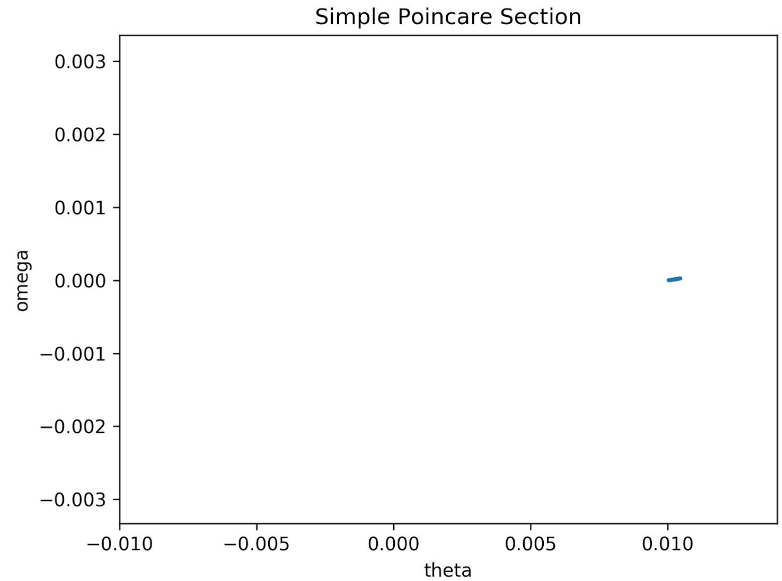
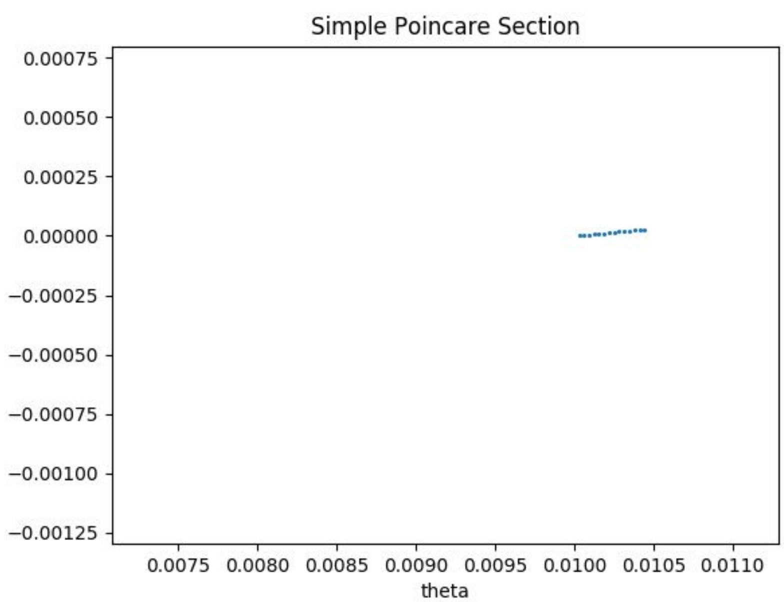
CSCI 4446 Spring 2019 PS6
Derek Wright
Resubmission

