Day 10 9-27

Section 3.2 continued.

Last time - Row reduction of the "Augmented Matrix"

Recall - The Augmented Matrix is a matrix that represents a system of equations

~~ex~~/

$$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

the Augmented matrix for this system is

$$\left[\begin{array}{cc:c} 2 & 3 & 1 \\ 4 & 7 & 2 \end{array}\right]$$

- Row-Reduction is a process of performing Row operations on a matrix to put it in Row-Reduced form

Row Reduction is good because it allows us to find solutions for any system (unlike matrix inversion) Also you can row-reduced following an algorithm so any computer can quickly do this for you.

ex/ Lets use row reduction to solve this system:

$$\begin{cases} x - y + 5z = -6 \\ 3x + 3y - z = 10 \\ x + 3y + 2z = 5 \end{cases} \rightarrow \begin{bmatrix} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{bmatrix}$$

now lets turn to a computer:

$$\rightarrow \left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

I

solution to our system is
$x = 1$
$y = 2$
$z = -1$

In 3.1 we saw there were 3 options for systems of 2 eq & 2 variables

1.) 1 sol.
2.) 0 sol.
3.) ∞ sol.

ex Inconsistent system $\rightarrow$ No solutions:

$$\begin{cases} x + y + z = 1 \\ 2x - y + z = 0 \\ 4x + y + 3z = 3 \end{cases} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & 0 \\ 4 & 1 & 3 & 3 \end{bmatrix}$$

Row Reduce $\rightarrow$
$$\begin{bmatrix} 1 & 0 & 2/3 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

as a system

$$\begin{cases} x + \frac{2}{3}z = 0 \\ y + \frac{1}{3}z = 0 \\ 0 = 1 \end{cases}$$

$0 \neq 1$
$\Rightarrow$ no solutions.

ex ∞ many solutions

$$\begin{cases} x + y + z = 1 \\ \frac{1}{4}x - \frac{1}{2}y + \frac{3}{4}z = 0 \\ x + 7y - 3z = 3 \end{cases} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1/4 & -1/2 & 3/4 & 0 \\ 1 & 7 & -3 & 3 \end{bmatrix}$$

Row-Reduce $\downarrow$

$$\begin{bmatrix} 1 & 0 & 5/3 & 2/3 \\ 0 & 1 & -2/3 & 1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

as a system

$$\begin{cases} x + 5/3\, z = 2/3 \\ y - \frac{2}{3}z = \frac{1}{3} \\ 0 = 0 \end{cases}$$

we know $0=0$ lets throw that out

$$\begin{cases} x + \frac{5}{3} z = \frac{2}{3} \\ y - \frac{2}{3} z = \frac{1}{3} \end{cases}$$

simplest way to write system.

$$x = \frac{2}{3} - \frac{5}{3} z$$

$$y = \frac{1}{3} + \frac{2}{3} z$$

$z =$ anything (in the domain of z)

This represents all of the solutions

$$\left(\frac{2}{3} - \frac{5}{3} z, \frac{1}{3} + \frac{2}{3} z, z \right)$$

A particular solution set $z =$ what you want

ex/ $z = 1$ our particular solution would be

$(-1, 1, 1)$ one of many solutions.

Other ways to have ∞ solutions:

ex/

$$\begin{cases} x + 3y + 2z - w = 6 \\ 2x + 6y + 6z + 3w = 16 \\ x + 3y - 2z - 11w = -2 \\ 2x + 6y + 8z + 8w = 20 \end{cases}$$

$$\begin{bmatrix} 1 & 3 & 2 & -1 & 6 \\ 2 & 6 & 6 & 3 & 16 \\ 1 & 3 & -2 & -11 & -2 \\ 2 & 6 & 8 & 8 & 20 \end{bmatrix} \xrightarrow{\text{Row Reduced}} \begin{bmatrix} 1 & 3 & 0 & -6 & 2 \\ 0 & 0 & 1 & 5/2 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

as a system this is

$$\begin{cases} x + 3y - 6w = 2 \\ z + \frac{5}{2}w = 2 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

