# Analysis III

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These notes were prepared between December 2025 and March 2027 **(Last update: December 23, 2025)**

If you find any mistakes or typos, please report them to **caccacpenguin@gmail.com**. I would really appreciate it.

I often use informal language to make the ideas easier to grasp, but it's important to keep in mind the formalism and not get too attached to the informal ideas. My goal is to make the material feel approachable, while still respecting the rigour that makes mathematics what it is.

I hope you find these notes helpful :D!

## Textbook Recommendations

These books will serve as our main references:

- Herbert Amann, Joachim Escher, Analysis III, Zweite Auflage, Birkhäuser-Verlag, 2008, Basel
- J. Elstrodt, Mass- und Integationtheorie, Springer-Spktrum, 2018.
- Otto Forster, Florian Lindemann, Analysis 3, 8. Auflage, 2017

Some other great resources.

- R. L. Schilling, Mass und Integal, De Gruyter, 2015
- Walter Rudin, Principles of Mathematical Analysis, 3rd. Ed.
- Walter Rudin, Real and Complex Analysis, 3rd. Ed.
- Klause Janich, Vektoranalysis
- John M. Lee, Introduction to Smooth Manifolds

### Other Notes

**Mathematics:**

- Foundation of Mathematics - *Logic, Set Theory and Proofs*
- Analysis I - *Analysis of Functions of Single Variable*

- Analysis II - *Differential Calculus of Several Variables*
- **Analysis III** - *Measure Theory, Integral Calculus, and Vector Analysis*
- Linear Algebra I - *Algebraic Foundations, Vector Spaces, Matrix and Eigenvalue Theory*
- Linear Algebra II - *Canonical Forms, Inner Product Space, Bilinear forms*
- Stochastic - *Probabilty, Statistics and Applications*
- Algebra - *Groups, Rings, Fields and Modules*
- Ordinary Differential Equations
- Function Theory - *Analysis of Functions of Complex Variables*
- Functional Analysis I - *General Theory of Functional Spaces*
- Functional Analysis II - *Advanced Theory of Functional Spaces*
- Geometry - *Foundations of Geometry and Topology*
- Probability Theory
- Differential Geometry
- Sympletic Geometry
- Riemannian Geometry
- Partial Differential Equations I
- Partial Differential Equations II
- Commutative Algebra
- Algebraic Number Theory - *Number Fields, Valuations and Adeles*
- Algebraic Topology - *Homotopy, Homology and Cohomology Theories*
- Stochastic Processes - *Markov Theory, Random Walks*
- Stochastic Analysis and Differential Equations
- Stochastic Partial Differential Equations - *Regularity Structures*
- Algebraic Geometry - *Varieties, Schemes and Sheaves*

**Mathematical Physics:**

- Classical Mechanics I - *Newton, Hamilton and Lagrangian formalism*
- Classical Mechanics II - *Sympletic Geometry, Qualitative Dynamics, Potential Theory, and Celestial Mechanics*

- Classical Mechanics III - *Continnum Field Theory on Infinite-Dimensional Manifolds and Euler-Poincaré Reduction*
- Quantum Mechanics I - *Spectral theory of unbounded operators on Hilbert space*
- Quantum Mechanics II - *Measurement and Supersymmetry*
- Classical Electrodynamics - $U(1)$ *Gauge Theory on Fibre Bundles*
- Statistical Physics - *Probabilitic Dynamical Systems and Ergodic Theory*
- Theory of Relativity - *Theory of Lorentzian and Semi-Riemannian Geometry*
- Quantum Field Theory I - *Path Integrals, EFT, Gauge Symmetry, QED, and The Standard Model*
- Quantum Field Theory II - *Axiomatic and Constructive QFT*
- Quantum Field Theory III - *Stochastic Quantisation and Regularity Structures in SPDEs*
- Mathematical Gauge Theory

# Contents

# 1 Introduction

Linear Algebra is one of the most well-understood theory of mathematics, it also has very wide spread applications to several fields beyong just the natural scinces, computer science, engineering, finance and social sciences. Modern age machine learning algorithm and neural network are written in matrices. It probably has more applications than even calculus.

Just like Analysis, the formal foundation for calculus, Linear Algebra is a not much of a new subject for most of us. We sure have had experiences of doing calculations with vectors, metrices and determinants which constitutes the matrix theory. Linear Algebra is really just the formal, general and abstract foundation for those concepts.

Though the concepts presented here would still be new to some of you, as will cover more computational techniques than done in high school.

It is also used for modern treatment of geometry and analysis which is what I do extensively in my notes, which is why the notes of Linear Algebra are extremely important for you to cover before moving on to anything.

There is a more general course of algebra sometimes called as *Abstract Algebra* and a lot of the concepts of linear algebra rely on the concepts from the general theory of algebraic structures. For that reason, my treatment mirrors the continental treatment of Europe and especially Germany. We will start from the elementary concepts of groups, rings, fields and modules and define vector spaces as modules over fields.

These notes present a rigorous and abstract treatment of the concepts of linear algebra in the elementary and more general infinite-dimensional cases. But we will not miss any computation techniques which can be used for several applications linear algebra has. My goal is always to give a comprehensive, historical, philosophical, and foundational treatment and show you the most elegant, modern formalism for mathematics.

The first volume covers the basic concepts typically presented in a one-semester course of vector spaces, linear maps, matrix theory and eigenvalue theory. We will also consider many applications, though primarily to the fields of theoretical physics and computer science. Applications to differential equations or other maths areas however are not considered here.

The **prerequisites** for this course are working knowledge of **logic, sets** and **proof techniques**, you can refer to my notes on *Foundations of Mathematics* for those topics. Those notes cover much more than we need for this course but you can skip some sections and cover the essentials. Apart from that, only high school mathematics is expected from the readers.

# 2 Measure Theory

## 2.1 Measurable Spaces

## 2.2 Measure

## 2.3 Outer Measure

## 2.4 Measurable Sets

## 2.5 The Lebesgue Measure

# 3 Integration

## 3.1 Measurable Functions

## 3.2 Integrable Functions

## 3.3 Convergence Theorems

## 3.4 The Lebesgue Space

## 3.5 The $n$-dimentional Bochner-Lebesgue Integral

## 3.6 Fubini's Theorem

## 3.7 Convolution

## 3.8 Transformation Theorem

## 3.9 Fourier Transformation

# 4 Manifolds

## 4.1 Submanifolds

## 4.2 Multilinear Algebra

## 4.3 The local theory of differential forms

## 4.4 Vector Fields and Differential forms

## 4.5 Riemannian Metric

## 4.6 Vector Analysis

# 5 Integration of Manifolds

## 5.1 Volume Measure

## 5.2 Integration of Differential forms

## 5.3 Stokes' Theorem