Computer Graphics (COMP0027) 2023/24

Rasterization

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Ray-tracing: photo-realistic, but not usually real-time

Goal: Photo-realism at interactive rate for VR/games

Summary

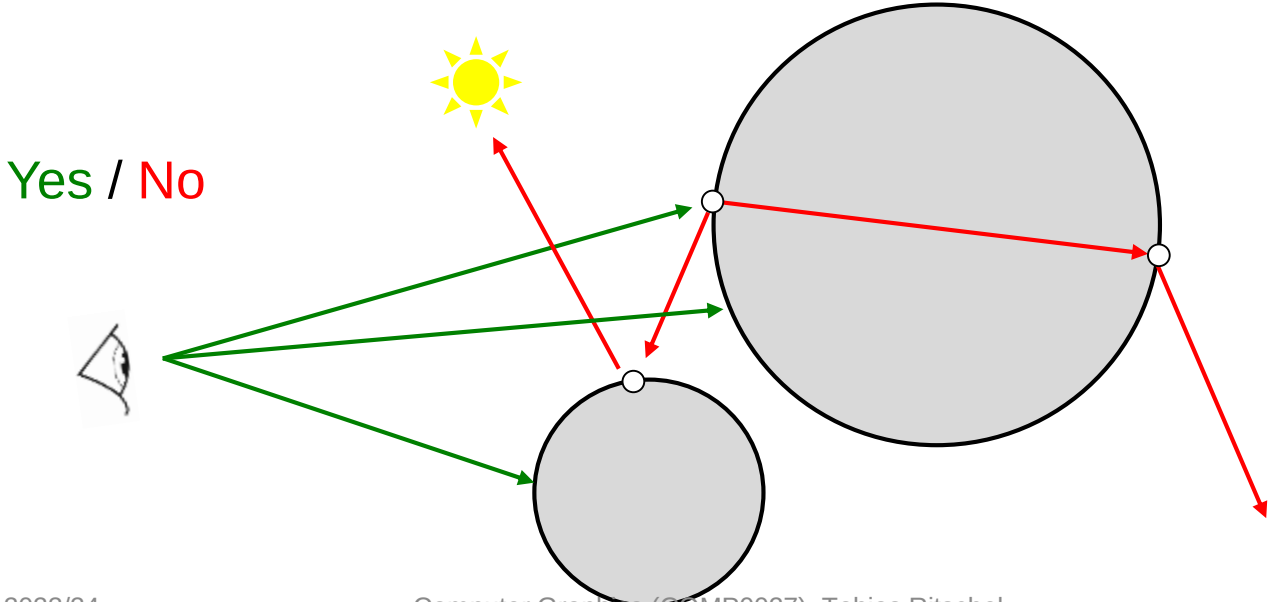
- Ray-tracing is elegant and general but slow
- Now we want to speed this up
- Main idea:
  - Don’t **trace** rays
  - **Project** primitives
- Creates many new challenges

Ray-tracing cost

- The process of casting rays is very slow
- Example
  - 10,000 triangles,
  - 1000x1000 pixels
  - Result in 1,000,000 primary rays to cast
  - Each one testing for intersection with the 10,000 triangles (and then reflections, shadow rays, etc…)

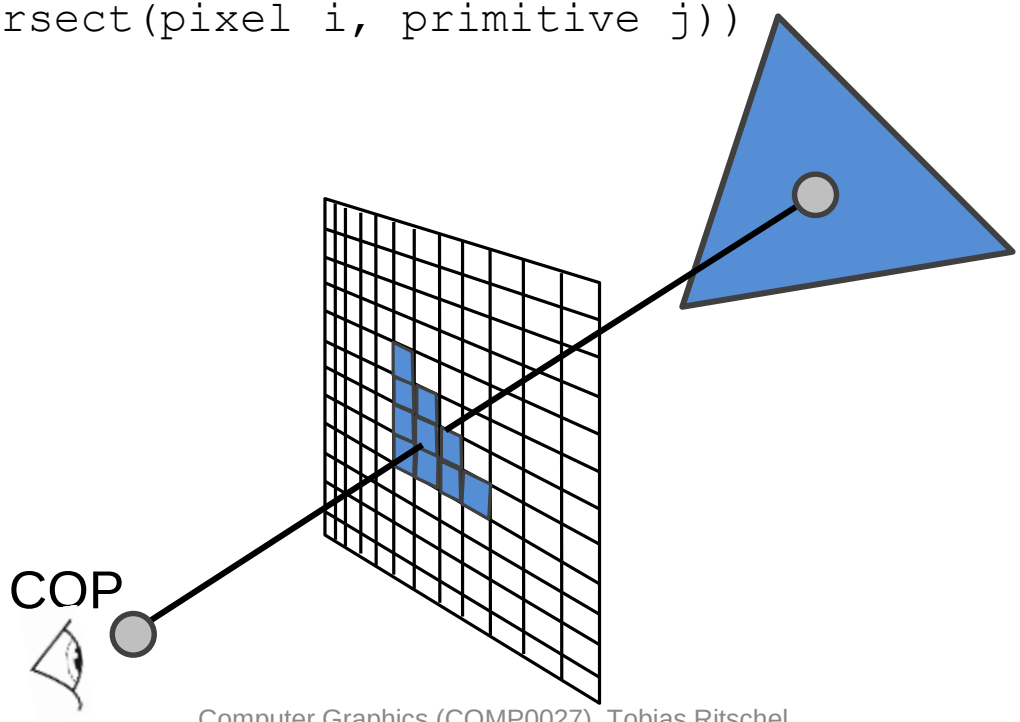
Simplifications from ray-tracing

- Rays only from one or a few specific points
- Only convex polygons
- No global illumination part (i.e. no recursion)



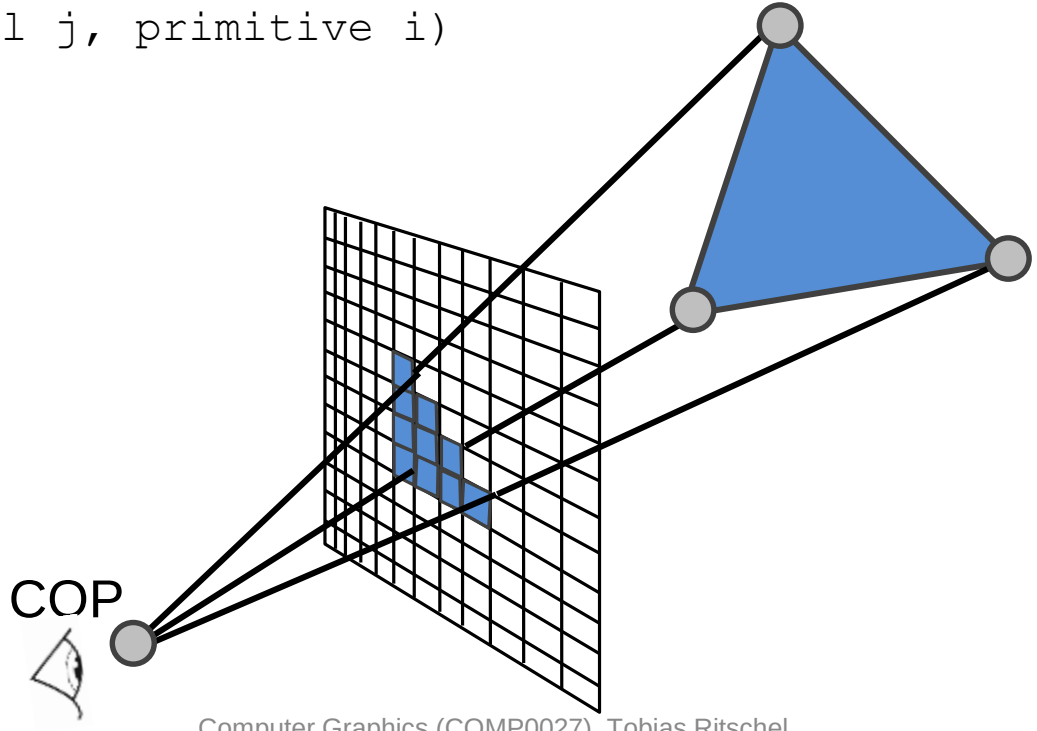
Ray-tracing

```
for all pixel rays i
 for all primitives j
  shade(intersect(pixel i, primitive j))
```

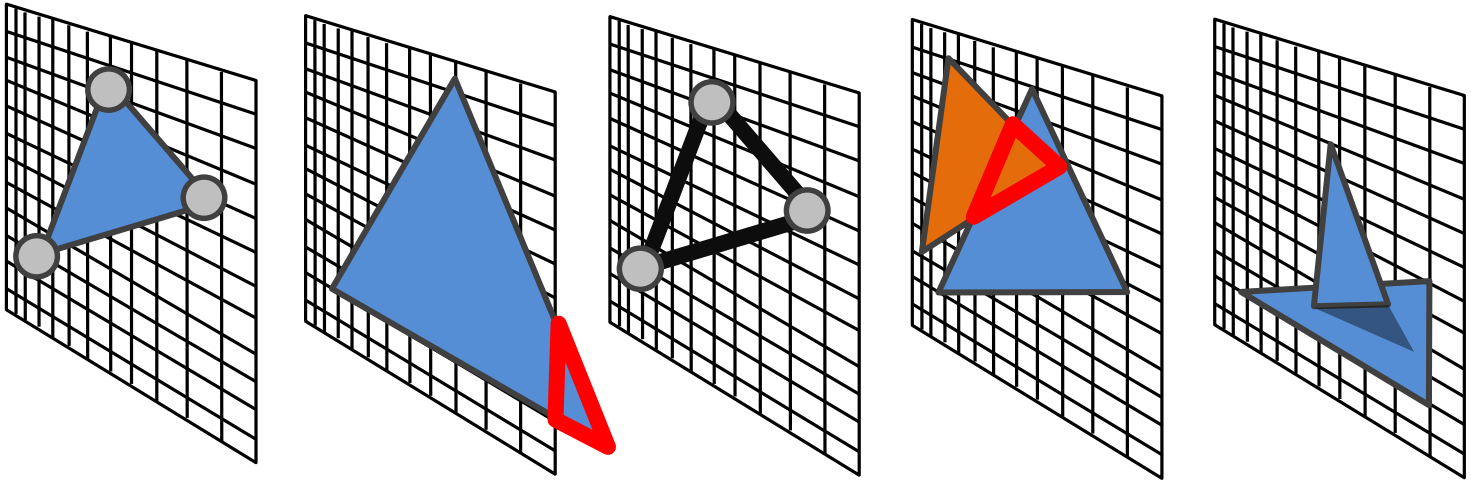


Rasterization

```
for all primitves i
 for all pixels j in projection of primitive i
  shade(pixel j, primitive i)
```



