Classical Mechanics I

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These notes were prepared between November 2025 and (tentitive) **(Last update: December 29, 2025)**, while I was self studying Mathematics and Mathematical Physics. If you find any mistakes or typos, please report them to **caccacpenguin@gmail.com**. I would really appreciate it.

I often use informal language to make the ideas easier to grasp, but it's important to keep in mind the formalism and not get too attached to the informal ones. My goal is to make the material feel approachable, while still respecting the rigor that makes mathematics what it is.

I hope you will find these notes helpful :D!

Textbook Recommendations

These books serve as our main references.

- Ralph Abraham, Jerrold E. Marsden, Foundations of Mechanics, 2nd. Ed., Addison-Wesley, 1987
- Jerrold E. Marsden, Tudor Ratiu, Introduction to Mechanics and Symmetry, 2nd. Ed., Springer, 1998
- V. I. Arnorld, Mathematical Methods of Classical Mechanics, 2nd. Ed., Springer, 1989

Some other great texts:

- L. D. Landau, E. M. Lifshitz, Mechanics
- Herbert Goldstein, Classical Mechanics, 2nd Ed. Addison-Wesley, 1980.
- Torsten Fließbach. Mechanik: Lehrbuch zur Theoretischen Physik I, 7. Auflage, 2015.
- H. Stephani, G. Kluge, Klassische Mechanik, Spektrum Verlag
- F. Scheck, Theoretische Physik 1: Mechanik, Springer-Verlag
- Jean-Louis Basdevant, Jean Dalibard, Mecanique

My Other Notes

Mathematics:

- Foundation of Mathematics - *Logic, Set Theory and Proofs*
- Analysis I - *Analysis of Functions of Single Variable*
- Analysis II - *Differential Calculus of Several Variables*
- Analysis III - *Measure Theory, Integral Calculus, and Vector Analysis*
- Linear Algebra I - *Algebraic Foundations, Vector Spaces, Matrix Theory and Eigenvalue Theory*
- Linear Algebra II - *Canonical Forms, Inner Product, Multilinear Algebra and Tensors*
- Stochastic - *Probabilty, Statistics and Applications*
- Algebra - *Groups, Rings, Fields and Modules*
- Ordinary Differential Equations
- Function Theory - *Analysis of Functions of Complex Variables*
- Functional Analysis I - *General Theory of Functional Spaces*
- Functional Analysis II - *Advanced Theory of Functional Spaces*
- Geometry - *Foundations of Geometry and Topology*
- Probability Theory
- Differential Geometry
- Sympletic Geometry
- Riemannian Geometry
- Partial Differential Equations I
- Partial Differential Equations II
- Commutative Algebra
- Algebraic Number Theory - *Number Fields, Valuations and Adeles*
- Topology - *Homotopy, Homology and Cohomology Theories*
- Algebraic Geometry - *Varieties, Schemes and Sheaves*

Mathematical Physics:

- **Classical Mechanics I** - *Newton, Hamilton and Lagrange*
- Classical Mechanics II - *Qualitative Dynamics, Potential Theory, and Celestial Mechanics*

- Quantum Mechanics I - *Canonical Quantisation, Path Integrals, Abstract Hilbert Space and Functional Analytic Foundations*
- Quantum Mechanics II - *Measurement*
- Classical Electrodynamics - *Differential forms and $U(1)$ Gauge Theory on Fibre Bundles*
- Statistical Physics - *Probabilitic Dynamical Systems and Ergodic Theory*
- Theory of Relativity - *Geometry of Lorentzian and pseudo-Riemannian Manifolds*
- Quantum Field Theories I - *Path Integrals, EFT, Gauge Symmetry , QED, and The Standard Model*
- Quantum Field Theories II - *Wightman Axioms, Osterwalder-Schrader Reconstruction, Renormalisation Group and Scalar Field Theories*
- Mathematical Gauge Theories

Contents

1 Introduction

Physics is one of the most fascinating fields of science, it's purpose is to predict the future and recontruct the past of every particle in the Universe. It has tremenduos applications especially to engineering but to chemistry, biophyics, neuroscience, cognitive science and much more. It is probably not wrong to say that physics is the most fundamental of all natural sciences.

