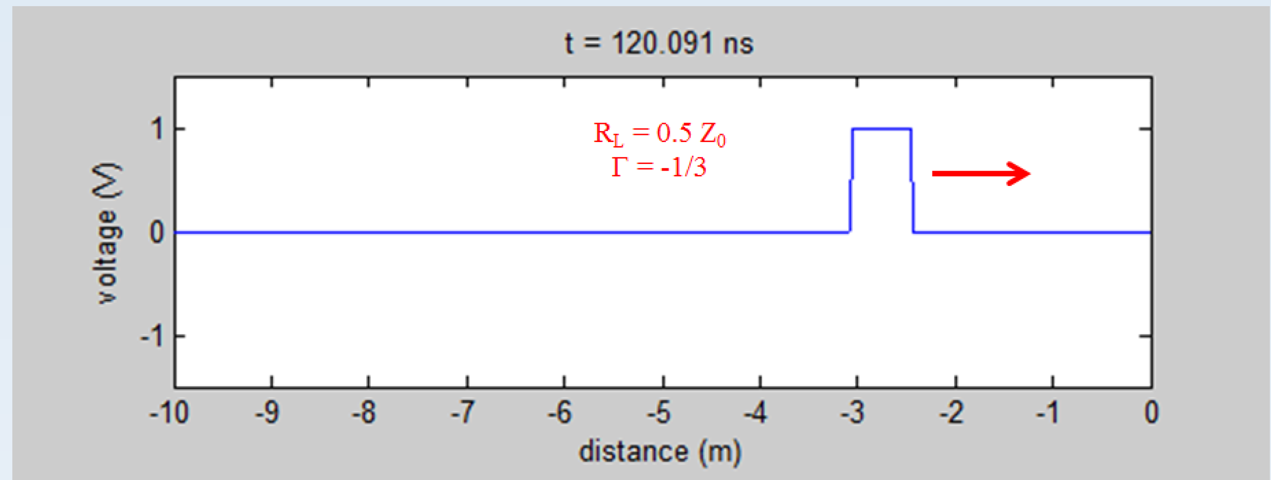

Figure 2a: Voltage pulse approaching the termination.

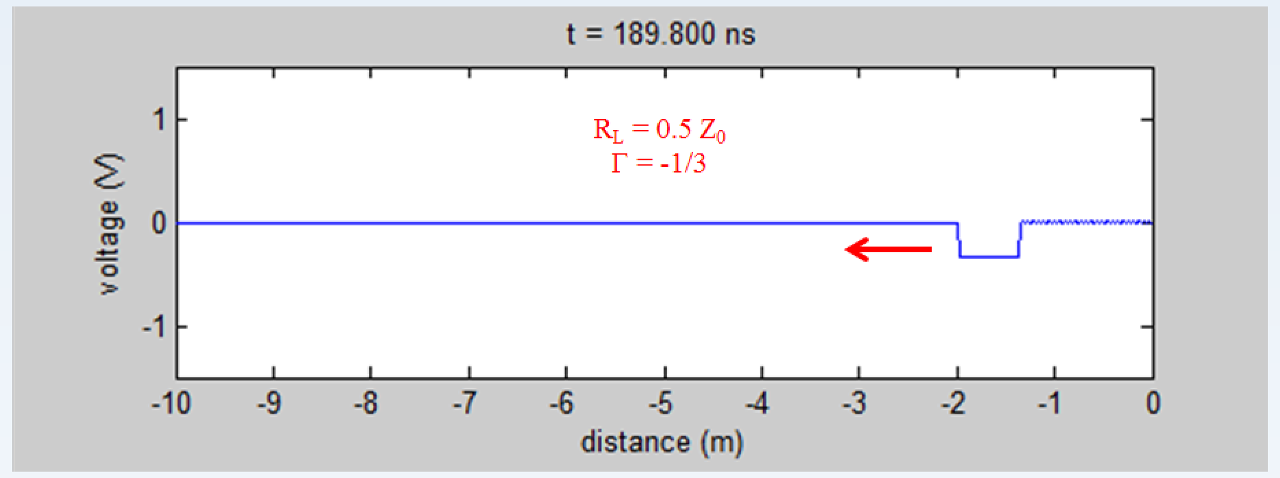

Figure 2b: Voltage pulse after reflecting off the termination. Note that the pulse has been inverted because $R_L < Z_0$ and it has been reduced to $1/3$ its original height (i.e. the reflection coefficient is $\Gamma = -1/3$)

- Higher-order derivative approximations did <u>not</u> improve accuracy. Generally, higher-order accuracy can be achieved by including additional non-adjacent nodes in the difference equation [1]. However, we discovered that wave phenomena do not benefit from this improved accuracy. For a given node, including extra non-adjacent nodes in the difference equation causes the node to "see" the wave ahead of time and therefore causes the node to respond too soon.
- Problems occur when simulating discontinuous media (non-uniform line parameters). In Discontinuous media, the wave propagation speed is not constant for the entire line. Interestingly, the ratio $\frac{\Delta z}{\Delta t}$ is directly related to the wave speed [2].

