COMP0130 Robot Vision and Navigation

Week 1 Seminar

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Friday 13 January 2023

Today's Session

- Week 1 Quiz and Exercises
- Week 1 Summary
- Questions from Students on the Lecture
- Preparing for Next Week

Week 1 Quiz Question 1

Which of these is not part of GNSS? (Select one)

1. GPS
2. Galileo
3. GLASNOST
4. QZSS
5. EGNOS




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Week 1 Quiz Question 2

Which of these best describes the operation of GNSS? (Select one)

1. GNSS receivers transmit signals to the satellites which compute their position
2. Satellites broadcast signals to receivers, which compute position using range measurements
3. Satellites broadcast signals to receivers, which compute position using signal timing measurements
4. Satellites broadcast signals to receivers, which compute position by triangulation

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Week 1 Quiz Question 3

Which of these pairs of measurements have independent errors and which have correlated errors?

1. A range measurement and an angle measurement
2. A north position solution and an east position solution
3. GNSS pseudo-ranges measured at different times

Week 1 Quiz Question 3

Which of these pairs of measurements have independent errors and which have correlated errors?

1. A range measurement and an angle measurement - **independent**
2. A north position solution and an east position solution - **correlated**
3. GNSS pseudo-ranges measured at different times – **it depends**
   - *Successive measurements from the same satellite are highly correlated, while measurements made hours apart are independent*

Week 1 Summary
What is GNSS?

GNSS = **G**lobal **N**avigation **S**atellite **S**ystem(s)

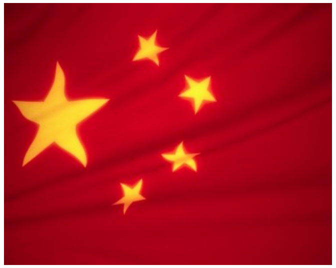
A generic term covering GPS and similar satellite navigation systems

Four global systems:

GPS
(United States)

GLONASS
(Russia)

Galileo
(European Union)



Beidou
(China)

And various regional augmentation systems

Most modern receivers use two or more of these systems

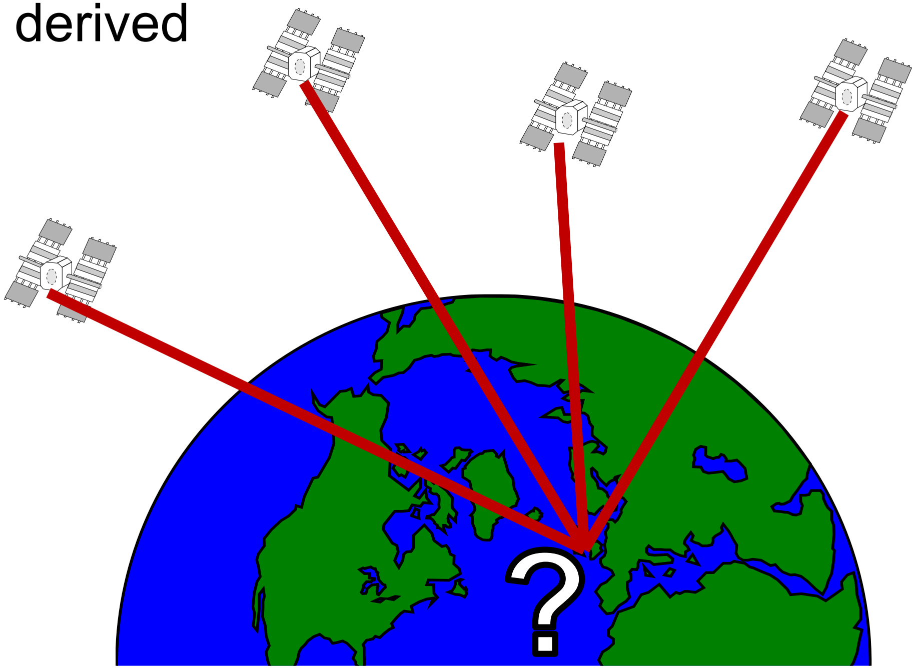
Week 1 Summary

How does GNSS work?

