The Exchange Protocol of Everlasting Options

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ABSTRACT
In this paper, we introduce a decentralized exchange protocol for everlasting options, a new type of derivative recently proposed to give traders long-term options exposure without the need of rolling positions. The protocol adopts the Proactive Market Making paradigm based on an analytical pricing formula of continuously funded everlasting options.

Contents

1. Introduction

1.1. *Option Basics*

Options are financial instruments that give investors the right (but not obligation) to buy or sell an underlying asset (i.e. underlier) at a fixed price (i.e. strike) on some set date in the future. There are two basic option types:

- Call options allow the holder to buy the underlying asset at strike.
- Put options allow the holder to sell the underlying asset at strike.

In practice, options are more often exercised by cash settlement, i.e. upon exercise the option writer (seller) pays the "payoff" to the option holder (buyer) with "cash". Therefore, options can be mathematically defined as an agreement stipulating the writer pays the holder the following amount upon exercise:

$$PAYOFF = \begin{cases} \max(S-K,0), & \text{for call option} \\ \max(K-S,0), & \text{for put option} \end{cases}$$

where S is the underlier's price and K is the strike.

Options are typically used by buyers for asymmetrical hedge, i.e. hedge against risks of underlier price moving to the unfavored direction without compromising the profits of the price moving to the favored direction. Whereas, option sellers usually try to make profits from collecting the option premiums by providing such one-sided protections to the buyers. Seeking one-sided protections and earning premiums by providing such protections are the typical motivations driving the demands and supplies of option markets.

Let's explain this with some specific examples. If Alice holds 1 BTC and would like to ensure that she would always be able to sell her position for at least $20000 per BTC, regardless of what happens to the market price of BTC, she could buy enough put options of BTC with a strike of $20000.

On the other hand, Bob predicts the price of BTC will not go below $20000 and would like to make profits from such a judgment, he could sell put options of BTC with a strike of $20000 to collect the option premiums. If the BTC price does stay above $20000 within the option time frame, Bob would walk away with the premiums as profit. However, if the BTC price goes below $20000 within the option time frame, Bob suffers the loss of paying $PAYOFF$ to the buyers.

In Alice's use case, the put options she holds will expire eventually. If she wants to keep her hedge, she will have to roll her options position forward, i.e., closing out the soon-to-expire position of puts and opening a new position of puts at the same strike with a later expiration. If Alice's expected hedging period is much longer than the expirations of the available options, this would be a burden and potentially costly process.

Option Pricing

Options can be priced using mathematical models such as the Black-Scholes[*1*] or Binomial pricing models. Most of these pricing models assume a *Geometric Brownian Motion* for the underlier price deviations and thus lead to the same results. The models assuming Geometric Brownian Motion for the underlier prices belong to the Black-Scholes pricing framework.

The famous Black-Scholes pricing formulae for European-style call and put options are as follows.

$$\begin{aligned} C &= N(d_1)S_t - N(d_2)Ke^{-rt} \\ P &= -N(-d_1)S_t + N(-d_2)Ke^{-rt} \end{aligned}$$

where

$$\begin{aligned} d_1 &= \frac{\ln \frac{S}{K} + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma\sqrt{t}} \\ d_2 &= d_1 - \sigma\sqrt{t} \end{aligned}$$

and

$$\begin{aligned} N &= \text{CDF of the normal distribution} \\ S &= \text{spot price of the underlier} \\ S &= \text{strike} \\ r &= \text{risk-free interest rate} \\ t &= \text{time to maturity} \\ \sigma &= \text{volatility of the underlier price} \end{aligned}$$

1.2. *Introducing Everlasting Options*

The recent "Everlasting Options" paper by White, D. *et al.*[*2*] has introduced a new type of financial derivative, *Everlasting Options*, inspired by the funding-fee-based paradigm introduced by BitMEX for the perpetual futures[*3*]. Perpetual futures have become extremely popular since their inception and brought to the industry a new category of derivatives, i.e. derivatives with positions maintained by regularly paying funding fees. Everlasting Options adopt this paradigm to options markets and have the potential to avoid the rolling issues and largely reduce the degree of liquidity fragmentation[*2*].

Perpetual futures as implemented in centralized exchanges (e.g. BitMEX) work as follows[3]: for every funding period (e.g. 1 day), those who are long the perpetual pay a funding fee to those who are short the perpetual. This funding fee is calculated as $(MARK - INDEX)$: the difference between the mark price (the trading price of the perpetual) and the index price (the price of the underlier)[1]. This funding fee mechanism causes the price of the perpetual to stay in line with the price of the underlier. One key parameter of the perpetual futures is the funding period, i.e. for how long the $(MARK - INDEX)$ funding fee is paid. The shorter the funding period is, the more fee the $(MARK - INDEX)$ discrepancy is charged for, and therefore the closer the mark price would stay with the index.

Everlasting Options work similarly: a long position is maintained by paying funding fees to a short position. However, in the case of everlasting options, the funding fee is charged as $(MARK - PAYOFF)$. Considering that theoretically $MARK$ should always be higher than $PAYOFF$, the funding fee should always be positive (i.e. long positions always pay short positions). For options, $(MARK - PAYOFF)$ has a specific financial significance - time value. Therefore, the mechanism of maintaining an everlasting option position has a very obvious financial significance too: option buyers pay the time values of the options associated with specific funding periods.

