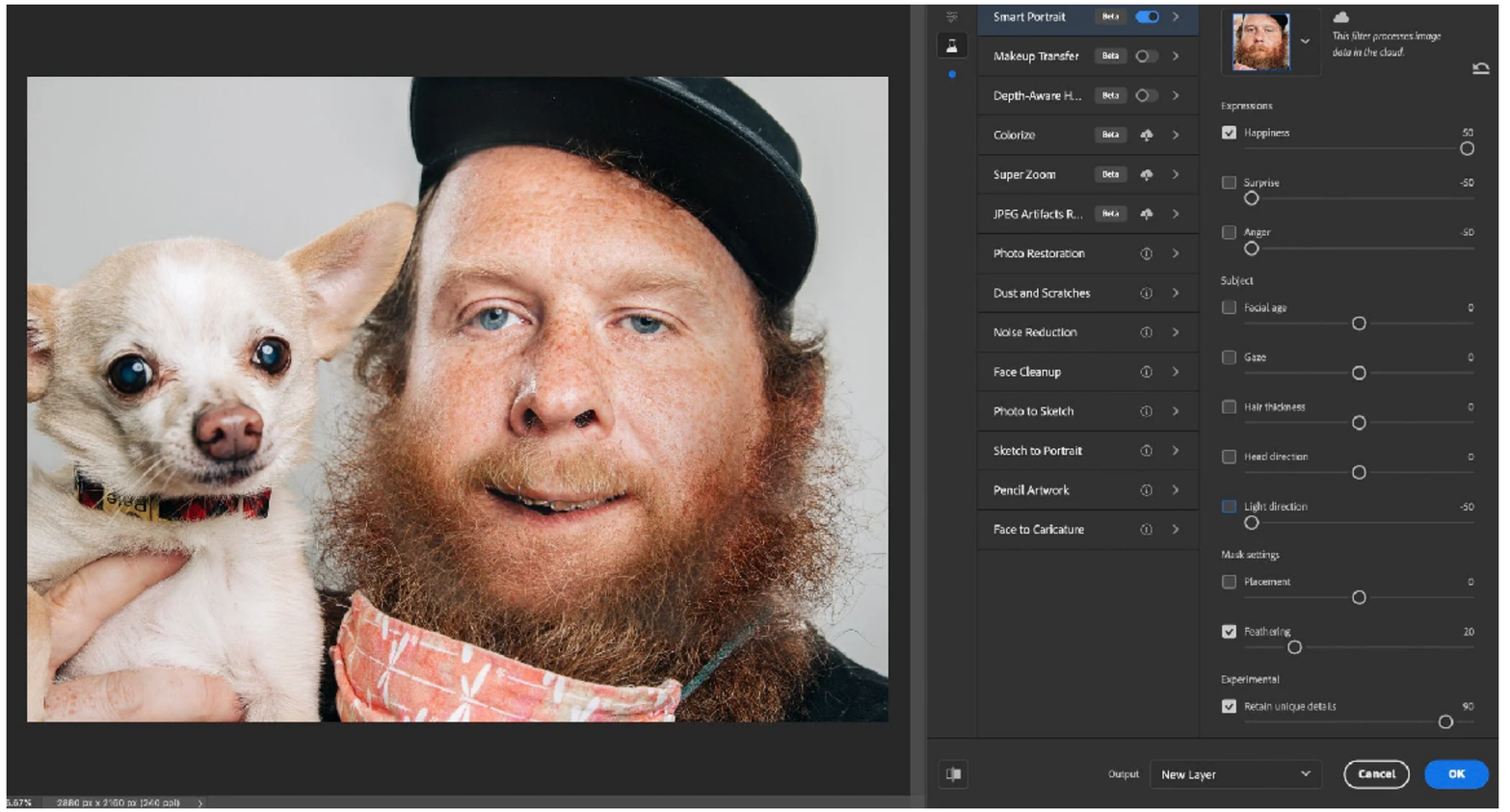
COMP0026: Image Processing

Image Transformations



[Popular Science]

Lectures will be Recorded

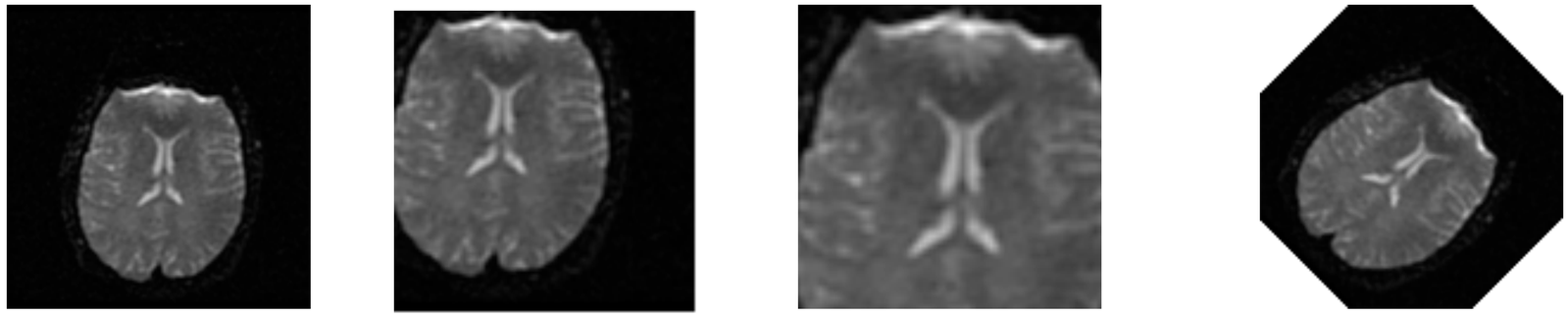
Outline

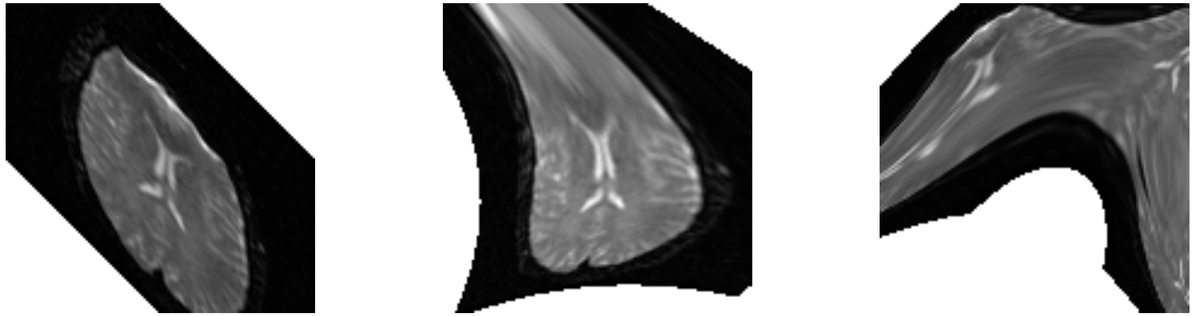
- Grey-level transformations
  - Histogram equalization

- Geometric transformations
  - Affine transformations
  - Interpolation
  - Warping and morphing

Geometric Transformations

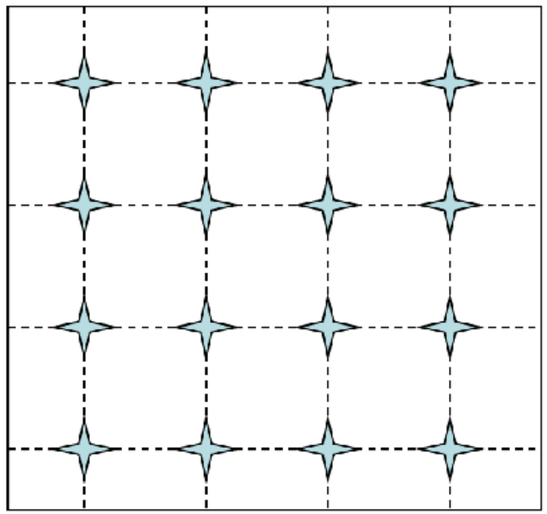
Change the location of image features

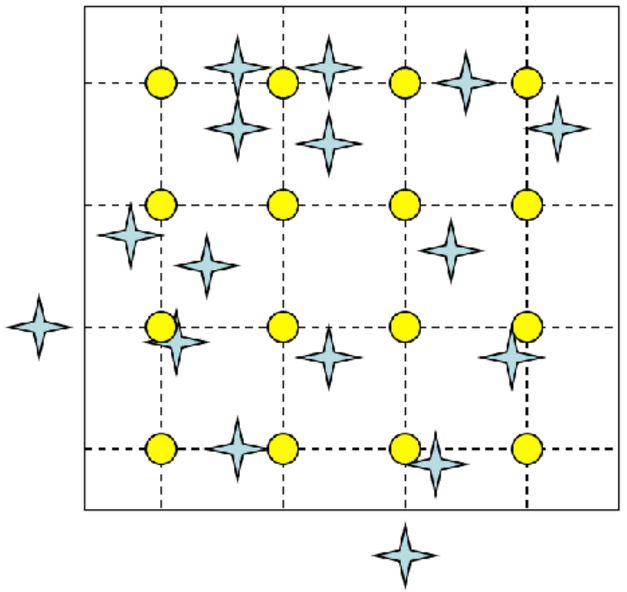




Geometric Transformations

- Mapping image f(u, v) to g(x, y) based on a given mapping function: x(u, v), y(u, v).
- Forward Mapping
  - For each point (u, v) in the original image, find the corresponding position (x, y) in the deformed image by the forward mapping function, and let g(x,y)=f(u,v).
  - What if the mapped position (x,y) is not an integer sample in the desired image?





Warping points are often non-integer samples

Many integer samples “o” are not assigned Values

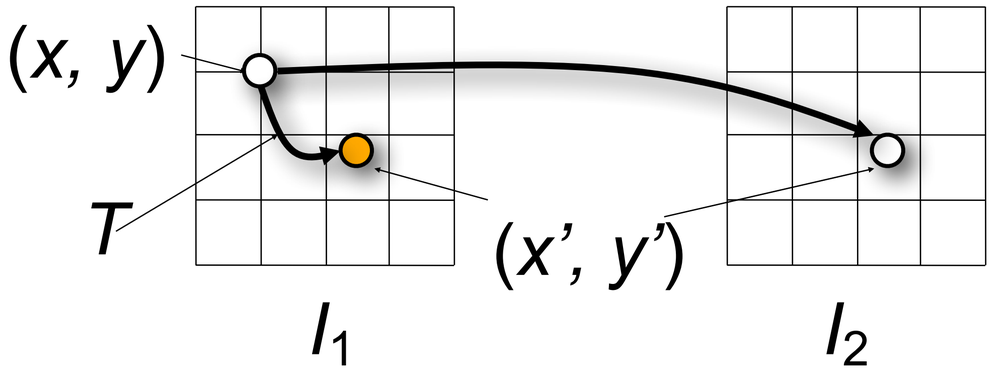
Geometric Transformations

- I_1 is the original image, I_2 is the warped image.

