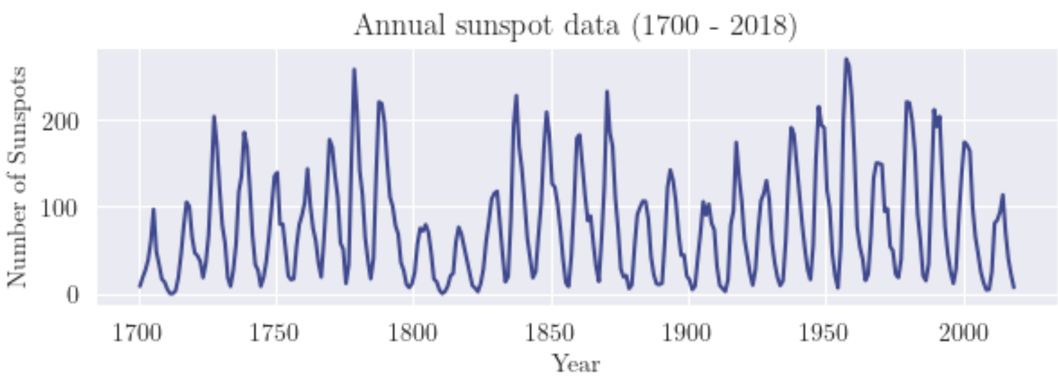
Question 8

This script determines the number of years in a solar cycle using annual sunspot data.

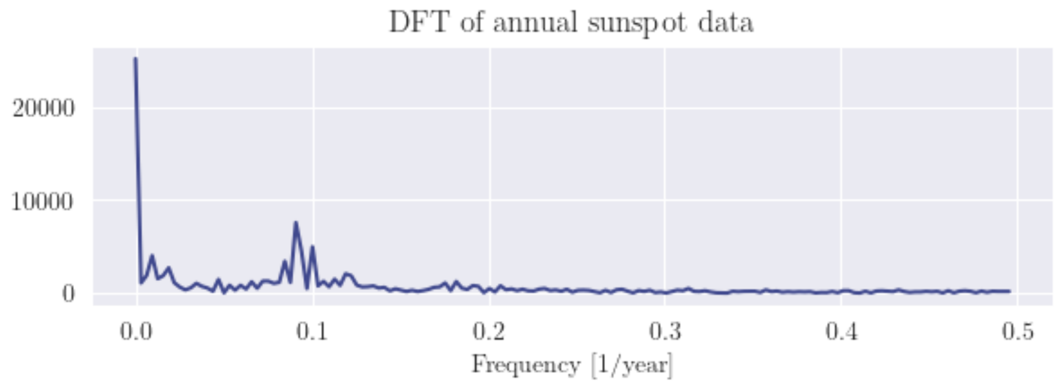
In [ ]:
```
from pathlib import Path

import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import seaborn as sns

from scipy.fft import fft, fftfreq

from config import A1_ROOT, SAVEFIG_CONFIG
```

In [ ]:
```
data = pd.read_csv(Path(A1_ROOT, "data", "SunspotData.csv"))
```

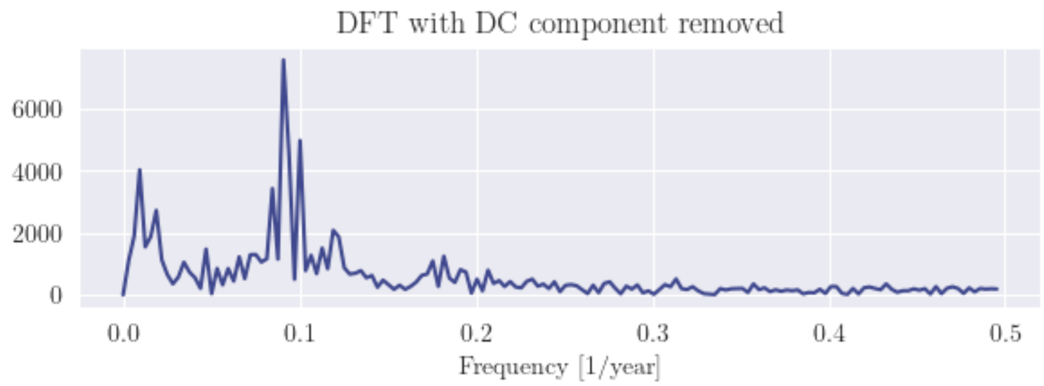
In [ ]:
```
# Plot the time series data
fig, ax = plt.subplots(figsize=(6, 2))
fig.tight_layout()

sns.lineplot(data, x="Year", y="Number of Sunspots", ax=ax)
ax.set_title("Annual sunspot data (1700 - 2018)")

fname = Path(A1_ROOT, "output", "q8_timeseries.png")
fig.savefig(fname, **SAVEFIG_CONFIG)
```



