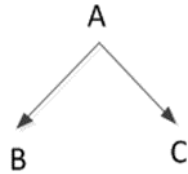


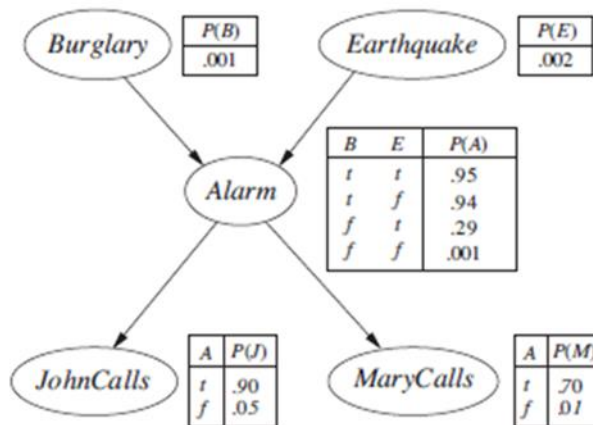
Home Assignment 6: Solution

Q1. [10] In the given Bayesian network where $P(a) = .4$, $P(b|a) = .3$, $P(b|\neg a) = .2$, $P(c|a) = .8$, $P(c|\neg a) = .5$, compute the **distribution** of $P(B | c)$.



$$\begin{aligned}
 \langle P(b | c), P(\sim b | c) \rangle &= \alpha \langle P(b, c), P(\sim b, c) \rangle && \text{where } \alpha = 1 / P(c) \\
 &= \alpha \langle \sum_a P(a, b, c), \sum_a P(a, \sim b, c) \rangle \\
 &= \alpha \langle \sum_a P(a)P(b|a)P(c|a), \sum_a P(a)P(\sim b|a)P(c|a) \rangle \\
 &= \alpha \langle P(a)P(b|a)P(c|a) + P(\sim a)P(b|\sim a)P(c|\sim a), P(a)P(\sim b|a)P(c|a) + P(\sim a)P(\sim b|\sim a)P(c|\sim a) \rangle \\
 &= \alpha \langle .4*.3*.8 + .6*.2*.5, .4*.7*.8 + .6*.8*.5 \rangle \\
 &= \alpha \langle .156, .464 \rangle \\
 &= \langle .156 / (.156 + .464), .084 / (.156 + .464) \rangle \\
 &= \langle 0.2516, 0.7484 \rangle
 \end{aligned}$$

Q2. [35] In the given Burglary Network, compute the followings.



(1) the distribution of $P(A)$, $P(J)$, $P(M)$, i.e. $(P(a), P(\neg a))$,

$$\begin{aligned}
 P(a) &= P(a, b, e) + P(a, \sim b, e) + P(a, b, \sim e) + P(a, \sim b, \sim e) \\
 &= P(a|b, e)P(b, e) + P(a|\sim b, e)P(\sim b, e) + P(a|b, \sim e)P(b, \sim e) + P(a|\sim b, \sim e)P(\sim b, \sim e) \\
 &= .95*0.001*0.002 + 0.29*0.999*0.002 + 0.94*0.001*0.998 + 0.001*0.999*0.998 \\
 &= 0.00252.
 \end{aligned}$$

$$\text{So, } \langle P(a), P(\sim a) \rangle = \langle 0.00252, 0.99748 \rangle$$

$$\begin{aligned}
 P(j) &= P(j, a) + P(j, \sim a) = P(j|a)P(a) + P(j|\sim a)P(\sim a) \\
 &= 0.9 * 0.00252 + 0.05 * 0.99748 \\
 &= 0.052142
 \end{aligned}$$

$$\text{So, } \langle P(j), P(\sim j) \rangle = \langle 0.052142, 0.947858 \rangle$$

$$\begin{aligned}
 P(m) &= P(m, a) + P(m, \sim a) = P(m|a)P(a) + P(m|\sim a)P(\sim a) \\
 &= 0.7 * 0.00252 + 0.01 * 0.99748 \\
 &= 0.011739
 \end{aligned}$$

