The *power set* of the set A, denoted by $\mathcal{P}(A)$, is the set of all subsets of A.

Example 2.6. Let $A = \{1, 2, 3\}$. Then

$$\mathcal{P}(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.$$

Since the empty set and the set A are subsets of A they are listed in $\mathcal{P}(A)$.

Problem 2.7. *Write the power set for the set* $\{\{1, 2\}, 3, \{\}\}$.

Problem 2.8. *How many elements are in the power set of a set containing exactly three elements?*

To show that a set A is a subset of the set B is suffices to show that every element in A is in B. One way to do this is to pick an arbitrary element in A, say x, and show that $x \in B$. Since x is an arbitrary element in A it is the case that all elements of A are elements of B.

Theorem 2.9. *For any set A, $A \subseteq A$ and $\{\} \subseteq A$.*

Proof. <u>Direct Proof</u>

For every $x \in A$ it is the case that $x \in A$. Therefore, by definition of subsets, $A \subseteq A$.

<u>Proof by Contradiction</u>

Suppose $\{\} \nsubseteq A$. By definition, there exists $x \in \{\}$ such that $x \notin A$. However, this contradicts that the empty set has no elements. This shows that $\{\} \nsubseteq A$ is false which implies that $\{\} \subseteq A$ is true. $\square$