Please note that perpetual futures and everlasting options are just two special cases of the general form of funding-fee-based perpetual derivative requiring one long position pays one short position $[MARK - I(S)]$ as funding fee, where $I(S)$ is a general intrinsic value function of the underlier price S. Some examples of intrinsic value functions are as follows:

$$
\begin{aligned}
I(S) &= S, && \text{for perpetual futures} \\
I(S) &= \max(S - K, 0), && \text{for everlasting calls} \\
I(S) &= \max(K - S, 0), && \text{for everlasting puts}
\end{aligned}
$$

Payment Frequency

One important parameter of everlasting options is payment frequency. Given that you need to pay a funding fee of $(MARK - PAYOFF)$ for a specific funding period (e.g. 1 day), the fee could be divided into many payments (like an installment plan). For example, one could pay one-half of the total funding fee twice a day, or 1/24 of the total funding fee on an hourly basis. Hereafter let's denote the payment frequency (i.e. the count of payments that one funding fee is divided into) as F. F could be from 1 to ∞.

Pricing Everlasting Options

By means of the no-arbitrage argument, [2] has proved a pricing framework for funding-fee-based perpetual derivatives:

$$E = \frac{1}{F}\left[\frac{F}{F+1}P_{t_1} + \left(\frac{F}{F+1}\right)^2 P_{t_2} + ...\right]$$

[1]Please note this funding fee mechanism of perpetual futures on centralized exchanges is defined differently from that of Deri perpetual futures[5], even though they have the same mathematical essence.

where F is the payment frequency and P_{t_i} is the price of the regular option with expiration of t_i (REGPRICE of t_i).

Continuously Funded Everlasting Options

We are especially interested in adopting this pricing method in DeFi scenarios, where funding is usually accrued on a per-block basis. This corresponds to a very large F. Mathematically, the cases of large F are more convenient to be treated as $F \to \infty$. When F converges to infinity, this leads to a special funding style - continuous funding. That is, the funding fee that one long contract should pay one short contract is accrued continuously. This is similar to how interests are accrued for continuously compounded interest rates.

Let's denote the funding period as T (e.g. 1 day) and payment interval as $\Delta t = T/F$ (e.g. 1 hour if $F = 24$). Then we can rewrite the formula above. Note that $t_i = i\Delta t = iT/F$ and $i = Ft_i/T$, we have

$$E = \Delta t \left[\left(\frac{1}{1+1/F} \right)^{Ft_1/T} P_{t_1} + \left(\frac{1}{1+1/F} \right)^{Ft_2/T} P_{t_2} + ... \right]$$

When $F \to \infty$, $\Delta t \to 0$, we have

$$\left(\frac{1}{1+1/F} \right)^{F} \to e^{-1}$$

And the summation converges to an integral

$$E = \int_0^\infty \frac{1}{T} e^{-t/T} P_t dt$$

Specifically, for everlasting call and put options, we have

$$\begin{aligned} C^{ever} &= \int_0^\infty \frac{1}{T} e^{-t/T} C(t) dt \\ P^{ever} &= \int_0^\infty \frac{1}{T} e^{-t/T} P(t) dt \end{aligned}$$

If we price $C(t)$ and $P(t)$ by the classic Black-Scholes pricing formula[*1*] with interest rate ignored (i.e. assuming 0 interest rate)[2]:

$$
\begin{aligned}
C(t) &= N\left(\frac{\ln(S/K)}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}\right) S - N\left(\frac{\ln(S/K)}{\sigma\sqrt{t}} - \frac{1}{2}\sigma\sqrt{t}\right) K \\
P(t) &= -N\left(-\frac{\ln(S/K)}{\sigma\sqrt{t}} - \frac{1}{2}\sigma\sqrt{t}\right) S + N\left(-\frac{\ln(S/K)}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}\right) K
\end{aligned}
$$

where

$$
\begin{aligned}
N &= \text{CDF of the normal distribution} \\
S &= \text{spot price of the underlier} \\
K &= \text{strike} \\
\sigma &= \text{volatility of } S
\end{aligned}
$$

then we can prove the following pricing formula for the continuously-funded everlasting options.

Pricing Formula of Continuously Funded Everlasting Options

Let's divide C^{ever} and P^{ever} into intrinsic value and time value:

$$
\begin{aligned}
C^{ever} &= \max(S-K, 0) + TimeValue_{call} \\
P^{ever} &= \max(K-S, 0) + TimeValue_{put}
\end{aligned}
$$

We can prove that the call and put options at the same strike have the same time value $TimeValue_{call} = TimeValue_{put} = V$, given by

$$
V = \begin{cases} \frac{K}{u}\left(\frac{S}{K}\right)^{-\frac{u-1}{2}}, & \text{if } S \geqslant K \\ \frac{K}{u}\left(\frac{S}{K}\right)^{\frac{u+1}{2}}, & \text{if } S < K \end{cases}
$$

where

$$
u = \sqrt{1 + \frac{8}{\sigma^2 T}}
$$

The details of derivations are laid out in the Appendix of this paper.

[2]Please note it would not be difficult to include interest rate in the calculation. However, as of the current stage of DeFi, risk-free interest rate itself is difficult to obtain. And it is mostly insignificant, especially relative to the volatility. Therefore, such an approximation would not cause non-negligible inaccuracy.