- Each satellite continuously transmits ranging signals
- GNSS user equipment measures signal arrival time, t_{sa}, from 4 (or more) satellites
- Each transmission time, t_{st}, is derived from the signal modulation

$$\rho = c(t_{sa} - t_{st})$$

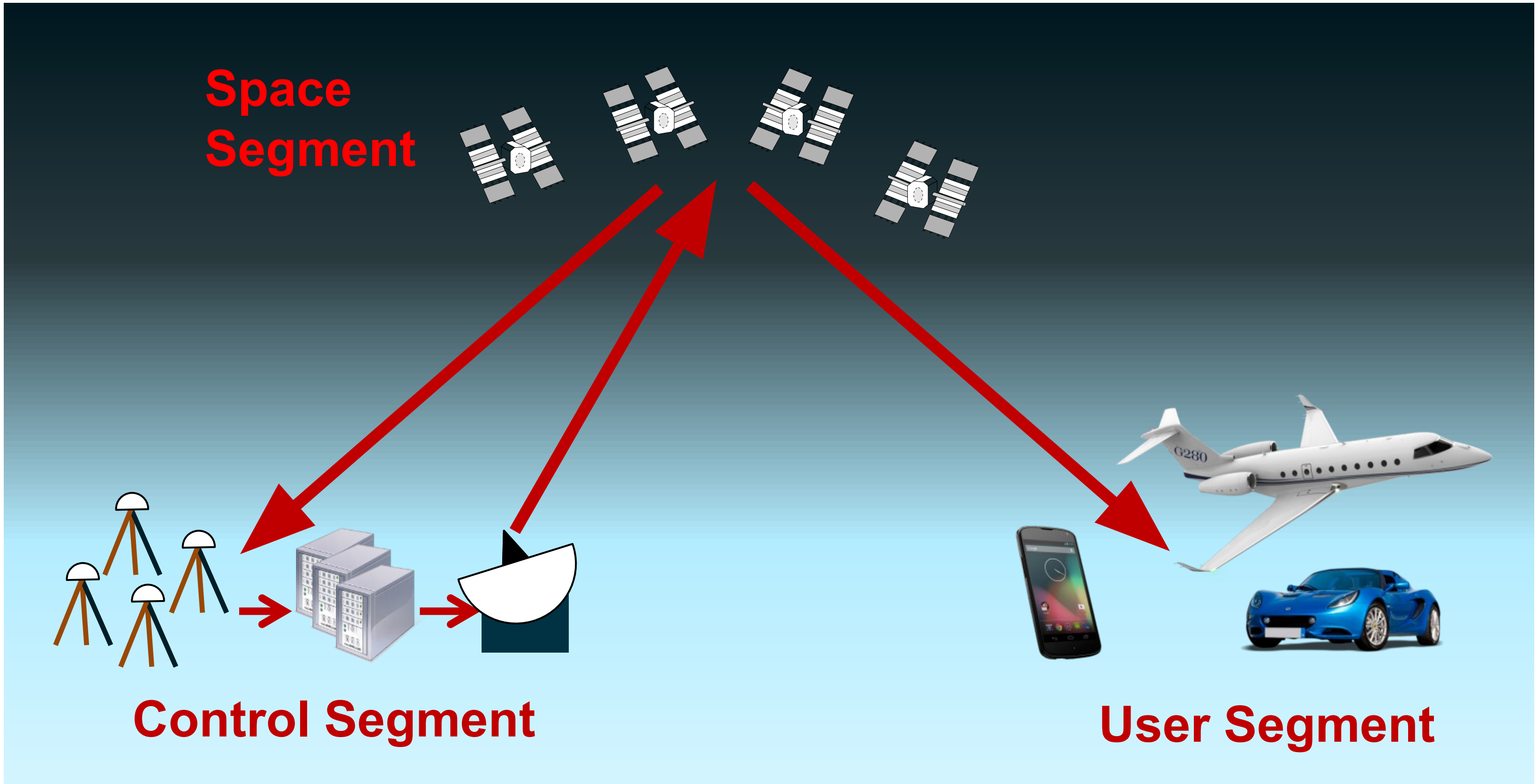
- This is pseudo-range, not range, because of unknown receiver clock offset
- With four pseudo-range measurements, the 3D user position and clock offset may be determined



