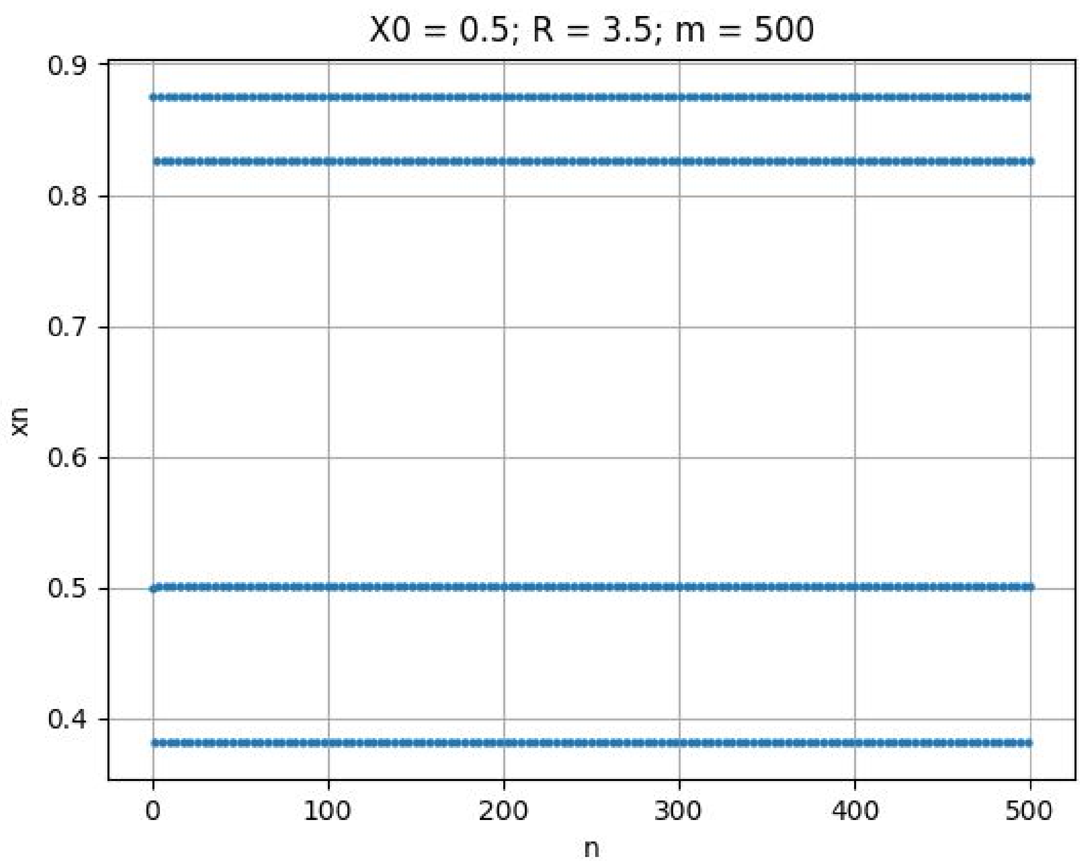
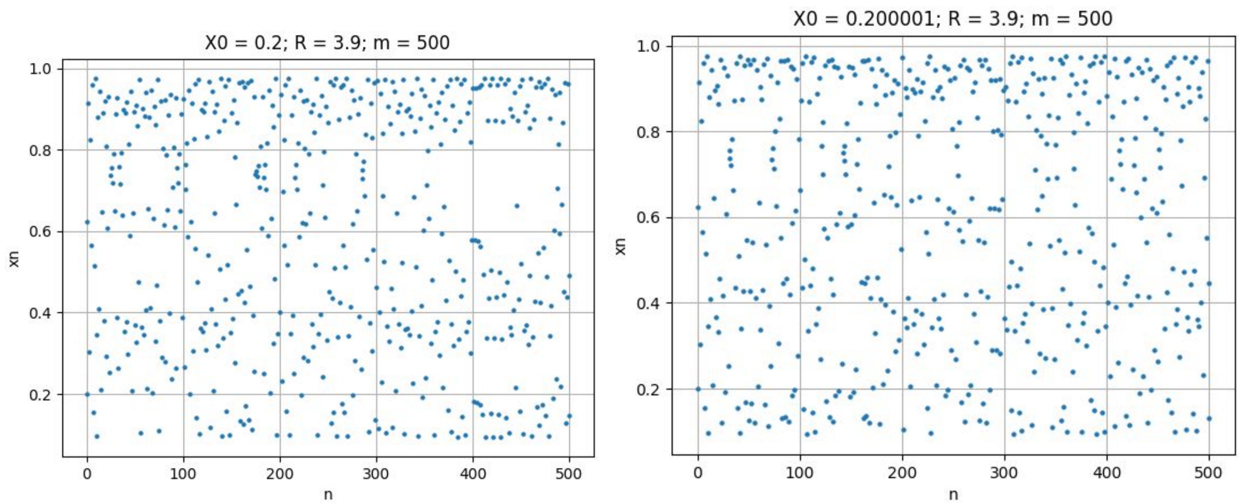
4.

Playing with R seemed to modify both the type of attractor (fixed point versus periodic orbit) of our logistic map system, as well as the location of that attractor. An R value of 2 results in a fixed point attractor at 0.5 for any starting conditions, but and R value of 3.3 gives a 2 cycle (0.48, .825), for example. An R value of 2.5 results in a fixed point at .6, while R = 3.5 results in a 4-cycle (.38, .50, .83, .88):



Each of these changes in attractor is a result of the system **bifurcating** because of the tweaking of the R parameter.