$$\text{So, } \langle P(m), P(\sim m) \rangle = \langle 0.011739, 0.988261 \rangle$$

$$\begin{aligned}
 (2) \quad P(j|b) &= P(j, b)/P(b) = [P(j, b, a) + P(j, b, \sim a)]/P(b) \\
 &= [P(j|a)P(a|b)P(b) + P(j|\sim a)P(\sim a|b)P(b)] / P(b) \\
 &= P(j|a)P(a|b) + P(j|\sim a)P(\sim a|b) \\
 &\quad \text{where } P(a|b)^{**} = P(a, b)/P(b) = [P(a, b, e) + P(a, b, \sim e)] / P(b) \quad ** \\
 &\quad = [P(a|b, e)P(b, e) + P(a|b, \sim e)P(b, \sim e)] / P(b) \\
 &\quad = [P(a|b, e)P(b)P(e) + P(a|b, \sim e)P(b)P(\sim e)] / P(b) \\
 &\quad = P(a|b, e)P(e) + P(a|b, \sim e)P(\sim e) \\
 &= P(j|a)[P(a|b, e)P(e) + P(a|b, \sim e)P(\sim e)] + P(j|\sim a)[P(\sim a|b, e)P(e) + P(\sim a|b, \sim e)P(\sim e)] \\
 &= 0.9 * (0.95 * 0.002 + 0.94 * 0.998) + 0.05 * (0.05 * 0.002 + 0.06 * 0.998) \\
 &= 0.849017
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad P(a|b)^{**} &= P(a|b, e)P(e) + P(a|b, \sim e)P(\sim e) \\
 &= 0.95 * 0.002 + 0.94 * 0.998 \\
 &= 0.94002
 \end{aligned}$$

$$(4) \quad P(j \text{ or } m | b)$$

Using De Morgan's law:

$$\begin{aligned}
 P(j \text{ or } m | b) &= P(\sim(\sim j, \sim m) | b) = 1 - P(\sim j, \sim m | b) \\
 &= 1 - [P(\sim j|a)P(\sim m|a)P(a|b) + P(\sim j|\sim a)P(\sim m|\sim a)P(\sim a|b)] \quad ** \\
 &= 1 - [0.1 * 0.3 * 0.94002 + .95 * 0.99 * 0.05998] \\
 &= 0.915388
 \end{aligned}$$

OR

$$\begin{aligned}
 P(j \text{ or } m | b) &= P(j | b) + P(m | b) - P(j, m | b) \\
 &\quad \{ \text{where } P(m|b) = P(m, b)/P(b) = [P(m, b, a) + P(m, b, \sim a)]/P(b) \\
 &\quad = [P(m|a)P(a|b)P(b) + P(m|\sim a)P(\sim a|b)P(b)] / P(b) \\
 &\quad = P(m|a)P(a|b) + P(m|\sim a)P(\sim a|b) \\
 &\quad = P(m|a)[P(a|b, e)P(e) + P(a|b, \sim e)P(\sim e)] \\
 &\quad + P(m|\sim a)[P(\sim a|b, e)P(e) + P(\sim a|b, \sim e)P(\sim e)] \\
 &\quad = 0.7 * (0.95 * 0.002 + 0.94 * 0.998) \\
 &\quad + 0.01 * (0.05 * 0.002 + 0.06 * 0.998) \\
 &\quad = 0.658614 \\
 &\quad \text{where } P(j, m | b) = P(j, m|a)P(a|b) + P(j, m|\sim a)P(\sim a|b) \\
 &\quad = P(j|a)P(m|a)P(a|b) + P(j|\sim a)P(m|\sim a)P(\sim a|b) \\
 &\quad = 0.9 * 0.7 * 0.94002 + .05 * 0.01 * 0.05998
 \end{aligned}$$

$$\begin{aligned}
 &= 0.849017 + 0.658614 - 0.592513 \\
 &= 0.912119
 \end{aligned}$$

$$\begin{aligned}
 (5) \ P(b|m) &= P(m|b)P(b) / P(m) = 0.658614 * 0.001 / 0.011739 \\
 &= 0.056105
 \end{aligned}$$

$$\begin{aligned}
 (6) \ P(e|\neg a) &= P(e, \sim a) / P(\sim a) \\
 &= [P(e, \sim a, b) + P(e, \sim a, \sim b)] / P(\sim a) \\
 &= [P(\sim a|b, e)P(b)P(e) + P(\sim a|\sim b, e)P(\sim b)P(e)] / P(\sim a) \\
 &= [0.05 * 0.01 * 0.002 + 0.71 * 0.999 * 0.002] / 0.99748 \text{ from (1)} \\
 &= 0.001423
 \end{aligned}$$

