1.

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dots \\ \dot{x}_n \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dots$$
$$\dot{x}_{n-1} = x_n$$
$$\dot{x}_n = a(x_1, x_2 \dots x_n, u)$$
$$\dot{x}_n = a_{n-1} q^{(n-1)} \dots a_1 \dot{q} + a_0 q + u$$

2i

$x_1 := q_1 \quad x_2 := \dot{q}_1$

$x_3 := q_2 \quad x_4 := \dot{q}_2$

$$\dot{x}_1 = x_2$$
$$\dot{x}_3 = x_4$$

$$2\dot{x}_2 + \dot{x}_4 + \sin x_1 = 0 \rightarrow \dot{x}_4 = -2\dot{x}_2 - \sin x_1 \quad (1)$$
$$\dot{x}_2 + 2\dot{x}_4 + \sin x_3 = 0 \rightarrow \dot{x}_2 = -2\dot{x}_4 - \sin x_3 \quad (2)$$

(2) → (1) $\dot{x}_4 = -2(-2\dot{x}_4 - \sin x_3) - \sin x_1$

$\dot{x}_4 = 4\dot{x}_4 + 2\sin x_3 - \sin x_1$

$-3\dot{x}_4 = 2\sin x_3 - \sin x_1$

$$\dot{x}_4 = -\frac{2}{3}\sin x_3 + \frac{1}{3}\sin x_1$$

(1) → (2) $\dot{x}_2 = -2(-2\dot{x}_2 - \sin x_1) - \sin x_3$

$\dot{x}_2 = 4\dot{x}_2 + 2\sin x_1 - \sin x_3$

$-3\dot{x}_2 = 2\sin x_1 - \sin x_3$

$$\dot{x}_2 = -\frac{2}{3}\sin x_1 + \frac{1}{3}\sin x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{2}{3}\sin x_1 + \frac{1}{3}\sin x_3 \\ 0 \\ -\frac{2}{3}\sin x_3 + \frac{1}{3}\sin x_1 \end{bmatrix}$$

2ii

(1) $\ddot{q}_1 + \dot{q}_2 + q_1^3 = 0$

(2) $\dot{q}_1 + \dot{q}_2 + q_2^5 = 0$

$x_1 = q_1 \quad x_2 = \dot{q}_1$

$x_3 = q_2$

$$\dot{x}_1 = x_2$$

(2) $x_2 + \dot{x}_3 + x_3^3 = 0$

$$\dot{x}_3 = -x_2 - x_3^3$$

(1) $\dot{x}_2 + \dot{x}_3 + x_1^3 = 0$

$\dot{x}_2 = -\dot{x}_3 - x_1^3$

$$\dot{x}_2 = x_2 + x_3^3 - x_1^3$$

3.

$x_1 := q_1 \quad x_2 := \dot{q}_1$

$x_3 := q_2 \quad x_4 := \dot{q}_2$

$$\dot{x}_1 = x_2 \qquad \dot{x}_3 = x_4$$
$$\dot{x}_2 = -x_4 - \sin x_1 + u$$
$$x_4 = -x_1 - x_3$$
$$y = x_1 + x_3$$

4.

$x_1(k) = q(k)$

$x_2(k) = q(k+1)$

$x_3(k) = q(k+2)$

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = x_3(k)$$
$$x_3(k+1) = -7x_3(k) - x_2(k) - 6x_1(k) + 7u(k)$$

5.

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = x_3(k)$$
$$\cdots\cdots$$
$$x_n(k+1) = a_{n-1}q(k+n-1) \cdots a_1 q(k+1) + a_0 q(k)$$

6. $x_1(k) = q_1(k) \rightarrow x_1(k+1) = x_2(k)$

$x_2(k) = q_1(k+1)$

$x_3(k) = q_2(k)$

$x_2(k+1) + x_3(k+1) + x_1(k) = u(k) \rightarrow x_2(k+1) = -x_3(k+1) - x_1(k) + u(k)$ ①

$x_2(k+1) - x_3(k+1) + x_3(k) = 0 \rightarrow x_3(k+1) = x_3(k) + x_2(k+1)$ ②

$$y(k) = x_2(k) + x_3(k)$$

① $x_2(k+1) = \left[-x_3(k) - x_2(k+1)\right] - x_1(k) + u(k)$ (② substituted)

$2x_2(k+1) = -x_3(k) - x_1(k) + u(k)$

$$x_2(k+1) = -\frac{1}{2}x_3(k) - \frac{1}{2}x_1(k) + \frac{1}{2}u(k)$$

② $x_3(k+1) = x_3(k)\left[-x_3(k+1) - x_1(k) + u(k)\right]$ (① substituted)

$2x_3(k+1) = x_3(k) - x_1(k) + u(k)$

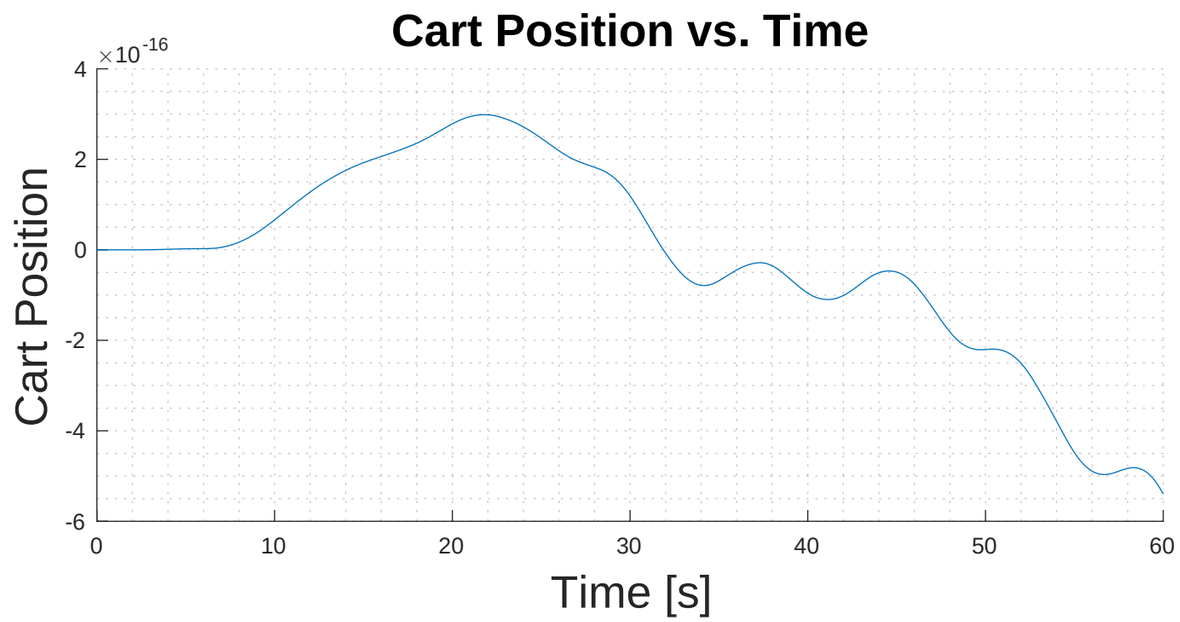
$$x_3(k+1) = \frac{1}{2}x_3(k) - \frac{1}{2}x_1(k) + \frac{1}{2}u(k)$$

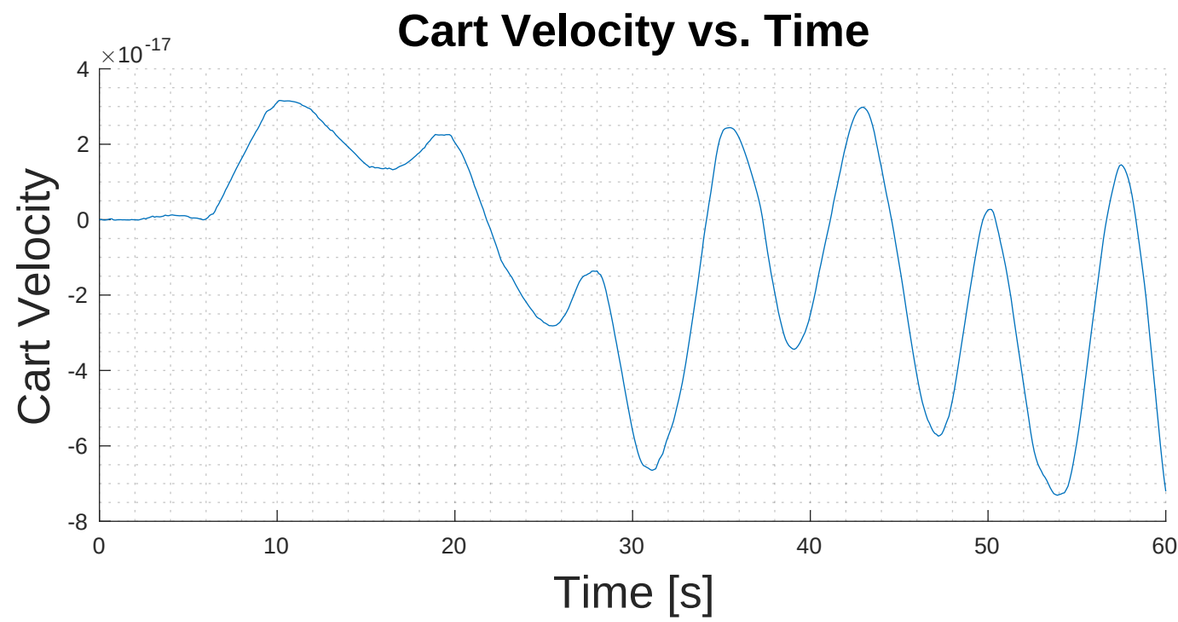
7.

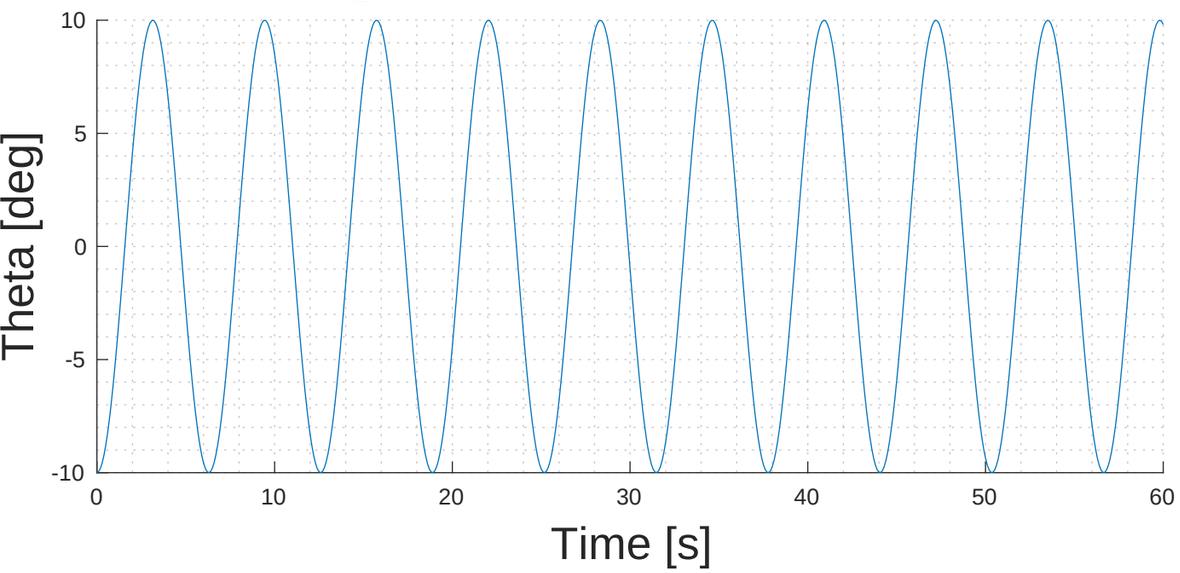
$$\underset{M}{\begin{bmatrix} m_0+m_1+m_2 & m_1 l_1 \cos\theta_1 & m_2 l_2 \cos\theta_2 \\ -m_1 l_1 \cos\theta_1 & m_1 l_1^2 & 0 \\ -m_2 l_2 \cos\theta_2 & 0 & m_2 l_2^2 \end{bmatrix}} \underset{\ddot{q}}{\begin{bmatrix} \ddot{y} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}} + \underset{G}{\begin{bmatrix} m_1 l_1 \sin\theta_1 \dot{\theta}_1^2 + m_2 l_2 \sin\theta_2 \dot{\theta}_2^2 \\ m_1 l_1 g \sin\theta \\ m_2 l_2 g \sin\theta_2 \end{bmatrix}} = \underset{Wu}{\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}}$$

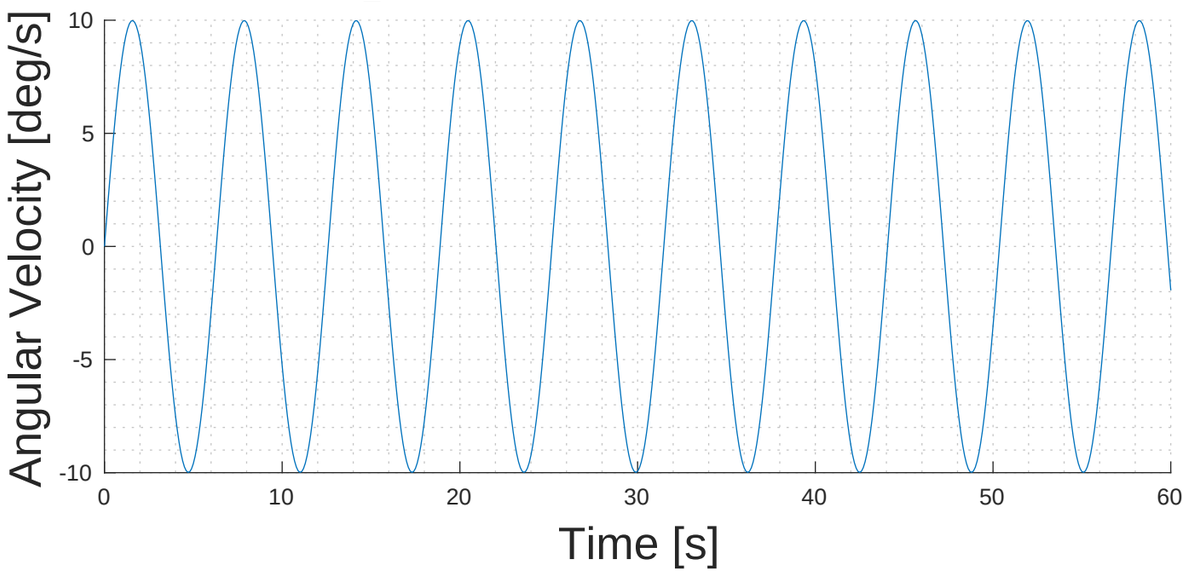
$$\begin{bmatrix} \ddot{y} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = INV\left(\begin{bmatrix} m_0+m_1+m_2 & m_1 l_1 \cos\theta_1 & m_2 l_2 \cos\theta_2 \\ -m_1 l_1 \cos\theta_1 & m_1 l_1^2 & 0 \\ -m_2 l_2 \cos\theta_2 & 0 & m_2 l_2^2 \end{bmatrix} \right) \left(\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} m_1 l_1 \sin\theta_1 \dot{\theta}_1^2 + m_2 l_2 \sin\theta_2 \dot{\theta}_2^2 \\ m_1 l_1 g \sin\theta_1 \\ m_2 l_2 g \sin\theta_2 \end{bmatrix} \right)$$

