12-04

Warm up: You flip 6 distinguishable coins. What is the probability that at least 2 are heads?

ex How many ways to flip exactly 3 heads

__ __ __ __ __ __

How many ways are there to choose 3 of these 6 blanks to fill in heads?

$$C(6,3) = \frac{6!}{3!\,3!} = \frac{\cancel{6}\cdot 5\cdot 4}{\cancel{3}\cdot\cancel{2}\cdot 1} = 20$$

Alt 1: 2 heads $= C(6,2) = \frac{6!}{4!\,2!} = \frac{6\cdot 5}{2} = 3\cdot 5 = 15$ outcomes

Alt 2: 3 heads = 20 outcomes

: 3   4 $= C(6,4) =$ 15 outcomes

: 4   5 $= C(6,5) =$ 6 outcomes

Alt 5 : 6 heads = 1 outcomes

- number of ways to flip at least 2 heads is $15 + 20 + 15 + 6 + 1 = 57$ way

- How many ways are there to flip 6 distinguishable coins?

$$n(S) = 2^6 = 64$$

Probability of flipping at least 2 heads is

$$\frac{57}{64}$$

How many ways to flip ~~0~~ at least 2 heads

The same thing as

Total ways to flip 6 coins − # ways to flip at most 1 head.

$64 - 7 = 57$

7.5 Condionala Probability and Independence

Nintendo ran ads for their latest Pokemon game and has a summary of the results

	saw the ad	Did not see the ad	Total
Purchased	100	200	300
Did not buy	200	1500	1700
Total	300	1700	2000

Did this ad work?

was it effective.

If some saw the ad were they more likely to buy than someone who did not see the ad?

- Probability they bought the game if they saw the ad? $\frac{\text{bought \& saw}}{\text{saw ad}} = \frac{100}{300} = 33\%$

- Probability they bought the game if they did not see the ad? $\frac{\text{bought \& did not see ad}}{\text{did not see the ad}} = \frac{200}{1700}$

12%

Two important related events (or unrelated)

A: event that a person buys the game

B: event that a person sees the ad.

The first probability we computed was the prob. a person bought the game <u>given that</u> they saw the ad.

"Probability of A given B" = $P(A|B)$

"given"

$P(A|B)$ is called a conditional probability.

$P(A|B) = .33$

~~$P(A'|B) = .12$~~ $P(A|B') = .12$

How do we calculate $P(A|B)$?

$$P(A|B) = \frac{\text{\# of people who saw the ad and bought the game}}{\text{\# of people who saw the ad}} = \frac{n(A\cap B)}{n(B)}$$

$$P(A|B) = \frac{n(A\cap B)}{n(B)} = \frac{n(A\cap B)/n(S)}{n(B)/n(S)} = \frac{P(A\cap B)}{P(B)}$$

ex/ Roll two distinguishable dice, fair.
what is the probability the sum of both numbers will be 8 given that the first number rolled is 3?

A: sum of # is 8 find $P(A|B)$

B: first # is 3 what is $n(A\cap B)$ and $n(B)$?

~~$A \cap B = 25$~~ $B = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$

$A \cap B = \{ \cancel{(3,5)}\ (3,5) \}$

~~$n(B)$~~ $P(A|B) = \dfrac{n(A \cap B)}{n(B)} = \dfrac{1}{6}$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ we can write as

$$P(A \cap B) = P(A|B)\,P(B)$$

ex/ If there is a 50% chance of rain (R) and a 20% chance of lightning <u>if it</u> rains (L), what is the probability of both happening?

