

**Problem 4.4** (The Division Algorithm). *Let  $a, b \in \mathbb{Z}, b \neq 0$ . Then there exist unique integers  $q$  and  $r$ , with  $0 \leq r < |b|$  such that  $a = qb + r$ .*

Hint: Try using proof by cases.

Case 1:  $a = 0, b \neq 0$ .

Case 2:  $a, b > 0$ .

Case 3:  $a > 0, b < 0$ .

Case 4:  $a < 0, b > 0$ .

Case 5:  $a < 0, b < 0$ .

It is worth noting that some books label Lemma 4.1 as the Division Algorithm. In the Division Algorithm  $q$  is the *quotient* and  $r$  is the *remainder* when  $a$  is divided by  $b$ .

**Example 4.5.** Find integers  $q$  and  $r$ , with  $0 \leq r < 20$  such that  $2,345 = -20q + r$ .