CSCI 4446 Spring 2019 PS2
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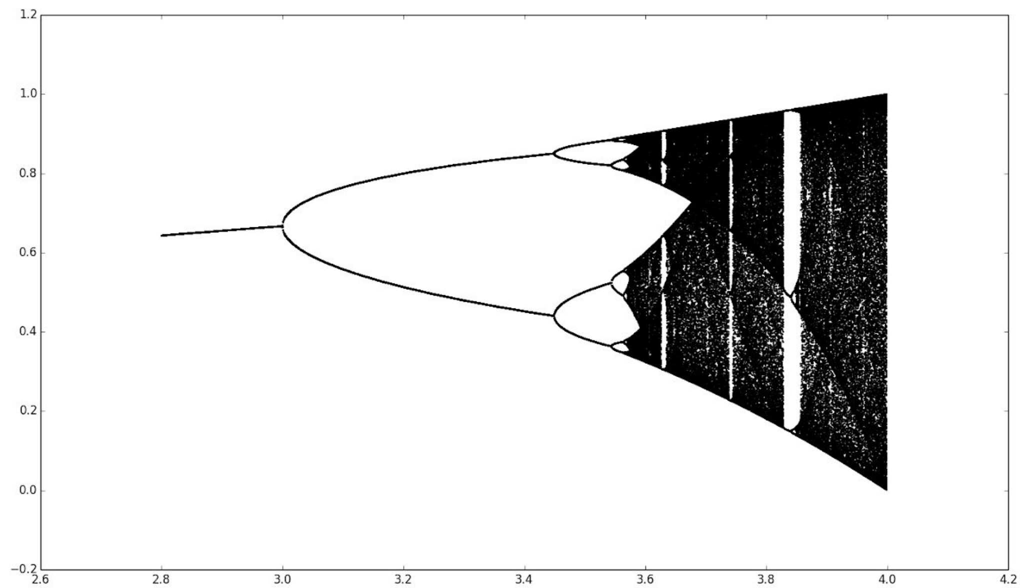
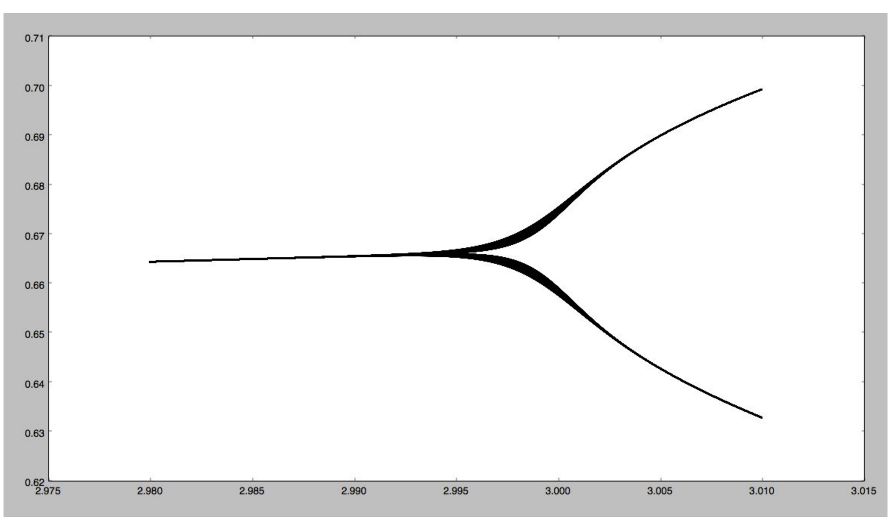
1.

Logistic Bifurcation Plot (xn vs R[2.8:4:.001])



2.

By running consecutive processes of smaller and smaller R ranges (example shown below), I compiled the table (also below) of the 3 Feigenbaum numbers I can compute with the first 4 bifurcation values.



Example of “zooming in” on the first bifurcation point.

Bifurc Number	R Value	Feigenbaum Ratio
0	0	-
1	2.9993	6.6621
2	3.4495	4.7640
3	3.5440	4.6097
4	3.5645	-

I'm unsure if we can count the R=0 point as a bifurcation point, and using it gives a Feigenbaum value of 6.66, which is well off of the known value of 4.66.

The next ratio draws closer, while the third overshoots. This is most likely due to the computation limitations on my machine and program, as well as the initial value I started with (x0 = 0.1; slightly different values are obtained with 0.2, for example). For these purposes, I am satisfied with the proximity of these values, but recognize that these values should converge to 4.66… as proven by Feigenbaum.

3.

