

Chapter 2

Sets and Relations

Sets

A *set* is a collection of elements. Sets are typically represented by a left curly brace before the first element of the list and a right curly brace after the last element of the list. A definition for the elements of a set can also be used to describe a set. The *empty set* is a set with no elements. The empty set can be denoted by \emptyset or $\{\}$.

Example 2.0.1. The following are sets.

$$\{1, 2, 3\} \quad \{\{1, w\}, \pi, x^2 + x, \text{'proofs'}\} \quad \{x | x^2 - 1 = 0\}$$

The symbols \in can be used to indicate that the element x is in the set A . Write $x \in A$. If the element x is not in the set A write, $x \notin A$. The symbols \setminus can be used to construct a set which is the difference between two set. For instance, the set containing elements in the set A which are not in the set B can be represented by $A \setminus B$.

Example 2.0.2. Let $A = \{1, 2, 3\}$ and $B = \{\{\}, 2, \{1, 2, 3\}\}$. Then $1 \in A$ and $A \in B$. Moreover $B \setminus A = \{1, 3, \emptyset, A\}$.

Problem 2.0.3. Write out the elements of the following sets.

1. $\{x | x^2 + 2x - 3 = 0\}$
2. $\{\{\}, 1, \{1, 2, 3\}\}$

Common Sets

- The natural numbers (denoted by \mathbb{N}) = $\{1, 2, 3, \dots\}$

- The integers (denoted by \mathbb{Z}) = $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- The rational numbers (denoted by \mathbb{Q}) = $\{\frac{m}{n} | m, n \in \mathbb{Z} \text{ and } n \neq 0\}$
- The real numbers (denoted by \mathbb{R}) is the set of all numbers
- The irrational numbers (denoted by \mathbb{I}) = $\mathbb{R} \setminus \mathbb{Q}$

Subsets

A set A is a *subset* of the set B , denoted by $A \subseteq B$, if and only if every element in A is an element in B . In this case, the set B is called *superset* of the set A . If A is not a subset of B write $A \not\subseteq B$.

Example 2.0.4. The set $\{1, 2, 3\} \subseteq \{1, 2, 3, 4\}$ and $\{\} \subseteq \{1, 2\}$.

Some of the common sets are subsets of each other.

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$$

Problem 2.0.5. Which of the common sets are supersets of \mathbb{I} ? Which of the common sets are subsets of \mathbb{I} ?