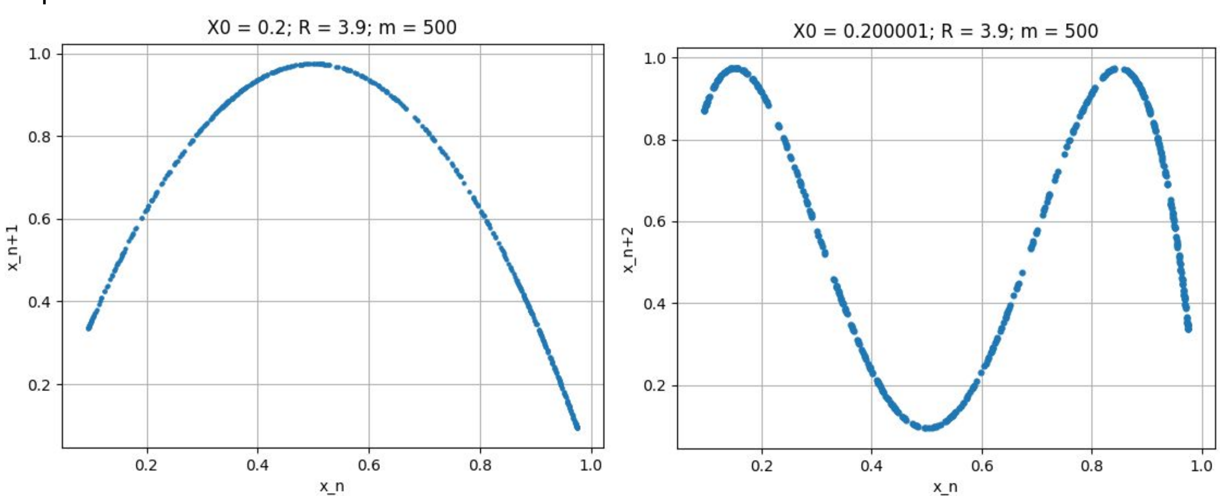
Chaos was observed at an R value of 3.9. To demonstrate this, two plots are shown below starting at **very** slightly different initial conditions. Notice the differences. They are subtle to see, but they exist, and show that the two sequences don’t closely follow each other or converge to a common attractor.



When R > 4, we see very unstable behavior. At larger n values, the plot quickly diverges. For example for one n = j value, $x_j = 15$, and $x_{j+1} = 10^7$. There is no finite attractor in this R range.

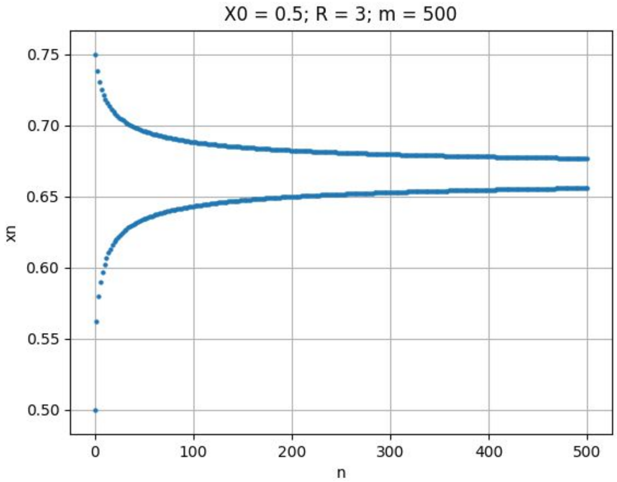
When R is fixed to 2.5, the initial conditions do not change the fixed point attractor. This means that the initial conditions are all in a “well of attraction” that settles on .6 (in this case). What does change with the initial conditions is the approach to the fixed point. A small value displays an over-damped behavior, never over-shooting the fixed point, and asymptotically approaching it. On the other side, a larger initial condition will create an under-damped behavior where the system overshoots the fixed point but settles on it eventually.

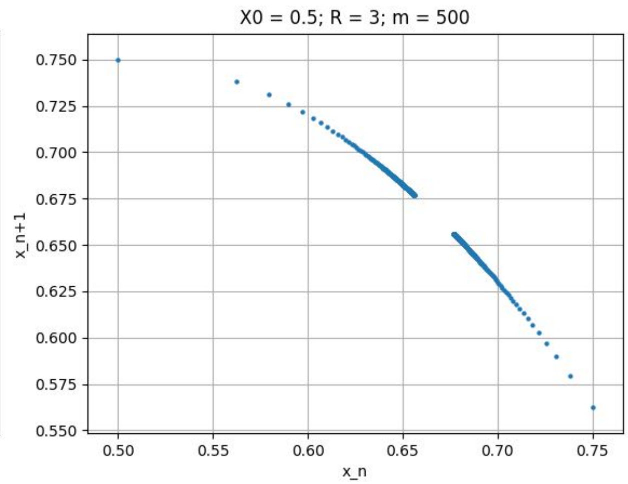
Some interesting plots I found include the structure of the chaotic system when plotted on return maps:



They are almost identical, but the curves are “filled in” in a different order.

Another interesting set of plots is this converging two-cycle:





They display very nicely the “keep out” region that it can never reach.