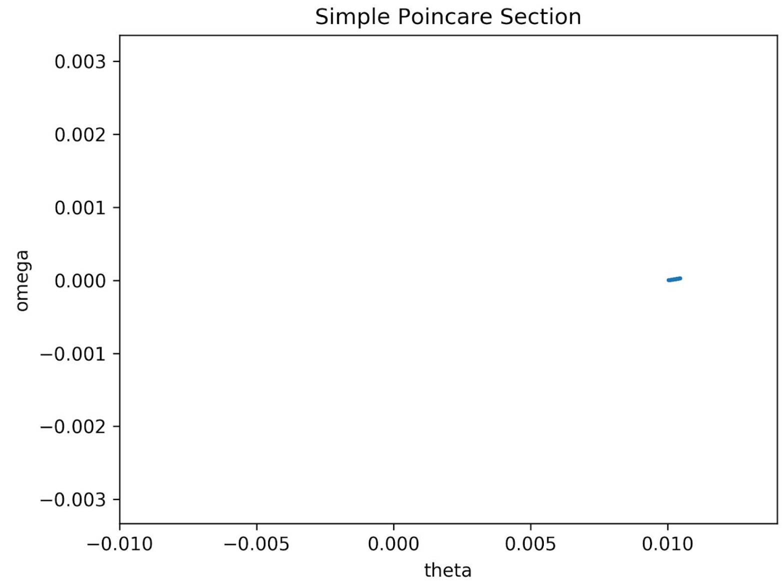
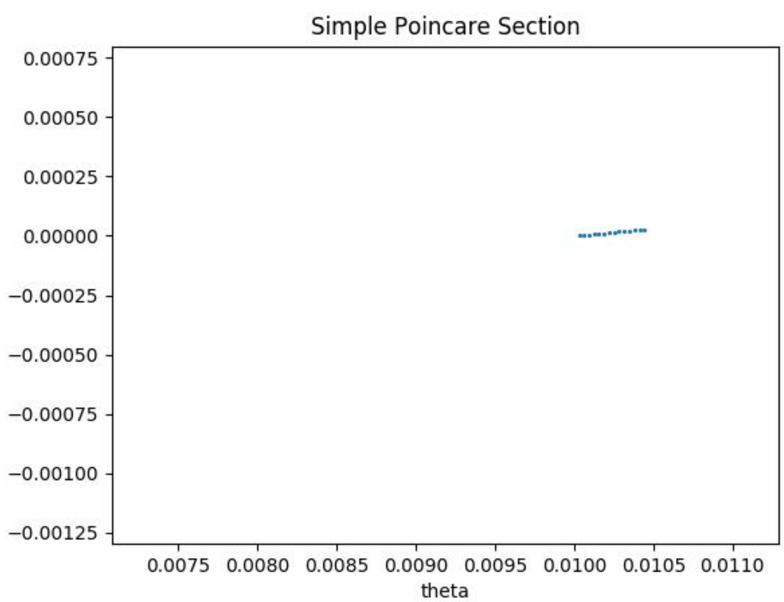
Derek Wright