Q3. [10] A diagnostic system is to be made for a dynamo-powered bicycle light using a Bayesian network. The variables are defined below:

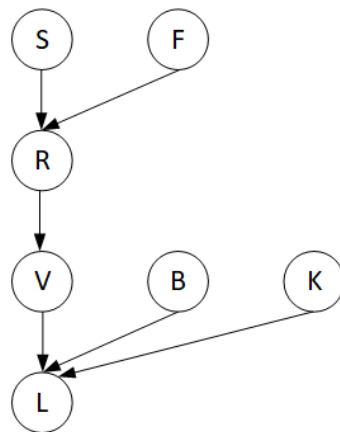
Variable	Value	Meaning
L	true / false	Light is on
S	dry, wet, snow	Street condition
F	true / false	Dynamo flywheel worn out
R	true / false	Dynamo sliding
V	true / false	Dynamo shows voltage
B	true / false	Light bulb OK
K	true / false	Cable OK

The following variables are independent: S, F, B, K.

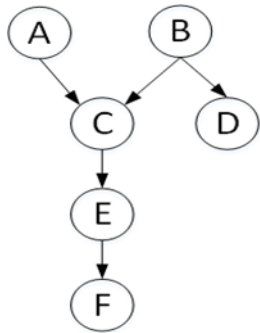
The following pairs of variables are independent: (R, B), (R, K), (V, B), (V, K)

and the following equation holds: $P(L | V, R) = P(L|V)$, $P(V | R, S) = P(V | R)$, $P(V | R, F) = P(V | R)$.

Draw the Bayesian network with the given variables in the Causal model.



Q4. [30] Consider the following belief network with Boolean variables and the following conditional probabilities:



$$\begin{aligned}
 P(a) &= 0.02 & P(d | b) &= 0.9 \\
 P(b) &= 0.01 & P(d | \neg b) &= 0.01 \\
 P(c | a, b) &= 0.5 & P(e | c) &= 0.88 \\
 P(c | a, \neg b) &= 0.99 & P(e | \neg c) &= 0.001 \\
 P(c | \neg a, b) &= 0.85 & P(f | e) &= 0.75 \\
 P(c | \neg a, \neg b) &= 0.0001 & P(f | \neg e) &= 0.01
 \end{aligned}$$

- (1) [15] a) Compute the conditional distribution $P(A | d, f)$ using Variable Elimination (VE).
b) Which variables are irrelevant to inference? First, prune irrelevant variables. Show the complete computation steps with the factors that are created for a given elimination ordering.

All the variables are relevant because they're ancestors of $\{A, D, F\}$.

$$\begin{aligned}
 P(A | d, f) &= \alpha P(A, d, f) & \text{where } \alpha &= 1 / P(d, f) \\
 &= \alpha \sum_b \sum_c \sum_e P(A, b, c, d, e, f) \\
 &= \alpha \sum_b \sum_c \sum_e P(A) P(b) P(c | A, b) P(d | b) P(e | c) P(f | e) \\
 &= \alpha P(A) \sum_b P(b) P(d | b) \sum_c P(c | A, b) \sum_e P(e | c) P(f | e) \\
 &= \alpha P(A) \sum_b P(b) P(d | b) \sum_c P(c | A, b) \sum_e f_4(E, C) f_5(f, E)
 \end{aligned}$$

