CSCI 4446 Spring 2019 PS5
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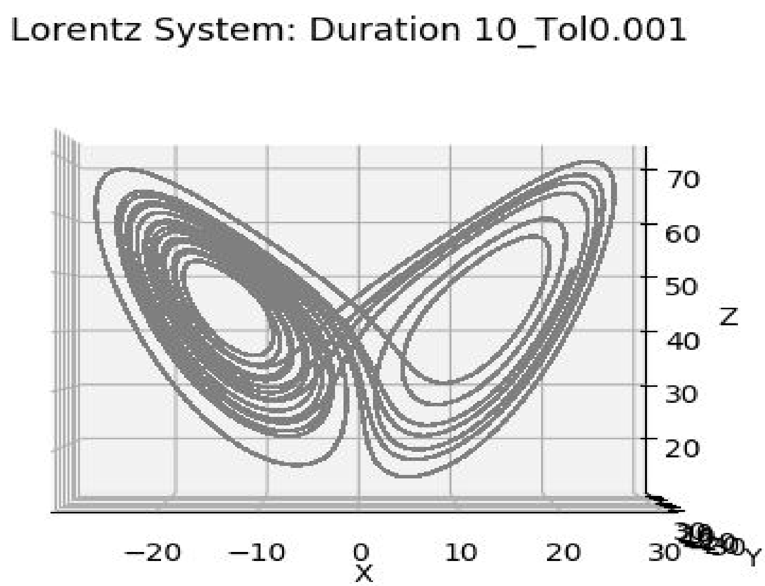
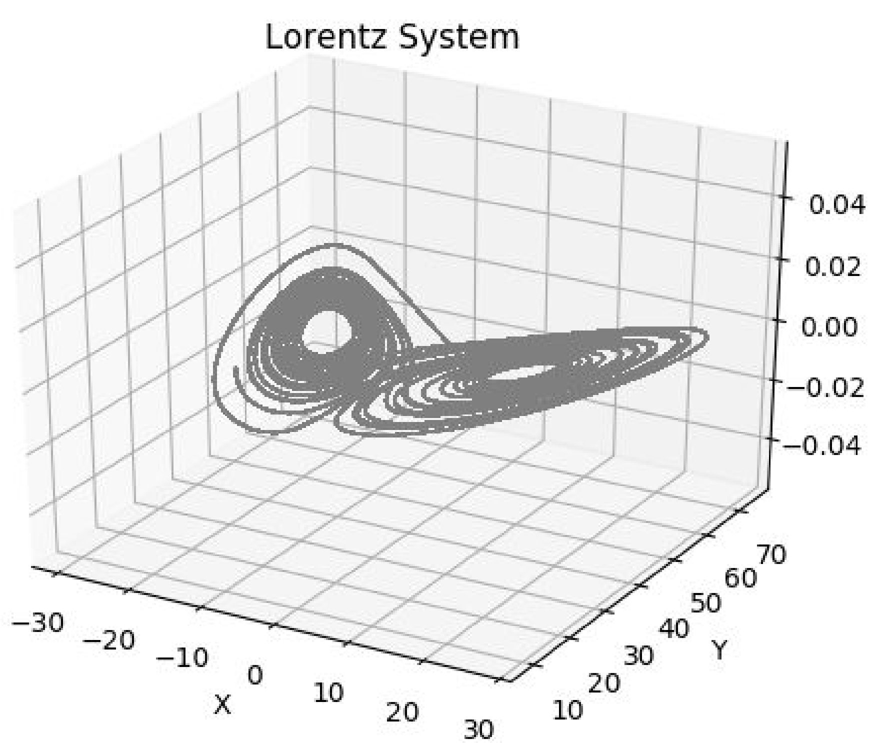
0. -- ; 1. --

2. a.