1. a



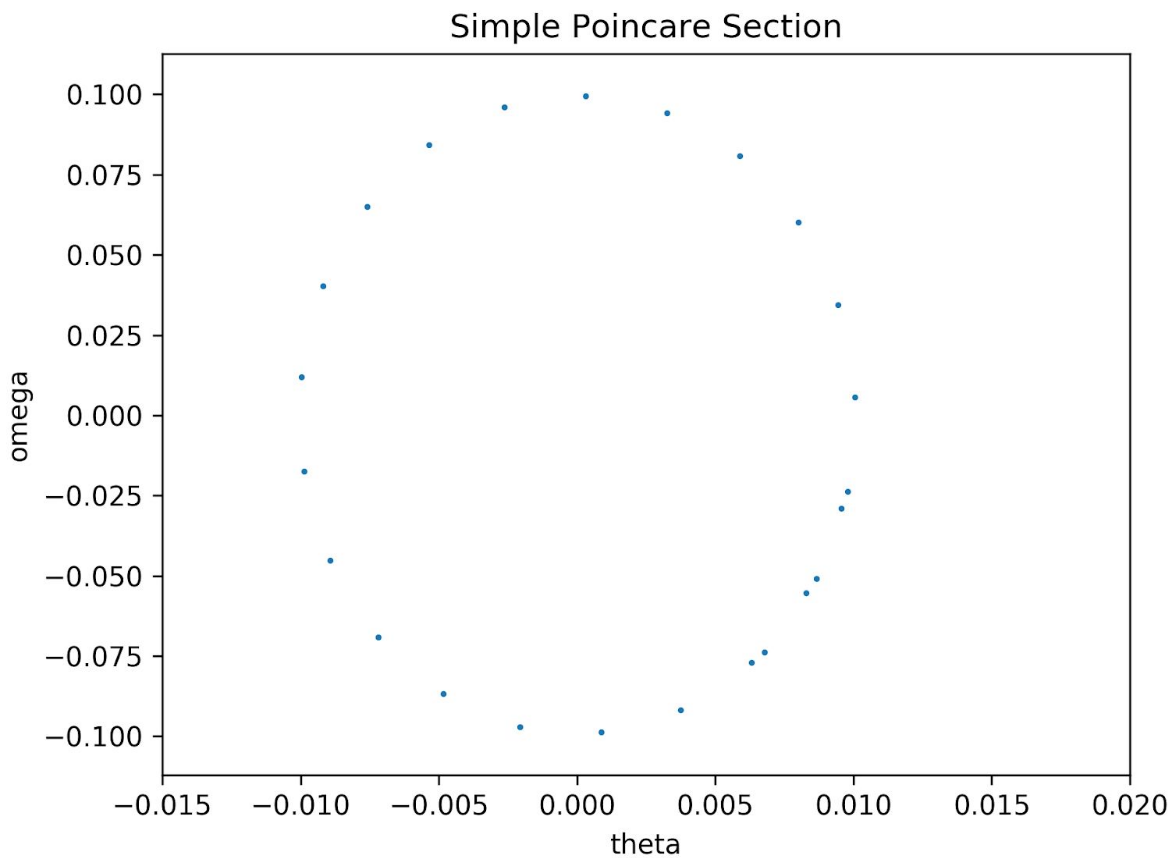
Above is the temporal cross section of the initial condition [0.01, 0], with $\Sigma : t = nT$ where *T* is the natural frequency of the pendulum. As you can see, all the dots are clustered around the starting point. A perfect solver would show a single dot at the same point, [0.01, 0]. This is because the natural frequency (in this case: ~π / 5) is the rate the pendulum passes the same state. We have some error as we look closer (notice the theta scale):



CSCI 4446 Spring 2019 PS6
Derek Wright
Resubmission

1b.

For this part, I multiplied the temporal plane by $\pi/3$. This is because this ratio is close to one, and is also irrationally related to the natural frequency, meaning I shouldn't get the same crosses in our section. I should see points spread out on the ellipse that would be this initial state's continuous trajectory. If I ran this for an arbitrarily long duration, I would get the full elliptical orbit of the fixed point at [0, 0].

