Chapter 2

Sets and Relations

Sets

A *set* is a collection of elements. Sets are typically represented by a left curly brace before the first element of the list and a right curly brace after the last element of the list. A definition for the elements of a set can also be used to describe a set. The *empty set* is a set with no elements. The empty set can be denoted by $\emptyset$ or $\{\}$.

Example 2.0.1. The following are sets.

$$\{1,2,3\} \quad \{\{1,w\},\pi,x^2+x,\text{'proofs'}\} \quad \{x|x^2-1=0\}$$

The symbols $\in$ can be used to indicate that the element x is in the set A. Write $x \in A$. If the element x is not in the set A write, $x \notin A$. The symbols $\setminus$ can be used to construct a set which is the difference between two set. For instance, the set containing elements in the set A which are not in the set B can be represented by $A \setminus B$.

Example 2.0.2. Let $A = \{1,2,3\}$ and $B = \{\{\},2,\{1,2,3\}\}$. Then $1 \in A$ and $A \in B$. Moreover $B \setminus A = \{1,3,\emptyset,A\}$.

Problem 2.0.3. *Write out the elements of the following sets.*

1. $\{x|x^2+2x-3=0\}$
2. $\{\{\},1,\{1,2,3\}\}$

Common Sets

- The natural numbers (denoted by $\mathbb{N}$) $= \{1,2,3,\dots\}$

- The integers (denoted by $\mathbb{Z}$) $= \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
- The rational numbers (denoted by $\mathbb{Q}$) $= \{\frac{m}{n} | m, n \in \mathbb{Z}$ and $n \neq 0\}$
- The real numbers (denoted by $\mathbb{R}$) is the set of all numbers
- The irrational numbers (denoted by $\mathbb{I}$) $= \mathbb{R} \setminus \mathbb{Q}$

Subsets

A set A is a *subset* of the set B, denoted by $A \subseteq B$, if and only if every element in A is an element in B. In this case, the set B is called *superset* of the set A. If A is not a subset of B write $A \not\subseteq B$.

Example 2.0.4. The set $\{1, 2, 3\} \subseteq \{1, 2, 3, 4\}$ and $\{\} \subseteq \{1, 2\}$.

Some of the common sets are subsets of each other.

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$$

Problem 2.0.5. *Which of the common sets are supersets of* $\mathbb{I}$*? Which of the common sets are subsets of* $\mathbb{I}$*?*