MATH 128B hw12

Problem 1

(a) See the attached plot.

(b) See the attached plot and note that the error function of h consists two line segments with different slopes. The slope of left line segment is -8.604069087504562e-01, and the slope of the other line segment is -2.575986786815643e+01. Also, the errors for h = 0.001, 0.01, and 0.1 are the following:

error3 =

2.383011191422021e+00,

error2 =

2.375267529243267e+00,

error1 =

5.687942110918837e-02.

(c) Note that, in this case, the result of Lax-Friedrichs Scheme is okay but not as satisfying as the result of Crank-Nicolson Scheme in part (a). From the graph, we can see the Lax-Friedrichs approximate solution differs from the exact solution at the peak of the Gaussian curve. One of the reason for this phenomenon is that we choose h = 0.1 and k = 0.09, and therefore, h is not equal to k. Recall that a = 1 in this problem, and the quantity (a * k / h) is less than one. The C.L.F condition guarantees the stability of the scheme. However, if we can choose h = k, then the C.L.F number is exactly one. Then the step sizes of our scheme match the speed of the wave, which makes the approximation more accurate.

Problem 2

See the plots attached.

(a) In the plot (2a), we can see most of the points (largest eigenvalues on the complex plane) are not inside the unit disk, but some are inside the disk. Therefore, the Lax-Wendroff Scheme is just conditionally stable. In this case, the stability of the Lax-Wendroff Scheme depends on the sigma we choose. Hence, we need to be very careful about the sigma we use.

(b) In the plot (2b), the distribution of the eigenvalues is quite symmetric about the vertical axis. Lots of points are in the unit disk, and the sigma's corresponding to those points lead to stable algorithms. In general, the Lax-Friedrichs Scheme is more stable than Lax-Wendroff.

(c) In the plot (2c), the distribution of the eigenvalues is very special because all the eigenvalues seem to lie on the top right of the circle. The scheme will be stable if the eigenvalues are on the unit circle or inside the unit disk. Hence, we can conclude that Crank-Nicolson is more stable than the previous two schemes and that Crank-Nicolson is stable whatever sigma we pick between zero and two.

Problem 3

(a) The proof part is handwritten. The plot is attached.

(b) The plot is attached. If we compare the approximate solution with h = 0.1 with the exact solution, we can see they basically have the same shape. In the graphs, both solutions are symmetric about the vertical line x = 0.5. However, the exact solution seems more concentrative at the extrema of the waves, while the approximate solution seem to disperse to the endpoints of x; i.e., x = 0 and x = 1. Overall, these two solutions are very close.

(c) The errors for h = 0.1, 0.01, and 0.001 are the following:

error1 = 3.141592653589794e-01

error2 = 3.141592653589661e-02

error3 = 3.141592653096267e-02.

The logarithmic plot is attached. Note that the line has two different slopes as follows:

leftSlope =

5.482157937485175e-10

rightSlope =

3.141592653589808e+00.