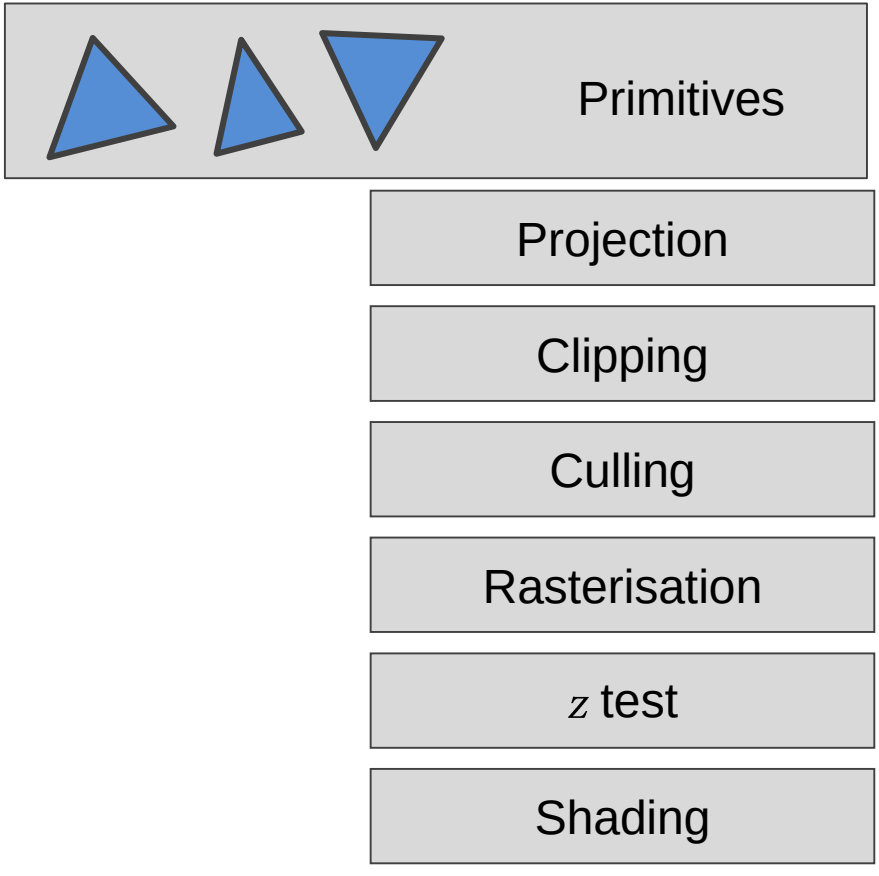
Challenges



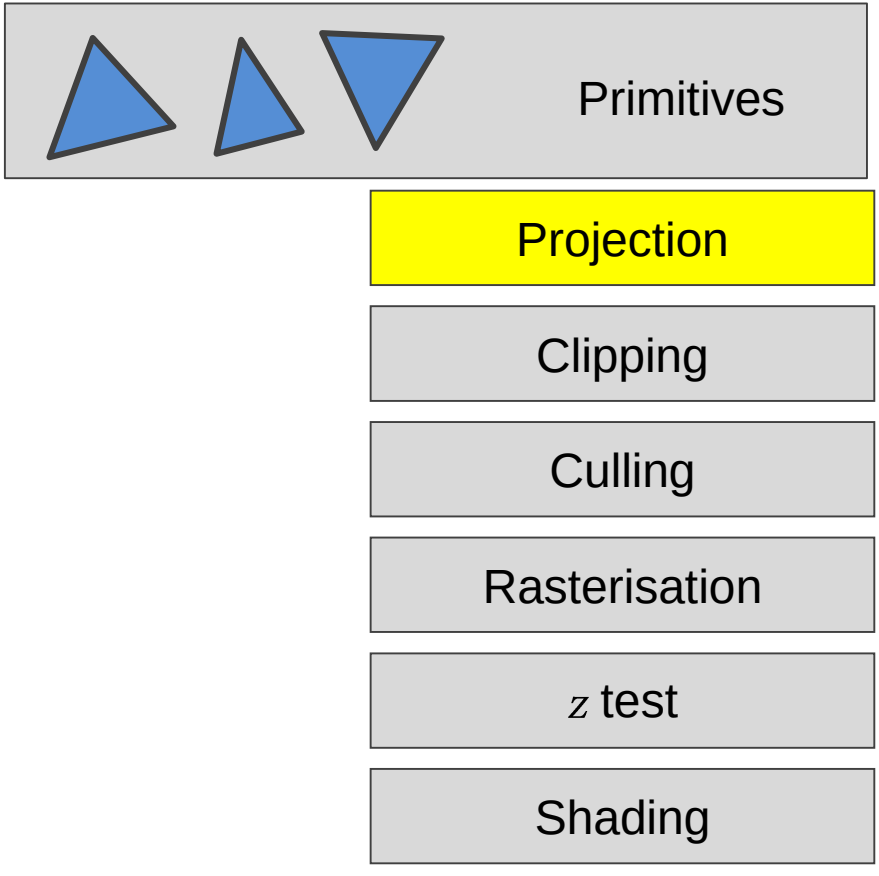
Projection Clipping Rasterization Visibility Shading

z-buffer

Pipeline


Primitives
Projection
Clipping
Culling
Rasterisation
z test
Shading

Pipeline


Primitives
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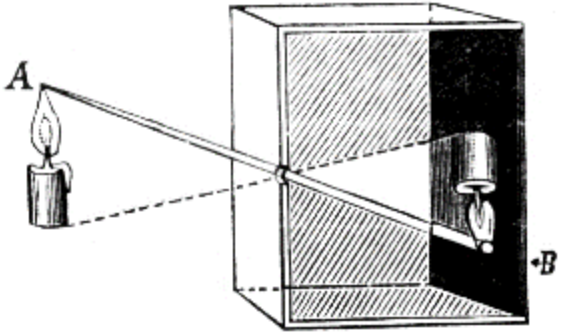
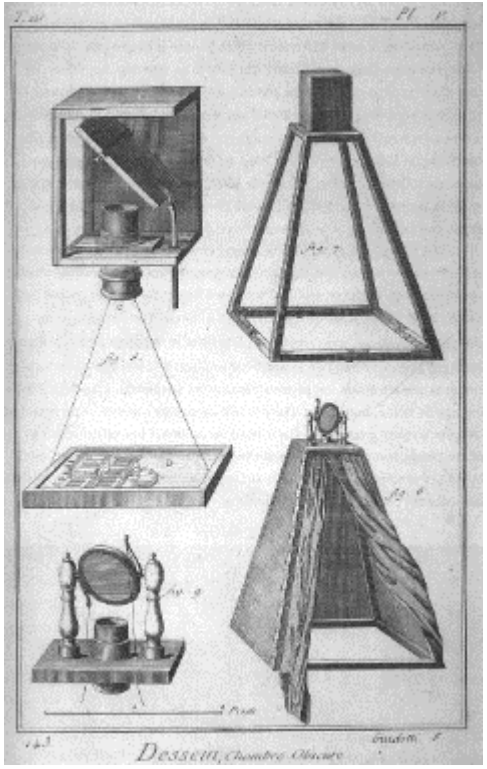
History of projection

- Camera Obscura
  - Earliest form of projection
  - Mo-Ti (470-390 BC)
  - Aristotle (384-322 BC)





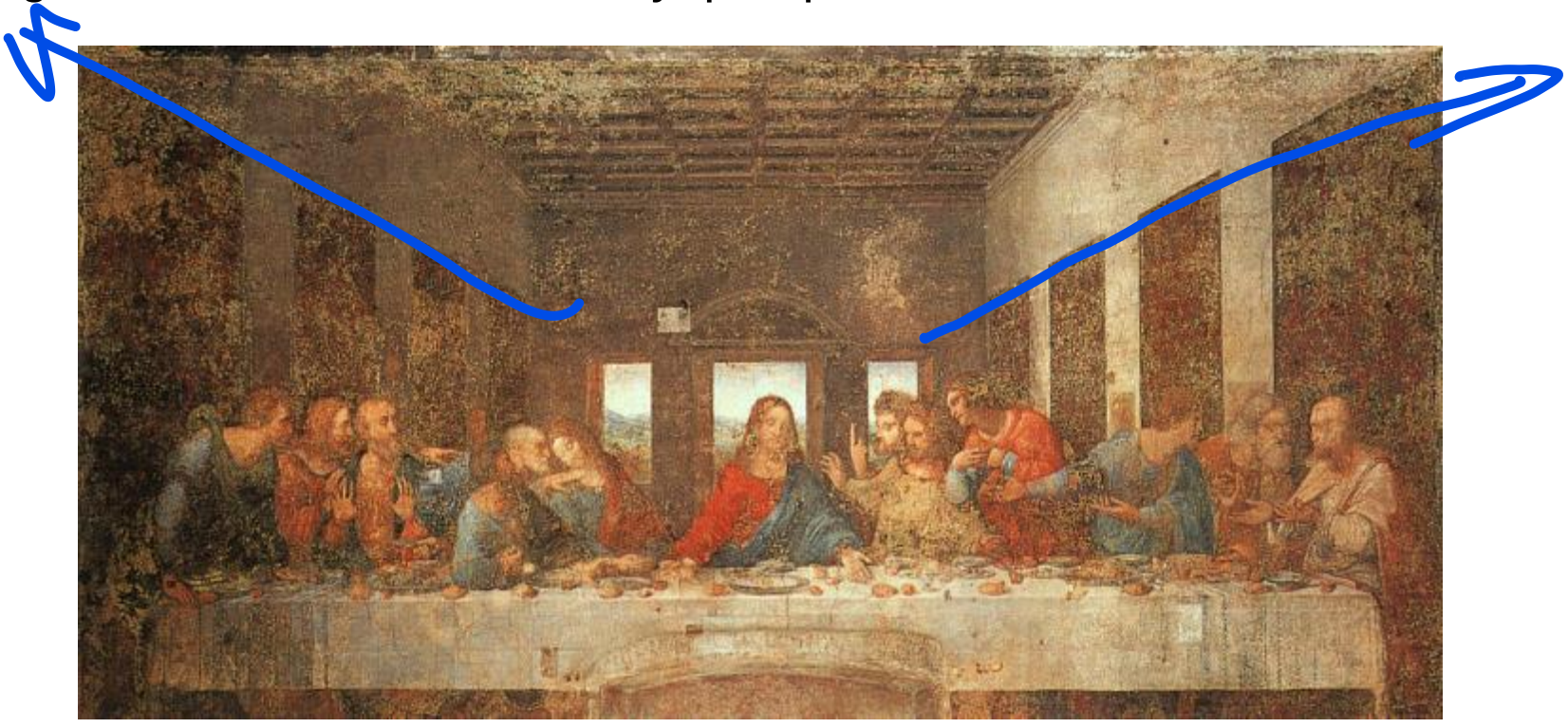
History of Projection

- Ancient Greeks knew the laws of perspective
- Renaissance: perspective is adopted by artists

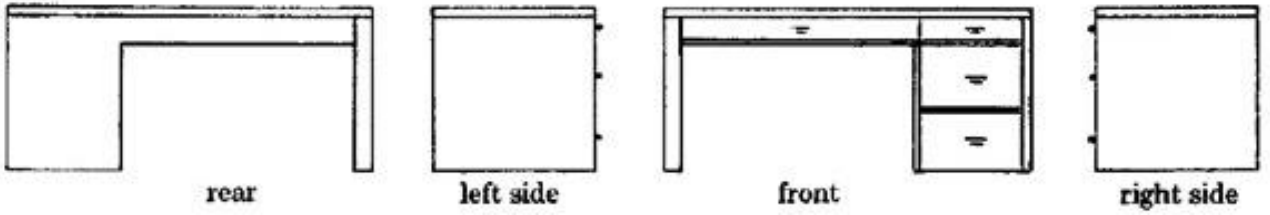
196. From an Apulian calyx-krater. *Würzburg*