where

$$f_5(f, E) = P(f | E) = (P(f | e), P(f | \neg e)) = (.75, .01)$$

$$\begin{aligned}
 f_4(E, C) &= P(E | C) = \begin{pmatrix} P(e | c) & P(e | \neg c) \\ P(\neg e | c) & P(\neg e | \neg c) \end{pmatrix} = \begin{pmatrix} .88 & .001 \\ .12 & .999 \end{pmatrix} \\
 &= \alpha P(A) \sum_b P(b) P(d | b) \sum_c P(c | A, b) f_6(f, C)
 \end{aligned}$$

where

$$\begin{aligned}
 f_6(f, C) &= \sum_e f_4(E, C) f_5(f, E) \\
 &= \sum_e \begin{pmatrix} .88 & .001 \\ .12 & .999 \end{pmatrix} (.75, .01) \\
 &= (.88 * .75 + .12 * .01, .001 * .75 + .999 * .01) \\
 &= (.6612, .01074) & = (P(f | c), P(f | \neg c))
 \end{aligned}$$

$$= \alpha P(A) \sum_b P(b)P(d|b) \sum_c f_7(C, A, B) f_6(f, C)$$

where

$$f_7(C, A, B) = P(c|A, b) = \begin{pmatrix} P(c|a, b) & P(c|a, \sim b) \\ P(c|\sim a, b) & P(c|\sim a, \sim b) \end{pmatrix} \begin{pmatrix} P(\sim c|a, b) & P(\sim c|a, \sim b) \\ P(\sim c|\sim a, b) & P(\sim c|\sim a, \sim b) \end{pmatrix}$$

$$= \begin{pmatrix} .5 & .99 \\ .85 & .0001 \end{pmatrix} \begin{pmatrix} .5 & .01 \\ .15 & .9999 \end{pmatrix}$$

$$= \alpha P(A) \sum_b P(b)P(d|b) f_8(f, A, B)$$

where

$$f_8(f, A, B) = \sum_c f_7(C, A, B) f_6(f, C)$$

$$= \sum_c \begin{pmatrix} P(c|a, b) & P(c|a, \sim b) \\ P(c|\sim a, b) & P(c|\sim a, \sim b) \end{pmatrix} \begin{pmatrix} P(\sim c|a, b) & P(\sim c|a, \sim b) \\ P(\sim c|\sim a, b) & P(\sim c|\sim a, \sim b) \end{pmatrix} (P(f|c), P(f|\sim c))$$

$$= \sum_c \begin{pmatrix} .5 & .99 \\ .85 & .0001 \end{pmatrix} \begin{pmatrix} .5 & .01 \\ .15 & .9999 \end{pmatrix} (.6612, .01074)$$

$$= \begin{pmatrix} .5 & .99 \\ .85 & .0001 \end{pmatrix} * 0.6612 + \begin{pmatrix} .5 & .01 \\ .15 & .9999 \end{pmatrix} * 0.01074$$

$$= \begin{pmatrix} .33597 & .654695 \\ .563631 & .010805 \end{pmatrix} = \begin{pmatrix} P(f|a, b) & P(f|a, \sim b) \\ P(f|\sim a, b) & P(f|\sim a, \sim b) \end{pmatrix}$$

$$= \alpha P(A) \sum_b f_{10}(B) f_9(d, B) f_8(f, A, B)$$

$$f_9(d, B) = (P(d|b), P(d|\sim b)) = (.9, .01)$$

$$f_{10}(B) = (P(b), P(\sim b)) = (.01, .99)$$

$$= \alpha P(A) f_{11}(A, d, f)$$

where

$$f_{11}(A, d, f) = \sum_b f_{10}(B) f_9(d, B) f_8(f, A, B)$$

$$= \sum_b (P(b), P(\sim b)) (P(d|b), P(d|\sim b)) \begin{pmatrix} P(f|a, b) & P(f|a, \sim b) \\ P(f|\sim a, b) & P(f|\sim a, \sim b) \end{pmatrix}$$

$$= \sum_b (.01, .99) (.9, .01) \begin{pmatrix} .33597 & .654695 \\ .563631 & .010805 \end{pmatrix}$$

$$= .01 * .9 * \begin{pmatrix} .33597 \\ .56363 \end{pmatrix} + .99 * .01 * \begin{pmatrix} .6547 \\ .01081 \end{pmatrix}$$

