Can Persistent Unobserved Heterogeneity in Returns-to-Wealth Explain Wealth Inequality?

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Abstract

Using a standard buffer stock model of saving, I ask whether wealth inequality can be explained by persistent unobserved heterogeneity in returns-to-wealth. Households in the model are ex-ante heterogeneous with respect to persistent returns-to-wealth types as recently measured by Fagereng et al. (2016). I find that the introduction of ex-ante persistent unobserved heterogeneity in returns-to-wealth is not sufficient to fit the wealth inequality in the US, although the fit is significantly better than a model with only ex-post heterogeneity. The introduction of a modest level of ex-ante heterogeneity in preferences, acting as a substitute to heterogeneity in returns-to-wealth, is sufficient to fit the wealth inequality in the US remarkably well.

Keywords: Wealth Distribution; Returns to Wealth; Heterogeneity; Inequality
JEL classification: D12, D31, D91, E21

1. Introduction

The recent literature on macroeconomic models with wealth inequality implications highlights the use of heterogeneous agents. The HANK model, which uses ex-post heterogeneity through income shocks, is a recent example. Ex-post heterogeneity, however, is not sufficient to fit the observed wealth inequality since it greatly exceeds permanent income inequality. Hence, there is a vast amount of literature on ex-ante heterogeneity. Introducing ex-ante heterogeneity through the time discount factor is a common application (e.g. Krusell and Smith (1998), Carroll et al. (2017)). However, discount factors are not observable variables.

There is a small line of literature that introduces ex-ante heterogeneity through rates of return, simultaneously with other layers of heterogeneity. Benhabib et al. (2015) estimate the moments of a stochastic process for the rate of returns and find it to be insufficient to match the wealth inequality without a bequest motive and income shocks. Ferreira and Guimaraes (2018), on the other hand, calibrate their model using the average returns data and show that returns and preference heterogeneity are sufficient to match the Gini index, which is a weaker condition than matching the wealth distribution. Fagereng et al. (2016), however, recently showed that the heterogeneity in the average returns across different wealth percentiles is highly positively correlated with the unobserved heterogeneity in returns-to-wealth, which persists across generations. They further show that the positive correlation between returns and wealth is mainly driven by the positive correlation between wealth and persistent unobserved heterogeneity. The main purpose of this paper is to explain the wealth distribution, at least partially, with an observed variable, which has been recently measured by Fagereng et al. (2016) and shown to be the source of the returns heterogeneity across different wealth percentiles. This paper further investigates whether the persistent unobserved returns-to-wealth heterogeneity and discount factor (or preference) heterogeneity are substitutes in explaining the wealth inequality.

I employ a buffer-stock saving model, derived from Carroll et al. (2017), in which the households face transitory and permanent idiosyncratic income shocks. I modify this model to incorporate ex-ante heterogeneity in returns-to-wealth and, first, omit the existing ex-ante heterogeneity in preferences. The returns-to-wealth values are calibrated with persistent unobserved heterogeneity in returns-to-wealth, as measured by Fagereng et al. (2016). I solve the problem of each household computationally and generate the consumption function of each household. Then, I construct the Lorenz curve implied by the model and compare it with the actual wealth distribution of the US, which is plotted with respect to 2004 Survey of Consumer Finances. The simulated Lorenz curve is not sufficient to fit the actual distribution while the introduction of a modest level of ex-ante heterogeneity in preferences is sufficient to fit the actual distribution remarkably well.

2. The Model

2.1. The Income Process

My model is a variant of the model of Carroll et al. (2017), which is a buffer-stock saving model with transitory and permanent income shocks. The household income is defined as:

$$y_t = p_t \xi_t W_t \tag{1}$$

where p_t is the permanent income, ξ_t is the transitory shock to income, and W_t is the aggregate wage rate which is assumed to be fixed in this partial-equilibrium framework. The transitory shock is defined as:

$$\xi_t = \begin{cases} \mu & \text{with probability } \wp \\ \ell\theta_t & \text{with probability } 1 - \wp \end{cases}$$

where μ represents unemployment insurance and is positive. $\wp$ is the probability of getting unemployed. ℓ is the number of hours worked and is assumed to be fixed. θ_t is a white noise. The permanent income follows a random walk, where ψ is the permanent shock to income:

$$p_t = p_{t-1}\psi_t \quad \text{where} \quad E_t[\psi_{t+n}] = 1 \ \ \forall n > 0 \tag{2}$$