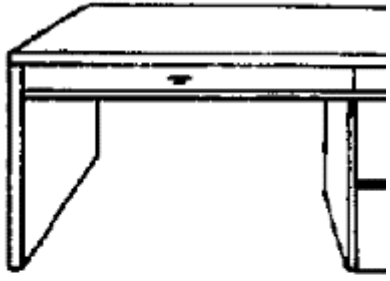
History of Projection

- High Renaissance, 15th century: perspective formalized



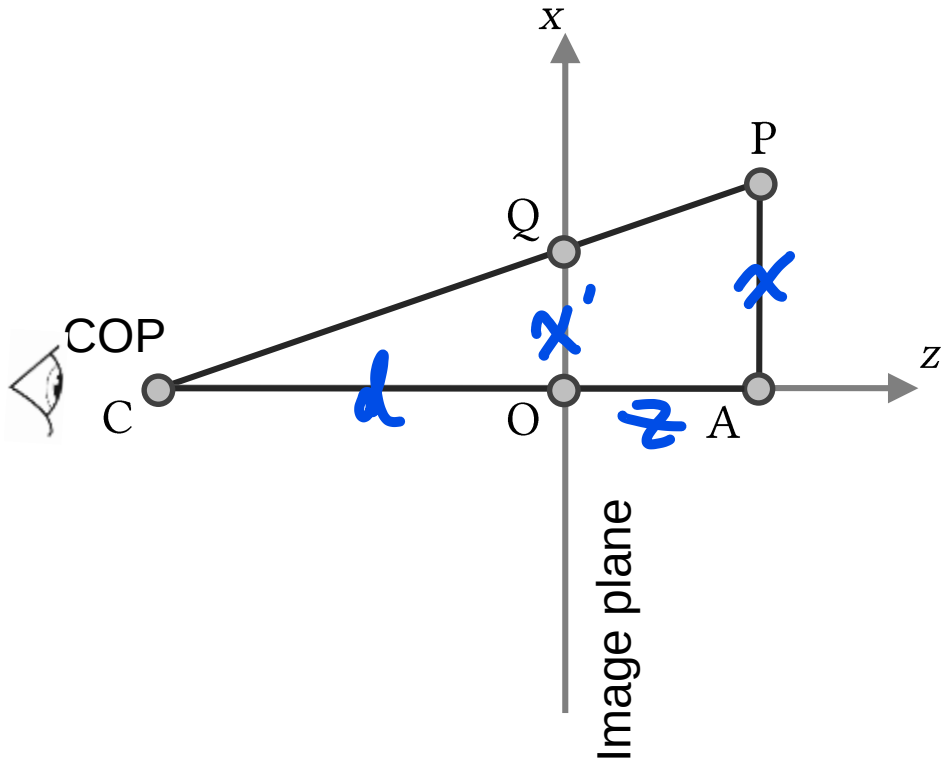
Orthographic vs. Perspective





Perspective projection

Perspective projection



We know:

$$QO/CO = PA/CA$$

For x, we define

$$x' = QO$$
$$x = PA$$
$$d = CO$$
$$z = AO$$

$$x'/d = x / (d + z)$$

so

$$x' = d\,x / (d + z)$$

E.g., for $d = 1$:

$$x' = x / (z + 1)$$

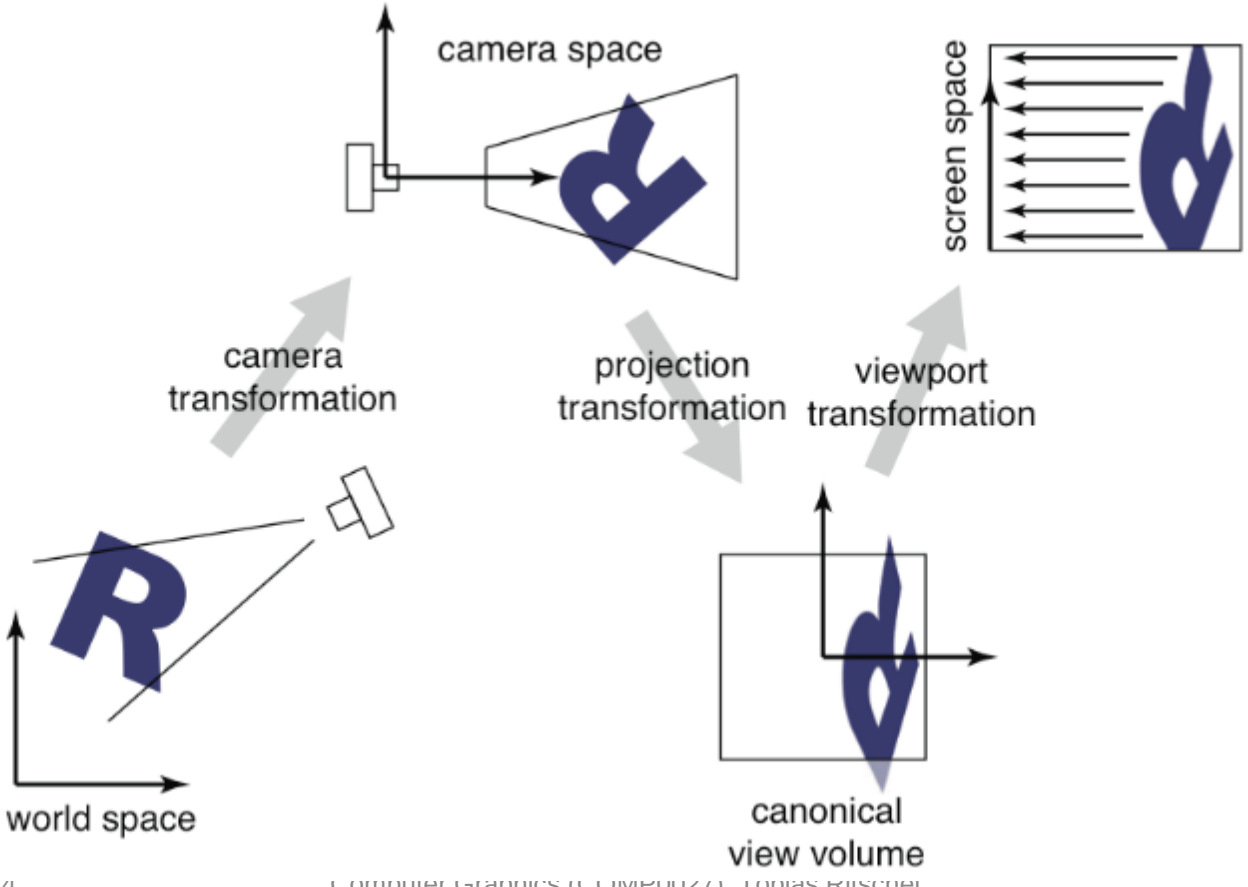
Perspective division

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

- Given a point $(x, y, z, 1)$
- Another good reason for 4x4 matrices
- Its transform is

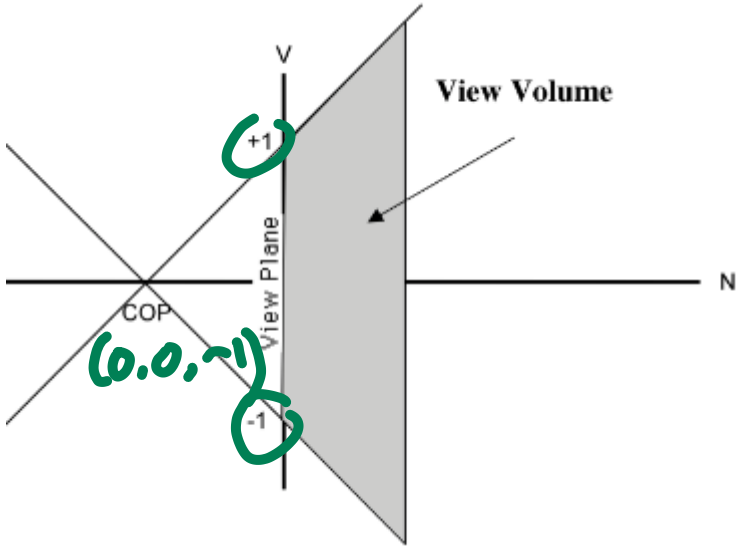
$$\begin{pmatrix} x \\ y \\ z \\ z+1 \end{pmatrix} = \begin{pmatrix} \frac{x}{z+1} \\ \frac{y}{z+1} \\ \frac{z}{z+1} \\ 1 \end{pmatrix}$$

Projection Pipeline



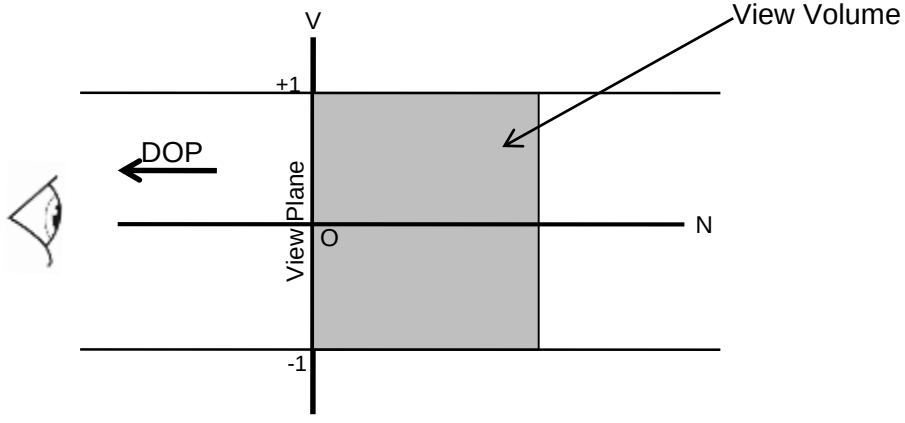
Canonical Frames 典型框架

- We use **canonical frames** as intermediate stages from which we know how to proceed
- Canonical Frame for Perspective Projection:
  - COP at $(0, 0, -1)$
  - View plane coincident with UV plane
  - Viewplane window bounded by -1 to $+1$



Canonical Frame for Parallel Projection

- Orthographic parallel projection
- Direction of projection (DOP) is $(0,0,-1)$
- View volume bounded by -1 and $+1$ on U and V
- And by 0 and 1 on the N axis
- $p' = (x, y, 0)$

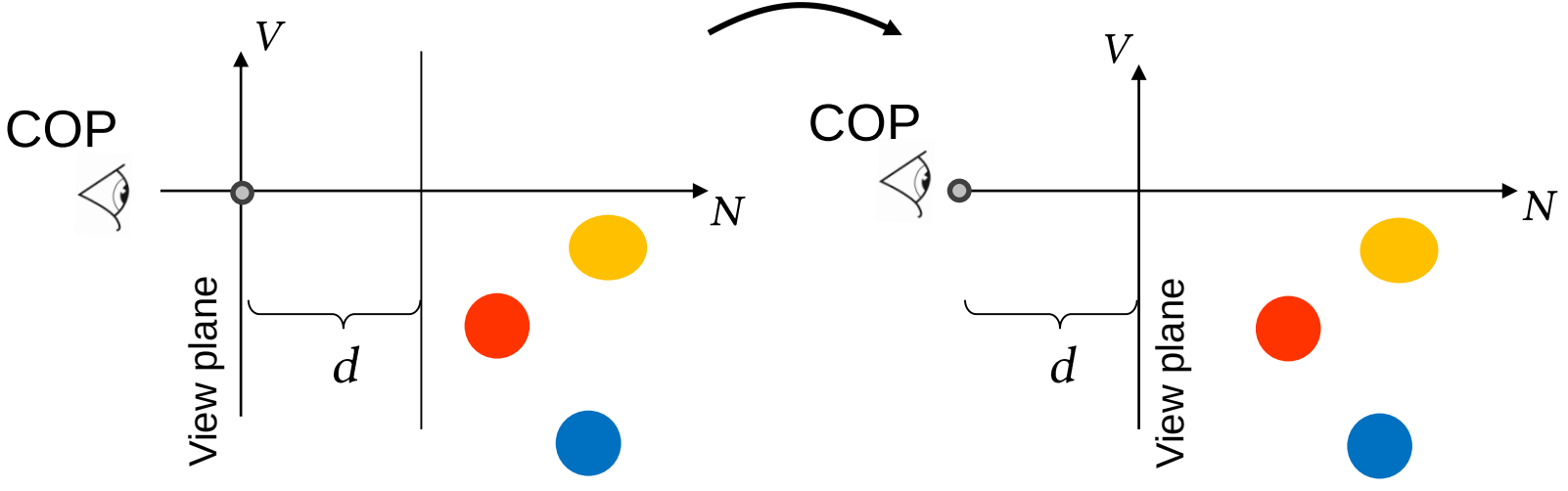


General Persp. to Canonical Persp.

