

# Chapter 3

## Functions

### Basic Terminology

Let  $A$  and  $B$  be sets. A *binary relation* from  $A$  to  $B$  is a subset of  $A \times B$ . A *function* (or *map*) from a set  $A$  to a set  $B$  is a binary relation, called  $f$ , from  $A$  to  $B$  with the property that

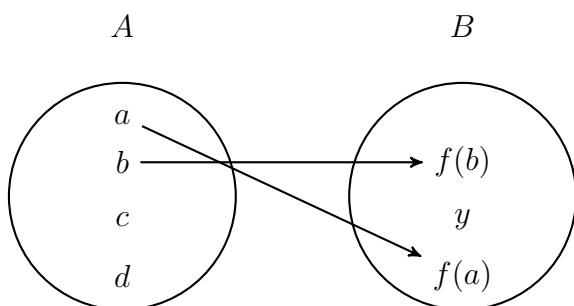
$$\forall a \in A \text{ there is exactly one } b \in B \text{ such that } (a, b) \in A \times B.$$

Write  $f : A \rightarrow B$  to indicate that the function  $f$  is being mapped from the set  $A$  to the set  $B$ . This notation should not be used if  $f$  is not a function. Write  $f : a \mapsto b$  whenever  $f(a) = b$ .

The following diagrams show a relation  $R$  which is not a function from  $A$  to  $B$  since all elements in  $A$  are not being mapped to  $B$ . Whereas, the relation  $f$  is a function from  $A$  to  $B$  even though multiple elements of  $A$  are being mapped to  $f(b)$ .

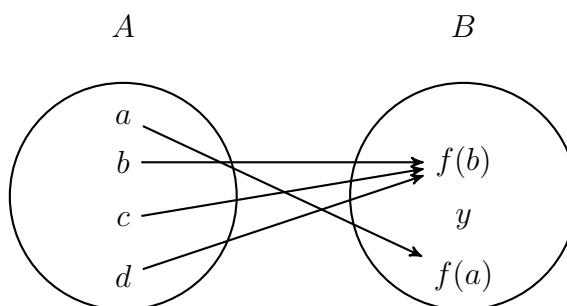
Relation and not a function

$$R = \{(a, f(a)), (b, f(b))\}$$

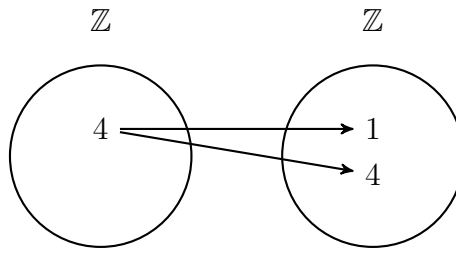


Relation and a function

$$f = \{(a, f(a)), (b, f(b)), (b, f(b)), (b, f(b))\}$$



**Example 3.0.1.** The set  $\{(a, b) \mid a, b \in \mathbb{N}, a/b \in \mathbb{Z}\}$  is a binary relation from  $\mathbb{N}$  to  $\mathbb{N}$ .



Note that the binary relation in Example 3.0.1 is not a function since both  $(4, 1), (4, 4)$  are in the relation.

**Example 3.0.2.** The set  $\{(x_1, x_2) \mid x_1, x_2 \in \mathbb{Z}, x_1^2 = x_2\} = f$  is a function. The function  $f$  could also be represented by  $f(x) = x^2$  for  $x \in \mathbb{Z}$ .