The Greeks

Given the pricing formula, it is easy to get the *Delta*, *Gamma* and *Vega* of time value:

$$\Delta = \frac{\partial V}{\partial S} = \begin{cases} -\frac{u-1}{2u}\left(\frac{S}{K}\right)^{-\frac{u+1}{2}} = -\frac{(u-1)}{2S}V, & \text{if } S > K \\ \frac{u+1}{2u}\left(\frac{S}{K}\right)^{\frac{u-1}{2}} = \frac{(u+1)}{2S}V, & \text{if } S < K \end{cases}$$

$$\Gamma = \frac{1}{4}(u^2-1)\frac{V}{S^2} = \frac{2}{\sigma^2 T}\frac{V}{S^2}$$

$$\nu = \frac{\partial V}{\partial u}\frac{\partial u}{\partial \sigma} = \begin{cases} \frac{K\left(1+\frac{u}{2}\ln\frac{S}{K}\right)}{u\sigma\left(1+\frac{T\sigma^2}{8}\right)}\left(\frac{S}{K}\right)^{-\frac{u-1}{2}} = \left(1+\frac{u}{2}\ln\frac{S}{K}\right)\left(1-\frac{1}{u^2}\right)\frac{V}{\sigma}, & \text{if } S > K \\ \frac{K\left(1-\frac{u}{2}\ln\frac{S}{K}\right)}{u\sigma\left(1+\frac{T\sigma^2}{8}\right)}\left(\frac{S}{K}\right)^{\frac{u+1}{2}} = \left(1-\frac{u}{2}\ln\frac{S}{K}\right)\left(1-\frac{1}{u^2}\right)\frac{V}{\sigma}, & \text{if } S < K \end{cases}$$

1.3. *Use Cases of Everlasting Options*

Case 1: Hedging

In Alice's use case mentioned in the previous subsection, if she were hedging her risk with everlasting put options, she would save a lot of hassle. She just needs to buy everlasting put options with a strike of \$20000 properly covering her spot portfolio and keeps paying the funding fee. As long as her long position is maintained (i.e. funding fee is paid on due), she remains protected from the downside market risk below \$20000. The protection takes place as follows: if the price of BTC went below \$20000, e.g. \$15000, the mark price of the put option would immediately go up by around \$5000 and Alice would have an unrealized PnL of \$5000 for the put option she holds. She could sell the everlasting put option to realize a profit of \$5000 to compensate for her loss due to the BTC price going to \$15000.

Table 1. Alice's hedged portfolio under different scenarios.

Scenario	BTC Price>\$20000	BTC Price=\$15000
EO Put unrealized PnL	0	+\$5000
Alice's portfolio value	BTC price	\$20000

Please note that, since there is no expiration, Alice cannot ”exercise” the everlasting put options. She needs to sell the everlasting options to realize her profit on it.

Case 2: Earning Premiums

On the other hand, in Bob's case, if he were selling some everlasting put options with a strike of \$20000 for his purpose, he would only need to keep sufficient collateral in his margin account. As long as his short position is maintained (i.e. his balance above maintenance margin requirement), he keeps collecting premium funding from the long positions. However, if the BTC price went below \$20000, Bob would immediately suffer an unrealized PnL of around -\$5000.

Table 2. Bob's short position under different scenarios.

Scenario	BTC Price>$20000	BTC Price=$15000
EO Put unrealized PnL	0	-$5000
Bob's overall PnL	funding fee collected	funding fee collected - $5000

Please note that, since there is no expiration, this unrealized PnL would only be "realized" upon the close of the position. This could be done by Bob himself or by a forced liquidation should Bob have failed to meet the maintenance margin requirement.

2. The Deri Everlasting Options

In Feb 2021, Deri Protocol was introduced as a decentralized protocol for users to exchange risk exposures precisely and capital-efficiently[*4*][*5*]. In this whitepaper we introduce an exchange protocol for everlasting options, as the option component of Deri protocol. Being able to facilitate the trading of both futures and options, Deri Protocol's mission of serving derivative trading on-chain is now comprehensive.

Following the AMM-based trading paradigm of Deri perpetual futures, Deri Protocol adapts the *Proactive Market Making*[*6*] framework for everlasting options. We name the adapted PMM for derivatives *Deri(vative) Proactive Market Making*, abbreviated as DPMM. Similar to that of Deri perpetual futures, liquidity pools under DPMM are also playing the roles of counterparties for traders. However, DPMM does not passively accept the oracle price it receives. Instead, it takes the oracle price as a guide to its built-in algorithm of price discovery. Before getting into what PMM is and how it is adapted for everlasting options, let's first explain why it is the choice among all the types of AMM.

2.1. *Rationale behind AMM Choice*

The AMM-based trading paradigm is popular in DeFi world, primarily because it provides an extremely simple way to organize liquidity for trading. Whereas the traditional way, i.e. orderbook, requires high expertise from the liquidity providers (i.e. the market makers). The easiness of liquidity-providing for trading is a revolutionary property of AMM. It does not just have a financial but also a social significance. But since this is beyond the scope of this whitepaper. We will only focus on the technical part of finance. The core function of AMM is price discovery. From an epistemological perspective, the price discovery of AMM is a process of obtaining knowledge (i.e. the prices). Therefore, AMM can be categorized per their price discovery procedures.

(1) *Ab intus* without *a priori* knowledge: the regular AMMs like constant product market makers (CPMM), e.g. Uniswap V2[*7*] or Balancer[*9*], determine their prices completely by the local transactions.
(2) *Ab intus* with *a priori* knowledge: the stableswap of Curve V1[*10*] is largely similar to CPMM except it adopts the *a priori* knowledge that $price = 1$.

(3) Guided by external information: AMMs like PMM, e.g. DODO, depend on oracle price input as a guide to its price discovery.