- We will apply **four** transformation matrices
- Each “corrects” one aspect of the projection
- Finally, we multiply them all together

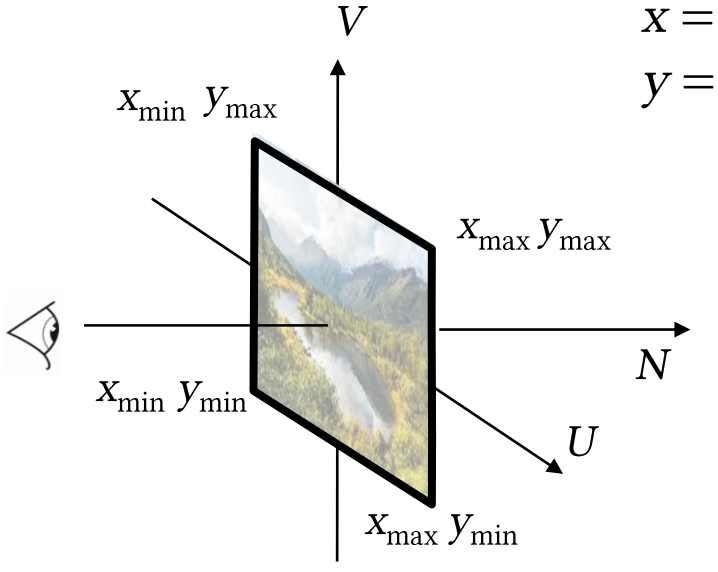
Step 1: Move to UV plane

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Recall the window

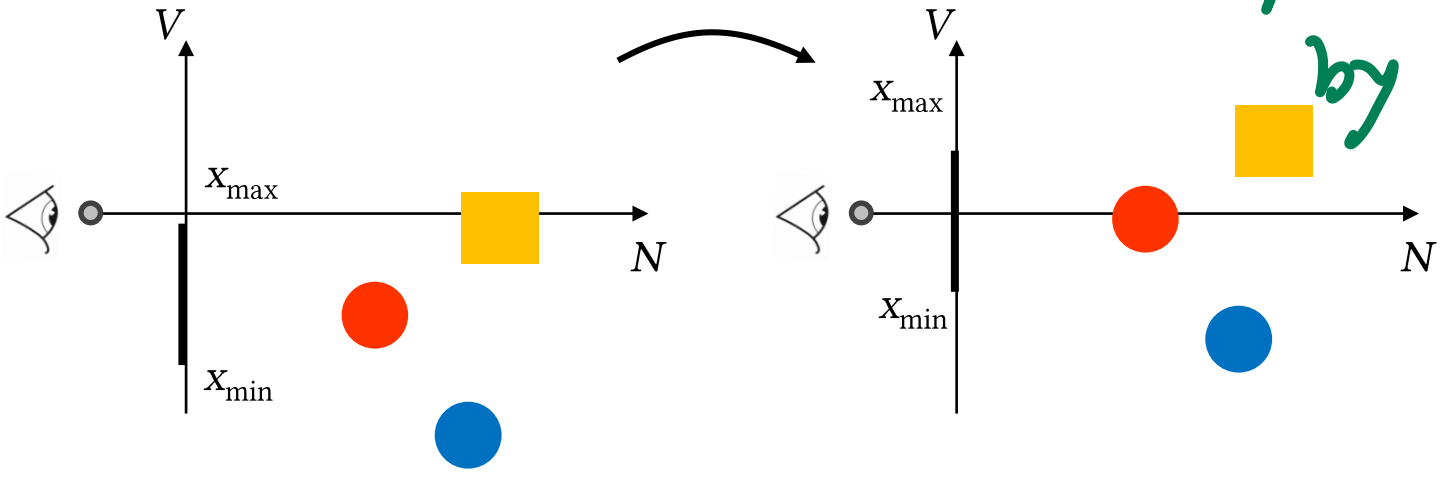
$$w = x_{\max} - x_{\min}$$
$$h = y_{\max} - y_{\min}$$
$$x = x_{\max} + x_{\min}$$
$$y = y_{\max} + y_{\min}$$



Step 2: Regular pyramid

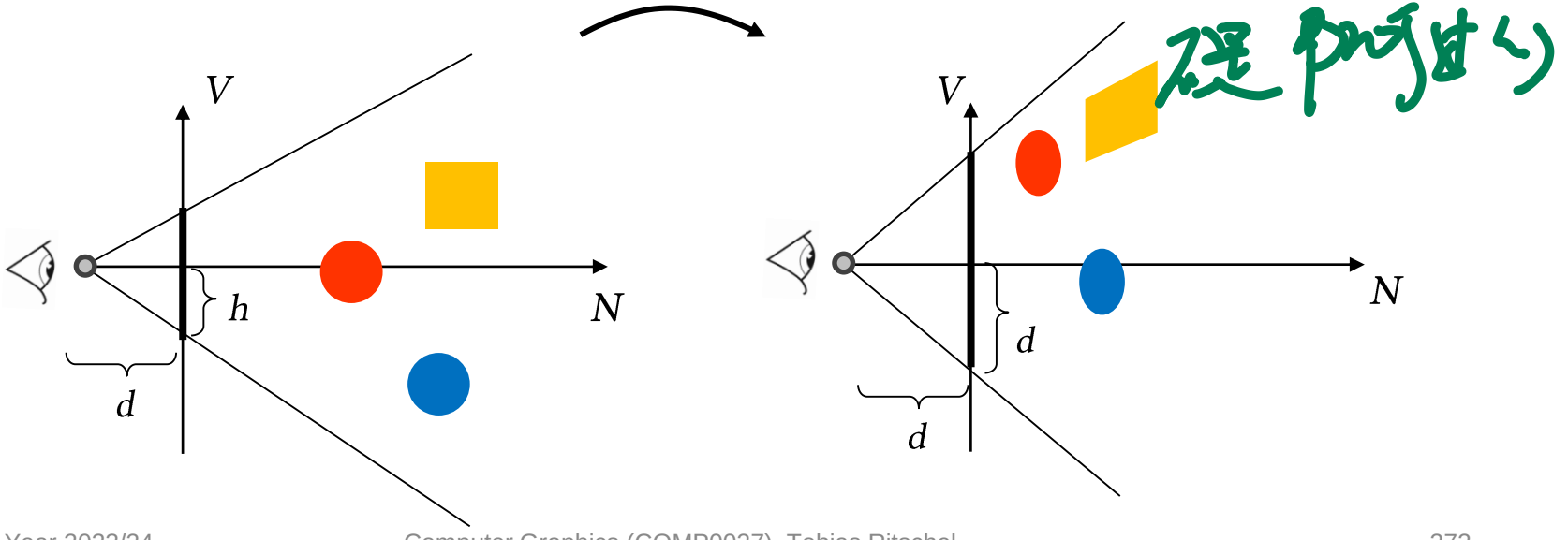
$$\begin{pmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$w = x_{\max} - x_{\min}$$
$$h = y_{\max} - y_{\min}$$
$$x = x_{\max} + x_{\min}$$
$$y = y_{\max} + y_{\min}$$

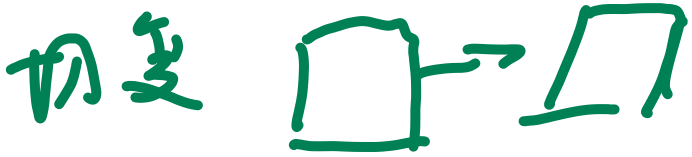


Step 3: Regular pyramid

This is not uniform scale: z did not change!

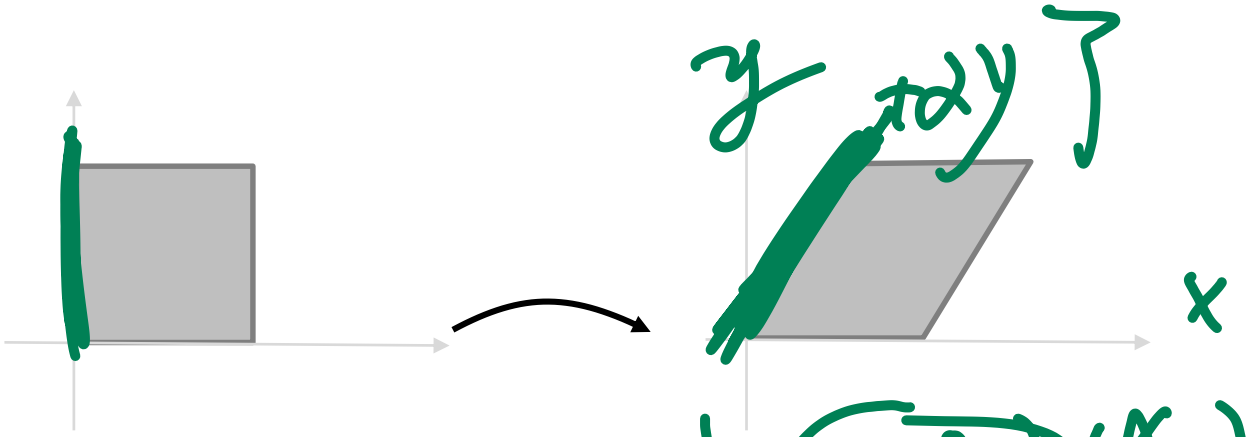


Shear



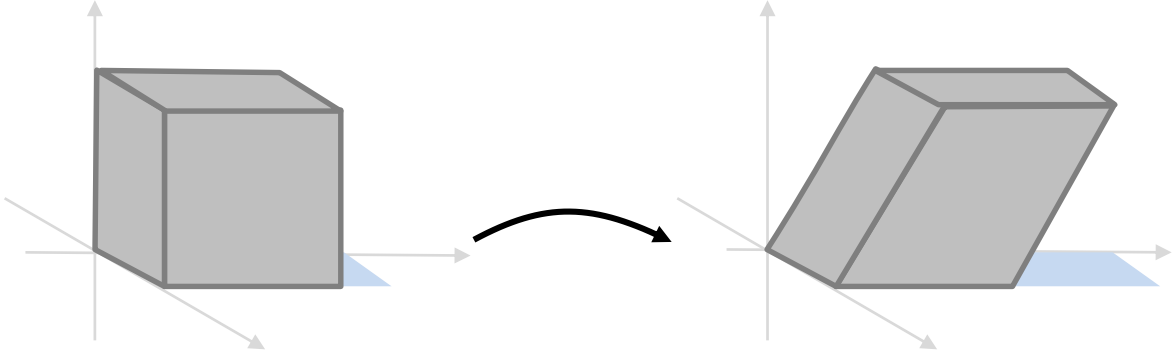
- This needs to be a **shear**
- If you want a regular pyramid to become a box ..
- A box has to become some other regular pyramid
- What is shear?