$$I_2(x', y') = I_1(x, y)$$

$$(x', y') \leftarrow T(x, y)$$

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$



Affine Transformations

translation, scaling, rotation, shear

$$x' = ax + by + t_x$$

$$y' = cx + dy + t_y$$

Affine Transformations

translation, scaling, rotation, shear

$$x' = ax + by + t_x$$

$$y' = cx + dy + t_y$$

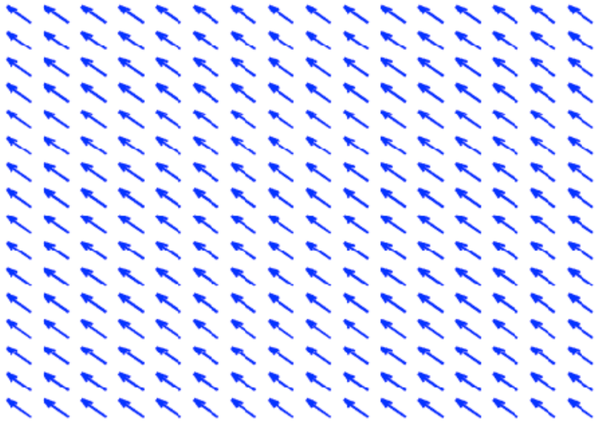
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

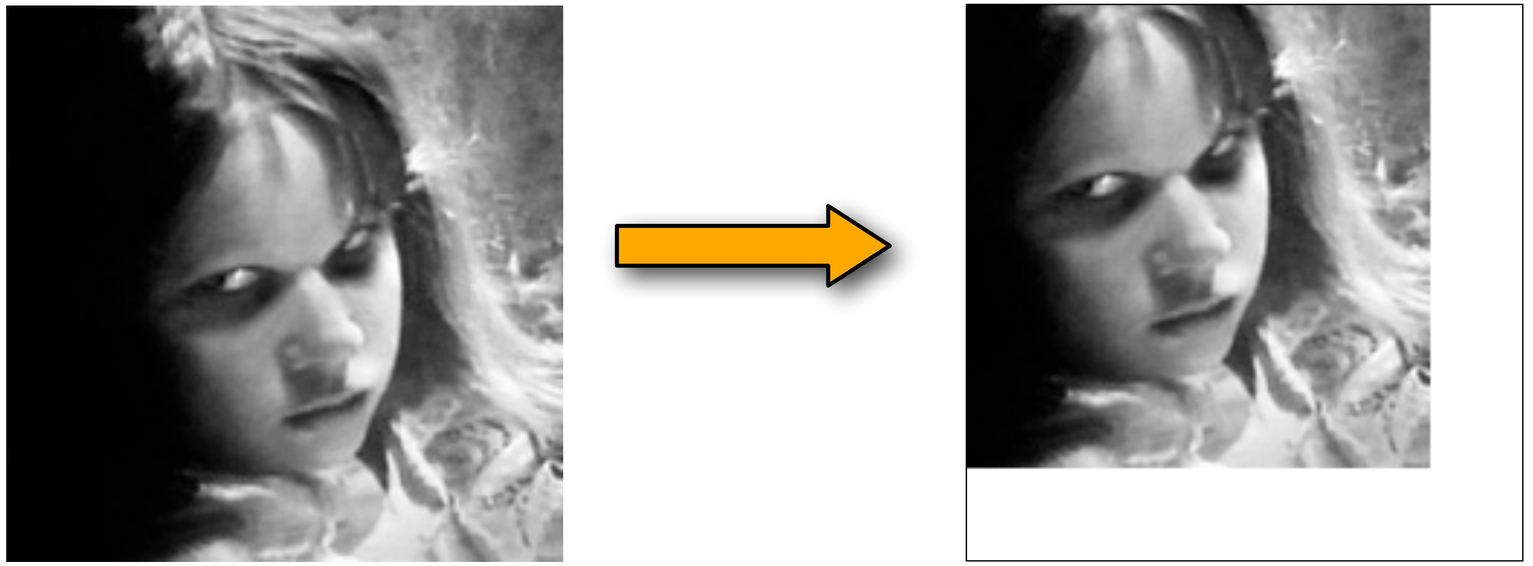
identity matrix (rotation + scaling + shear?)

Translation

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

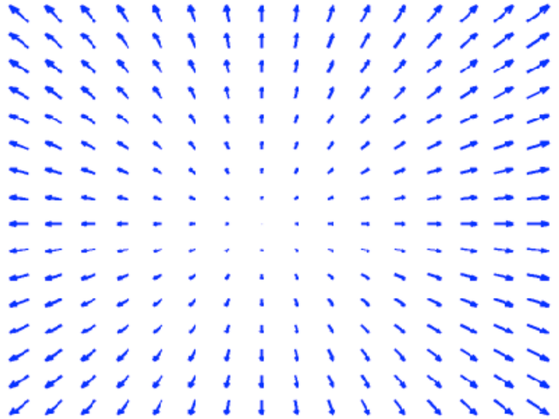




Scaling

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

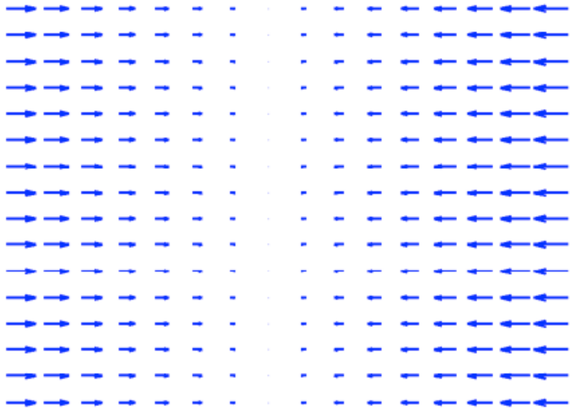
$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} -w/2 \\ -h/2 \end{pmatrix}$$

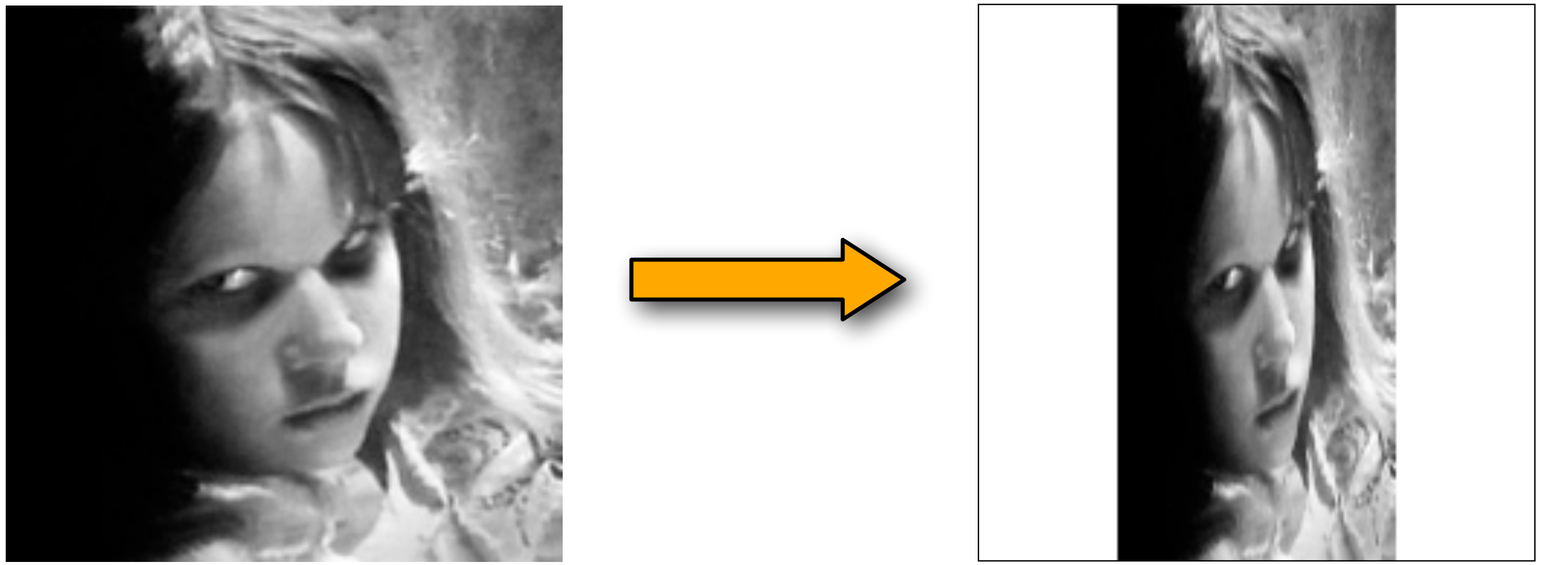


Stretch

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} -w/4 \\ 0 \end{pmatrix}$$

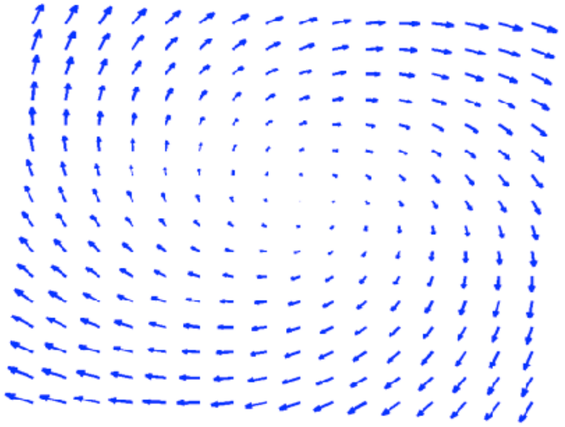




Rotation

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad \theta = \pi/4$$

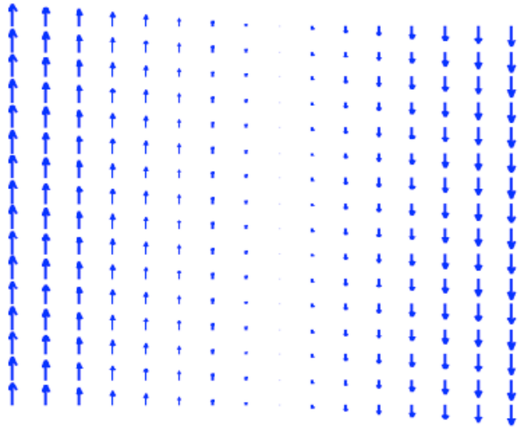
$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} -(W(\cos\theta - 1) + H\sin\theta)/2 \\ (W\sin\theta - H(\cos\theta - 1)/2) \end{pmatrix}$$



Shear (along y axis)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix}$$

$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} 0 \\ -sH/2 \end{pmatrix}$$



Homogeneous Coordinates

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Remember:

$$x' = ax + by + t_x$$
$$y' = cx + dy + t_y$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

Homogeneous Coordinates

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

“Subsequent” operations are inserted here,
by pre-multiplying

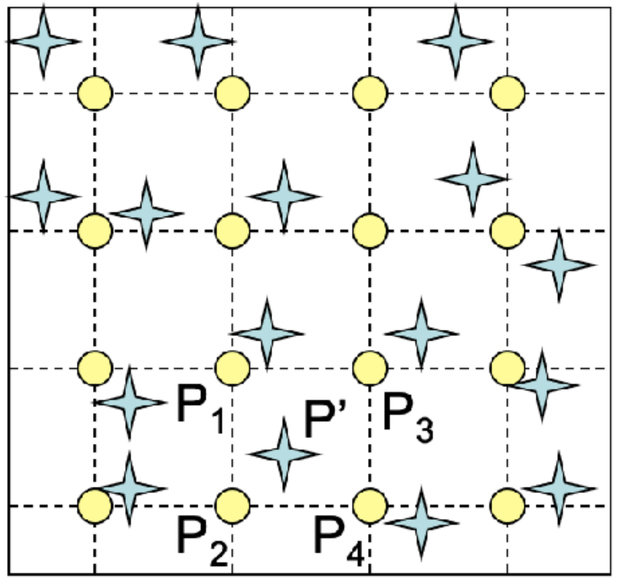
Implementation

Always use the inverse transformation:

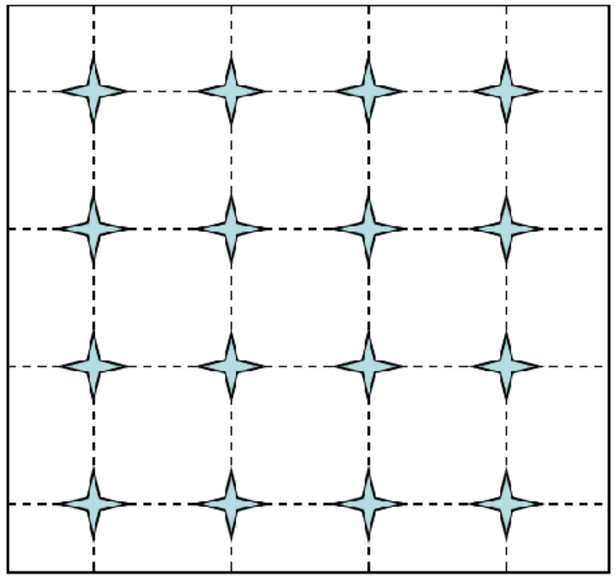
```
function warpedImage = transform(image, T)
    [W,H] = size(image);
    warpedImage = zeros(W,H);
    invT = invert(T);
    for x=1:W
        for y=1:H
            warpedImage(x,y)=image(invT(x,y));
        end
    end
```

Interpolation

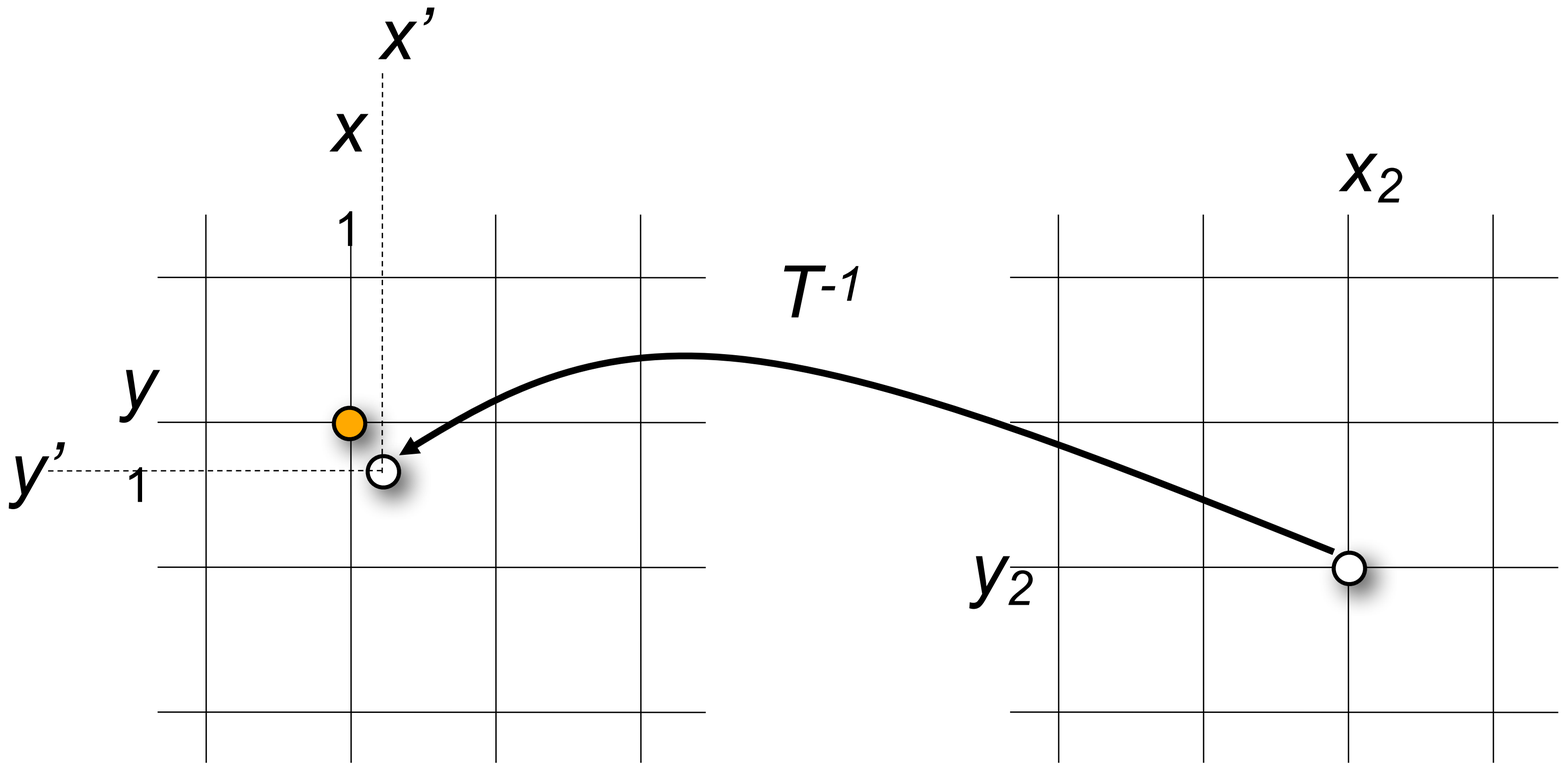
- **Usually,** $(x', y') \leftarrow T^{-1}(x, y)$ **are not integer coordinates.**
- **We estimate** $I_1(x', y')$ **by** *interpolation* **from surrounding positions.**



P' will be interpolated from P_1, P_2, P_3, and P_4



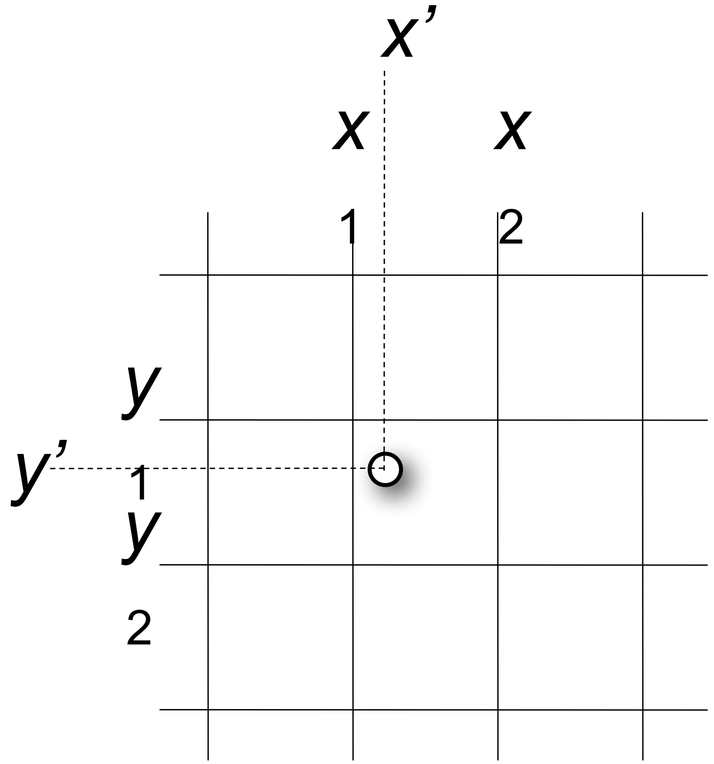
Nearest Neighbour Interpolation



Take the value at the nearest grid point:

$$I_1(x', y') = I_2(x_2, y_2)$$

Bilinear interpolation

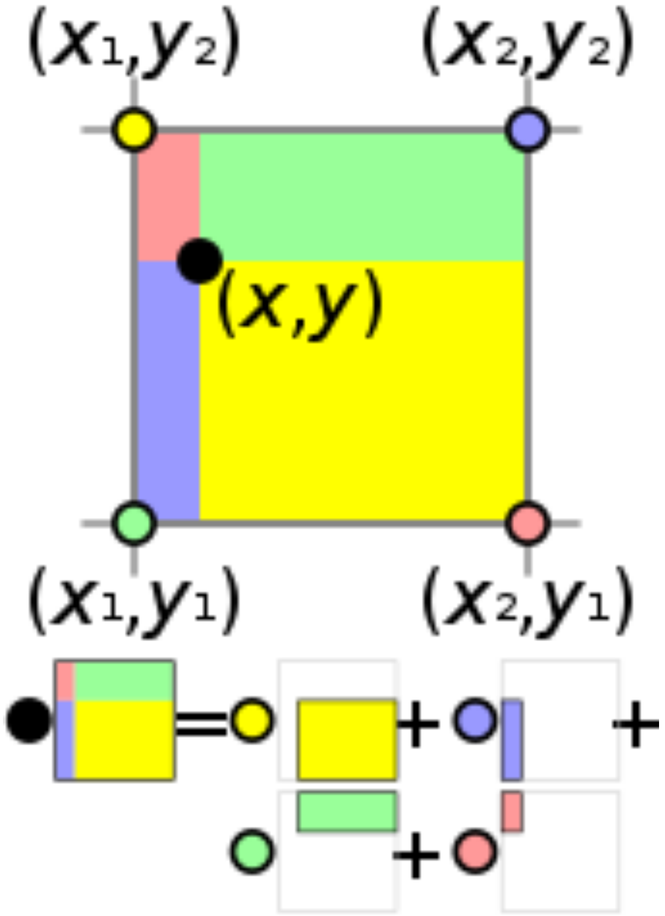


Weighted average from neighboring grid points:

$$I_1(x', y') = \Delta x\, \Delta y\, I_1(x_2, y_2)$$
$$+ \Delta x\, (1 - \Delta y)\, I_1(x_2, y_1)$$
$$+ (1 - \Delta x)\, \Delta y\, I_1(x_1, y_2)$$
$$+ (1 - \Delta x)\, (1 - \Delta y)\, I_1(x_1, y_1)$$