$$= \begin{pmatrix} .003024 \\ .005073 \end{pmatrix} + \begin{pmatrix} .006481 \\ .000107 \end{pmatrix}$$

$$= \begin{pmatrix} .009505 \\ .00518 \end{pmatrix} = \begin{pmatrix} P(f, d|a) \\ P(f, d|\sim a) \end{pmatrix}$$

$$= \alpha f_{12}(A) f_{11}(A, d, f)$$

where

$$f_{12}(A) = (P(a), P(\sim a)) = (.02, .98)$$

$$= \alpha (.02, .98) \begin{pmatrix} .009505 \\ .00518 \end{pmatrix}$$

$$= \alpha (.00019, .005076) = (P(f, d, a), P(f, d, \sim a))$$

$$= \left(\frac{.00019}{(.00019 + .005076)}, \frac{.005076}{(.00019 + .005076)} \right) \quad \text{where } \alpha = 1 / P(d, f)$$

$$= (.036099, .963901) = < P(a | d, f), P(\sim a | d, f) >$$

(2) [15] c) Compute $P(e | b)$ using VE. a) Which variables are irrelevant to inference? b) Which factors can be reused from (1)? d) Show the factors that are different from those in (1).

a) Irrelevant variables D and F

b) The reused factors

$$f1(C, e) = P(e|C) = (P(e|c), P(e|\sim c)) = (.88, .001) \quad \text{- part of f4 in 1) with e.}$$

$$f2(A, C, b) = \begin{pmatrix} P(c|a,b) & P(\sim c|a,b) \\ P(c|\sim a,b) & P(\sim c|\sim a,b) \end{pmatrix} = \begin{pmatrix} .5 & .5 \\ .85 & .15 \end{pmatrix} \quad \text{- part of f7 in 1) with b.}$$

$$c) P(e | b) = \alpha P(e, b) \quad \text{where } \alpha = 1/P(b)$$

$$= \alpha \sum_a \sum_c \sum_d \sum_f P(a, b, c, d, e, f)$$

$$= \alpha \sum_a \sum_c P(a, b, c, e) \quad \text{because D and F are irrelevant.}$$

$$= \alpha \sum_a \sum_c P(A)P(b)P(C|A, b)P(e|C)$$

$$= \alpha P(b) \sum_a P(A) \sum_c P(C|A, b) P(e|C)$$

$$= \alpha P(b) \sum_a P(A) \sum_c f2(A, C, b) f1(C, e)$$

$$\text{where } f1(C, e) = P(e|C) = (P(e|c), P(e|\sim c)) = (.88, .001) \quad \text{- part of f4 in 1) with e.}$$

$$f2(A, C, b) = \begin{pmatrix} P(c|a,b) & P(\sim c|a,b) \\ P(c|\sim a,b) & P(\sim c|\sim a,b) \end{pmatrix} = \begin{pmatrix} .5 & .5 \\ .85 & .15 \end{pmatrix} \quad \text{- part of f7 in 1) with b.}$$

$$= \alpha P(b) \sum_a P(A) f3(A, b)$$

where

$$f3(A, b) = \sum_c f2(A, C, b) f1(C, e)$$

$$= \sum_c \begin{pmatrix} P(c|a,b) & P(\sim c|a,b) \\ P(c|\sim a,b) & P(\sim c|\sim a,b) \end{pmatrix} (P(e|c), P(e|\sim c))$$

$$= \sum_c \begin{pmatrix} .5 & .5 \\ .85 & .15 \end{pmatrix} (.88, .001)$$

$$= \begin{pmatrix} .5 \\ .85 \end{pmatrix} * .88 + \begin{pmatrix} .5 \\ .15 \end{pmatrix} * .001$$

$$= \begin{pmatrix} .4405 \\ .74815 \end{pmatrix} = \begin{pmatrix} P(e|a,b) \\ P(e|\sim a,b) \end{pmatrix}$$

$$= \alpha P(b) f4(b)$$

$$= f4(b) \quad \text{since } \alpha = 1/P(b)$$

$$= \sum_a P(A) f3(A, b)$$

$$= \sum_a (P(a), P(\sim a)) \begin{pmatrix} P(e|a,b) \\ P(e|\sim a,b) \end{pmatrix}$$

$$= \sum_a (.02, .98) \begin{pmatrix} .4405 \\ .74815 \end{pmatrix}$$

$$= .02 * .4405 + .98 * .74815$$

$$= .821287 = P(e | b)$$

Q5. [15] In the given Bayesian network, compute the following probabilities.