Interlude: Shear 2D



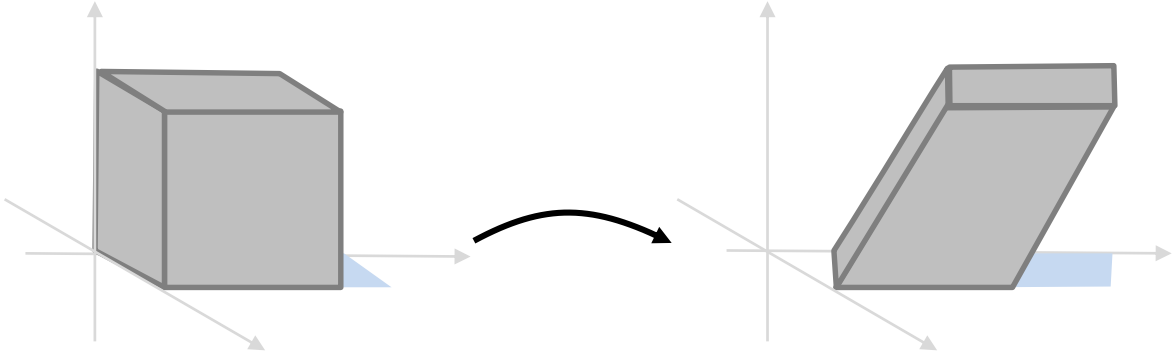
$$\begin{pmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Interlude: Shear 3D



$$\begin{pmatrix} 1 & a & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Interlude: Shear 3D

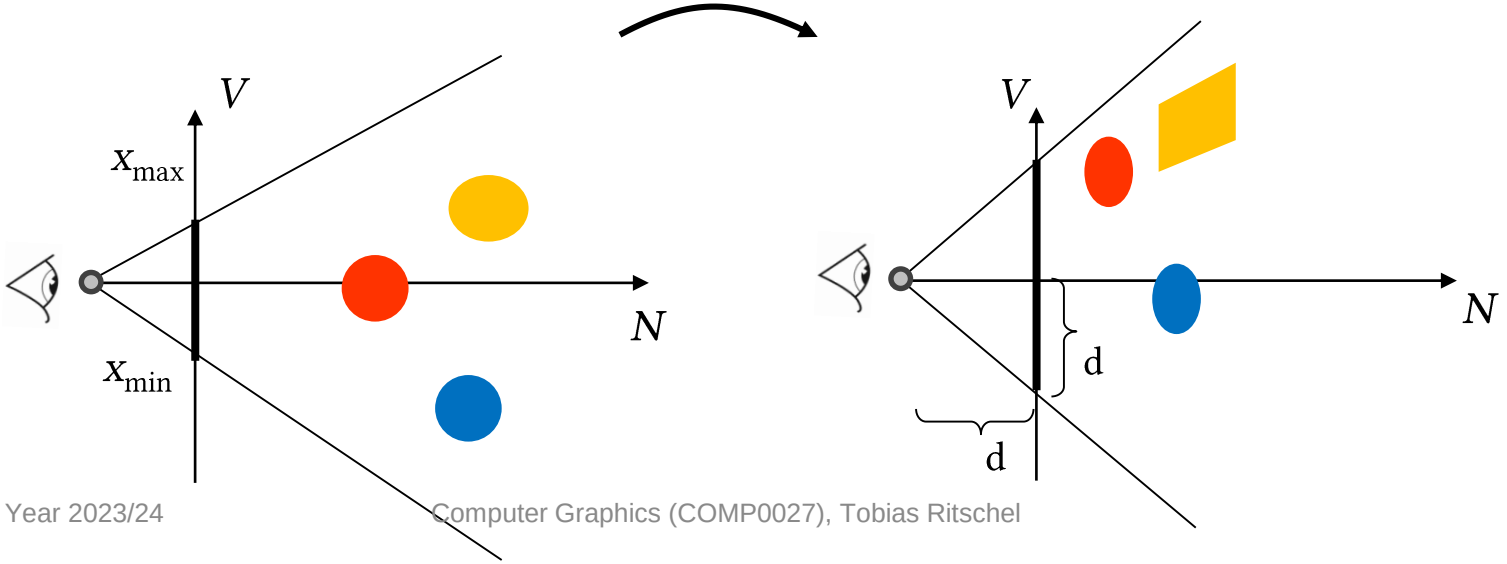


$$\begin{pmatrix} 1 & a & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Step 3: Regular pyramid

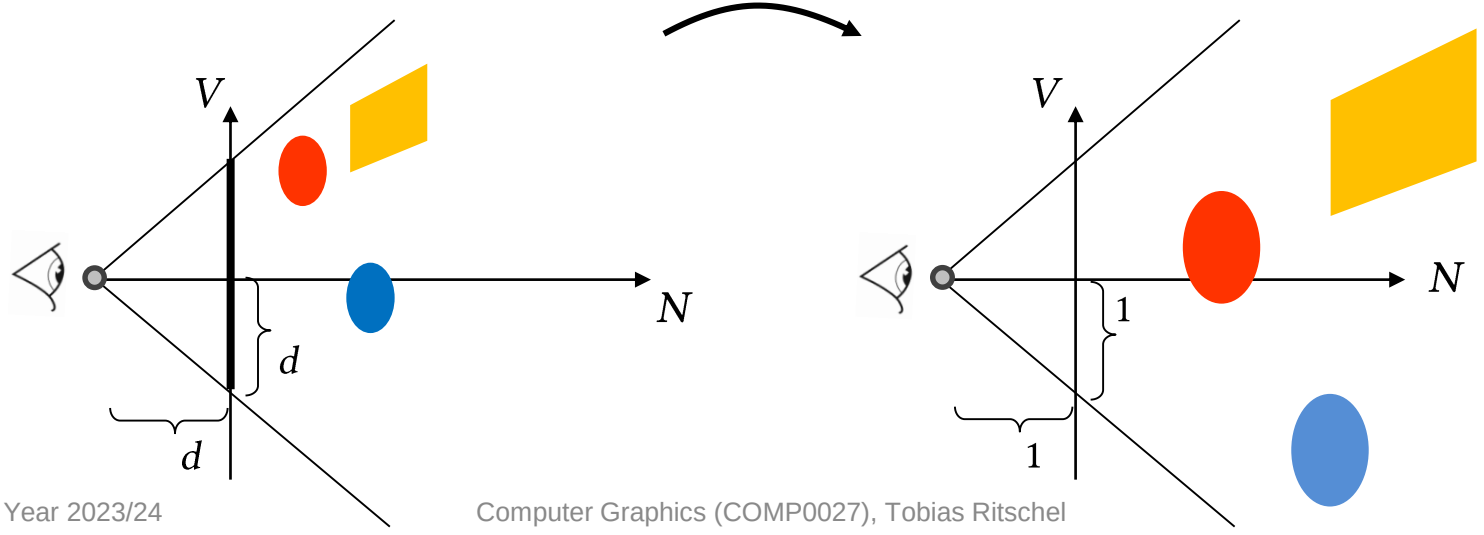
$$\begin{pmatrix} d/w & 0/w & -x/w & 0/w \\ 0/h & d/h & -y/h & 0/h \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$w = x_{\max} - x_{\min}$$
$$h = y_{\max} - y_{\min}$$
$$x = x_{\max} + x_{\min}$$
$$y = y_{\max} + y_{\min}$$



Step 4: Scale by $1/d$

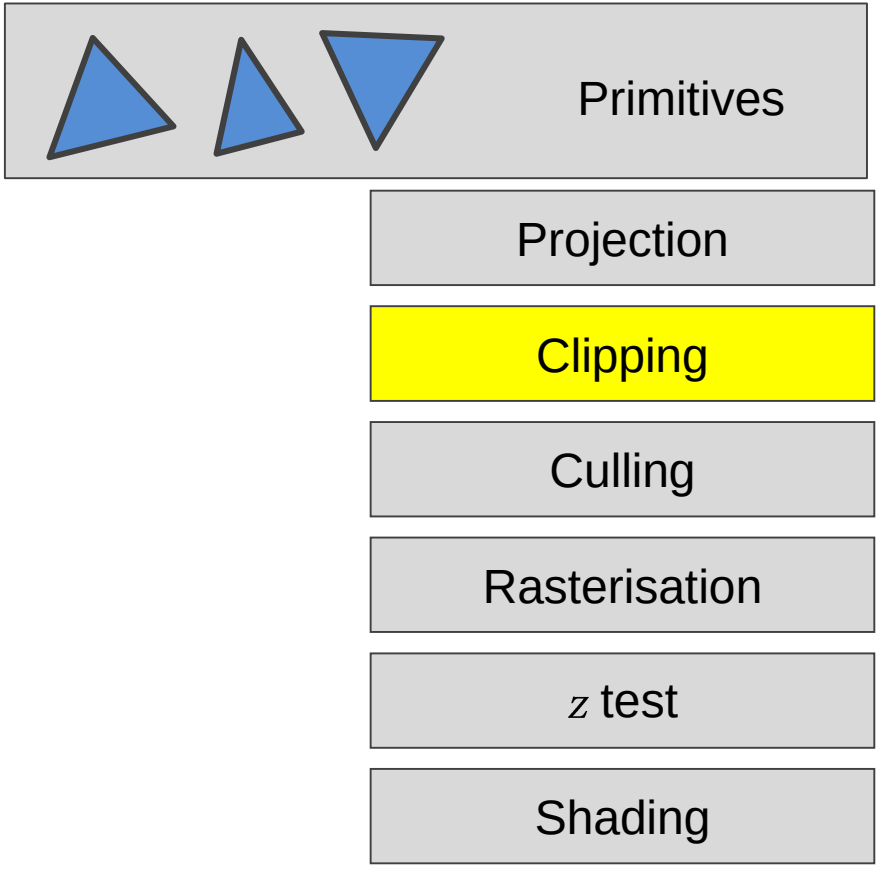
$$\begin{pmatrix} 1/d & 0 & 0 & 0 \\ 0 & 1/d & 0 & 0 \\ 0 & 0 & 1/d & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Two simplifications

1. The COP might not fall on the z axis
   - In our model it does
   - Required in professional photography
   - Required in virtual reality (Stereo, HMDs)
2. Might want to re-scale z to fit a range
   - Will get back to this adjustment later

Pipeline


Primitives
Projection
Clipping
Culling
Rasterisation
z test
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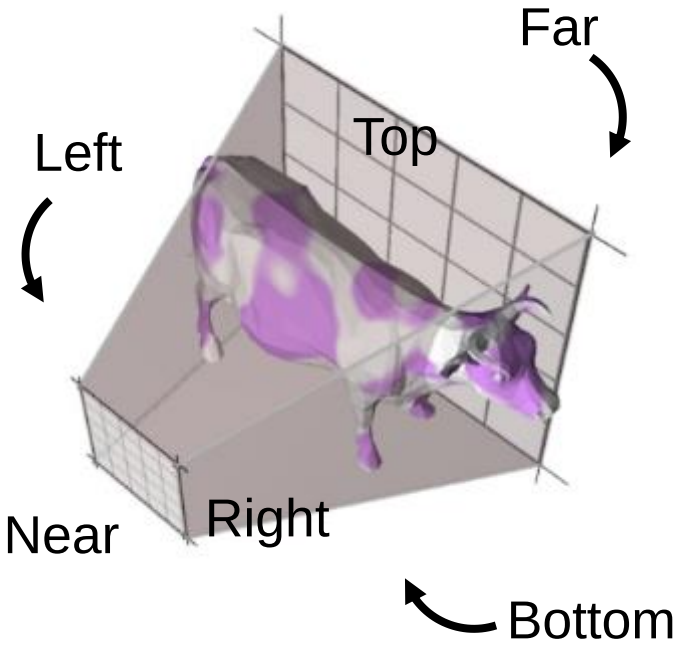
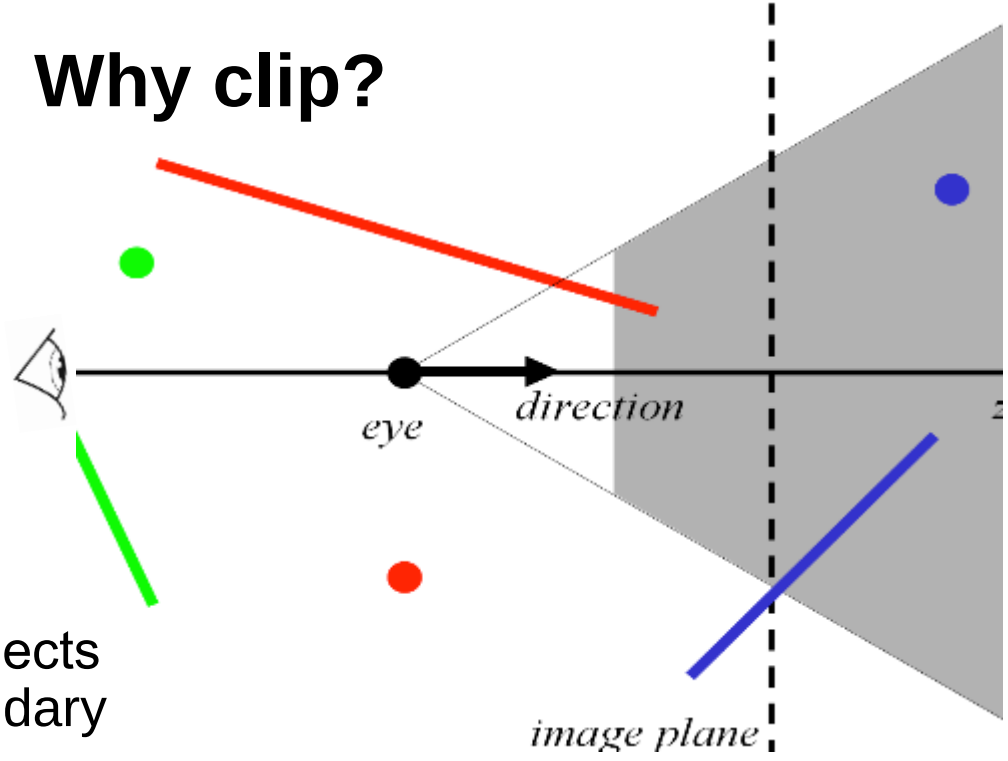
Clipping

- Eliminate portions of objects outside the viewing frustum
- View frustum
  - Boundaries of the image plane projected in 3D
  - A near & far clipping plane
- User may define additional clipping planes



Why clip?



