

## Double implication ( $Q \leftrightarrow P$ )

The statement “ $P$  implies  $Q$ ” and “ $Q$  implies  $P$ ” is a double implication. Other forms of a double implication are “ $P \leftrightarrow Q$ ”, “ $P \Leftrightarrow Q$ ”, “ $P$  iff  $Q$ ”, where ‘iff’ means ‘if and only if’.

**Example 1.0.4.** (Double implication) The value of  $x$  is greater than 0 if and only if  $x^2$  is greater than 0.

## Negation ( $\neg P$ )

The negation of  $P$  is not  $P$ . The notation for the negation of  $P$  is denoted by  $\neg P$ .

**Example 1.0.5.** (Negation) Let  $P$  : The value of  $x$  is greater than 0. Then  $\neg P$  : The value of  $x$  is less than or equal to 0.

## Contrapositive ( $\neg Q \rightarrow \neg P$ )

The contrapositive of an implication  $P \rightarrow Q$  is an equivalent statement of the following form  $\neg Q \rightarrow \neg P$ .

**Example 1.0.6.** The following statement is the contrapositive of the statement from example Example 1.0.2. If  $x^2$  is less than or equal to 0 then  $x$  is less than or equal to 0.

**Problem 1.0.7.** *Is the statement “Let  $x$  be a real number.” a mathematical statement?*

**Solution 1.0.8.** This is not a mathematical statement because it does not have a truth value. Statements similar to the statement “Let  $x$  be a real number.” are *commands* and are typically used to define variables.

## Quantifiers

Expressions that quantify statements. Common quantifiers are “for all” and “there exists” denoted by  $\forall$  and  $\exists$  respectively.

**Example 1.0.9.** (Quantifiers)

- For all integers  $x$ ,  $x^2 \geq 0$ . Here the expression “For all” quantifies for which integers the statement  $x^2 \geq 0$  is true.
- There exists an integer  $x$  such that  $x^2 - 1 = 0$ .
- For all  $x$  there exists  $y$  such that  $x$  is less than  $y$ .