2.2. The Problem of the Household

The economy consists of a finite number of agents.[1] Below is the standard infinite-horizon problem of each household whose consumption function is obtained as the solution.

$$\max_{c(m_t)} \ \mathbb{E}_t \sum_{n=0}^{\infty} (\not{D}\beta)^n u(c(m_{t+n})) \quad \text{subject to}$$

$$a_t = m_t - c(m_t) \tag{3}$$

$$k_{t+1} = a_t / (\not{D}\psi_{t+1}) \tag{4}$$

[1]Following Carroll et al. (2017), the number of simulated agents per preference type is 2,000.

$$m_{t+1} = ((1-\delta) + r_t)k_{t+1} + \xi_{t+1} \tag{5}$$

where $\not{D}$ is the survival probability, i.e. the inverse of dying probability. The constraints specify how asset holdings, a_t, market resources, m_t, and the capital stock, k_{t+1}, change in time (see Carroll et al. (2017) for details). Unlike Carroll et al. (2017), I do not impose a non-negativity constraint on asset holdings. Instead, I use the sign of the asset holdings to determine whether the household is a net lender or net borrower, each of which faces a different level of interest rate, i.e. R_{boro} and R_{save} where $R_{boro} \geq R_{save}$.

$$(1 + r_t) = R_t = \begin{cases} R_{boro} & a_t < 0 \\ R_{save} & a_t \geq 0 \end{cases}$$

While R_{boro} is a scalar, R_{save} is a 15-dimensional vector since households are ex-ante heterogeneous with respect to their persistent returns-to-wealth types. Each element of this vector represents a wealth percentiles' persistent unobserved returns.

2.3. Adding Preference Heterogeneity

The model described above can be solved with and without ex-ante heterogeneity of preferences. The former case assumes a common time discount factor for all households. Instead of exogenously defining this common β, following the papers in the special issue of the Journal of Economic Dynamics and Control (JEDC, 2010) on solution methods for the Krusell-Smith model, this paper uses the unique $\bar{\beta}$, which equates the capital-to-output ratio to its steady-state value under perfect foresight.

The case with ex-ante preference heterogeneity defines more than one preference types and follows the technique introduced by Carroll et al. (2017). A uniform distribution around $\bar{\beta}$ is constructed by adding and subtracting a dispersion variable, ∇, which is chosen as the value that minimizes the distance between the actual and simulated Lorenz curves. The measure of distance between two Lorenz curves follows from Castaneda et al. (2003), where authors minimize the difference between the net worth held by certain percentiles. I target the net worth of the 20th, 40th, 60th, 80th percentiles, following Carroll et al. (2017). Formally, $\bar{\beta}$ and ∇ are obtained by solving the following problem:

$$\{\bar{\beta}, \nabla\} = \underset{\{\beta', \nabla'\}}{\operatorname{argmin}} \left(\sum_{i=20,40,60,80} (w_i(\beta', \nabla') - \omega_i)^2 \right)^{1/2}$$

$$\text{subject to } K/Y = K_{PF}/Y_{PF}$$

where w_i is the simulated net worth of the 20th, 40th, 60th, 80th wealth percentiles and ω_i is their actual net worth. The distance minimized in the argument will be referred to as the 'Lorenz distance'. The PF subscript stands for perfect foresight. In order to convert the constructed uniform distribution into a vector of preference types, I use a symmetric discrete approximation to the uniform distribution. The calibration of all parameters is given in Table 1 along with their sources. The requirements articulated in Carroll (2011) for an infinite-horizon buffer-stock model with uncertainty to have a solution are satisfied for all parameter values used in my model.