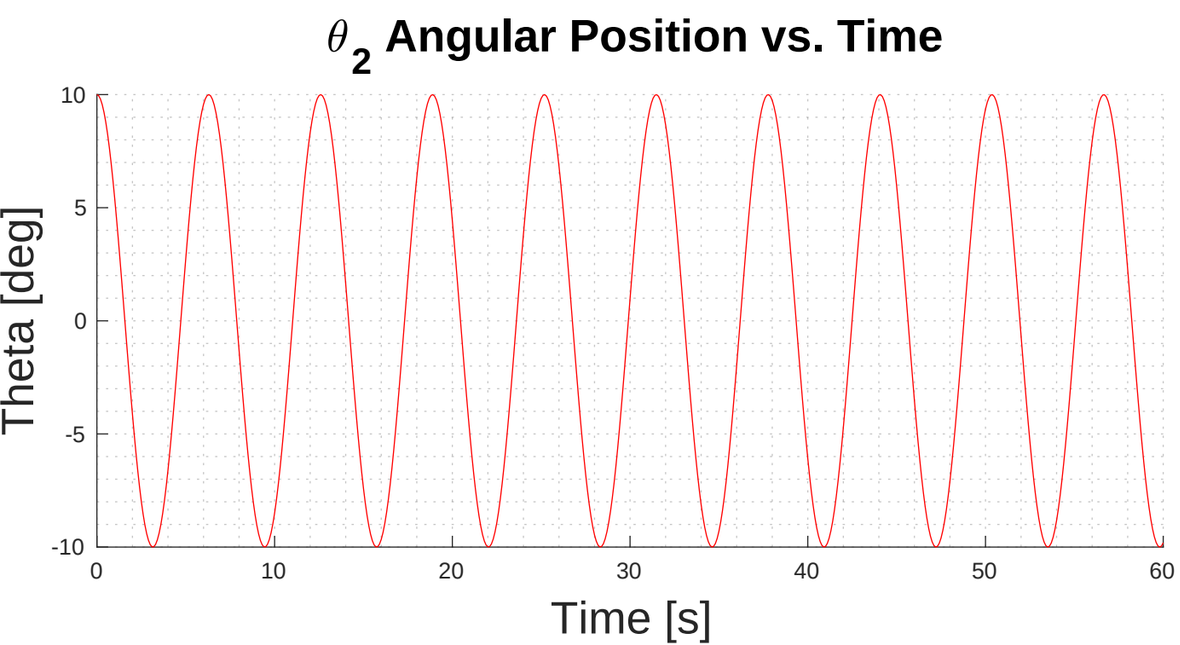
P1 & IC1

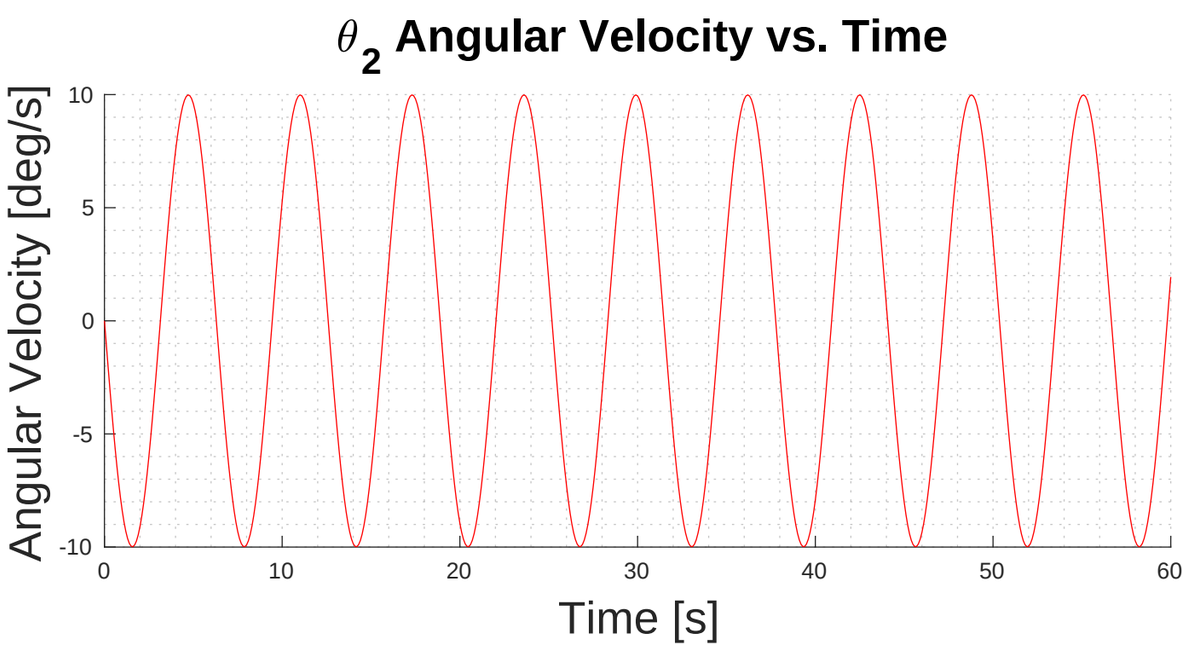

Cart Position vs. Time
×10^-16
Cart Position
Time [s]


Cart Velocity vs. Time
×10^-17
Cart Velocity
Time [s]

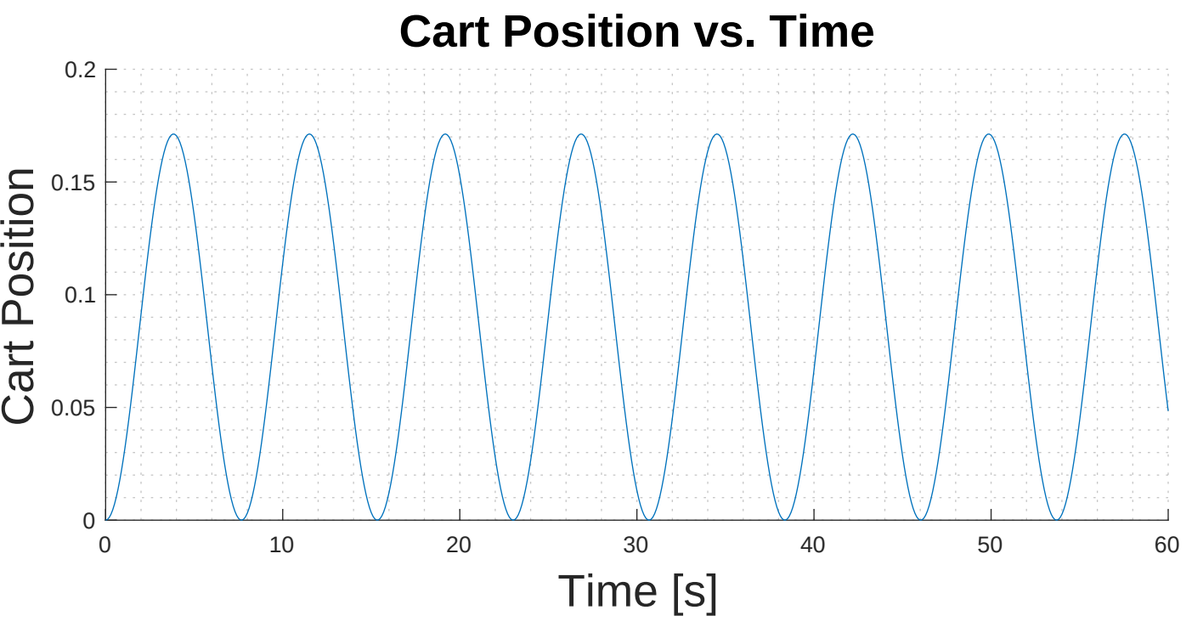

θ1 Angular Position vs. Time
Theta [deg]
Time [s]

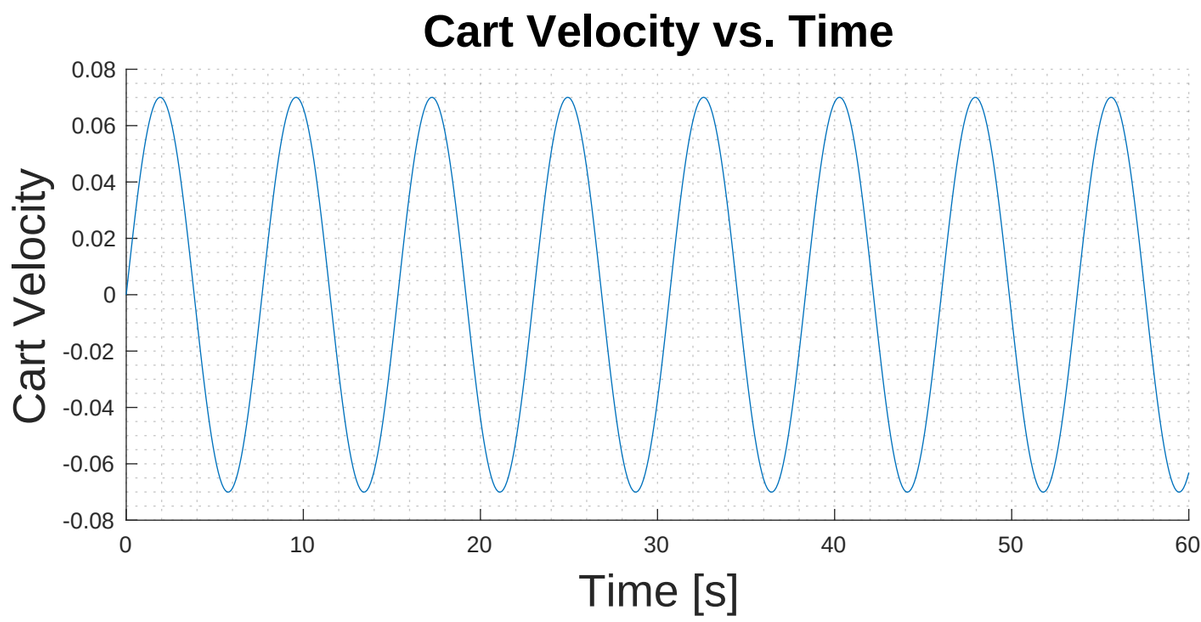

θ1 Angular Velocity vs. Time
Angular Velocity [deg/s]
Time [s]

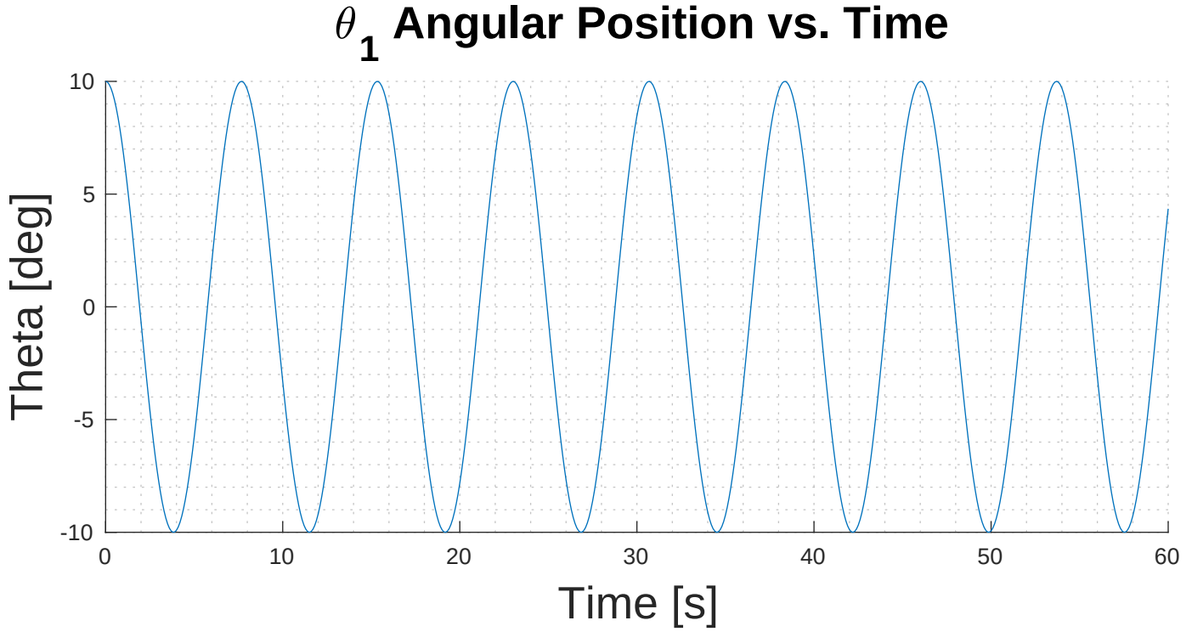

θ2 Angular Position vs. Time
Theta [deg]
Time [s]

