1.

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdots \\ \dot{x}_n \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\cdots$$
$$\dot{x}_{n-1} = x_n$$
$$\dot{x}_n = a(x_1, x_2 \ldots x_n, u)$$
$$\dot{x}_n = a_{n-1} q^{(n-1)} \ldots a_1 \dot{q} + a_0 q + u$$

2i

$x_1 := q_1 \quad x_2 := \dot{q}_1$

$x_3 := q_2 \quad x_4 := \dot{q}_2$

$$\dot{x}_1 = x_2$$
$$\dot{x}_3 = x_4$$

$$2\dot{x}_2 + \dot{x}_4 + \sin x_1 = 0 \rightarrow \dot{x}_4 = -2\dot{x}_2 - \sin x_1 \quad ①$$
$$\dot{x}_2 + 2\dot{x}_4 + \sin x_3 = 0 \rightarrow \dot{x}_2 = -2\dot{x}_4 - \sin x_3 \quad ②$$

② → ①

$$\dot{x}_4 = -2(-2\dot{x}_4 - \sin x_3) - \sin x_1$$
$$\dot{x}_4 = 4\dot{x}_4 + 2\sin x_3 - \sin x_1$$
$$-3\dot{x}_4 = 2\sin x_3 - \sin x_1$$
$$\dot{x}_4 = -\frac{2}{3}\sin x_3 + \frac{1}{3}\sin x_1$$

① → ②

$$\dot{x}_2 = -2(-2\dot{x}_2 - \sin x_1) - \sin x_3$$
$$\dot{x}_2 = 4\dot{x}_2 + 2\sin x_1 - \sin x_3$$
$$-3\dot{x}_2 = 2\sin x_1 - \sin x_3$$
$$\dot{x}_2 = -\frac{2}{3}\sin x_1 + \frac{1}{3}\sin x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{2}{3}\sin x_1 + \frac{1}{3}\sin x_3 \\ 0 \\ -\frac{2}{3}\sin x_3 + \frac{1}{3}\sin x_1 \end{bmatrix}$$

2ii

① $\ddot{q}_1 + \dot{q}_2 + q_1^3 = 0$

② $\dot{q}_1 + \dot{q}_2 + q_2^3 = 0$

$x_1 = q_1 \quad x_2 = \dot{q}_1$

$x_3 = q_2$

$$\dot{x}_1 = x_2$$

②

$$x_2 + \dot{x}_3 + x_3^3 = 0$$
$$\dot{x}_3 = -x_2 - x_3^3$$

①

$$\dot{x}_2 + \dot{x}_3 + x_1^3 = 0$$
$$\dot{x}_2 = -\dot{x}_3 - x_1^3$$
$$\dot{x}_2 = x_2 + x_3^3 - x_1^3$$

3.

$x_1 := q_1 \quad x_2 := \dot{q}_1$

$x_3 := q_2 \quad x_4 := \dot{q}_2$

$$\boxed{\begin{aligned} &\dot{x}_1 = x_2 \qquad \dot{x}_3 = x_4 \\ &\dot{x}_2 = -x_4 - \sin x_1 + u \\ &x_4 = -x_1 - x_3 \\ &y = x_1 + x_3 \end{aligned}}$$

4.

$x_1(k) = q(k)$

$x_2(k) = q(k+1)$

$x_3(k) = q(k+2)$

$$\boxed{\begin{aligned} &x_1(k+1) = x_2(k) \\ &x_2(k+1) = x_3(k) \\ &x_3(k+1) = -7x_3(k) - x_2(k) - 6x_1(k) + 7u(k) \end{aligned}}$$

5.

$$\boxed{\begin{aligned} &x_1(k+1) = x_2(k) \\ &x_2(k+1) = x_3(k) \\ &\quad \cdots\cdots \\ &x_n(k+1) = a_{n-1}q(k+n-1) \ldots a_1 q(k+1) + a_0 q(k) \end{aligned}}$$

6. $x_1(k) = q_1(k) \longrightarrow \boxed{x_1(k+1) = x_2(k)}$

$x_2(k) = q_1(k+1)$

$x_3(k) = q_2(k)$

$x_2(k+1) + x_3(k+1) + x_1(k) = u(k) \rightarrow x_2(k+1) = -x_3(k+1) - x_1(k) + u(k)$ (1)

$x_2(k+1) - x_3(k+1) + x_3(k) = 0 \rightarrow x_3(k+1) = x_3(k) + x_2(k+1)$ (2)

$$\boxed{y(k) = x_2(k) + x_3(k)}$$

(1) $x_2(k+1) = \overset{(2)}{\left[-x_3(k) - x_2(k+1)\right]} - x_1(k) + u(k)$

$2x_2(k+1) = -x_3(k) - x_1(k) + u(k)$

$$\boxed{x_2(k+1) = -\tfrac{1}{2}x_3(k) - \tfrac{1}{2}x_1(k) + \tfrac{1}{2}u(k)}$$

(2) $x_3(k+1) = x_3(k)\overset{(1)}{\left[-x_3(k+1) - x_1(k) + u(k)\right]}$

$2x_3(k+1) = x_3(k) - x_1(k) + u(k)$

$$\boxed{x_3(k+1) = \tfrac{1}{2}x_3(k) - \tfrac{1}{2}x_1(k) + \tfrac{1}{2}u(k)}$$

7.

$$
\begin{array}{ccccccc}
M & \ddot{q} & & G & & W\,u \\
\begin{bmatrix} m_0+m_1+m_2 & m_1 l_1 \cos\theta_1 & m_2 l_2 \cos\theta_2 \\ -m_1 l_1 \cos\theta_1 & m_1 l_1^2 & 0 \\ -m_2 l_2 \cos\theta_2 & 0 & m_2 l_2^2 \end{bmatrix} & \begin{bmatrix} \ddot{y} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} & + & \begin{bmatrix} m_1 l_1 \sin\theta_1 \dot{\theta}_1^2 + m_2 l_2 \sin\theta_2 \dot{\theta}_2^2 \\ m_1 l_1 g \sin\theta \\ m_2 l_2 g \sin\theta_2 \end{bmatrix} & = & \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}
\end{array}
$$

$$
\begin{bmatrix} \ddot{y} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = INV\left(\begin{bmatrix} m_0+m_1+m_2 & m_1 l_1 \cos\theta_1 & m_2 l_2 \cos\theta_2 \\ -m_1 l_1 \cos\theta_1 & m_1 l_1^2 & 0 \\ -m_2 l_2 \cos\theta_2 & 0 & m_2 l_2^2 \end{bmatrix}\right)\left(\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} m_1 l_1 \sin\theta_1 \dot{\theta}_1^2 + m_2 l_2 \sin\theta_2 \dot{\theta}_2^2 \\ m_1 l_1 g \sin\theta_1 \\ m_2 l_2 g \sin\theta_2 \end{bmatrix}\right)
$$