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1.a.

```
A = [-1 2001;-1 0];
eig(A)
```

This system is asymptotically stable because the real part of the eigen values are negative. Even though there are large imaginary components and there is damped oscillation, the negative real parts guarantee that the system will converge over time

```
ans =
```

```
-0.5000 +44.7297i
-0.5000 -44.7297i
```

1.b.

```
A = [-1 0; 0 1];
eig(A)
```

This system is unstable because all roots of a system need to be negative in order for the system to be asymptotically stable

```
ans =
```

```
-1
1
```

1.c.

```
A = [1i 1; 0 1i];
eig(A)
```

The system is marginally stable because it has neither positive or negative roots, and only has imaginary components. The system will oscillate forever.

However, this A matrix is defective, and $R(\lambda) = 0$ which also defines this system as unbounded or unstable

```
ans =
```

```
0.0000 + 1.0000i
0.0000 + 1.0000i
```

1.d.

```
A = [-2 0;0 0.5];
eig(A)
```

Criteria for stability in discrete systems:
 $|\lambda| \leq 1$
And if $|\lambda| = 1$, λ must be non-defective

```
ans =
```

Since absolute value of -2 is 2, which is greater than 1, system is unstable

```
-2.0000
0.5000
```

T Q ↴ ↵



2.

With MATLAB:

```
A = [0.5 1 -0.5 0; -1 0.5 0 -0.5; 0.5 0 -0.5 1; 0 0.5 -1 -0.5];
eig(A)
```

ans =

```
0.0000 + 1.0000i
0.0000 - 1.0000i
-0.0000 + 1.0000i
-0.0000 - 1.0000i
```

With MATLAB, the eigen vectors of matrix A are purely imaginary.
That means the system is marginally stable.

A is not defective.

By hand:

$$\lambda_A = \det(A - \lambda I) = 0$$

$$A = \begin{bmatrix} \frac{1}{2} - \lambda & & & & \\ & \frac{1}{2} - \lambda & & & \\ & & -\frac{1}{2} - \lambda & & \\ & & & -\frac{1}{2} - \lambda & \\ & & & & \end{bmatrix}$$

$$\begin{aligned} \det(A - \lambda I) &= (0.5 - \lambda) \left[(0.5 - \lambda) \left[(-0.5 - \lambda)(0.5 - \lambda) - (-1) \right] + \left(-\frac{1}{2} \right) (0 - (0.5 - \lambda)(-0.5)) \right] \\ &\quad - 1 \left(-1 \left[(-0.5 - \lambda)(-0.5 - \lambda) - (-1) \right] - 0 + (-0.5) ((0.5)(-1)) \right] \\ &\quad - 0.5 \left(-1 \left[0 - (0.5)(1) - (0.5)((0.5)(-0.5 - \lambda)) - 0 \right] - \frac{1}{2}(0.5)(0.5) \right] \end{aligned}$$

Solve for lambda...

Then, if $\lambda < 0$, the system is stable.If $\lambda > 0$, the system is unstable.If $\lambda = \text{imaginary}$, the system is marginally stable.If there are a mix of positive and negative λ 's, all λ 's must be negative to be stable.

Answer should match the matlab answer above, marginally stable



$$3.a. \ddot{q} + (\dot{q}-1)|\dot{q}-1| + 2\sin q = 0$$

$$\cos(\theta) = 1 \text{ when } \theta = 0$$

$$\sin(\theta) \approx \theta \text{ when } \theta = 0$$

$$q^0 = \pi/6 \quad \ddot{q} = 0 \quad \dot{q} = 0$$

$$\left[\frac{\partial f}{\partial x_1} \quad \frac{\partial f}{\partial x_2} \right] \rightarrow f = \ddot{q} = -(\dot{q}-1)|\dot{q}-1| - 2\sin q = \ddot{x}_2 = -(-1)|-1| - 2\sin x_1$$

