# Chapter 1

# Proofs

## Mathematical Statements

A *mathematical statement* is an English statement that has a truth value.

**Types of Statements** Compound statements, Implications, Double implications, Converse of an Implications, Negations, and Quantifiers.

### Compound Statement ($P$ and $Q$)

A *compound statement* is a statement constructed from two statements joined be the words "and" or "or".

**Example 1.0.1** (Compound Statement)**.** Let $x$ be a real number.

$P$ : Then number $x$ is greater than 3.

$Q$ : Then number $x$ is even.

$P$ and $Q$ : The number $x$ is greater than 3 **and** $x$ is even.

$P$ and $Q$ : The number $x$ is greater than 3 **or** $x$ is even.

**Question:** What are the truth values of $P$ and $Q$ and $P$ and $Q$?

1. If $x = 6$? $P$ and $Q$, $P$ or $Q$ are true.
2. If $x = 5$? $P$ and $Q$ is false. $P$ or $Q$ is true.
3. If $x < 3$? $P$ and $Q$ is false. , $P$ or $Q$ depends on the value of $x$.

## Implication ($P \to Q$)

The mathematical statement "$P$ implies $Q$" is an implication where $P$ is the hypothesis and $Q$ is the conclusion. Other forms of an implication are "If $P$ then $Q$.", " $P \to Q$", and "$P \Rightarrow Q$".

**Example 1.0.2.** If $x$ is greater than 0, then $x^2$ is greater than 0. Here "$x$ is greater than 0" is the hypothesis and "$x^2$ is greater than 0" is the conclusion.

## Converse of an implication ($Q \to P$)

The converse of " $P$ implies $Q$" is " $Q$ implies $P$".

**Example 1.0.3. Converse of previous example**

If $x^2$ is greater than 0, then $x$ is greater than 0.