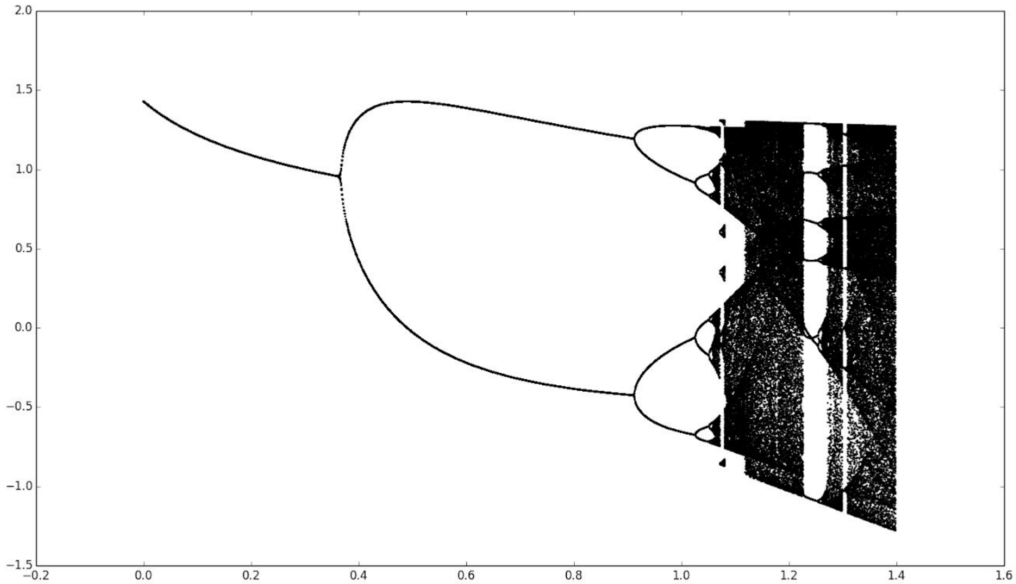
The first step in this process was to recognize that we can write:

$$x_{k+1} = bx_{k-1} + 1 - ax_k^2 \qquad (1)$$

This tells us that we need two seed values, or initial conditions, because we have no way of relating x_{k-1} to x_k by themselves.

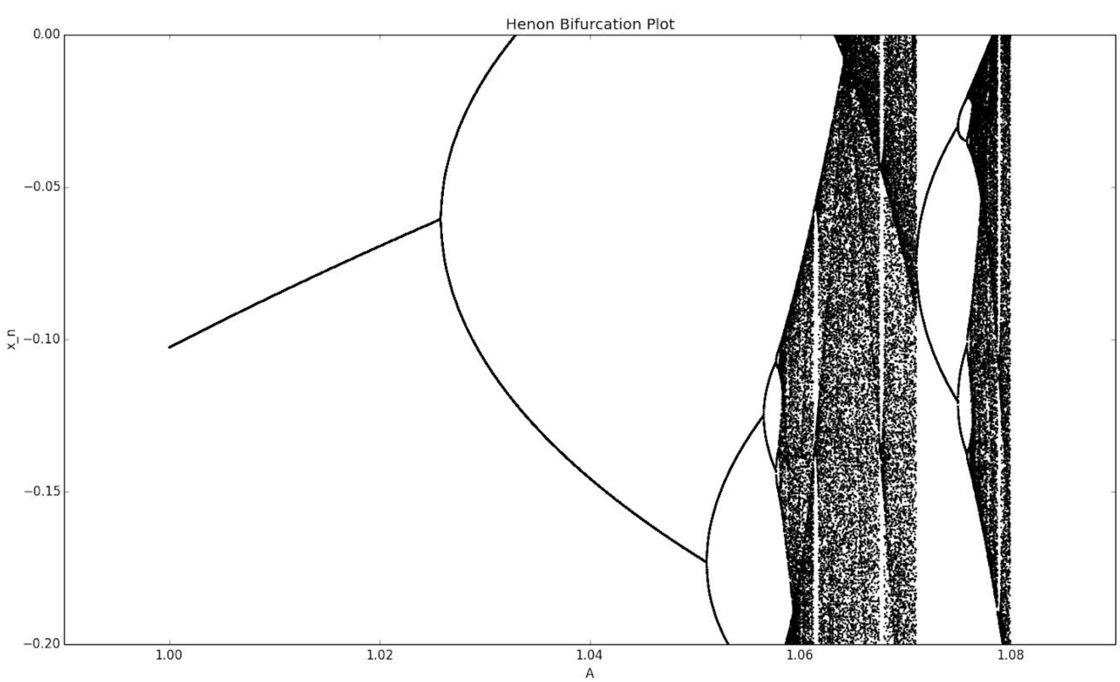
Plotting this using seed values of 0.1, 0.1, b = 0.3, and varying a over [0,1.4]:

Hénon Bifurcation Plot (xn vs a[0:1.4:.001])



Note: I am interested by the “floating” islands found in this plot. Examples I found online were conflicting; some had the islands while some did not. One video showed the islands emerge after b is brought higher than 0.34. If the grader has any insights to both the islands as well as the discrepancies I found, please share in the feedback!

Zooming in on the bifurcations, and producing the table:



Example of zooming in. Notice this plot starts at the 3rd bifurcation shown above. Love the detail!

Bifurc	**A Value**	**Feigenbaum Ratio**
1	0.367	-
2	0.91005	4.6916
3	1.0258	3.7802
4	1.05642	-

4.

Feigenbaum proved the 4.66 ratio for all 1-D maps with quadratic maxima, i.e:

$$x_{n+1} = f(x_n) \sim -x_n^2 \tag{2}$$

The Hénon map clearly has a quadratic maxima. But, is a function of BOTH the current state and the previous state. I.e.:

$$x_{n+1} = f(x_n,\ x_{n-1}) \tag{3}$$

Because of this, I'd say that the Hénon map is 2D, and although the Feigenbaum ratio *might* hold for higher dimensions, the proof is not there to say it holds with any confidence. **No, the numbers need not be the same**. This makes me feel better about the 3.78 I found experimentally, knowing that it need not be 4.66.