1. a

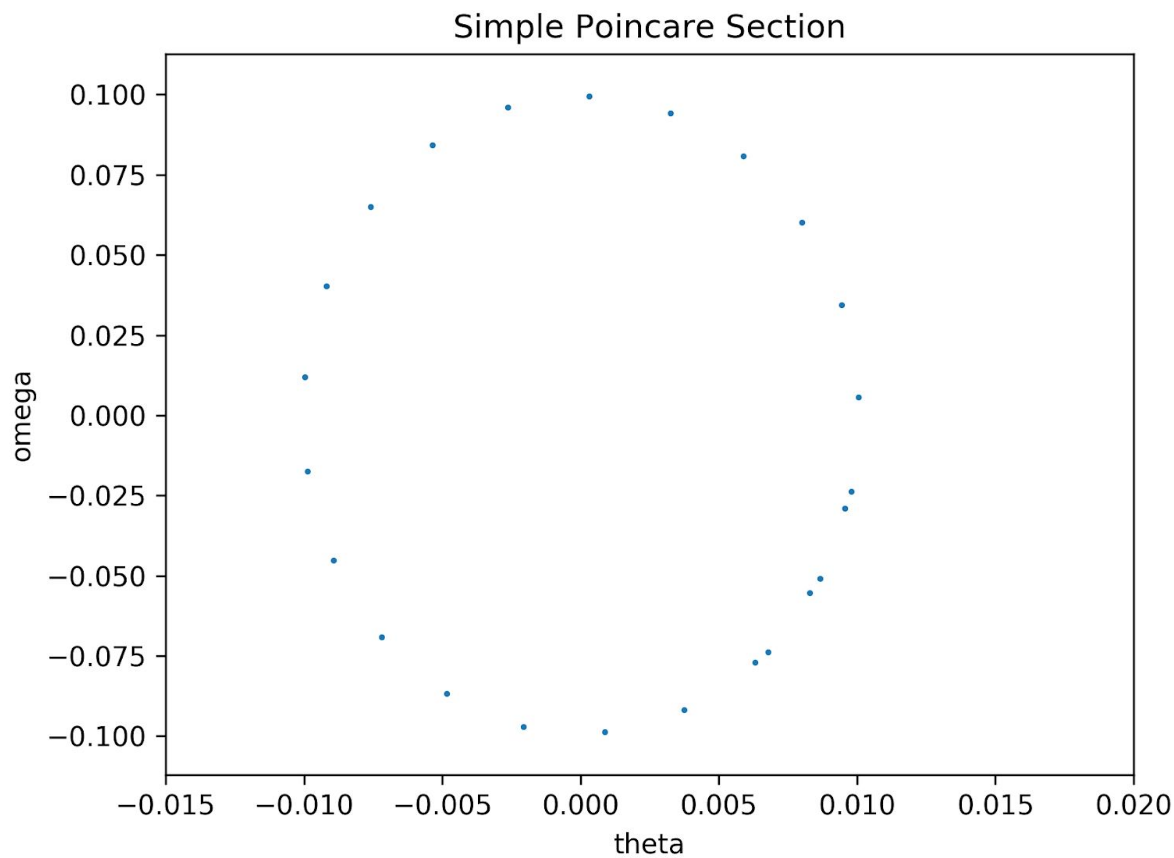


Above is the temporal cross section of the initial condition [0.01, 0], with $\Sigma : t = nT$ where *T* is the natural frequency of the pendulum. As you can see, all the dots are clustered around the starting point. A perfect solver would show a single dot at the same point, [0.01, 0]. This is because the natural frequency (in this case: ~π / 5) is the rate the pendulum passes the same state. We have some error as we look closer (notice the theta scale):



1b.

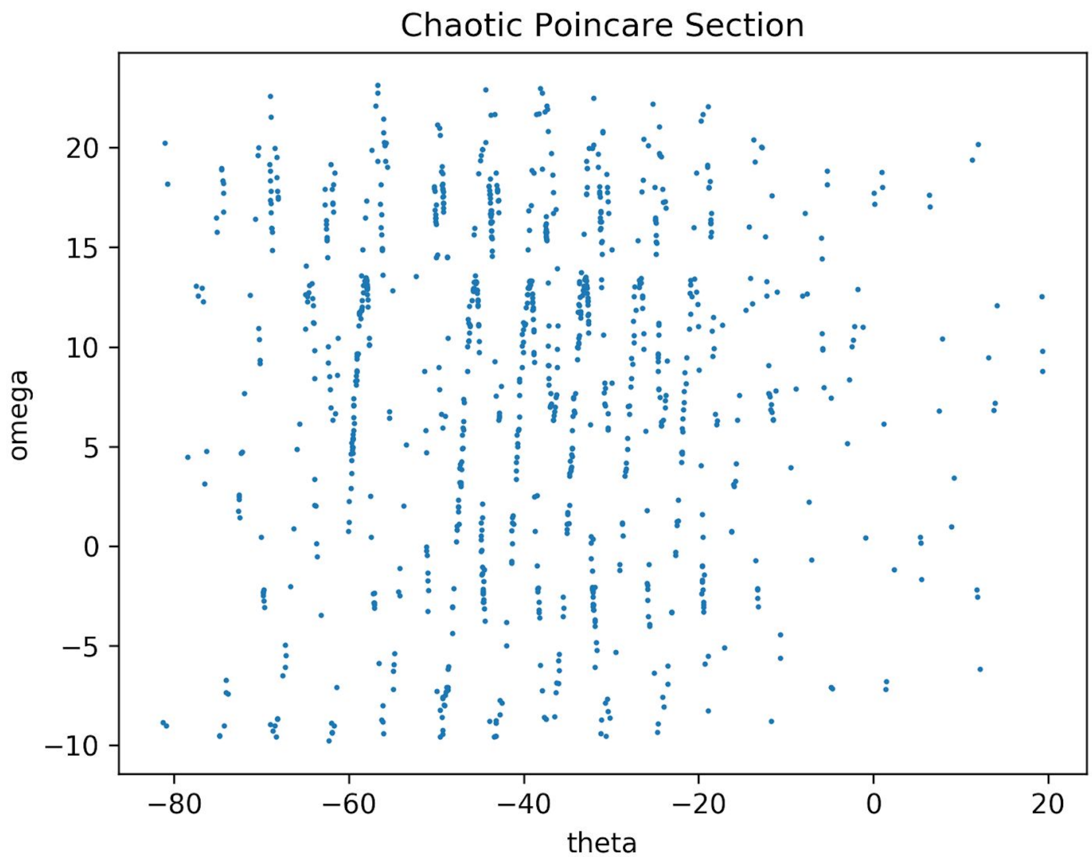
For this part, I multiplied the temporal plane by $\pi/3$. This is because this ratio is close to one, and is also irrationally related to the natural frequency, meaning I shouldn't get the same crosses in our section. I should see points spread out on the ellipse that would be this initial state's continuous trajectory. If I ran this for an arbitrarily long duration, I would get the full elliptical orbit of the fixed point at [0, 0].



