Double implication ($Q \leftrightarrow P$)

The statement "“P implies Q” and “Q implies P”' is a double implication. Other forms of a double implication are “ $P \leftrightarrow Q$”, “$P \Leftrightarrow Q$”, “P iff Q”, where ‘iff’ means ‘if and only if’.

Example 1.4. (Double implication) The value of x is greater than 0 if and only if x^2 is greater than 0.

Negation ($\neg P$)

The negation of P is not P. The notation for the negation of P is denoted by $\neg P$

Example 1.5. (Negation) Let P : The value of x is greater than 0. Then $\neg P$: The value of x is less than or equal to 0.

Contrapositive ($\neg Q \rightarrow \neg P$)

The contrapositive of an implication $P \rightarrow Q$ is an equivalent statement of the following form $\neg Q \rightarrow \neg P$.

Example 1.6. The following statement is the contrapositive of the statement from example Example 1.2. If x^2 is less than or equal to 0 then x is less than or equal to 0.

Problem 1.7. *Is the statement “Let x be a real number.” a mathematical statement?*

Solution 1.8. This is not a mathematical statement because it does not have a truth value. Statements similar to the statement “Let x be a real number.” are *commands* and are typically used to define variables.

Quantifiers

Expressions that quantify statements. Common quantifiers are “for all” and “there exists” denoted by $\forall$ and $\exists$ respectively.

Example 1.9. (Quantifiers)

- For all integers x, $x^2 \geq 0$. Here the expression “For all” quantifies for which integers the statement $x^2 \geq 0$ is true.
- There exists an integer x such that $x^2 - 1 = 0$.
- For all x there exists y such that x is less than y.