Mathematical Logic

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These notes were prepared between January 2026 and (tentative) **(Last update: January 4, 2026)**.
If you find any mistakes or typos, please report them to **caccacpenguin@gmail.com**. I would really appreciate it.
I often use informal language to make the ideas easier to grasp. My goal is to make the material feel approachable, while still respecting the rigor that makes mathematics what it is.
I hope you will find these notes helpful :D!

References

These notes are closely based on H. Schwichtenberg's lecture notes from WiSe 2025-26 on *Mathematische Logik* given at Mathematisches Institut, Ludwig-Maximilians-Universität, München. These books can serve as our main resources.

- Stanley S. Wainer, H. Schwichtenberg, Proofs and Computations
- A.S. Troelstra, H. Schwichtenberg, Basic Proof Theory, 2nd. Ed., 2000.
- Dirk van Dalen, Logic and Structure, 5th Edition, 2013.
- Heinz-Dieter Ebbinghaus, Jörg Flum, Wolfgang Thomas, Einführung in die mathematische Logik, 6. Auflage, 2018.

Other great resources and furthur readings:

- Stephen Cole Kleene, Introduction to Metamathematics, 1971.
- Joseph R. Shoenfield, Mathematical Logic, 1967.
- Joseph Mileti, Modern Mathematical Logic, 2013.
- Haskell B. Curry, Foundations of Mathematical Logic, 1977.
- Herbert B. Enderton, A Mathematical Introduction to Logic, 2nd. Ed., 2001.

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1 Introduction

This course is an introduction to mathematical logic and the foundations of mathematics. It covers both proof theory and model theory, with a focus on formal systems, their syntax and semantics, and the relationship between them.

The **contents** of this course is minimal logic and the integration of classical and intuitionistic logic; Gentzen's natural deduction calculus. Semantics and completeness in first-order predicate logic. Foundations of computability: Church's thesis and the undecidability of predicate logic. Gödel's incompleteness theorem for extensions of elementary number theory. Construction and structure of the number systems.

We'll be using Minlog, developed by Schwichtenberg's logic team at LMU Munich to formalise proofs and not something like Lean or Coq (cry more). The reason for this is because it's a low-level language so you're really learn the foundations of proof assistants well(but prof approval is more imp obv). To put that in perspective, if Lean or Coq are Python then Minlog is C. yes, youre bouta get cooked!!

The **prerequisites** for this course is some background in basic logic and set theory which you can find in the lecture notes on *Mathematical Foundations*.

2 Proof Theory

3 Recursion Theory

4 Gödel's Theorems

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