The choice of specific AMM type largely depends on the trading characteristics. First of all, if there is some *a priori* knowledge, it is better to adopt it and hence take the second approach (*ab intus* with *a priori* knowledge). Curve has become the dominant AMM for stablecoins mainly because it makes good use of the *a priori* knowledge about the price and thus achieves a superb financial efficiency. However, what the stableswap of Curve deals with is a very special scenario. In more general cases, *a priori* knowledge is usually unavailable. For general cases, if the price discrepancy with external markets is not a major concern, then the first type of AMM would be an elegant solution. This is usually the case when an AMM has authority or dominance in the price discoveries across all markets. For spot trading, it is usually so for those "long-tail" pairs, which are primarily (or even solely) traded on a single AMM, e.g. Uniswap. Nevertheless, when an AMM does not dominate the price discoveries across all markets, then the price discrepancy with external markets becomes a problem. For the first type of AMM, there is no “built-in” mechanism to synchronize the price with external markets. Instead, such synchronization is handled by a “plug-in” mechanism: traders take arbitrage and consequently converge the price to external markets. In general, such a “plug-in” mechanism is financially inefficient compared to built-in mechanisms. The so-called *impermanent loss* is the cost for such inefficiency. In other words, the AMM (or it’s really the liquidity providers) pays the arbitrageurs to synchronize the prices at the cost of "impermanent loss". Therefore, when an AMM has neither *a priori* knowledge nor dominance of price discoveries across all markets, the third approach, i.e. adopting guide by external information, might be a pragmatic choice.

There are two special cases worthy of special discussion.

Curve V2[*11*] extends its application to non-stablecoins by introducing an "internal oracle" to dynamically repeg the stableswap curve. The knowledge about "repegging" (given by "internal oracle") is learned from past experiences (*a posteriori* but *ab intus*), rather than *a priori*. So the AMM of Curve V2 for non-stablecoins belongs to the first category.

Uniswap V3[*8*] adopts a very special external mechanism to optimize the price synchronization: range orders. When LPs provide liquidity with specified "ranges", they implicitly bring in a lot of external market information to guide the price discovery, which substantially enhances the financial efficiency of the price synchronization. In other words, instead of solely depending on the arbitrageurs, Uniswap V3 lets the liquidity providers help synchronize the price, at zero cost. (Our discussion is from a financial perspective so gas cost is not in consideration.) However, similar to the orderbook paradigm, this requires relatively high expertise from the liquidity providers. Such a dependency on the guide by external information makes Uniswap V3 more of a third-type AMM.

In this whitepaper, we are especially interested in the case of derivative trading, in which price synchronization is a core issue. For derivative trading, what is happening in external markets is a major concern, simply because the value of a derivative depends on the underlier price, by definition. Therefore, the third approach might play a more important role in derivative trading.

It is worth noting that some of the DeFi derivative trading protocols do choose the first approach and leave the price synchronization to the external arbitrage mechanism. While such an arbitrage mechanism would eventually synchronize the price, it is financially inefficient. This is the major drawback of such derivative AMM.

As a representative of the third type, the price discovery of PMM[*6*] converges to the external markets much faster than the first approach (the regular AMM). This is our rationale behind choosing PMM for Deri everlasting options. In the following subsection, let's briefly explain the mechanism of PMM and our adapted version, DPMM.

2.2. *Proactive Market Making*

Proactive Market Making (PMM) was introduced by DODO as a universal liquidity framework[*6*]. Essentially, PMM adopts two parameters i and k to concisely parameterize (and thus compress the information of) virtual orderbooks. A liquidity pool working under a PMM algorithm executes trades with users as if it provides an orderbook, against which the users' market orders are filled. This virtual orderbook is parameterized by two parameters: i representing the initial mid price of the orderbook and k controlling the shape of the orderbook (and thus the depth at each bid and ask price).

In practice, parameter i is fed by an oracle to reflect the input of external market information and parameter k is hand-picked by the controller to determine the shape of the virtual orderbook.

2.3. *DPMM-based Everlasting Options*

With the system parameter k specified, PMM is essentially an algorithm to output a price curve for spot exchange, given oracle price i, the token balances in equilibrium state Q_0 and B_0, and the current inventory increment ΔB (or ΔQ). The price curve is to keep the 3 variables (mark price, Q, B) consistent during the transactions. In the case of derivatives, there are two differences:

- Price curve formula:
  There is no real spot exchange. The "inventory" being dealt with is the derivative positions, which can be mapped to a virtual surplus or shortage of the spot. Then the price curve is to keep a new set of 3 variables (mark price, pool liquidity, pool net position) consistent.

- Pricer and oracle:
  The input i is fed by a theoretical pricer, instead of a simple oracle. Besides the contract parameters of an everlasting option, the pricer takes two inputs: spot price S and volatility σ, which it reads from oracles. The pricer calculates the theoretical prices for the options with the analytical pricing formula introduced in Section 1.

With these adaptions, we rename our algorithm as *Deri(vative) Proactive Market Making*, abbreviated as DPMM, to emphasize the derivative-oriented nature of our PMM paradigm.

Architecture

The chart below illustrates the architecture of Deri Protocol's implementation of everlasting options with the DPMM-based paradigm.

