Synthetic Differences in Differences Simulations

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Simulations

Parameters

- Number of Ls: 1
- Draws per L: 250
- Number of Units: 200
- Number of Control Units: 198
- Number of Times: 200
- Number of pre-treatment Times: 198
- Rank of L: 10
- Autocorrelation Parameter: 0.6
- True Effect Size: 3
- Error Type: gaussian
- Error Variance (if Gaussian error): 1
- Degrees of freedom (if t-error): 5
- Exchangable: FALSE
- Penalized: TRUE
- Rank Estimation Method: threshold
- Scaling for L: 1

Simulation Description

In this simulation we sample 1 signal matricies $L \in \mathbb{R}^{200\times 200}$ of rank 10, and for each L we sample 250 Ys such that for each Y $E(Y_{ij}) = L_{ij} + \tau W_{ij}$ where W is known, and $\tau = 3$ (in the case of the t-distribution, we ensure the median of each cell is $L_{ij} + \tau W_{ij}$). We generate L so that the rows and columns are not exchangable, and we estimate the weights in SDID using penalized regressions. For estimating the rank of L we are using a threshold method. From each corresponding pair of (L, Y) an estimate of τ is generated via method i, which we call $\hat{\tau}_{(L,Y),i}$. To evaluate the performance of method i we calculate the following RMSE:

$$RMSE_i = \sqrt{\frac{1}{250}\sum_{(L,Y)} (\hat{\tau}_{(L,Y),i} - \tau)^2}$$

Our Method vs Competitors, Fixed Parameters

Results

Signal to Noise Ratio

```
## [1] 26.20078
```

Rmse for DID

```
## [1] 1.204644
```

Se for Rmse for DID

```
## [1] 0
```

Rmse for SC

```
## [1] 0.6783659
```

Se for Rmse for SC

```
## [1] 0
```

Rmse for our Method (Explicit Tau)

```
## [1] 0.6890854
```

Se for Rmse for our Method (Explicit Tau)

```
## [1] 0
```

Rmse for SDID

```
## [1] 0.6771128
```

Se for Rmse for SDID

```
## [1] 0
```

RMSE For Our Method (Not Explicit Tau)

```
## [1] 0.6934425
```

Se for Rmse for Our Method (Not Explicit Tau)

```
## [1] 0
```

RMSE For Oracle (Perfect L)

```
## [1] 0.6436312
```

RMSE For Oracle (Perfect L)

```
## [1] 0
```

Matrix Bias vs Reduction in Variance due to Averaging

For more general designs of W (like the block design scheme considered here) we allow a block in the bottom right hand corner of W to be non-zero. When implementing our method, we have two competing effects on estimation:

- The bias that's introduced by making more of the Y_{ij}s zero.
- The help we get with estimating τ by being able to average over cells (because we assume tau) is the same for all units and times.

It would appear that accurracy increases for estimating τ to a point, and then decreases when the bias introduced by replacement of cells with 0 in Y becomes too great.