$$\Delta x = x' - x_1, \qquad \Delta y = y' - y_1$$

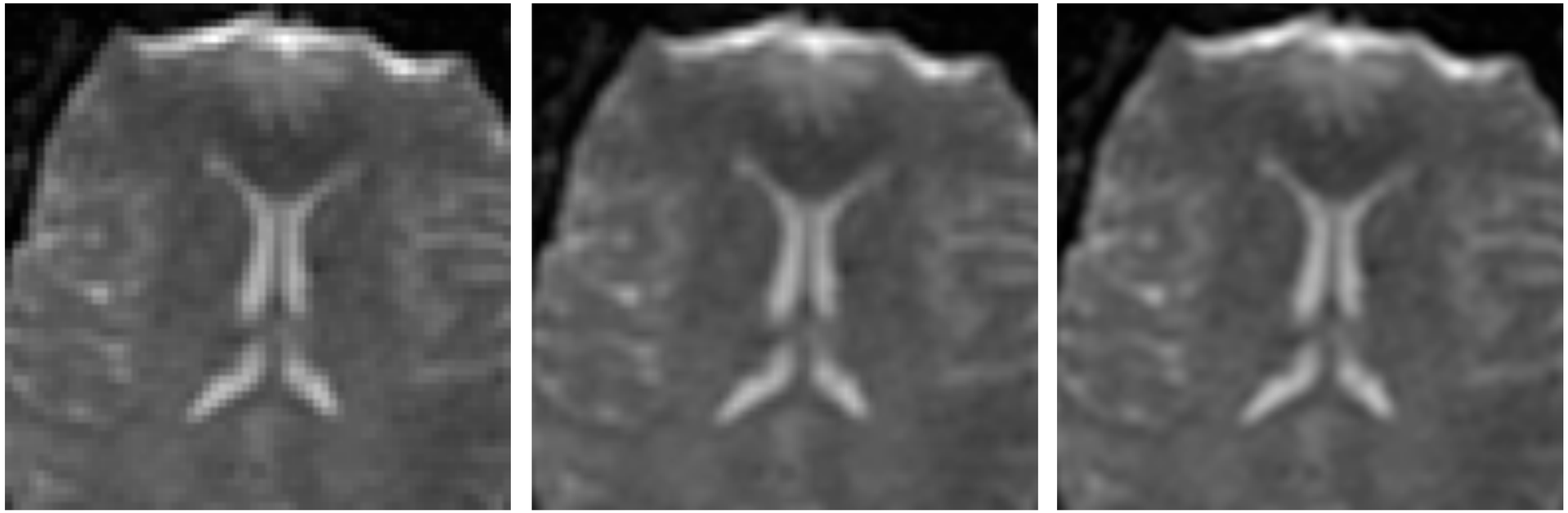
Bilinear interpolation



Higher Order Interpolation

- Quadratic interpolation fits a bi-quadratic function to a 3x3 neighbourhood of grid points
- Cubic interpolation fits a bi-cubic function to a 4x4 neighbourhood.

Interpolation Comparison



Nearest neighbour

Bilinear

Cubic

Polynomial Warps

- **Polynomial transformations, e.g., quadratic transformations:**

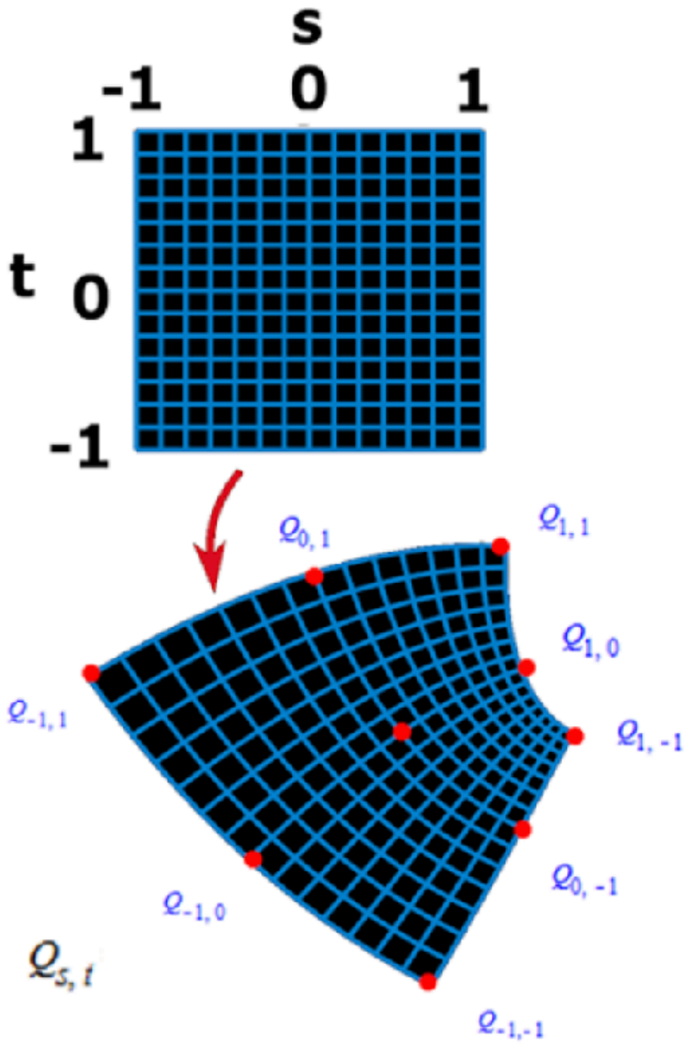
$$x' = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2$$

$$y' = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2$$

- **Affine transformations map lines to lines.**
- **They are *first-order* polynomial transformations.**
- **Higher-order polynomials can bend lines.**

Quadratic Warp

Quadratic Warp



Control Point Warps

- **Moves some pixels to specified locations.**
- **Interpolate the displacement at intermediate positions.**
- **Find a polynomial warp *P* that maps:**

$$(x_1, y_1) \rightarrow (x'_1, y'_1)$$
$$(x_2, y_2) \rightarrow (x'_2, y'_2)$$
$$\vdots$$
$$(x_m, y_m) \rightarrow (x'_m, y'_m)$$

Control point warps

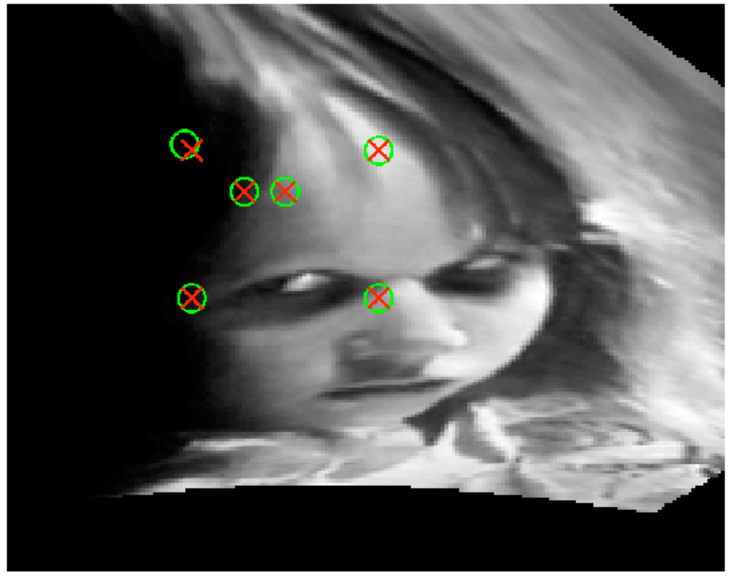
Least squares estimate of *P* is

$$\begin{pmatrix} x'_1 & y'_1 \\ x'_2 & y'_2 \\ \vdots & \vdots \\ x'_m & y'_m \end{pmatrix} = \begin{pmatrix} 1 & x_1 & y_1 & x_1^2 & x_1 y_1 & y_1^2 \\ 1 & x_2 & y_2 & x_2^2 & x_2 y_2 & y_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_m & y_m & x_m^2 & x_m y_m & y_1^2 \end{pmatrix} \begin{pmatrix} a_0 & b_0 \\ a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \\ a_4 & b_4 \\ a_5 & b_5 \end{pmatrix}$$

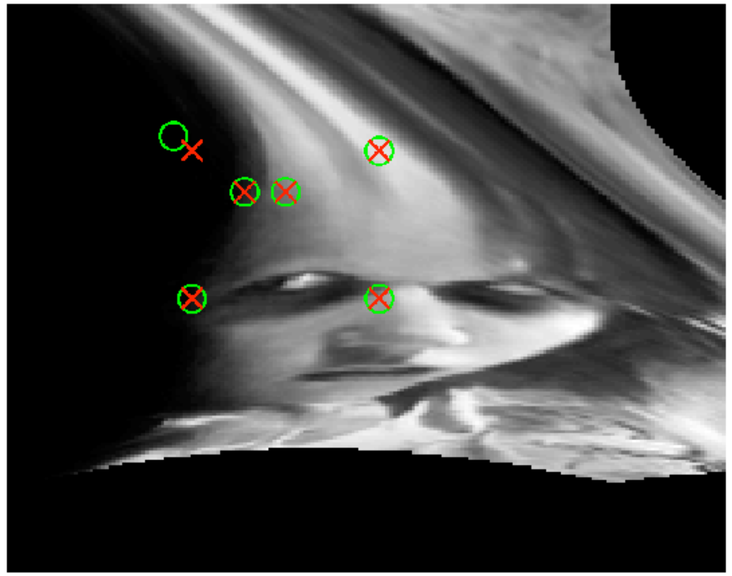
$$A = XP \qquad (m \geq 6)$$

$$P = (X^T X)^{-1} X^T A$$

Example – Two Pixel Displacement



Five Pixel Displacement



Over-determined

Applications

- Special effects
  - Film industry
  - Computer games

- Image registration
  - Medical imaging
  - Security

Image Morphing

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Image Morphing

How do we get the *i*-th image in the sequence?