Indeed, we see the behavior we expected. The first wrap-around can be seen, and the dots are offset from the first pass.

1c.

Below is the Poincaré section for A = 0.92, alpha = .75 *sqrt(98). These values were found in hw5.



This is mostly what I would expect: A large variance of points and no two that exactly cross at the same point. However, there is some clear structure to this temporal cross section. This is explored in part d.

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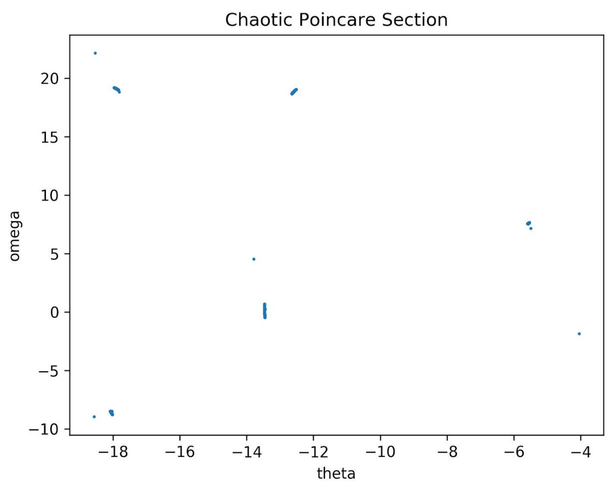
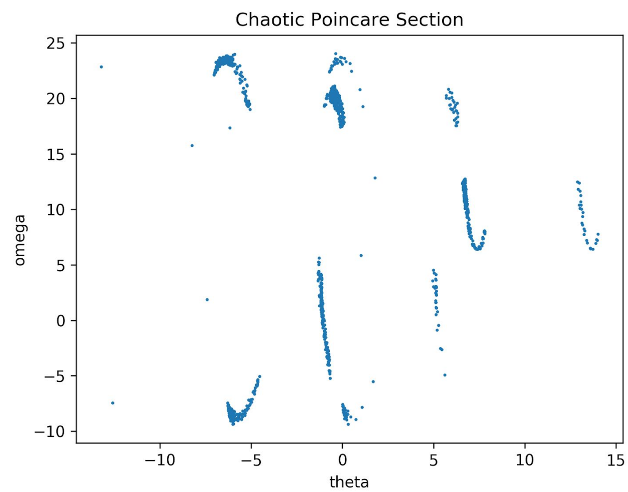
1d.

The behavior of the temporal Poincaré section as I adjusted the step size of my RK4 algorithm acted in a somewhat similar manner to varying the parameters of the pendulum system. That is, I would see chaotic sections bifurcate into quasi-periodic ones, and vice versa. Two sequences of just this are shown below.

