

Analysis III

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These notes were prepared between December 2025 and (tentative) (**Last update: January 4, 2026**)

If you find any mistakes or typos, please report them to caccacpenguin@gmail.com. I would really appreciate it.

I often use informal language to make the ideas easier to grasp, but it's important to keep in mind the formalism and not get too attached to the informal ideas. My goal is to make the material feel approachable, while still respecting the rigour that makes mathematics what it is.

I hope you find these notes helpful :D!

Textbook Recommendations

These books will serve as our main references:

- Herbert Amann, Joachim Escher, Analysis III, Zweite Auflage, Birkhäuser-Verlag, 2008, Basel
- J. Elstrodt, Mass- und Integrationstheorie, Springer-Spektrum, 2018.
- Otto Forster, Florian Lindemann, Analysis 3, 8. Auflage, 2017

Some other great resources.

- R. L. Schilling, Mass und Integral, De Gruyter, 2015
- Walter Rudin, Principles of Mathematical Analysis, 3rd. Ed.
- Walter Rudin, Real and Complex Analysis, 3rd. Ed.
- Klaus Janich, Vektoranalysis
- John M. Lee, Introduction to Smooth Manifolds

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1 Introduction

2 Measure Theory

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5 Integration of Manifolds

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5.2 Integration of Differential forms

5.3 Stokes' Theorem