$$x_1 = \delta_2, \quad x_2 = \delta \dot{q}, \quad \dot{x}_1 = x_2$$

$$\ddot{x}_2 = -(|x_2-1| - 2\sin x_1)$$

$$\left[\frac{\partial f}{\partial x_1} = -2(\pi/6) = -\frac{\pi}{3} \right]$$

$$\text{for } x_2 > 1, |x_2-1| = (x_2-1) \rightarrow f = -(x_2-1)^2 - 2\sin x_1$$

$$\left[\frac{\partial f}{\partial x_2} = -2(x_2-1) \right]$$

$$\text{for } x_2 < 1, |x_2-1| = -(x_2-1) \rightarrow f = (x_2-1)^2 - 2\sin x_1$$

$$\left[\frac{\partial f}{\partial x_2} = 2(x_2-1) \right]$$

$$x_1 = q^0 = \pi/6 \quad x_2 = \dot{q} = 0$$

$$\text{So, for } x_2 = \dot{q} > 0 \rightarrow A = \begin{bmatrix} 0 & 1 \\ -\pi/3 & -2(0-1) \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -\pi/3 & 2 \end{bmatrix} \rightarrow \text{eig}(A_1) = \frac{1+0.2172i}{1-0.2172i}$$

$$\text{for } x_2 = \dot{q} < 0 \rightarrow A_2 = \begin{bmatrix} 0 & 1 \\ -1.732 & -2 \end{bmatrix} \rightarrow \text{eig}(A_2) = \frac{-1+0.8556i}{-1-0.8556i}$$

For \dot{q} greater than 0, the system is unstable as there is a positive root.

For \dot{q} less than 0, the system is asymptotically stable as there are only negative real roots.

$$3.b. \quad \dot{x}_1 = e^{x_1} x_2 - x_1^3 \quad x_1 = q_1$$

$$\dot{x}_2 = -q_1 \cos q_2 \quad x_2 = \dot{q}_2$$

$$f_1 \rightarrow \dot{x}_1 = e^{x_1} x_2 - x_1^3$$

$$f_2 \rightarrow \dot{x}_2 = -x_1 \cos x_2$$

$$3.b. \quad \dot{q}_1 = e^{x_1} q_2 - q_1^3 \quad x_1 = q_1 \\ \dot{q}_2 = -q_1 \cos q_2 \quad x_2 = q_2$$

$$f_1 \rightarrow \dot{x}_1 = e^{x_1} x_2 - x_1^3$$

$$f_2 \rightarrow \dot{x}_2 = -x_1 \cos x_2 \quad q_1^e = q_2^e = 0$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} e^{x_1} x_2 - 3x_1^2 & e^{x_1} \\ -\cos x_2 & x_1 \sin x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\text{eig}(A) = i, -i$$

System is marginally stable because the roots are imaginary with no real parts.
A is not defective.

$$3.c. \quad \ddot{q}_1 = q_2 \quad x_1 = q_1 \quad \dot{x}_1 = x_2 \leftarrow f_1 \\ \ddot{q}_2 = \sin q_1 \quad x_2 = \dot{q}_1 \\ \dot{x}_2 = x_3 \quad \leftarrow f_2 \\ \ddot{x}_4 = \sin x_1 \quad \leftarrow f_4 \quad q_1 = q_2 = 0 \\ \dot{x}_3 = q_2 \quad \dot{x}_4 = \dot{q}_2$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \dots & \dots & \dots \\ \frac{\partial f_3}{\partial x_1} & & & \\ \frac{\partial f_4}{\partial x_1} & & & \dots \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \cos x_1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$\text{eig}(A) =$

```
ans =
-1.0000 + 0.0000i
0.0000 + 1.0000i
0.0000 - 1.0000i
1.0000 + 0.0000i
```

System is unstable because there is a root with a positive real part.



$$x(k) \approx 0$$

$$x_1(k) = 0 \quad x_2(k) = 0$$

$$\cos(x(k)) \approx 1$$

$$\sin(x(k)) \approx x(k)$$

4. $x_1(k+1) = x_1(k)^3 + \sin(x_2(k))$

Higher order term

$$x_1(k+1) = x_2(k)$$

$$x_2(k+1) = -\frac{1}{2} \cos(x_2(k)) x_1(k) + x_2(k)^3$$

H.O.T.

$$x_2(k+1) = -\frac{1}{2} x_1(k)$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.5 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$\text{eig}(A) =$

```
ans =
0.0000 + 0.7071i
0.0000 - 0.7071i
```

System is marginally stable because it only has purely imaginary roots



5.

