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1.a. A = [-1 2001;-1 0]; eig(A)

ans =

-0.5000 + 44.7297i-0.5000 -44.7297i

This system is asymptotically stable because the real part of the eigen values are negative. Even though there are large imaginary components and there is damped oscillation, the negative real parts guarantee that the system will converge over time

1.6.  $A = [-1 \ 0; \ 0 \ 1];$ eig(A)

ans =

1

This system is <u>unstable</u> because all roots of a system need to be negative in order for the system to be asymptotically stable

1.c. A = [1i 1; 0 1i];eig(A)

ans =

0.0000 + 1.0000i 0.0000 + 1.0000i The system is marginally stable because it has neither positive or negative roots, and only has imaginary components. The system will oscillate forever.

However, this A matrix is defective, and  $R(\lambda) = 0$  which also defines this system as unbounded or unstable

1.8.  $A = [-2 \ 0; 0 \ 0.5];$ eig(A)

0.5000

Criteria for stability in discrete systems:  $|\lambda| \ll 1$ And if  $|\lambda| = 1$ ,  $\lambda$  must be non-defective

ans =

-2.0000

Since absolute value of -2 is 2, which is greater than 1, system is unstable



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2.

With MATLAB:

 $A = [0.5 \ 1 \ -0.5 \ 0; \ -1 \ 0.5 \ 0 \ -0.5; \ 0.5 \ 0 \ -0.5 \ 1; 0 \ 0.5 \ -1 \ -0.5];$  eig(A)

ans =

0.0000 + 1.0000i

0.0000 - 1.0000i

-0.0000 + 1.0000i

-0.0000 - 1.0000i

With MATLAB, the eigen vectors of matrix A are purely imaginary. That means the system is marginally stable.

A is not defective.

By hand:

$$A = \begin{bmatrix} \frac{1}{2} - \lambda & \frac{1}{2}$$

$$det (A-\lambda I) = (0.5-\lambda)(0.5-\lambda)(0.5-\lambda)(-0.5-\lambda)(-0.5-\lambda) - (-1) + (-\frac{1}{2})(0-(0.5-\lambda)(-0.5))$$

$$-1(-1)(-0.5-\lambda)(-0.5-\lambda) - (-1) - 0 + (-0.5)(0.5)(-1))$$

$$-0.5(-1)(0-(0.5)(1) - (0.5)((0.5)(-0.5-\lambda) - 0) - \frac{1}{2}(0.5)(0.5)$$

Solve for lambda...

Then, if  $\lambda$ <0, the system is stable.

If  $\lambda$ >0, the system is unstable.

If  $\lambda$  = imaginary, the system is marginally stable.

If there are a mix of positive and negative  $\lambda$ 's, all  $\lambda$ 's must be negative to be stable.

Answer should match the matlab answer above, marginally stable

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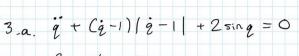
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$$e^{e} = \pi/6 \quad \dot{e} = 0 \quad \dot{e} = 0$$

$$\frac{\partial f}{\partial x_{1}} \xrightarrow{\partial f} \rightarrow f = \hat{q} = -(\hat{q}_{-1})|\hat{q}_{-1}| - 2\sin q = 8\hat{q} = -(-1)|-1| - 28q$$

$$x_{1} = 8q, \quad x_{2} = 8\hat{q}, \quad x_{1} = x_{2}$$

$$\hat{q}_{1} = -(x_{2} - 1)|x_{2} - 1| - 2\sin x_{1}$$

$$\frac{\partial f}{\partial x_1} = -2(\frac{\pi}{6}) = -\frac{\pi}{3}$$

$$\begin{cases}
f_{\text{or}} \times_{2} > (\int_{1} |x_{2} - 1| = (x_{2} - 1)) \\
\frac{\partial f}{\partial x_{2}} = -2(x_{2} - 1)
\end{cases}$$

$$\begin{cases}
\frac{\partial f}{\partial x_{2}} = -2(x_{2} - 1)
\end{cases}$$

for 
$$X_{2} < 1$$
,  $|x_{2}-1| = -(x_{2}-1) \rightarrow f = (x_{2}-1)^{2} - 2\sin x$ ,  $\frac{2f}{dx_{2}} = 2(x_{2}-1)$ 

$$x_1 = e^e = \pi/6$$
  $x_2 = e^e = 0$ 

So, for 
$$x_2 = \dot{a} > 0 \Rightarrow A = \begin{bmatrix} 0 \\ -\pi/3 \\ -2(0-1) \end{bmatrix}$$

$$A_{i} = \begin{bmatrix} 0 & 1 \\ -\pi/3 & 2 \end{bmatrix} \rightarrow eig(A_{i}) = \begin{cases} 1 + 0.2172; \\ 1 - 0.2172; \end{cases}$$

For  $\vec{q}$  greater than 0, the system is unstable as there is a positive root.

For  $m{q}$  less than 0, the system is asympotically stable as there are only negative real roots.

3.6. 
$$\dot{\varrho}_1 = e^{\varrho_1} \varrho_2 - \varrho_1^3$$

$$f_1 \rightarrow \chi_1 = e^{\chi_1} \chi_2 - \chi_1^3$$

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3.6. 
$$\dot{\varrho}_1 = e^{\varrho_1} \varrho_2 - \varrho_1^3$$

 $X_{i} = 2$ 

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \\ \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1^2 & e^{x_1} \\ -\cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1 & e^{x_1} \\ -\cos x_1 & \cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1 & e^{x_1} \\ -\cos x_1 & \cos x_2 & \cos x_1 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1 & e^{x_1} \\ -\cos x_1 & \cos x_2 & \cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1 & e^{x_1} \\ -\cos x_1 & \cos x_1 & \cos x_2 & \cos x_1 & \cos x_2 \end{bmatrix} = \begin{bmatrix} e^{x_1} \times_2 - 3x_1 & e^{x_1} \\ -\cos x_1 & \cos x_1 & \cos x_2 & \cos x_1 & \cos x_2 & \cos x_$$

fz > X2 = - X, (65 X2





eig (A) = i, -i

System is marginally stable because the roots are imaginary with no real parts. A is not defective.

3.c. 
$$\frac{2}{2} = \frac{2}{5} \ln \frac{2}{1}$$

$$\dot{x}_2 = \dot{x}_3 \in f_2$$

$$A = \begin{cases} 2f_1 & 3f_1 & 2f_1 & 3f_1 \\ \frac{1}{\delta x_1} & \frac{1}{\delta x_2} & \frac{1}{\delta x_3} & \frac{1}{\delta x_4} \end{cases}$$

$$A = \begin{cases} 2f_3 & \frac{1}{\delta x_1} & \frac{1}{\delta x_2} & \frac{1}{\delta x_3} & \frac{1}{\delta x_4} \\ \frac{1}{\delta x_1} & \frac{1}{\delta x_1} & \frac{1}{\delta x_2} & \frac{1}{\delta x_3} & \frac{1}{\delta x_4} \end{cases}$$

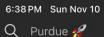
$$Cos x_1 & 0 & 0 & 0 & 0$$

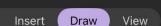
$$Cos x_1 & 0 & 0 & 0 & 0$$

eig(A) =

- -1.0000 + 0.0000i
- 0.0000 1.0000i
- 1.0000 + 0.0000i

System is unstable because there is a root with a positive real part.











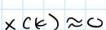












$$\begin{bmatrix} \chi_1(k+1) \\ \chi_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.5 & 0 \end{bmatrix} \begin{bmatrix} \chi_1(k) \\ \chi_2(k) \end{bmatrix}$$