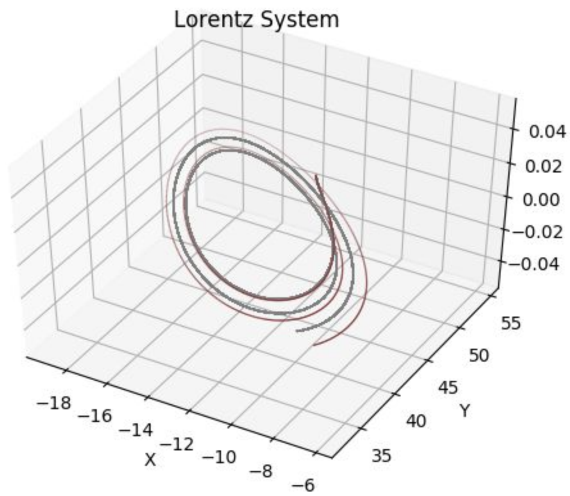


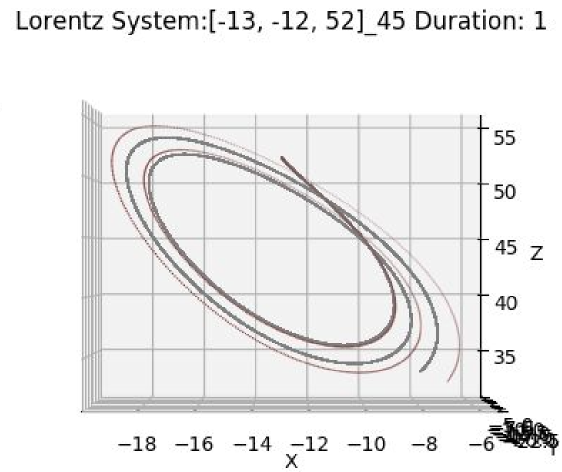
The Lorentz system with I.C. = [-13, -12, 52]. My adaptive RK4 solver used a duration of 10, and a L-infinity tolerance of 0.001.



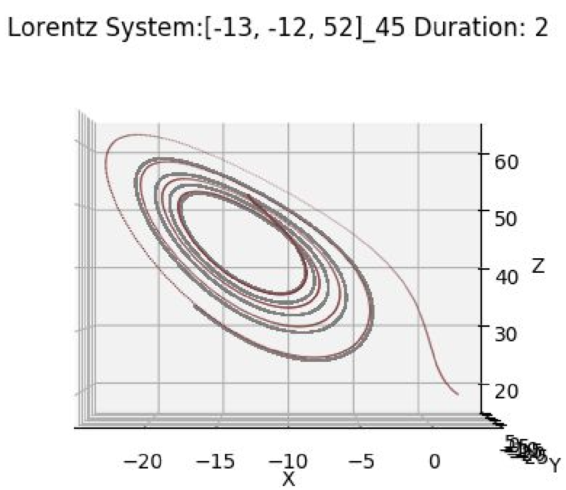
2. b

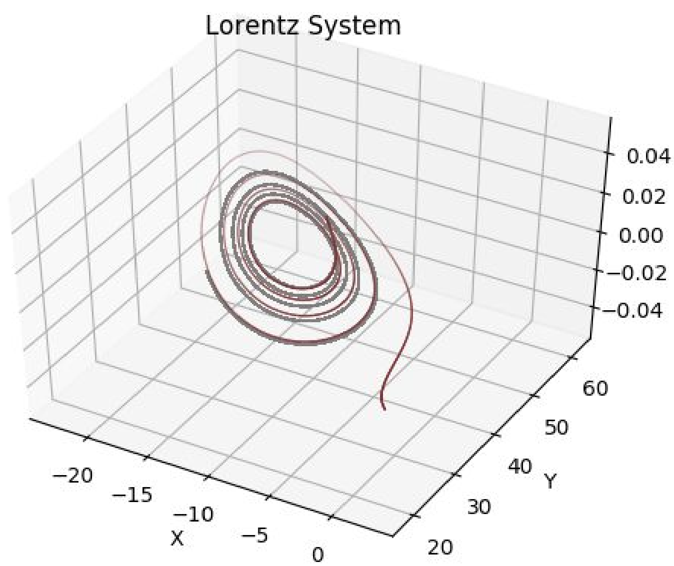
Below are a couple plots of the Lorentz system using the same starting point, but using two different solvers. The gray is the new, adaptive solver while the read trajectory is the non-adaptive solver from last week.



