

11-13

Warm-up Poker Hands

4 suits,
spades, hearts,
clubs, diamonds.

A, 2, 3, ..., 10, J, Q, K
13 cards in each suit

- a.) In 5-card Poker, how many possible hands are there?
- b.) How many Full Houses with 3 10's and 2 queens?
- c.) How many Full Houses?

b.) How many different ways to get 3 10's and 2 queens?

$$\bullet C(4,3) = \frac{4!}{(4-3)!3!} = 4 \text{ ways to choose 3 10's}$$

$$\bullet C(4,2) = \frac{4!}{(4-2)!2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = \frac{2 \cdot 3}{1} = 6 \text{ ways to choose 2 queens}$$

$$\Rightarrow 4 \cdot 6 = 24 \text{ ways to get a full house with 3 10's \& 2 Q's}$$

a.) 2,598,960 hands possible in 5-card poker. $C(52,5)$

- c.) Step 1: Pick a denomination: $C(13,1) = 13$ options
Step 2: Choose 3 of 4 cards: $C(4,3) = 4$ options
Step 3: Choose our next denomination: $C(12,1) = 12$ options
Step 4: Choose 2 of 4 cards: $C(4,2) = 6$ options

$$13 \cdot 4 \cdot 12 \cdot 6 = 3,744 \text{ possible full houses.}$$

Chapter 7: Probability

7.1 Sample Spaces & Events

A sample space is the set of all possible outcomes of an experiment.

ex/ Experiment: Flipping a coin and seeing which side faces up

Sample space: $S = \{H, T\}$

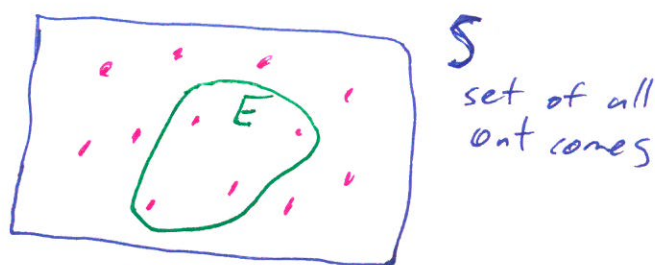
Outcomes are possible results of an experiment

ex/ Experiment: Rolling die and observing top number.

Outcomes: 1, 2, 3, 4, 5, 6

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

An Event is a subset of our sample space.



E: subset of S
outcomes we are
interested in.

ex/ Roll a die

$S = \{1, 2, 3, 4, 5, 6\}$

Interested in event where ~~result~~ resulting # is odd

$E = \{1, 3, 5\}$

ex/ Experiment: Select a city that starts with the letter "J"

Sample Space: $S = \{x \mid x \text{ is a city starting with the letter "J"}\}$

Event: The city is Johannesburg

$$E = \{ \text{Johannesburg} \}$$

- Events are subsets of our sample space

ex/ Experiment: Roll two distinguishable dice.

$$S = \{(1,1), (1,2), \dots, (6,6)\}$$

Event: when the numbers rolled add up to 1

$$E = \{ \} = \emptyset$$

Mutually Exclusive Events

events are mutually exclusive when their intersection is empty. Two things that cannot happen at the same time.

ex/ Experiment: Roll two distinguishable dice

$$S = \{(1,1), \dots, (6,6)\}$$

Event E: The numbers rolled are the same

Event F: The sum of the numbers rolled is odd

Mutually exclusive

- Remember that events are sets
 - we can take unions either/or
 - we can take intersections and
 - we can take complements not

7.2 Relative Frequency

Motivation: want to test if a coin is fair.

How to check: Flip it a lot of times.

ex / flip a coin 100 times

53 heads & 47 tails.

$\frac{53}{100} = .53$ is our relative frequency
or estimated probability

Had an experiment - flipping a coin repeated N times and we observe the outcome from ¹⁰⁰ $S = \{H, T\}$ looking for Event $E = \{H\}$

Number of times that E occurs is called the frequency $fr(E) = 53$

The relative frequency is the frequency of our event divided by the number times we did the experiment.

$$\frac{fr(E)}{N} = \frac{53}{100} = .53$$

ex Auctions on eBay

We take a survey of 50 paintings on eBay and observe the Bid Price

Bid Price	\$0-9.99	\$10-49.99	\$50-99.99	$\geq \$100$
Frequency	6	23	15	6

Say our experiment pick a painting and observe the price.

What is the relative frequency distribution?

- The relative frequency for each individual outcome.

Bid Price	\$0-9.99	\$10-49.99	\$50-99.99	$\geq \$100$
Relative frequency	$\frac{6}{50} = .12$	$\frac{23}{50} = .46$	$\frac{15}{50} = .30$	$\frac{6}{50} = .12$

$$\text{note: } .12 + .46 + .30 + .12 = 1$$

Question: What is the relative frequency that a painting in this survey has a bid price of less than \$50

Solution: # of paintings less than \$50 is $6 + 23 = 29$
and $\frac{29}{50} = .58 = \frac{\text{\# of paintings} < \$50}{\text{\# of paintings in survey}}$

Also we could just add the relative frequencies from our distribution $.12 + .46 = .58$

ex Table of 200 authors surveyed by a publishing company.

	New Authors	Established Authors	Total
Successful	12	44	56
Unsuccessful	38	106	144
Total	50	150	200

Find the relative frequency that

a.) Established and successful

$$\frac{44}{200} = .22$$

b.) Unsuccessful and new

$$\frac{38}{200} = .19$$

c.) Successful

$$\frac{56}{200} = .28$$

d.) A successful author is established.

Previously sample space was authors surveyed
Now it is successful authors surveyed.

$$\frac{44}{56} =$$