Landmarks

Image Registration

- Determines the transformation that aligns two similar images

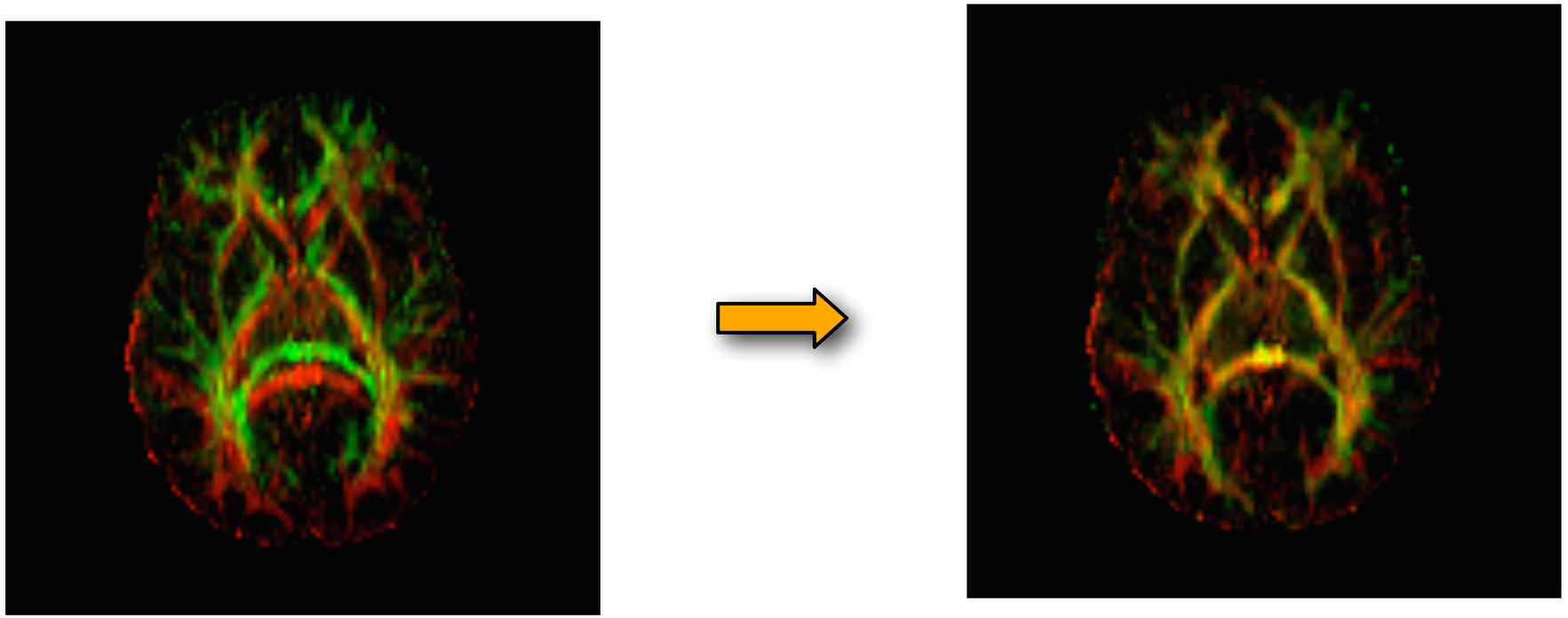


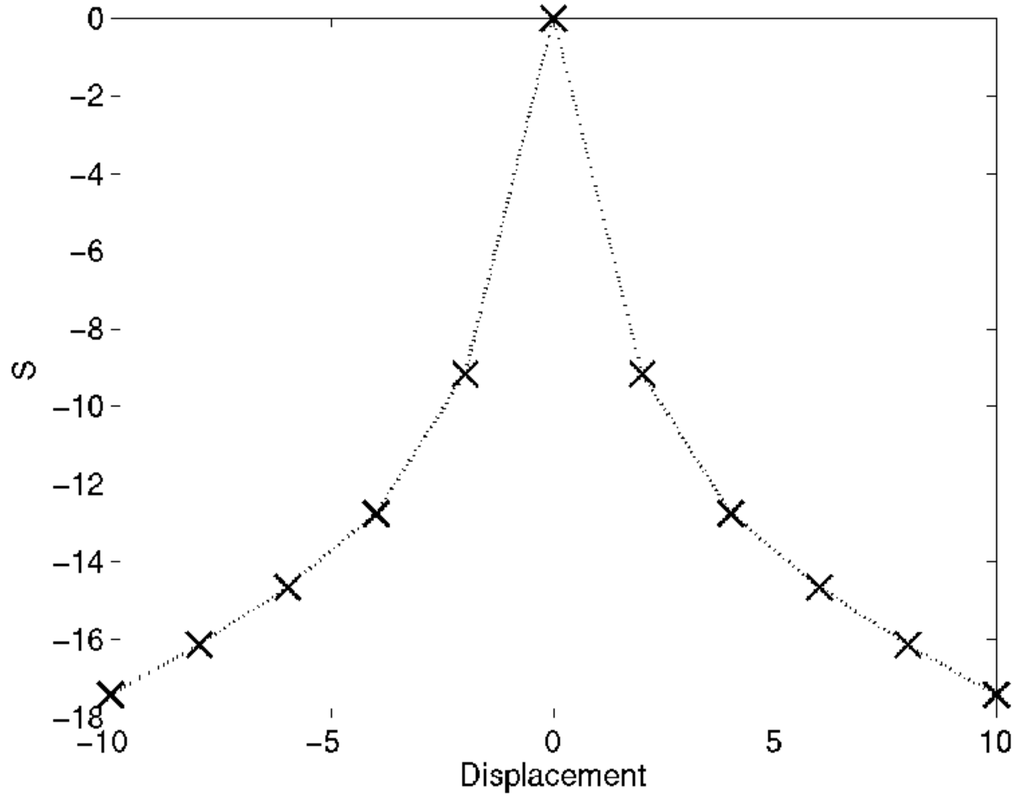
Image Similarity

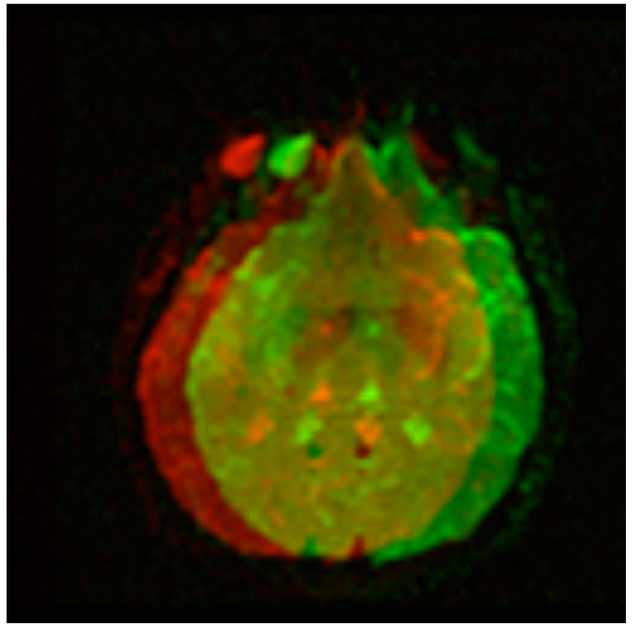
- **We find the transformation that maximises the similarity of the images.**
- **A simple similarity measure is:**

$$S(I_1, I_2) = -\left(\sum_{\mathbf{x} \in I_1} (I_1(\mathbf{x}) - I_2(\mathbf{x}))^2 \right)^{1/2}$$

- **Many others are used including the *cross-correlation* and *mutual information*.**

Registration Example





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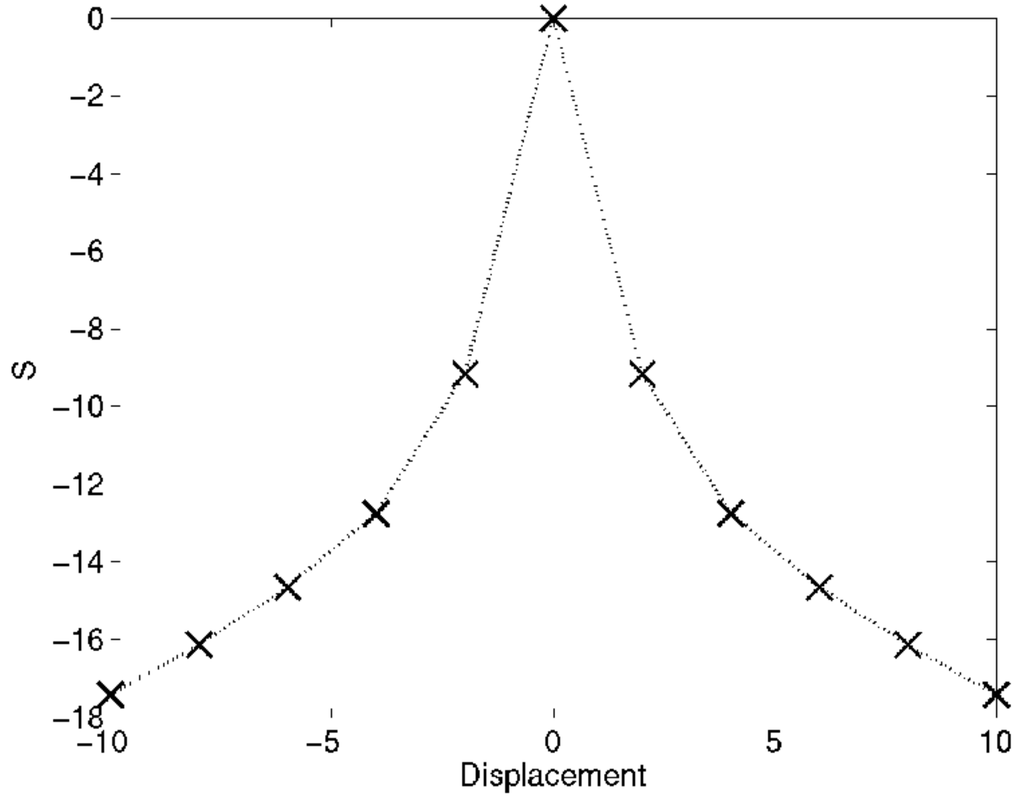
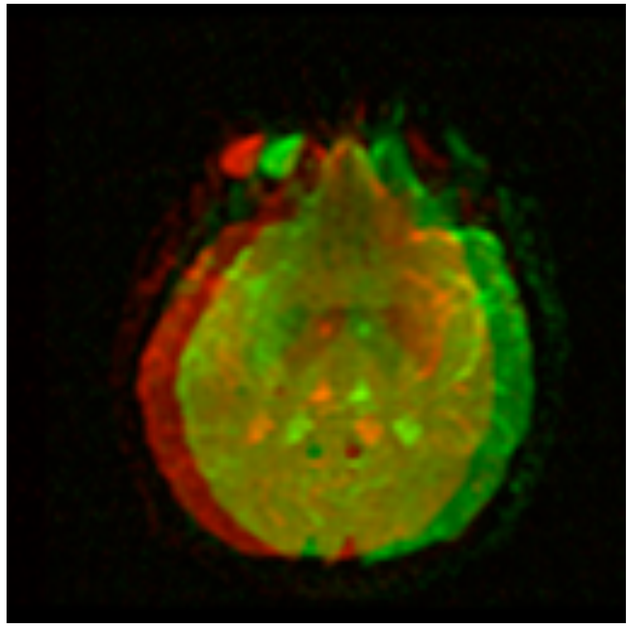
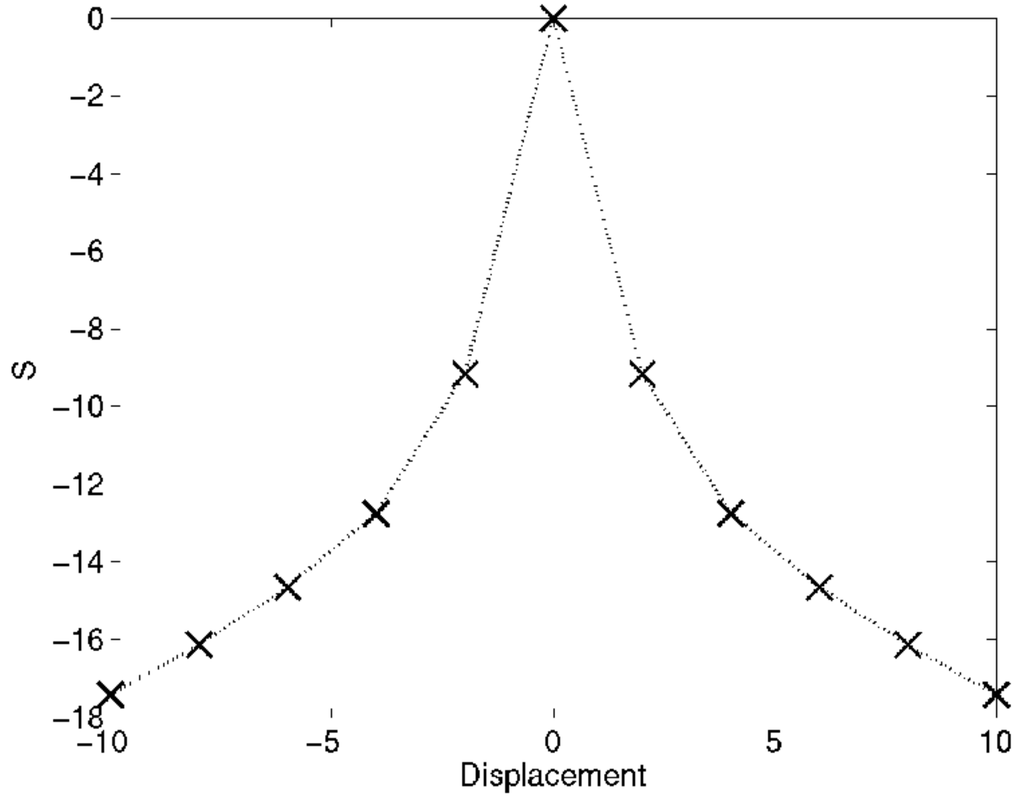
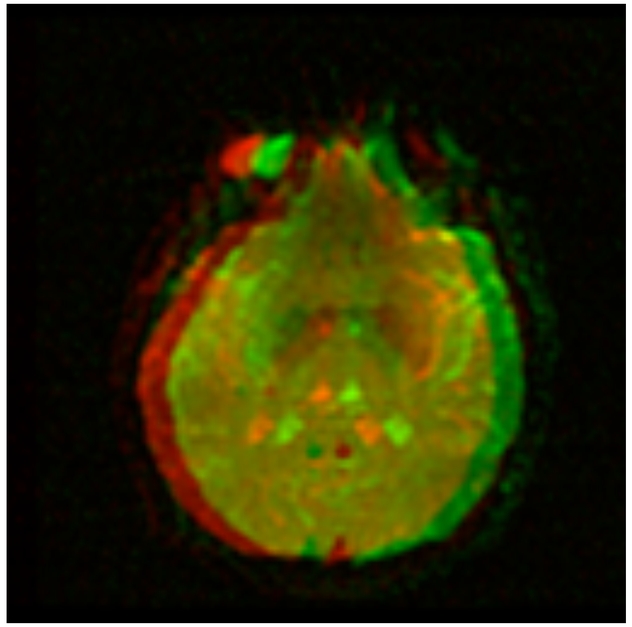