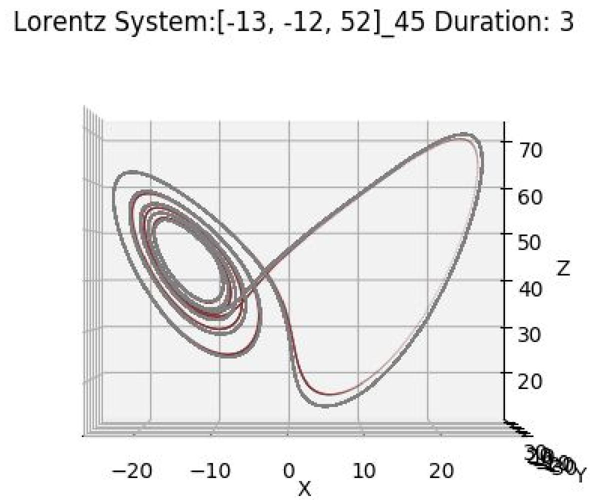


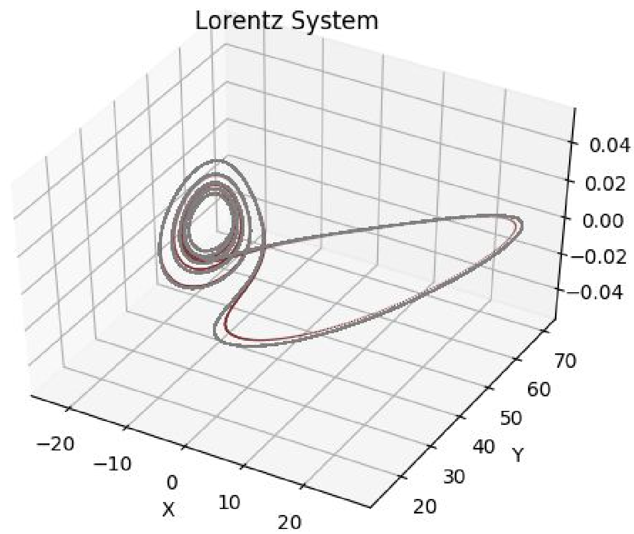
Above is for a 1 second time duration, and you can already see that the solvers are not the same. For the most part, though, they are at a similar point in time -- they have done about the same amount of loops.





Above is the same picture, but progressed 2 seconds. Here it is clear to see that the red (non-adaptive) trajectory has diverged from the adaptive. The adaptive trajectory chooses to go around its fixed point one more time before transitioning to the second lobe of the classic attractor of the system.





One final pair to show that they do agree fairly well in macro-structure, but as we saw at lower levels, there are wide discrepancies if you are measuring them at the same time.

The adaptive solver is not evenly spaced in time. I know this because I start the solver at an initial h of 0.001. Here are the results of the average time step using a L-infinity tolerance of 0.001:

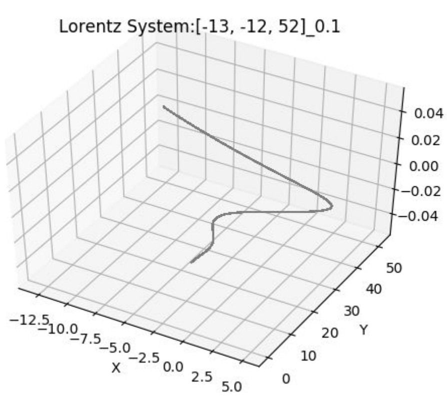
t_mean: 3.98422247898e-06
t_old_mean: 0.001

Clearly if the adaptive solver were evenly spaced in time, its mean would look the same as the old_mean.