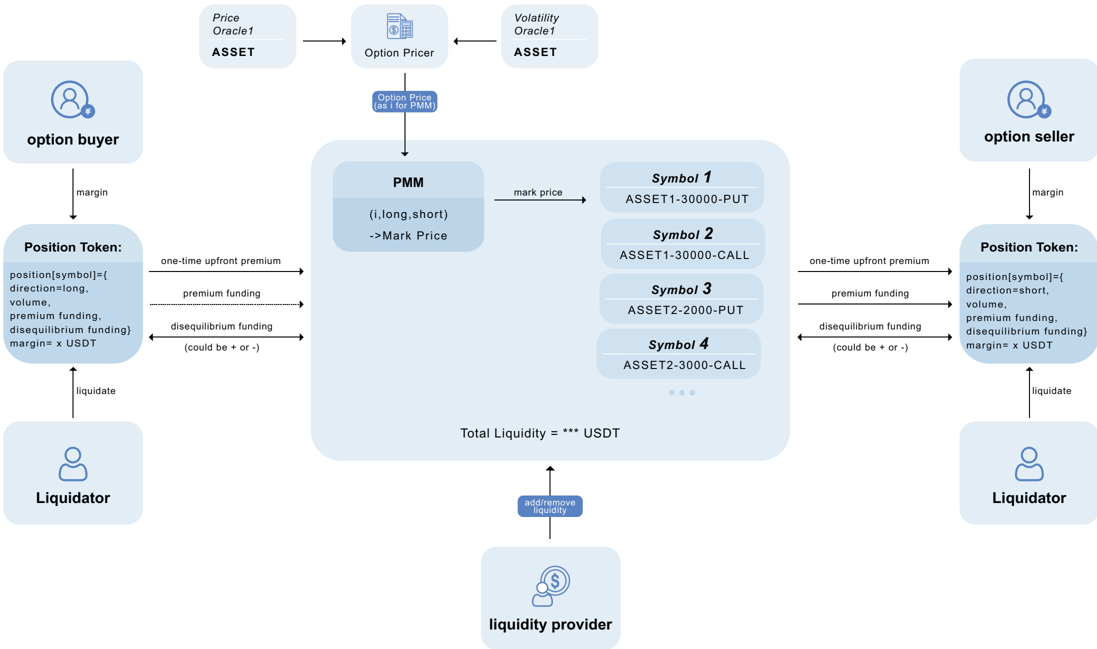

Figure 1. Architecture of the PMM-based trading protocol of everlasting options

2.4. *Funding Fee for Delta Risk*

In general, the long and short positions of the users would not be balanced, and consequently the liquidity pool would not be in a state of equilibrium. The liquidity pool will bear a market risk (more specifically, Delta risk in the option terminology) for the non-zero net position it holds. On the other hand, such a net position would cause the mark price to change accordingly, per the DPMM price curve, which consequently causes an increase or decrease of the funding fee. This extra part of the funding fee is corresponding to the Delta risk borne by the liquidity pool[3].

2.5. *Margin and Liquidation*

In-the-money everlasting options behave similarly to perpetual futures. Thus the maintenance and initial margin requirements are defined similarly. However, the margin

[3]In the case of perpetual futures, the whole funding fee is corresponding to the Delta risk

requirement for out-of-money everlasting options should be lowered since they have lower deltas, depending on the degree of out-of-money (i.e. out-of-money ratio).

With a benchmark margin ratio of 5%, We define the margin requirements as follows.

$$MM = \begin{cases} 0.05, & \text{for in-the-money} \\ \max(0.05(1 - 3 * R_{OTM}), 0.005), & \text{for out-of-money} \end{cases}$$

where 0.5 is the regular maintenance margin ratio, 0.05 is the minimum maintenance margin ratio, and R_{OTM} is the out-of-money ratio, given by

$$R_{OTM} = \begin{cases} \max((strike - spot)/strike, 0), & \text{for call options} \\ \max((spot - strike)/strike, 0), & \text{for put options} \end{cases}$$

Similarly, With a benchmark margin ratio of 10%, initial margin is defined as follows.

$$IM = \begin{cases} 0.1, & \text{for in-the-money} \\ max(0.1(1 - 3 * R_{OTM}), 0.01), & \text{for out-of-money} \end{cases}$$

Forced Liquidation of Positions
When the dynamic balance of a position token fails to meet the maintenance margin requirement, it could be liquidated by calling the *liquidate()* function of the smart contract by anybody (the liquidator). The handle of liquidation is largely similar to that of Deri perpetual futures. The liquidator pays the gas and shares the remaining value of the position with the liquidity pool. Please note that liquidation is against a whole position token, which might contain positions of several options. That is, when a position token gets liquidated, all the positions contained in this NFT are closed.

2.6. *Liquidity Consolidation*

One of the advantages of everlasting options over the classic ones is that there is no liquidity fragmentation by different expirations. However, if implementing everlasting options with the classical orderbook-based paradigm, it still suffers liquidity fragmentation by put/call and different strikes. Whereas the DPMM (or in general, AMM) paradigm further avoids such fragmentation. That is, put and call everlasting options at different strikes could be traded against one single liquidity pool. Moreover, even options of different underliers could be traded against the same liquidity pool and thus share the liquidity. By such liquidity consolidation, the DPMM-based paradigm could achieve an optimal capital efficiency for the everlasting option trading.