Week 1 Summary

The Segments of GNSS

Each GNSS comprises three segments



Week 1 Summary

GNSS User Equipment

User Antenna
Converts GNSS signals from radio to electrical

→

Receiver Hardware
Downconverts, Samples & Correlates Signals

→

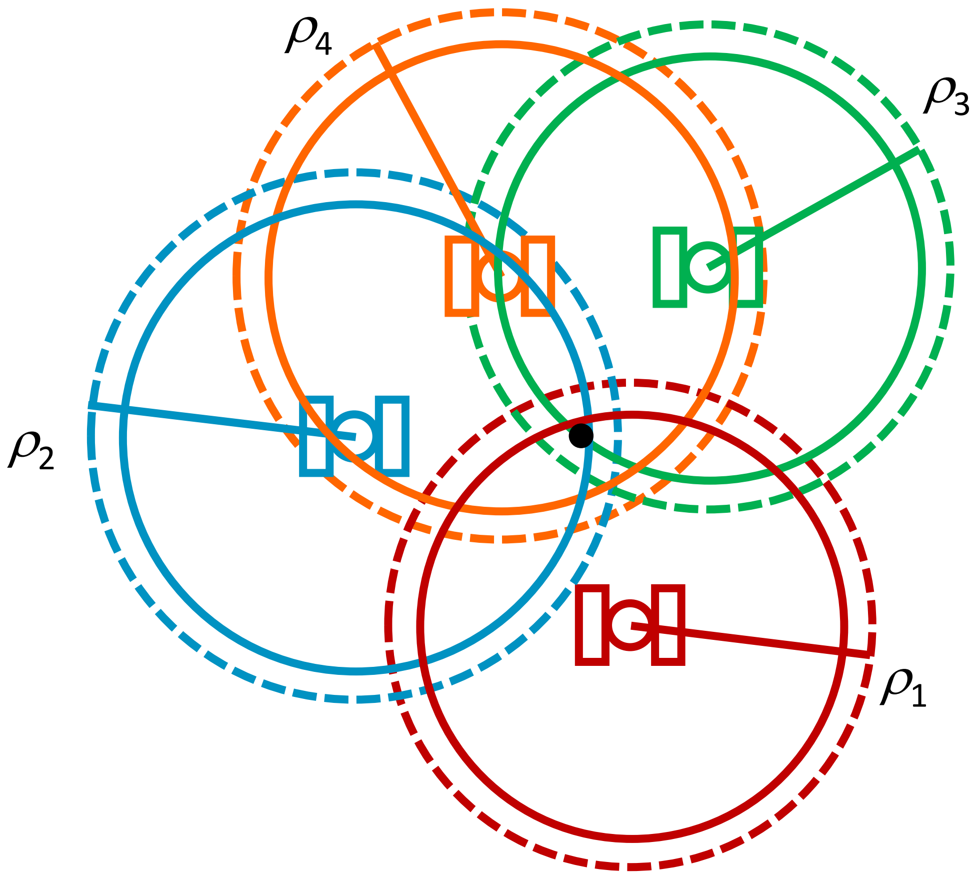
Ranging Processor
Acquires and tracks the signals

