Composition

Let $f : A \to B$ and $g : B \to C$. The function $g \circ f$ from A to C is the composition of g and f. The function is defined in the following way, $(g \circ f)(a) = g(f(a))$ where $a \in A$.

Problem 3.0.12. *Using the definition of function composition, verify that $g \circ f$ is a function from A to C.*

Example 3.0.13. Let $f = \{(1, a), (2, b), (3, c)\}$ and $g = \{(a, x), (b, y), (c, z)\}$. Then $g \circ f = \{(1, x), (2, y), (3, z)\}$.

Problem 3.0.14. *Let $A = \{1, 2, 3, 4\}, B = \{a, b, c, d\}$ and $C = \{r, s, t, u, v\}$ and define the functions $f : A \to B$ and $g : B \to C$ by*

$$f = \{(1, b), (2, d), (3, a), (4, a)\} \text{ and } g = \{(a, u), (b, r), (c, r), (d, s)\}.$$

Determine $g \circ f$ and $(g \circ f)(1)$.

Theorem 3.0.15. *The functions $f : A \to B$ and $g : B \to A$ are inverses of each other if and only if*

$$(g \circ f)(a) = a \text{ and } (f \circ g)(b) = b$$

for all $a \in A$ and for all $b \in B$.