

**Theorem 4.17.** *Let  $a, b \in \mathbb{Z}$ . Then there exists integers  $m$  and  $n$  such that  $\gcd(a, b) = ma + nb$ .*

**Example 4.18.** Find  $m$  and  $n$  such that  $630m + 196n = \gcd(630, 196)$ . In Example 4.15 it was shown that  $\gcd(630, 196) = 14$ . Hence,

$$630 = 3(196) + 42$$

$$196 = 4(42) + 28$$

$$42 = 1(28) + 14$$

$$28 = 2(14) + 0.$$

Rewriting the previous equations we get,

$$630 - 3(196) = 42$$

$$196 - 4(42) = 28$$

$$42 - 1(28) = 14.$$

Now use backwards substitution to get,

$$42 - 1(196 - 4(42)) = 14 \quad \text{plug in expression}$$

$$5(42) - 1(196) = 14 \quad \text{simplify}$$

$$5(630 - 3(196)) - 1(196) = 14 \quad \text{plug in expression}$$

$$5(630) - 4(196) = 14 \quad \text{simplify}$$

Therefore  $m = 5$  and  $n = -4$ . It is important to write  $m = 5$  and not  $-4$  and  $n = -4$  and not  $n = 4$ .

**Problem 4.19.** *Find integers  $m$  and  $n$  such that  $-19m + 119n = 1$ .*