Truth Tables

Truth tables help us determine the validity of a statement. Truth tables give us a way to construct equivalent statements.

Let P and Q be mathematical statements. A table listing the possible truth values of each statement is called a truth table. For simplicity, instead of writing the word "and"("or") to join to statements we will used the symbols $\wedge(\vee)$ respectively.

Example 1.0.10. The following is a truth table which can be used to determine the value of the statement $P \wedge Q$.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Example 1.0.11. Below are the truth values of some frequently used statements.

P	Q	$\neg P$	$P \vee Q$	$P \to Q$
T	T	F	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

Consider the more complexed statement $\neg Q \to \neg P$ which is the contrapositive of $P \to Q$. A truth table can be used to show that the two statements are actually equivalent. To do this we must show that the two statements have the exact same true values which are independent of the values of P and Q.

Example 1.0.12. The statements $\neg Q \to \neg P$ and $P \to Q$ are equivalent statements because they have the same values in their columns.

P	Q	$\neg P$	$\neg Q$	$P \to Q$	$\neg Q \to \neg P$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

In some cases, proving an equivalent statements may be easier than proving the actual statement.

Problem 1.0.13. *Use a truth table to show that* $(P \to Q) \wedge (Q \to R) \to (P \to R)$.