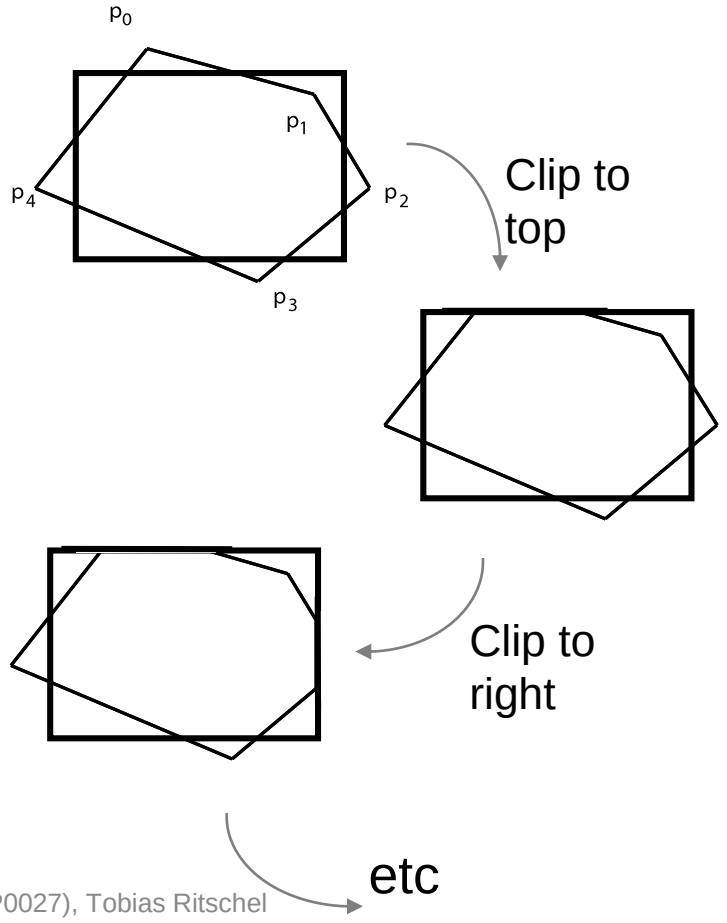
- Avoid degeneracies
  - Don't draw stuff behind the eye
  - Avoid division by 0 and overflow
- Efficiency
  - Don't waste time on objects outside the image boundary
- Other graphics applications (often non-convex)
  - Hidden-surface removal, Shadows, Picking, Binning, CSG (Boolean) operations (2D & 3D)

Clipping Summary

- It’s the process of finding the exact part of a polygon lying inside the view volume
- To maintain consistency, clipping of a polygon should result in a polygon, not a sequence of partially unconnected lines
- We will first look at two different 2D solutions and then extend one to 3D

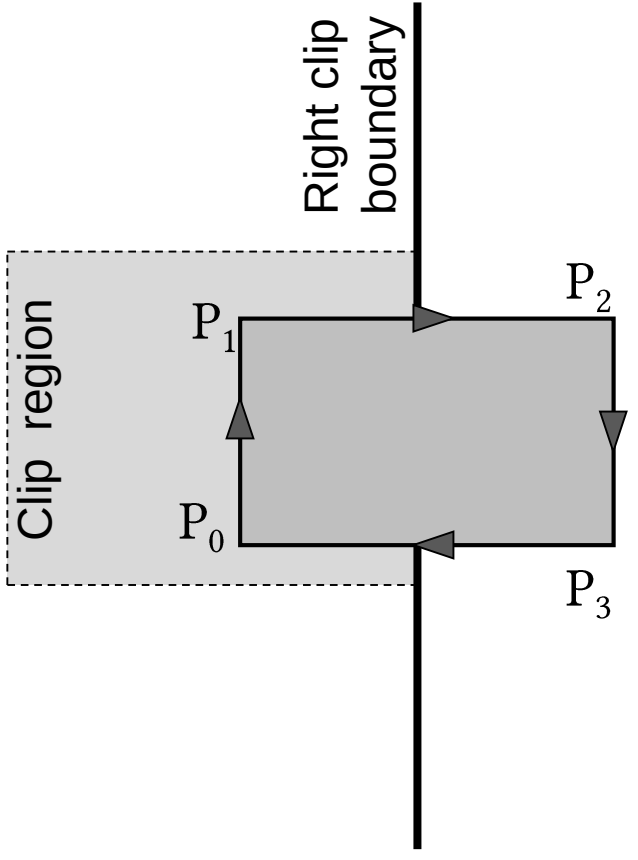
Sutherland-Hodgman Algorithm

- Clip the polygon against each boundary of the clip region successively
- Result is possibly NULL if polygon is outside
- Can be generalised to work for any polygonal clip region, not just rectangular



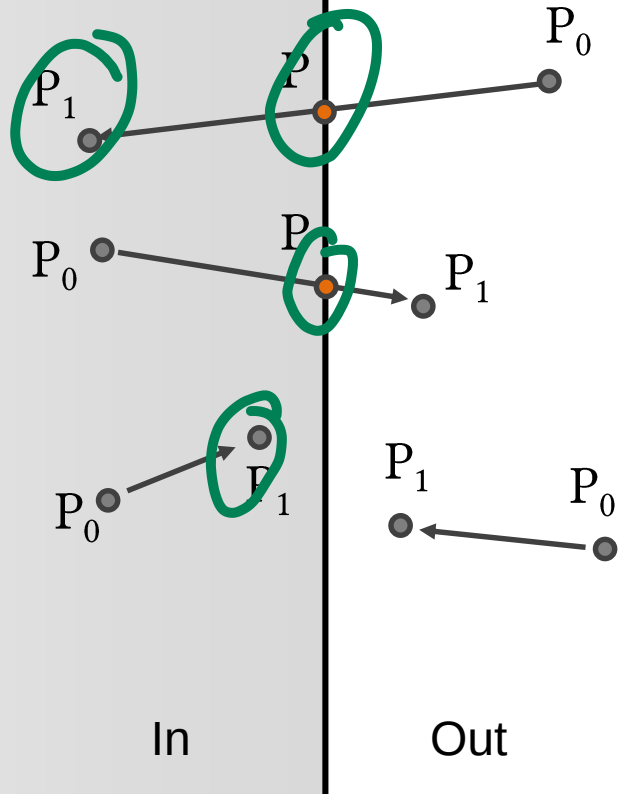
Clipping To A Region

- To find the new polygon
  - iterate through each of the polygon edges and construct a new sequence of points
  - starting with an empty sequence
  - for each edge there are 4 possible cases to consider



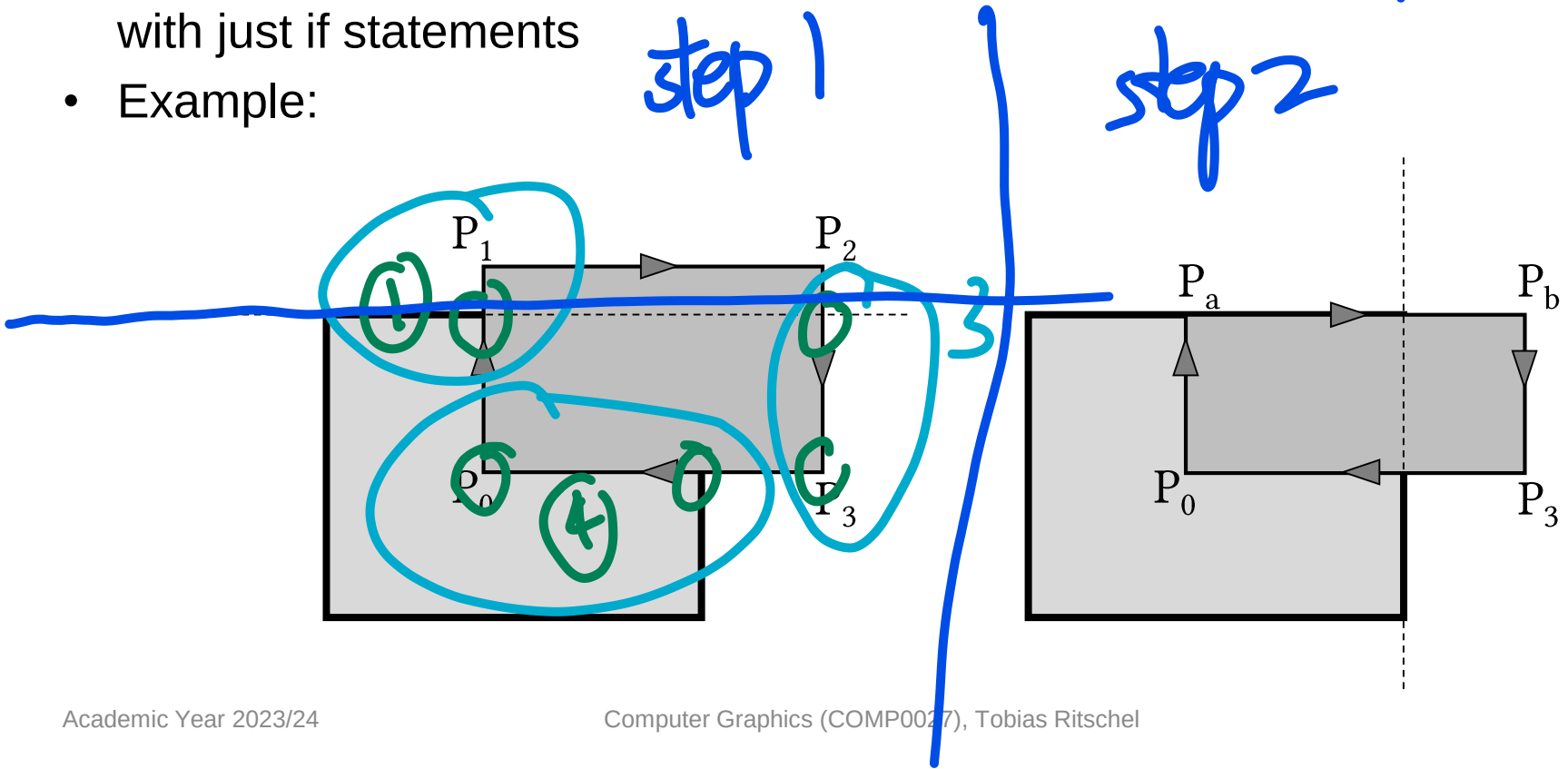
Clipping polygon edge against boundary

- Given an edge P_0, P_1 we have 4 case. Can …
  - enter the clip region, add P and P_1
  - leave the region, add only P
  - be entirely outside, do nothing
  - be entirely inside, add only P_1
- Where P is the point of intersection