Influence of N_0 on Performance

```
## [1] -0.2039807
## [1] 0.7195122
## [1] -0.05663118
## [1] -0.5269894
## [1] 0.1435216
## [1] 0.83326
## [1] -0.06466425
## [1] -0.4278573
## [1] 0.6958552
## [1] -0.707359
## [1] -0.2440663
## [1] 0.795717
## [1] 0.6932442
## [1] -0.299534
## [1] -0.09428365
## [1] 0.4201124
## [1] -0.7678519
## [1] 1.182507
## [1] 0.6653593
## [1] -0.3836112
## [1] -0.3241891
## [1] -0.1604156
## [1] 0.270872
## [1] -0.2320172
## [1] 0.2090906
## [1] 0.4941801
## [1] 0.332352
## [1] -0.04124149
```

```
## [1] -0.4624517
## [1] 0.5132839
## [1] 0.8412458
## [1] 0.6391494
## [1] -0.05173623
## [1] 0.6382357
## [1] 0.009712651
## [1] 0.1212436
## [1] 0.7361735
## [1] -0.09765762
## [1] 0.2697572
## [1] 0.3214128
## [1] -0.08847875
## [1] -0.2081526
## [1] -0.2414864
## [1] 0.2222339
## [1] 0.174658
## [1] 0.6837716
## [1] 0.1321924
## [1] -0.5887319
## [1] -0.2000268
## [1] -0.9555766
## [1] 0.004608481
## [1] -0.00839127
## [1] -0.401
## [1] 0.06617315
## [1] 0.9045605
## [1] 0.6023738
## [1] 0.04622338
## [1] 0.5145162
## [1] 0.2266985
## [1] 0.5085384
## [1] 0.565814
## [1] -0.7042746
## [1] 0.09886062
## [1] -0.5247472
## [1] 0.5037181
## [1] 0.2221042
## [1] 0.9019207
## [1] 0.5077473
## [1] 0.2637817
## [1] 0.4751997
## [1] -0.3144943
## [1] 0.2110552
## [1] -0.5920474
## [1] 0.2242799
## [1] -0.1736182
## [1] -0.1122982
## [1] 0.06531051
## [1] 0.1232541
## [1] -0.02183714
## [1] -0.1270582
## [1] 0.1749804
## [1] -0.1050326
```

```
## [1] 0.3243671
## [1] 0.1575845
## [1] -0.2605136
## [1] -0.400678
## [1] -0.2147045
## [1] 0.007729831
## [1] 0.04407834
## [1] 0.6498893
## [1] 0.1721173
## [1] -0.1301031
## [1] 0.06980732
## [1] 0.1238042
## [1] -0.2366367
## [1] -0.217564
## [1] 0.3195094
## [1] -0.3389511
## [1] 0.497933
## [1] 0.3601702
## [1] 0.04853112
## [1] -0.8204143
## [1] -0.2535806
## [1] 0.7654289
## [1] -0.2525585
## [1] -0.1468845
## [1] -0.2021198
## [1] 0.3097541
## [1] -0.6081058
## [1] 0.1990355
## [1] 0.1061568
## [1] 0.07451322
## [1] -0.1628335
## [1] 0.8107455
## [1] -0.2203342
## [1] -0.1340047
## [1] 0.7165682
## [1] 0.6110271
## [1] 0.6370051
## [1] 0.03769879
## [1] -0.2193037
## [1] 0.154598
## [1] 0.2374744
## [1] -0.008784863
## [1] 0.1029134
## [1] 0.1941017
## [1] 0.3900188
## [1] 0.1793854
## [1] 0.3535745
## [1] 1.064086
## [1] 0.1899473
## [1] -0.1235129
## [1] 0.4047439
## [1] -0.1256022
## [1] -0.1310526
## [1] 0.1938749
```

```
## [1] 0.02326905
## [1] -0.222512
## [1] 0.2002994
## [1] 0.2870124
## [1] 0.2262261
## [1] -0.2950809
## [1] -0.2865117
## [1] 0.1554272
## [1] 0.7059537
## [1] 0.08616393
## [1] -0.1031193
## [1] -0.08778265
## [1] 0.1037392
## [1] -0.1092388
## [1] 0.5886639
## [1] 0.1556349
## [1] 0.5842693
## [1] 0.2049357
## [1] -0.05956916
## [1] 0.4310127
## [1] 0.06306535
## [1] 0.04020987
## [1] 0.01870615
## [1] 0.3631591
## [1] -0.05633292