3. Summary

Since its launch, Deri Protocol has been working well for several months with various trading pools deployed on multiple blockchain networks to serve traders' hedging and speculating demands. It has proved that such a framework is generally effective and efficient for users to exchange risk exposures. Now we extend this framework from linear derivatives, i.e. futures, to nonlinear ones, i.e. options. To maximize liquidity consolidation and capital efficiency, we extend and implement the recently proposed everlasting options. Our approach is based on an analytical pricing formula of continuously funded everlasting options. Following the AMM-based tradition since Deri V1, we choose to base our implementation on the *Deri Proactive Market Making* framework, an adapted version of PMM. This is a prudent choice after a thorough review and analysis of the existing AMM types.

The everlasting options implemented as a decentralized protocol is one of the pioneering DeFi primitives. That is, unlike the other DeFi derivatives that were transplanted from the centralized financial world, the Deri everlasting options is a new species native to the DeFi world. The DeFi world will not authentically prosper until plenty of such native species thrive.

Acknowledgements

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References

(1) Black, Fischer; Scholes, Myron (1973). *"The Pricing of Options and Corporate Liabilities"*. Journal of Political Economy. 81 (3): 637–654.
(2) White, Dave; Bankman-Fried, Sam (2021). *"Everlasting Options"*. https://www.paradigm.xyz/2021/05/everlasting-options/
(3) *"Perpetual Contracts Guide"*. https://www.bitmex.com/app/perpetualContractsGuide
(4) *"The Derivative Exchange Protocol"*. https://github.com/deri-finance/whitepaper/blob/master/deri_whitepaper.pdf
(5) *"Deri V2: The Derivative Exchange Protocol with Extreme Capital Efficiency"* https://github.com/deri-finance/whitepaper/blob/master/deri_v2_whitepaper.pdf
(6) *"Proactive Market Making Algorithm"*. https://dodoex.github.io/docs/docs/pmm
(7) Adams, Hayden; Zinsmeister, Noah; Robinson, Dan (2020). *"Uniswap v2 Core"*. https://uniswap.org/whitepaper.pdf
(8) Adams, Hayden; Zinsmeister, Noah; Salem, Moody; Keefer, River; Robinson, Dan (2021). *"Uniswap v3 Core"*. https://uniswap.org/whitepaper-v3.pdf
(9) Martinelli, Fernando; Mushegian, Nikolai (2019). *"A non-custodial portfolio manager, liquidity provider, and price sensor"* https://balancer.fi/whitepaper.pdf
(10) Egorov, Michael (2019). *"StableSwap - efficient mechanism for Stablecoin liquidity"*. https://curve.fi/files/stableswap-paper.pdf

(11) Egorov, Michael (2021). *"Automatic market-making with dynamic peg"*. https://curve.fi/files/crypto-pools-paper.pdf

Appendices

Appendix A. Pricing Continuously Funded Everlasting Options

Starting from the Black-Scholes pricing formula with interest rate ignored, we have

$$
\begin{aligned}
C(t) &= N\left(\frac{\ln(S/K)}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}\right) S - N\left(\frac{\ln(S/K)}{\sigma\sqrt{t}} - \frac{1}{2}\sigma\sqrt{t}\right) K \\
P(t) &= -N\left(-\frac{\ln(S/K)}{\sigma\sqrt{t}} - \frac{1}{2}\sigma\sqrt{t}\right) S + N\left(-\frac{\ln(S/K)}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}\right) K
\end{aligned}
$$

where

$$
\begin{aligned}
N &= \text{CDF of the normal distribution} \\
S &= \text{spot price of the underlyer} \\
K &= \text{strike} \\
\sigma &= \text{volatility of } S
\end{aligned}
$$

Let $\tau = \frac{t}{T}$, $a = \frac{ln(S/K)}{\sigma\sqrt{T}}$, $b = \frac{1}{2}\sigma\sqrt{T}$, then we can rewrite the Black-Scholes pricing formula for call options as

$$
C(t) = C(T\tau) = N\left(\frac{a}{\sqrt{\tau}} + b\sqrt{\tau}\right) S - N\left(\frac{a}{\sqrt{\tau}} + b\sqrt{\tau}\right) K
$$

and

$$
C^{ever} = \int_0^{\infty} e^{-\tau} C(T\tau) d\tau
$$

Let's rescale and still use the letter t in the place of τ, we have:

$$
\begin{aligned}
C^{ever} &= \frac{S}{\sqrt{2\pi}} \int_0^{\infty} e^{-t} \left[\int_{-\infty}^{\frac{a}{\sqrt{t}}+b\sqrt{t}} e^{-\frac{y^2}{2}} dy\right] dt - \frac{K}{\sqrt{2\pi}} \int_0^{\infty} e^{-t} \left[\int_{-\infty}^{\frac{a}{\sqrt{t}}-b\sqrt{t}} e^{-\frac{y^2}{2}} dy\right] dt \\
&= SI_1 - KI_2
\end{aligned}
$$

with

$$
\begin{aligned}
I_1 &= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-t} \left[\int_{-\infty}^{\frac{a}{\sqrt{t}}+b\sqrt{t}} e^{-\frac{y^2}{2}} dy\right] dt \\
&= -\frac{1}{\sqrt{2\pi}} \int_0^{\infty} d(e^{-t})[...] \\
&= -\frac{1}{\sqrt{2\pi}} e^{-t}[...]|_0^{\infty} + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-t} d[...]
\end{aligned}
$$