→

Navigation Processor
Computes position solution

Images from the manufacturers

GNSS Positioning Geometry



The position solution is determined by the *ranges*

With GNSS, we have *pseudo*-ranges, due to the receiver clock error

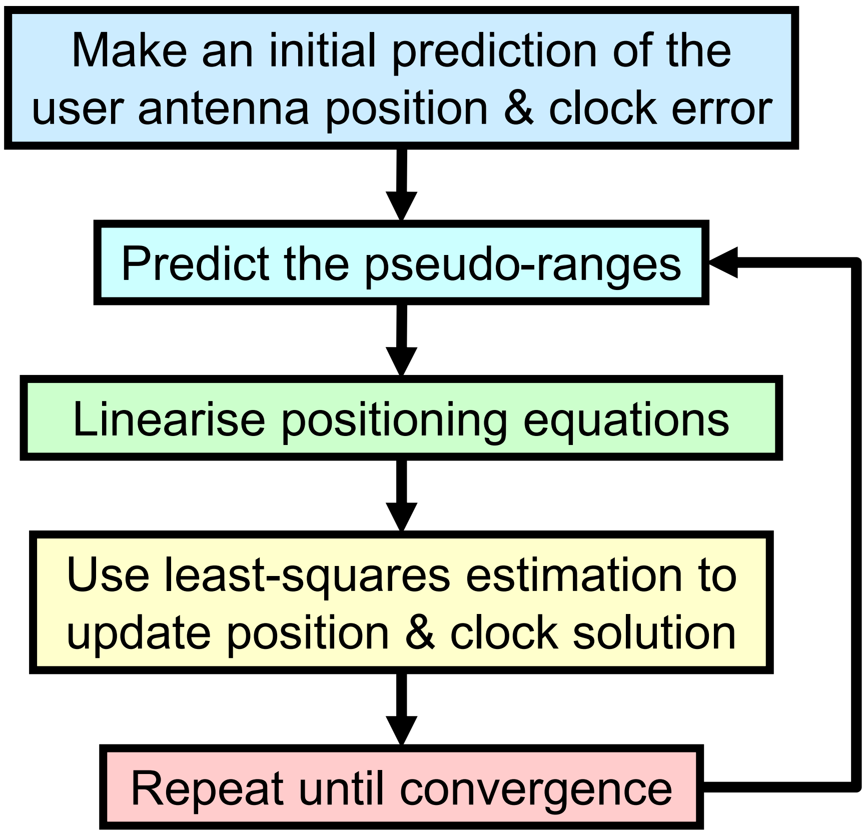
A 4th satellite is needed to determine this

An unknown correction must be applied to the pseudo-ranges to obtain spheres that intersect

In practice, the receiver solves nonlinear simultaneous equations

Week 1 Summary

Single-Epoch Positioning using Least-Squares



- The more accurate the predicted antenna position, the smaller the linearisation errors will be
- Iteration may be needed if the error in the predicted position is more than a few km
  - *New prediction = old solution*
- Large predicted clock errors do not affect the position solution

Week 1 Summary

Positioning Equations

Predict pseudo-ranges:

Predicted user position

Predicted receiver clock offset

$$\hat{\rho}_{a,C}^{s-} = \sqrt{\left(x_{Is}^{I} - \hat{x}_{ea}^{e-}\right)^2 + \left(y_{Is}^{I} - \hat{y}_{ea}^{e-}\right)^2 + \left(z_{Is}^{I} - \hat{z}_{ea}^{e-}\right)^2} + \delta\hat{\rho}_{c}^{a-} \qquad s \in 1,2\dots m$$

Satellite positions (known)

Linearise positioning equations:

$$\begin{pmatrix} \tilde{\rho}_{a,C}^{1} - \hat{\rho}_{a,C}^{1-} \\ \tilde{\rho}_{a,C}^{2} - \hat{\rho}_{a,C}^{2-} \\ \vdots \\ \tilde{\rho}_{a,C}^{m} - \hat{\rho}_{a,C}^{m-} \end{pmatrix} \approx \mathbf{H} \begin{pmatrix} \hat{x}_{ea}^{e+} - \hat{x}_{ea}^{e-} \\ \hat{y}_{ea}^{e+} - \hat{y}_{ea}^{e-} \\ \hat{z}_{ea}^{e+} - \hat{z}_{ea}^{e-} \\ \delta\hat{\rho}_{c}^{a+} - \delta\hat{\rho}_{c}^{a-} \end{pmatrix} \qquad \mathbf{H} = \begin{pmatrix} -\hat{u}_{a1,x}^{e-} & -\hat{u}_{a1,y}^{e-} & -\hat{u}_{a1,z}^{e-} & 1 \\ -\hat{u}_{a2,x}^{e-} & -\hat{u}_{a2,y}^{e-} & -\hat{u}_{a2,z}^{e-} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\hat{u}_{am,x}^{e-} & -\hat{u}_{am,y}^{e-} & -\hat{u}_{am,z}^{e-} & 1 \end{pmatrix}$$