In [ ]:
```
# Plot the DFT
N = data["Year"].count()
f = fftfreq(N, 1)[:N//2]
H = np.abs(fft(data["Number of Sunspots"].to_numpy())[:N//2])

fig, ax = plt.subplots(figsize=(6, 2))
fig.tight_layout()

sns.lineplot(x=f, y=H, ax=ax)

ax.set_title("DFT of annual sunspot data")
ax.set_xlabel("Frequency [1/year]")

fname = Path(A1_ROOT, "output", "q8_rawdft.png")
fig.savefig(fname, **SAVEFIG_CONFIG)
```

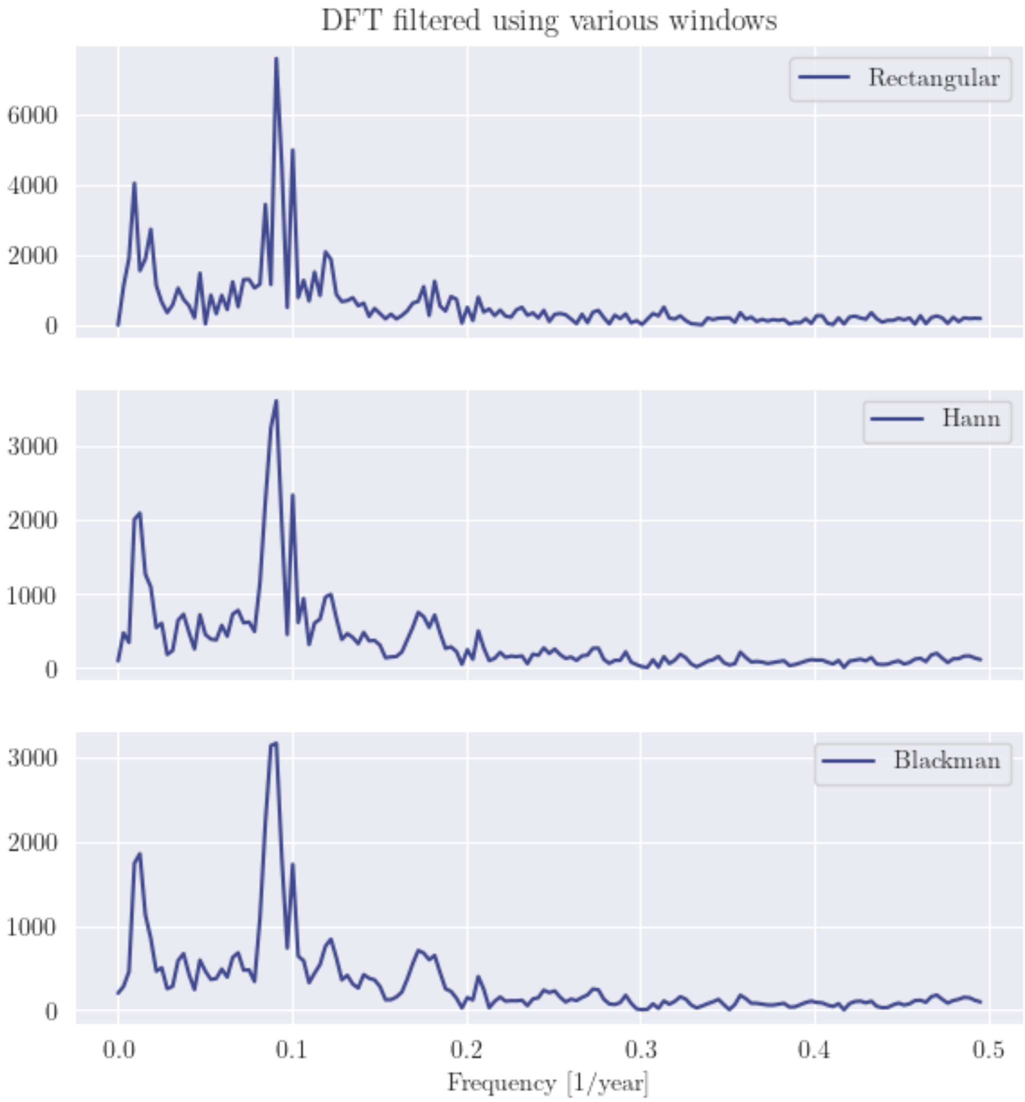


In [ ]:
```
# Remove the DC component and plot the DFT again
x = (data["Number of Sunspots"] - data["Number of Sunspots"].mean()).to_numpy()
H = np.abs(fft(x)[:N//2])

fig, ax = plt.subplots(figsize=(6, 2))
fig.tight_layout()

sns.lineplot(x=f, y=H, ax=ax)

ax.set_title("DFT with DC component removed")
ax.set_xlabel("Frequency [1/year]")

fname = Path(A1_ROOT, "output", "q8_nodcdft.png")
fig.savefig(fname, **SAVEFIG_CONFIG)
```



In [ ]:
```
# Try filtering the signal using each of: blackman, boxcar, and hann windows
from scipy.signal.windows import blackman, boxcar, hann

fig, axs = plt.subplots(3, figsize=(6, 6), sharex=True)
fig.tight_layout()

# 1. Boxcar (rectangular) window
x_boxcar = x * boxcar(N)
H_boxcar = np.abs(fft(x_boxcar)[:N//2])
sns.lineplot(x=f, y=H_boxcar, ax=axs[0], label="Rectangular")

# 2. Hann window
x_hann = x * hann(N)
H_hann = np.abs(fft(x_hann)[:N//2])
sns.lineplot(x=f, y=H_hann, ax=axs[1], label="Hann")

# 3. Blackman window
x_blackman = x * blackman(N)