Computing the A matrices of the linearized two pendulum two cart system with L1 and L2 properties, and then computing the eigen values:

A =

```

    0      0      0   1.0000      0      0
    0      0      0      0   1.0000      0
    0      0      0      0      0   1.0000
 -0.2500  -0.5000  -0.5000  -2.5000      0      0
 -0.2500  -1.5000  -0.5000  -2.5000      0      0
 -0.2500  -0.5000  -1.5000  -2.5000      0      0

```

L1

```

-1.9694 + 0.0000i
-0.1095 + 0.0000i
-0.2105 + 1.0557i
-0.2105 - 1.0557i
-0.0000 + 1.0000i
-0.0000 - 1.0000i

```

We see that for L1, there are only negative real part roots but also imaginary components. Therefore, the system is stable

A =

```

    0      0      0   1.0000      0      0
    0      0      0      0   1.0000      0
    0      0      0      0      0   1.0000
 -0.2500  -0.5000  -0.5000  -2.5000      0      0
  0.2500   1.5000   0.5000   2.5000      0      0
  0.2500   0.5000   1.5000   2.5000      0      0

```

L2

```

-2.8174
 1.1380
-0.1097
-0.7109
-1.0000
 1.0000

```

We see that for L2, there are positive real part roots, therefore the system is unstable.

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HW 9 P1

a

```
clear;clc;close all

% A = [-1 1;0 -1];
% eig(A)

% A = [-1 1;-1 -1];
% eig(A)

A = [-1 2001;-1 0];
[V, D] = eig(A)

rankV = rank(V);
sizeA = size(A, 1); % Number of rows/columns of A
if rankV < sizeA
    disp('The matrix A is defective.');
else
    disp('The matrix A is not defective.');
end

V =
```

```
0.9998 + 0.0000i  0.9998 + 0.0000i
0.0002 + 0.0223i  0.0002 - 0.0223i
```

D =

```
-0.5000 +44.7297i  0.0000 + 0.0000i
0.0000 + 0.0000i  -0.5000 -44.7297i
```

The matrix A is not defective.

b

```
clear;clc;close all

A = [-1 0; 0 1];
[V, D] = eig(A)

rankV = rank(V);
sizeA = size(A, 1); % Number of rows/columns of A
if rankV < sizeA
    disp('The matrix A is defective.');
else
    disp('The matrix A is not defective.');
end
```

V =

$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

D =

$$\begin{matrix} -1 & 0 \\ 0 & 1 \end{matrix}$$

The matrix A is not defective.

c

```
clear;clc;close all

A = [1i 1; 0 1i];
[V, D] = eig(A)

rankV = rank(V);
sizeA = size(A, 1); % Number of rows/columns of A
if rankV < sizeA
    disp('The matrix A is defective.');
else
    disp('The matrix A is not defective.');
end
```

V =

$$\begin{matrix} 1.0000 & 1.0000 \\ 0 & -0.0000 \end{matrix}$$

D =

```
0.0000 + 1.0000i  0.0000 + 0.0000i
0.0000 + 0.0000i  0.0000 + 1.0000i
```

The matrix A is defective.

d

```
clear;clc;close all

A = [-2 0;0 0.5];
[V, D] = eig(A)

rankV = rank(V);
sizeA = size(A, 1); % Number of rows/columns of A
if rankV < sizeA
    disp('The matrix A is defective.');
else
    disp('The matrix A is not defective.');
end
```

$V =$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$D =$

$$\begin{pmatrix} -2.0000 & 0 \\ 0 & 0.5000 \end{pmatrix}$$

The matrix A is not defective.

