Conditions for Classical Behaviour from Quantum Mechanics

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Abstract

It has recently been shown that coarse-grained spin measurements can explain the transition from quantum mechanics to classical physics for a large set of Hamiltonians. Here we extend these results to photonic and mechanical resonator coherent states. We analyse the difference between non-invasive (classical) measurements and sharp or coarse-grained quantum measurements. We give a quantitative criterion for noninvasive measurability and apply it to a set of states of interest, as well as recents experiments. We conjecture that in general either sharp quantum measurements or non-classical time evolution are required to violate macrorealism.

Macrorealism

Macrorealism per se (MRps):
At any time, system is in exactly one definite macroscopic state

Noninvasive measurability (NIM):
Measurement doesn't disturb
a) the state of the system (NIM$_a$)
b) its subsequent evolution (NIM$_b$)

MRps + NIM$_a$ + NIM$_b$ = MR

¬ NIM$_a$ → ¬ MR

¬ NIM$_b$ → ¬ MR

"CGM" + "class. H" → MR

¬ "CGM" → ¬ MR

"nonclass. H" → ¬ MR

Consider Husimi Q-distribution:

$$Q(\alpha) = \mathcal{N}\langle\alpha|\hat\rho|\alpha\rangle$$
$$Q_m(\alpha) = \frac{1}{w_m}\langle\alpha|\hat M_m^\dagger\hat\rho\hat M_m|\alpha\rangle$$
$$\bar Q(\alpha) = \sum_m w_m Q_m(\alpha) = \mathcal{N}\sum_m\langle\alpha|\hat M_m^\dagger\hat\rho\hat M_m|\alpha\rangle$$

NIM$_a$: coarse-grained measurements (CGM) satisfy:

$$Q(\alpha)\approx\bar Q(\alpha)$$

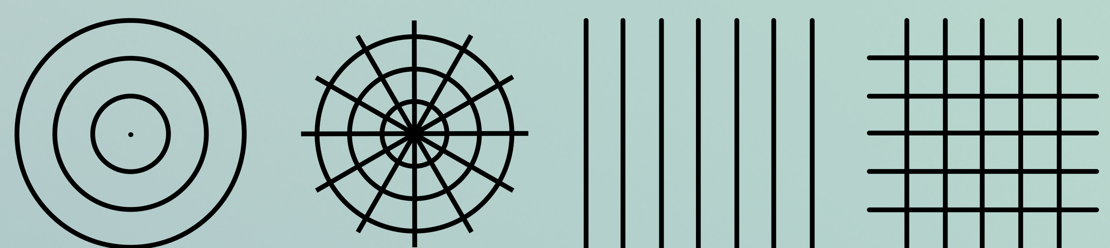
NIM$_b$: coarse-grained measurements + classical time evolution satisfy:

Sufficient condition for Macrorealism ("No Signalling In Time")

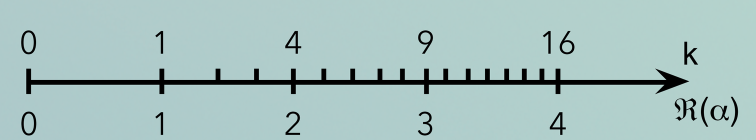
$$Q(\alpha,t_2)\approx\bar Q_{t_1}(\alpha,t_2) = \mathcal{N}\sum_m\langle\alpha|\hat U^\dagger(\Delta t)\hat M_m^\dagger\hat\rho(t_1)\hat M_m\hat U(\Delta t)|\alpha\rangle \quad \forall t_1,t_2,\hat\rho$$

Coarse-Grained Measurements

In coherent state space:



In Fock space:



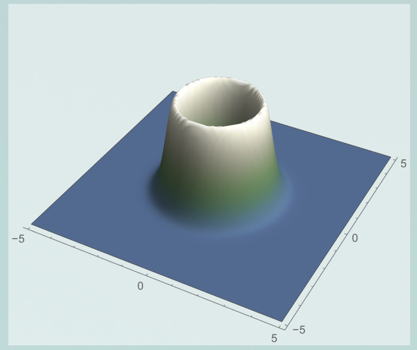
In position / momentum space: Gaussian quadrature measurement