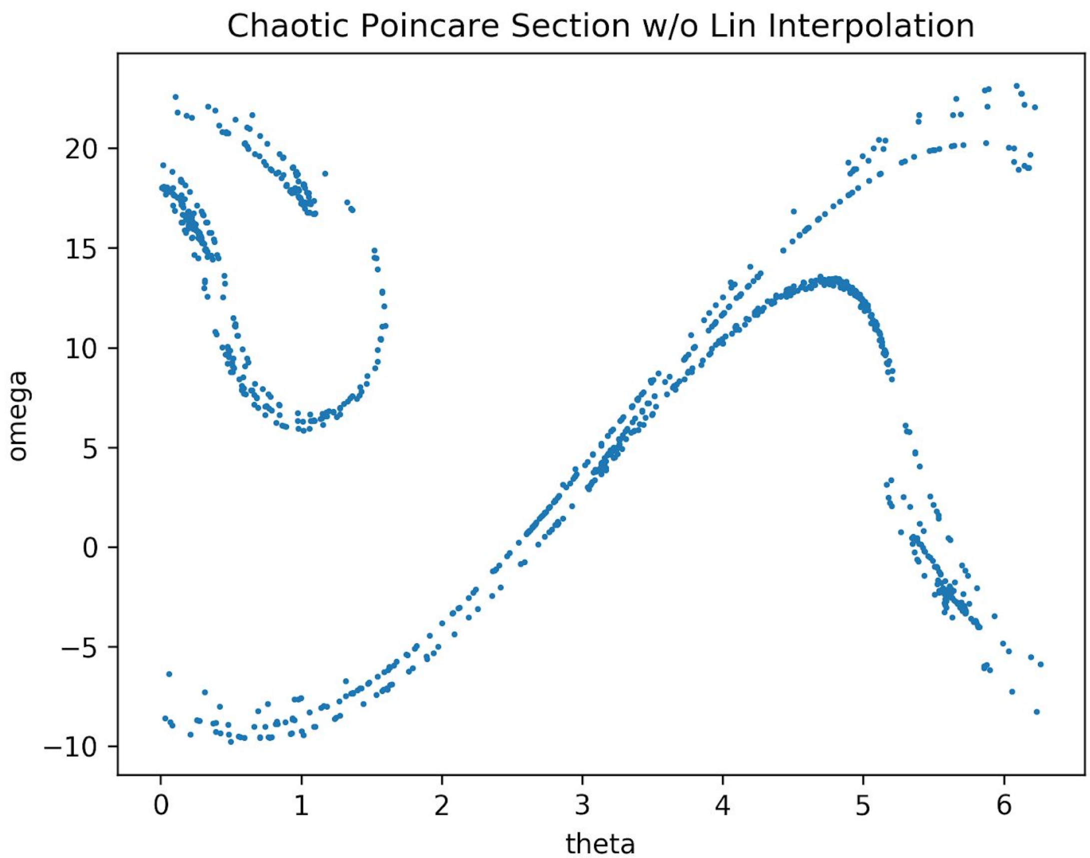


Indeed, we see the behavior we expected. The first wrap-around can be seen, and the dots are offset from the first pass.

CSCI 4446 Spring 2019 PS6
Derek Wright
Resubmission

1c.

Below is the Poincaré section for A = 0.92, alpha = .75 *sqrt(98). These values were found in hw5 to be chaotic. The section is different from my first submission in that I remembered to mod-2π each crossing.



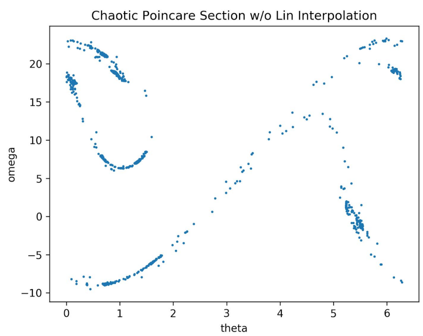
This is mostly what I would expect: A large variance of points and no two that exactly cross at the same point. However, there is some clear structure to this temporal cross section. This is explored in part d.

CSCI 4446 Spring 2019 PS6
Derek Wright
Resubmission

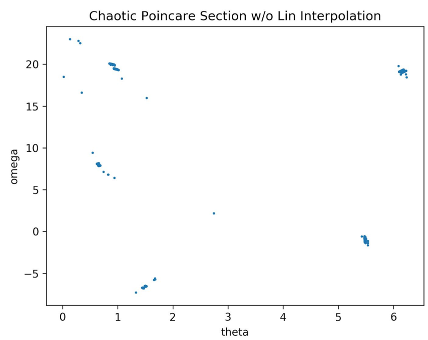
1d.