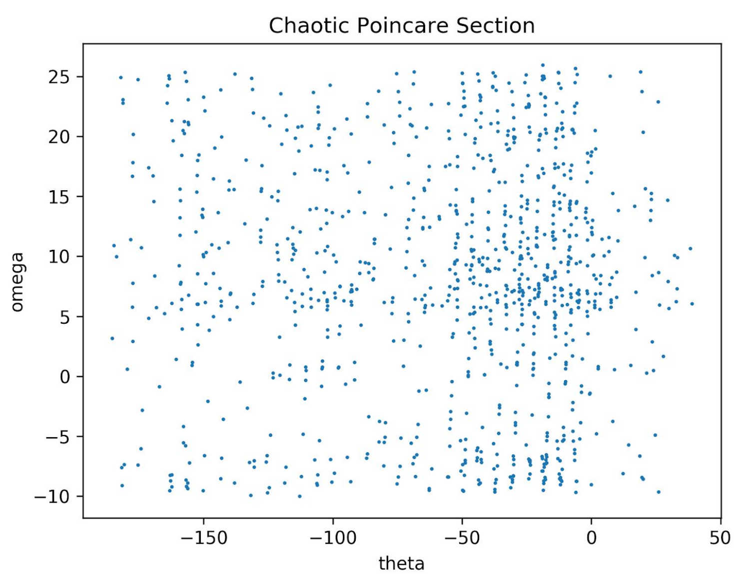


Tstep = 0.007



Tstep = 0.013

Tstep = 0.025



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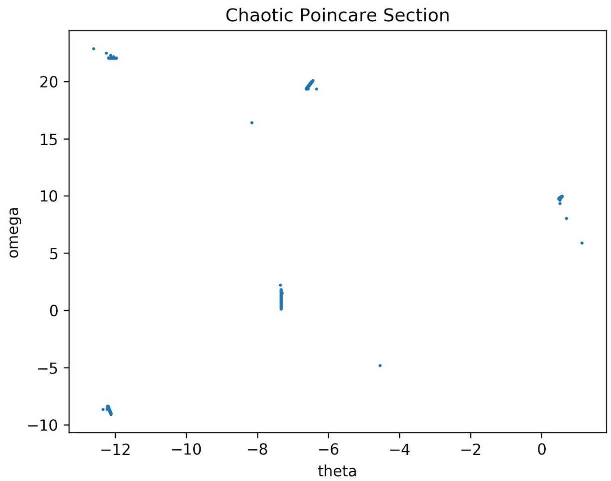
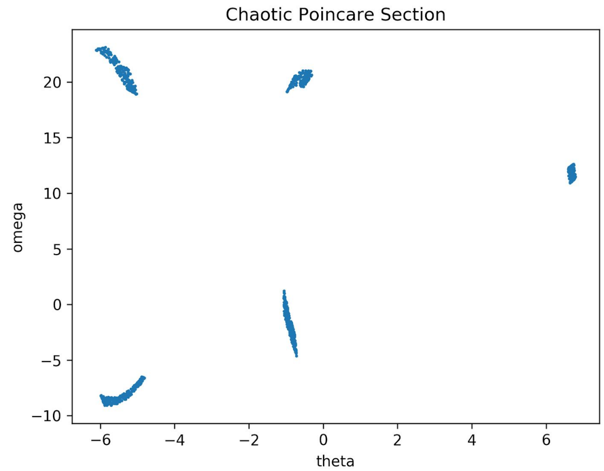
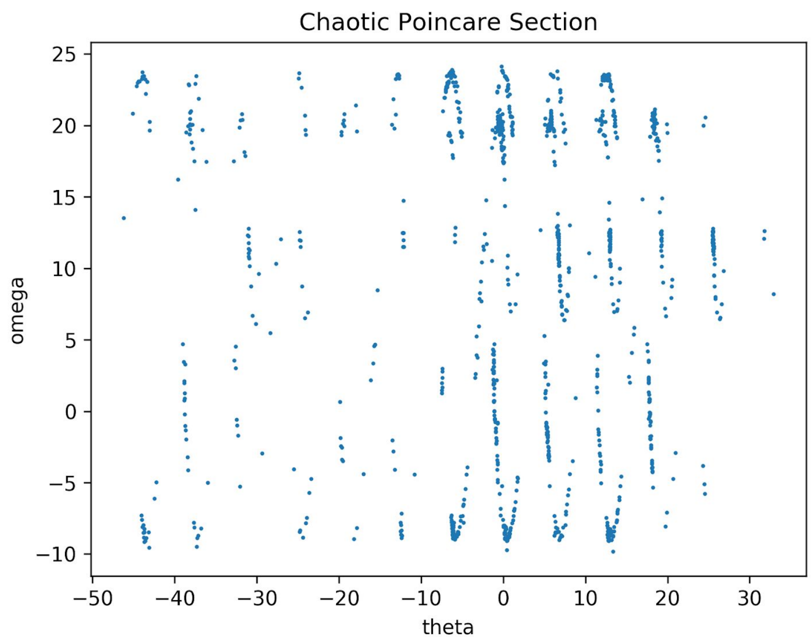