(1) [15] Compute the probability that an elderly female who is over 60 gets no cancer (i.e. benign).

$$P(C=\text{malignant} | \text{Age} > 60, \text{Gender}=\text{female})$$

$$= \alpha \sum_e \sum_s P(C=\text{malignant}, \text{old}, \text{female}, e, s)$$

where e = Exposure2Toxic, s = Smoking

$$= \alpha \sum_e \sum_s P(C=\text{malignant} | e, s) P(e | \text{old}) P(s | \text{old}, \text{female}) P(\text{old}) P(\text{female})$$

$$\begin{aligned}
&= \alpha P(\text{old}) P(\text{female}) \sum_e \sum_s P(e | \text{old}) P(C=\text{malignant} | e, s) P(s | \text{old}, \text{female}) \\
&\quad \text{i.e. } P(C=\text{malignant} | \text{old}, \text{female}) \\
&= \alpha P(\text{old}) P(\text{female}) [P(e | \text{old}) P(\text{malignant} | e, s) P(s | \text{old}, \text{female}) \\
&\quad + P(e | \text{old}) P(\text{malignant} | e, \sim s) P(\sim s | \text{old}, \text{female}) \\
&\quad + P(\sim e | \text{old}) P(\text{malignant} | \sim e, s) P(s | \text{old}, \text{female}) \\
&\quad + P(\sim e | \text{old}) P(\text{malignant} | \sim e, \sim s) P(\sim s | \text{old}, \text{female})] \\
&= \alpha P(\text{old}) P(\text{female}) [.7*.9*.1 + .7*.6*.9 + .3*.6*.1 + .3*.1*.9] \\
&= \alpha P(\text{old}) P(\text{female}) *.0.486 \quad \text{where } P(C=\text{malignant} | \text{old female}) = 0.486 \\
&= \alpha' *.0.486 \quad \text{where } \alpha' = \alpha P(\text{old}) P(\text{female}) = 1
\end{aligned}$$

Similarly,

$$\begin{aligned}
P(C=\text{benign} | \text{old}, \text{female}) &= \alpha \sum_e \sum_s P(C=\text{benign}, \text{old}, \text{female}, e, s) \\
&= \alpha P(\text{old}) P(\text{female}) \sum_e \sum_s P(e | \text{old}) P(C=\text{benign} | e, s) P(s | \text{old}, \text{female}) \\
&\quad \text{i.e. } P(C=\text{benign}) \\
&= \alpha P(\text{old}) P(\text{female}) * P(C=\text{benign} | \text{old female}) \\
&= \alpha' *.514 \quad \text{where } P(C=\text{benign} | \text{old}, \text{female}) = 1 - P(\text{malignant} | \text{old}, \text{female})
\end{aligned}$$

Thus,

$$\begin{aligned}
&< P(\text{malignant} | \text{old}, \text{female}), P(\text{benign} | \text{old}, \text{female}) > \\
&= < .486 / (.486 + .514), .514 / (.486 + .514) > \\
&= < .486, .514 >
\end{aligned}$$