$P(R) = .5$

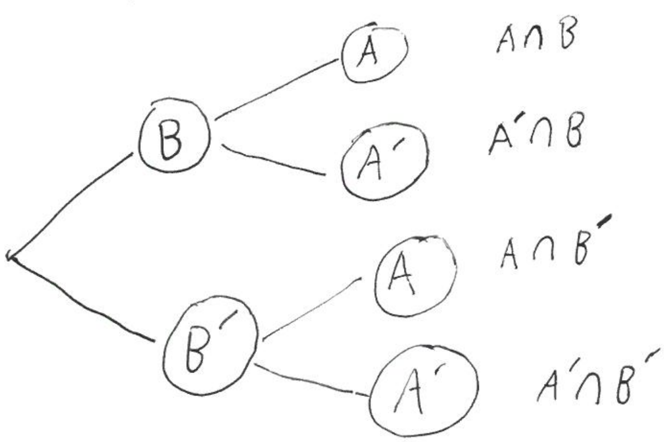
$P(L|R) = .2$

$$P(R \cap L) = P(L|R)\,P(R)$$
$$= .2 \times .5 = .1 \text{ or } 10\%$$

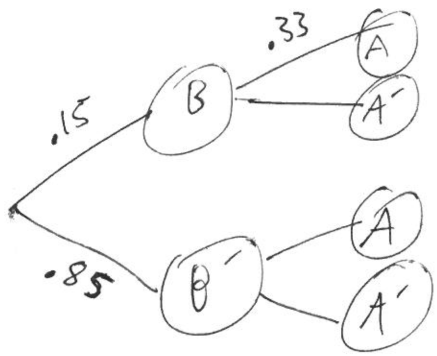
Tree diagrams:

A: buy the game

B: saw the ad.



To find $P(A \cap B)$ we travel up to B node and from there to the A node.



$P(B) = .15$

$P(B') = .85$

The Probability of Traveling from B to A

$P(A | B) = .33$

what is the probability $P(A \cap B)$?

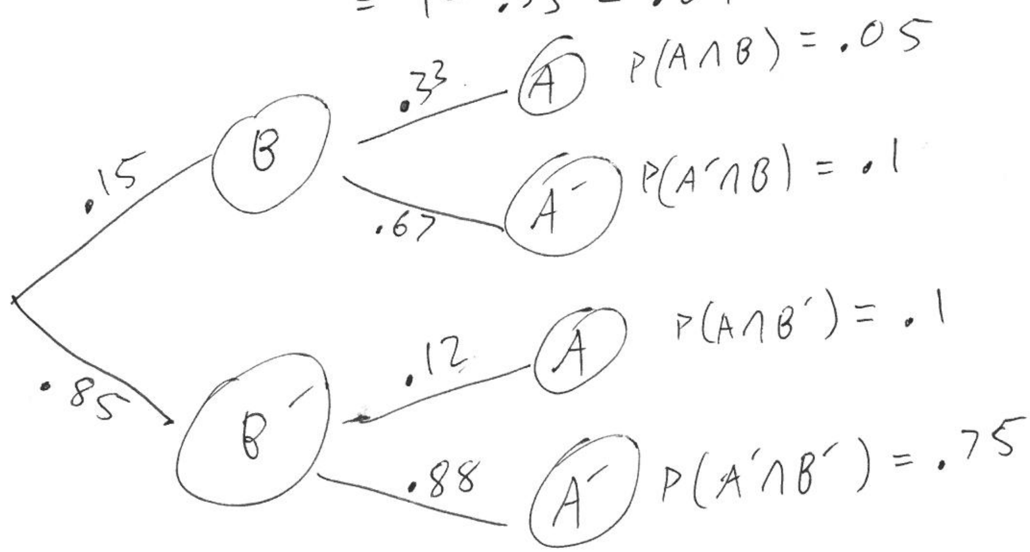
$$P(A \cap B) = P(A|B)P(B)$$
$$= .33 \times .15 = .05$$

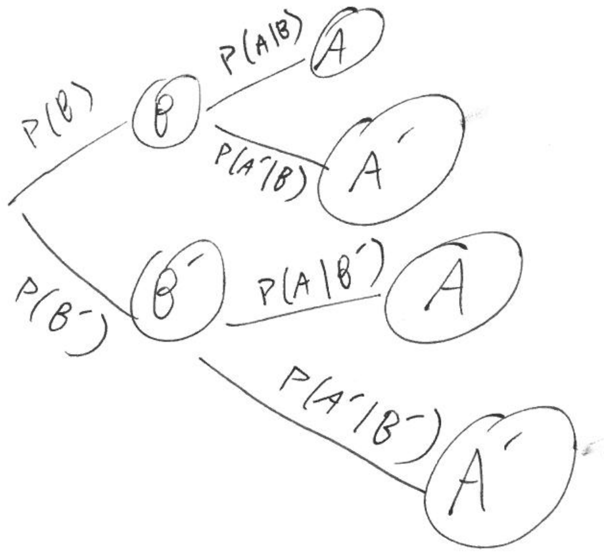
what is $P(A'|B)$?