So, in order to simulate a transmission line with discontinuous wave speed, either Δz or Δt needs to be altered for a portion of the line in order to maintain accuracy. Further research is required to determine how to accomplish this practically. However, a special case was observed where the line parameters are discontinuous but the wave speed remains constant for the entire line (see Figure 3).

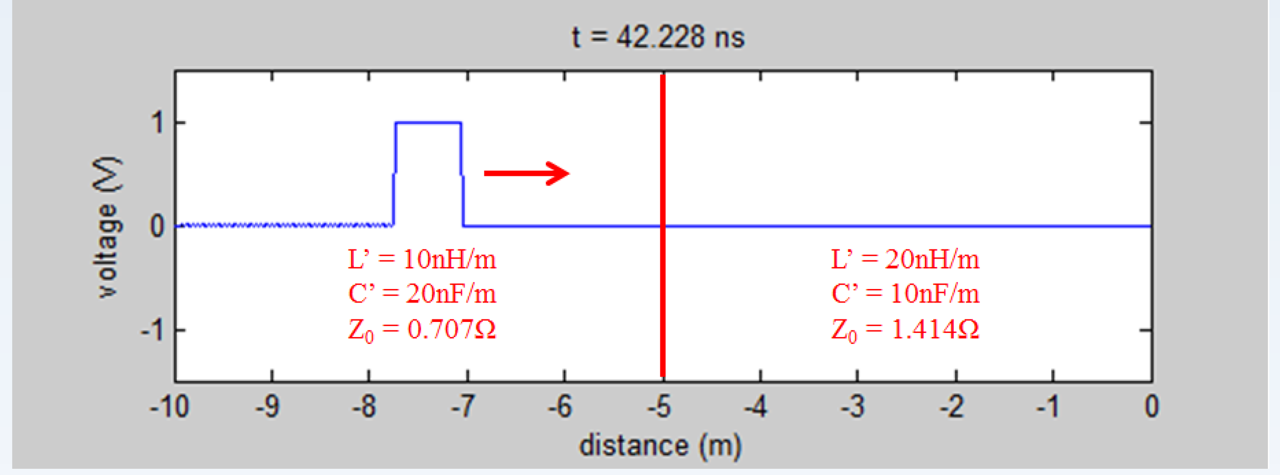

Figure 3a: Voltage pulse approaching the impedance discontinuity.

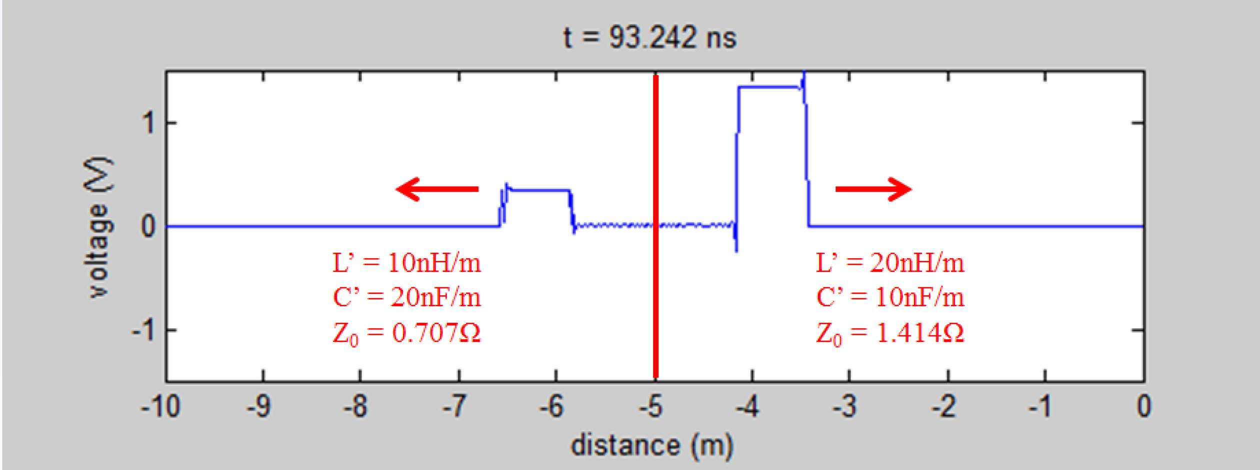

Figure 3b: Reflection coefficient is $\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = \frac{1.414 - 0.707}{1.414 + 0.707} = 1/3$. Transmission coefficient is $T = \Gamma + 1 = 4/3$. However, note that the wave speed is $u_p = \frac{1}{\sqrt{L'C'}}$, which is constant for the entire line.

References

[1] Chapra, Steven C. Applied Numerical Methods with MATLAB for Engineers and Scientists. Boston [etc.: McGraw-Hill Higher Education, 2008. Print.

[2] Gedney, Stephen D. Introduction to the Finite-Difference Time-Domain (FDTD) Method for Electromagnetics. 1st ed. Morgan & Claypool, 2011. Print.

[3] Ulaby, Fawwaz T. *Fundamentals of Applied Electromagnetics*. 5th ed. Upper Saddle River, NJ: Pearson Prentice Hall, 2007. Print.

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