OR

$$\begin{aligned}
P(C=\text{benign} | \text{Age} > 60, \text{Gender}=\text{female}) &= \alpha \sum_e \sum_s P(C=\text{benign}, \text{old}, \text{female}, e, s) \\
&\quad \text{where } e = \text{Exposure2Toxic}, s = \text{Smoking} \\
&= \alpha \sum_e \sum_s P(C=\text{malignant} | e, s) P(e | \text{old}) P(s | \text{old}, \text{male}) P(\text{old}) P(\text{male}) \\
&= \alpha P(\text{old}) P(\text{female}) \sum_e \sum_s P(e | \text{old}) P(C=\text{benign} | e, s) P(s | \text{old}, \text{female}) \\
&\quad \text{since } \alpha = 1 / P(\text{old}, \text{female}) = 1 / P(\text{old}) P(\text{female}) \\
&= \sum_e \sum_s P(e | \text{old}) P(C=\text{benign} | e, s) P(s | \text{old}, \text{female}) \\
&= [P(e | \text{old}) P(\text{benign} | e, s) P(s | \text{old}, \text{female}) \\
&\quad + P(e | \text{old}) P(\text{benign} | e, \sim s) P(\sim s | \text{old}, \text{female}) \\
&\quad + P(\sim e | \text{old}) P(\text{benign} | \sim e, s) P(s | \text{old}, \text{female}) \\
&\quad + P(\sim e | \text{old}) P(\text{benign} | \sim e, \sim s) P(\sim s | \text{old}, \text{female})] \\
&= [.7*.1*.1 + .7*.4*.9 + .3*.4*.1 + .3*.9*.9] = .514
\end{aligned}$$

- (2) [15, optional] Compute the probability that an elderly female patient with low Serum Calcium to have no cancer (i.e. benign).

Compute $P(C=\text{benign} | \text{SC} = \text{low}, \text{Age} > 60, \text{Gender}=\text{female})$.

Method A.

$$\begin{aligned}
&P(C=\text{benign} | \text{SC}=\text{low}, \text{Age} > 60, \text{Gender}=\text{female}) \\
&= \alpha P(C=\text{benign}, \text{SC}=\text{low}, \text{old}, \text{female}) \quad \text{where } \alpha = P(\text{SC}=\text{low}, \text{old}, \text{female}) \\
&= \alpha P(\text{SC}=\text{low} | C=\text{benign}, \text{old}, \text{female}) P(C=\text{benign}, \text{old}, \text{female}) \\
&\quad // \text{ Since SC is conditionally independent of its non-descendants (i.e. Age, Gender) given its} \\
&\quad \text{parent, Cancer, } P(\text{SC}=\text{low} | \text{benign}, \text{old}, \text{female}) = P(\text{SC}=\text{low} | \text{malignant})
\end{aligned}$$

$$\begin{aligned}
&= \alpha P(\text{SC}=\text{low} \mid \text{C}=\text{benign})P(\text{C}=\text{benign} \mid \text{old, female})P(\text{old, female}) \\
&= \alpha P(\text{SC}=\text{low} \mid \text{C}=\text{benign})P(\text{C}=\text{benign} \mid \text{old, female})P(\text{old})P(\text{female}) \\
&= \alpha' P(\text{SC}=\text{low} \mid \text{C}=\text{benign})P(\text{C}=\text{benign} \mid \text{old, female}) \quad \text{where } \alpha' = \alpha \cdot P(\text{old})P(\text{female}) \\
&= \alpha' 0.5 \cdot 0.514 \quad \text{where } P(\text{C}=\text{benign} \mid \text{old, female}) = 0.514 \text{ from (1)} \\
&= 0.257\alpha'
\end{aligned}$$

Similarly,

$$\begin{aligned}
&P(\text{C}=\text{malignant} \mid \text{SC}=\text{low, old, female}) \\
&= \alpha' P(\text{SC}=\text{low} \mid \text{C}=\text{malignant})P(\text{C}=\text{malignant} \mid \text{old, female}) \quad \text{where } \alpha' = \alpha \cdot P(\text{old})P(\text{female}) \\
&= \alpha' 0.1 \cdot 0.486 \quad \text{where } P(\text{C}=\text{malignant} \mid \text{old, female}) = 0.486 \text{ from (1)} \\
&= 0.0486 \alpha'
\end{aligned}$$

After normalization,

$$\begin{aligned}
&< P(\text{C}=\text{benign} \mid \text{SC}=\text{low, old, female}), P(\text{C}=\text{malignant} \mid \text{SC}=\text{low, old, female}) > \\
&= < 0.257\alpha', 0.0486 \alpha' > \\
&= < 0.257/(0.257 + 0.0486), 0.0486/(0.257 + 0.0486) > \\
&= < \mathbf{0.841}, 0.159 >
\end{aligned}$$

Method B.

