Operations on Sets

The *union* of two sets A and B, denoted by $A \cup B$, is the set containing elements from A or B. The *intersection* of two sets A and B, denoted by $A \cap B$, is the set of elements which are in A and B.

Example 2.12.

$$A = \{1, 2, 4, 6\} \quad B = \{1, 3, 5, 6\}$$

$$A \cap B = \{1, 6\} \quad A \cup B = \{1, 2, 3, 4, 5, 6\}$$

Problem 2.13. *Let* $A = \{a, b, c\}$ *and* $B = \{A, b, 3\}$. *Find* $A \cup B$ *and* $A \cap B$.

The union of multiple sets can be generalized in the following way.

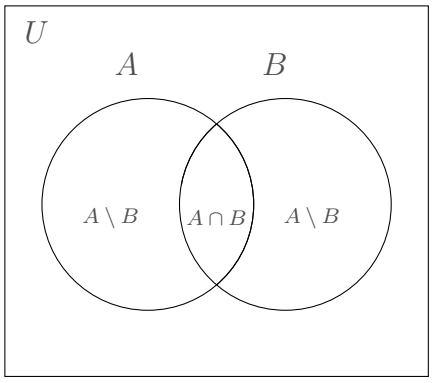
Notation

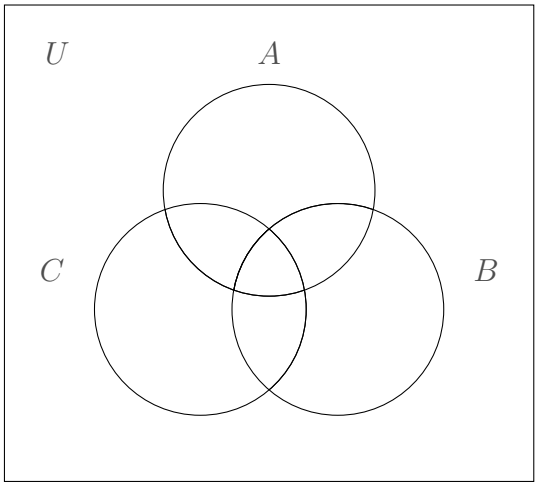
Union of n Sets	Intersection of n Sets
$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$	$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$

The *complement* of a set A with respect to the superset U, denoted by A^c, is the set containing all elements of U which are not in A.

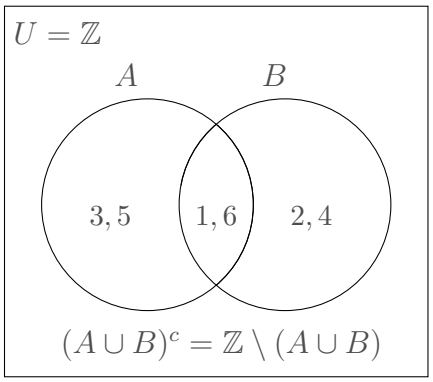
Venn Diagram

A *Venn diagram* is a diagram which shows the relationship between an element x in a set A with another set B.





Example 2.14. Consider the following set define in Example 2.12. The following Venn diagram show lists all elements from all set.



Problem 2.15. *Make a Venn diagram for the sets* $A = \{1, 2, 3\}$, $B = \{1, 4, 5\}$, *and* $C = \{2, 5, 7\}$.

The *Cartesian product* of the set A and B, denoted by $A \times B$, is the set

$$\{(a, b) \mid a \in A \text{ and } b \in B\}.$$

Note that the Cartesian product of two sets is a set of order pairs. Hence $(a, b) \in A \times B$ does not imply that $(b, a) \in A \times B$. Also,

$$A^n = \underbrace{A \times A \times \cdots \times A}_{n \text{ times}} = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A \, , i \in \{1, 2, \ldots, n\}\}.$$

Example 2.16. Consider the sets $A = \{1, 2\}$ and $B = \{x, y, z\}$. Then the Cartesian product $A \times B$ is

$$\{(1, x), (1, y), (1, z), (2, x), (2, y)(2, z)\}$$

and

$$A^3 = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}.$$

Problem 2.17. *Let* A *and* B *be the sets defined in Example 2.16. Find* $B \times A$ *and* B^2.