Assignment 5: Propositional and First Order Logic in Reasoning

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Deadline: 11.11.2022, 23:59 hrs

Overview

This is a problem set for you to gain experience with propositional logic and First-Order Logic (FOL) by solving many small problems. Refer to the textbook (**Artificial Intelligence: A Modern Approach, 4rd ed. (Global edition)**) for reference, specially Chapters 7, 8 and 9.

1 Models and entailment in propositional logic

1.1 Modelling

For each of the following statements, determine whether it is true or not by building the complete model (truth table). *Hint: you can add the full statement as a column in the table by replacing the "entails" ($\models$) symbol by an "implies" ($\implies$) symbol.*

a) $\neg A \land \neg B \models \neg B$

b) $\neg A \lor \neg B \models \neg B$

c) $\neg A \land B \models A \lor B$

d) $A \implies B \models A \iff B$

e) $(A \implies B) \iff C \models A \lor \neg B \lor C$

f) $(\neg A \implies \neg B) \land (A \land \neg B)$ is satisfiable

g) $(\neg A \iff \neg B) \land (A \land \neg B)$ is satisfiable

1.2 Trouble in the lab

The Bio Space Lab orbiting Earth suffered an electrical anomaly, leading to severe damage in some of its sectors. One of the most affected sectors was Sector 4 (VANN), which holds aquatic and marine specimens of our planet in its tanks. HQ has assigned you the task to implement a control system for the 32 tanks in the sector while maintenance and repairs are being carried out. Each tank has three sensors:

(S_1) A **motion detection** sensor to know if the tank is unoccupied or if there are specimens inside

(S_2) A **toxicity level** detector

(S_3) An **electrical charge** meter

For Sensors S_2 and S_3, the signal is marked as 'high' after a certain threshold.

Tanks send updates to the control mechanism once every minute, using an 8-byte packet that looks like this:

$$\textcolor{red}{010}\textcolor{blue}{00111}$$

where the first three bits (in red above) represent the reading of the sensors (motion, toxicity and electrical charge; in this order). The remaining five bits (in blue above) are used for representing the ID of the tank (in binary encoding).

HQ has notified you that a tank gate should be closed if **any** of the following situations arise:

(C_1) If species inside are safe (low toxicity and low electric charge).

(C_2) If the tank is unoccupied and water toxicity is high.

(C_3) If the electrical charge level in a tank is marked as 'dangerous', no matter if it is inhabited or its toxicity.

For the remaining cases, energy should be saved and normal flow is to be expected. Your task is to design a control system using model checking to close tank gates whenever an 8-bit packet arrives:

a) Generate the vocabulary of the system

b) Express all closing conditions (C_1, C_2, C_3) for the gates as propositional logic statements

c) Write the complete model (truth table) of the system

d) Write an example packet and explain what the packet means.

2 Resolution in propositional logic

2.1 Conjunctive Normal Form

Convert each of the following sentences to their Conjunctive Normal Form (CNF).

a) $A \lor (B \land C \land \neg D)$

b) $\neg(A \implies \neg B) \land \neg(C \implies \neg D)$

c) $\neg((A \implies B) \land (C \implies D))$

d) $(A \land B) \lor (C \implies D)$

e) $A \iff (B \implies \neg C)$

2.2 Inference in propositional logic

Consider the following knowledge:

If the weather is both Sunny and Warm, then I Enjoy. If the weather is both Warm and Nice (not Raining), then I pick up Berries. If it is Raining, then I won't pick up Berries. If it is Raining, then I will get wet. It is Warm. It is Raining. It is Sunny.

Build a knowledge base and **prove or disprove** the following statements using **resolution**:

(Q_1) I won't pick up Berries

(Q_2) I will Enjoy

(Q_3) I will get wet

Hint: we suggest you use either complete words or the initial letter of each condition as your symbols.

3 Representation in First-Order Logic (FOL)

Question taken from AIMA 3rd edition.

