Homework Suggested Problems

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sec 0.1 : {4b,5k,6b,7c}
sec 0.2 : {29}
sec 2.1 : {3c,6a,11,14b}
sec 2.2 : {5b,8c,12b}
sec 3.1 : {11 correction:floor(x) + n = floor(x+n) and n is an integer,31a}
sec 3.2 : {20a,23b,23c,24c}
sec 3.3 : {6,22b}
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Problems from the Notes

Problem 1. *Use a truth table to show that* $(P \to Q) \wedge (Q \to R) \to (P \to R)$.

Problem 2. *Prove that if* x *is odd then* x^2 *is odd.*

Problem 3. *Let* x *be an integer. Prove that* $x^2 - 3x + 9$ *is odd.*

Problem 4. *Prove that if* x^2 *is even then* x *is even.*

Problem 5. *Prove that no odd integer can be expressed as the sum of three even integers.*

Problem 6. *Disprove the following statement. For all positive integers* x*, if* $\frac{x(x+1)}{2}$ *is odd then* $\frac{(x+1)(x+2)}{2}$ *is odd.*

Problem 7. *Is the statement "Let* x *be a real number." a mathematical statement?*

Problem 8. *Write out the elements of the following sets.*

1. $\{x | x^2 + 2x - 3 = 0\}$
2. $\{\{\}, 1, \{1, 2, 3\}\}$

Problem 9 (The Division Algorithm)**.** *Let* $a, b \in \mathbb{Z}, b \neq 0$*. Then there exist unique integers* q *and* r*, with* $0 \leq r < |b|$ *such that* $a = qb + r$.

Problem 10. *Which of the common sets are supersets of* $\mathbb{I}$*? Which of the common sets are subsets of* $\mathbb{I}$*?*

Problem 11. *Write the power set for the set* $\{\{1, 2\}, 3, \{\}\}$.

Problem 12. *How many elements are in the power set of a set containing exactly three elements?*

Problem 13. *Let* $A, B,$ *and* C *be sets such that* $A \in B$*. Prove that if* $B \subseteq C$ *then* $A \in C$.

Problem 14. *Let* $A = \{a, b, c\}$ *and* $B = \{A, b, 3\}$*. Find* $A \cup B$ *and* $A \cap B$.

Problem 15. *Make a Venn diagram for the sets* $A = \{1, 2, 3\}$*,* $B = \{1, 4, 5\}$*, and* $C = \{2, 5, 7\}$.

Problem 16. *Prove that for any set* A *and* B*,* $(A \cap B)^c = A^c \cup B^c$.

Problem 17. *Suppose* $a, b,$ *and* c *are integers such that* $c \mid a$ *and* $c \mid b$*. Show that* $c \mid (ax + yb)$ *for any integers* x *and* y.

Problem 18. *Let* $A = \{1, 2, 3\}$ *and* $B = \{a, b, c, d\}$*. Give an example of a relation from* A *to* B *containing exactly three elements such that the relation is not a function from* A *to* B.

Problem 19. *Let* $A = \{a, b, c, d\}$ *and* $B = \{x, y, z\}$*. Then* $f\{(a, y), (b, z), (c, y), (d, z)\}$ *is a function from* A *to* B*. Determine* $\operatorname{dom} f$ *and* $\operatorname{rng} f$.

Problem 20. *Show that the function* $\{(x_1, x_2) \mid x_1, x_2 \in \mathbb{Z}, x_1^2 = x_2\}$ *is one-to-one.*

Problem 21. *Using the definition of function composition, verify that $g \circ f$ is a function from A to C.*

Problem 22. *Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$ and $C = \{r, s, t, u, v\}$ and define the functions $f : A \to B$ and $g : B \to C$ by*

$$f = \{(1, b), (2, d), (3, a), (4, a)\} \text{ and } g = \{(a, u), (b, r), (c, r), (d, s)\}.$$

Determine $g \circ f$ and $(g \circ f)(1)$.

Problem 23. *Suppose f is a one-to-one and onto function from $\mathbb{N} \to \mathbb{Z}$. Prove that the function g from $\mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{Z}$ defined by $g : (m, n) \mapsto (m, f(n))$ is one-to-one and onto.*