It's lowkey deadass that our universe can be understood by using mathematics rather than say poetry. However, it is important to distinguish the mathematical models from the real universe.

Many people treat mathematics as the reality. We see the phrase "the universe is written in the language of mathematics" being thrown around a lot. However, the fact is: our theories are just mathematical models of the reality. There is a difference between these two statements. The difference is that our physical interpretation of the mathematical concepts come from the experimental observations, but it is not reflected in the formalism.

Mathematical physics is just family of mathematical models, we apply those models to certain situations when the conditions are met, and to properly understand when and where our models apply we need to understand their foundations and study the axiom system and the model formally.

This change of philosophical perspective will help us understand why our models don't work at certain situations and directly target those problems rather than just trying new objects and hoping it would work.

However, reality is not a mathematical structure either because there are some things that our models alone cannot explain without something outside the system, our axioms themselves come from empirical observations and to make sense of where the models apply you need physical interpretation which is not part of the formalism.

There's no shame about admitting that we don't understand nature we just use models. We don't really understand what the "physical aspect" of our models are, what are forces? What really are quantum particles? These philosophical questions are hard to answer even though our models work spectacularly and we make predictions and experimentally confirm them up to several decimal places.

Our formal system mostly focuses on the syntax, the semantics is not there. For example, quantum mechanics tells us that an electrons is: A representation of the Poincaré group, state in a Hilbert space, with eigenvalue of mass, spin, parity. But this is just the mathematical object. The formal system says nothing about the physical electron. You need semantics for physical interpretation, ontology for what exists physically and epistemology for how we know it (the experimental part).

And the biggest issue of Physics, as it exists today, is: syntactically hand-wavy, semantically incomplete, ontologically ambiguous, and absent epistemology.

You might be worried, thinking, "*If physical theories are just mathematics then isn't it kind of sad? How are we to talk about atoms and elementary particles if we can't precisely model them?*"

Indeed, it does seem like we can't talk about atoms and elementary particles, if you take physical interpretation completely out of the picture, then there would be no point in calling this physics. We study physics because we want to understand nature.

But this is not true, we can talk about these phenomena if we consider the semantics of our theories. The point of the above discussion was to strictly seperate the mathematical formalism from the physical interpretation and not to through away the physical nature of the theories altogether.

Physicists don't really care about language and precision, they just want to make predictions; but to truly understand physical theories we must be clear about what we are talking about and what we are not.

Our approach will be to study the mathematical models rigorously and understand where they can be applied to solve pracical problems. The focus is on conceptual understanding, formal rigour and mathematical precision. But we won't neglact practical applications either, and we'll solve plenty of problems as well.

These notes are not self-contained, you need elite ball knowledge, Analysis I and II, Linear Algebra I and II, Differential Topology and some Differential Geometry. Typically an elementary course on classical mechanics is assumed for this course however there is no need for you to already be comfortable with variational principles and Hamiltonian systems, we'll start from the Newtonian formalism and take everything step-by-step. Only elementary understand of Newtonian Mechanics is expected.

In the first volume, we'll focus on analytical dynamics of Hamiltonian and Lagrangian systems, the Symplectic form, symmetries and the Hamilton-Jacobi Theory. Special Relativity will not be covered here, it will be covered in a combined notes with General Relativity in *Theory of Relativity.*

2 Newtonian Mechanics

In the first chapter we'll review Galileo and Newton's formalism and the three axioms along with some basic concepts of mechanics, but we'll do it more rigorously than elementary courses. These concepts will be use in every subsequent course. Understanding them is crucial to understanding almost all of physics.

2.1 Galilean Space-time

First we examine some experimental fact which are the basis for our axiom system and mechanics.

Definition 2.1. Space is a three-dimentional, oriented Euclidean space $\mathbb{E}$.

