

# Synthetic Differences in Differences Simulations

*Joshua Derenski*

## Simulations

### Parameters

- Number of Ls: 1
- Draws per L: 100
- Number of Units: 100
- Number of Control Units: 95
- Number of Times: 200
- Number of pre-treatment Times: 198
- Rank of L: 10
- Autocorrelation Parameter: 0
- True Effect Size for Constant Effect: 10
- Error Type: gaussian
- Error Variance (if Gaussian error): 16
- Degrees of freedom (if t-error): 5
- Exchangable: FALSE
- Penalized: TRUE
- Rank Estimation Method: threshold
- Scaling for  $L$ : 5
- Treatment Effect Type: up\_down
- Treatment Design: block\_treatment
- Lag Structure (if using lagged adoption structure): random
- Average Treatment Length (if using lagged adoption structure, with random adoption): 2
- Maximum lag: 3

## Simulation Description

In this simulation we sample 1 signal matrices  $L \in \mathbb{R}^{100 \times 200}$  of rank 10, and for each  $L$  we sample 100  $Y$ s such that for each  $Y$   $E(Y_{ij}) = L_{ij} + \tau W_{ij}$  where  $W$  is known, and  $\tau = 10$  (in the case of the t-distribution, we ensure the median of each cell is  $L_{ij} + \tau W_{ij}$ ). We generate  $L$  so that the rows and columns are not exchangeable, and we estimate the weights in SDID using penalized regressions. For estimating the rank of  $L$  we are using a threshold method. From each corresponding pair of  $(L, Y)$  an estimate of  $\tau$  is generated via method  $i$ , which we call  $\hat{\tau}_{(L,Y),i}$ . To evaluate the performance of method  $i$  we calculate the following mse:

$$mse_i = \sqrt{\frac{1}{100} \sum_{(L,Y)} (\hat{\tau}_{(L,Y),i} - \tau)^2}$$

## Our Method vs Competitors, Fixed Parameters

### Results

#### Signal to Noise Ratio

```
## [1] 21.79198
```

#### mse for DID

```
##      mse
## 64.09489
```

#### Se for mse for DID

```
##    se_mse
## 2.095585
```

#### mse for SC

```
##      mse
## 5.497954
```

#### Se for mse for SC

```
##    se_mse
## 0.5328966
```

#### mse for our Method (Explicit Tau)

```
##      mse
## 3.412313
```

Se for mse for our Method (Explicit Tau)

```
##      se_mse
## 0.3729328
```

mse for SDID

```
##      mse
## 4.771616
```

Se for mse for SDID

```
##      se_mse
## 0.501873
```

mse For Our Method (Not Explicit Tau)

```
##      mse
## 3.412324
```

Se for mse for Our Method (Not Explicit Tau)

```
##      se_mse
## 0.3729317
```

mse For Oracle (Perfect L)

```
##      mse
## 2.756558
```

mse For Oracle (Perfect L)

```
##      se_mse
## 0.2697047
```

## Matrix Bias vs Reduction in Variance due to Averaging

For more general designs of  $W$  (like the block design scheme considered here) we allow a block in the bottom right hand corner of  $W$  to be non-zero. When implementing our method, we have two competing effects on estimation:

- The bias that's introduced by making more of the  $Y_{ij}$ s zero.
- The help we get with estimating  $\tau$  by being able to average over cells (because we assume  $\tau$  is the same for all units and times).

It would appear that accuracy increases for estimating  $\tau$  to a point, and then decreases when the bias introduced by replacement of cells with 0 in  $Y$  becomes too great.

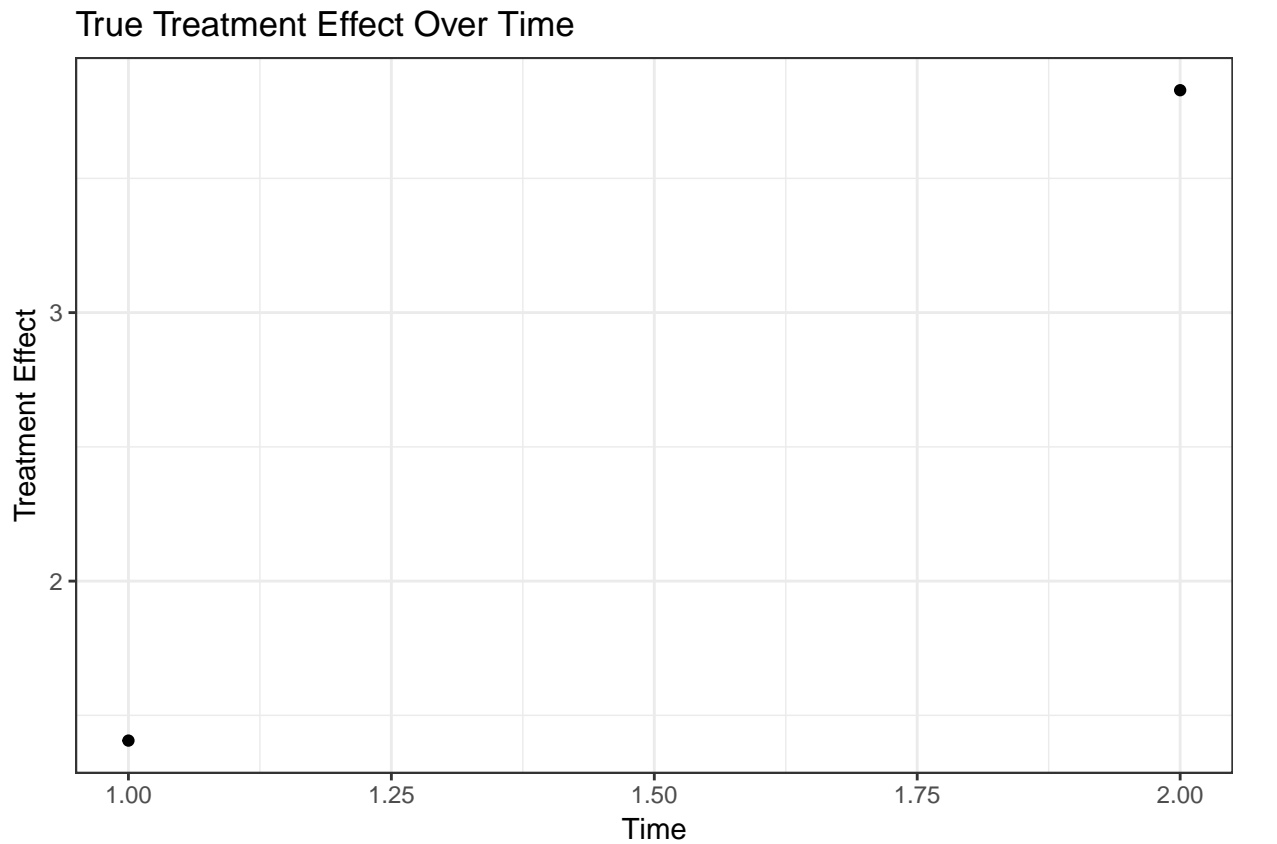
Influence of  $N_0/N$  on Performance

Influence of  $\rho$  on Performance

Influence of  $\tau$  on Performance

Influence True Rank on Performance

Influence of Rank Error on Performance



```
#quiz_1 <- c(12,11,10,12,12,10,10,12,12,9,13,4,12,9,9,13,12,12,8,10,13,13,11,13,13,7,13,13,  
#           13,13,5,5,2,11,12,9,9,11,12,3,13,11,12,11,9,12,13)
```