

2-Branch Conditional Expressions: $(\text{if } B \text{ Exp}_1 \text{ Exp}_2)$

$\text{Eval}(B, \alpha)$

Structured operational semantics

if $\text{Eval}(B) = \perp_v$ then \perp_v

else if $\text{Eval}(B) = \text{true}$ then $\text{Eval}(\text{Exp}_1)$

else $\text{Eval}(\text{Exp}_2)$

Boolean

Expression

True

False

Case

Case

The state α is implicitly given by the values of the function parameters

Multi-Branch Conditional Expressions: $(\text{cond}(B_1 \text{ Exp}_1) \dots (B_n \text{ Exp}_n))$

$B_i, 1 \leq i \leq n = \text{Boolean Expression}$

$\text{Exp}_i, 1 \leq i \leq n = \text{Expression to be evaluated if } B_i \Rightarrow \text{true}$

Any B_i may be keyword else which always evaluates to true.

if $\text{Eval}(B_1) = \perp_v$ then \perp_v

else if $\text{Eval}(B_1) = \text{true}$, then $\text{Eval}(\text{Exp}_1)$

else if $\text{Eval}(B_2) = \perp_v$, then \perp_v

= true, then $\text{Eval}(\text{Exp}_2)$

\vdots

else if $\text{Eval}(B_n) = \perp_v$ then \perp_v

= true, then $\text{Eval}(\text{Exp}_n)$

Function Definitions

$(\text{define } (\text{funName } p_1 \dots p_n) \text{ Exp}), n \geq 0$

↑

keyword

formal
parameters

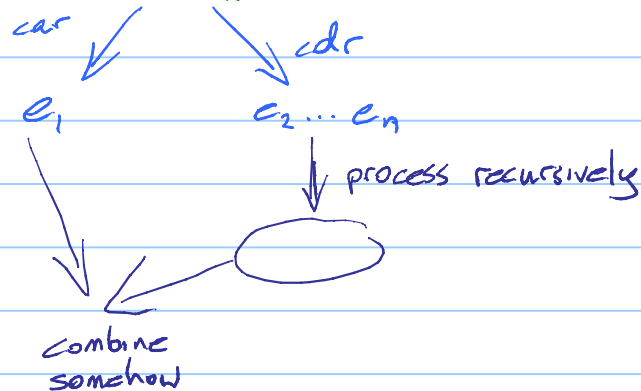
body expression

$(\text{define } (\text{square } x) (* x x))$

$(\text{define } (\text{fact } n) (\text{if } (= n 0) 1 (* n (\text{fact } (- n 1)))))$

Recursive Processing of Lists

- Process bottom case - process lists of 0, 1, 2 elements
- Recursive case - $(e_1 e_2 \dots e_n)$



(define (length list) ; return the # of elements in list
(if (null? list) 0 built-in function to see if list is ()
 (+ 1 (length (cdr list)))))

(length '(1 2 3)) \Rightarrow

(+ 1 (length '(2 3))) \Rightarrow

(+ 1 (+ 1 (length '(3)))) \Rightarrow

(+ 1 (+ 1 (+ 1 (length '())))) \Rightarrow

(+ 1 (+ 1 (+ 1 0))) \Rightarrow

(+ 1 (+ 1 (+ 1 1))) \Rightarrow

(+ 1 2) \Rightarrow

3

$(e_1 \dots e_n)(e'_1 \dots e'_m) \Rightarrow (e_1 \dots e_n e'_1 \dots e'_m)$

(define (append list 1 list 2) .

(if (null? list 1) list 2

(cons (car list 1) (append (cdr list 1) list 2)

)

)

)

$(\text{append } '(1\ 2) '(a\ b)) \Rightarrow$
 $(\text{cons } 1 (\text{append } '(2) '(a\ b))) \Rightarrow$
 $(\text{cons } 1 (\text{cons } 2 (\text{append } '(1) '(a\ b)))) \Rightarrow$
 $(\text{cons } 1 (\text{cons } 2 '(a\ b))) \Rightarrow$
 $(\text{cons } 1 '(2\ a\ b)) \Rightarrow$
 $(1\ 2\ a\ b)$

In Lisp, all operators and user-defined functions can be passed as actual parameters of functions

$(\text{define } (\text{map } f \text{ list}) ; \text{ apply unary operator/function } f \text{ to each element of list}$
 $(\text{if } (\text{null? list}) ()$
 $(\text{cons } (f (\text{car list})) (\text{map } f (\text{cdr list})))$
 $)$
 $)$

$(\text{map square } '(1\ 2\ 3)) \Rightarrow$
 $(\text{cons } 1 (\text{map square } '(2\ 3))) \Rightarrow$
 $(\text{cons } 1 (\text{cons } 4 (\text{map square } '(3)))) \Rightarrow$
 $(\text{cons } 1 (\text{cons } 4 (\text{cons } 9 (\text{map square } '(1))))) \Rightarrow$
 $(\text{cons } 1 (\text{cons } 4 (\text{cons } 9 '())) \Rightarrow$
 $(\text{cons } 1 (\text{cons } 4 '(9))) \Rightarrow$
 $(\text{cons } 1 '(4\ 9)) \Rightarrow$
 $(1\ 4\ 9)$

$(\text{map length } '(a\ b) (1\ 2\ 3) (\text{c d f})) \Rightarrow (2\ 3\ 3)$

$(\text{define } (\text{filter } f \text{ list}) ; f \text{ is a boolean unary function, drop from list all elements}$
 $\text{for which } f \text{ returns false}$

$(\text{cond } ((\text{null? list}) ())$
 $(\text{if } (\text{f } (\text{car list})) (\text{cons } (\text{car list}) (\text{filter } f (\text{cdr list})))$
 $(\text{B}_2 \quad \text{Exp}_2)$

$$\begin{aligned}
 & \underbrace{(\text{else } (\text{filter } f (\text{cdr list})))}_{B_3} \underbrace{(\text{filter number? } '(a\ 3\ 4\ 5.6\ \#3/6\ a\ 10.5e-5))}_{Exp_3} \\
 & \Rightarrow (3\ 4\ 5.6\ 10.5e-5)
 \end{aligned}$$

(define (accumulate f list init-val) ; Given a list $= (x_1, \dots, x_n)$ and a binary operator/function f ,
 ; return $f(x_2, f(x_2, \dots, f(x_n, \text{init-val}) \dots))$
 if (null? list) init-val
 (f (car list) (accumulate f (cdr list) init-val))
)

(define (sum-list list) (accumulate + list 0)) (sum-list '(1 4 5)) $\Rightarrow 1+4+5=10$
 (define (product-list list) (accumulate * list 1))
 (define (append-list list) (accumulate append list '()))
 (define (or-list list) (accumulate or list false))
 (define (and-list list) (accumulate and list true))