# Synthetic Differences in Differences Simulations

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### **Simulations**

#### **Parameters**

• Number of Ls: 1

• Draws per L: 1

• Number of Units: 100

• Number of Control Units: 98

• Number of Times: 200

• Number of pre-treatment Times: 198

• Rank of L: 10

• Autocorrelation Parameter: 0

• True Effect Size for Constant Effect: 10

• Error Type: gaussian

• Error Variance (if Gaussian error): 16

• Degrees of freedom (if t-error): 5

• Exchangable: FALSE

• Penalized: TRUE

• Rank Estimation Method: threshold

• Scaling for L: 5

• Treatment Effect Type: up\_down

# Simulation Description

In this simulation we sample 1 signal matricies  $L \in \mathbb{R}^{100 \times 200}$  of rank 10, and for each L we sample 1 Ys such that for each  $Y E(Y_{ij}) = L_{ij} + \tau W_{ij}$  where W is known, and  $\tau = 10$  (in the case of the t-distribution, we ensure the median of each cell is  $L_{ij} + \tau W_{ij}$ ). We generate L so that the rows and columns are not exchangable, and we estimate the weights in SDID using penalized regressions. For estimating the rank of L we are using a threshold method. From each corresponding pair of (L, Y) an estimate of  $\tau$  is generated via method i, which we call  $\hat{\tau}_{(L,Y),i}$ . To evaluate the performance of method i we calculate the following mse:

$$mse_i = \sqrt{\frac{1}{1} \sum_{(L,Y)} (\hat{\tau}_{(L,Y),i} - \tau)^2}$$

# Our Method vs Competitors, Fixed Parameters

#### Results

```
Signal to Noise Ratio
## [1] 21.79198
mse for DID
##
       mse
## 1177.033
Se for mse for DID
## se_mse
##
      NA
mse for SC
##
       mse
## 175.3094
Se for mse for SC
## se_mse
##
   NA
mse for our Method (Explicit Tau)
##
       mse
## 10.70531
Se for mse for our Method (Explicit Tau)
## se_mse
##
   NA
mse for SDID
      mse
## 33.0073
Se for mse for SDID
## se_mse
##
      NA
```

```
mse For Our Method (Not Explicit Tau)
##
        mse
## 10.70533
Se for mse for Our Method (Not Explicit Tau)
## se_mse
##
       NA
mse For Oracle (Perfect L)
##
         mse
## 0.8634898
mse For Oracle (Perfect L)
## se_mse
##
       NA
```

# Matrix Bias vs Reduction in Variance due to Averaging

For more general designs of W (like the block design scheme considered here) we allow a block in the bottom right hand corner of W to be non-zero. When implementing our method, we have two competing effects on estimation:

- The bias that's introduced by making more of the  $Y_{ij}$ s zero.
- The help we get with estimating  $\tau$  by being able to average over cells (because we assume tau) is the same for all units and times.

It would appear that accurracy increases for estimating  $\tau$  to a point, and then decreases when the bias introduced by replacement of cells with 0 in Y becomes too great.

Influence of  $N_0/N$  on Performance

Influence of  $\rho$  on Performance

Influence of  $\tau$  on Performance

Influence True Rank on Performance

## Influence of Rank Error on Performance

## True Treatment Effect Over Time