Definition 2.2. Time is described by a one-dimensional, oriented Euclidean space, represented by the continnum $\mathbb{R}$ of the real numbers, using the order $\leq$ on $\mathbb{R}$ as orientation and the usual distances between numbers.

Definition 2.3. Galilean Spacetime is a four-dimensional affine space $\mathcal{G} = (\tilde{\mathcal{P}}, \mathcal{V}, +)$, consisting of a set $\tilde{\mathcal{P}} = \mathbb{R} \times \mathcal{P}$ with $\mathcal{P} = \mathbb{R}^3$ of point-like spacetime events, a vector space $\mathcal{V} = \mathbb{R} \times \mathbb{E}$ and an addition operation, together with a linear form $\Delta\colon \mathcal{V} \to \mathbb{R}$, whose kernel $\mathbb{E}_0 \coloneqq ker\Delta \coloneqq \{v \in \mathcal{V} | \Delta(v) = 0\}$ is endowed with a Euclidean inner product, making $\mathbb{E}_0$ a three-dimensional Euclidean space.

Newtonian mechanics is the study of particles and systems of particles in a 3-dimensional Euclidean space.

2.2 Galilean Group

Newtonian mechanics is all about particles but what exactly *are* "particles"? We are told that: particles are small objects that can be treated as point masses. However this definition is oversimplified and not precise. In reality, there is no such thing as a particle, no object has "zero dimensions" and treating them as such is only an application of the object but not the true definition of the object itself.

Below I provide a formal definition of a particle:

Definition 2.4. A **particle** is a point in an affine space associated with a scalar, $m \in \mathbb{R}^+$, called mass of the particle.

Remark 2.5. In the framework of Newtonian mechanics, particles have *only* two properties, its position and its mass. Asking about any other property such as charge, spin, shape, size or colour of the particle is meaningless. These properties are not defined in this framework and hence cannot be discussed.

Technically, we're being unclear about what we mean by *mass* here. For now, we assume that mass is understood intuitively as a measure of the amount of matter in the particle, for now we treat it as a coupling constant, a positive real number. We will make these concepts more precise later on.

You may ask, "*Okay, but what does a particle physically represent?*"

A particle is a model of small objects whose dimensions are negligible compared to the length scales involved in the problem.

We can *apply* this model to physical objects only when the conditions are appropriate. For example, when studying the motion of planets around the sun, we can treat the planets and the sun as particles because their sizes are negligible compared to the distances between them. When talking about planets as particles we can only talk about it's position and mass at a certain instance and not about its magnetic field strength, the amount of water on the planet or the colour of the planet.

However, when studying the motion of a real object it is not clear how to apply this model. For example, when studying the motion of a car, we cannot meaningfully assign position to a spinning, flying car and treat it as a particle, do we assign the position at the trunk or at the stearing wheels? However, we'll see in future sections how almost all objects can be modelled as a particle if we consider the motion of its center of mass.

You might feel, "*This abstract idea of a particle is a bit difficult to grasp.*" Of course it can be, to make it easier, think of a particle not as a physical object but pair of two numbers (a, b) where a represents the position of the particle and b represents its mass. Formally we don't represent particles in this way, it's just a simple analogy to help you seperate the intuition from the abstract concept.

You might also wonder, "*Why do we even need this abstract definition of a particle? Why can't we just do physics in the intuitive way like we used to?*"

The reason for why we're being so strict about seperating the formalism and the interpretation is because our intuition can carry us only so far. When we study more advanced topics, such as quantum mechanics or general relativity, concepts get highly abstract and our intuition often fails us. Developing a habit of thinking precisely and clearly will be helpful in the long run.

3 Hamiltonian Mechanics

Definition 3.1. Let $\mathbb{E}$ be a finite-dimensional real vector space and $\omega \in L^2(\mathbb{E}, \mathbb{R})$ a bilinear form on E, so w: E X E+R. We say that w is nondegenerate

4 Lagrangian Mechanics