H = Measurement Matrix

$$\mathbf{u}_{as}^{e-} = \frac{\mathbf{C}_{e}^{I}\mathbf{r}_{es}^{e} - \hat{\mathbf{r}}_{ea}^{e-}}{\hat{r}_{as}^{-}} \qquad s \in 1,2\dots m$$

Least-Squares Position Solution (Unweighted)

$$\begin{pmatrix} \hat{x}_{ea}^{e+} \\ \hat{y}_{ea}^{e+} \\ \hat{z}_{ea}^{e+} \\ \delta\hat{\rho}_{c}^{a+} \end{pmatrix} = \begin{pmatrix} \hat{x}_{ea}^{e-} \\ \hat{y}_{ea}^{e-} \\ \hat{z}_{ea}^{e-} \\ \delta\hat{\rho}_{c}^{a-} \end{pmatrix} + \left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}} \begin{pmatrix} \tilde{\rho}_{a,C}^{1} - \hat{\rho}_{a,C}^{1-} \\ \tilde{\rho}_{a,C}^{2} - \hat{\rho}_{a,C}^{2-} \\ \vdots \\ \tilde{\rho}_{a,C}^{m} - \hat{\rho}_{a,C}^{m-} \end{pmatrix}$$

We can use a similar approach for the velocity:

$$\begin{pmatrix} \hat{v}_{ea,x}^{e+} \\ \hat{v}_{ea,y}^{e+} \\ \hat{v}_{ea,z}^{e+} \\ \delta\hat{\dot{\rho}}_{c}^{a+} \end{pmatrix} = \begin{pmatrix} \hat{v}_{ea,x}^{e-} \\ \hat{v}_{ea,y}^{e-} \\ \hat{v}_{ea,z}^{e-} \\ \delta\hat{\dot{\rho}}_{c}^{a-} \end{pmatrix} + \left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}} \left.\begin{pmatrix} \tilde{\dot{\rho}}_{a,C}^{1} - \hat{\dot{\rho}}_{a,C}^{1-} \\ \tilde{\dot{\rho}}_{a,C}^{2} - \hat{\dot{\rho}}_{a,C}^{2-} \\ \vdots \\ \tilde{\dot{\rho}}_{a,C}^{m} - \hat{\dot{\rho}}_{a,C}^{m-} \end{pmatrix}\right\}$$

Difference between measured and predicted pseudo-range rates

$$\hat{\dot{\rho}}_{aj}^{-} = \hat{\mathbf{u}}_{as}^{e-\mathrm{T}}\left[\mathbf{C}_{e}^{I}\left(\hat{\mathbf{v}}_{es}^{e} + \mathbf{\Omega}_{ie}^{e}\hat{\mathbf{r}}_{es}^{e}\right) - \left(\hat{\mathbf{v}}_{ea}^{e-} + \mathbf{\Omega}_{ie}^{e}\hat{\mathbf{r}}_{ea}^{e-}\right)\right] + \delta\hat{\dot{\rho}}_{c}^{a-}$$

