T1: Mechanics

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References

These books serve as our main references.

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1 Newtonian Mechanics

We begin with a review of Newtonian mechanics.

1.1 Newton's Axioms

Definition 1.1 (Particle)**.** A **particle** is a mass point, the only information associated with it is its mass m and its position $r(t)$ at time t. A free particle is a particle on which no forces act.

Axiom 1.1 (**N1**: Inertial Frames)**.** *There exists a reference frame (RF) called **inertial frames** (IR) in which a free particle moves with $\dot{r}(t) = v(t) = v_0$, where v_0 is a constant vector.*

Remark 1.2.

1. Axiom 1.1 does not hold for all RF. Ex: rotating RF, accelerating RF, etc.
2. Physics equations have the same form in all IRs.
3. Axiom 1.1 is not a corollary of Axiom 1.2 with $K = 0$, it defines an IR.

Axiom 1.2 (**N2**: Dynamics)**.** *In an IR, the motion of a particle is governed by Newton's second law:*

$$\boxed{K = \frac{dp}{dt}, \quad \textit{with}, \quad p = mv.} \tag{1}$$

Remark 1.3.

1. K is the force acting on the particle, defined as mass times acceleration.
2. The mass m of a particle is the proportionality constant to compare accelerations of different particles for same applied force.
3. Axiom 1.2 gives a statement about the trajectory of a particle.
4. Axiom 1.2 holds only in non-relativistic velocities ($v \ll c$).

Axiom 1.3 (**N3**: Action = Reaction)**.**

$$\boxed{K_{\textit{reaction}} = -K_{\textit{action}}.} \tag{2}$$

Remark 1.4.

1. Only valid for forces along the line connecting two particles.
2. For multiples forces, the superposition principle holds: $K = \sum_i K_i$.
3. Only valid for non-relativistic cases, since it implies instantaneous reaction, which contradicts principle of relativity (nothing propagates faster than light).

Example 1.5 (Solution of Equation 1 for 1-dim)**.** Let, $K = -\partial_x U(x)$, be a conservative field, where $U(x)$ is the potential and the particle moves on the x-axis. By (1), $m\ddot{x} = -\partial_x U(x)$, multiply by $\dot{x}$ on both sides: $m\ddot{x}\dot{x} = -\dot{x}\partial_x U(x)$. By chain rule, we have $\frac{m}{2}\frac{d}{dt}(\dot{x})^2 = -\frac{d}{dt}U(x)$. Integrating both sides w.r.t t, we get: $\frac{m}{2}\int dt \frac{d}{dt}(\dot{x})^2 = -\int dt \frac{d}{dt}U(x) + E \Rightarrow \frac{m}{2}(\dot{x})^2 = -U(x(t)) + E$. Rearranging, we get the total energy:

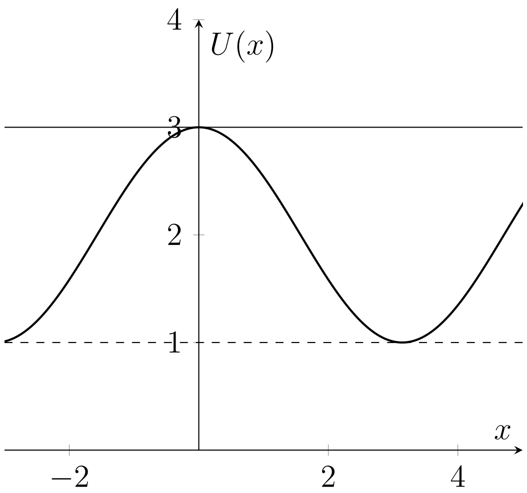
$$\boxed{E = \frac{1}{2}m\dot{x}^2 + U(x).} \tag{3}$$

The differential equation for $x(t)$, $\frac{dx}{dt} = \sqrt{\frac{2}{m}(E - U(x))}$ solved by separation of variables:

$$t - t_0 = \sqrt{\frac{m}{2}}\int_{x_0}^{x} \frac{dx'}{\sqrt{E - U(x')}}, \tag{4}$$

the initial conditions, $x(t_0) = x_0$ and $\dot{x}(t_0) = v_0$ determine $E = \frac{1}{2}mv_0^2 + U(x_0)$. The graph of $U(x)$ makes a classical potential well for stable equilibrium points, where the particle oscillates back and forth with a time period:

$$T = 2\int_{x_1}^{x_2} \frac{dx'}{\sqrt{\frac{m}{2}(E - U(x'))}}. \tag{5}$$



1.2 Conservation Laws

Initially formulated using Axiom 1.2, later using Lagrangians for symmetry.

Theorem 1.6 (Conservation of Momentum)**.**

$$\boxed{K = 0 \quad \overset{(1)}{\Longrightarrow} \quad \frac{d}{dt}p = 0 \quad \Longrightarrow \quad p = const.} \tag{6}$$

Definition 1.7 (Angular Momentum)**.** $l = r \times p$.

Definition 1.8 (Torque)**.** $M = r \times K$, where, $\dot{l} = M$ (N2 for rotation).

From these definitions, it follows:

Theorem 1.9 (Conservation of Angular Momentum)**.**

$$\boxed{M = 0 \quad \Longrightarrow \quad \frac{d}{dt}l = 0 \quad \Longrightarrow \quad l = const.} \tag{7}$$

Definition 1.10 (Work)**.** Work done by external force K upon a particle m moving from point 1 to 2 along a path C corresponds to the energy transfered by the force field to m.

$$\boxed{A_{1\to 2} = \int_C dr \cdot K.} \tag{8}$$

Definition 1.11 (Kinetic energy)**.** From calculations in Example 1.5,

$$\boxed{T = \frac{1}{2}m\dot{r}^2.} \tag{9}$$

1.3 System of Particles

1.4 Intertial System

1.5 Accelerated Reference Frames

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