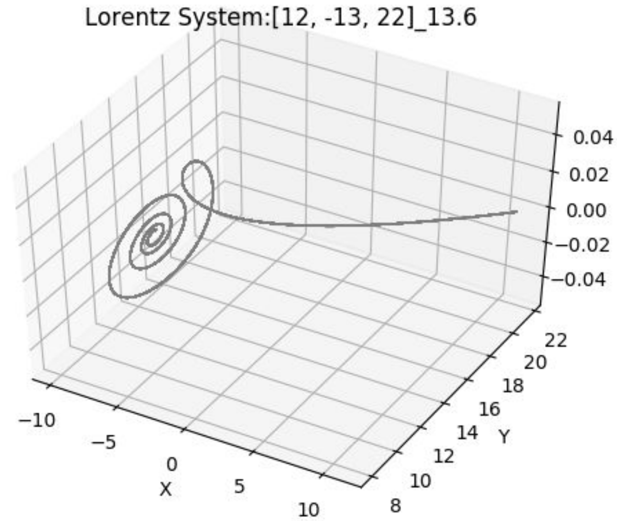
2. c

Exploring the effects of the parameter *r* as we hold *a* and *b* constant produced the following observations:

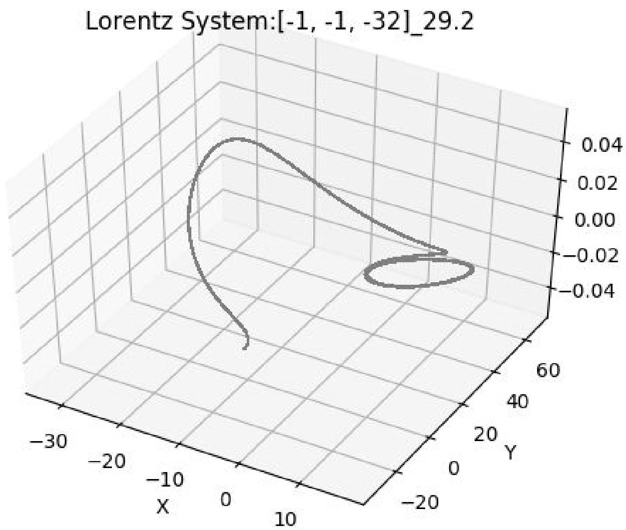
- For small r (between 0 and 1), the system converged to a stable fixed point in a spiralling fashion, no matter the initial condition.

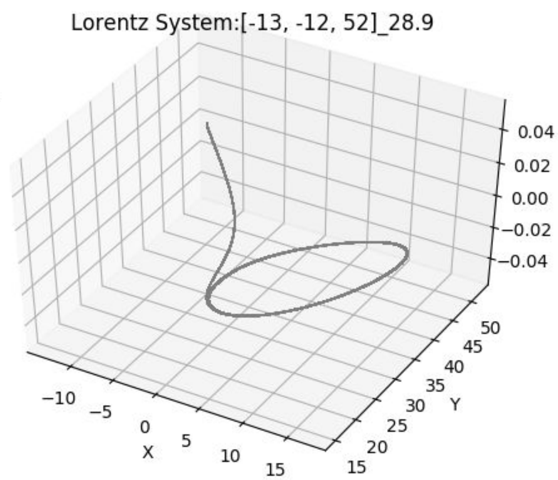


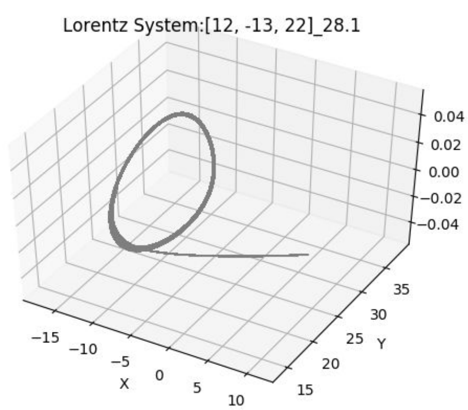
- For r between 13 and 14, we could see the first forms of one lobe of the attractor:



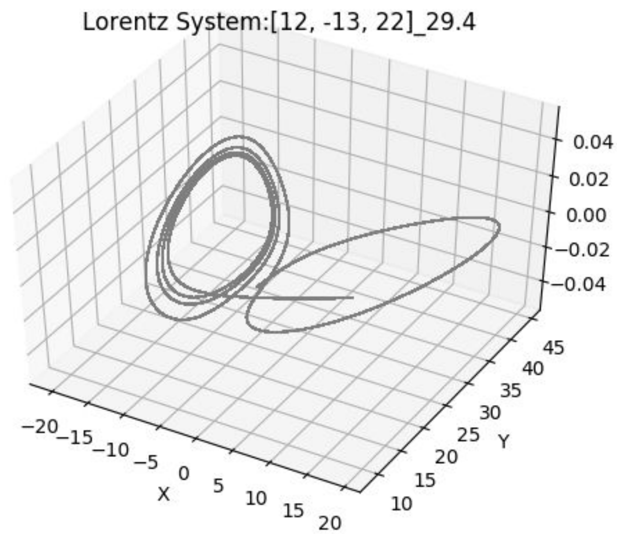
- For r between 23 and 30, some interesting things can be seen. The first I'll show is this unstable periodic orbit:

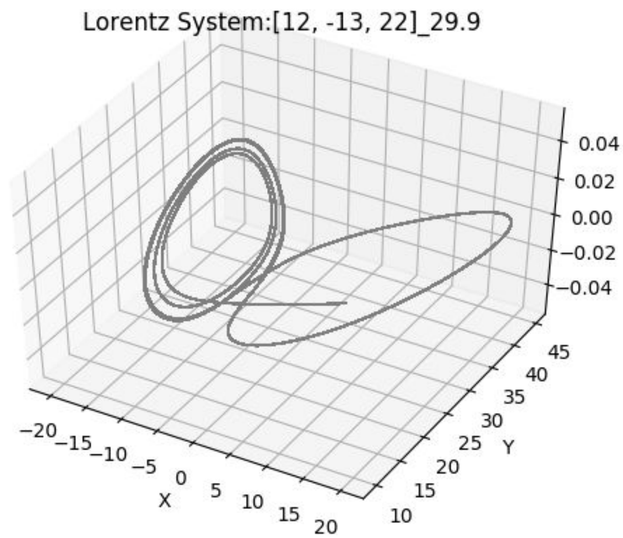




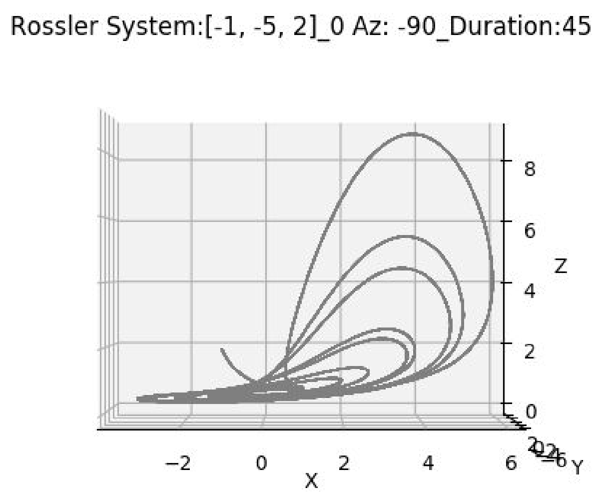


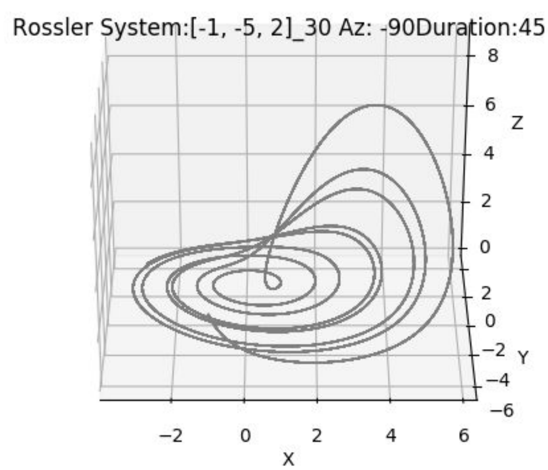
- Next we see a bifurcation onto the second lobe once we get above ~29.4:

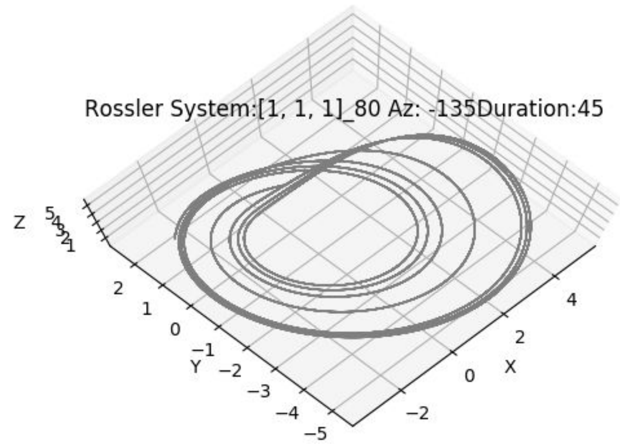


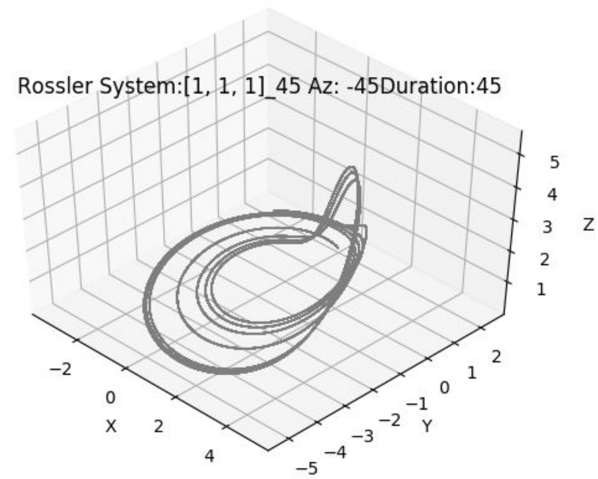


3. Below are some Rossler system images with varying projection planes as well as initial conditions.



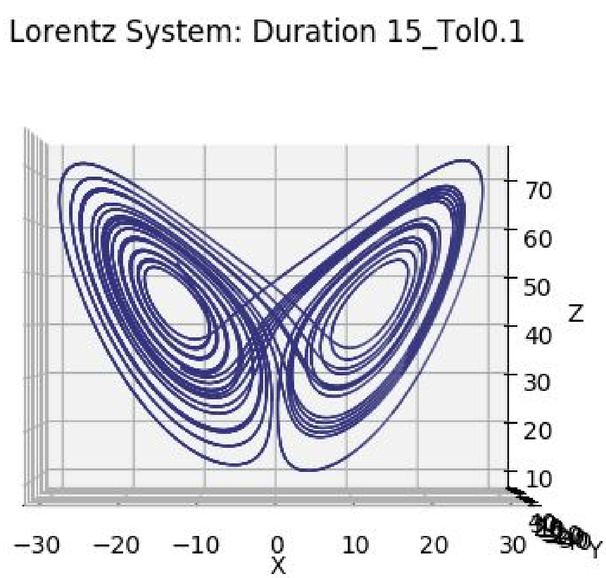


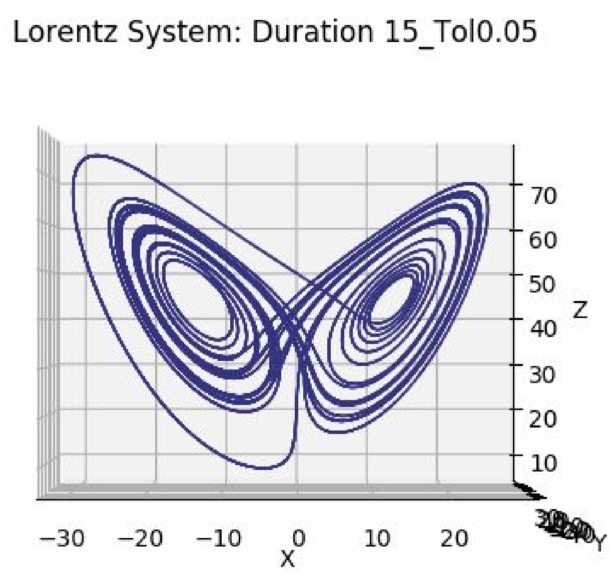




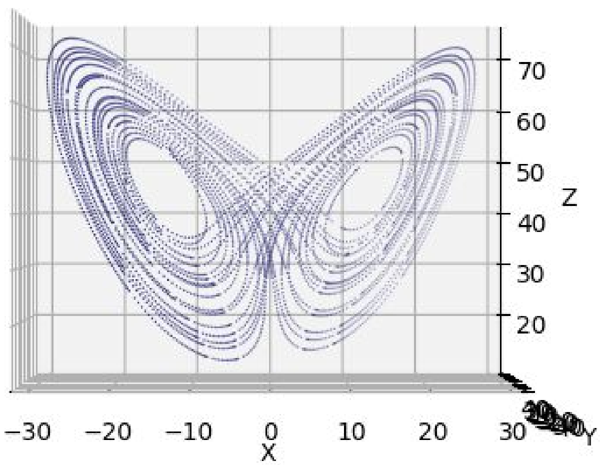
4.

Next, we varied the tolerance in order to test the effects on the trajectory. I found the overall structure to be surprisingly resilient to changes in tolerance. The graphs below get more spacious, since my h step increases