Chapter 4

Integers

The Division Algorithm

Let $\emptyset \neq A \subseteq \mathbb{R}$ and $x \in A$. Then x is the *least element* of A if $x \leq b$, for all $b \in A$.

Let $S \subseteq A$ where $S \neq \emptyset$. Then A is *well-ordered* if every S has a least element.

(Well-Ordering Principle) The set of natural numbers is well-ordered. In other words, any nonempty subset of $\mathbb{N}$ contains a least element.

Lemma 4.1. *Let $a, b \in \mathbb{N}$. Then there are unique nonnegative integers q and r with $0 \leq r < b$ such that*

$$a = qb + r.$$

Example 4.2. Consider the integers 11 and 5. Then $11 = 2(5) + 1$. Here $a = 11$, $b = 5$, $q = 2$, and $r = 1$. Notice that $0 \leq r < b$.

Problem 4.3. *Find integers q and r as in Lemma 4.1 for the integers $a = 51$ and $b = 7$.*