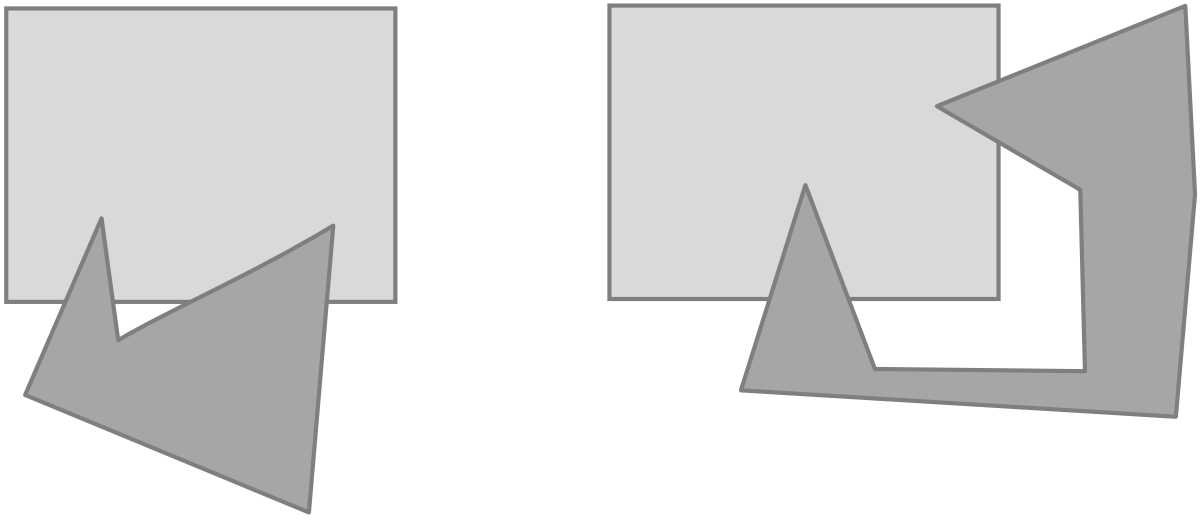
Still the Sutherland-Hodgman

- We can determine which of the 4 cases and also the point of intersection with just if statements
- Example:



Sutherland-Hodgman Problem

- Clipping is an example where convex polys make our lives easier
- Concave: Weiler-Atherton algorithm

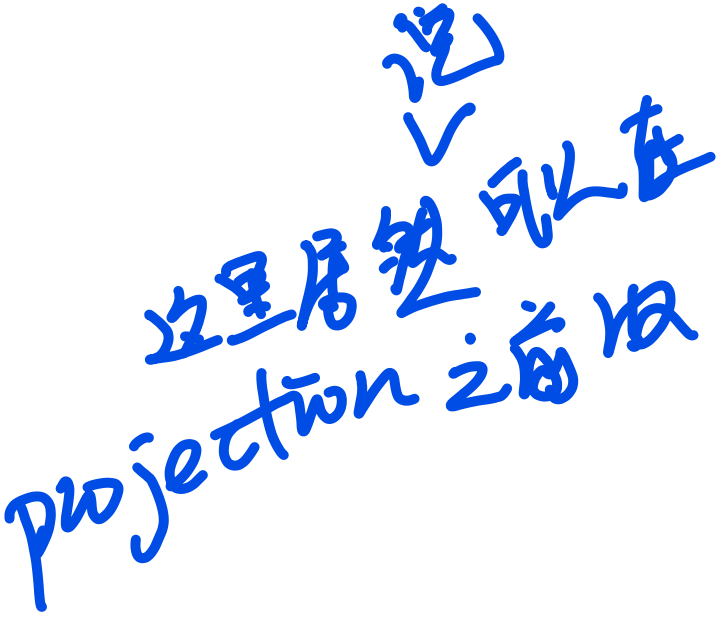


Clipping Polygons in 3D

- The Sutherland-Hodgman can easily be extended to 3D
  - Clipping boundaries are 6 planes instead of 4 lines
  - Intersection calculation is done by comparing an edge to a plane instead of edge to edge

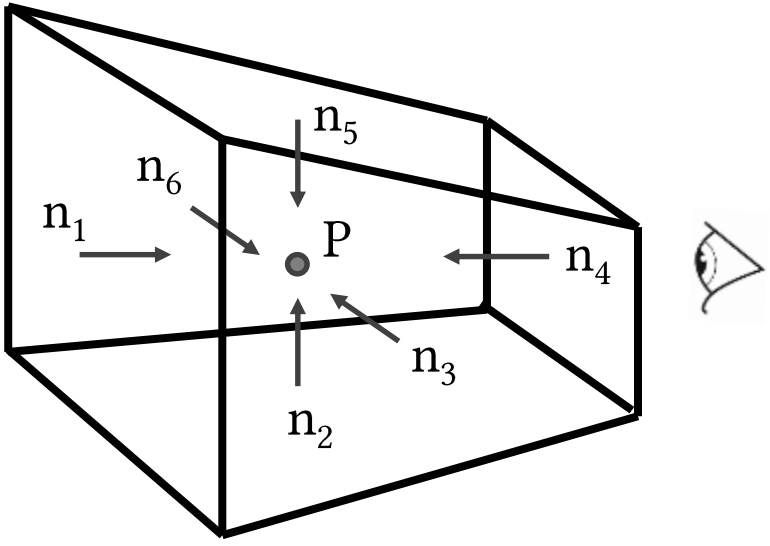
When to clip?

- Before perspective transform in 3D space
  - Use the equation of 6 planes
  - Natural, not too degenerate
- In homogeneous coordinates after perspective transform (Clip space)
  - **Before** perspective divide (4D space, weird w values)
  - Canonical, independent of camera
  - The simplest to implement in fact
- In the transformed 3D screen space after perspective division (canonical par.)
  - Problem: objects in the plane of the camera



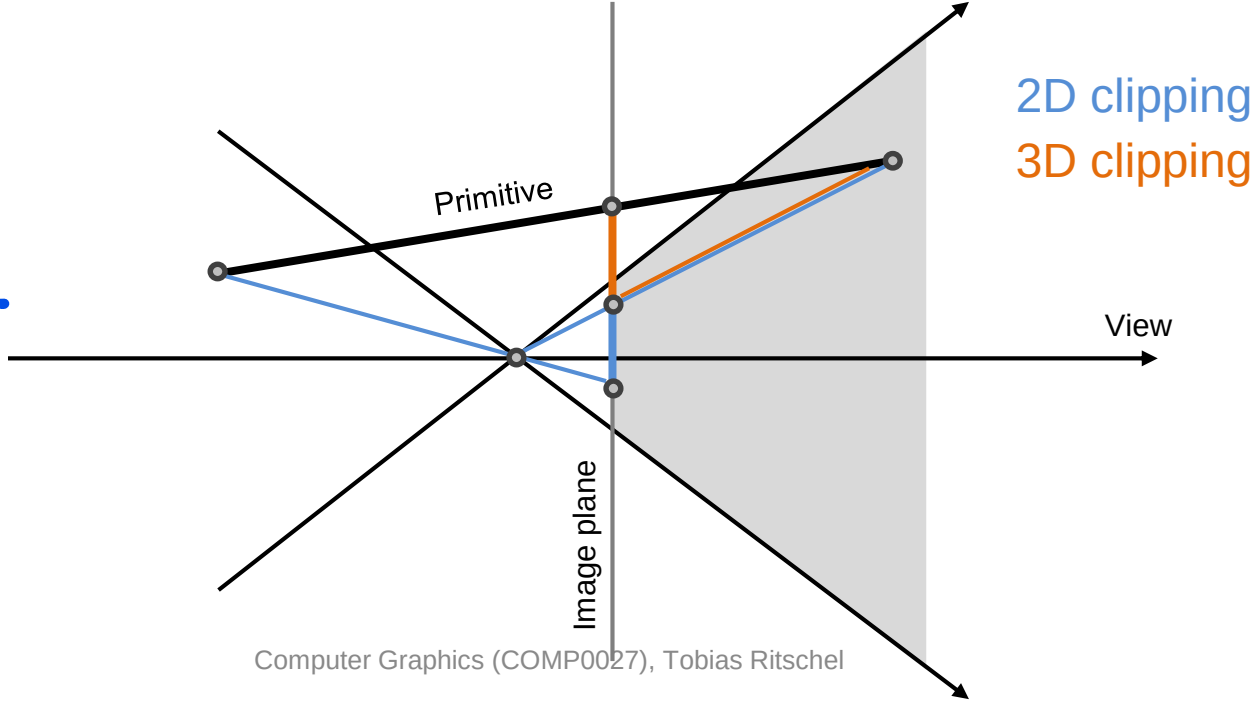
Clipping with respect to View Frustum

- Test against each of the 6 planes
  - Normals oriented towards the interior
- Clip / cull / reject point P if any $<\mathbf{n}_i, \mathbf{p}> < d_i$



Clipping After Projecting

- When we have an edge that extends from the front to behind the COP, then if we clip after projection (which in effect is what e.g. the PS 1 did) we might get wrong results



A note on implementation

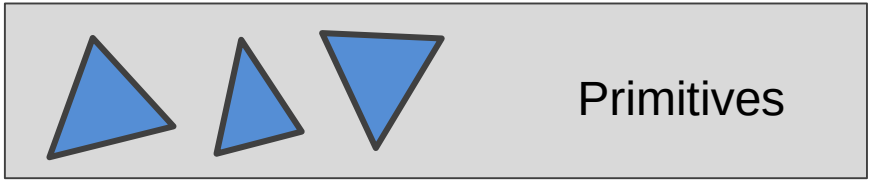
- Vanilla implementation uses dynamic data structure

```
iterate windowVertices edges {
 iterate primitiveVertices
  if(…)
   primitiveVertcies.add(new vertex)
 if(…)
   primitiveVertcies.remove(new vertex)
}}
```

- Better, without

```
for MAX_EDGES edges {
 for MAX_CLIPPED_VERTICES {
  if(…)
   primitiveVertcies[vertexCount++](new vertex)
 if(…)
   vertexCount++
}}
```

Pipeline



Primitives

Projection

Clipping

Culling

Rasterisation

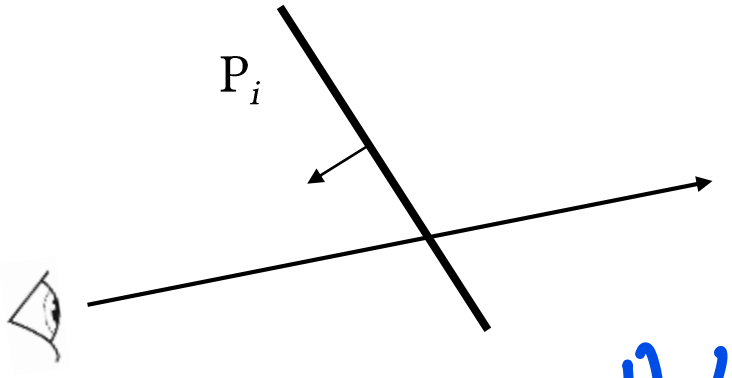
z test

Shading

(optional)

Idea

- Optional speed-up
- Remove primitives P_i not facing the camera
- Polygons whose normal does not face the viewpoint, are not rendered



Back Face Culling

- Thus if we have the plane equation

$$l(x, y, z) = ax + by + cz - d = 0$$

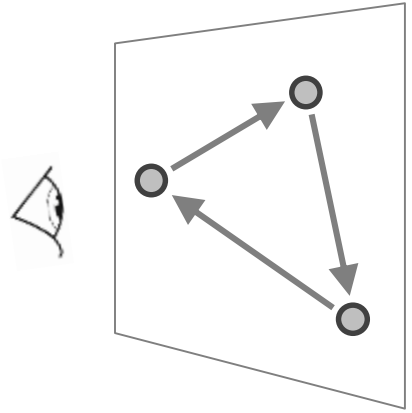
- and the COP (c_x, c_y, c_z), then

$$l(c_x, c_y, c_z) > 0$$

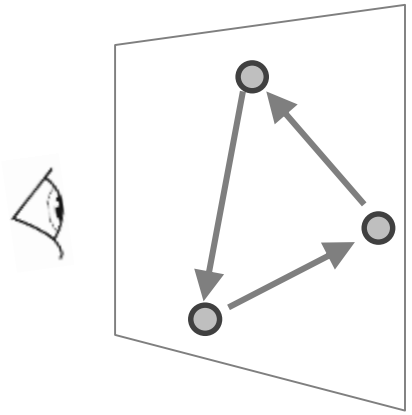
- if the COP is in front of the polygon
- Otherwise: cull!

