

## Composition

Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$ . The function  $g \circ f$  from  $A$  to  $C$  is the composition of  $g$  and  $f$ . The function is defined in the following way,  $(g \circ f)(a) = g(f(a))$  where  $a \in A$ .

**Problem 3.12.** *Using the definition of function composition, verify that  $g \circ f$  is a function from  $A$  to  $C$ .*

**Example 3.13.** Let  $f = \{(1, a), (2, b), (3, c)\}$  and  $g = \{(a, x), (b, y), (c, z)\}$ . Then  $g \circ f = \{(1, x), (2, y), (3, z)\}$ .

**Problem 3.14.** Let  $A = \{1, 2, 3, 4\}$ ,  $B = \{a, b, c, d\}$  and  $C = \{r, s, t, u, v\}$  and define the functions  $f : A \rightarrow B$  and  $g : B \rightarrow C$  by

$$f = \{(1, b), (2, d), (3, a), (4, a)\} \text{ and } g = \{(a, u), (b, r), (c, r), (d, s)\}.$$

Determine  $g \circ f$  and  $(g \circ f)(1)$ .

**Theorem 3.15.** *The functions  $f : A \rightarrow B$  and  $g : B \rightarrow A$  are inverses of each other if and only if*

$$(g \circ f)(a) = a \text{ and } (f \circ g)(b) = b$$

*for all  $a \in A$  and for all  $b \in B$ .*