The behavior of the temporal Poincaré section as I adjusted the step size of my RK4 algorithm acted in a somewhat similar manner to varying the parameters of the pendulum system. That is, I would see chaotic sections "bifurcate" into differing structures. What this does **not** mean is that the changing of the time step affects the dynamics of the system. I am merely attempting to express how the variations of the graphs appear to the beholder -- there is definite structure as well as definite patterns that I see when changing the time step, just as there are when changing any other parameter of the system. I believe in my original submission I was using terminology too similar to that used when describing dynamics of a system, and that is where the confusion arose.

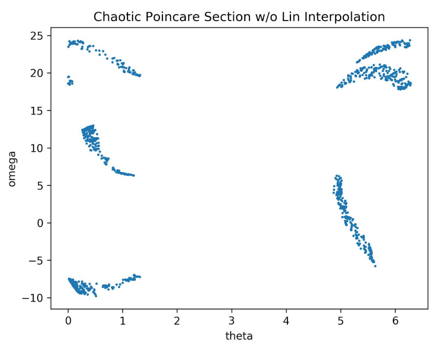
Now that I have heard of the Los Alamos spring/mass chain experiments (Fermi, et. al.), I would now note that I am seeing something similar to the "collection and re-dispersion of energy" that that team observed. This is displayed in the following sequence.