Tstep = 0.096



Tstep = 0.0125

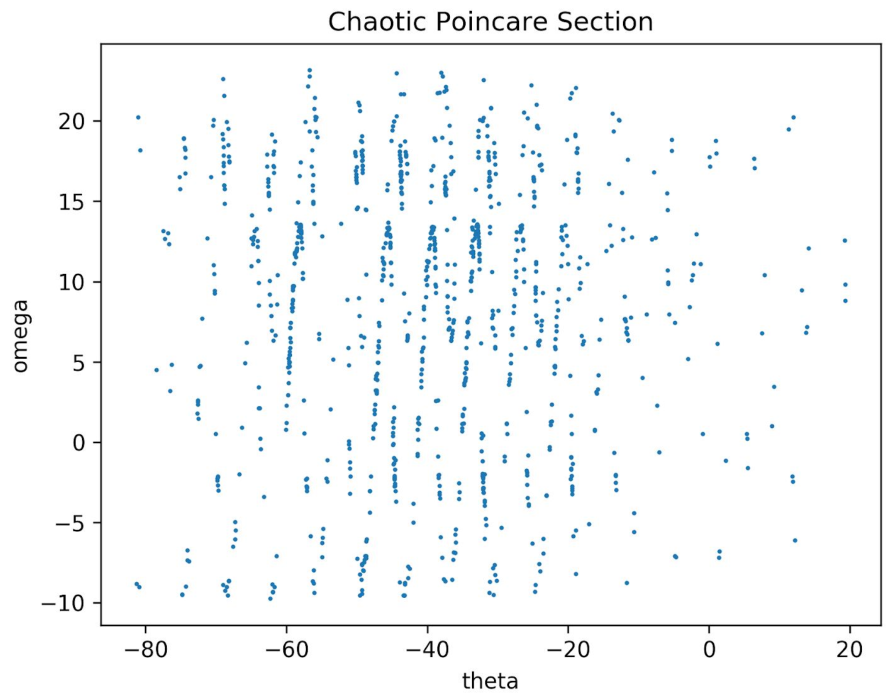


Tstep = 0.0134

This was an interesting exploratory. We can now see the structure of the sections as we vary the time step and it is quite interesting. The sections seem to branch off into a sinusoidal shape, before exhibiting a chaotic section once the sinusoids become too densely packed.
Of note here is that these effects are produced only by the error in the RK4 algorithm with different time steps. They do not reflect a real parameter we could tweak to get these results.

2a.

For this part, I now interpolate a point halfway between the two points on the trajectory that straddle my temporal section. This is in contrast to the method before, where I simply took the point that was "behind" the plane of section (t < nT).
Tstep = 0.001



This section is nearly identical to the simple one from **(1c).** The difference was to the hundredth or thousands of a radian or radian/sec. This reflects the smoothness of this system, even under chaotic conditions. There simply is not enough time in between these steps to see a large deviation.
I might also say that this result reflects the fact that both of the methods used are not very good estimators of the cross section. A better way would be to take the exact time step to ensure we land right on the plane of section, as described in the problem set and in class.

Repeating **(1d)**, I saw similar results. Sinusoid structure amid bifurcation of quasi-periodic orbits into densely packed, chaotic sections. On such sequence is shown below.