P2

```
clear;clc;close all

A = [0.5 1 -0.5 0; -1 0.5 0 -0.5; 0.5 0 -0.5 1;0 0.5 -1 -0.5];
[V, D] = eig(A)

rankV = rank(V);
sizeA = size(A, 1); % Number of rows/columns of A
if rankV < sizeA
    disp('The matrix A is defective.');
else
    disp('The matrix A is not defective.');
end

V =
```

$$\begin{pmatrix} 0.5000 + 0.0000i & 0.5000 + 0.0000i & -0.5000 - 0.0000i & -0.5000 + 0.0000i \end{pmatrix}$$

$$\begin{array}{cccc} -0.0000 + 0.5000i & -0.0000 - 0.5000i & 0.0000 - 0.5000i & 0.0000 + 0.5000i \\ 0.5000 + 0.0000i & 0.5000 - 0.0000i & -0.5000 + 0.0000i & -0.5000 + 0.0000i \\ -0.0000 + 0.5000i & -0.0000 - 0.5000i & 0.0000 - 0.5000i & 0.0000 + 0.5000i \end{array}$$

$D =$

$$\begin{array}{cccc} 0.0000 + 1.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 0.0000 - 1.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & -0.0000 + 1.0000i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & -0.0000 - 1.0000i \end{array}$$

The matrix A is not defective.

P3.a.

```
clear;clc;close all

% A1 = [0 1;-1.732 2];
A1 = [0 1;-pi/3 2];
[V, D] = eig(A1)

rankV = rank(V);
sizeA1 = size(A1, 1); % Number of rows/columns of A
if rankV < sizeA1
    disp('The matrix A1 is defective.');
else
    disp('The matrix A1 is not defective.');
end

A2 = [0 1;-1.732 -2];
[V, D] = eig(A2)

rankV = rank(V);
sizeA2 = size(A2, 1); % Number of rows/columns of A
if rankV < sizeA2
    disp('The matrix A2 is defective.');
else
    disp('The matrix A2 is not defective.');
end

V =
0.6830 - 0.1484i  0.6830 + 0.1484i
0.7152 + 0.0000i  0.7152 + 0.0000i

D =
1.0000 + 0.2172i  0.0000 + 0.0000i
0.0000 + 0.0000i  1.0000 - 0.2172i
```

The matrix A1 is not defective.

V =

$$\begin{array}{cc} -0.4597 - 0.3933i & -0.4597 + 0.3933i \\ 0.7962 + 0.0000i & 0.7962 + 0.0000i \end{array}$$

D =

$$\begin{array}{cc} -1.0000 + 0.8556i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & -1.0000 - 0.8556i \end{array}$$

The matrix A2 is not defective.

P3.b.

```
clear;clc;close all
```

```
A = [0 1;-1 0];
[V, D] = eig(A)
```

```
rankV = rank(V);
sizeA = size(A, 1); % Number of rows/columns of A
if rankV < sizeA
    disp('The matrix A is defective.');
else
    disp('The matrix A is not defective.');
end
```

V =

$$\begin{array}{cc} 0.7071 + 0.0000i & 0.7071 + 0.0000i \\
0.0000 + 0.7071i & 0.0000 - 0.7071i \end{array}$$

D =

$$\begin{array}{cc} 0.0000 + 1.0000i & 0.0000 + 0.0000i \\
0.0000 + 0.0000i & 0.0000 - 1.0000i \end{array}$$

The matrix A is not defective.