## [1] -0.3119538
## [1] 0.1250593
## [1] -0.5022807
## [1] 0.3128777
## [1] 0.06953299
## [1] -0.1982678
## [1] 0.2963315
## [1] -0.1011037
## [1] 0.5776724
## [1] 0.1237639
## [1] 0.2094847
## [1] 0.1352989
## [1] -0.2397246
## [1] -0.06259032
## [1] 0.1760737
## [1] -0.6255642
## [1] 0.4515513
## [1] 0.01920969
## [1] -0.2402775
## [1] -0.2870878
## [1] -0.216646
## [1] -0.382227
## [1] -0.3942864
## [1] 0.6074818
## [1] 0.4057714
## [1] 0.8574719
## [1] -0.06160816
## [1] -0.3884304
## [1] 0.1712166
```

```
## [1] -0.08651217
## [1] -0.09727396
## [1] 0.7103244
## [1] -0.2565313
## [1] 0.1454227
## [1] -0.3077864
## [1] -0.04769269
## [1] 0.08875627
## [1] -0.2923331
## [1] -0.4858675
## [1] 0.4935831
## [1] 0.3024652
## [1] -0.02159273
## [1] 0.4364967
## [1] 0.5147575
## [1] 0.322549
## [1] -0.2736788
## [1] 0.3644818
## [1] 0.3433073
## [1] -0.1043795
## [1] 0.2915686
## [1] 0.1594594
## [1] 0.7139994
## [1] 0.133533
## [1] -0.06726681
## [1] 0.1716104
## [1] 0.1528767
## [1] 0.0882284
## [1] -0.01035058
## [1] -1.122196
## [1] -0.3826578
## [1] 1.270137
## [1] -0.1605231
## [1] 0.8484648
## [1] 0.1748202
## [1] 0.352516
## [1] 0.09045742
## [1] 0.2357301
## [1] -0.6395599
## [1] 0.6031732
## [1] 0.2139171
## [1] 0.4398134
## [1] 0.3144097
## [1] 0.2173168
## [1] 0.04008059
## [1] 1.064085
## [1] 0.1350856
## [1] -0.32459
## [1] -1.000215
## [1] 0.4537493
## [1] 0.2864288
## [1] -0.2052973
## [1] 0.5925247
## [1] 1.437646
```

```
## [1] -0.2998278
## [1] 0.1643766
## [1] 0.4576775
## [1] 0.2868437
## [1] 0.2782599
## [1] 0.6557518
## [1] -0.5556144
## [1] -0.362163
## [1] -0.2590868
## [1] -0.3842533
## [1] 0.2392134
## [1] -0.7512287
## [1] 0.3796152
## [1] -0.08157948
## [1] 0.5891527
## [1] -0.303388
## [1] -0.2984651
## [1] 0.4062701
## [1] -0.1139865
## [1] 0.1129085
## [1] -0.2341154
## [1] -0.2533716
## [1] 0.1020335
## [1] 0.1615105
## [1] -0.5319894
## [1] 0.2363553
## [1] -0.1232761
## [1] -0.4495679
## [1] 0.2283413
## [1] -0.004660812
## [1] 0.130542
## [1] 0.2198495
## [1] -0.2830829
## [1] -0.4262981
## [1] 0.05393468
## [1] 0.1952424
## [1] -0.1485305
## [1] -0.1982348
## [1] 0.05857227
## [1] 0.1738032
## [1] 0.1397535
## [1] 0.4401267
## [1] 0.1058227
## [1] 0.2768031
## [1] 0.6135386
## [1] -0.2470039
## [1] 0.5581605
## [1] -0.2859161
## [1] -0.5927713
## [1] 0.1241094
## [1] -0.2830712
## [1] 0.2628602
## [1] -0.04743769
## [1] -0.02724908
```

```
## [1] -1.048613
## [1] -0.09149421
## [1] 0.2636221
## [1] -0.07341446
## [1] 0.3947719
## [1] -0.1374101
## [1] 0.2863224
## [1] -0.2096094
## [1] 0.2441385
## [1] 0.1771919
## [1] -0.2530407
## [1] -0.9346603
## [1] -0.4466615
## [1] -0.1444036
## [1] -0.1559847
## [1] 0.0350832
## [1] 0.1885982
## [1] -0.8715572
## [1] -0.3064275
## [1] -0.2668368
## [1] 0.05858632
## [1] -0.05073725
## [1] -0.1853645
## [1] -0.3219654
## [1] 0.4715288
## [1] 0.2287414
## [1] 