Notes on proper motions

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Chapter 1

Proper motions in ra/dec

The relation between ra/dec (α, δ) and galactocentric coordinates (l, b) is given by

$$\begin{pmatrix} r\cos l\cos b \\ r\sin l\cos b \\ r\sin b \end{pmatrix} = A'_G \begin{pmatrix} r\cos\alpha\cos\delta \\ r\sin\alpha\cos\delta \\ r\sin\delta \end{pmatrix} \tag{1.0.1}$$

where

$$A_G = \begin{pmatrix} -0.0548755604 & 0.4941094279 & -0.8676661490 \\ -0.8734370902 & -0.4448296300 & -0.1980763734 \\ -0.4838350155 & 0.7469822445 & 0.4559837762 \end{pmatrix} \tag{1.0.2}$$

and $A'_G = A^{\mathrm{T}}_G$.

1.1 U, V, W to $v_r, \mu_l\cos b, \mu_b$

Taking the derivatives of the left side of (1.0.1) we get

$$\begin{aligned} \begin{pmatrix} U \\ V \\ W \end{pmatrix} &= \begin{pmatrix} v_r\cos l\cos b - r\mu_l\cos b\sin l - r\mu_b\cos l\sin b \\ v_r\sin l\cos b + r\mu_l\cos b\cos l - r\mu_b\sin l\sin b \\ v_r\sin b + r\mu_b\cos b \end{pmatrix} \\ &= M_{UVW,PMlb} \begin{pmatrix} v_r \\ r\mu_l\cos b \\ r\mu_b \end{pmatrix} \end{aligned} \tag{1.1.1}$$

where

$$M_{UVW,PMlb} = \begin{pmatrix} \cos l\cos b & -\sin l & -\cos l\sin b \\ \sin l\cos b & \cos l & -\sin l\sin b \\ \sin b & 0 & \cos b \end{pmatrix} \tag{1.1.2}$$

Finally, if the proper motions are in units of mas/yr and the radii are in kpc this gives

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = M_{UVW,PMlb} \begin{pmatrix} v_r \\ kr\mu_l \cos b \\ kr\mu_b \end{pmatrix} \tag{1.1.3}$$

where $k = 4.74047$. Note that the inverse of $M_{UVW,PMlb}$ is just its transpose.

1.2 U, V, W to $v_r, \mu_\alpha \cos\delta, \mu_\delta$

Taking derivatives of the right side of (1.0.1) we get

$$\begin{aligned} \begin{pmatrix} U \\ V \\ W \end{pmatrix} &= A'_G \begin{pmatrix} v_r \cos\alpha\cos\delta - r\mu_\alpha \cos\delta\sin\alpha - r\mu_\delta \cos\alpha\sin\delta \\ v_r \sin\alpha\cos\delta + r\mu_\alpha \cos\delta\cos\alpha - r\mu_\delta \sin\alpha\sin\delta \\ v_r \sin\delta + r\mu_\delta \cos\delta \end{pmatrix} \\ &= A'_G M_{UVW,PM\alpha\delta} \begin{pmatrix} v_r \\ r\mu_\alpha \cos\delta \\ r\mu_\delta \end{pmatrix} \end{aligned} \tag{1.2.1}$$

where

$$M_{UVW,PM\alpha\delta} = \begin{pmatrix} \cos\alpha\cos\delta & -\sin\alpha & -\cos\alpha\sin\delta \\ \sin\alpha\cos\delta & \cos\alpha & -\sin\alpha\sin\delta \\ \sin\delta & 0 & \cos\delta \end{pmatrix} \tag{1.2.2}$$

and if the radii are in kpc and the proper motions are in mas/yr

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = A'_G M_{UVW,PM\alpha\delta} \begin{pmatrix} v_r \\ kr\mu_\alpha \cos\delta \\ kr\mu_\delta \end{pmatrix} \tag{1.2.3}$$

where $k = 4.74047$.

1.3 $v_r, \mu_\alpha \cos\delta, \mu_\delta$ to $v_r, \mu_l \cos b, \mu_b$

We can also relate the proper motions to each other by equating both expressions equal to (U, V, W) which gives

$$M_{UVW,PMlb} \begin{pmatrix} v_r \\ kr\mu_l \cos b \\ kr\mu_b \end{pmatrix} = A'_G M_{UVW,PM\alpha\delta} \begin{pmatrix} v_r \\ kr\mu_\alpha \cos\delta \\ kr\mu_\delta \end{pmatrix} \tag{1.3.1}$$

where $k = 4.74047$.

1.4 U, V, W to $v_r, \mu_{\phi 1} \cos\phi_2, \mu_{\phi 2}$

If we have a set of coordinates (ϕ_1, ϕ_2) which are related to (α, δ) by

$$\begin{aligned} \begin{pmatrix} r\cos\phi_1\cos\phi_2 \\ r\sin\phi_1\cos\phi_2 \\ r\sin\phi_2 \end{pmatrix} &= R_{\phi 1\phi 2,\alpha\delta} \begin{pmatrix} r\cos\alpha\cos\delta \\ r\sin\alpha\cos\delta \\ r\sin\delta \end{pmatrix} \\ &= R_{\phi 1\phi 2,\alpha\delta} A_G \begin{pmatrix} r\cos l\cos b \\ r\sin l\cos b \\ r\sin b \end{pmatrix} \end{aligned} \tag{1.4.1}$$

To relate this to UVW we multiply both sides by the inverse of $R_{\phi 1\phi 2,\alpha\delta} A_G$ and take the derivative which gives

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = A_G^{-1} R_{\phi 1\phi 2,\alpha\delta}^{-1} M_{UVW,PM\phi 1\phi 2} \begin{pmatrix} v_r \\ kr\mu_{\phi 1}\cos\phi_2 \\ kr\mu_{\phi 2} \end{pmatrix} \tag{1.4.2}$$

where $M_{UVW,PM\phi 1\phi 2}$

$$M_{UVW,PM\phi 1\phi 2} = \begin{pmatrix} \cos\phi_1\cos\phi_2 & -\sin\phi_1 & -\cos\phi_1\sin\phi_2 \\ \sin\phi_1\cos\phi_2 & \cos\phi_1 & -\sin\phi_1\sin\phi_2 \\ \sin\phi_2 & 0 & \cos\phi_2 \end{pmatrix} \tag{1.4.3}$$

1.5 Solar reflex motion

To compute the solar reflex motion in any coordinates, you transform between U, V, W and $v_r, \mu_{\phi 1}, \mu_{\phi 2}$ for a body at rest with respect to the galaxy, i.e. its $(U, V, W) = -\vec{v}_{\rm lsr}$. You then subtract these proper motions from the standard $v_r, \mu_{\phi 1}, \mu_{\phi 2}$.

1.6 Converting proper motion between frames without radial velocity

It is also possible to convert the proper motion between frames (e.g. $(\mu_\alpha\cos\delta, \mu_\delta)$ to $(\mu_l\cos b, \mu_b)$. This can be done by setting the radial velocity and radius to any particular value (e.g. 0 km/s, 1 kpc), and doing the conversions above. Of course, the 3d velocities are meaningless and must be ignored: the answer can only be trusted once converted into proper motions in a new frame.

Alternatively, one can derive the rotations by taking relations like (1.3.1) and working out the relation between the proper motions in the two frames.

1.7 Geometric distortion

For extended objects, there is a difference in the proper motion and radial velocity across the object just from geometric distortion. This is because even if the object is a solid body, there will be differences in proper motion and radial velocity across the object to account for how it changes in apparent size as it moves.

To derive this, we assume that the object is moving