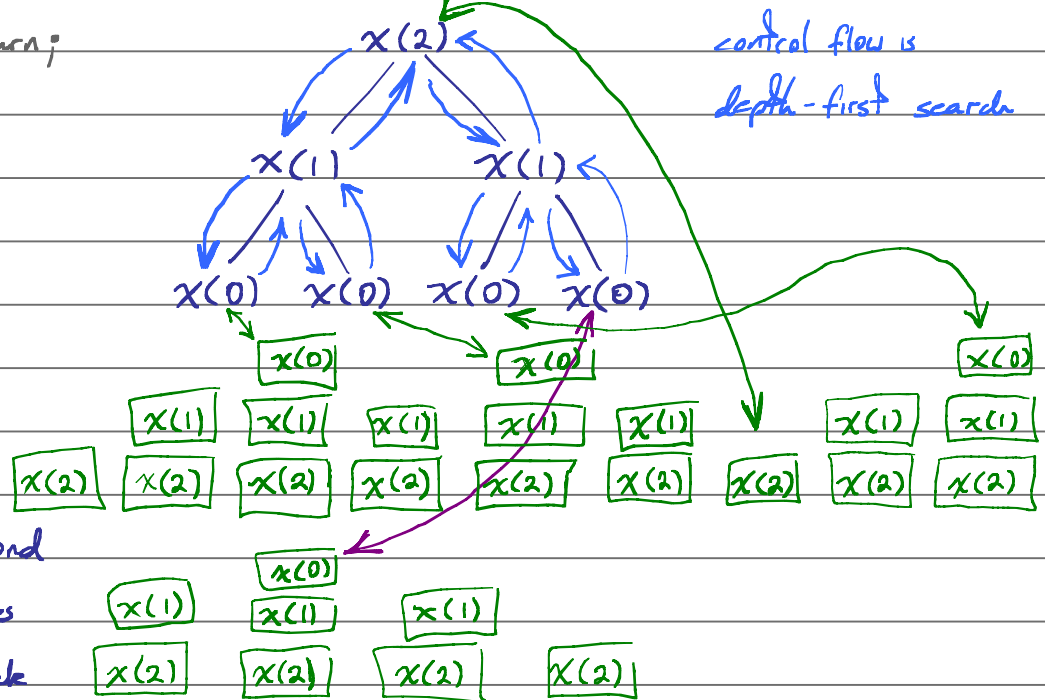


An example of free recursion:

```
void f(int n)
{
    if (n == 0) return;
    else
    {
        f(n-1);
        f(n-1);
    }
    return;
}
```

Binary Tree Recursion



- The peaks in the runtime stacks correspond to the deepest leaf nodes
- The # of A.R.s at peak size is approximately equal to the height of the tree of function calls

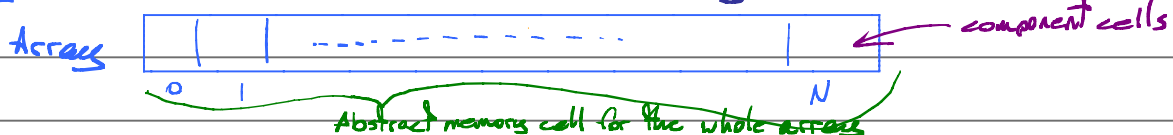
Memory Mapping and Memory Management

- Arrays
- Class Objects
 - Dynamic binding of method functions, inheritance, polymorphism
- Heap memory for dynamically created data objects.
- Garbage collection - automatically reclaims data objects not used.

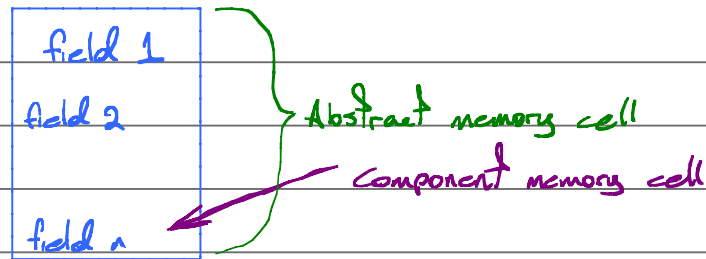
Abstract Memory cells = • regions of real/virtual main memory that holds well-defined data values/objects.

