

Synthetic Differences in Differences Simulations

Joshua Derenski

Simulations

Parameters

- Number of L s: 1
- Draws per L : 150
- Number of Units: 100
- Number of Control Units: 90
- Number of Times: 200
- Number of pre-treatment Times: 190
- Rank of L : 10
- Autocorrelation Parameter: 0
- True Effect Size: 10
- Error Type: gaussian
- Error Variance (if Gaussian error): 16
- Degrees of freedom (if t-error): 5
- Exchangable: FALSE
- Penalized: TRUE
- Rank Estimation Method: threshold
- Scaling for L : 5

Simulation Description

In this simulation we sample 1 signal matrices $L \in \mathbb{R}^{100 \times 200}$ of rank 10, and for each L we sample 150 Y s such that for each Y $E(Y_{ij}) = L_{ij} + \tau W_{ij}$ where W is known, and $\tau = 10$ (in the case of the t-distribution, we ensure the median of each cell is $L_{ij} + \tau W_{ij}$). We generate L so that the rows and columns are not exchangable, and we estimate the weights in SDID using penalized regressions. For estimating the rank of L we are using a threshold method. From each corresponding pair of (L, Y) an estimate of τ is generated via method i , which we call $\hat{\tau}_{(L,Y),i}$. To evaluate the performance of method i we calculate the following mse:

$$mse_i = \sqrt{\frac{1}{150} \sum_{(L,Y)} (\hat{\tau}_{(L,Y),i} - \tau)^2}$$

Our Method vs Competitors, Fixed Parameters

Results

Signal to Noise Ratio

```
## [1] 20.47774
```

mse for DID

```
##      mse
## 22.10732
```

Se for mse for DID

```
##    se_mse
## 0.341953
```

mse for SC

```
##      mse
## 1.715073
```

Se for mse for SC

```
##    se_mse
## 0.09885186
```

mse for our Method (Explicit Tau)

```
##      mse
## 2.659727
```

Se for mse for our Method (Explicit Tau)

```
##    se_mse
## 0.129504
```

mse for SDID

```
##      mse
## 0.3304593
```

Se for mse for SDID

```
##    se_mse
## 0.04201004
```

mse For Our Method (Not Explicit Tau)

```
##      mse
## 0.343545
```

Se for mse for Our Method (Not Explicit Tau)

```
##      se_mse
## 0.04267445
```

mse For Oracle (Perfect L)

```
##      mse
## 0.1639212
```

mse For Oracle (Perfect L)

```
##      se_mse
## 0.02102221
```

Matrix Bias vs Reduction in Variance due to Averaging

For more general designs of W (like the block design scheme considered here) we allow a block in the bottom right hand corner of W to be non-zero. When implementing our method, we have two competing effects on estimation:

- The bias that's introduced by making more of the Y_{ij} s zero.
- The help we get with estimating τ by being able to average over cells (because we assume τ is the same for all units and times).

It would appear that accuracy increases for estimating τ to a point, and then decreases when the bias introduced by replacement of cells with 0 in Y becomes too great.

Influence of N_0 on Performance

Influence of ρ on Performance

Influence of τ on Performance

```
## [1] 0
## [1] 2
## [1] 4
## [1] 6
## [1] 8
## [1] 10
## [1] 12
```

```
## [1] 14
## [1] 16
## [1] 18
## [1] 20
## [1] 22
## [1] 24
## [1] 26
## [1] 28
## [1] 30
```

Influence of Rank Error on Performance