Synthetic Differences in Differences Simulations

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Simulations

Parameters

• Number of Ls: 1

• Draws per L: 100

• Number of Units: 100

• Number of Control Units: 90

• Number of Times: 200

• Number of pre-treatment Times: 190

• Rank of L: 10

• Autocorrelation Parameter: 0

• True Effect Size: 10

• Error Type: gaussian

• Error Variance (if Gaussian error): 16

• Degrees of freedom (if t-error): 5

• Exchangable: FALSE

• Penalized: TRUE

• Rank Estimation Method: threshold

• Scaling for L: 5

Simulation Description

In this simulation we sample 1 signal matricies $L \in \mathbb{R}^{100 \times 200}$ of rank 10, and for each L we sample 100 Ys such that for each Y $E(Y_{ij}) = L_{ij} + \tau W_{ij}$ where W is known, and $\tau = 10$ (in the case of the t-distribution, we ensure the median of each cell is $L_{ij} + \tau W_{ij}$). We generate L so that the rows and columns are not exchangable, and we estimate the weights in SDID using penalized regressions. For estimating the rank of L we are using a threshold method. From each corresponding pair of (L, Y) an estimate of τ is generated via method i, which we call $\hat{\tau}_{(L,Y),i}$. To evaluate the performance of method i we calculate the following mse:

$$mse_i = \sqrt{\frac{1}{100} \sum_{(L,Y)} (\hat{\tau}_{(L,Y),i} - \tau)^2}$$

Our Method vs Competitors, Fixed Parameters

Results

```
Signal to Noise Ratio
## [1] 21.79198
mse for DID
##
        mse
## 13.87091
Se for mse for DID
##
     se_mse
## 0.3452591
mse for SC
##
        mse
## 0.4154084
Se for mse for SC
##
      se_mse
## 0.0559525
mse for our Method (Explicit Tau)
##
        mse
## 0.2384414
Se for mse for our Method (Explicit Tau)
##
       se_mse
## 0.03338038
mse for SDID
## 0.264084
Se for mse for SDID
       se_mse
## 0.03485523
```

```
mse For Our Method (Not Explicit Tau)

## mse
## 0.2627483

Se for mse for Our Method (Not Explicit Tau)

## se_mse
## 0.03799581

mse For Oracle (Perfect L)

## mse
## 0.1688268

mse For Oracle (Perfect L)

## se_mse
## 0.02999922
```

Matrix Bias vs Reduction in Variance due to Averaging

For more general designs of W (like the block design scheme considered here) we allow a block in the bottom right hand corner of W to be non-zero. When implementing our method, we have two competing effects on estimation:

- The bias that's introduced by making more of the Y_{ij} s zero.
- The help we get with estimating τ by being able to average over cells (because we assume tau) is the same for all units and times.

It would appear that accurracy increases for estimating τ to a point, and then decreases when the bias introduced by replacement of cells with 0 in Y becomes too great.

Influence of N_0 on Performance

Influence of ρ on Performance

Influence of τ on Performance

Influence of Rank Error on Performance