P3.c.

```
clear;clc;close all
```

```
A = [0 1 0 0; 0 0 1 0; 0 0 0 1; 1 0 0 0];
[V, D] = eig(A)
```

```
rankV = rank(V);
sizeA = size(A, 1); % Number of rows/columns of A
```

```

if rankV < sizeA
    disp('The matrix A is defective.');
else
    disp('The matrix A is not defective.');
end

V =
0.5000 + 0.0000i  0.5000 + 0.0000i  0.5000 - 0.0000i  0.5000 + 0.0000i
-0.5000 + 0.0000i  0.0000 + 0.5000i  0.0000 - 0.5000i  0.5000 + 0.0000i
0.5000 + 0.0000i  -0.5000 + 0.0000i  -0.5000 + 0.0000i  0.5000 + 0.0000i
-0.5000 + 0.0000i  0.0000 - 0.5000i  0.0000 + 0.5000i  0.5000 + 0.0000i

D =
-1.0000 + 0.0000i  0.0000 + 0.0000i  0.0000 + 0.0000i  0.0000 + 0.0000i
0.0000 + 0.0000i  0.0000 + 1.0000i  0.0000 + 0.0000i  0.0000 + 0.0000i
0.0000 + 0.0000i  0.0000 + 0.0000i  0.0000 - 1.0000i  0.0000 + 0.0000i
0.0000 + 0.0000i  0.0000 + 0.0000i  0.0000 + 0.0000i  1.0000 + 0.0000i

```

The matrix A is not defective.

P4

```

clear;clc;close all

A = [0 1; -0.5 0];
[V, D] = eig(A)

rankV = rank(V);
sizeA = size(A, 1); % Number of rows/columns of A
if rankV < sizeA
    disp('The matrix A is defective.');
else
    disp('The matrix A is not defective.');
end

V =
0.8165 + 0.0000i  0.8165 + 0.0000i
0.0000 + 0.5774i  0.0000 - 0.5774i

D =
0.0000 + 0.7071i  0.0000 + 0.0000i
0.0000 + 0.0000i  0.0000 - 0.7071i

```

The matrix A is not defective.

P5

```
clear;clc;close all

c = 5;
k = 0.5;

% Define symbolic variables
syms m0 m1 m2 l1 l2 thetal_e theta2_e g real
% Define the matrix M
M = [m0 + m1 + m2, -m1*l1*cos(thetal_e), -m2*l2*cos(theta2_e);
      -m1*l1*cos(thetal_e), m1*l1^2, 0;
      -m2*l2*cos(theta2_e), 0, m2*l2^2];

% Invert the matrix
K = [-k 0 0;
      0 -m1*l1*g*cos(thetal_e) 0;
      0 0 -m2*l2*g*cos(theta2_e)];

C = [-c 0 0; 0 0 0; 0 0 0];

Phi = [1; 0; 0];

A_top = [zeros(3), eye(3)];
A_bottom = [M\K, M\C];
A = [A_top; A_bottom];

B = [0; 0; 0; M\Phi];

A = double( subs(A, {m0, m1, m2, l1, l2, g, thetal_e, theta2_e}, {2, 1, 1, 1,
1, 1, 0, 0}));
B = double( subs(B, {m0, m1, m2, l1, l2, g, thetal_e, theta2_e}, {2, 1, 1, 1,
1, 1, 0, 0}));
disp('L1')
disp(eig(A))
% A = double( subs(A, {m0, m1, m2, l1, l2, g, thetal_e, theta2_e}, {2, 1, 1,
1, 1, 1, pi, pi}))
% B = double( subs(B, {m0, m1, m2, l1, l2, g, thetal_e, theta2_e}, {2, 1, 1,
1, 1, 1, pi, pi}));
% disp('L2')
% disp(eig(A))

% Simulate Linearized system

% Initial conditions
y0 = 0; % Initial position of the cart
y_dot0 = 0; % Initial velocity of the cart
theta0 = 1 * pi/180; % Initial angle of the pendulum (45 degrees)
theta_dot0 = 0; % Initial angular velocity of the pendulum
initial_conditions = [y0; y_dot0; 0; theta0; theta_dot0; 0];

% Time span
```

```

t_span = [0 60]; % Simulate for 10 seconds

options = odeset('RelTol', 1e-10,'AbsTol',1e-15);
[t, X] = ode45(@(t, x, A, B) linearizedTwoPendulumTwoCart(t, x, A, B),
t_span, initial_conditions,options, A, B);

a = figure();
subplot(2,1,1);
hold on
grid minor
plot(t, X(:,1), 'LineWidth',2); % Cart position
title('Cart Position Over Time');
xlabel('Time (s)');
ylabel('Position (y)');
ax = gca;
ax.FontSize = 16;
subplot(2,1,2);
hold on
grid minor
plot(t, X(:,3)*180/pi, 'LineWidth',2); % Pendulum angle
title('Pendulum Angle Over Time');
xlabel('Time (s)');
ylabel('Angle [degrees]');
a.Position = [100 100 1400 1000];
ax = gca;
ax.FontSize = 16;

A =