If we are at the B node there are only two ways to go: to A or to A'

$$P(A'|B) + P(A|B) = 1$$

so $P(A'|B) = 1 - P(A|B)$
$$= 1 - .33 = .67$$



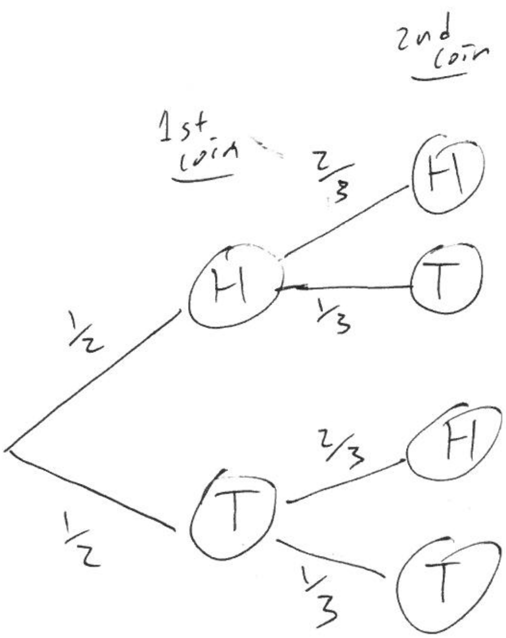


ex Unfair coins.

We toss two coins, the first is fair and the second is twice as likely to land on heads than tails.

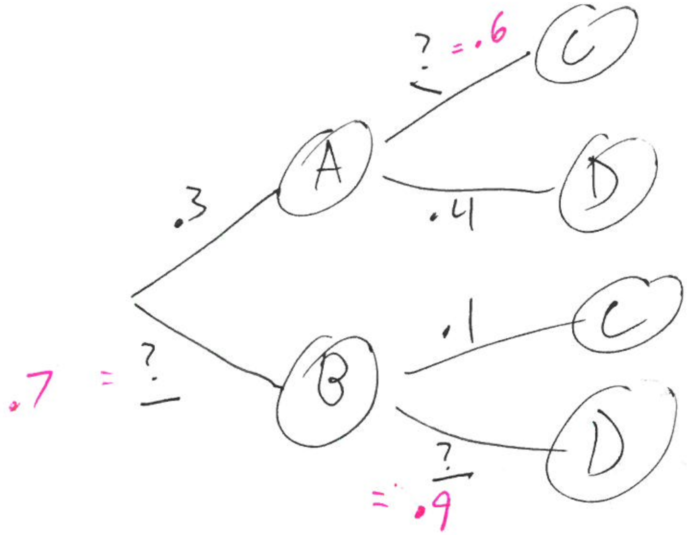
First coin	Second coin
$P(H) = \frac{1}{2}$	$P(H) = 2x = \frac{2}{3}$
$P(T) = \frac{1}{2}$	$P(T) = x = \frac{1}{3}$

$$2x + x = 1$$
$$3x = 1 \Rightarrow x = \frac{1}{3}$$



$$P(HH) = \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{3}$$
$$P(HT) = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$$
$$P(TH) = \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{3}$$
$$P(TT) = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$$

ex/ Fill in the missing quantities:



$P(A \cap C) = \underline{?} = .3 \times .6$

$P(A \cap D) = \underline{?} = .3 \times .4$

$P(B \cap C) = \underline{?} = .7 \times .1$

$P(B \cap D) = \underline{?} = .7 \times .9$