Linear Algebra I

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Preface

Linear Algebra is one of the most thoroughly developed areas of mathematics, with wide-ranging applications across natural scinces, computer science, engineering, finance and the social sciences. In particular, modern machine learning methods and neural network heavily rely on linear algebra. It is also essential for the modern treatment of geometry and analysis.

Just as *Analysis* prvide the formal foundation for calculus, linear algebra is the generalised, rigorous foundation for concepts like vectors, matrices and linear systems. We have all had experiences of doing calculations involving these objets, albeit informally.

Many concepts in linear algebra rely on more general algebraic structures, sometimes studied in *Abstract Algebra*. We shall therefore begin with the basic notions of groups, rings, and fields before defining vector spaces over fields.

This first volume covers groups, rings, fields, polynomials and modules; finite-dimentional vector spaces; bases and dimension; systems of linear equations; linear mappings and matrices; determinants; eigenvalues and eigenvectors.

Applications are primarily considered to theoretical physics (quantum mechanics and relativity) and computer science. Applications to differential equations or other areas of mathematics are not considered here.

The **prerequisites** for this course is high school mathematics and working knowledge of **logic, sets** and **proof techniques**, you can refer to the notes on *Mathematical Foundations* for those topics.

These notes were prepared between October 2025 and (tentative) **(Last update: January 28, 2026)**.

References

These books will serve as our main references:

- Klaus Jänich, Lineare Algebra, Springer-Verlag, 1994.
- Siegfried Bosch, Lineare Algebra, 5. Auflage, Springer-Verlag, 2014, Heidelberg.
- B.L. van der Waerden, Modern Algebra (Vol I)

Some other recommendations:

- Kenneth Hoffman and Ray Kunze, Linear Algebra, 2nd. Edition

- James B. Carell. Groups, Matrices, and Vector Spaces
- Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence. Linear Algebra. 4th. Ed.
- Sheldon Axler. Linear Algebra Done Right, 4th. Edition
- Serge Lang. Linear Algebra. 3rd. Edition
- Saunders MacLane. G. Birkhoff. Algebra. 1967
- Michael Artin. Algebra

Contents

1 Algebraic Structures

Our treatment of linear algebra begins with the discussion of the fundamental algebraic structures of gropus, rings, fields and modules. These structures are defined axiomatically in terms of one or more binary operations satisfying certain properties.

1.1 Groups

1.2 Homomorphisms

1.3 Rings

1.4 Fields

1.5 Polynomials

1.6 Modules

2 Vector Spaces

2.1 Vector Spaces

2.2 Basis and Dimensions

2.3 Direct Sums

3 Matrices and Systems of Linear Equations

3.1 Matrix Multiplication

3.2 Systems of Linear Equations

3.3 Matrices and Elementary Row Operations

3.4 Row-Reduced Echelon Matrices

3.5 Invertible Matrices

4 Linear Maps

4.1 Linear Maps

4.2 Quotient Space

4.3 Dual Space