- an abstract memory cell is compound if it consists of component abstract memory cells

ex=

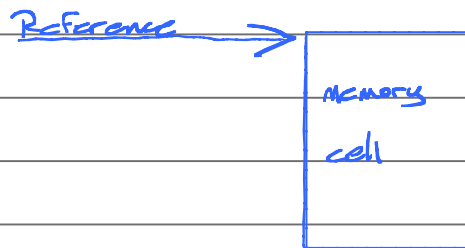


class object
Record Structure
'struct' type in C/C++

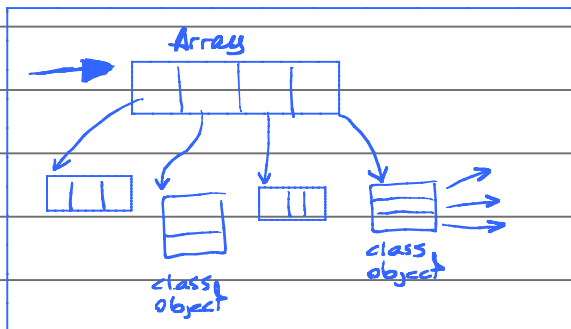
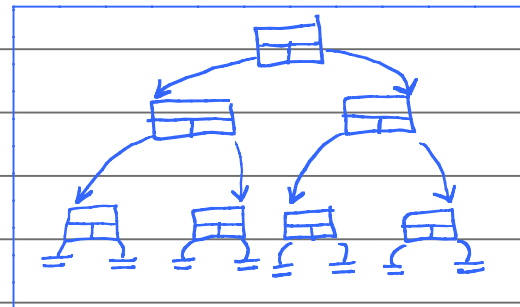
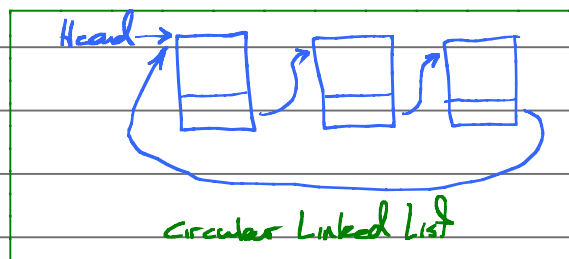


Reference / Pointer / Link

Physically, a reference (pointer, link) is the address of an abstract memory cell



References are crucial for dynamic creation of linked data structures



Memory Mapping OF Arrays

- $[a_1, \dots, b_1, \dots, a_n, \dots, b_n]$ denotes an anonymous n -dimensional array
- $a_i \leq b_i$ for each $1 \leq i \leq n$
- The k^{th} index ranges from a_k through b_k
- $[i_1, \dots, i_n]$ denotes $[i_1, \dots, i_n]^{\text{th}}$ element ($a_k \leq i_k \leq b_k$ for each $k: 1 \leq k \leq n$)

Contiguous Allocation

- n -dimensional $\xrightarrow{\text{how to map}}$ linear memory
- generalized row-major order $[a_1, \dots, b_1, \dots, a_n, \dots, b_n]$ is structured as 1-dimensional array $[a_1, \dots, b_1]$ of elements which are each $(n-1)$ -dimensional

array $[a_2 \dots b_2, \dots, a_n \dots b_n]$. Recursively, $[a_2 \dots b_2, \dots, a_n \dots b_n]$ is structured as a 1-dimensional array $[a_2 \dots b_2]$ of elements which are each $(n-2)$ -dimensional arrays $[a_3 \dots b_3, \dots, a_n \dots b_n]$, and so on.

$[0 \dots 1, 0 \dots 1, 0 \dots 1]$

<u>Row-Major</u>	<u>Column-Major</u>	<u>Symmetry Law</u> = $[i_1, \dots, i_n]$ - element of $[a_1 \dots b_1, \dots, a_n \dots b_n]$ in row-major order is in the same position as $[i_n, \dots, i_1]$ - element of $[a_n \dots b_n, \dots, a_1 \dots b_1]$ in column-major order
0 0 0	0 0 0	<u>BA</u> : <u>Base address</u> = the start address of the memory cell allocated to $[a_1, \dots, a_n]$ <u>ES</u> : <u>Element Size</u> = the size of each memory cell allocated to array elements, measured by the # of addressable memory units (e.g. bytes)
0 0 1	1 0 0	
0 1 0	0 1 0	
0 1 1	1 1 0	
1 0 0	0 0 1	
1 0 1	1 0 1	
1 1 0	0 1 1	
1 1 1	1 1 1	

$$\text{Address}([i_1, \dots, i_n]) = \text{BA} + \text{Rank}([i_1, \dots, i_n]) \times \text{ES}$$

where $\text{rank}([i_1, \dots, i_n]) = \text{the \# of elements preceding } [i_1, \dots, i_n]$

1-dimensional $[a_1 \dots b_1]$ = no difference between row-major and column major
 $\text{rank}([i_1]) = i_1 - a_1$

$$\text{address}([i_1]) = \text{BA} + (i_1 - a_1) \times \text{ES}$$

$$= \underbrace{\text{BA} - a_1 \times \text{ES}}_{\substack{\text{virtual base address} \\ \text{(virtual origin)}}} + i_1 \times \text{ES}$$