Synthetic Differences in Differences Simulations

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Simulations

Parameters

- Number of Ls: 1

- Draws per L: 100

- Number of Units: 100

- Number of Control Units: 95

- Number of Times: 200

- Number of pre-treatment Times: 190

- Rank of L: 10

- Autocorrelation Parameter: 0

- True Effect Size for Constant Effect: 10

- Error Type: gaussian

- Error Variance (if Gaussian error): 16

- Degrees of freedom (if t-error): 5

- Exchangable: FALSE

- Penalized: TRUE

- Rank Estimation Method: threshold

- Scaling for L: 5

- Treatment Effect Type: decay

- Treatment Design: lagged_adoption

- Lag Structure (if using lagged adoption structure): random

- Average Treatment Length (if using lagged adoption structure, with random adoption): 6

- Maximum lag: 3

Simulation Description

In this simulation we sample 1 signal matricies $L \in \mathbb{R}^{100 \times 200}$ of rank 10, and for each L we sample 100 Ys such that for each Y $E(Y_{ij}) = L_{ij} + \tau W_{ij}$ where W is known, and $\tau = 10$ (in the case of the t-distribution, we ensure the median of each cell is $L_{ij} + \tau W_{ij}$). We generate L so that the rows and columns are not exchangable, and we estimate the weights in SDID using penalized regressions. For estimating the rank of L we are using a threshold method. From each corresponding pair of (L, Y) an estimate of τ is generated via method i, which we call $\hat{\tau}_{(L,Y),i}$. To evaluate the performance of method i we calculate the following mse:

$$mse_i = \sqrt{\frac{1}{100} \sum_{(L,Y)} (\hat{\tau}_{(L,Y),i} - \tau)^2}$$

Our Method vs Competitors, Fixed Parameters

Results

Signal to Noise Ratio

```
## [1] 20.556
```

mse for DID

```
##      mse
## 8.514083
```

Se for mse for DID

```
##    se_mse
## 0.3202701
```

mse for SC

```
##      mse
## 10.89639
```

Se for mse for SC

```
##    se_mse
## 0.3895714
```

mse for our Method (Explicit Tau)

```
##      mse
## 7.434963
```

Se for mse for our Method (Explicit Tau)

```
##     se_mse
## 0.4022735
```

mse for SDID

```
##      mse
## 91.70307
```

Se for mse for SDID

```
##   se_mse
## 2.061336
```

mse For Our Method (Not Explicit Tau)

```
##      mse
## 7.434959
```

Se for mse for Our Method (Not Explicit Tau)

```
##     se_mse
## 0.4022732
```

mse For Oracle (Perfect L)

```
##      mse
## 9.187401
```

mse For Oracle (Perfect L)

```
##     se_mse
## 0.3338231
```

Matrix Bias vs Reduction in Variance due to Averaging

For more general designs of W (like the block design scheme considered here) we allow a block in the bottom right hand corner of W to be non-zero. When implementing our method, we have two competing effects on estimation:

- The bias that's introduced by making more of the Y_{ij}s zero.

- The help we get with estimating τ by being able to average over cells (because we assume tau) is the same for all units and times.

It would appear that accurracy increases for estimating τ to a point, and then decreases when the bias introduced by replacement of cells with 0 in Y becomes too great.
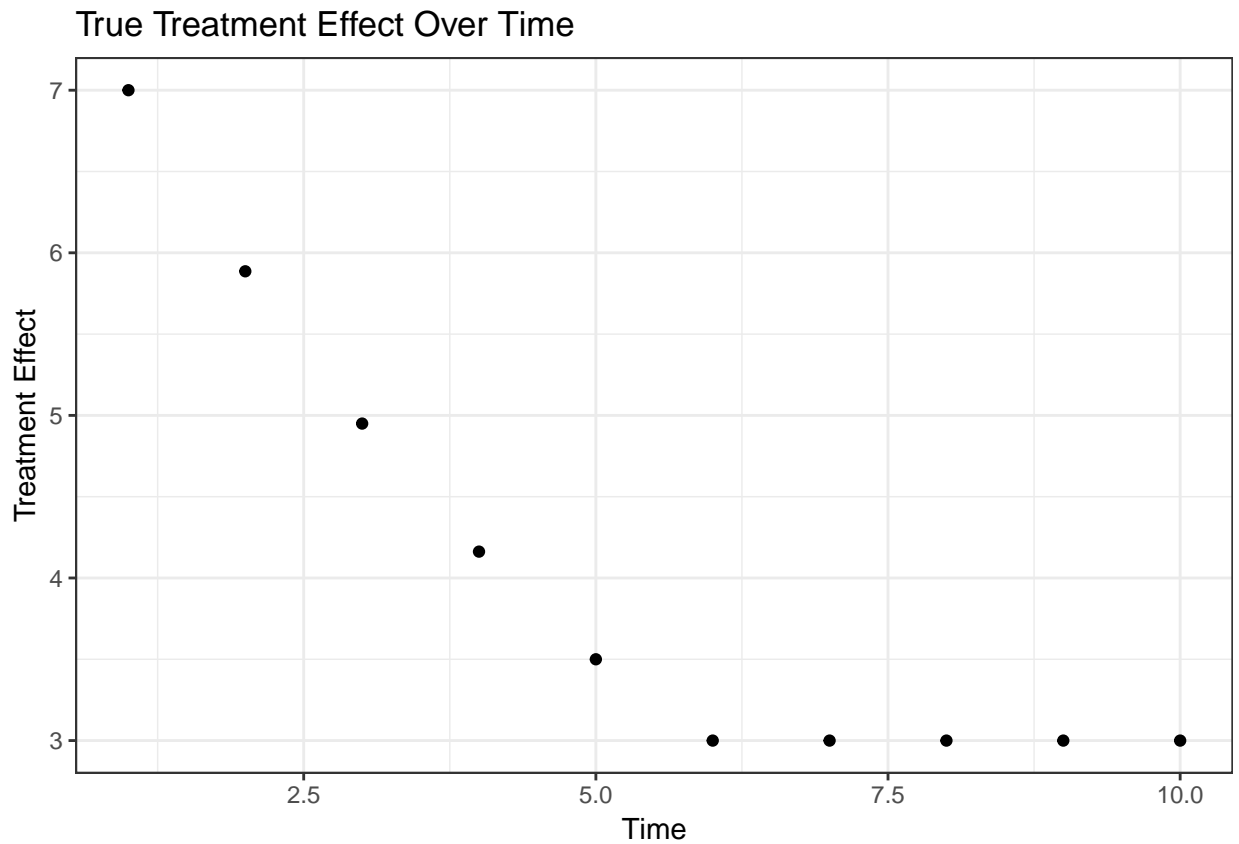
Influence of N_0/N on Performance

Influence of ρ on Performance

Influence of τ on Performance

Influence True Rank on Performance

Influence of Rank Error on Performance

True Treatment Effect Over Time


```
#quiz_1 <- c(12,11,10,12,12,10,10,12,12,9,13,4,12,9,9,13,12,12,8,10,13,13,11,13,13,7,13,13,
#            13,13,5,5,2,11,12,9,9,11,12,3,13,11,12,11,9,12,13)
```