H_blackman = np.abs(fft(x_blackman)[:N//2])
sns.lineplot(x=f, y=H_blackman, ax=axs[2], label="Blackman")

axs[0].set_title("DFT filtered using various windows")
axs[2].set_xlabel("Frequency [1/year]")
```

```
fname = Path(A1_ROOT, "output", "q8_windowed.png")
fig.savefig(fname, **SAVEFIG_CONFIG)
```



Find maxima using coarse and fine peak search algorithm[1].

In [ ]:
```
from typing import Tuple

def coarse_search(x: np.array, T: float, k: float = 2,
        clip: Tuple[float, float] = None) -> float:
    """
    Performs the coarse search algorithm described by Rife and Boorstyn [1].
    The factor `k` is equivalent to (M/N) in [1].
    """
    z = np.concatenate([k * x, np.zeros(len(x) * (k - 1))])
    f = fftfreq(len(z), T)[:len(z)//2]
    B = np.abs(fft(z)[:len(z)//2])
    B = B if not clip else B[(f >= clip[0]) & (f <= clip[1])]
    f = f if not clip else f[(f >= clip[0]) & (f <= clip[1])]
    return f[(B == B.max())][0]

rough_max_1 = coarse_search(x_blackman, T=1, k=2, clip=(0.00, 0.05))
rough_max_2 = coarse_search(x_blackman, T=1, k=2, clip=(0.05, 0.15))

print("Coarse search:", [rough_max_1, rough_max_2], "[1/years]")
print("              ", [1/rough_max_1, 1/rough_max_2], "[years]")
```

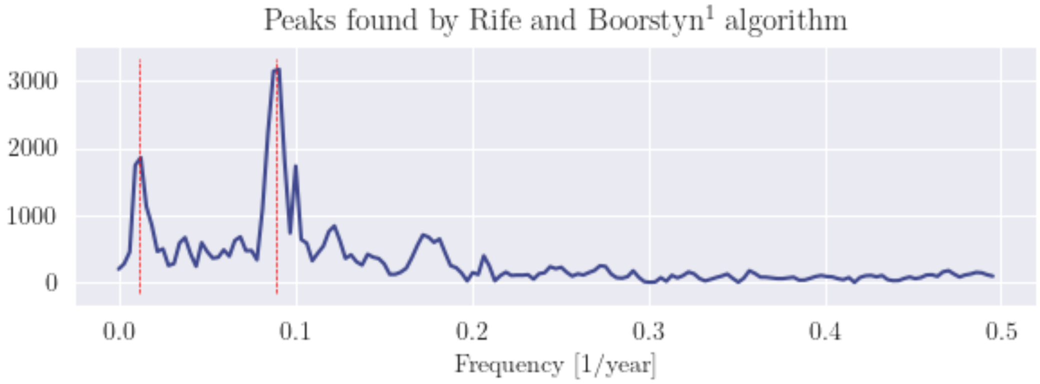
```
Coarse search: [0.0109717868338558, 0.08934169278996865] [1/years]
               [91.14285714285714, 11.192982456140351] [years]
```