0.01054023
## [1] -0.4166465
## [1] -0.2022193
## [1] -0.2558802
## [1] -0.110267
## [1] 0.2097315
## [1] -0.1603177
## [1] -0.341711
## [1] 0.1733211
## [1] -0.2692339
## [1] -0.2062526
## [1] -0.1919132
## [1] 0.1000357
## [1] 0.02432825
## [1] -0.2749606
## [1] -0.05613106
## [1] -0.2556364
## [1] -0.04641764
## [1] 0.1021246
## [1] -0.1833675
## [1] -0.1396304
## [1] 0.3992461
## [1] -0.2939984
## [1] 0.2066052
## [1] 0.0150033
## [1] -0.1621733
## [1] -0.06212306
## [1] -1.117875
```

```
## [1] -0.2501767
## [1] -0.6781537
## [1] 0.05353574
## [1] -0.3511788
## [1] -0.1509057
## [1] -0.2387915
## [1] 0.1457488
## [1] 0.3233149
## [1] -0.2640785
## [1] 0.3530212
## [1] 0.7339306
## [1] 0.6925909
## [1] -0.54351
## [1] 0.1270901
## [1] -0.01583642
## [1] 0.3637648
## [1] -0.2486105
## [1] -0.3855262
## [1] -0.8367163
## [1] -0.2093756
## [1] -0.4843479
## [1] 0.2758725
## [1] -0.04382105
## [1] -0.2465833
## [1] -0.3770708
## [1] -0.6749184
## [1] 0.06143775
## [1] 0.3113895
## [1] 0.06909051
## [1] -0.5070597
## [1] -0.4625027
## [1] -0.6521201
## [1] 0.5282303
## [1] -0.5799434
## [1] -0.6934053
## [1] -0.2103438
## [1] 0.02069452
## [1] -0.5349489
## [1] -0.08365993
## [1] -0.7424709
## [1] -0.431116
## [1] -0.4355495
## [1] 0.02905554
## [1] -0.2951045
## [1] -0.3945242
## [1] -0.4258655
## [1] 0.2002047
## [1] 0.01979739
## [1] -0.2504366
## [1] -0.6834015
## [1] -0.2990372
## [1] -0.03192165
## [1] 0.1493461
## [1] -0.3455947
```

```
## [1] -0.4214745
## [1] -0.8600915
## [1] -0.181074
## [1] -0.3059192
## [1] -0.2233165
## [1] -0.08407188
## [1] 0.2664694
## [1] -0.09431359
## [1] -0.5631972
## [1] -0.008606553
## [1] 0.3548539
## [1] -1.11217
## [1] 0.007780138
## [1] -0.8696998
## [1] -0.2666659
## [1] -0.1876261
## [1] 0.03656011
## [1] 0.2887257
## [1] -0.1948171
## [1] -0.1141402
## [1] -0.1948747
## [1] -0.194602
## [1] -0.1037466
## [1] 0.1223057
## [1] -0.3843698
## [1] 0.3431784
## [1] 0.005350837
## [1] -0.4971244
## [1] -0.3262205
## [1] -0.4215327
## [1] -0.07926979
## [1] 0.008967069
## [1] -0.2010574
## [1] 0.332316
## [1] 0.1443136
## [1] 0.3702423
## [1] -0.6932433
## [1] -0.9793074
## [1] -0.3970582
## [1] -0.8720684
## [1] 0.06890825
## [1] 0.1962395
## [1] 0.2535722
## [1] 0.02355294
## [1] -0.103686
## [1] 0.1189783
## [1] 0.4354006
## [1] -0.6011046
## [1] -0.4082511
## [1] -0.5198496
## [1] -0.6199803
## [1] 0.3739449
## [1] -0.2023651
## [1] 0.05135154
```

```
## [1] -0.7283189
## [1] 0.1620272
## [1] 0.3474183
## [1] 0.1694853
## [1] 0.2775011
## [1] -0.6193428
## [1] 0.1559681
## [1] -0.4525128
## [1] 0.1846122
## [1] 0.2848629
## [1] -0.3411741
## [1] 0.4260148
## [1] -0.2434127
## [1] -0.4050463
## [1] 0.7311479
## [1] 0.2493267
## [1] 0.2670204
## [1] 0.3933382
## [1] -0.3911601
## [1] -0.179706
## [1] 0.1119947
## [1] 0.2369495
## [1] -0.9142438
## [1] -0.3239715
## [1] -0.03871544