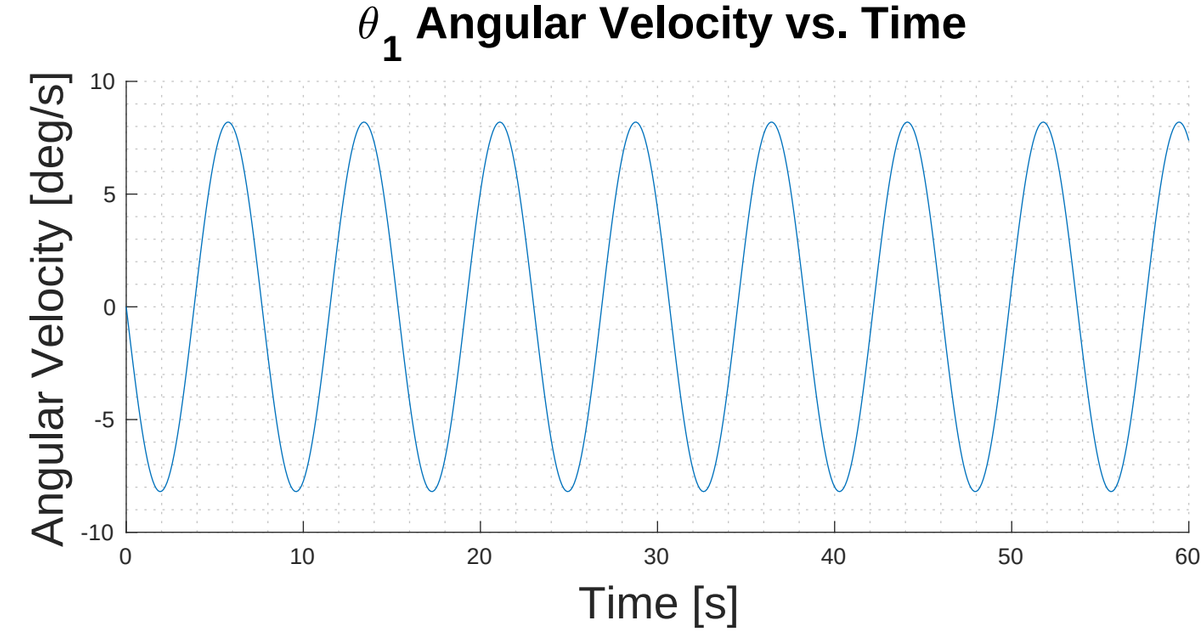

θ2 Angular Velocity vs. Time
Angular Velocity [deg/s]
Time [s]

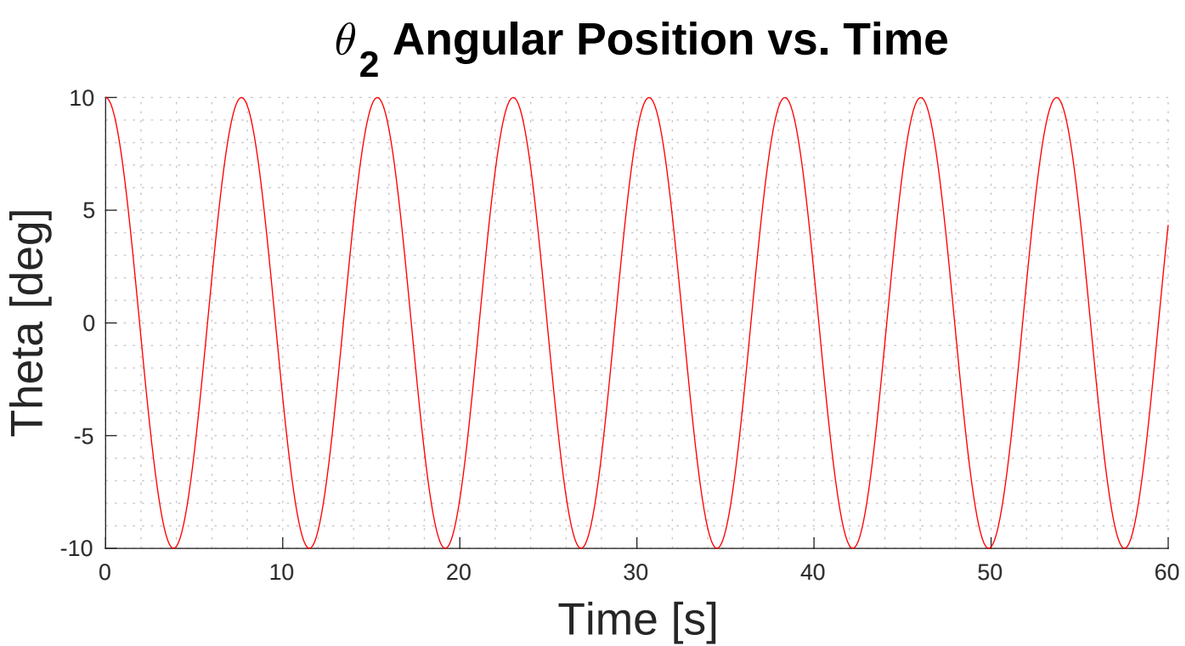
P1 & IC2

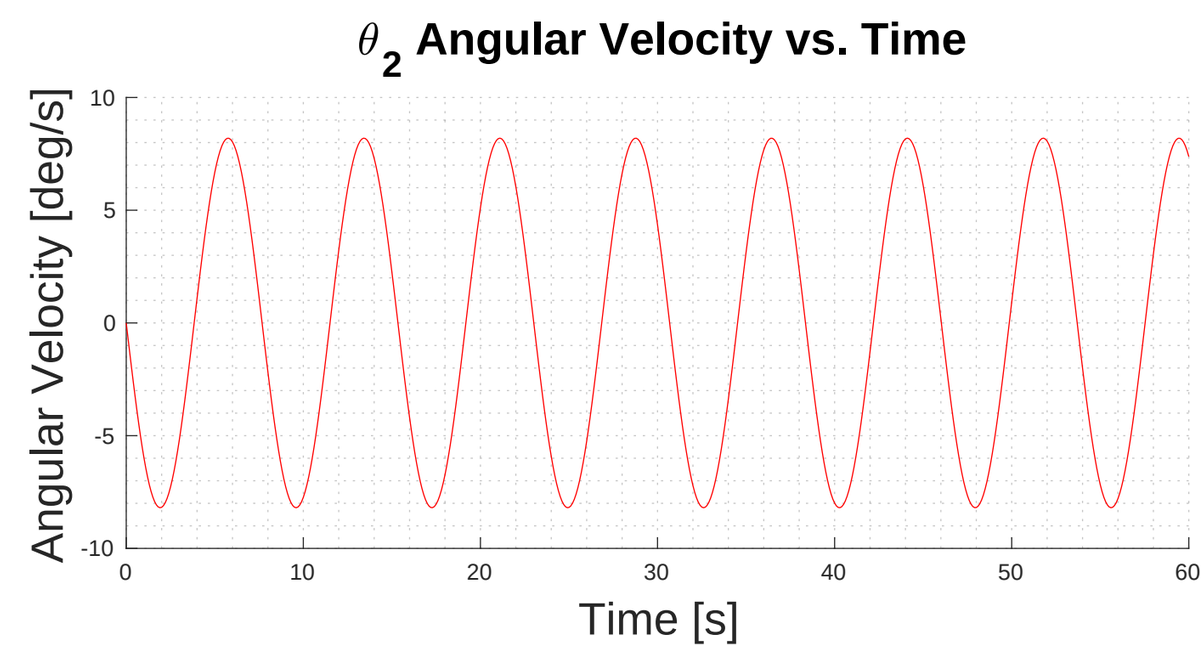

Cart Position vs. Time
Cart Position
Time [s]
0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
60


Cart Velocity vs. Time
Cart Velocity
Time [s]
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
10
20
30
40
50
60


θ1 Angular Position vs. Time
Theta [deg]
Time [s]
10
5
0
-5
-10
0
10
20
30
40
50
60

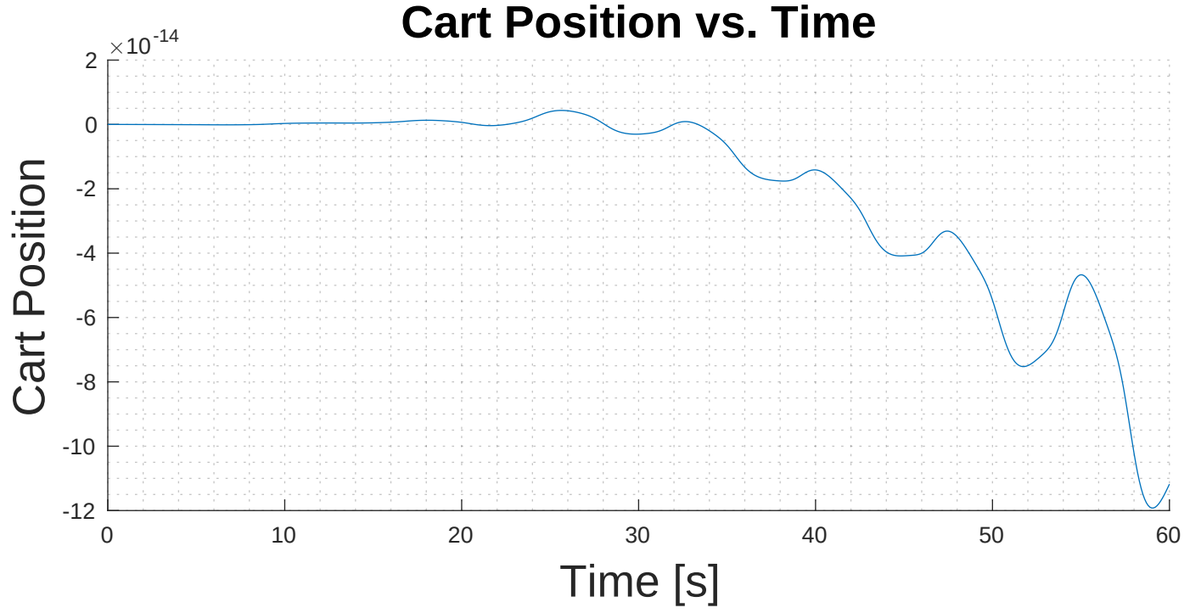

θ1 Angular Velocity vs. Time
Angular Velocity [deg/s]
Time [s]
10
5
0
-5
-10
0
10
20
30
40
50
60


θ2 Angular Position vs. Time
Theta [deg]
Time [s]
10
5
0
-5
-10
0
10
20
30
40
50
60

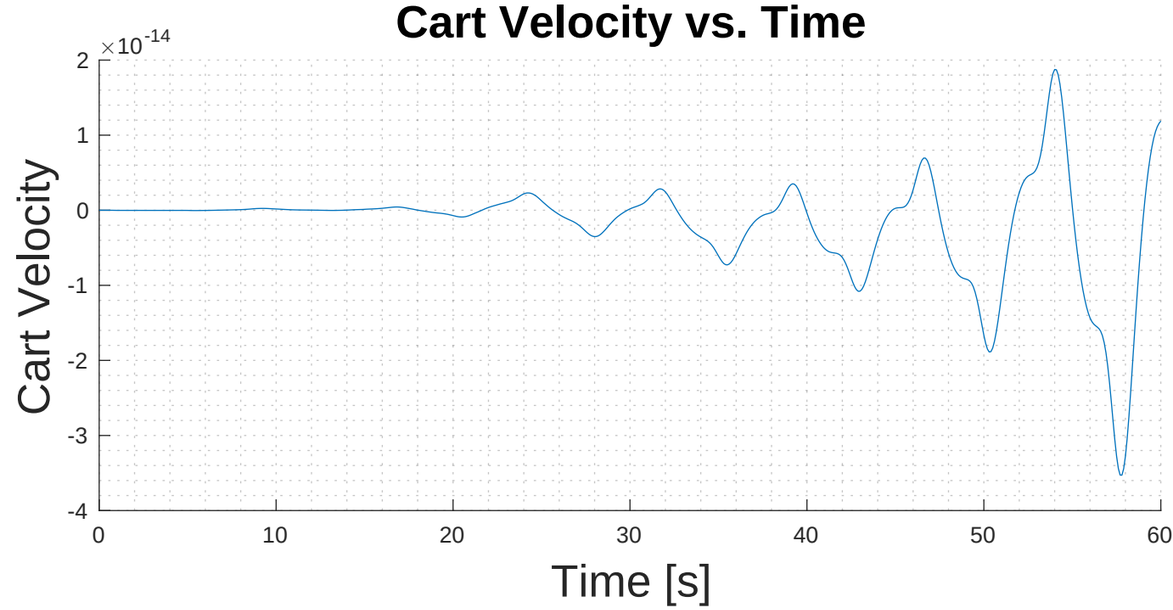

θ2 Angular Velocity vs. Time
Angular Velocity [deg/s]
Time [s]
10
5
0
-5
-10
0
10
20
30
40
50
60

P1 & IC3

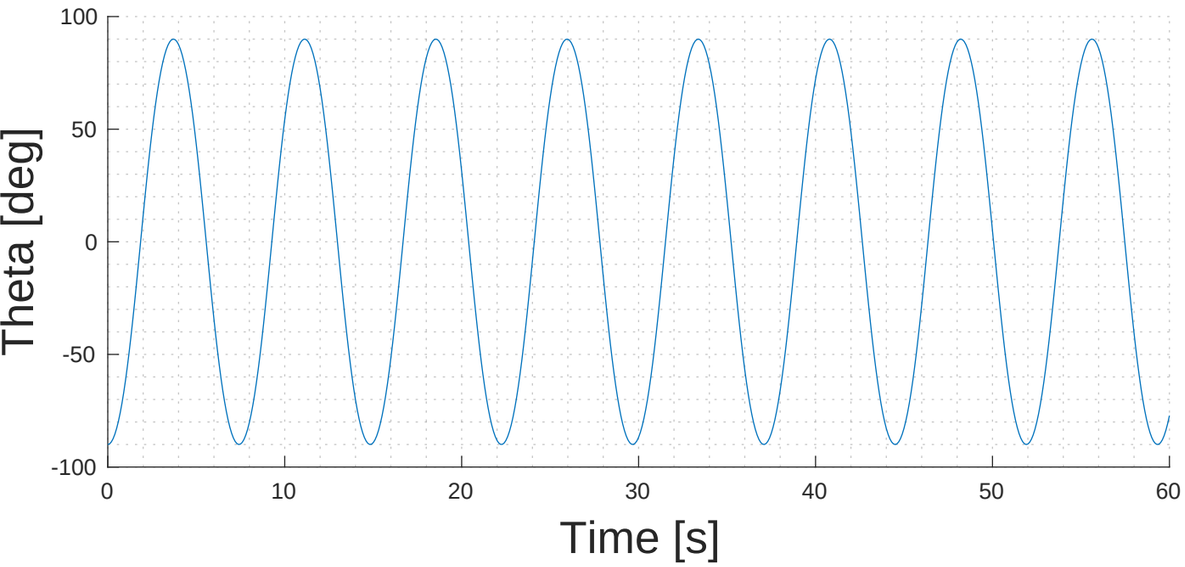
Cart Position vs. Time



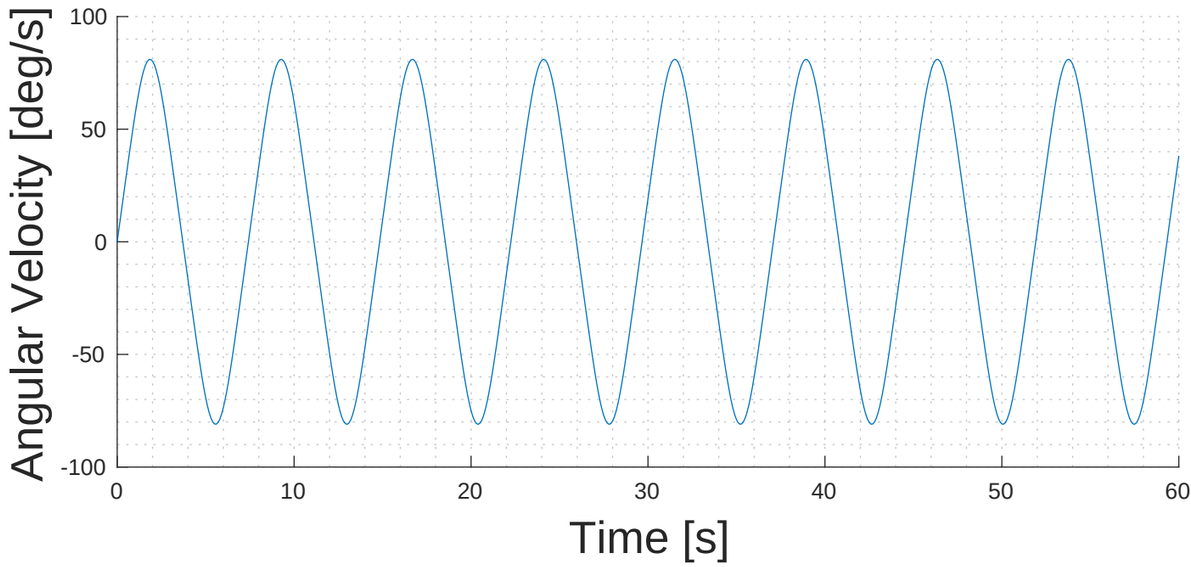
Cart Velocity vs. Time



θ_1 Angular Position vs. Time



θ_1 Angular Velocity vs. Time

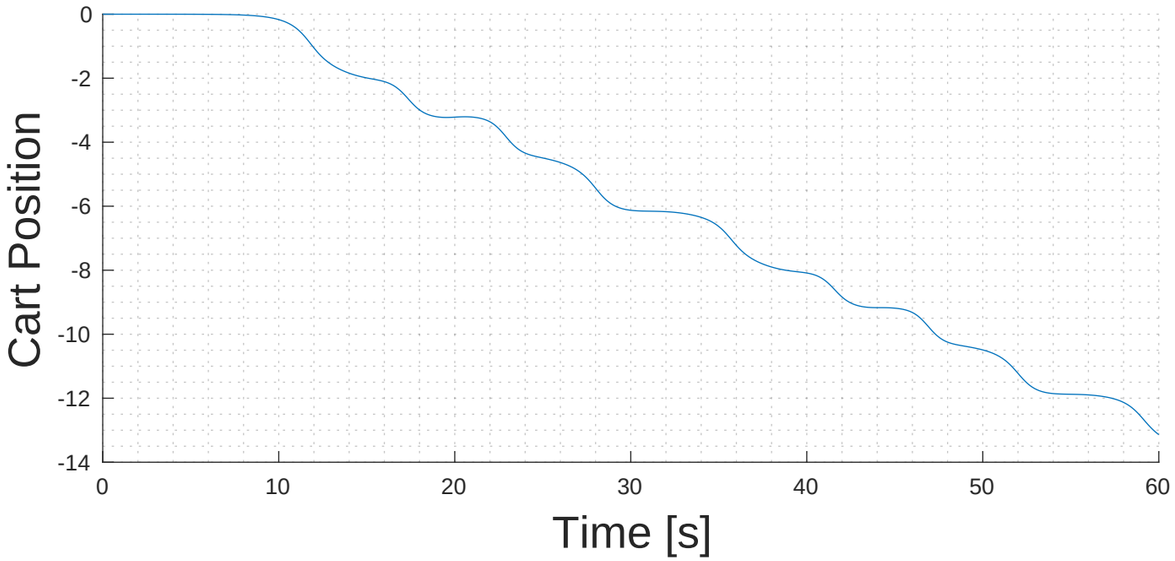


θ_2 Angular Position vs. Time

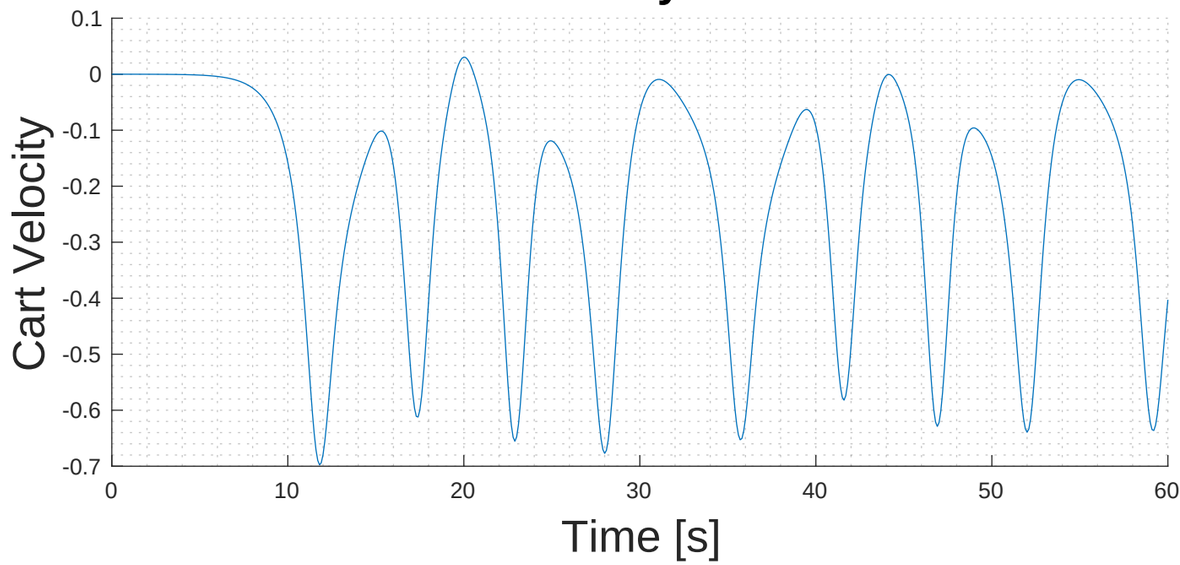
θ_2 Angular Velocity vs. Time

P1 & IC7

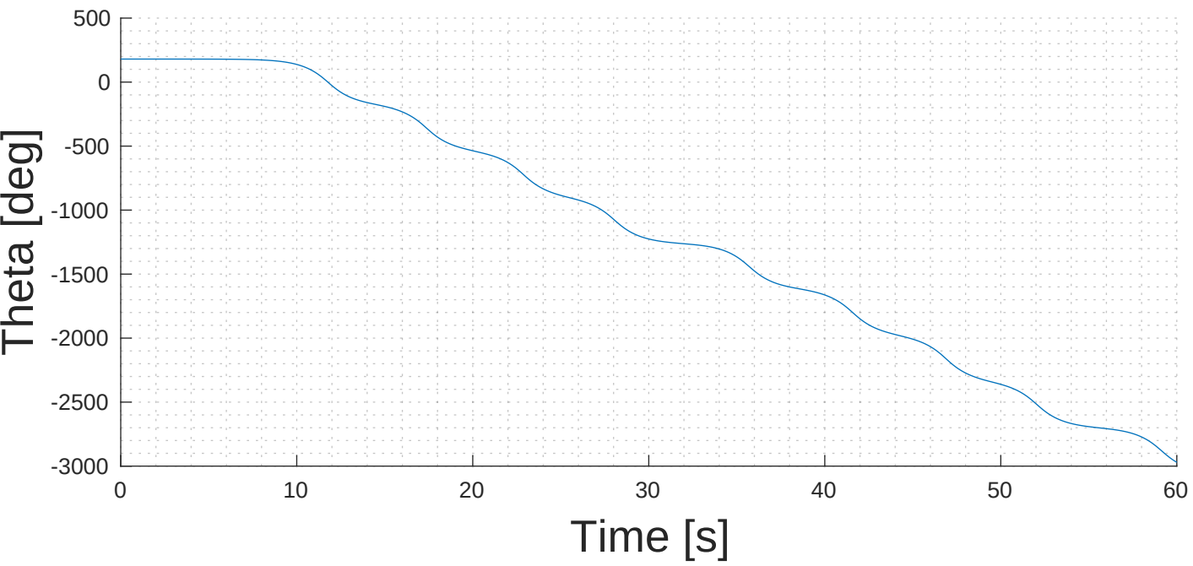
Cart Position vs. Time



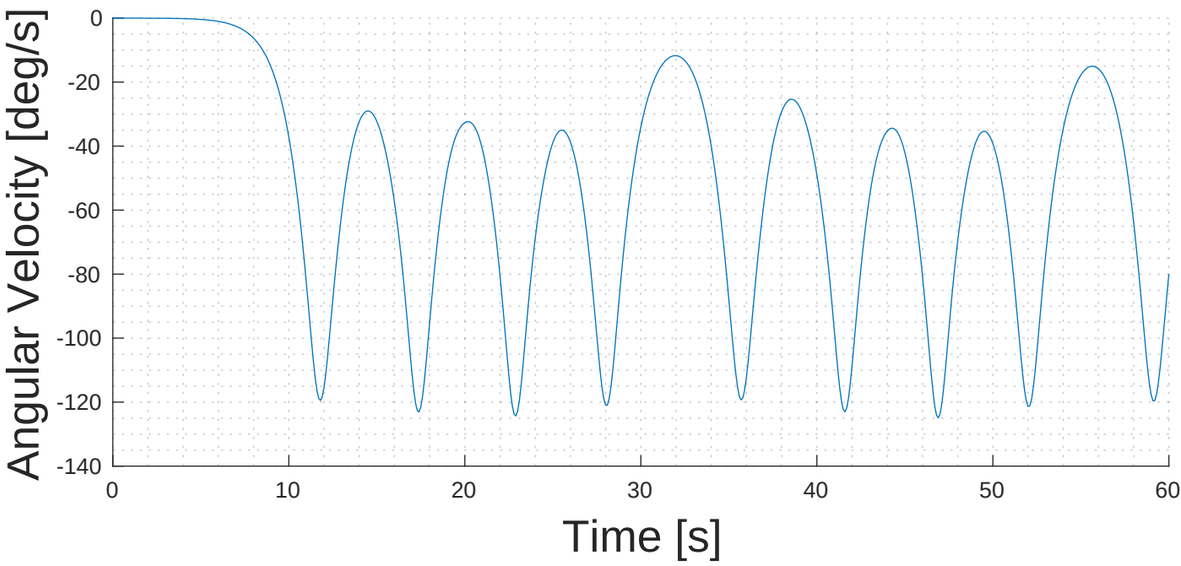
Cart Velocity vs. Time



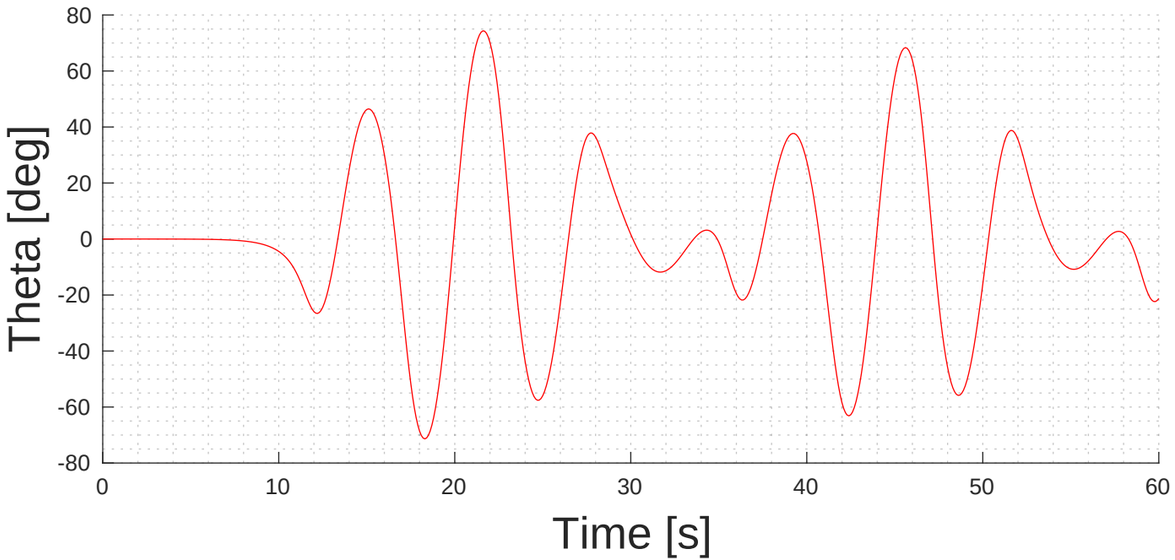
θ_1 Angular Position vs. Time



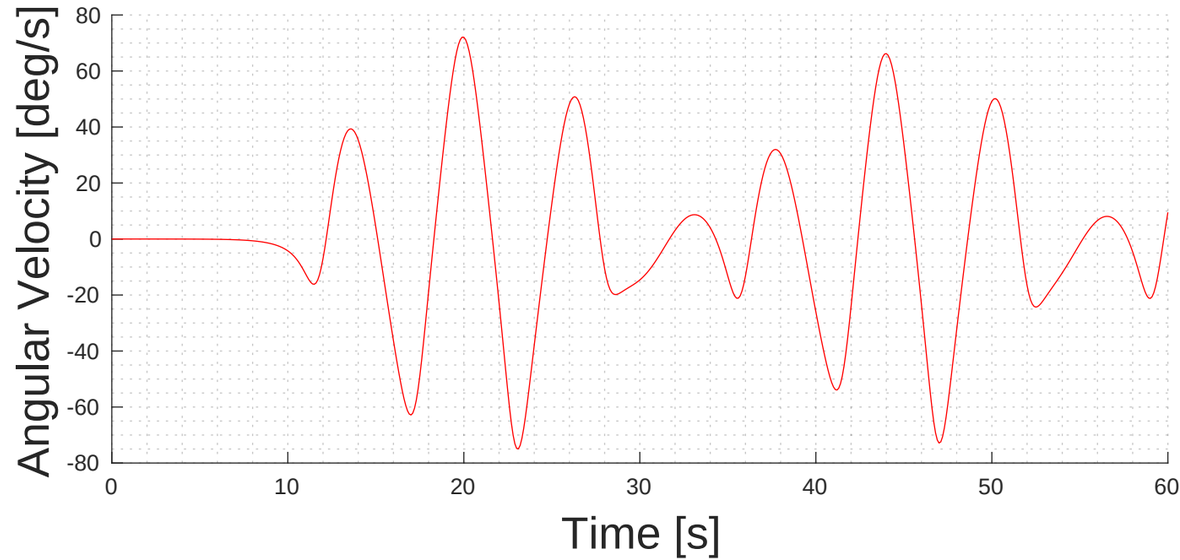
θ_1 Angular Velocity vs. Time



θ_2 Angular Position vs. Time

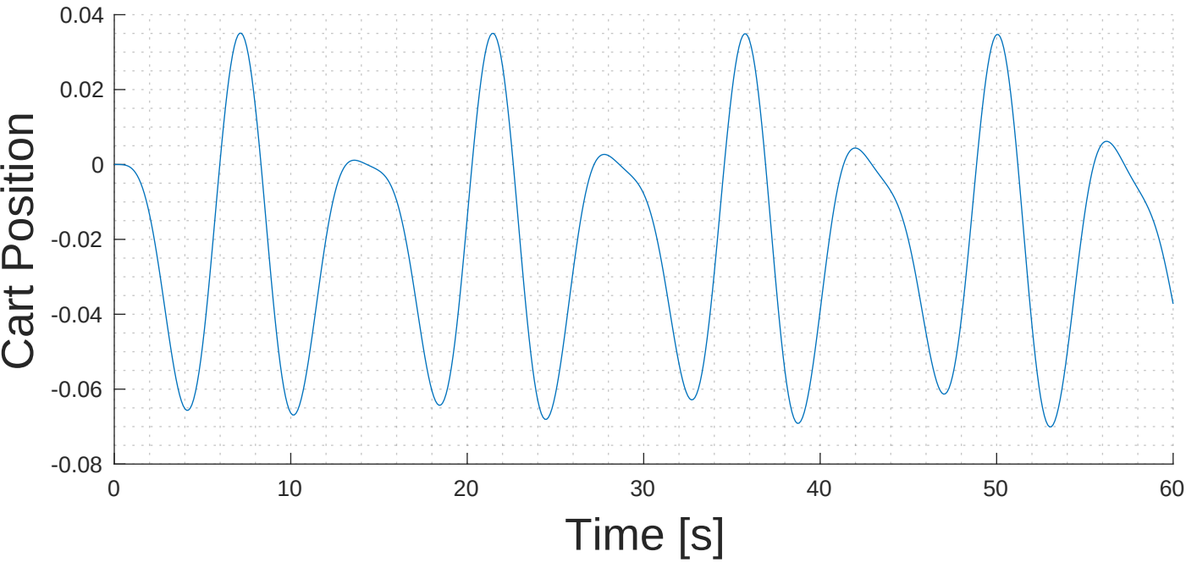


θ_2 Angular Velocity vs. Time

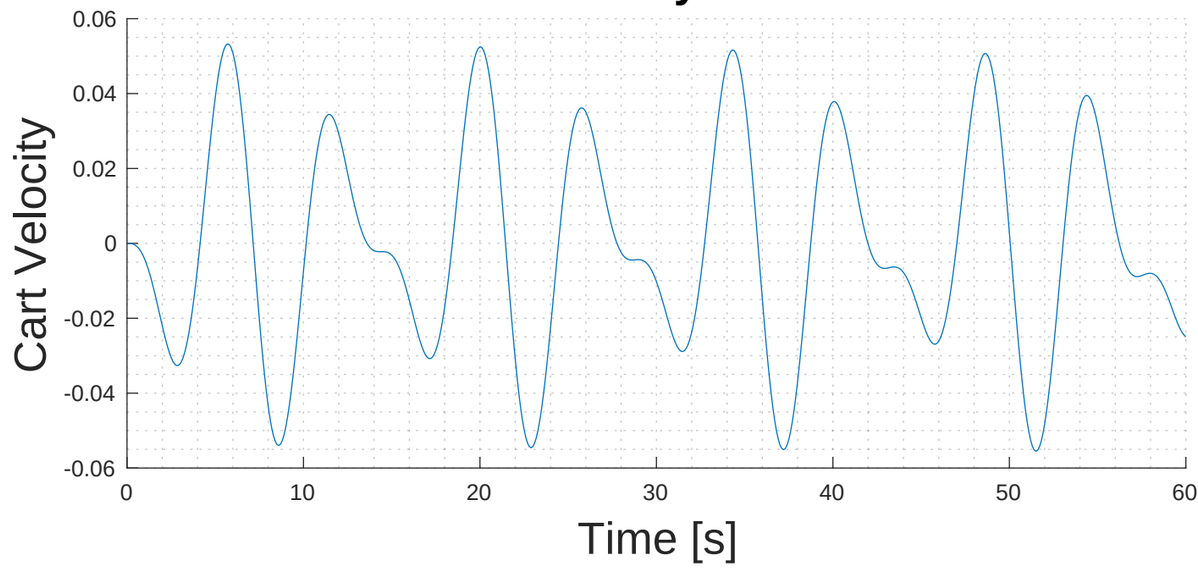


P4 & IC1

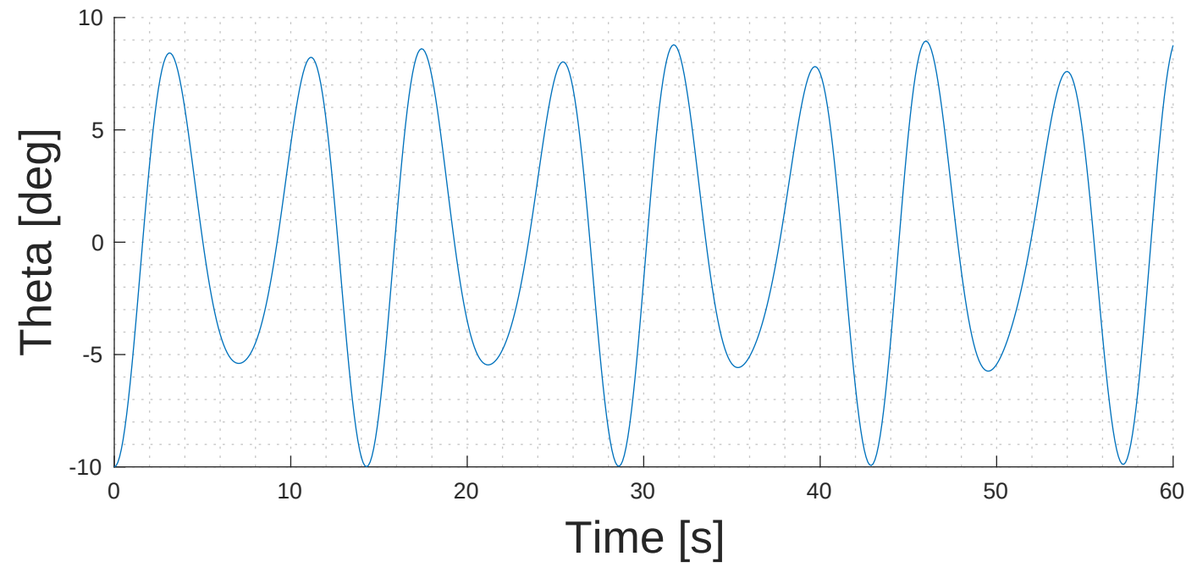
Cart Position vs. Time



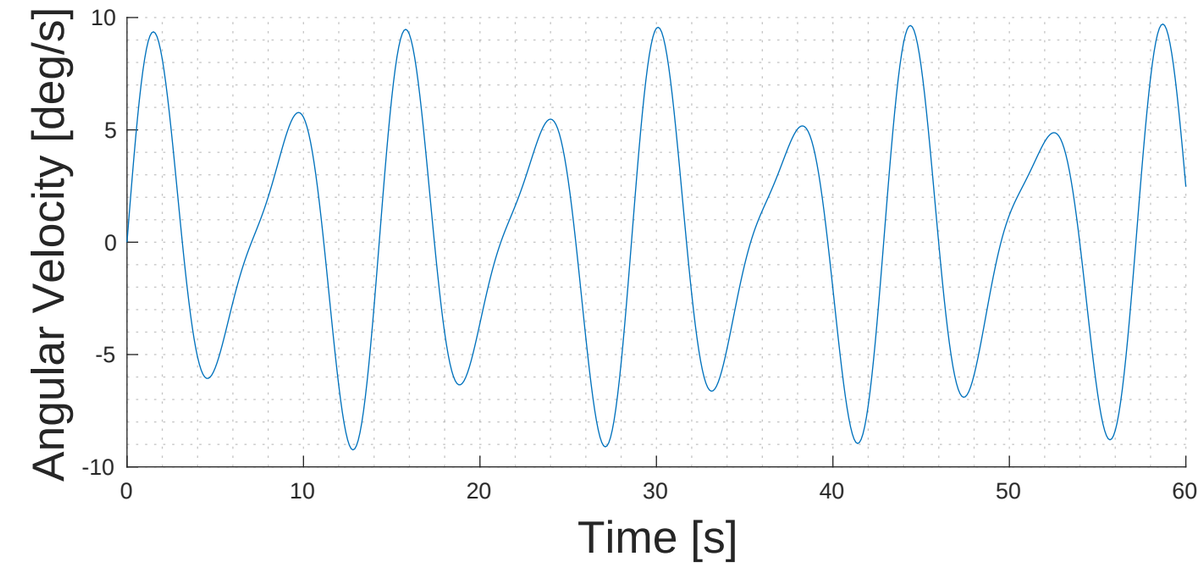
Cart Velocity vs. Time



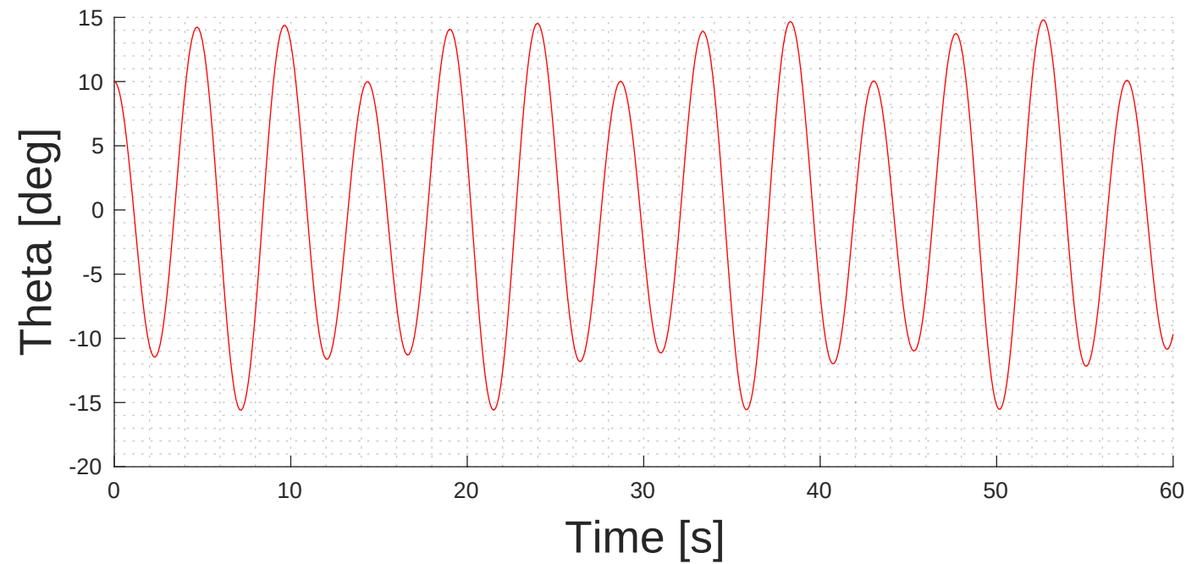
θ_1 Angular Position vs. Time



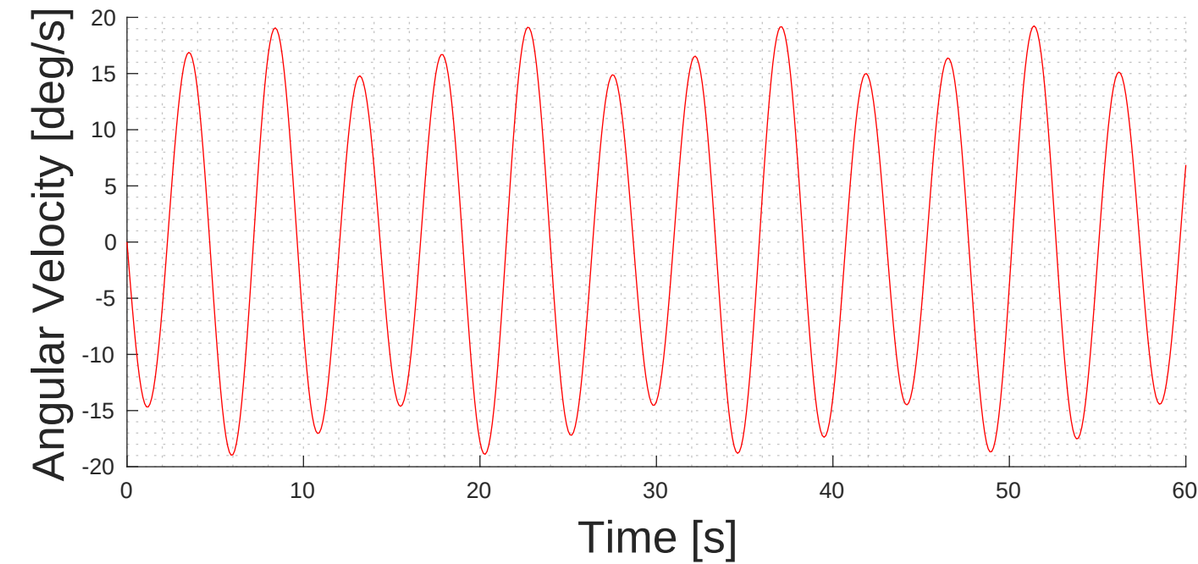
θ_1 Angular Velocity vs. Time



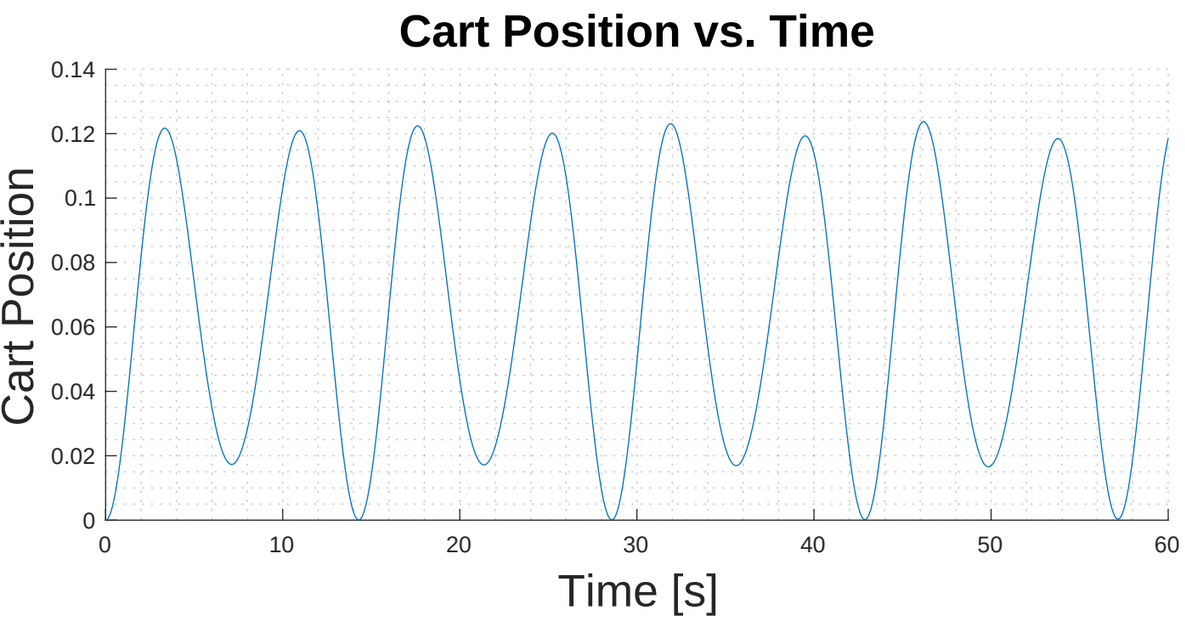
θ_2 Angular Position vs. Time

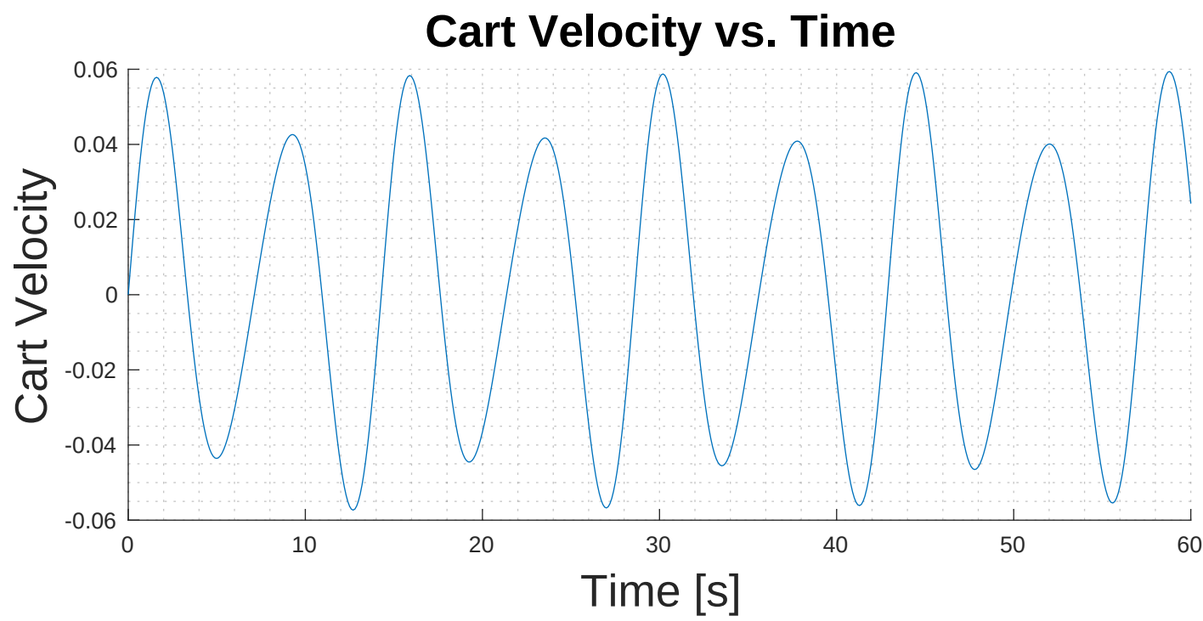


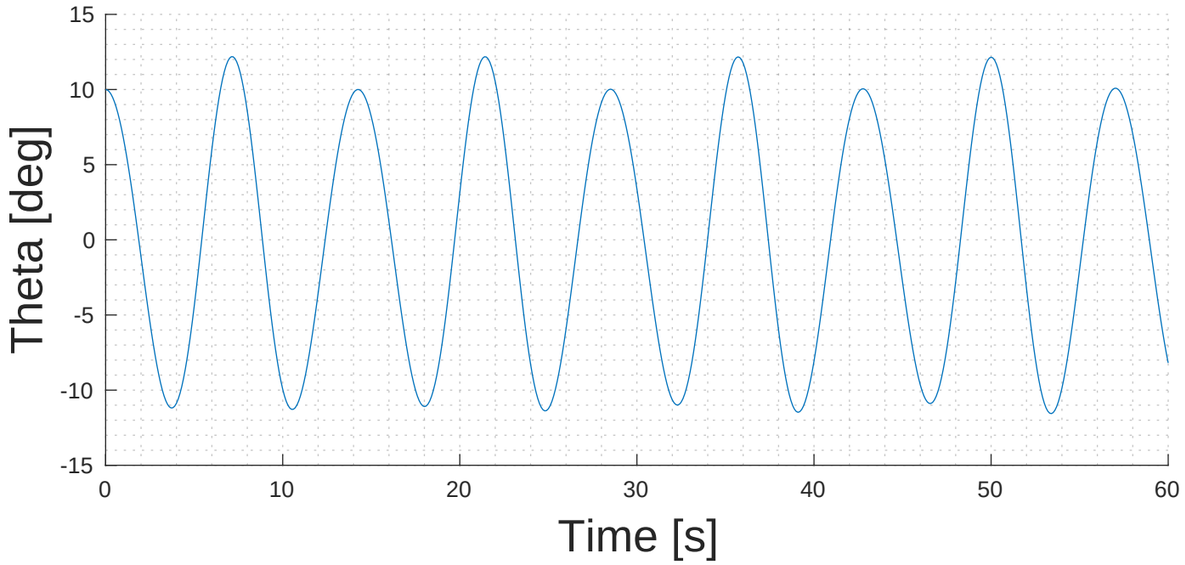
θ_2 Angular Velocity vs. Time

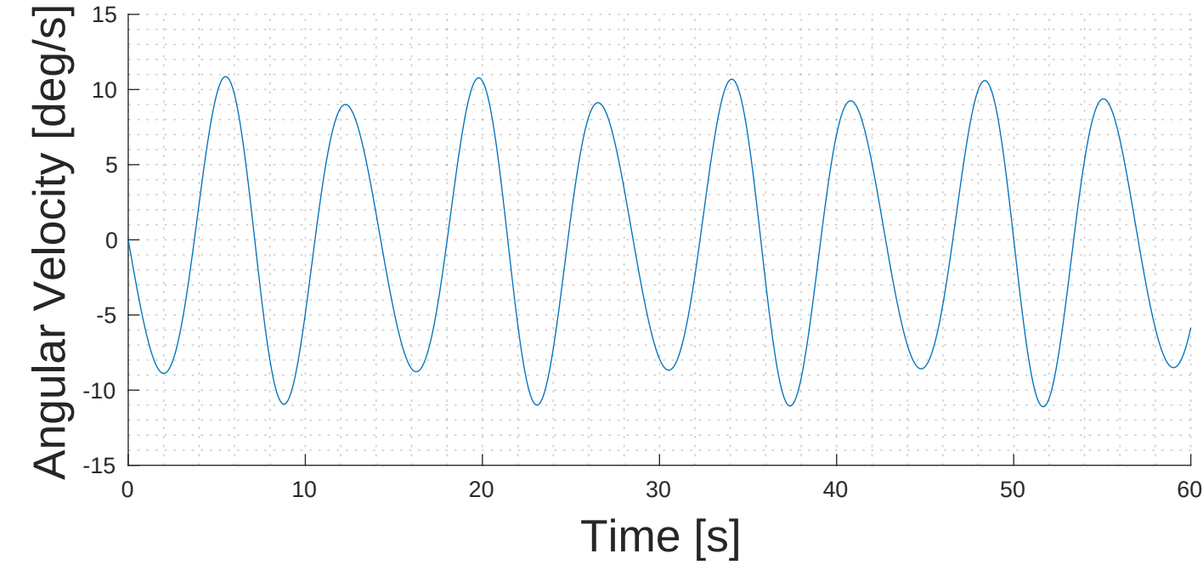


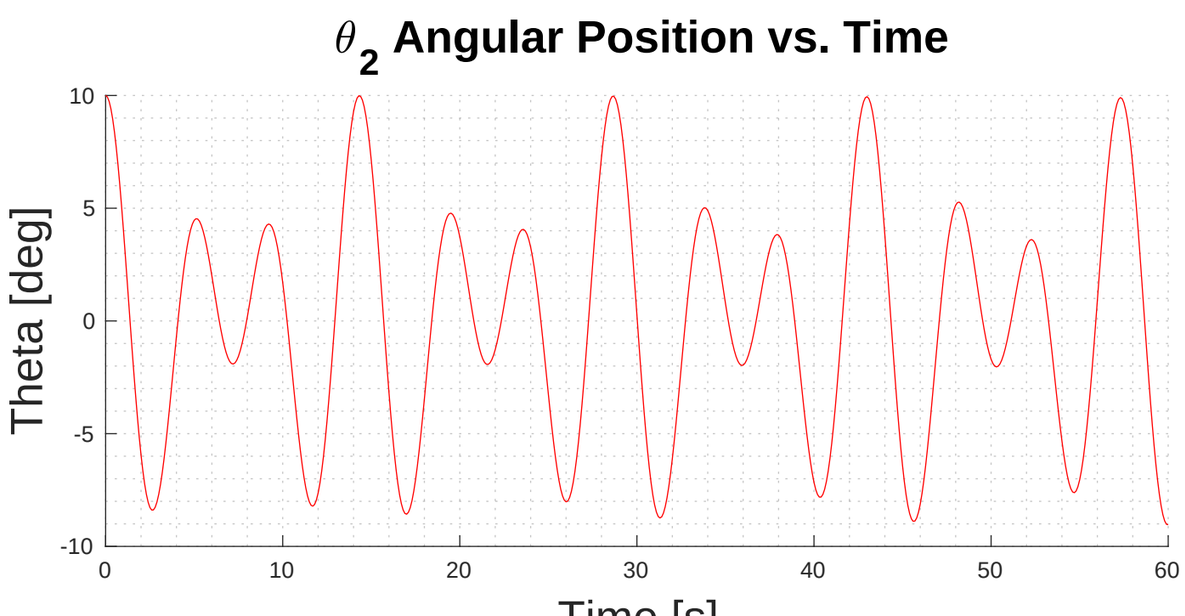
P4 & IC2

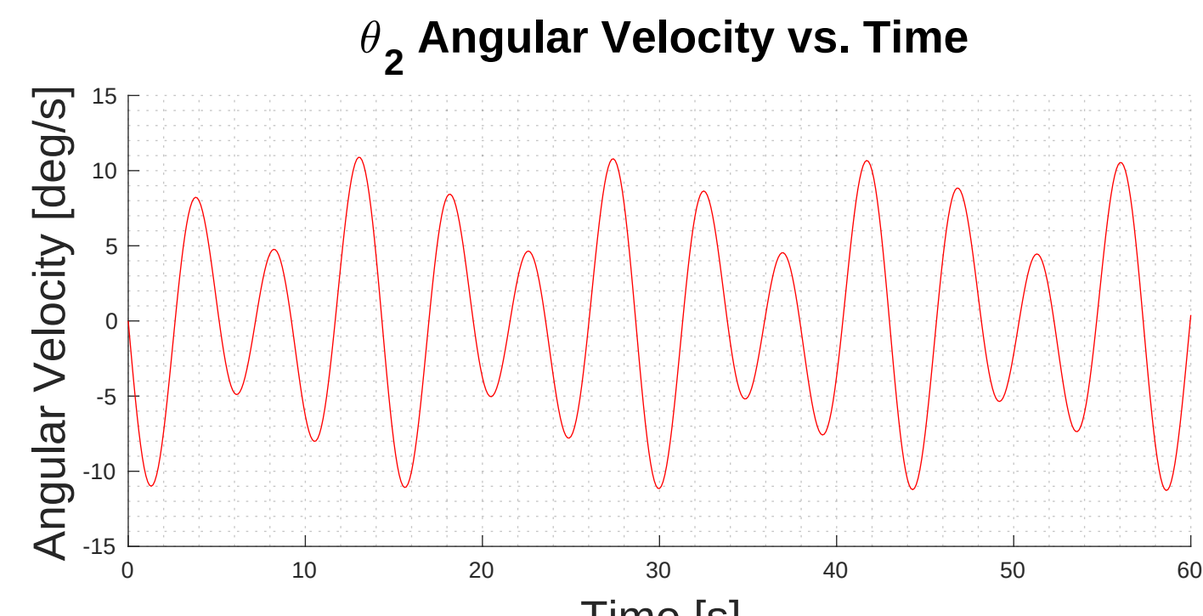

Cart Position vs. Time
Cart Position
Time [s]
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
10
20
30
40
50
60


Cart Velocity vs. Time
Cart Velocity
Time [s]
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0
10
20
30
40
50
60


θ1 Angular Position vs. Time
Theta [deg]
Time [s]
15
10
5
0
-5
-10
-15
0
10
20
30
40
50
60

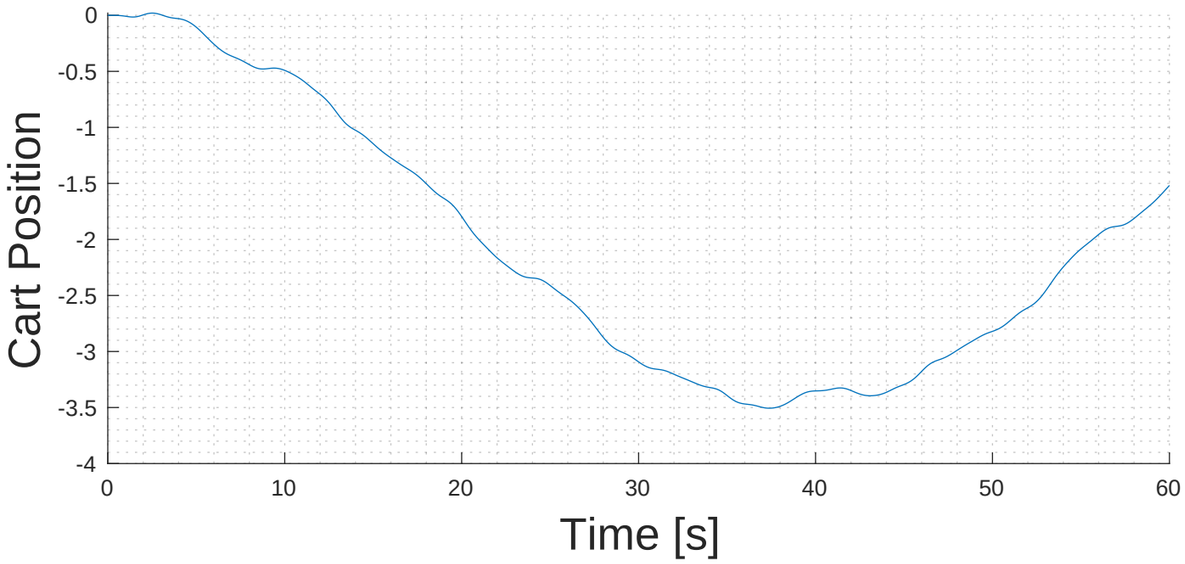

θ1 Angular Velocity vs. Time
Angular Velocity [deg/s]
Time [s]
15
10
5
0
-5
-10
-15
0
10
20
30
40
50
60


θ2 Angular Position vs. Time
Theta [deg]
Time [s]
10
5
0
-5
-10
0
10
20
30
40
50
60

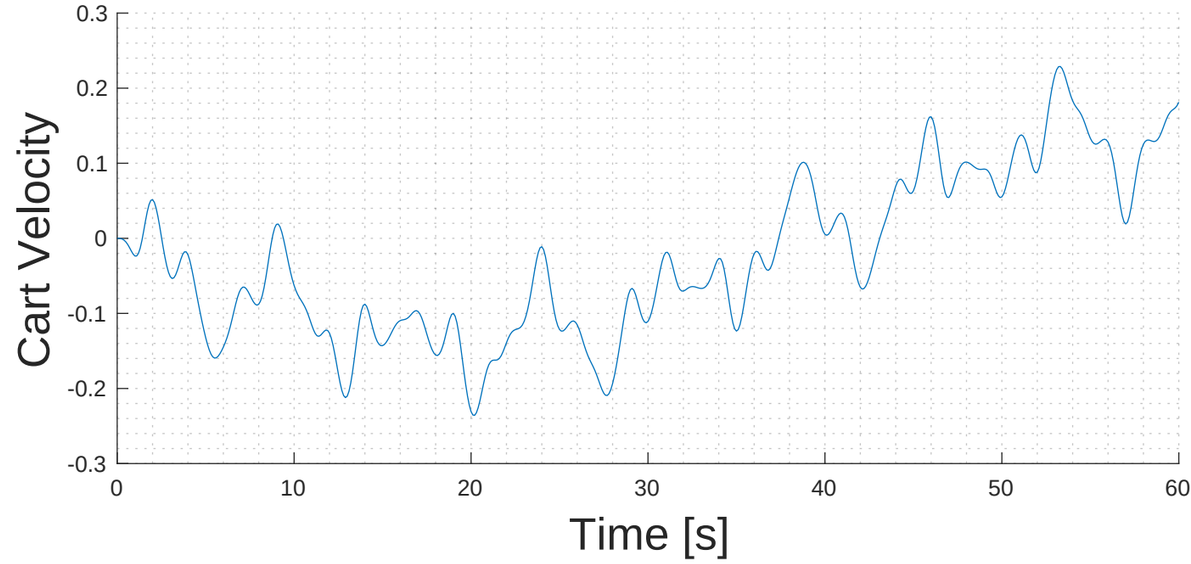

θ2 Angular Velocity vs. Time
Angular Velocity [deg/s]
Time [s]
15
10
5
0
-5
-10
-15
0
10
20
30
40
50
60

P4 & IC3

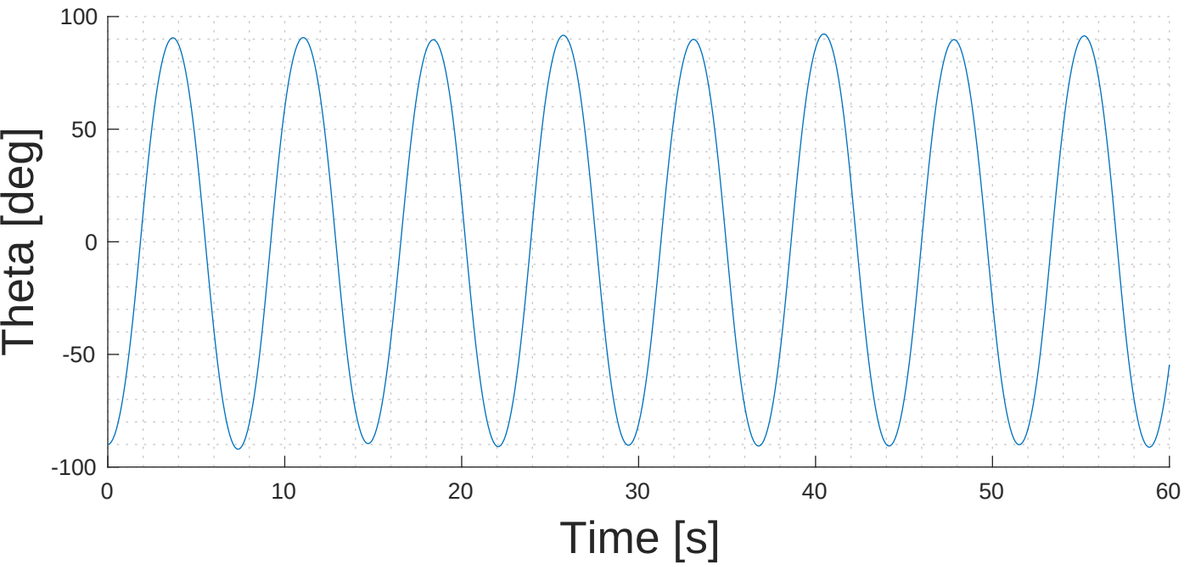
Cart Position vs. Time



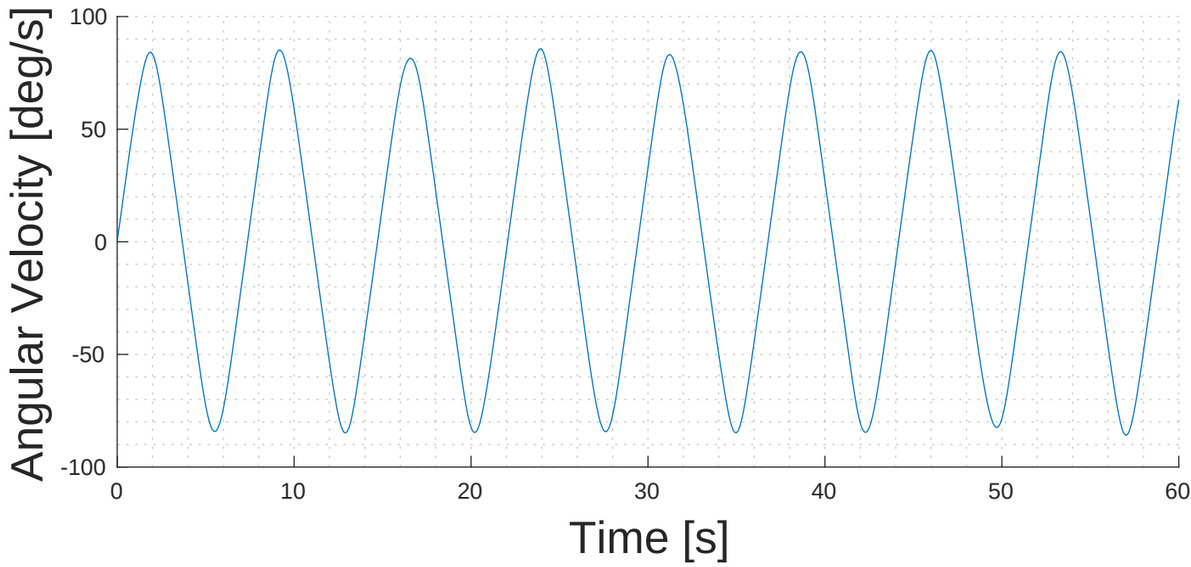
Cart Velocity vs. Time



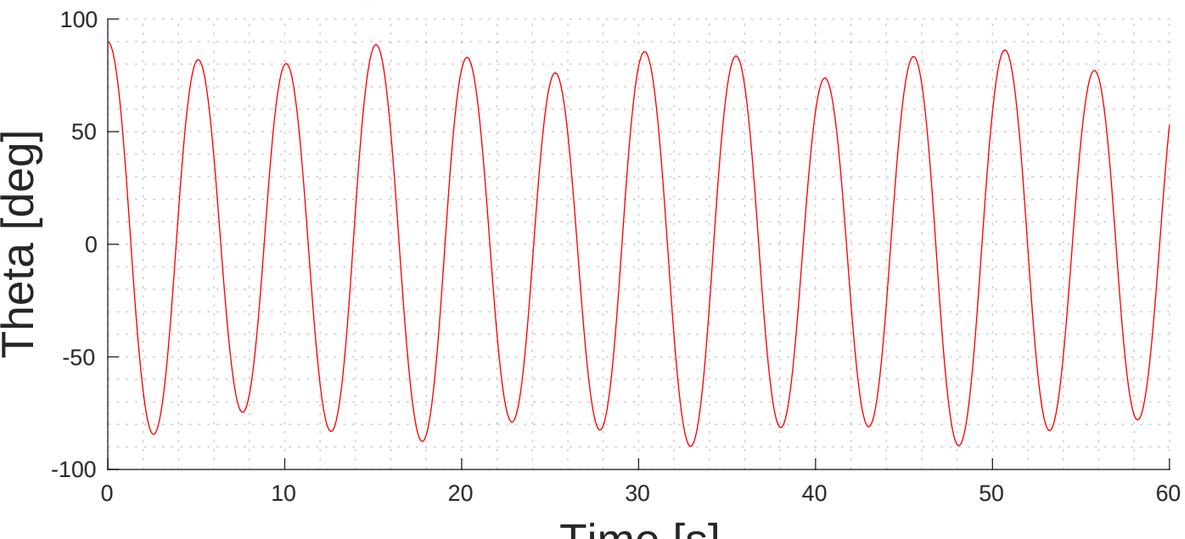
θ_1 Angular Position vs. Time



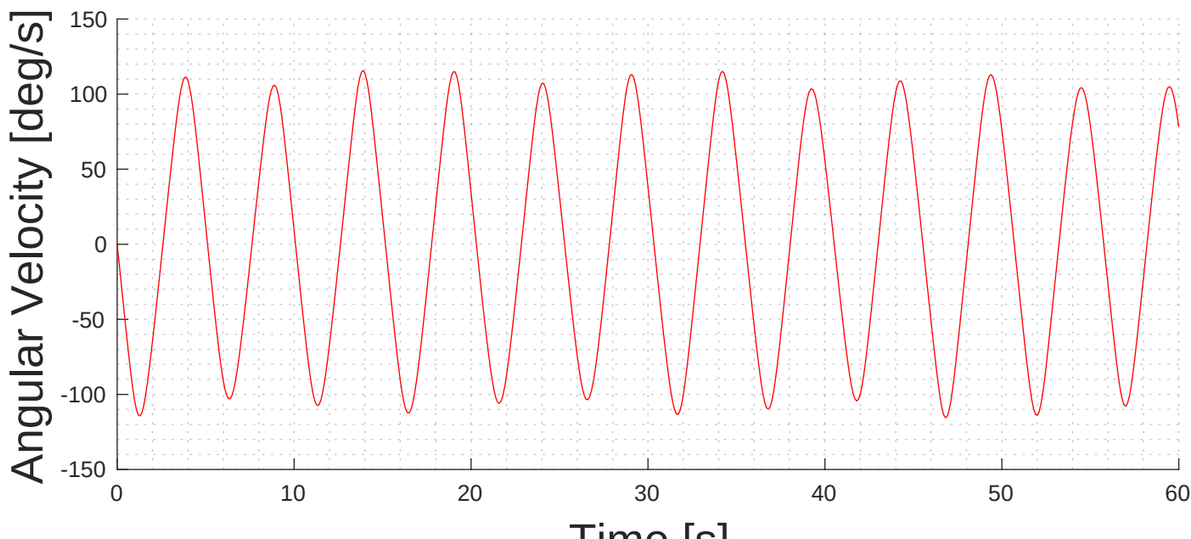
θ_1 Angular Velocity vs. Time



θ_2 Angular Position vs. Time

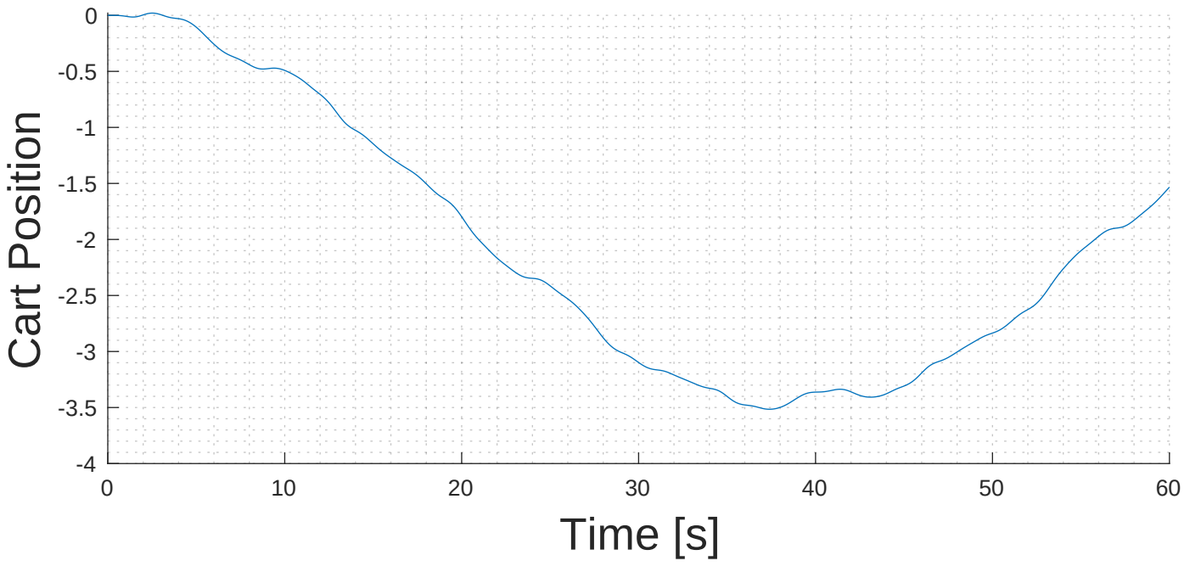


θ_2 Angular Velocity vs. Time

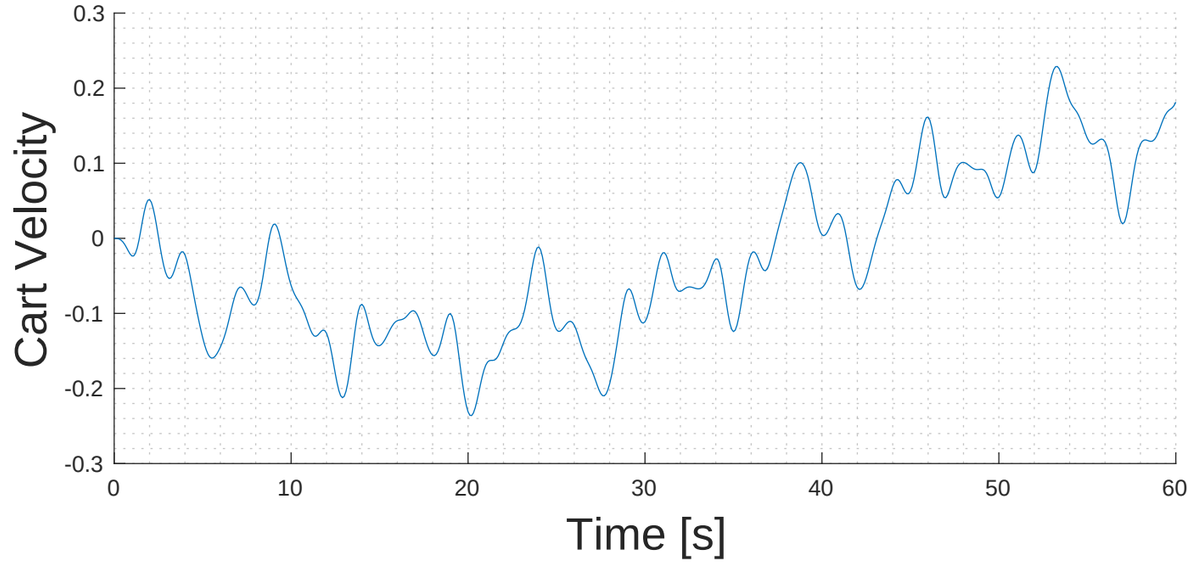


P4 & IC4

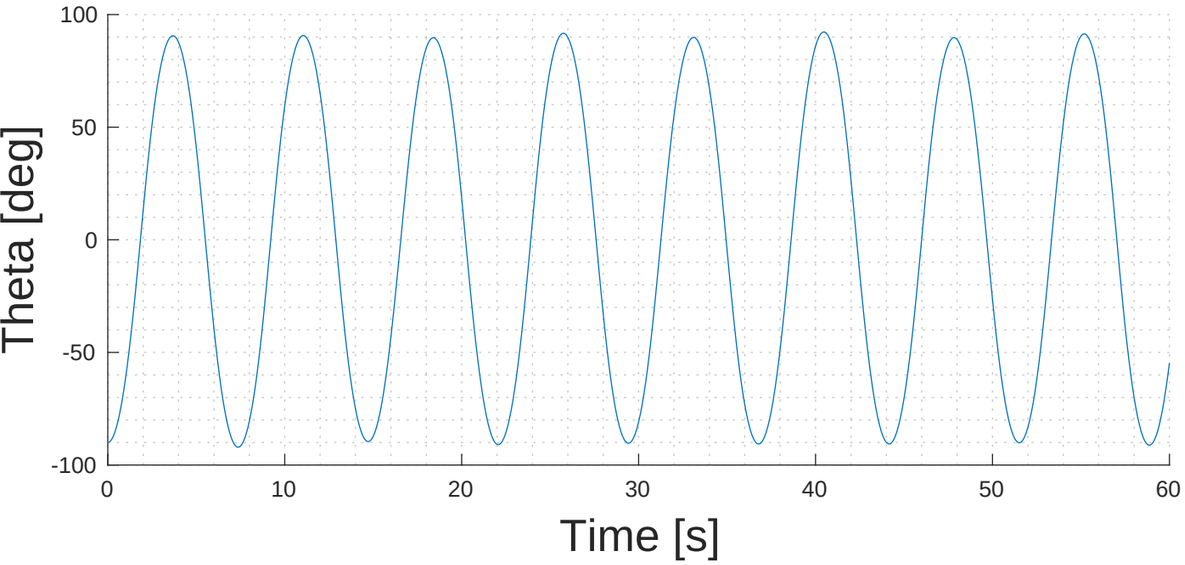
Cart Position vs. Time



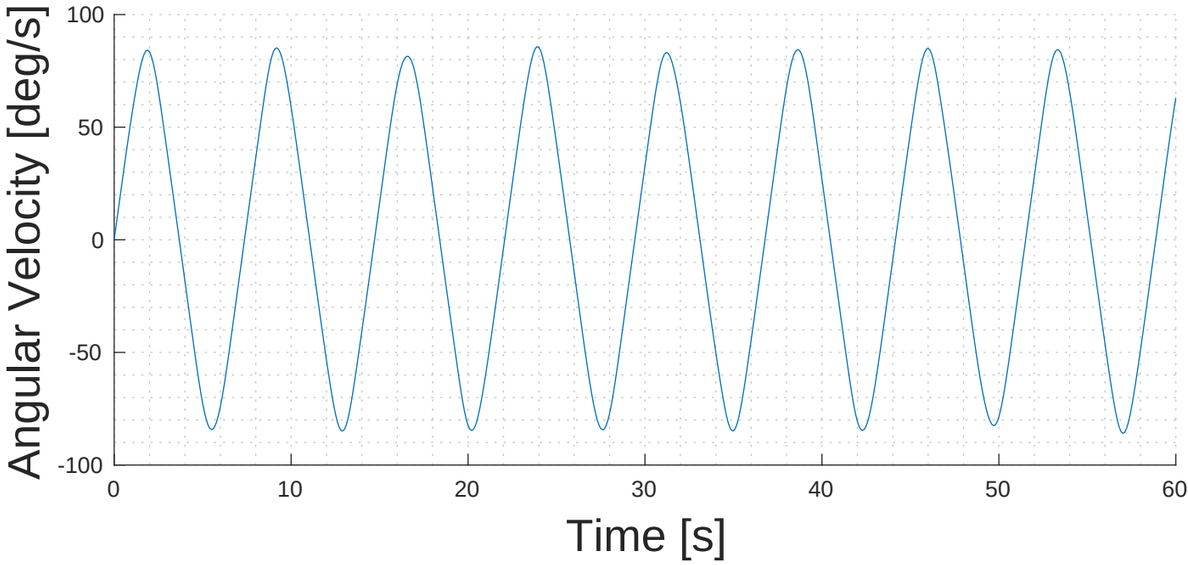
Cart Velocity vs. Time



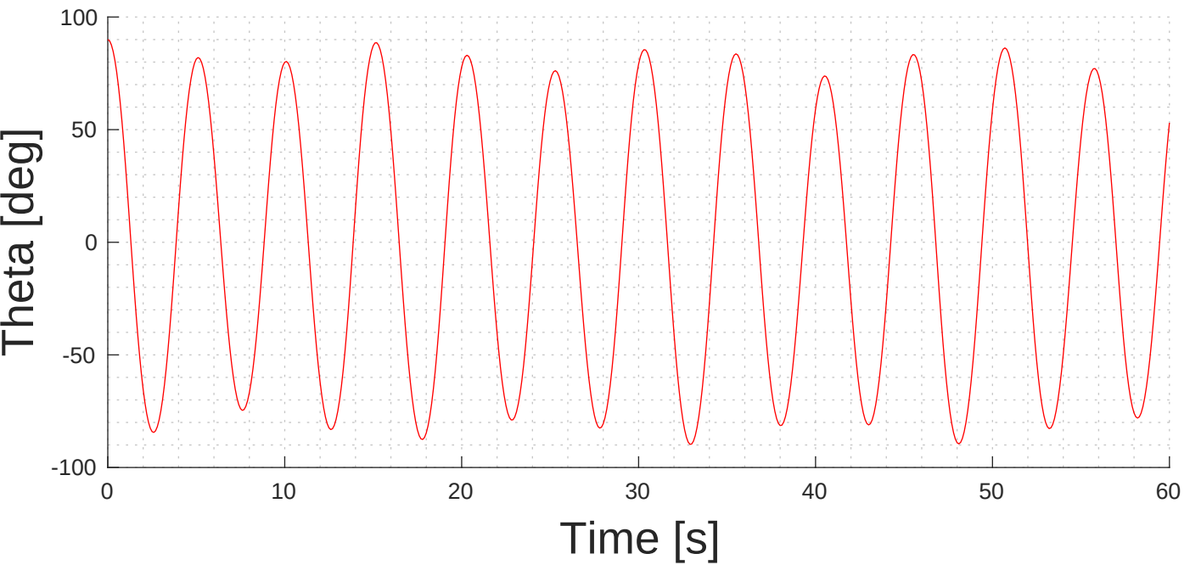
θ_1 Angular Position vs. Time



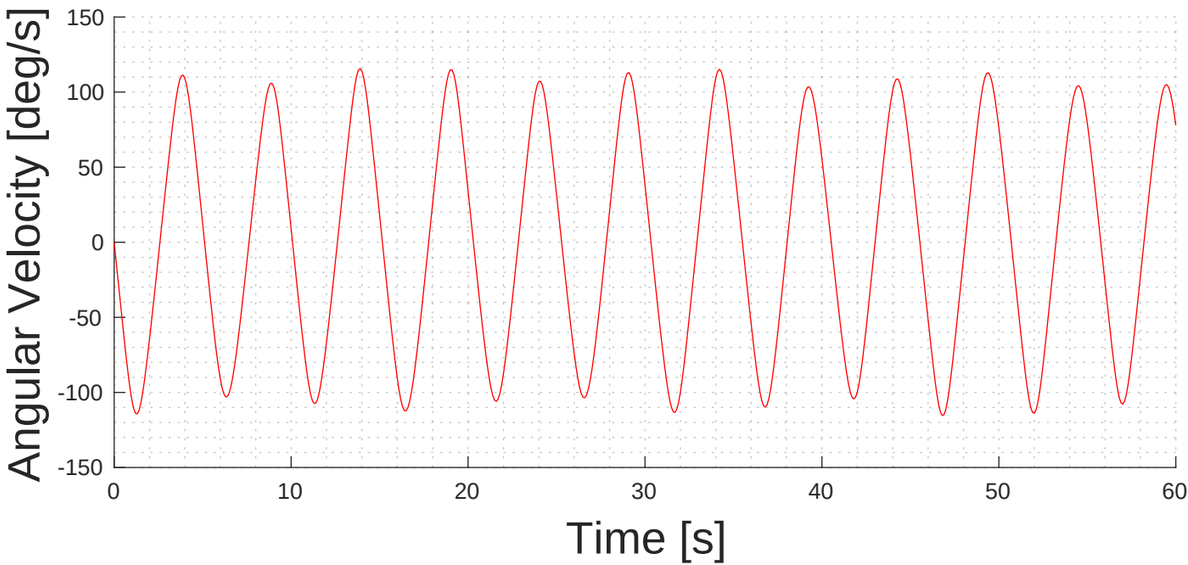
θ_1 Angular Velocity vs. Time



θ_2 Angular Position vs. Time



θ_2 Angular Velocity vs. Time



```
function dx = doublePendulumSS(t,x)

m0 = 2; % Mass of cart
m1 = 1; % Mass of pendulum 1
m2 = 1; % Mass of pendulum 2
l1 = 1; % Length of rod 1
l2 = 0.5; % Length of rod 2
g  = 1; % Gravity

y = x(1);
y_dot = x(2);
th1 = x(3);
th1_dot = x(4);
th2 = x(5);
th2_dot = x(6);

A = [   m0+m1+m2         m1*l1*cos(th1)     m2*l2*cos(th2);
     -m1*l1*cos(th1)        m1*l1^2               0;
     -m2*l2*cos(th2)           0                m2*l2^2];

B = -[m1*l1*sin(th1)*th1_dot^2 + m2*l2*sin(th2)*th2_dot^2;
                   m1*l1*g*sin(th1);
                   m2*l2*g*sin(th2)];

q = A\B;

dx = [y_dot; q(1); th1_dot; q(2); th2_dot; q(3)];

end
```

```
Not enough input arguments.

Error in doublePendulumSS (line 10)
y = x(1);
```

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```
clear;clc;close all;

tfinal = 10;
dt = 0.1;
t = 0:dt:tfinal;

th1_init = -90.0 * pi/180;
th2_init = 90 * pi/180;

x0 = [0 0 th1_init 0 th2_init 0];

options = odeset('RelTol', 1E-12, 'AbsTol', 1E-12, 'InitialStep',
 dt, 'MaxStep', dt);
[t,x] = ode45(@doublePendulumSS, t, x0,options);

close all;

% a = figure();
% sgtitle('P4 & IC4','FontSize',30)
% subplot(3,2,1)
% hold on
% grid minor
% plot(t,x(:,1))
% xlabel('Time [s]','FontSize',20)
% ylabel('Cart Position','FontSize',20)
% title('Cart Position vs. Time','FontSize',20)
%
% subplot(3,2,2)
% hold on
% grid minor
% plot(t,x(:,2))
% xlabel('Time [s]','FontSize',20)
% ylabel('Cart Velocity','FontSize',20)
% title('Cart Velocity vs. Time','FontSize',20)
%
% subplot(3,2,3)
% hold on
% grid minor
% plot(t,x(:,3).*180/pi)
% xlabel('Time [s]','FontSize',20)
% ylabel('Theta [deg]','FontSize',20)
% title('\theta_1 Angular Position vs. Time','FontSize',20)
%
% subplot(3,2,4)
% hold on
% grid minor
% plot(t,x(:,4).*180/pi)
% xlabel('Time [s]','FontSize',20)
% ylabel('Angular Velocity [deg/s]','FontSize',20)
% title('\theta_1 Angular Velocity vs. Time','FontSize',20)
%
% subplot(3,2,5)
```

```
% hold on
% grid minor
% plot(t,x(:,5).*180/pi,'r')
% xlabel('Time [s]','FontSize',20)
% ylabel('Theta [deg]','FontSize',20)
% title('\theta_2 Angular Position vs. Time','FontSize',20)
%
% subplot(3,2,6)
% hold on
% grid minor
% plot(t,x(:,6).*180/pi,'r')
% xlabel('Time [s]','FontSize',20)
% ylabel('Angular Velocity [deg/s]','FontSize',20)
% title('\theta_2 Angular Velocity vs. Time','FontSize',20)

a.Position = [100 100 1400 1000];
```

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