Chapter 3

Functions

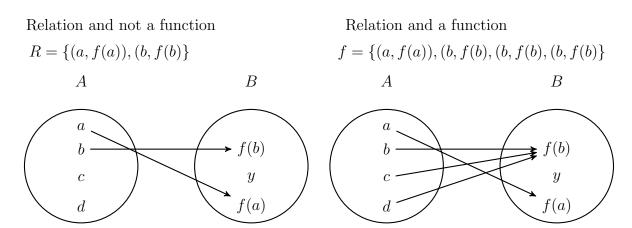
Basic Terminology

Let A and B be sets. A binary relation from A to B is a subset of $A \times B$. A function (or map) from a set A to a set B is a binary relation, called f, from A to B with the property that

 $\forall a \in A \text{ there is exactly one } b \in B \text{ such that } (a, b) \in A \times B.$

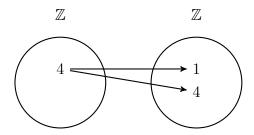
Write $f: A \to B$ to indication that the function f is being mapped from the set A to the set B. This notation should not be used if f is not a function. Write $f: a \mapsto b$ whenever f(a) = b.

The following diagram show a relation R which is not a function from A to B since all elements in A are not being mapped to B. Whereas, the relation f is a function from A to B even though multiple elements of A are being mapped to f(b).



Example 3.0.1. The set $\{(a,b) | a,b \in \mathbb{N}, a/b \in \mathbb{Z}\}$ is a binary relation from \mathbb{N} to \mathbb{N} .

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Note that the binary relation in Example 3.0.1 is not a function since both (4,1),(4,4) are in the relation.

Example 3.0.2. The set $\{(x_1, x_2) | x_1, x_2 \in \mathbb{Z}, x_1^2 = x_2\} = f$ is a function. The function f could also be represented by $f(x) = x^2$ for $x \in \mathbb{Z}$.