Since Cancer is conditionally independent of Age and Gender given its Markov Blanket (Serum Calcium), $P(\text{C}=\text{benign} \mid \text{SC} = \text{low, Age} > 60, \text{Gender}=\text{female}) = P(\text{C}=\text{benign} \mid \text{SC} = \text{low})$. So, we can simply compute $P(\text{C}=\text{benign} \mid \text{SC}=\text{low})$.

$$\begin{aligned}
P(\text{C}=\text{malignant} \mid \text{SC} = \text{low}) &= \alpha P(\text{SC}=\text{low} \mid \text{C}=\text{malignant}) P(\text{malignant}) = \alpha \cdot \mathbf{.1} * \mathbf{.486} = \mathbf{.0486} \alpha \\
P(\text{C}=\text{benign} \mid \text{SC} = \text{low}) &= \alpha P(\text{SC}=\text{low} \mid \text{C}=\text{benign}) P(\text{benign}) = \alpha \cdot .5 * .514 = .257\alpha
\end{aligned}$$

where $P(\text{SC} = \text{low} \mid \text{C}=\text{malignant}) = .1$ and $P(\text{SC} = \text{low} \mid \text{C}=\text{benign}) = .5$ in CPT and $P(\text{malignant} \mid \text{old, male}) = .486$ in (1) and $P(\text{benign} \mid \text{old, male}) = .514$ in 1)

Thus,

$$\begin{aligned}
&< P(\text{C}=\text{malignant} \mid \text{SC} = \text{low}), P(\text{C}=\text{benign} \mid \text{SC} = \text{low}) > \\
&= < .0486\alpha, .257\alpha > = < .0486/(.0486+.257), .257/(.0486+.257) > = < \mathbf{.159}, \mathbf{.841} >
\end{aligned}$$

Method C. $P(\text{C}=\text{malignant} \mid \text{SC} = \text{low, Age} > 60, \text{Gender}=\text{female})$

$$= \alpha P(\text{SC}=\text{low} \mid \text{C}=\text{malignant, old, female}) P(\text{malignant} \mid \text{old, female})$$

Since SC is conditionally independent of its non-decendants (i.e. Age, Gender) given its parent, Cancer,

$$\begin{aligned}
P(\text{SC} = \text{low} \mid \text{malignant, old, female}) &= P(\text{SC}=\text{low} \mid \text{malignant}) \\
&= \alpha P(\text{SC}=\text{low} \mid \text{C}=\text{malignant}) P(\text{malignant} \mid \text{old, female}) \\
&= \alpha \cdot \mathbf{.1} * .486 = \mathbf{.0486} \alpha
\end{aligned}$$

Similarly,

$$\begin{aligned}
P(\text{benign} \mid \text{old, female}) &= \alpha P(\text{C} = \text{benign} \mid \text{old, female}) * P(\text{SC} = \text{low} \mid \text{C} = \text{benign}) \\
&= \alpha P(\text{SC} = \text{low} \mid \text{C} = \text{benign}) \cdot P(\text{C} = \text{benign} \mid \text{old, female}) *
\end{aligned}$$

$$= \alpha .5 * 514$$

$$= .257\alpha$$

After normalization of $\langle .0486\alpha, .257\alpha \rangle = \langle .159, .841 \rangle$

Method D.

$$P(SC = \text{low} | \text{old, female}) = \sum_{\text{cancer}} P(SC=\text{low} | \text{Cancer, old, female}) * P(\text{Cancer} | \text{old, female})$$

$$= P(SC=\text{low} | \text{malignant, old, female})P(\text{malignant} | \text{old, female})$$

$$+ P(SC=\text{low} | \text{benign, old, female}) P(\text{benign} | \text{old, female})$$

$$= .1 * .486 + .5 * (.514) = .0486 + .257 = .3056$$

$$P(\text{benign} | SC=\text{low, old, female}) = (.1 * .486) / .3056 = .841$$