Week 1 Summary

Outlier Detection using Normalised Residuals

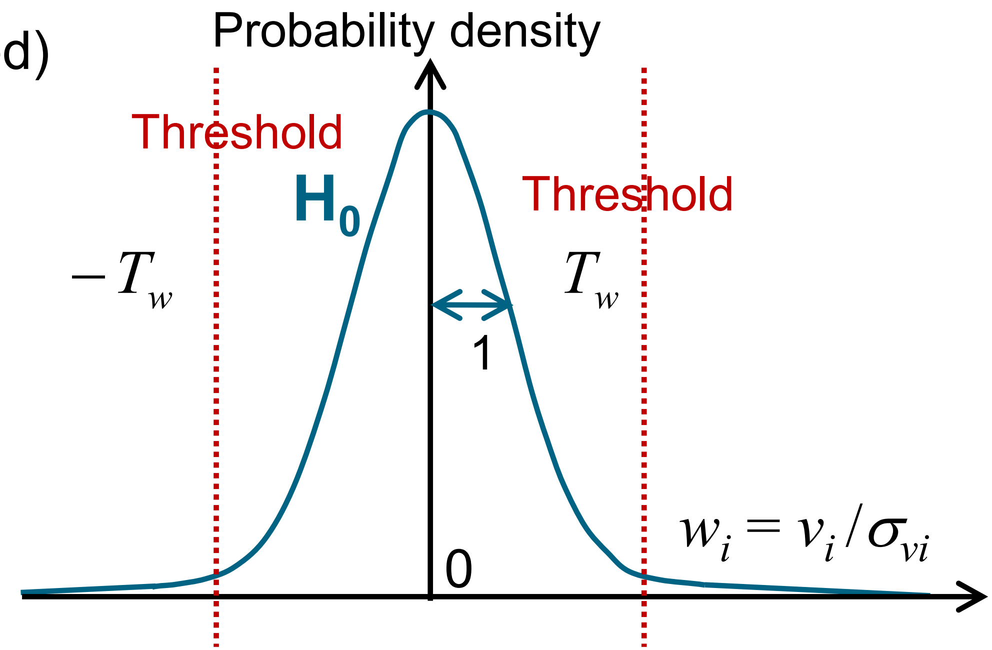
1. Calculate Residuals (unweighted)

$$\mathbf{v} = \left[\mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}} - \mathbf{I}_m \right] \delta\mathbf{z}^-$$

2. Calculate Residual Covariance

$$\mathbf{C}_\mathbf{v} = \left[\mathbf{I}_m - \mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}} \right] \sigma_\rho^2$$

Measurement error variance



3. Calculate Normalised Residuals

$$w_i = \frac{v_i}{\sigma_{vi}}$$

Root diagonal of $\mathbf{C}_\mathbf{v}$

4. Compare them with a threshold

$|w_i| \le T_w$: No fault assumed

$|w_i| > T_w$: Fault assumed – reject measurement

Questions on the Lectures from the Audience

Preparing for Next Week Monday Workshop

In **Workshop 1**, you will compute your own GNSS position solution using least-squares estimation

- Simulated data and some source code is available on Moodle
- You will need to write the rest of the source code
- A full mathematical description of the algorithms and answers for testing your code are provided

You can start work immediately if you want to

Help will be available during the Monday lab session

- Check your timetable to see if you have been allocated to the morning group or the afternoon group

Preparing for Next Week Monday Workshop

In **Workshop 1**, you will compute your own GNSS position solution using least-squares estimation

There are four tasks

1. Compute a position solution from the first set of measurements at time 0, working step by step
2. Compute single-epoch position solutions for the remaining times
3. Detect any errors in the measurement data
4. Compute a velocity solution

Preparing for Next Week Lectures 2A and 2B

Before Friday 20 January: Watch the recordings of Lectures 2A & 2B

Lecture 2A: GNSS Errors and Advanced Techniques

- Understand GNSS error sources, limitations and performance
- Show how Advanced Techniques can be used to improve performance

Lecture 2B: The Kalman Filter and its use for GNSS

- Introduce sequential least-squares estimation to efficiently process measurements made at different times
- Introduce the Kalman filter for estimating time-varying states
- Apply the Kalman filter to GNSS positioning

Final Questions and Comments

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