Registration Example





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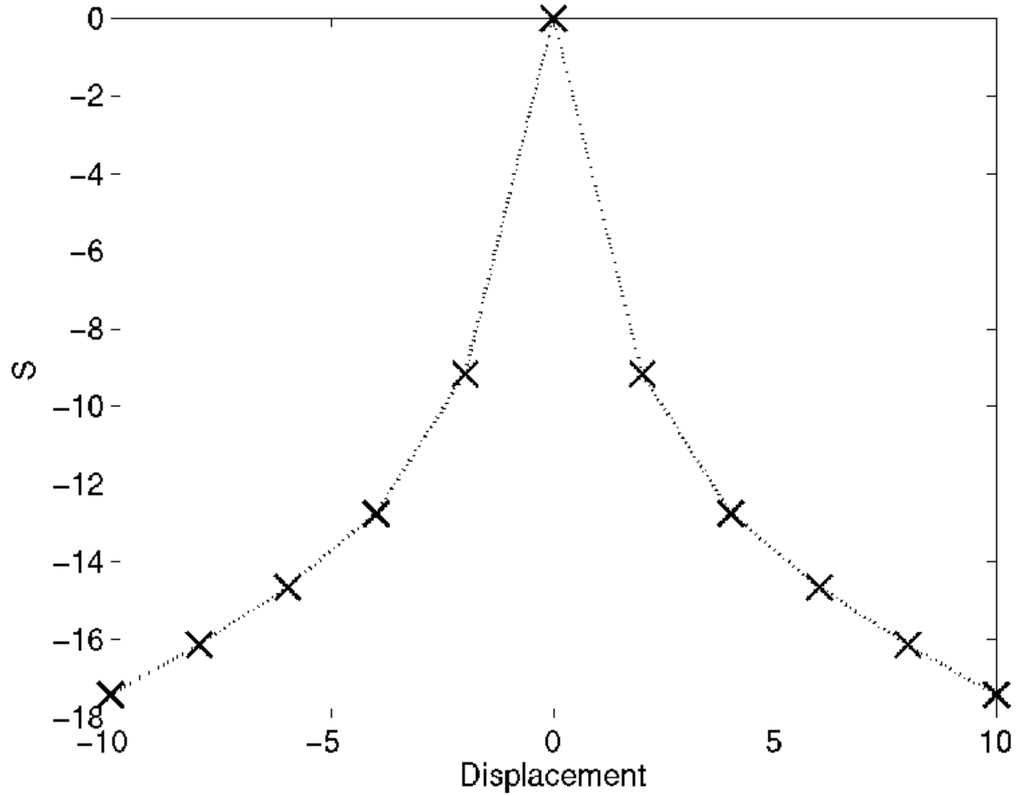
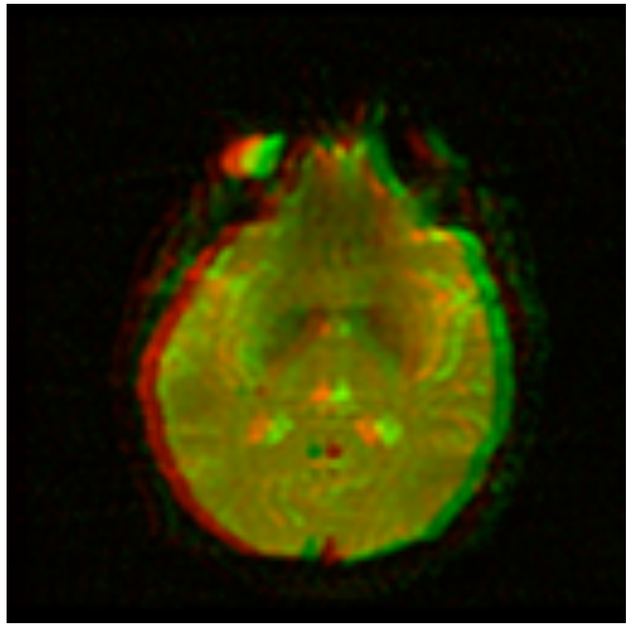
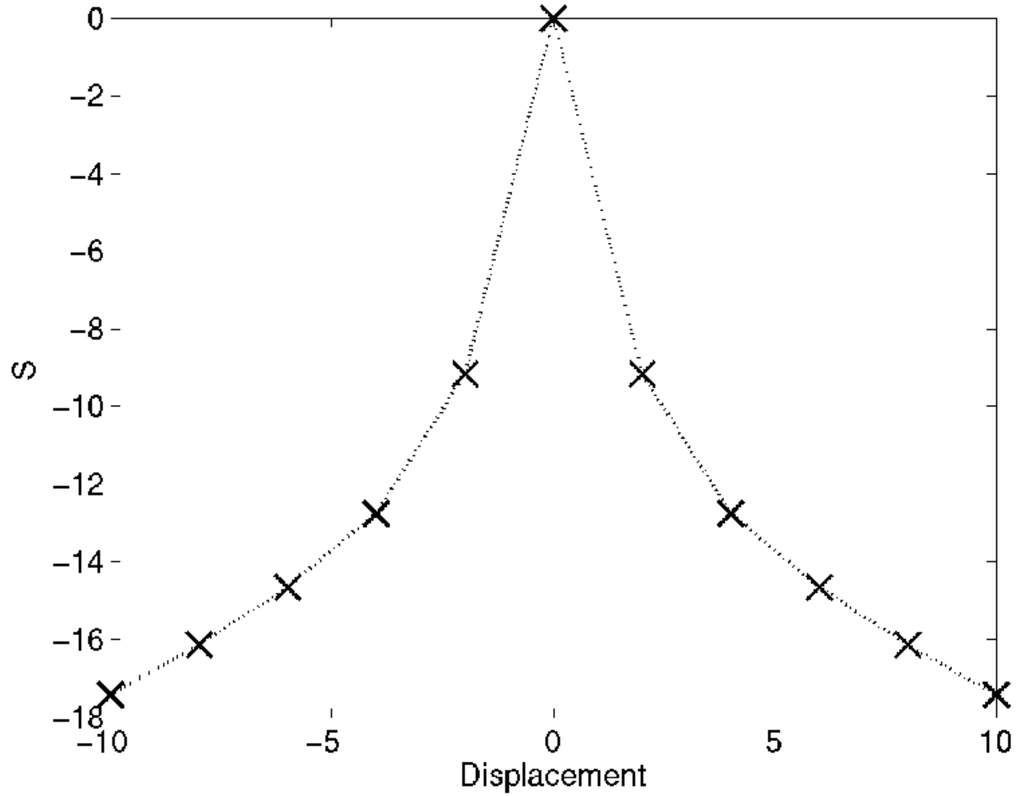
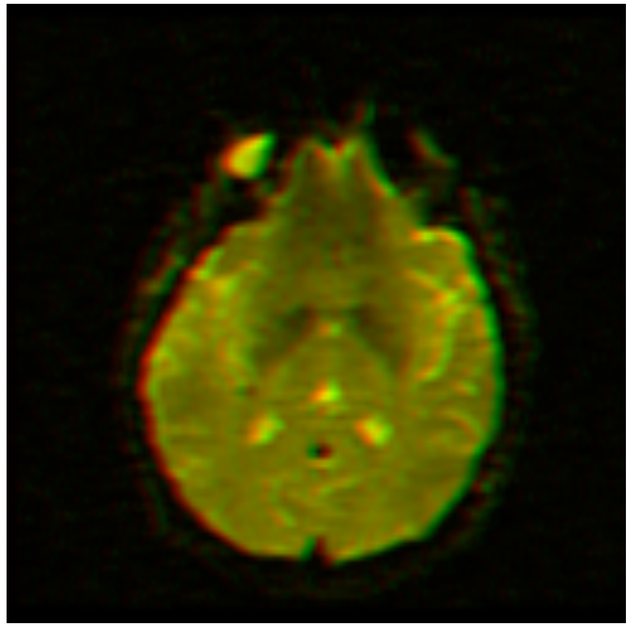




