Theorem 4.17. *Let $a, b \in \mathbb{Z}$. Then there exists integers m and n such that* $\gcd(a, b) = ma + nb$.

Example 4.18. Find m and n such that $630m + 196n = \gcd(630, 196)$. In Example 4.15 it was shown that $\gcd(630, 196) = 14$. Hence,

$$\begin{aligned} 630 &= 3(196) + 42 \\ 196 &= 4(42) + 28 \\ 42 &= 1(28) + 14 \\ 28 &= 2(14) + 0. \end{aligned}$$

Rewriting the previous equations we get,

$$\begin{aligned} 630 - 3(196) &= 42 \\ 196 - 4(42) &= 28 \\ 42 - 1(28) &= 14. \end{aligned}$$

Now use backwards substitution to get,

$$\begin{aligned} 42 - 1(196 - 4(42)) &= 14 && \text{plug in expression} \\ 5(42) - 1(196) &= 14 && \text{simplify} \\ 5(630 - 3(196)) - 1(196) &= 14 && \text{plug in expression} \\ 5(630) - 4(196) &= 14 && \text{simplify} \end{aligned}$$

Therefore $m = 5$ and $n = -4$. It is important to write $m = 5$ and not -4 and $n = -4$ and not $n = 4$.

Problem 4.19. *Find integers m and n such that $-19m + 119n = 1$.*