

# Chapter 4

## Integers

### The Division Algorithm

Let  $\emptyset \neq A \subseteq \mathbb{R}$  and  $x \in A$ . Then  $x$  is the *least element* of  $A$  if  $x \leq b$ , for all  $b \in A$ .

Let  $S \subseteq A$  where  $S \neq \emptyset$ . Then  $A$  is *well-ordered* if every  $S$  has a least element.

**(Well-Ordering Principle)** The set of natural numbers is well-ordered. In other words, any nonempty subset of  $\mathbb{N}$  contains a least element.

**Lemma 4.0.1.** *Let  $a, b \in \mathbb{N}$ . Then there are unique nonnegative integers  $q$  and  $r$  with  $0 \leq r < b$  such that*

$$a = qb + r.$$

**Example 4.0.2.** Consider the integers 11 and 5. Then  $11 = 2(5) + 1$ . Here  $a = 11$ ,  $b = 5$ ,  $q = 2$ , and  $r = 1$ . Notice that  $0 \leq r < b$ .

**Problem 4.0.3.** *Find integers  $q$  and  $r$  as in Lemma 4.0.1 for the integers  $a = 51$  and  $b = 7$ .*