

# Chapter 2

## Sets

### Sets

A *set* is a collection of elements. Sets are typically represented by a left curly brace before the first element of the list and a right curly brace after the last element of the list. A definition for the elements of a set can also be used to describe a set. The *empty set* is a set with no elements. The empty set can be denoted by  $\emptyset$  or  $\{\}$ .

**Example 2.1.** The following are sets.

$$\{1, 2, 3\} \quad \{\{1, w\}, \pi, x^2 + x, \text{'proofs'}\} \quad \{x | x^2 - 1 = 0\}$$

The symbols  $\in$  can be used to indicate that the element  $x$  is in the set  $A$ . Write  $x \in A$ . If the element  $x$  is not in the set  $A$  write,  $x \notin A$ . The symbols  $\setminus$  can be used to construct a set which is the difference between two set. For instance, the set containing elements in the set  $A$  which are not in the set  $B$  can be represented by  $A \setminus B$ .

**Example 2.2.** Let  $A = \{1, 2, 3\}$  and  $B = \{\{\}, 2, \{1, 2, 3\}\}$ . Then  $1 \in A$  and  $A \in B$ . Moreover  $B \setminus A = \{\emptyset, A\}$ .

**Problem 2.3.** Write out the elements of the following sets.

1.  $\{x | x^2 + 2x - 3 = 0\}$
2.  $\{\{\}, 1, \{1, 2, 3\}\}$

### Common Sets

- The natural numbers (denoted by  $\mathbb{N}$ ) =  $\{1, 2, 3, \dots\}$

- The integers (denoted by  $\mathbb{Z}$ ) =  $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- The rational numbers (denoted by  $\mathbb{Q}$ ) =  $\{\frac{m}{n} | m, n \in \mathbb{Z} \text{ and } n \neq 0\}$
- The real numbers (denoted by  $\mathbb{R}$ ) is the set of all numbers
- The irrational numbers (denoted by  $\mathbb{I}$ ) =  $\mathbb{R} \setminus \mathbb{Q}$

## Subsets

A set  $A$  is a *subset* of the set  $B$ , denoted by  $A \subseteq B$ , if and only if every element in  $A$  is an element in  $B$ . In this case, the set  $B$  is called *superset* of the set  $A$ . If  $A$  is not a subset of  $B$  write  $A \not\subseteq B$ .

**Example 2.4.** The set  $\{1, 2, 3\} \subseteq \{1, 2, 3, 4\}$  and  $\{\} \subseteq \{1, 2\}$ .

Some of the common sets are subsets of each other.

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$$

**Problem 2.5.** Which of the common sets are supersets of  $\mathbb{I}$ ? Which of the common sets are subsets of  $\mathbb{I}$ ?