

316 10-22-09

Semantics of expressions given by $\text{Eval}(E, \alpha)$

Semantics of expressions given by $M(S', \alpha)$

assignment $x_i = E$

$$M(x_i = E, \{ \langle x_1, v_1 \rangle, \dots, \langle x_n, v_n \rangle \}) =$$

if $(v_e = \perp_v)$ then \perp_s

else $\{ \langle x_1, v_1 \rangle, \dots, \langle x_i, v_e \rangle, \dots, \langle x_n, v_n \rangle \}$

where $v_e = \text{Eval}(E, \{ \langle x_1, v_1 \rangle, \dots, \langle x_n, v_n \rangle \})$

if (B) S_1 else S_2

$$M(\text{if}(B) S_1 \text{ else } S_2, \alpha) =$$

if $(\text{Eval}(B, \alpha) = \perp_v)$ then \perp_s

else if $(\text{Eval}(B, \alpha) = \text{true})$ then $M(S_1, \alpha)$

else $M(S_2, \alpha)$

if (B) S

$$M(\text{if}(B) S, \alpha) =$$

if $(\text{Eval}(B, \alpha) = \perp_v)$ then \perp_s

else if $(\text{Eval}(B, \alpha) = \text{true})$ then $M(S, \alpha)$

else α

while (B) do S

$$M(\text{while}(B) \text{ do } S, \alpha) =$$

if $(\text{Eval}(B, \alpha) = \perp_v)$ then \perp_s

else if $(\text{Eval}(B, \alpha) = \text{false})$ then α

else if $(M(S, \alpha) = \perp_s)$ then \perp_s

else $M(\text{while}(B) \text{ do } S, M(S, \alpha))$ //iterate the loop from the new state $M(S, \alpha)$

do S while (B)

$$M(\text{do } S \text{ while } (B), \alpha) =$$

if $(M(S, \alpha) = \perp_s)$ then \perp_s

else if $(\text{Eval}(B, M(S, \alpha)) = \perp_v)$ then \perp_s

else if $(\text{Eval}(B, M(S, \alpha)) = \text{false})$ then $M(S, \alpha)$

else $M(\text{do } S \text{ while } (B), M(S, \alpha))$

$\alpha \rightarrow S_1; S_2; \dots; S_n \rightarrow ?$
 $M(S_n, \dots M(S_2, M(S_1, \alpha)) \dots)$

α_0
 $S_1: x=3;$
 α_1
 $S_2: a=2;$
 α_2
 $S_3: \text{if}(x < a) \quad \boxed{x = x + a}; \text{ else } \boxed{a = x + a}$
 α_3

$\alpha_0 = \text{initial program state} = \{ \langle x, 1 \rangle, \langle a, 1 \rangle \}$

$\alpha_1 = M(S_1, \alpha_0) = \{ \langle x, 3 \rangle, \langle a, 1 \rangle \}$

$\alpha_2 = M(S_2, \alpha_1) = \{ \langle x, 3 \rangle, \langle a, 2 \rangle \}$

$\alpha_3 = M(S_3, \alpha_2) ?$

$\text{Eval}(x < a, \alpha_2) = 3 < 2 = \text{false}$

$\alpha_3 = M(S_3, \alpha_2) = M(S_5, \alpha_2) = \{ \langle x, 3 \rangle, \langle a, 5 \rangle \}$

α_0
 $S_1: x=3;$
 α_1
 $S_2: a=2;$
 α_2
 $S_3: \text{while}(x < 5) \text{ do } \boxed{x = x + a}$
 α_3

$\alpha_0, \alpha_1, \alpha_2$ are the same as before

$\alpha_3 = M(S_3, \alpha_2) ?$

$\text{Eval}(x < 5, \alpha_2) = 3 < 5 = \text{true}$

$\alpha_4 = M(S_4, \alpha_2) = \{ \langle x, 5 \rangle, \langle a, 2 \rangle \}$

$\alpha_3 = M(S_3, \alpha_2) = M(S_3, \alpha_4)$

$\text{Eval}(x < 5, \alpha_4) = 5 < 5 = \text{false}$

$M(S_3, \alpha_4) = \alpha_4$

$\alpha_3 = M(S_3, \alpha_2) = M(S_3, \alpha_4) = \alpha_4$

- $\text{Eval}(E, \alpha)$ and $M(S, \alpha)$ can be implemented by a program (i.e. by recursive functions.)
- E and S are implemented by explicit parse trees.
- Data structures for program state α
 - List or vector of $\langle x_i, v_i \rangle$ pairs
 - More efficient table structures like Hash Tables.

- Resulting program is pure interpreter

Semantics By Equivalence = Applicable to any semantic description method (operational, denotational, axiomatic, etc.)

do S while(B) is equivalent to: S; while(B) do S

for(init_statement; B; inc_statement) S is equivalent to:

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inc_statement;
while(B) do
{
  S;
}
inc_statement;

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Controlling Function Calls by Runtime Stack

