Operations on Sets

The *union* of two sets A and B, denoted by $A \cup B$, is the set containing elements from A or B. The *intersection* of two sets A and B, denoted by $A \cap B$, is the set of elements which are in A and B.

Example 2.0.12.

$$A = \{1, 2, 4, 6\}$$
 $B = \{1, 3, 5, 6\}$

$$A \cap B = \{1, 6\}$$
 $A \cup B = \{1, 2, 3, 4, 5, 6\}$

Problem 2.0.13. *Let* $A = \{a, b, c\}$ *and* $B = \{A, b, 3\}$. *Find* $A \cup B$ *and* $A \cap B$.

The union of multiple sets can be generalized in the following way.

Notation

Union of n Sets

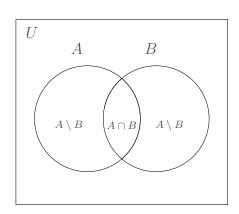
Intersection of n Sets

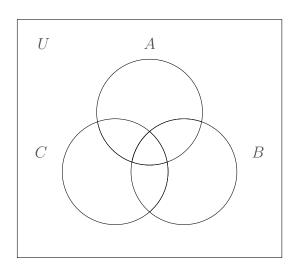
$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n \quad \bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

The *complement* of a set A with respect to the superset U, denoted by A^c , is the set containing all elements of U which are not in A.

Venn Diagram

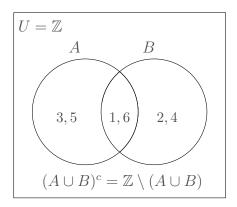
A Venn diagram is a diagram which shows the relationship between an element x in a set A with another set B.





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Example 2.0.14. Consider the following set define in Example 2.0.12. The following Venn diagram show lists all elements from all set.



Problem 2.0.15. Make a Venn diagram for the sets $A = \{1, 2, 3\}$, $B = \{1, 4, 5\}$, and $C = \{2, 5, 7\}$.

The Cartesian product of the set A and B, denoted by $A \times B$, is the set

$$\{(a,b) \mid a \in A \text{ and } b \in B\}.$$

Note that the Cartesian product of two sets is a set of order pairs. Hence $(a, b) \in A \times B$ does not imply that $(b, a) \in A \times B$. Also,

$$A^{n} = \underbrace{A \times A \times \cdots \times A}_{n \text{ times}} = \{(a_{1}, a_{2}, \dots, a_{n}) \mid a_{i} \in A, i \in \{1, 2, \dots, n\}\}.$$

Example 2.0.16. Consider the sets $A = \{1, 2\}$ and $B = \{x, y, z\}$. Then the Cartesian product $A \times B$ is

$$\{(1,x),(1,y),(1,z),(2,x),(2,y)(2,z)\}$$

and

$$A^3 = \{(1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2)\}.$$

Problem 2.0.17. Let A and B be the sets defined in Example 2.0.16. Find $B \times A$ and B^2 .