Registration Example





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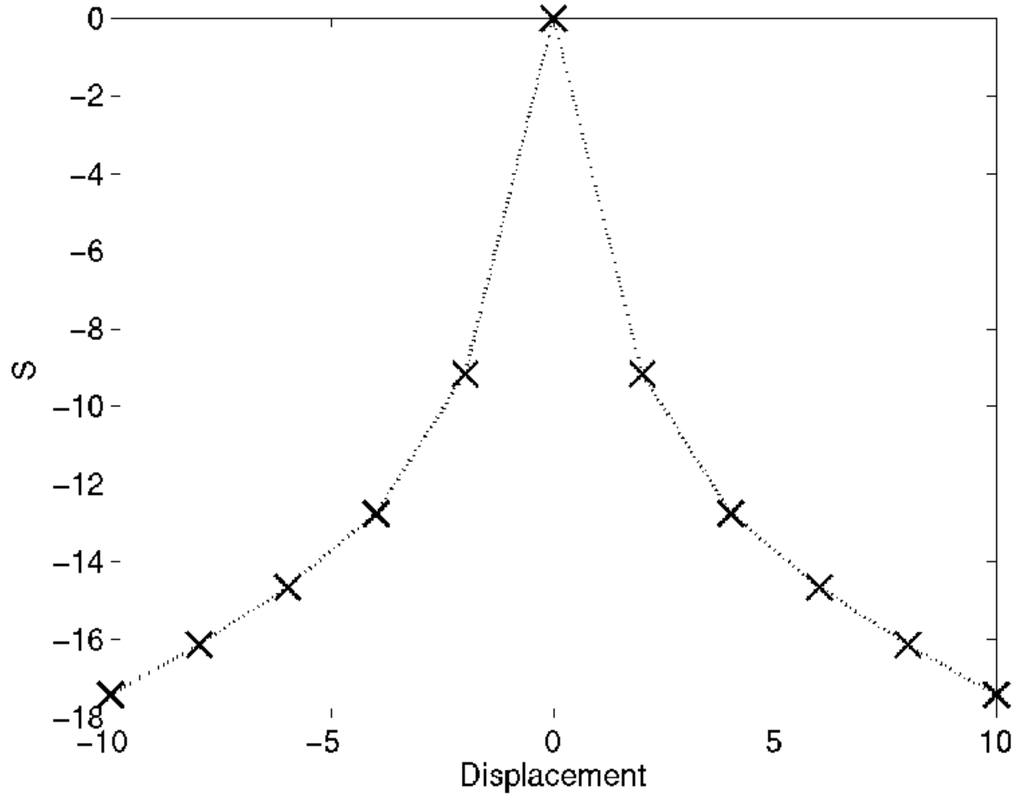
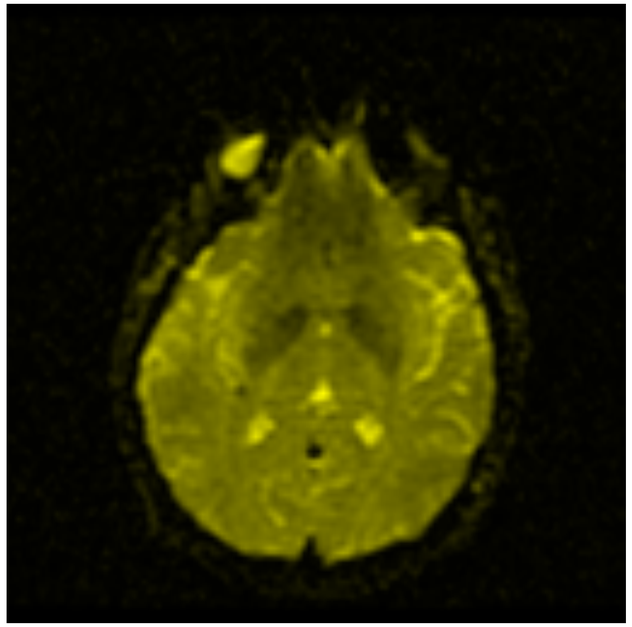
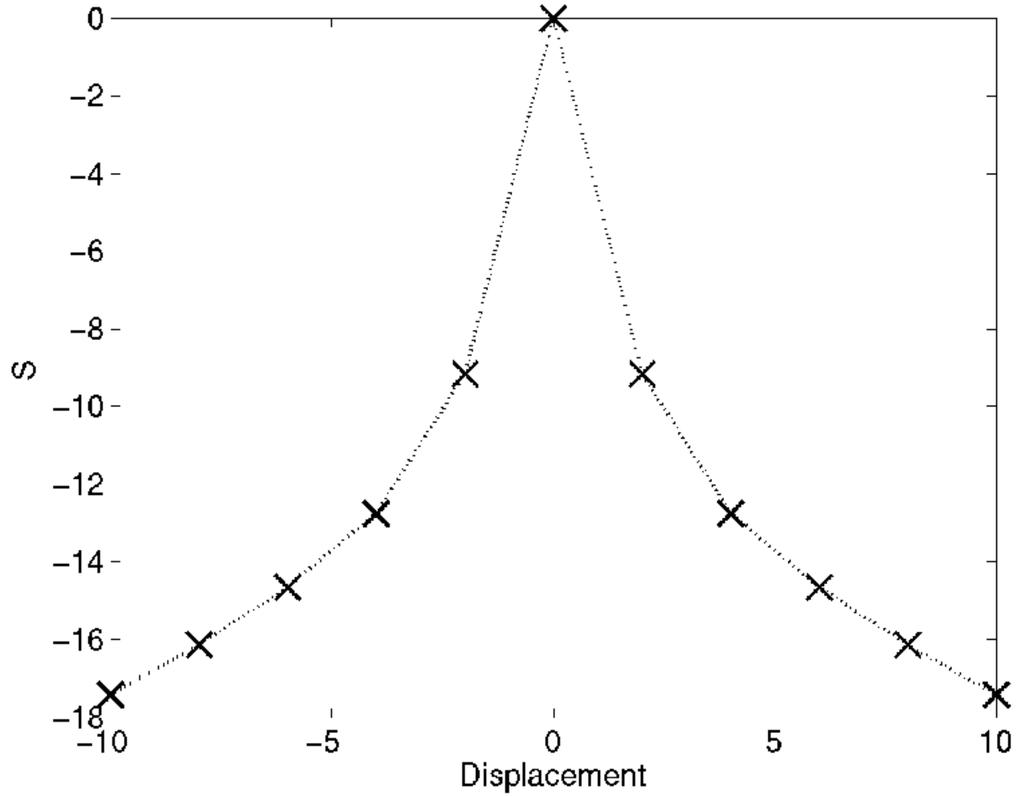
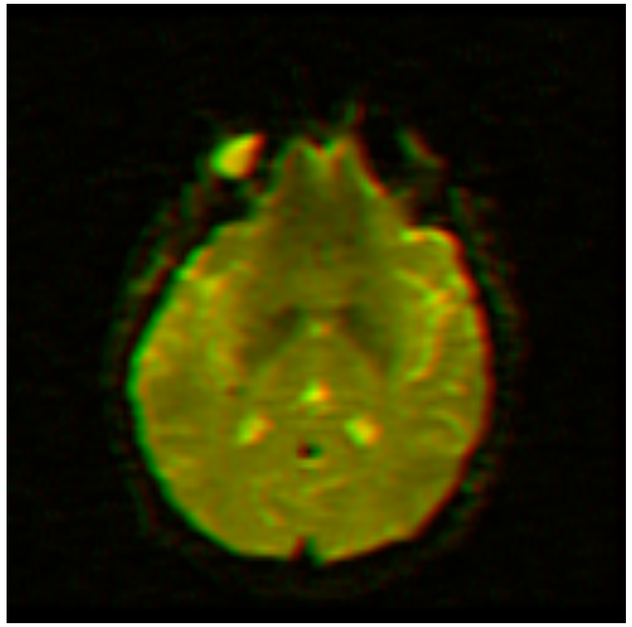


