Geometric Methods in Computer Vision

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Congruence and Curvature

- What makes objects congruent?
- Transformation groups
  - Special Euclidean group
  - Equi-affine group
- Invariants - properties of curves which do not change under transformation
  - Euclidean curvature: κ
  - Equi-affine curvature: μ

tri.png

Euclidean versus Equiaffine Invariants

$\gamma = x(t), y(t)$ is a parametric curve.

Euclidean	Equiaffine
$s(t) = \int_0^t \lvert\gamma_\tau\rvert d\tau$	$\alpha(t) = \int_0^t (\gamma_\tau(\tau) \times \gamma_{\tau\tau}(\tau))^{\frac{1}{3}} d\tau$
curvature: $\kappa(s) = \gamma_{ss}$	curvature: $\mu(\alpha) = \gamma_{\alpha\alpha\alpha} \times \gamma_{\alpha\alpha}$
$T' = \kappa(s)N$	$T'' = \mu(\alpha)T$
$\gamma'(s) = T(s)$	$\gamma'(\alpha) = T(\alpha)$

The last two equation in each case allow reconstruction of γ from curvatures.

Power Series Method for Solving $T'' = \mu(\alpha)T$

- $\mu(\alpha) = \sum_{n=0}^{\infty} g_n \alpha^n$
  - $\mu(\alpha)$ must be analytic
  - all g_n's are known
- $T = \sum_{n=0}^{\infty} b_n \alpha^n$
- $T'' = \sum_{n=2}^{\infty} n(n-1) b_n \alpha^{n-2}$
- By substituting the above series into $T'' = \mu(\alpha)T$, you can solve for the b_n coefficients that make up the series solution to T
- Integrating T results in γ

Curves γ Reconstructed from Different $\mu(\alpha)$

alpha.png

$\mu(\alpha) = \alpha$

-alpha.png

$\mu(\alpha) = -\alpha$

newalpha2.png

-alphasquared.png

Picard Iteration for $\mu(\alpha) = -1$

Recall that for constant $\mu(\alpha) < 0$, the reconstructed curve becomes an ellipse. For $\mu(\alpha) = -1$, and $\begin{bmatrix} T \\ N \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the Picard iterations approximate to a circle in the following way:

What Picard Iterations Tell Us

As we continue to iterate our system, the iterated curves converge to our solution reconstruction. Similarly, our parameterization converges to the solution parameterization.

- $\gamma_1(t)$
- $\gamma_6(t)$
- $\gamma_{12}(t)$
- $\gamma_{18}(t)$

Euclidean Signature

$$\{(\kappa(s), \kappa'(s))\}$$

SignatureExample.png

ThumbPrint.jpg

Data is Noisy

DataIsNoisy.png

The Hausdorff Metric

HausdorffPicture.png

Signature as a Phase Portrait

UVectorField.png

The Tube Neighborhood

TubeDetailed.png

In Practice

SignatureSqueeze.png

Smooth Deformations of Curvature

- We can smoothly deform curvatures by adding a "bump" function
- Use bump functions to construct κ_n such that $|\sin(s) - \kappa_n(s)| \leq \frac{\pi}{n}$ for all s and κ_n is smooth and periodic
- $\kappa_n(s)$ converges uniformly to $\sin(s)$

bumpedSin,3,3.pdf κ_3 bumpedSin,3,10.pdf κ_{10}

- Call Γ_κ the curve reconstructed from curvature $\kappa(s)$ with initial conditions $\alpha_0 = x_0 = y_0 = 0$.

Reconstructions of $\sin(s)$ and $\kappa_3(s)$ on $[0, 12\pi]$

Reconstructed From Sine Curvature

From Sine Curvature.pdf

n = 3.pdf

Reconstructions of $\kappa_5(s)$ and $\kappa_{10}(s)$

$= 5.\mathrm{pdf}$

n = 5.pdf

n = 10.pdf

Noninteger n

n = 3,7.pdf

= 3,7.pdf

n = 4,5.pdf