$x = 2 + 6w - 3y$

$y =$ arbitrary

$z = 2 - \frac{5}{2}w$

$w =$ arbitrary

write solutions as ordered quadruple

$$\left(2+6w-3y,\ y,\ 2-\tfrac{5}{2}w,\ w\right)$$

we can use row reduction when we have a different # of equations than the # of variables

ex/

$$\begin{cases} x + y = 1 \\ 13x - 26y = -11 \\ 26x - 13y = 2 \end{cases}$$

$$\begin{bmatrix} 1 & 1 \\ 13 & -26 \\ 26 & -13 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 2 \end{bmatrix}$$

no inverse

Augmented matrix:

$$\left[\begin{array}{cc:c} 1 & 1 & 1 \\ 13 & -26 & -11 \\ 26 & -13 & 2 \end{array}\right] \xrightarrow[\text{Reduce}]{\text{Row}} \begin{bmatrix} 1 & 0 & 5/3 \\ 0 & 1 & 8/13 \\ 0 & 0 & 0 \end{bmatrix}$$

$x = 5/3$

$y = 8/13$

☆ WE CAN ALWAYS ROW REDUCE!

Section 3.3 Applications of systems of equations:

Focus → Translating word problems into linear systems.

ex 1 Resource Allocation

PineOrange : 2q pineapple 2q orange
Pine kiwi : 3q pine 1q kiwi
Orange kiwi : 3q orange 1q kiwi

800 q pineapple
650 q orange
350 q kiwi

How many gallons of each blend should we make?

x → PO
y → PK
z → OK

$$\begin{aligned} 2x + 3y &= 800 \quad \text{pineapple} \\ 2x + 3z &= 650 \quad \text{orange} \\ y + z &= 350 \quad \text{kiwi} \end{aligned}$$

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 2 & 3 & 0 & 800 \\ 2 & 0 & 3 & 650 \\ 0 & 1 & 1 & 350 \end{array}\right]$$

RR

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 200 \\ 0 & 0 & 1 & 150 \end{array}\right] \to \begin{aligned} x &= 100 \\ y &= 200 \\ z &= 150 \end{aligned}$$

Note: Wolfram Alpa uses { } to write matrices

$$\{\{1,2\},\{3,4\}\} \longleftrightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

ex/ Airline Airplane Purchases

$x \to$ # Airbus purchased
$y \to$ # 767's
$z \to$ # Dreamliners

x | seats ~~250~~ 320
  | costs ~~$125m~~ $200m

y | seats 250
  | costs $125m

z | seats 275
  | costs $200

* total cost is $3,100m
* Twice as many Dreamliners as 767
* want to seat 4800 people

step 1 # of people

$$320x + 250y + 275z = 4800$$

step 2 cost

$$200x + 125y + 200z = 3100$$

step 3 Ratio of 767's to Dreamliners

twice as many z as y

then for every y there are 2z

ratio: $\frac{z}{y} = \frac{2}{1}$

$$z = 2y$$

~~2y=z~~

$y = 2z$

If unsure of right way to write ratio → plug in easy #'s to check.

lets say $y = 2$

then z should be $= 4$

plug into $2y = z$ & $y = 2z$

$2 \cdot 2 = 4$ ✓ $\quad$ $2 = 2(4)$ ✗

final equation is

$$2y = z \quad \text{or} \quad 2y - z = 0$$

$$\begin{bmatrix} 320 & 250 & 275 & 4800 \\ 200 & 125 & 200 & 3100 \\ 0 & 2 & -1 & 0 \end{bmatrix}$$

$\downarrow$ RR

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 8 \end{bmatrix} \rightarrow \begin{matrix} x = 5 \\ y = 4 \\ z = 8 \end{matrix}$$

ex 3 Traffic flow

x = # cars hourly on Allen St.

y = # cars hourly on Baker

z = # cars hourly on Coal

Use notion that Traffic In = Traffic Out