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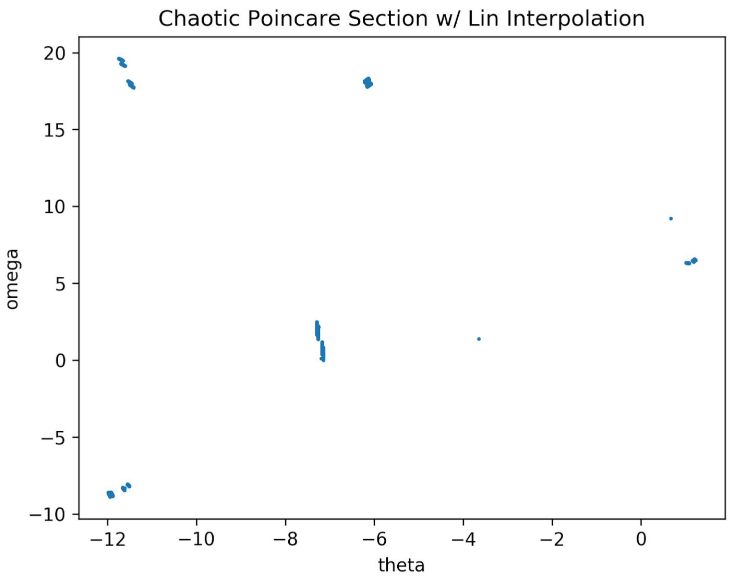
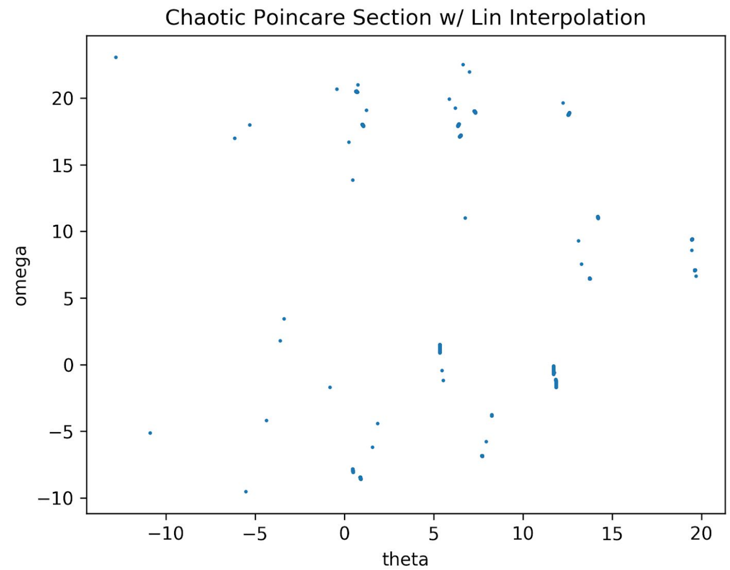
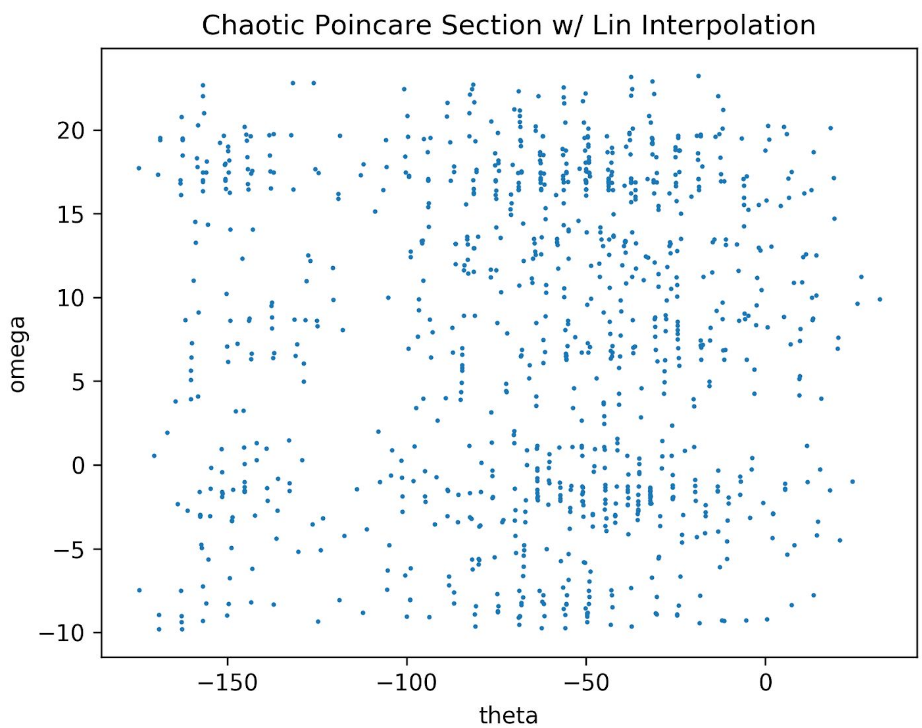
Tstep = 0.005



T step = 0.00375



Tstep = 0.0025



BONUS

This is the irrational temporal plane iterated for a lot more points than in **(1a)**. It's not perfect, of course, but shows a nice outward spiral shape, as well as exhibiting the whole elliptical trajectory we would expect: