

**Theorem 2.18.** *For any sets  $A$  and  $B$ ,*

$$(A \cup B)^c = A^c \cap B^c$$

*Proof.* We want to show that  $(A \cup B)^c \subseteq A^c \cap B^c$  and  $A^c \cap B^c \subseteq (A \cup B)^c$ .

1.  $(A \cup B)^c \subseteq A^c \cap B^c$

Let  $x \in (A \cup B)^c$ . Then  $x \notin A$  and  $x \notin B$ . Hence  $x \in A^c$  and  $x \in B^c$ . Thus  $x \in A^c \cap B^c$ . Therefore  $(A \cup B)^c \subseteq A^c \cap B^c$ .

Notice that this proof is nice and boring but it gets the job done.

2.  $A^c \cap B^c \subseteq (A \cup B)^c$

Let  $x \in A^c \cap B^c$ . Then  $x \in A^c$  and  $x \in B^c$  which implies  $x \notin A$  and  $x \notin B$ . Hence  $x \notin A \cup B$  and it follows that  $x \in (A \cup B)^c$ . Therefore  $A^c \cap B^c \subseteq (A \cup B)^c$ .

This proof has a little more flavor than the former but it is still kept simple.

□

**Problem 2.19.** *Prove that for any set  $A$  and  $B$ ,  $(A \cap B)^c = A^c \cup B^c$ .*