tstep = 0.00423



tstep = 0.0045



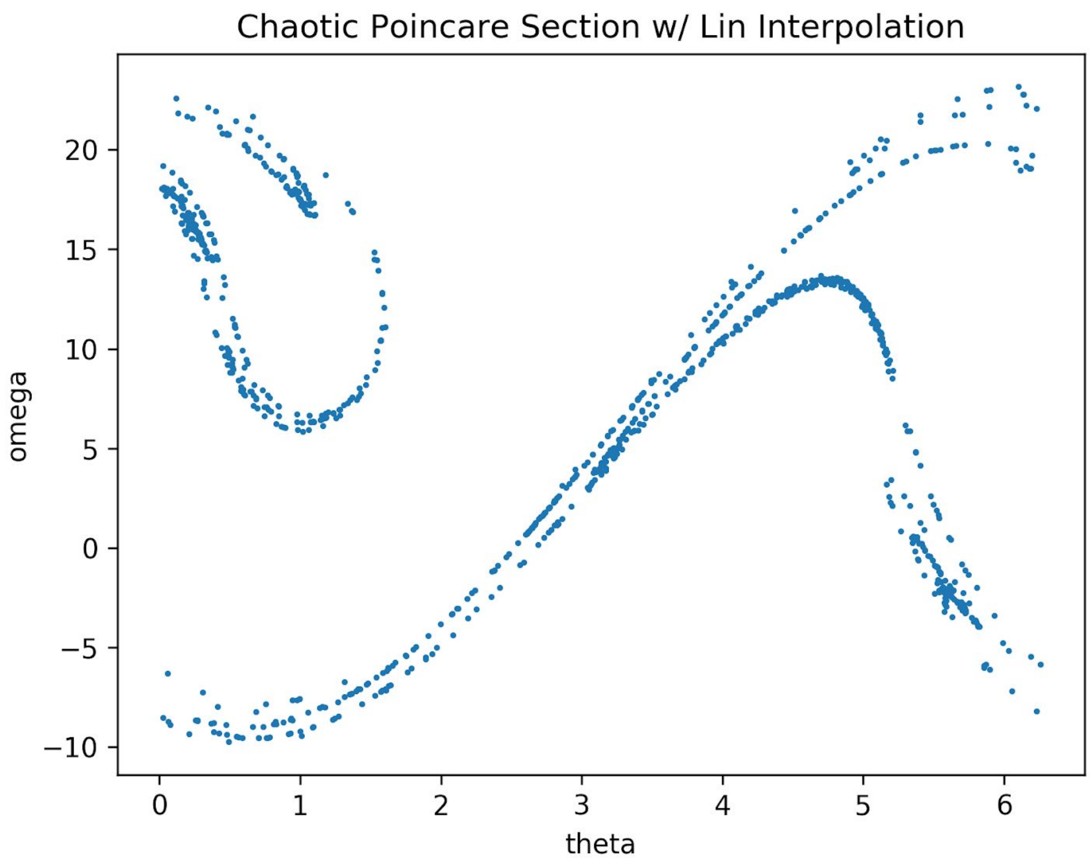
tstep = 0.017

CSCI 4446 Spring 2019 PS6
Derek Wright
Resubmission

The reason for the changes in the sections, in this case, are a result of being either closer or farther from the temporal plane of section, based on the time-step size. So no dynamics are changing, and I am not tweaking any system parameters. Ideally the section would be the collection of points right on the temporal plane, but that would be a different algorithm, and would need to generate the proper points, rather than work with constant trajectory data like we are here.

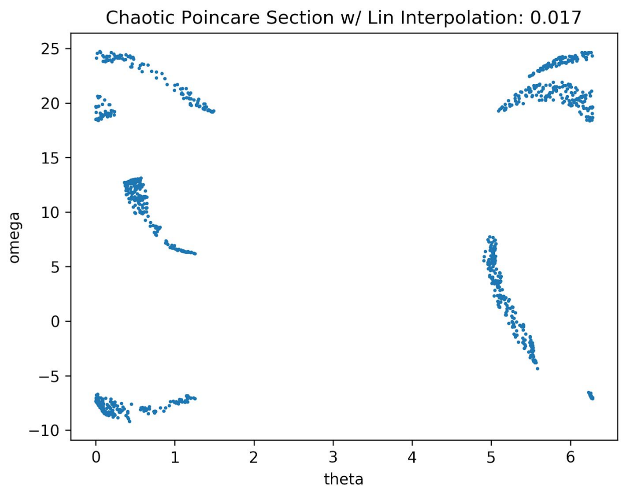
2a.

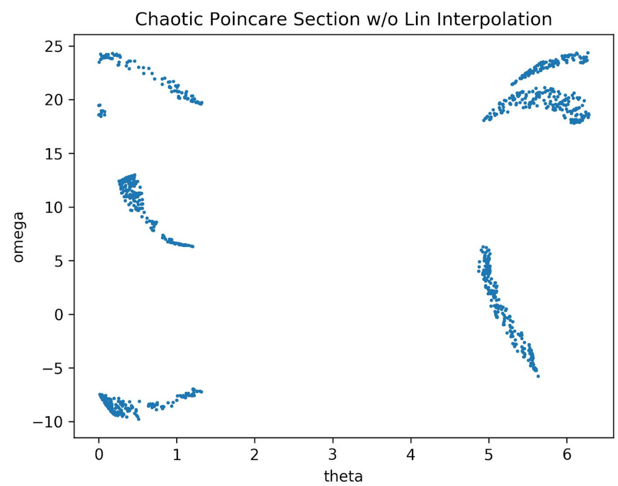
For this part, I now interpolate a point halfway between the two points on the trajectory that straddle my temporal section. This is in contrast to the method before, where I simply took the point that was "behind" the plane of section (t < nT).
Tstep = 0.001



This section is nearly identical to the simple one from **(1c).** The difference was to the hundredth or thousands of a radian or radian/sec. Another pair of plots is shown below for a tstep = 0.017, where you can see more differences. In general, though, the change in theta for every time step was simply too small for us to see a large difference -- i.e. the interpolated value is very close to the point "behind" the plane of section.

CSCI 4446 Spring 2019 PS6
Derek Wright
Resubmission





The differences here are chiefly seen in the upper left “island” and the bottom right, in which a separate island formed.