## [1] -0.5887818
## [1] -0.1075988
## [1] 0.314923
## [1] 0.087131
## [1] -0.08842168
## [1] -0.06724121
## [1] -0.06216149
## [1] -0.1884414
## [1] 0.1659758
## [1] -0.8642999
## [1] -0.6643879
## [1] -0.2445553
## [1] 0.2582546
## [1] -0.7735187
## [1] 0.1393467
## [1] 0.1620741
## [1] 0.134652
## [1] 0.07413939
## [1] 0.2280916
## [1] 0.001853233
## [1] 0.1258658
## [1] -0.199471
## [1] 0.04793822
## [1] -0.4341439
## [1] -0.1655872
## [1] 0.3397794
## [1] 0.05680641
## [1] 0.2563605
## [1] -0.04551845
```

```
## [1] -0.02821389
## [1] 0.09279649
## [1] 0.3451779
## [1] -0.1613471
## [1] -0.4703735
## [1] -0.006958541
## [1] 0.1385824
## [1] -0.2547227
## [1] -0.1398486
## [1] 0.2409935
## [1] -0.01050944
## [1] 0.165938
## [1] 0.03448893
## [1] -0.4685141
## [1] -0.4978263
## [1] -0.01849712
## [1] -0.3432755
## [1] 0.3009777
## [1] 0.1299662
## [1] 0.1693878
## [1] -0.1892977
## [1] -0.1721301
## [1] 0.6028943
## [1] -0.200717
## [1] 0.07528287
## [1] 0.302394
## [1] 0.2207064
## [1] 0.5086655
## [1] -0.3383349
## [1] -0.09560425
## [1] 0.2743364
## [1] 0.1363274
## [1] -0.3348578
## [1] -0.2017623
## [1] -0.1123653
## [1] 0.2158728
## [1] -0.4649177
## [1] 0.05569025
## [1] 0.0947728
## [1] 0.4491932
## [1] -0.1921081
## [1] -0.5367735
## [1] 0.5605546
## [1] 0.1704214
## [1] -0.2584508
## [1] 0.52852
## [1] -0.04720111
## [1] 0.1403687
## [1] -0.3996894
## [1] 0.5737586
## [1] -0.06037196
## [1] -0.004956556
## [1] -0.4850916
## [1] -0.6149888
```

```
## [1] -0.323543
## [1] 0.2017813
## [1] 0.357443
## [1] -0.15565
## [1] 0.3719785
## [1] 0.2453643
## [1] 0.2414792
## [1] 0.3492217
## [1] -0.03211963
## [1] -0.1815356
## [1] 0.2184741
## [1] 0.2273859
## [1] 0.287767
## [1] -0.5951485
## [1] 0.3946391
## [1] -0.4071934
## [1] -0.1487819
## [1] 0.2232466
## [1] -0.5930849
## [1] 0.03046991
## [1] -0.2883265
## [1] 0.3624743
## [1] -0.3176194
## [1] 0.1957215
## [1] -0.7956968
## [1] -0.2603587
## [1] -0.2472499
## [1] 0.2015193
## [1] 0.273335
## [1] -0.4353134
## [1] 0.2574038
## [1] 0.5316876
## [1] 0.1070165
## [1] 0.3753665
## [1] -0.1856889
## [1] 0.1001487
## [1] -0.03566984
## [1] 0.03639096
## [1] -0.03855125
## [1] 0.2290779
## [1] -0.3276434
## [1] -0.2777489
## [1] -0.4601315
## [1] 0.4322947
## [1] -0.2796941
## [1] -0.168509
## [1] 0.2498436
## [1] -0.5775589
## [1] -0.419634
## [1] 0.7556864
## [1] -0.587093
## [1] 0.2426851
## [1] 0.3426368
## [1] 0.2625114
```

```
## [1] -0.07222597
## [1] -0.1764287
## [1] 0.09364548
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## [1] -0.1401218
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## [1] 0.330142