Parameter	Notation	Value	Source
Borrowing rate	R_{boro}	12.5%	Author's calculation[2]
Coefficient of relative risk aversion	ρ	1	JEDC (2010)
Depreciation rate	δ	0.025	JEDC (2010)
Time worked per capita	ℓ	10/9	JEDC (2010)
Wage	W	2.37	JEDC (2010)
Capital-to-output ratio (steady state)	K/Y	10.26	JEDC (2010)
Probability of dying	D	0.00625	Carroll et al. (2017)
Variance of log transitory shock	σ_θ^2	0.04	Carroll (1992)
Variance of log permanent shock	σ_ψ^2	0.04 × 1/11	Carroll (1992)
Probability of getting unemployed	$\wp$	0.07	Average in JEDC (2010)

Note: The model is calibrated at a quarterly frequency.

Table 1: Parameter Values

[2]Mode of the monthly prime loan rate in 2004, the year of the employed Survey of Consumer Finances, is converted into quarterly basis with compounding. The monthly prime loan rate data is retrieved from Board of Governors of the Federal Reserve System (2017).

3. Results and Discussion

Figure 1 illustrates the distribution of wealth implied by the model with idiosyncratic income shocks and 15 returns-to-wealth types. The returns-to-wealth types are calibrated using the persistent unobserved component of the returns of different wealth percentiles, truncating both tails of the wealth distribution at 10%.[3] The simulated Lorenz curve has a much better fit with a Lorenz distance of 7.96, compared to the case with only ex-post heterogeneity, which has a Lorenz distance of 42.74 and is presented in Figure 2. The fit is also better than the earlier studies that introduce returns-to-wealth heterogeneity through the estimation of a stochastic process or the calibration of a returns vector using the average returns data. However, it still implies a more egalitarian wealth distribution than the actual distribution.

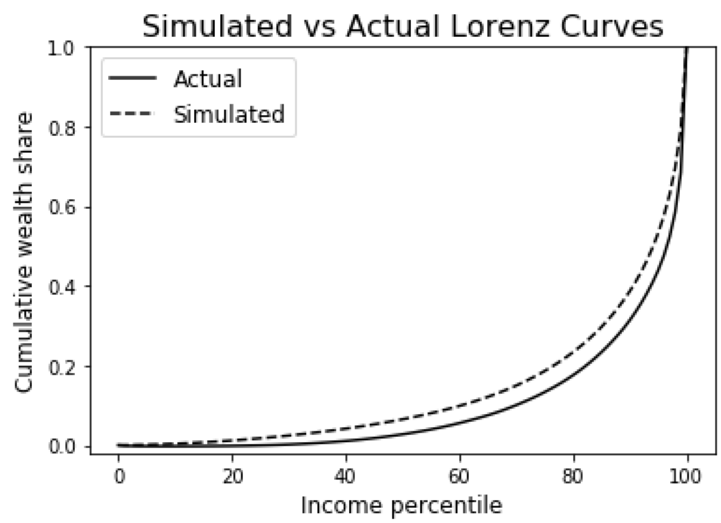

Figure 1: Distribution of Net Worth with Returns-to-Wealth Heterogeneity

[3]Figure 11 in Fagereng et al. (2016) shows the decomposition of average returns by wealth percentile. I truncate the persistent unobserved component of the returns of the top and bottom 10% since the empirically high returns of the top 10% makes the simulated distribution of wealth more inegalitarian than the actual Lorenz curve. Besides, higher estimated returns have larger confidence intervals. The fixed effects function maps the 10th and 90th wealth percentiles to roughly -0.7% and 0.9%. Since the difference between different wealth percentiles' persistent returns are of interest, I shift the return rates up by 2% to avoid negative and near-zero interest rates. The calibrated returns-to-wealth vector is [1.3%, 1.5%, 1.6%, 1.7%, 1.8%, 1.9%, 1.9%, 2.0%, 2.1%, 2.2%, 2.3%, 2.3%, 2.4%, 2.6%, 2.9%].