Noninvasive Measurability NIM$_a$

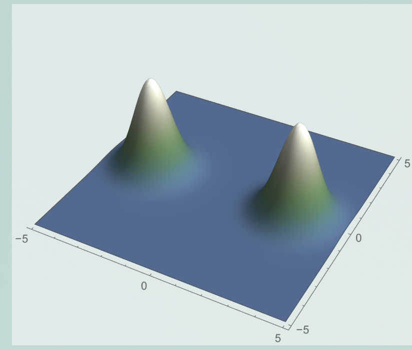
States of interest:

$$\hat\rho = |\psi\rangle\langle\psi|$$
$$|\psi\rangle = \begin{cases}|k\rangle\\ |\beta\rangle\\ \mathcal{N}(\beta)(|\beta\rangle+|-\beta\rangle)\\ \hat D(\beta)(|0\rangle+|1\rangle)/\sqrt2\end{cases}$$

Mixed states described by sum of Qs



Fock state k=3



$|\beta\rangle+|-\beta\rangle$

Coherent state POVMs:

$$\hat P_\alpha = |\alpha\rangle\langle\alpha|,\ \hat\rho = |\beta\rangle\langle\beta|:\ V = \frac{2\sqrt2}{3}\approx 0.943$$

This gives a lower bound for:

$$\hat P_m = \int_m |\alpha\rangle\langle\alpha|,\ \hat\rho = |\beta\rangle\langle\beta|:\ V \gtrsim 0.943$$

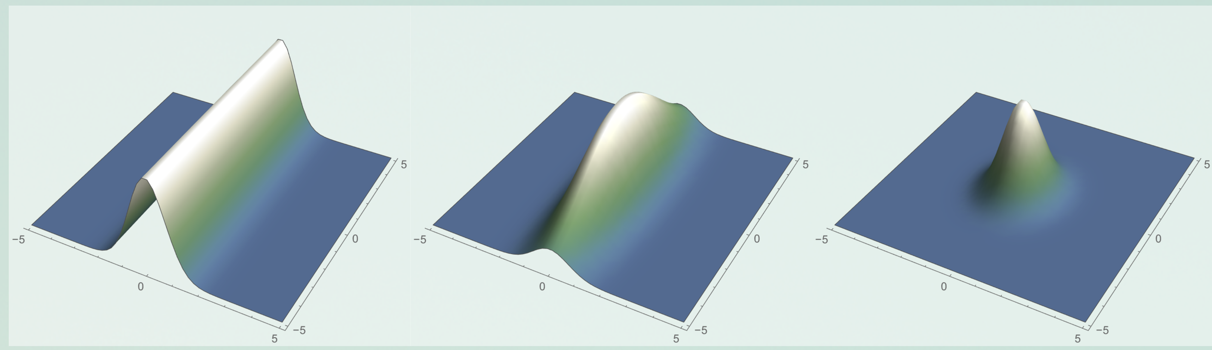
Measure of overlap between Q functions:
Bhattacharyya coefficient

$$V = \int d\alpha\ \sqrt{Q(\alpha)\bar Q(\alpha)}\in[0,1]$$

NIM$_a$ → V close to 1 for all states

Quadrature measurements of $\hat\rho = |\beta\rangle\langle\beta|$

$$\hat M_\Delta(x) = \frac{1}{(\Delta\pi)^{1/4}}e^{-\frac{1}{2\Delta}(x-\hat x)^2}$$
$$V = \left(\frac{8\Delta(1+2\Delta)}{(1+4\Delta)^2}\right)^{1/4}$$
$$\lim_{\Delta\to0} V = 0$$
$$\lim_{\Delta\to\infty} V = 1$$