according to the tolerance, but the general shape of the attractor is still discernible for large values of tolerance.

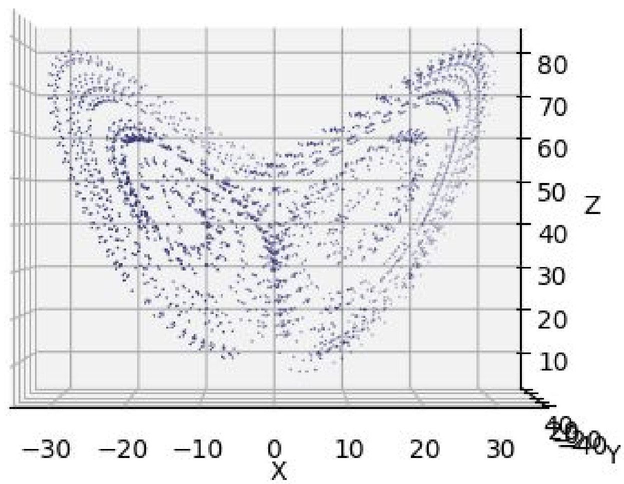




Lorentz System: Duration 15_Tol0.9

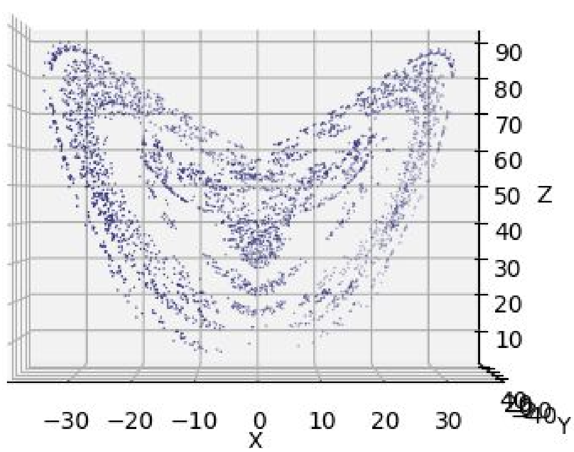


Lorentz System: Duration 45_Tol4



At a Tolerance value of 9, things start to get weird. Below is clearly a Cheshire Cat with its evil smile. After this point, the trajectory begins to diverge from the attractor never to be seen again.

Lorentz System: Duration 100_Tol9



Lorentz System: Duration 100_Tol11