```

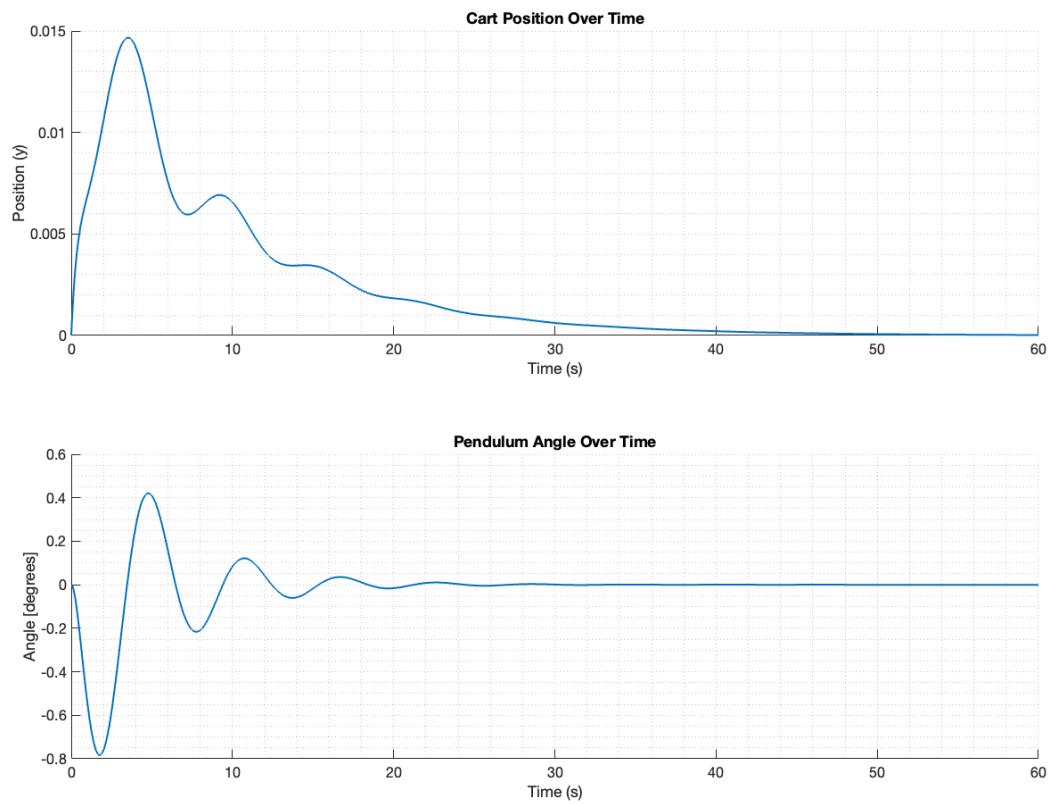
0	0	0	1.0000	0	0
0	0	0	0	1.0000	0
0	0	0	0	0	1.0000
-0.2500	-0.5000	-0.5000	-2.5000	0	0
-0.2500	-1.5000	-0.5000	-2.5000	0	0
-0.2500	-0.5000	-1.5000	-2.5000	0	0

L1

```

-1.9694 + 0.0000i
-0.1095 + 0.0000i
-0.2105 + 1.0557i
-0.2105 - 1.0557i
-0.0000 + 1.0000i
-0.0000 - 1.0000i

```



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..... 1

```
function dx = linearizedTwoPendulumTwoCart(t,x,A,B)
```

```
u = 0;
```

```
dx = A * x + B * u;
```

```
end
```

Not enough input arguments.

Error in linearizedTwoPendulumTwoCart (line 5)

```
dx = A * x + B * u;
```

```
^
```

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