In [ ]:
```
from typing import Callable

def root_secant(f: Callable, x0: float, x1: float, maxiter: int = 100) -> float:
    """
    Performs the secant method root finding algorithm on the function `f`(x),
    with initial guesses `x0` and `x1`. The initial guesses should be close to
    the desired zero. Returns the root found.
    """
    def stop_condition_met(xi: float, xj: float) -> bool:
        """
        Conditions to terminate the root search and return the latest value.
        """
        return (xj == 0) or np.isclose(xi, xj, rtol=0, atol=5e-4)

    while ((maxiter := maxiter - 1) > 0):
        x0 = x1 - f(x1) * (x1 - x0) / (f(x1) - f(x0))
        if (stop_condition_met(x1, x0)):
            return x0
        x1 = x0 - f(x0) * (x0 - x1) / (f(x0) - f(x1))
        if (stop_condition_met(x0, x1)):
            return x1

    raise RuntimeError("secant method did not converge")

def fine_search(x: np.array, T: float, w0: float) -> float:
    """
    Performs the fine search algorithm described by Rife [2].

    Parameters:
        x - set of discrete time observations of length N
        T - sampling period of x
        w0 - initial guess at frequency maximising A(w)

    Returns:
        Fine approximation of frequency maximising A(w).
    """
    step = 1 / (5 * N * T)

    A = lambda w: sum(x[n] * np.exp(-1j * n * w * T) for n in range(N)) / N
    B = lambda w: np.abs(A(w))
    dB = lambda w: (B(w + step / 2) - B(w - step / 2)) / step
    dBf = lambda f: dB(2 * np.pi * f)

    step *= np.sign(dBf(w0))

    p = w0
    while (np.sign(dBf(p0 := p)) == np.sign(dBf(p := p0 + step))):
        continue

    return root_secant(dBf, p0, p)

fine_max_1 = fine_search(x_blackman, T=1, w0=rough_max_1)
fine_max_2 = fine_search(x_blackman, T=1, w0=rough_max_2)

print("Fine search:", [fine_max_1, fine_max_2], "[1/years]")
print("            ", [1/fine_max_1, 1/fine_max_2], "[years]")
```

```
Fine search: [0.011152801870022704, 0.08947384225381592] [1/years]
             [89.6635672052842, 11.176450846530528] [years]
```

In [ ]:
```
# Display the Blackman-windowed signal with identified frequencies overlayed
fig, ax = plt.subplots(figsize=(6, 2))
fig.tight_layout()

sns.lineplot(x=f, y=H_blackman, ax=ax)

# Draw vertical lines at identified frequencies
```

```
ylim = ax.get_ylim()
ax.vlines(x=[fine_max_1, fine_max_2], ymin=ylim[0], ymax=ylim[1], color="r",
    lw=0.5, ls="--")

ax.set_title("Peaks found by Rife and Boorstyn$^1$ algorithm")
ax.set_xlabel("Frequency [1/year]")

fname = Path(A1_ROOT, "output", "q8_searchresults.png")
fig.savefig(fname, **SAVEFIG_CONFIG)
```



References

[1] D. C. Rife and R. R. Boorstyn, "Single-tone parameter estimation from discrete-time observations," *IEEE Trans. Inf. Theory*, vol. IT-20, no. 5, pp. 591-598, Sep., 1974.

[2] D. C. Rife, "Digital tone parameter estimation in the presence of Gaussian noise," Ph.D. dissertation, Polytech. Inst. Brooklyn, Brooklyn, N.Y., Jun. 1973.

[3] Wikipedia. "Secant method." Wikipedia.org. https://en.wikipedia.org/wiki/Secant_method (accessed Aug. 14, 2023).