Chapter 1

Proofs

Mathematical Statements

A *mathematical statement* is an English statement that has a truth value.

Types of Statements Compound statements, Implications, Double implications, Converse of an Implications, Negations, and Quantifiers.

Compound Statement (P and Q)

A *compound statement* is a statement constructed from two statements joined be the words "and" or "or".

Example 1.1 (Compound Statement)**.** Let x be a real number.

P : Then number x is greater than 3.

Q : Then number x is even.

P and Q : The number x is greater than 3 **and** x is even.

P and Q : The number x is greater than 3 **or** x is even.

Question: What are the truth values of P and Q and P and Q?

1. If $x = 6$? P and Q, P or Q are true.
2. If $x = 5$? P and Q is false. P or Q is true.
3. If $x < 3$? P and Q is false. , P or Q depends on the value of x.

Implication ($P \to Q$)

The mathematical statement "P implies Q" is an implication where P is the hypothesis and Q is the conclusion. Other forms of an implication are "If P then Q.", "$P \to Q$", and "$P \Rightarrow Q$".

Example 1.2. If x is greater than 0, then x^2 is greater than 0. Here "x is greater than 0" is the hypothesis and "x^2 is greater than 0" is the conclusion.

Converse of an implication ($Q \to P$)

The converse of " P implies Q" is " Q implies P".

Example 1.3. Converse of previous example

If x^2 is greater than 0, then x is greater than 0.