The first part of I_1:

$$-\frac{1}{\sqrt{2\pi}}e^{-t}[...]|_0^\infty = \frac{1}{\sqrt{2\pi}}e^{-t}\left[\int_{-\infty}^{\frac{a}{\sqrt{t}}+b\sqrt{t}} e^{-\frac{y^2}{2}}dy\right]_{t=0} - \frac{1}{\sqrt{2\pi}}e^{-t}\left[\int_{-\infty}^{\frac{a}{\sqrt{t}}+b\sqrt{t}} e^{-\frac{y^2}{2}}dy\right]_{t=\infty}$$

within which the second item always converges to 0:

$$e^{-t}\left[\int_{-\infty}^{\frac{a}{\sqrt{t}}+b\sqrt{t}} e^{-\frac{y^2}{2}}dy\right]_{t=\infty} = 0$$

If $S > K, a > 0$, when $t \to 0$, $\frac{a}{\sqrt{t}} + b\sqrt{t} \to \infty$, we have:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{a}{\sqrt{t}}+b\sqrt{t}} e^{-\frac{y^2}{2}}dy \to \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}dy = 1$$

If $S < K, a < 0$, when $t \to 0$, $\frac{a}{\sqrt{t}} + b\sqrt{t} \to -\infty$, we have:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{a}{\sqrt{t}}+b\sqrt{t}} e^{-\frac{y^2}{2}}dy \to \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\infty} e^{-\frac{y^2}{2}}dy = 0$$

Second part of I_1

$$\begin{aligned}\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-t}d[...] &= \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-t}e^{-\frac{1}{2}\left(\frac{a}{\sqrt{t}}+b\sqrt{t}\right)^2}\frac{d}{dt}\left(\frac{a}{\sqrt{t}}+b\sqrt{t}\right)dt \\ &= \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}\left(\frac{a}{\sqrt{t}}+b\sqrt{t}\right)^2-t}\left(-\frac{a}{2}t^{-3/2}+\frac{b}{2}t^{-1/2}\right)dt\end{aligned}$$

let $\sqrt{t} = x$

$$\begin{aligned}&= \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}\left(\frac{a}{x}+bx\right)^2-x^2}\left(-\frac{a}{2}x^{-3}+\frac{b}{2}x^{-1}\right)2xdx \\ &= \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}\left(\frac{a}{x}+bx\right)^2-x^2}\left(-ax^{-2}+b\right)dx = A\end{aligned}$$

Similarly,

$$\begin{aligned}I_2 &= \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-t}\left[\int_{-\infty}^{\frac{a}{\sqrt{t}}-b\sqrt{t}} e^{-\frac{y^2}{2}}dy\right]dt \\ &= -\frac{1}{\sqrt{2\pi}}\int_0^\infty d\left(e^{-t}\right)[...] = -\frac{1}{\sqrt{2\pi}}e^{-t}[...]|_0^\infty + \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-t}d[...]\end{aligned}$$

The first part of I_2:

$$-\frac{1}{\sqrt{2\pi}}e^{-t}[...]|_0^\infty = \frac{1}{\sqrt{2\pi}}e^{-t}\left[\int_{-\infty}^{\frac{a}{\sqrt{t}}-b\sqrt{t}} e^{-\frac{y^2}{2}}dy\right]_{t=0} - \frac{1}{\sqrt{2\pi}}e^{-t}\left[\int_{-\infty}^{\frac{a}{\sqrt{t}}-b\sqrt{t}} e^{-\frac{y^2}{2}}dy\right]_{t=\infty}$$

within which the second item always converges to 0:

$$\frac{1}{\sqrt{2\pi}}e^{-t}\left[\int_{-\infty}^{\frac{a}{\sqrt{t}}-b\sqrt{t}} e^{-\frac{y^2}{2}}dy\right]_{t=\infty} = 0$$

If $S > K, a > 0$, when $t \to 0$, $\frac{a}{\sqrt{t}} - b\sqrt{t} \to \infty$, we have:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{a}{\sqrt{t}}-b\sqrt{t}} e^{-\frac{y^2}{2}}dy \to \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}dy = 1$$

If $S < K, a < 0$, when $t \to 0$, $\frac{a}{\sqrt{t}} - b\sqrt{t} \to -\infty$, we have:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{a}{\sqrt{t}}+b\sqrt{t}} e^{-\frac{y^2}{2}}dy \to \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\infty} e^{-\frac{y^2}{2}}dy = 0$$

Second part of I_2:

$$\begin{aligned}\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-t}d[...] &= \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-t}e^{-\frac{1}{2}\left(\frac{a}{\sqrt{t}}-b\sqrt{t}\right)^2}\frac{d}{dt}\left(\frac{a}{\sqrt{t}}-b\sqrt{t}\right)dt \\ &= \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}\left(\frac{a}{x}-bx\right)^2-x^2}\left(-ax^{-2}-b\right)dx = B\end{aligned}$$

Then we have:

$$C^{ever} = \begin{cases} S(1+A) - K(1+B), & \text{if } S > K \\ SA - KB, & \text{if } S < K \end{cases}$$

where

$$A = \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}\left(\frac{a}{x}+bx\right)^2-x^2}\left(-ax^{-2}+b\right)dx$$

$$B = \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}\left(\frac{a}{x}-bx\right)^2-x^2}\left(-ax^{-2}-b\right)dx$$

Or we can rewrite C^{ever} as the following, which is naturally divided into the intrinsic value (the first part) and the time value (the second part):

$$C^{ever} = \max(S - K, 0) + (SA - KB)$$

With a similar derivation, we have the following for everlasting put options:

$$P^{ever} = \max(K - S, 0) + (SA - KB)$$

Note that the everlasting call and put options have the same time value:

$$V = SA - KB$$

Now let's derive A and B.