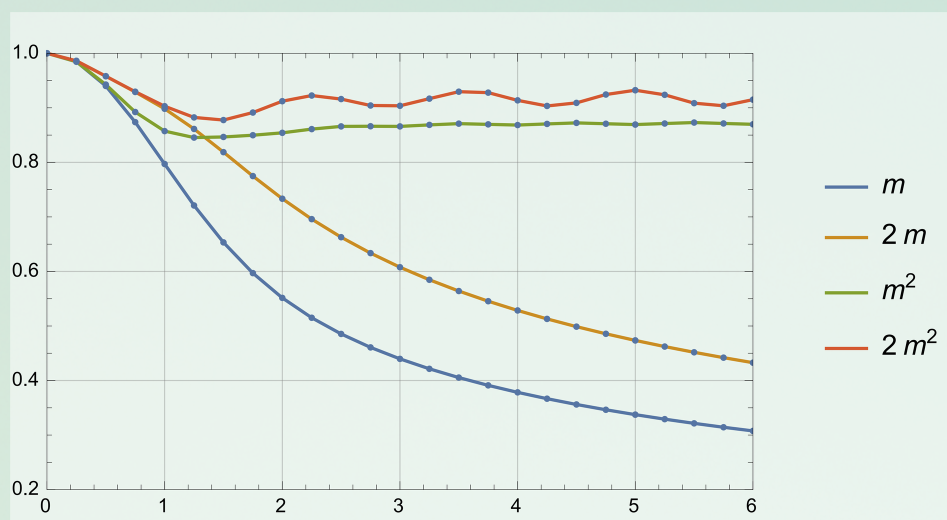


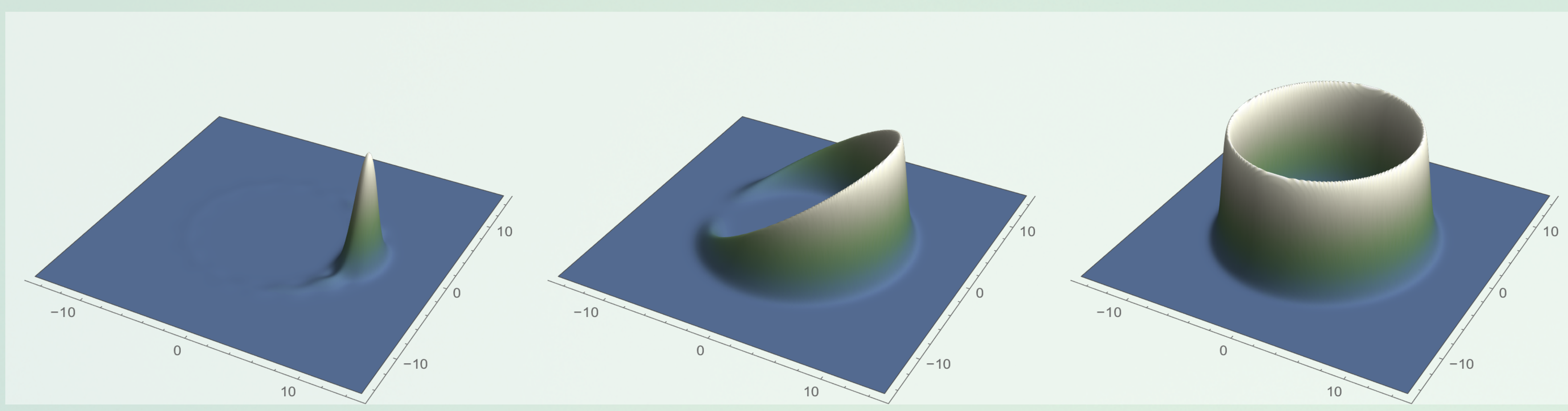
$\Delta = 0$ $\Delta = 0.03$ $\Delta = 1$

Fock measurements of $\hat\rho = |\beta\rangle\langle\beta|$

$$\hat M_m = \sum_k\begin{cases}|k\rangle\langle k| & \text{if } b(m)\le k< b(m+1)\\ 0 & \text{else}\end{cases}$$

Measurements
- sharp (a states per bin):
  $b(m) = am$
- coarse-grained (am states per bin):
  $b(m) = am^2$





