Operations on Sets

The *union* of two sets A and B, denoted by $A \cup B$, is the set containing elements from A or B. The *intersection* of two sets A and B, denoted by $A \cap B$, is the set of elements which are in A and B.

Example 2.0.12.

$$A = \{1, 2, 4, 6\} \quad B = \{1, 3, 5, 6\}$$

$$A \cap B = \{1, 6\} \quad A \cup B = \{1, 2, 3, 4, 5, 6\}$$

Problem 2.0.13. *Let* $A = \{a, b, c\}$ *and* $B = \{A, b, 3\}$*. Find* $A \cup B$ *and* $A \cap B$*.*

The union of multiple sets can be generalized in the following way.

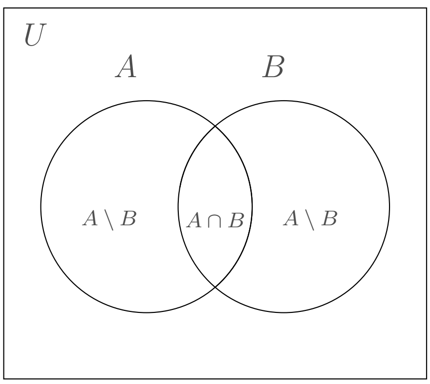
Notation

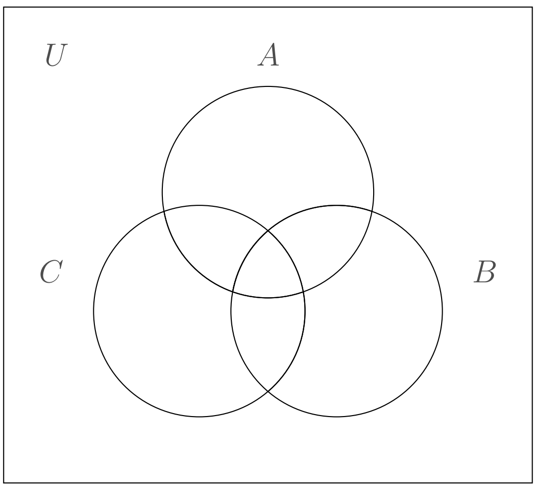
Union of n Sets	Intersection of n Sets
$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$	$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$

The *complement* of a set A with respect to the superset U, denoted by A^c, is the set containing all elements of U which are not in A.

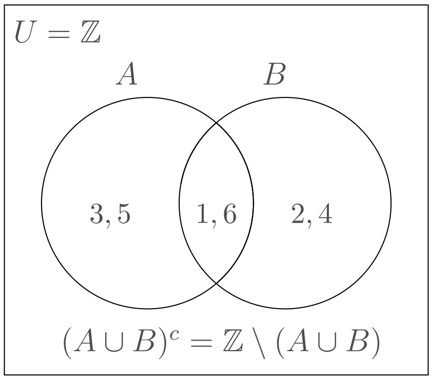
Venn Diagram

A *Venn diagram* is a diagram which shows the relationship between an element x in a set A with another set B.





Example 2.0.14. Consider the following set define in Example 2.0.12. The following Venn diagram show lists all elements from all set.



Problem 2.0.15. *Make a Venn diagram for the sets* $A = \{1, 2, 3\}$*,* $B = \{1, 4, 5\}$*, and* $C = \{2, 5, 7\}$*.*

The *Cartesian product* of the set A and B, denoted by $A \times B$, is the set

$$\{(a, b) \mid a \in A \text{ and } b \in B\}.$$

Note that the Cartesian product of two sets is a set of order pairs. Hence $(a, b) \in A \times B$ does not imply that $(b, a) \in A \times B$. Also,

$$A^n = \underbrace{A \times A \times \cdots \times A}_{n \text{ times}} = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A\,, i \in \{1, 2, \ldots, n\}\}.$$

Example 2.0.16. Consider the sets $A = \{1, 2\}$ and $B = \{x, y, z\}$. Then the Cartesian product $A \times B$ is

$$\{(1, x), (1, y), (1, z), (2, x), (2, y)(2, z)\}$$

and

$$A^3 = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}.$$

Problem 2.0.17. *Let A and B be the sets defined in Example 2.0.16. Find $B \times A$ and B^2.*

Theorem 2.0.18. *For any sets A and B,*

$$(A \cup B)^c = A^c \cap B^c$$

Proof. We want to show that $(A \cup B)^c \subseteq A^c \cap B^c$ and $A^c \cap B^c \subseteq (A \cup B)^c$.

1. $(A \cup B)^c \subseteq A^c \cap B^c$

Let $x \in (A \cup B)^c$. Then $x \notin A$ and $x \notin B$. Hence $x \in A^c$ and $x \in B^c$. Thus $x \in A^c \cap B^c$. Therefore $(A \cup B)^c \subseteq A^c \cap B^c$.

Notice that this proof is nice and boring but it gets the job done.

2. $A^c \cap B^c \subseteq (A \cup B)^c$

Let $x \in A^c \cap B^c$. Then $x \in A^c$ and $x \in B^c$ which implies $x \notin A$ and $x \notin \cap B$. Hence $x \notin A \cap B$ and it follows that $x \in (A \cap B)^c$. Therefore $A^c \cap B^c \subseteq (A \cup B)^c$.

This proof has a little more flavor than the former but it is still kept simple.

$\square$

Problem 2.0.19. *Prove that for any set A and B, $(A \cap B)^c = A^c \cup B^c$.*