$$\begin{aligned}
\sqrt{2\pi}A &= \int_0^\infty e^{-\frac{1}{2}(\frac{a}{x}+bx)^2-x^2}(-ax^{-2}+b)dx \\
&= e^{-ab}\int_0^\infty e^{-\frac{1}{2}(\frac{a}{x^2}+b_1x^2)}(-ax^{-2}+b)dx \\
&= e^{-ab}\int_0^\infty e^{-\frac{1}{2}(\frac{a}{x^2}+b_1x^2)}\left(ad\left(\frac{1}{x}\right)+bdx\right) \\
&= e^{-ab}\int_0^\infty e^{-\frac{1}{2}\left[\left(\frac{ab_1}{y}\right)^2+y^2)\right]}\left(ab_1d\frac{1}{y}+\frac{b}{b_1}dy\right) \\
&= e^{-ab}\int_0^\infty e^{-\frac{1}{2}\left[\left(\frac{c}{y}\right)^2+y^2)\right]}\left(d\left(\frac{c}{y}\right)+\frac{b}{b_1}dy\right)
\end{aligned}$$

where

$$b_1 = \sqrt{b^2+2}, y = b_1x, c = ab_1$$

Let $f(c) = \int_0^\infty e^{-\frac{1}{2}\left[\frac{c^2}{y^2}+y^2\right]}dy$. When $a > 0, c > 0$, it's easy to see:

$$f(c) = \int_0^\infty e^{-\frac{1}{2}\left[\frac{c^2}{y^2}+y^2\right]}dy = \int_{\frac{c}{y}\to\infty}^{\frac{c}{y}\to 0} e^{-\frac{1}{2}\left[\frac{c^2}{y^2}+y^2\right]}d\left(\frac{c}{y}\right) = -\int_0^\infty e^{-\frac{1}{2}\left[\frac{c^2}{y^2}+y^2\right]}d\left(\frac{c}{y}\right)$$

Therefore we can calcualte $f(c)$ as follows.

$$\begin{aligned}
f(c) &= \frac{1}{2}\int_0^{\infty} e^{-\frac{1}{2}\left[\frac{c^2}{y^2}+y^2\right]} d\left(y-\frac{c}{y}\right) \\
&= \frac{e^{-c}}{2}\int_0^{\infty} e^{-\frac{1}{2}\left(y-\frac{c}{y}\right)^2} d\left(y-\frac{c}{y}\right) \\
&= \frac{e^{-c}}{2}\int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} dz \\
&= e^{-c}\sqrt{\frac{\pi}{2}}
\end{aligned}$$

Then we have

$$\begin{aligned}
A &= \frac{1}{\sqrt{2\pi}} e^{-ab}\left[-f(c)+\frac{b}{b_1}f(c)\right] \\
&= -\frac{1}{2}e^{-a(b+b1)}\left(1-\frac{b}{b_1}\right) \\
&= -\frac{1}{2}e^{-w(1+u)}\left(1-\frac{1}{u}\right) \\
&= -\frac{1}{2}\left(\frac{S}{K}\right)^{-\frac{u+1}{2}}\left(1-\frac{1}{u}\right)
\end{aligned}$$

where

$$w = ab = \frac{1}{2}\ln\frac{S}{K}, u = b_1/b = \sqrt{1+\frac{8}{\sigma^2 T}}$$

Let $A = g(a,b)$, we have

$$\begin{aligned}
B = g(a,-b) &= -\frac{1}{2}e^{-a(-b+b1)}\left(1+\frac{b}{b_1}\right) = -\frac{1}{2}e^{w(1-u)}\left(1+\frac{1}{u}\right) \\
&= -\frac{1}{2}\left(\frac{S}{K}\right)^{\frac{1-u}{2}}\left(1+\frac{1}{u}\right)
\end{aligned}$$

$$V = SA - KB = \frac{K}{u}\left(\frac{S}{K}\right)^{-\frac{u-1}{2}}$$

Similarly, when $a<0, c<0$, we have

$$f(c) = \int_0^{\infty} e^{-\frac{1}{2}\left[\frac{c^2}{y^2}+y^2\right]} dy = e^{c}\sqrt{\frac{\pi}{2}}$$

$$A = \frac{1}{\sqrt{2\pi}} c^{-ab} \left(1 + \frac{b}{b_1}\right) f(c) = \frac{1}{2} e^{a(b_1 - b)} \left(1 + \frac{b}{b_1}\right) = \frac{1}{2} e^{w(u-1)} \left(1 + \frac{1}{u}\right)$$

$$B = \frac{1}{\sqrt{2\pi}} c^{-ab} \left(1 + \frac{b}{b_1}\right) f(c) = \frac{1}{2} e^{a(b_1 + b)} \left(1 - \frac{b}{b_1}\right) = \frac{1}{2} e^{w(u+1)} \left(1 - \frac{1}{u}\right)$$

$$V = SA - KB = \frac{K}{u} \left(\frac{S}{K}\right)^{\frac{u+1}{2}}$$

The case of $a = 0$ needs to be treated separately:

$$\begin{aligned} A &= \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}(b+2)^2 x^2} b dx \\ &= \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2} b_1^2 x^2} \frac{b}{b_1} d(b_1 x) \\ &= \frac{1}{2u} \end{aligned}$$

$$B = -\frac{1}{2u}$$

$$V = SA - KB = \frac{K}{u}$$

Note that V is continuous at $a = 0$, i.e. the point of ATM.