m states in bin m, $b(m) = m^2$
Coarse in coherent space

2 states per bin, $b(m) = 2m$
Sharp in coherent space

1 state per bin, $b(m) = m$
Sharp in coherent space

Case Study: De Martini et al, PRL 2008

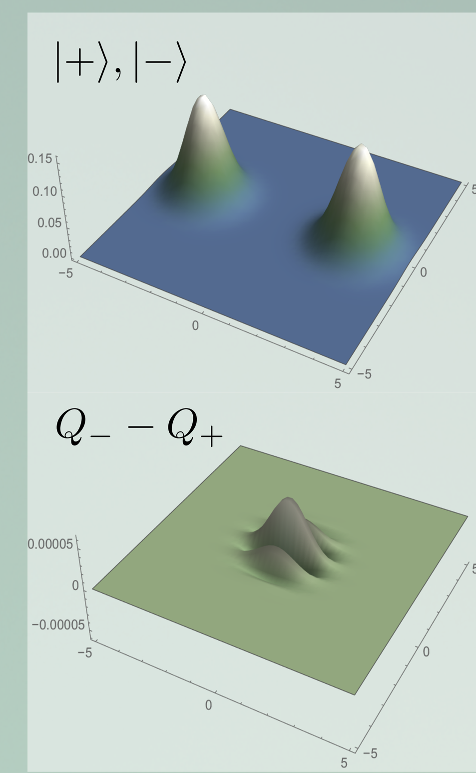
Superposition of two polarisation states:

$$|\Phi^\phi\rangle = \sum_{i,j=0}^{\infty}\gamma_{ij}|(2i+1)\phi;(2j)\phi^\perp\rangle$$
$$|\Psi\rangle = (|\Phi^\phi\rangle+|\Phi^{\phi\perp}\rangle)/\sqrt2$$

Simplified model:

$$|+\rangle\propto\sum_{k=0}^{\infty}|2k\rangle\langle 2k|\beta\rangle$$
$$|-\rangle\propto\sum_{k=0}^{\infty}|2k+1\rangle\langle 2k+1|\beta\rangle$$
$$\langle+|-\rangle = 0,\ Q_+\approx Q_-$$

Orthogonal, but V very close to 1
No distinct Q function despite large photon number



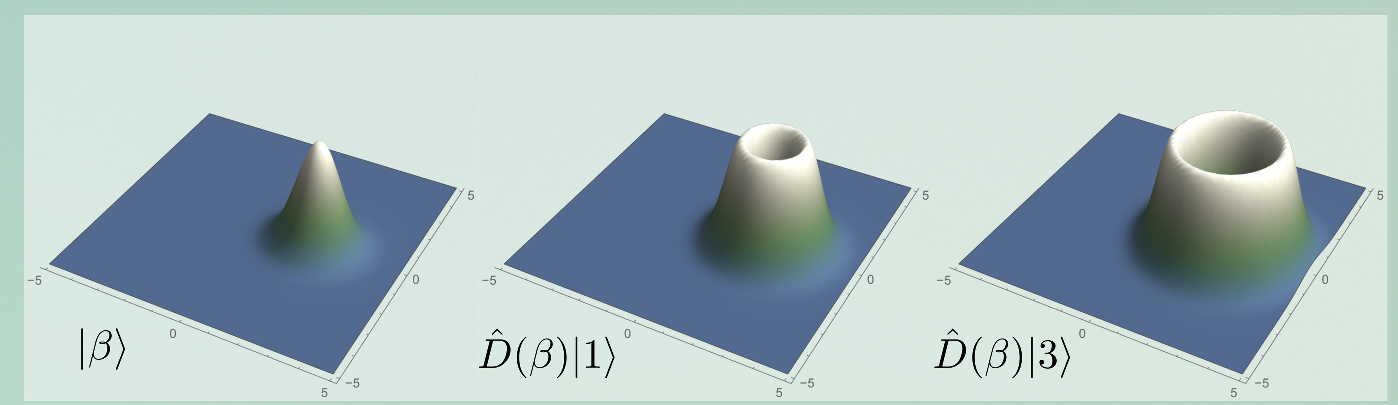
Case Study: Simon et al, NPhys 2013

Superposition of displaced Fock states:

$$|\psi\rangle = \hat D(\beta)(|0\rangle+|k=1\rangle)/\sqrt2$$

V decreases for large k

$$V_{k=1}\approx 0.886$$
$$V_{k=3}\approx 0.543$$



Superposition of moderately distinct states
Could be improved by displacing more than 1 photon

Next Steps

- Other states of interest
- Consider NIM$_b$ (time evolution)
- Single combined condition on measurements and Hamiltonians
- Free particles (c.f. research done by P. Busch et al.)

References

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