**Theorem 2.18.** *For any sets $A$ and $B$,*

$$(A \cup B)^c = A^c \cap B^c$$

*Proof.* We want to show that $(A \cup B)^c \subseteq A^c \cap B^c$ and $A^c \cap B^c \subseteq (A \cup B)^c$.

1. $(A \cup B)^c \subseteq A^c \cap B^c$

Let $x \in (A \cup B)^c$. Then $x \notin A$ and $x \notin B$. Hence $x \in A^c$ and $x \in B^c$. Thus $x \in A^c \cap B^c$. Therefore $(A \cup B)^c \subseteq A^c \cap B^c$.

Notice that this proof is nice and boring but it gets the job done.

2. $A^c \cap B^c \subseteq (A \cup B)^c$

Let $x \in A^c \cap B^c$. Then $x \in A^c$ and $x \in B^c$ which implies $x \notin A$ and $x \notin \cap B$. Hence $x \notin A \cap B$ and it follows that $x \in (A \cap B)^c$. Therefore $A^c \cap B^c \subseteq (A \cup B)^c$.

This proof has a little more flavor than the former but it is still kept simple.

$\square$

**Problem 2.19.** *Prove that for any set $A$ and $B$, $(A \cap B)^c = A^c \cup B^c$.*