Chapter 3

Functions

Basic Terminology

Let A and B be sets. A *binary relation* from A to B is a subset of $A \times B$. A *function* (or *map*) from a set A to a set B is a binary relation, called f, from A to B with the property that

$$\forall a \in A \text{ there is exactly one } b \in B \text{ such that } (a, b) \in A \times B.$$

Write $f : A \to B$ to indication that the function f is being mapped from the set A to the set B. This notation should not be used if f is not a function. Write $f : a \mapsto b$ whenever $f(a) = b$.

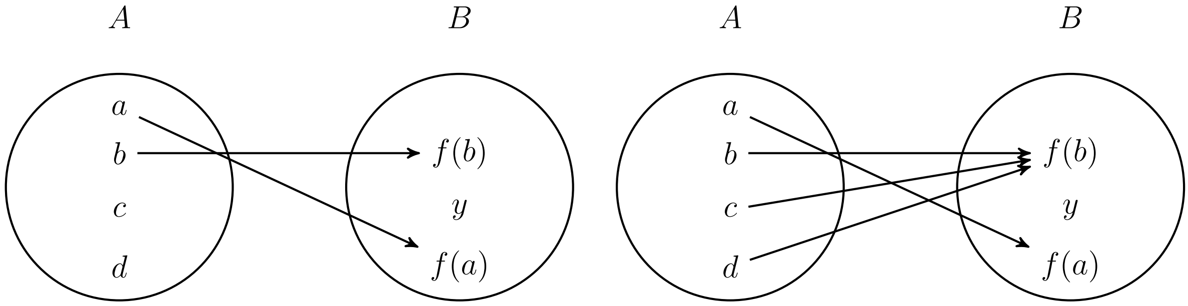
The following diagram show a relation R which is not a function from A to B since all elements in A are not being mapped to B. Whereas, the relation f is a function from A to B even though multiple elements of A are being mapped to $f(b)$.

Relation and not a function

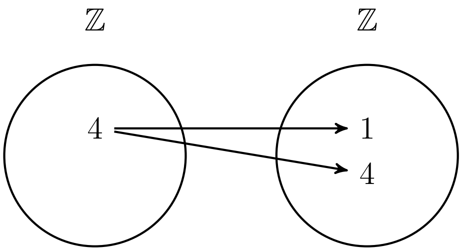
$R = \{(a, f(a)), (b, f(b)\}$

Relation and a function

$f = \{(a, f(a)), (b, f(b), (b, f(b), (b, f(b)\}$



Example 3.1. The set $\{(a, b) \mid a, b \in \mathbb{N}, a/b \in \mathbb{Z}\}$ is a binary relation from $\mathbb{N}$ to $\mathbb{N}$.



Note that the binary relation in Example 3.1 is not a function since both $(4, 1), (4, 4)$ are in the relation.

Example 3.2. The set $\{(x_1, x_2) \,|\, x_1, x_2 \in \mathbb{Z}, x_1^2 = x_2\} = f$ is a function. The function f could also be represented by $f(x) = x^2$ for $x \in \mathbb{Z}$.