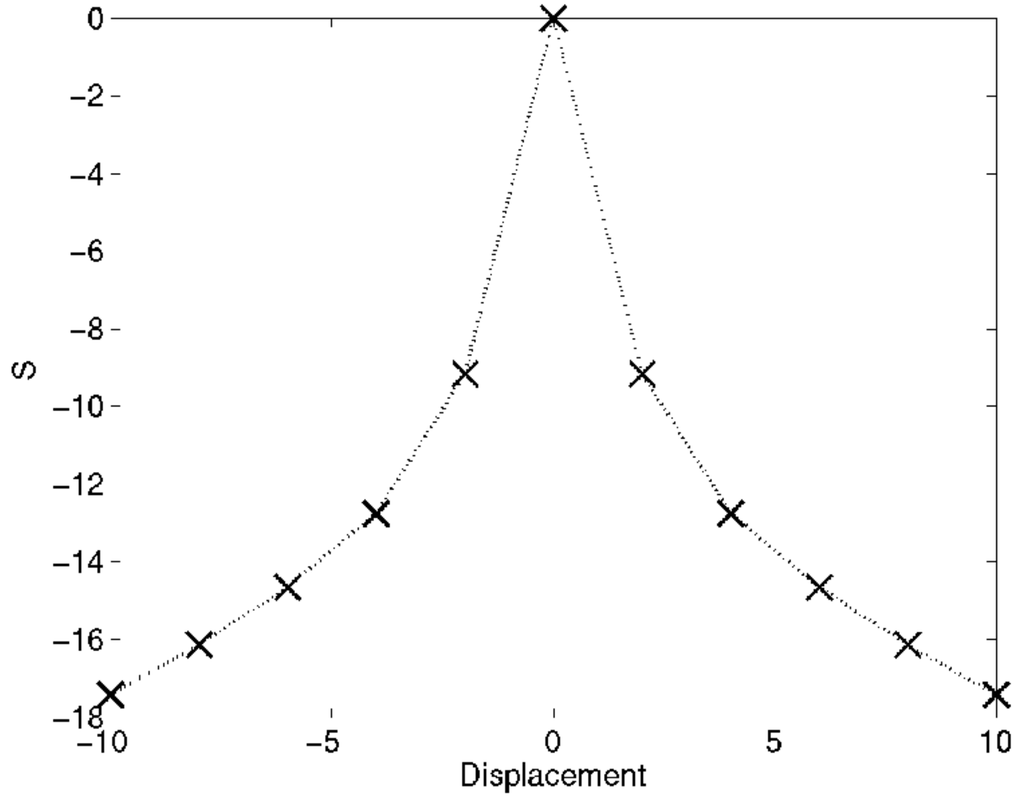


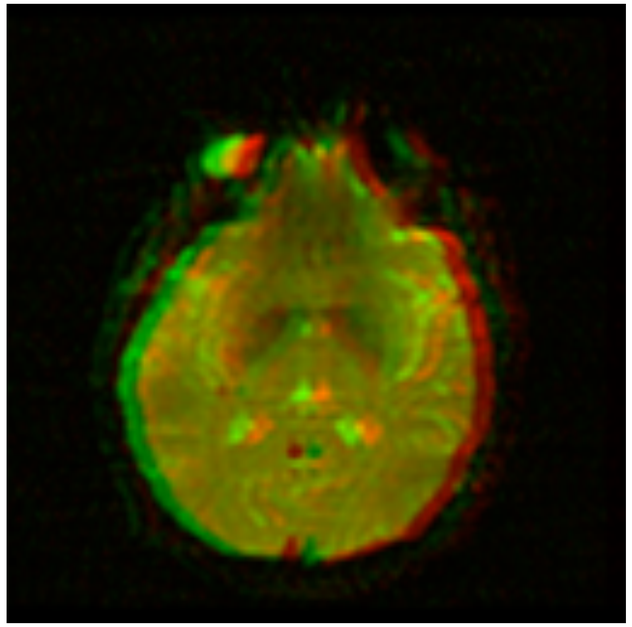
Registration Example





Registration Example





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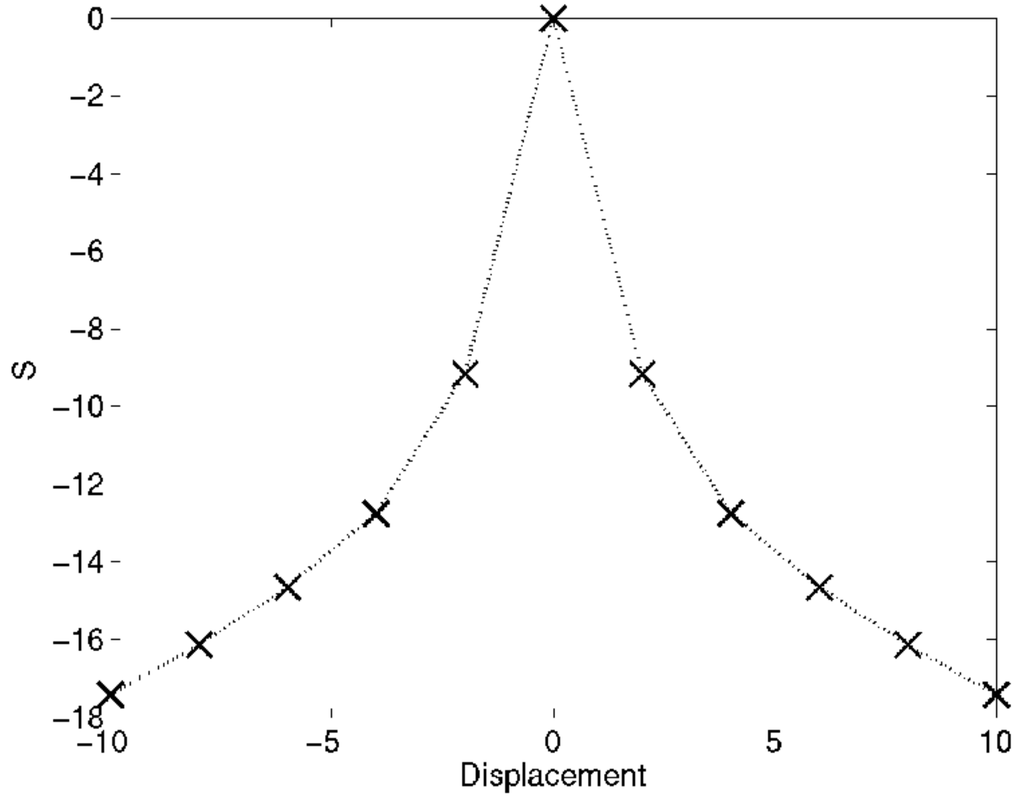
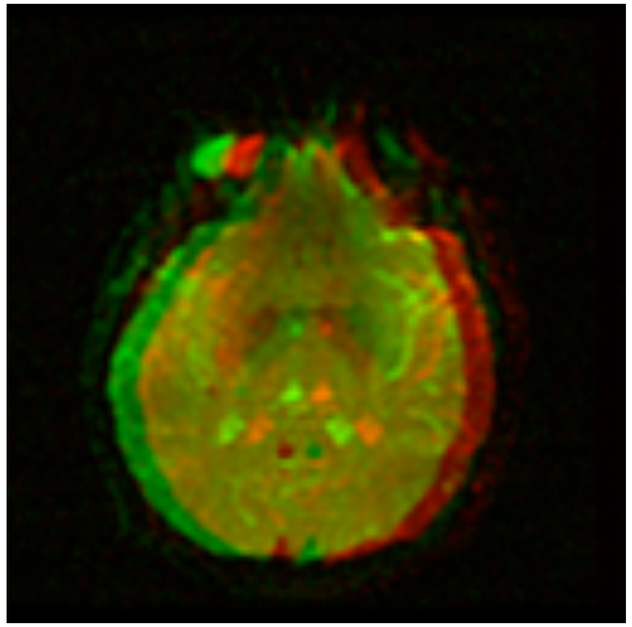
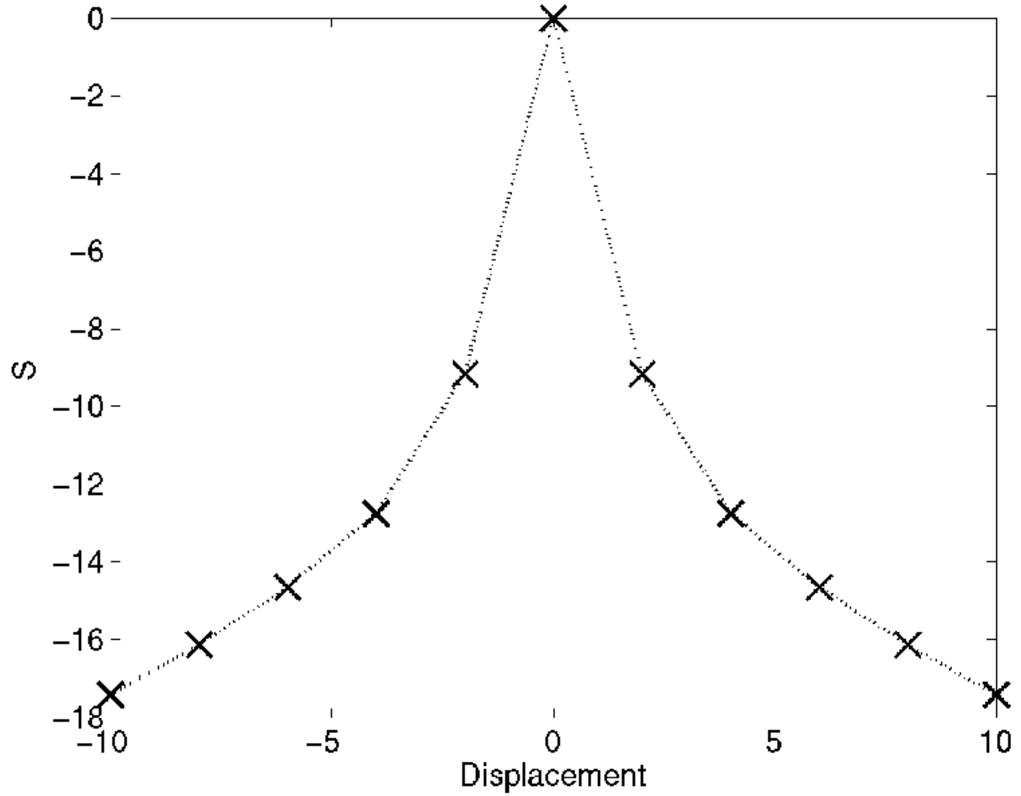
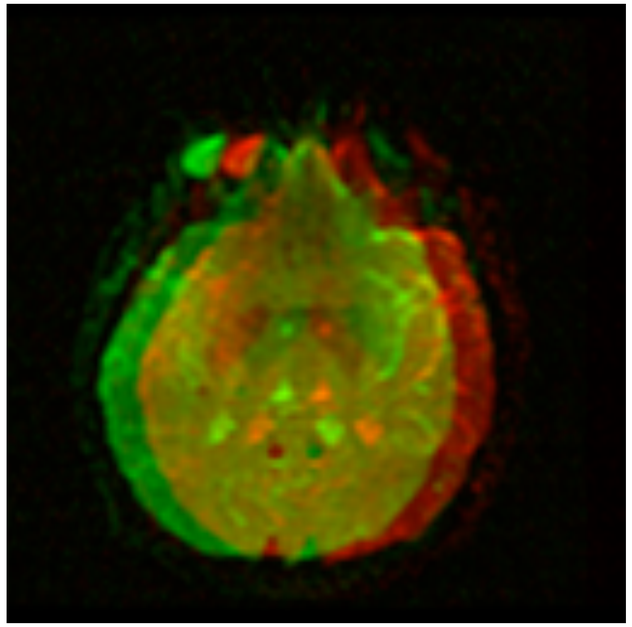


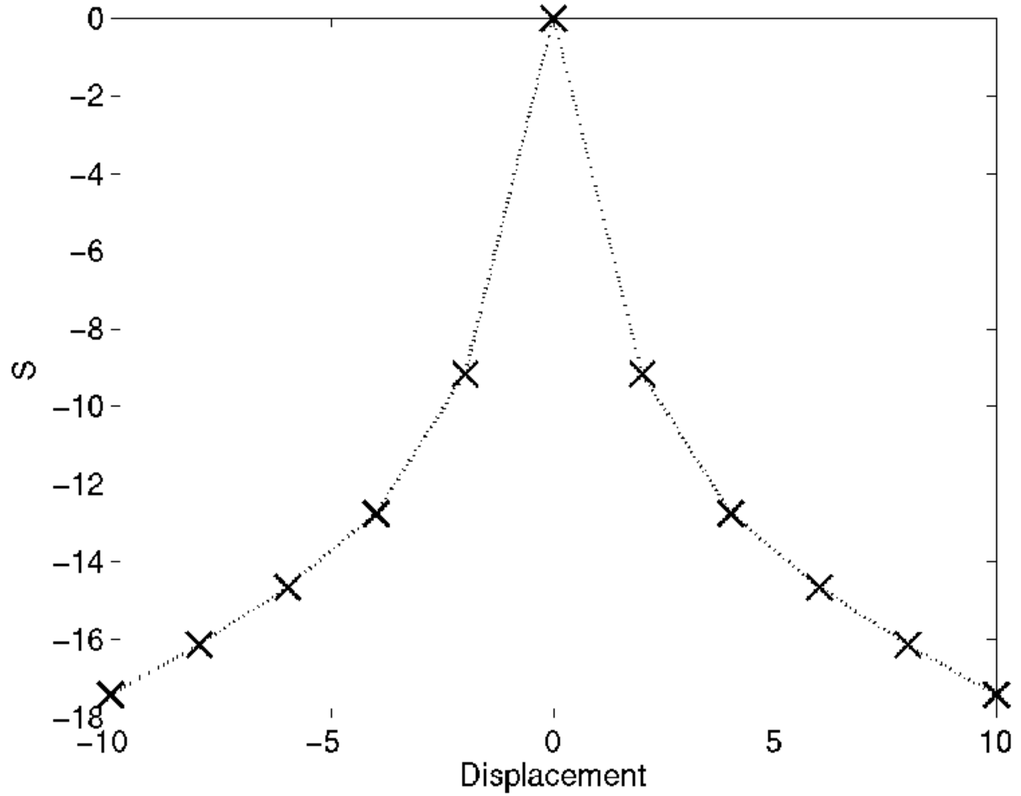


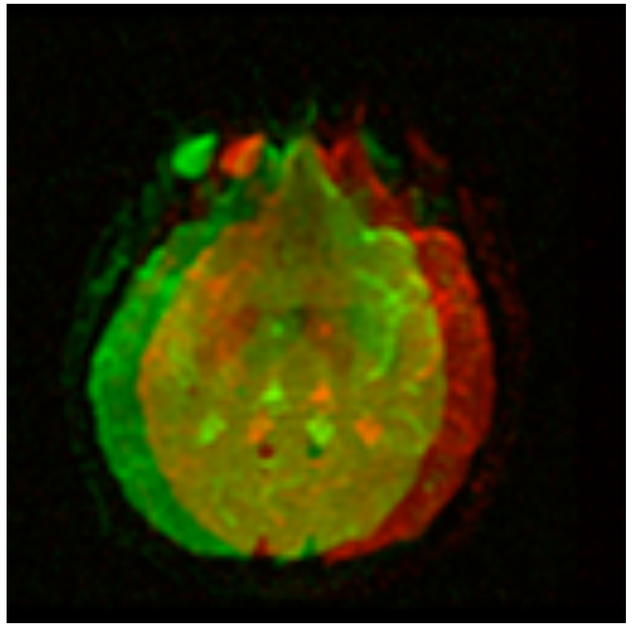
Registration Example





Registration Example





Registration

- **We have to search for the transformation the minimizes S.**
- **Simple in the 1D example.**
- **For more complex transformations, we use *numerical optimization* (see the Matlab function `fminunc` for example).**

Feature-Based Image Metamorphosis

Beier and Neely

[*Feature-Based Image Metamorphosis*, SIGGRAPH, 1992]

Explicit Correspondences

Individual face

Average face

$$u = \frac{(X - P) \cdot (Q - P)}{\| Q - P \|^2} \quad (1)$$

$$v = \frac{(X - P) \cdot Perpendicular\,(Q - P)}{\| Q - P \|} \quad (2)$$

$$X' = P' + u \cdot (Q' - P') + \frac{v \cdot Perpendicular\,(Q' - P')}{\| Q' - P' \|} \quad (3)$$