One-to-One Correspondence and the Cardinality of Sets

A *one-to-one correspondence* is a function that is both one-to-one and onto. Sets A and B have the *same cardinality* if and only if there exists a one-to-one correspondence from A to B. This is denoted by $|A| = |B|$. The *cardinality* of a set A, denoted by $|A|$, is the number of elements in A.

If $|A|$ is a natural number then A is a finite set. Otherwise A in an infinite set.

Example 3.16. Let $A = \{1, 2\}$ and $B = \{a, b\}$. Then $f = \{(1, a), (2, b)\}$ is a one-to-one correspondence from the set A to B. This shows that $|A| = |B|$.

In Example 3.16, counting the number of elements in B is easy. However, it is possible to determine the number of elements in B by counting the number of elements in A. This is the case since there is an one-to-one correspondence from A to B. In the next example it might not seem obvious that the two set have the same cardinality.

Example 3.17. Consider the two set $\mathbb{N}$ and $\mathbb{N} \cup \{0\}$. Define the function f to be a function from $\mathbb{N}$ to $\mathbb{N}$ such that $f : a \mapsto a - 1$. Then f^{-1} defined by $f^{-1} : a \mapsto a + 1$ is a function from $\mathbb{N} \to \mathbb{N} \cup \{0\}$. Therefore f is a bijection which implies that f is one-to-one correspondence from $\mathbb{N} \to \mathbb{N} \cup \{0\}$. This shows that $|\mathbb{N}| = |\mathbb{N} \cup \{0\}|$.