## Synthetic Differences in Differences Simulations

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#### **Simulations**

#### **Parameters**

• Number of Ls: 1

• Draws per L: 100

• Number of Units: 100

• Number of Control Units: 98

• Number of Times: 200

• Number of pre-treatment Times: 198

• Rank of L: 10

• Autocorrelation Parameter: 0.6

• True Effect Size: 10

• Error Type: gaussian

• Error Variance (if Gaussian error): 1

• Degrees of freedom (if t-error): 5

• Exchangable: FALSE

• Penalized: TRUE

• Rank Estimation Method: threshold

• Scaling for L: 1

## Simulation Description

In this simulation we sample 1 signal matricies  $L \in \mathbb{R}^{100 \times 200}$  of rank 10, and for each L we sample 100 Ys such that for each  $Y E(Y_{ij}) = L_{ij} + \tau W_{ij}$  where W is known, and  $\tau = 10$  (in the case of the t-distribution, we ensure the median of each cell is  $L_{ij} + \tau W_{ij}$ ). We generate L so that the rows and columns are not exchangable, and we estimate the weights in SDID using penalized regressions. For estimating the rank of L we are using a threshold method. From each corresponding pair of (L, Y) an estimate of  $\tau$  is generated via method i, which we call  $\hat{\tau}_{(L,Y),i}$ . To evaluate the performance of method i we calculate the following RMSE:

$$RMSE_i = \sqrt{\frac{1}{100} \sum_{(L,Y)} (\hat{\tau}_{(L,Y),i} - \tau)^2}$$

## Our Method vs Competitors, Fixed Parameters

# Results Signal to Noise Ratio ## [1] 17.44909 Rmse for DID ## [1] 6.832237 Se for Rmse for DID ## [1] 0 Rmse for SC ## [1] 2.877996 Se for Rmse for SC ## [1] 0 Rmse for our Method (Explicit Tau) ## [1] 0.7457445 Se for Rmse for our Method (Explicit Tau) ## [1] 0 Rmse for SDID ## [1] 1.554131 Se for Rmse for SDID ## [1] 0 RMSE For Our Method (Not Explicit Tau) ## [1] 0.8741724 Se for Rmse for Our Method (Not Explicit Tau)

## [1] 0

### RMSE For Oracle (Perfect L)

## [1] 0.5660541

RMSE For Oracle (Perfect L)

**##** [1] 0

### Matrix Bias vs Reduction in Variance due to Averaging

For more general designs of W (like the block design scheme considered here) we allow a block in the bottom right hand corner of W to be non-zero. When implementing our method, we have two competing effects on estimation:

- The bias that's introduced by making more of the  $Y_{ij}$ s zero.
- The help we get with estimating  $\tau$  by being able to average over cells (because we assume tau) is the same for all units and times.

It would appear that accurracy increases for estimating  $\tau$  to a point, and then decreases when the bias introduced by replacement of cells with 0 in Y becomes too great.

Influence of  $N_0$  on Performance

Influence of  $\rho$  on Performance

Influence of  $\tau$  on Performance

Influence of Rank Error on Performance