The set A and B are equal, denote by $A = B$, whenever every element of A is in B and every element of B is in A. A common technique to prove that two sets are equal is to show that they are subsets of each other.

Theorem 2.0.10. *Let A and B be sets. Then $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.*

Since Theorem 2.0.10 is a double implication it is necessary to prove two implications. While this can be done in a linear fashion(The proof only uses double implications.), it is not always easy to construct a proof which only uses double implications.

Proof. This proof breaks the double implication into two implications and prove them individually.

($\Rightarrow$) We want to show, if $A = B$ then $A \subseteq B$ and $B \subseteq A$.

Direct Proof

Suppose $A = B$. Let $x \in A$. Then $x \in B$ since $A = B$. By definition $A \subseteq B$. Now let $x \in B$. Then $x \in A$ since $A = B$. Therefore $B \subseteq A$.

($\Leftarrow$) We want to show, if $A \subseteq B$ and $B \subseteq A$ then $A = B$.

Direct Proof

Suppose $A \subseteq B$ and $B \subseteq A$. Then every element in A is in B and every element in B is in A. By definition, $A = B$. $\square$

Problem 2.0.11. *Let A, B, and C be sets such that $A \in B$. Prove that if $B \subseteq C$ then $A \in C$.*