3.1 Predicates

Consider the following vocabulary:

1. $Occupation(p, o)$ is a predicate where person p has occupation o
2. $Customer(p_1, p_2)$ is a predicate where person p_1 is a customer of person p_2
3. $Boss(p_1, p_2)$ is a predicate where person p_1 is a boss of person p_2
4. $Doctor, Surgeon, Lawyer, Actor$ are constants denoting an occupation
5. $Emily, Joe$ are constants denoting people

Use the symbols above to **write the following statements in FOL**:

a) Emily is either a surgeon or a lawyer

b) Joe is an actor, but he also holds another job

c) All surgeons are doctos

d) Joe does not have a lawyer (i.e. he's not a customer of any lawyer)

e) Emily has a boss who is a lawyer

f) There exists a lawyer all of whose customers are doctors

g) Every surgeon has a lawyer

3.2 Functions as predicates

Arithmetic assertions can be written using FOL. Use the predicates ($<, \leq, \neq, =$), the usual arithmetic operations ($+, -, \times, /$) as function symbols, biconditionals to create new predicates, and integer number constants to **express the following statements in FOL**:

a) $Divisible(x, y)$: an integer number x is *divisible* by y if there is some integer z less than x such that $x = z \times y$.

b) $Even(x)$: a number is *even* if and only if it is divisible by 2.

c) $Odd(x)$: a number is *odd* if it is not divisible by 2.

d) $Odd(x)$: a number is *odd* if it is the result of summing 1 to an *even* number.

e) $Prime(x)$: a number is *prime* if is divisible only by itself.

f) There is only one even prime number.

g) Every integer number is equal to a product of prime numbers (*Hint: you can use $\prod_{i=1}^{k} p_k$ to express a product of numbers, or use $\ldots$ to express a repeating pattern like $p_1, \ldots, p_n$ meaning p_1, p_2, p_3 and on until p_n).*

4 Resolution in FOL

A new niche market has been discovered and some of your friends had the idea of developing a girl group K-Pop recommender system. Instead of using machine learning (as they think doing so may include bias in the data), they decided that going for a rule-based system was a better approach. This is what they said about it:

- All users who like GG are known as $Sone$, and therefore all $Sone$ like GG
- All users who like RV are known as $Reveluvs$, and therefore all $Reveluvs$ like RV
- All users who like BP are known as $Blinks$, and therefore all $Blinks$ like BP
- All users who like both $Dance$ and $Ballads$ will always like CH
- All users who like both $Drama$ and $Ballads$ will always like HE
- For all users who identify as $Sone$, the following holds:
  - If they like $Electro$, they will always like DJH
  - If they like $Drama$, they will always like SEO
  - If they like $Ballads$, they will always like TAE

You start to think if going with Machine Learning could result in less bias than using such rules, but you love your friends and carry on with their business idea. Your task is then to:

a) **Generate the knowledge base** by converting each rule to symbolic form.

b) Using **resolution**, **prove or disprove** (by showing the complete process) that if a new user u_1 is a fan of GG and identifies as $Reveluv$, then TAE will be a good recommendation.

c) Considering the same user u_1, **prove or disprove** that HE will be a good recommendation.

d) Given what you know, if another user u_2 claims to be a $Sone$, a $Reveluv$, a $Blink$, and likes $Drama$; what are the possible artists and genre recommendations the system will provide?

Hint: Consider using predicates to indicate that a user likes something (e.g. $Electro(x)$) instead of using $likes(x, y)$ as a function.

Deliverables and recommendations

You must upload a **single** PDF containing the **typeset** equations, formulas and/or diagrams of your solutions.

- Include **natively digital** equations and diagrams. Do **not** upload scans or photos of hand-written solutions as these will be ignored by the TAs
- You can typeset your equations in Microsoft Word/Google Docs. If you're feeling adventurous, try LaTeX. Overleaf is a great place to start.