Problem 4.0.4 (The Division Algorithm)**.** Let $a, b \in \mathbb{Z}, b \neq 0$. Then there exist unique integers q and r, with $0 \leq r < |b|$ such that $a = qb + r$.

Hint: Try using proof by cases.

Case 1: $a = 0, b \neq 0$.

Case 2: $a, b > 0$.

Case 3: $a > 0, b < 0$.

Case 4: $a < 0, b > 0$.

Case 5: $a < 0, b < 0$.

It is worth noting that some books label Lemma 4.0.1 as the Division Algorithm. In the Division Algorithm q is the *quotient* and r is the *remainder* when a is divided by b.

Example 4.0.5. Find integers q and r, with $0 \leq r < 20$ such that $2,345 = -20q + r$.