## [1] -0.07996157
## [1] -0.03721941
## [1] 0.3006615
## [1] 0.8793768
## [1] 0.03326888
## [1] -0.4488161
## [1] -0.2872698
## [1] -0.01792615
## [1] 0.6626469
## [1] -0.7742802
## [1] -0.007484712
## [1] -0.2544165
## [1] -0.06215282
## [1] 0.09844006
## [1] 0.4489919
## [1] 0.4602623
## [1] -0.102306
## [1] 0.1854191
## [1] -0.229
## [1] 0.2205036
## [1] 0.3453601
## [1] -0.4416097
## [1] 0.1508541
## [1] -0.0188673
## [1] 0.7559581
## [1] -0.3945488
## [1] 0.01049295
## [1] 0.08851992
## [1] 0.1027883
## [1] -0.3818984
## [1] -0.05876261
## [1] 0.9659611
## [1] 0.04005865
## [1] -0.3234483
## [1] 0.3217594
## [1] -0.02250111
## [1] -0.3212328
## [1] 1.018702
## [1] -0.1548695
## [1] 0.1736406
## [1] 0.718585
## [1] 0.4919544
## [1] 0.7912655
## [1] 0.3919165
## [1] -0.2983083
## [1] -0.2902371
## [1] 0.1841659
## [1] 0.1290685
## [1] -0.03683344
```

```
## [1] -0.3674829
## [1] 0.3837232
## [1] -0.04205589
## [1] 0.2572362
## [1] -0.00120518
## [1] 0.1858412
## [1] 0.4157702
## [1] -0.1332222
## [1] -0.4412962
## [1] -0.06885445
## [1] -0.0355114
## [1] 0.5312175
## [1] -0.1481761
## [1] 0.1422009
## [1] 0.2853976
## [1] 0.0669
## [1] -0.050314
## [1] 0.6011492
## [1] 0.5122592
## [1] 0.8229658
## [1] -0.07086207
## [1] -0.2273213
## [1] 0.3235392
## [1] -0.007569987
## [1] -0.07067595
## [1] 0.1225126
## [1] -0.07672162
## [1] 0.5420257
## [1] -0.6464027
## [1] -0.1363704
## [1] -0.04877282
## [1] 0.5271079
## [1] 0.6738619
## [1] 0.5051127
## [1] -0.4658726
## [1] 0.4992466
## [1] -0.09255235
## [1] 0.6106585
## [1] 0.1574658
## [1] 0.08127126
## [1] 0.1541631
## [1] -0.0399838
## [1] 0.02740505
## [1] -0.6765445
## [1] -0.477534
## [1] -0.3697522
## [1] -0.02520784
## [1] 0.5036117
## [1] -0.4307889
## [1] 0.2058887
## [1] -0.3881365
## [1] 0.05148572
## [1] 0.6291541
## [1] -0.194594
```

```
## [1] -0.07418616
## [1] 0.02058307
## [1] 0.3037934
## [1] -0.2973399
## [1] -0.2437354
## [1] -0.06106434
## [1] -0.06457873
## [1] -0.3422788
## [1] -0.676104
## [1] -0.1463646
## [1] -0.1664002
## [1] 0.1681696
## [1] 0.3012727
## [1] 0.5092097
## [1] 0.3696911
## [1] 0.3472658
## [1] 0.2006372
## [1] -0.2368828
## [1] 0.02921696
## [1] 0.2589532
## [1] -0.05203487
## [1] -0.2271057
## [1] 0.5148939
## [1] -0.1788177
## [1] 0.6051233
## [1] -0.4465206
## [1] -0.4224013
## [1] -0.1287017
## [1] -0.6421
## [1] 0.6222869
## [1] -0.005643392
## [1] -0.039988
## [1] -0.1222778
## [1] 0.5934945
## [1] -0.4550586
## [1] -0.05927648
## [1] 0.2838955
## [1] 0.2009212
## [1] -0.474778
## [1] -0.200564
## [1] -0.353556
## [1] -0.4502523
## [1] -0.2510376
## [1] 0.6356998
## [1] -0.0126192
## [1] 0.5393121
## [1] -0.1777517
## [1] -0.2496952
## [1] 0.6923445
## [1] 0.391077
## [1] -0.3872142
## [1] 0.2301165
## [1] 0.3590331
## [1] 0.190325
```

```
## [1] -0.05043921
## [1] 0.2017992
## [1] 0.1207632
## [1] 0.6254874
## [1] 0.08605561
## [1] 0.3980876
## [1] -1.107866
## [1] 0.194075
## [1] 0.240726
## [1] -0.3666158
## [1] 0.5000759
## [1] 0.1702489
## [1] 0.1422808
## [1] 0.8804593
```

Influence of ρ on Performance

Influence of τ on Performance

Largest Eigenvalue of L

```
## [1] 1130
```