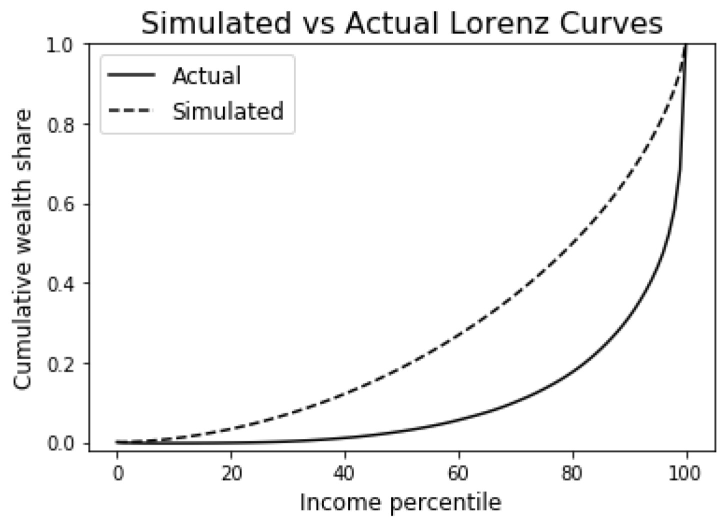

Figure 2: Distribution of Net Worth without Returns-to-Wealth Heterogeneity

Since returns-to-wealth inequality is not sufficient to match the actual wealth inequality, I introduce another source of ex-ante heterogeneity. Figure 3 plots the implied wealth distribution of the model, where there are three preference types and 15 returns-to-wealth types along with idiosyncratic income shocks. The endogenously found $\bar{\beta}$ is 0.9807 and the optimal spread parameter ∇ is 0.0040. Thus, the implied β distribution is [0.9767, 0.9847] and the three preference types are 0.9780, 0.9807 and 0.9834. The difference between the most patient and least patient types is roughly 2% at an annual frequency. The introduction of three preference types with a very modest degree of heterogeneity matches the actual distribution with a Lorenz distance of 2.81. Table 2 shows that this fit is almost as good as the only preference heterogeneity case covered in Carroll et al. (2017). Although both cases have an observationally equivalent fit, the case without returns-to-wealth heterogeneity requires a much larger variation between the most patient and least patient types, 5% at an annual frequency.

	With Returns Hetero.	Without Returns Hetero.
With Preference Hetero.	2.81	2.71
Without Pref. Hetero.	7.96	42.74

Table 2: Lorenz Distances

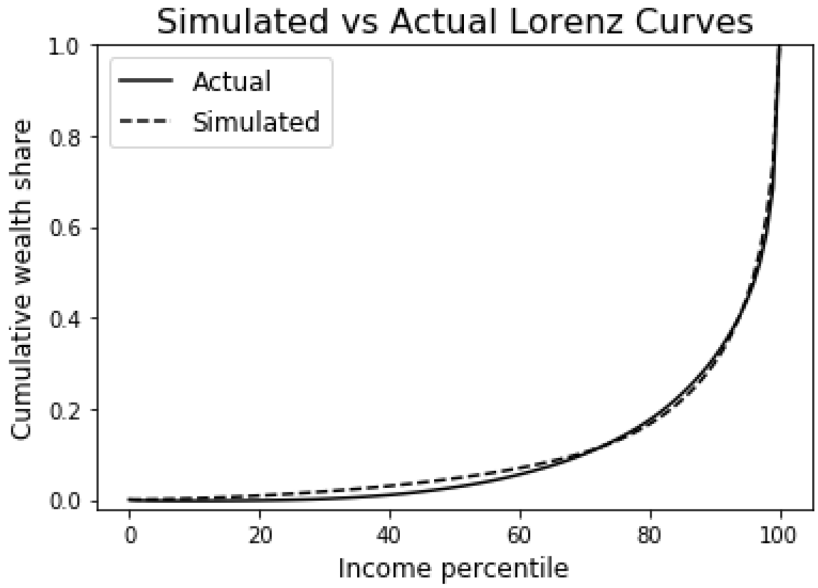

Figure 3: Distribution of Net Worth with Returns-to-Wealth and Preference Heterogeneity

4. Conclusion

This paper shows that wealth inequality can, in part, be explained by ex-ante persistent unobserved heterogeneity in returns-to-wealth, which is recently shown to derive the positive correlation between wealth and returns. A small variation in preferences with three types is needed to match the wealth distribution in the US with a standard buffer-stock model. Thus, this paper suggests a practical method where ex-ante and ex-post heterogeneities, both of which are empirically verifiable, are sufficient to fit the wealth inequality in the US, with a small variation in preferences, which is not empirically verifiable. This is a novel contribution to the theoretical macro literature, which matches the wealth inequality in order to assess the effects of policy shocks.

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