Clockwise/counter-clockwise

Typically done without normals: All polygons that are not counter-clockwise after projection are culled



Clockwise: Culled



Counter clockwise: Not culled

Limitation

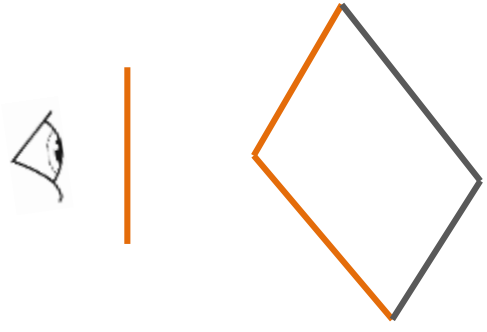
Does not ensure correct visible surfaces determination, unless there is only a single convex object

Convex: Works

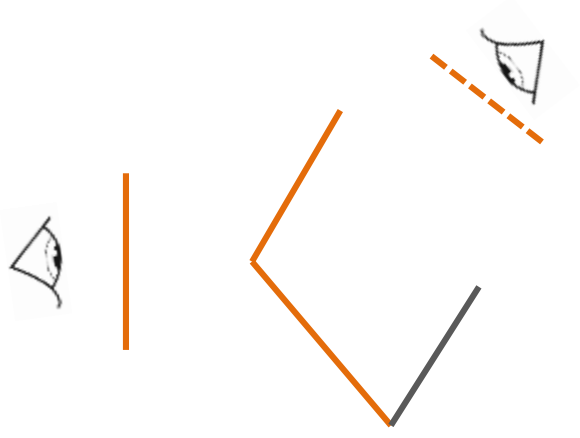
Concave: Does not work

Limitation

Only reliable if backsides are never required. Therefore, does not work for non-closed polytopes.



Closed polytop: Works

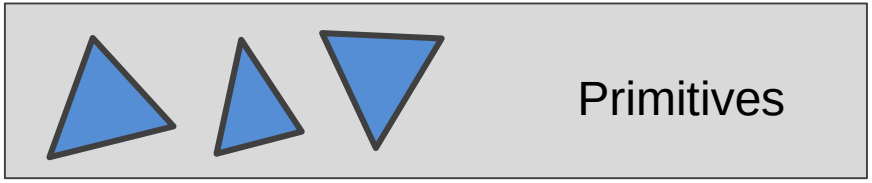


Non-closed polytop: Does not work

Culling recap

- Culling can easily & quickly remove (some!) invisible polygons from the pipeline
- Still some (partially) invisible ones remain

Pipeline



Primitives

Projection

Clipping

Culling

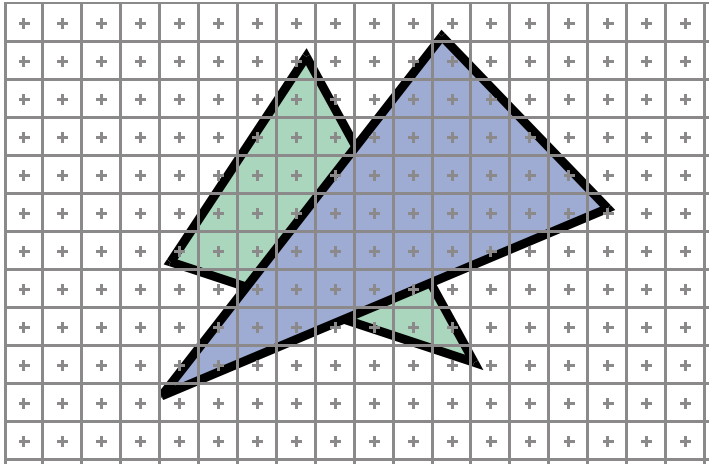
Rasterisation

z test

Shading

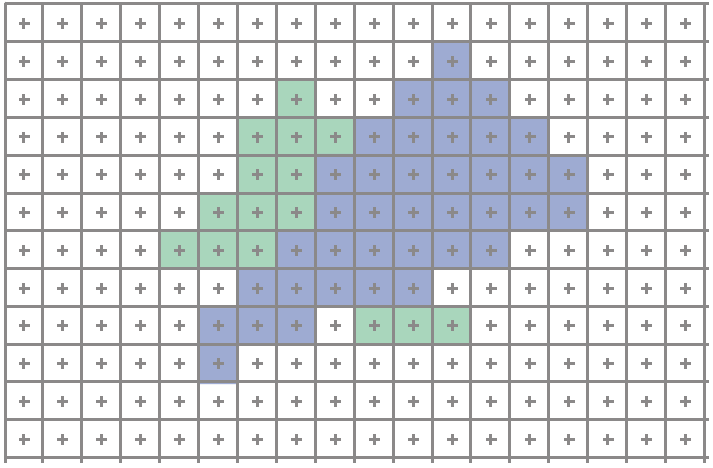
2D Scan Conversion

- Primitives are continuous; screen is discrete
  - Triangles defined by discrete set of vertices
  - But they define a continuous area on screen



2D Scan Conversion

- Solution: compute discrete approximation
- Scan Conversion (Rasterization):
  algorithms for efficient generation of the samples comprising this approximation



Naïve Filling Algorithm

- Find a point inside the polygon
- Do a **flood fill:**
  - Keep a stack of points to be tested
  - When the stack is not empty
    - Pop the top point (Q)
    - Test if Q is inside or outside
      - If Inside, colour Q, push neighbours of Q if not already tested
      - If outside discard
    - Mark Q as tested

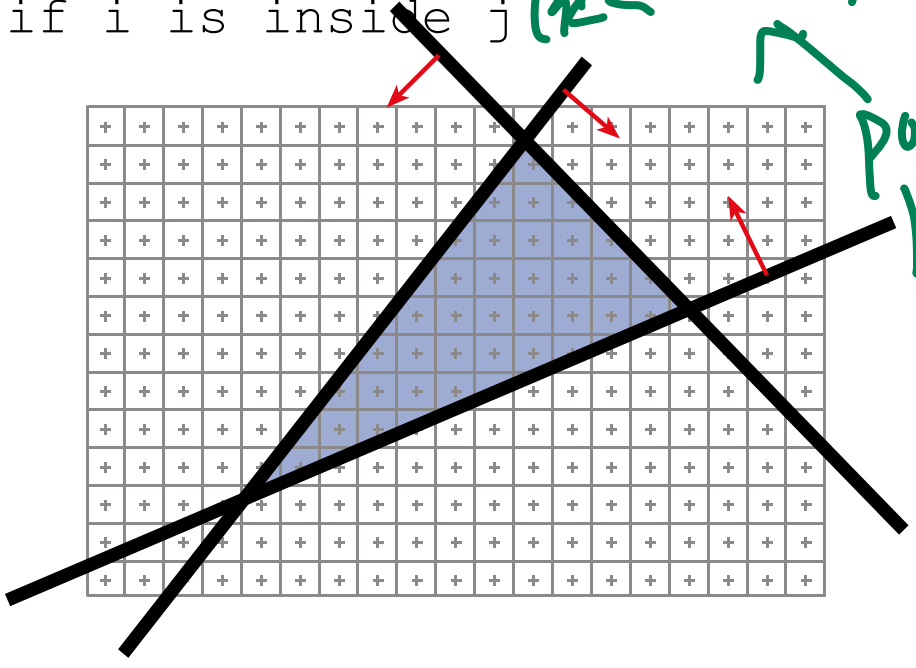
2 BFS

Critique 评论

- Horribly slow
  - Explicit in/out test at every point
  - But still very common in paint packages!
  - Only solid color
- Stack might be very deep
- Want to do this
  - Without much memory
  - In parallel

Brute Force Solution for Triangles

```
For all pixels i
  For all edges j
    Test if i is inside j
```



Half-Space Test Reminder

- For each edge compute line equation (analogue to plane equation):

$$L_i(x, y) = a_i x + b_i y + c_i$$

- If $L_i(x, y) > 0$ point in **positive** half-space
- If $L_i(x, y) < 0$ point in **negative** half-space
- If all $L_{1,2,3}(x, y) \geq 0$ point inside triangle

Half-Space Test Reminder

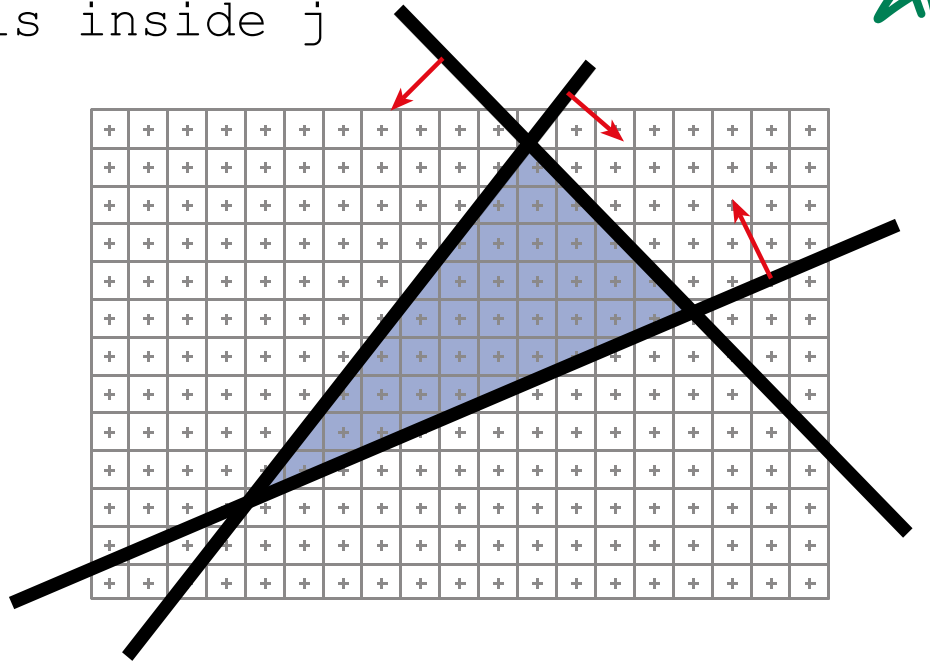
- For each edge compute line equation (analogue to plane equation):

$$L_i(x, y) = a_i x + b_i y + c_i$$

- Example:
  - Assuming $(2, 2)$ to $(4, 7)$,
  - The slope is $\mathrm{d}y/\mathrm{d}x = 5/2 = 2.5$
  - Hence $y - 7 = 2.5(x - 4)$,
  - So $L_1(x, y) = 2.5x - y - 3.$

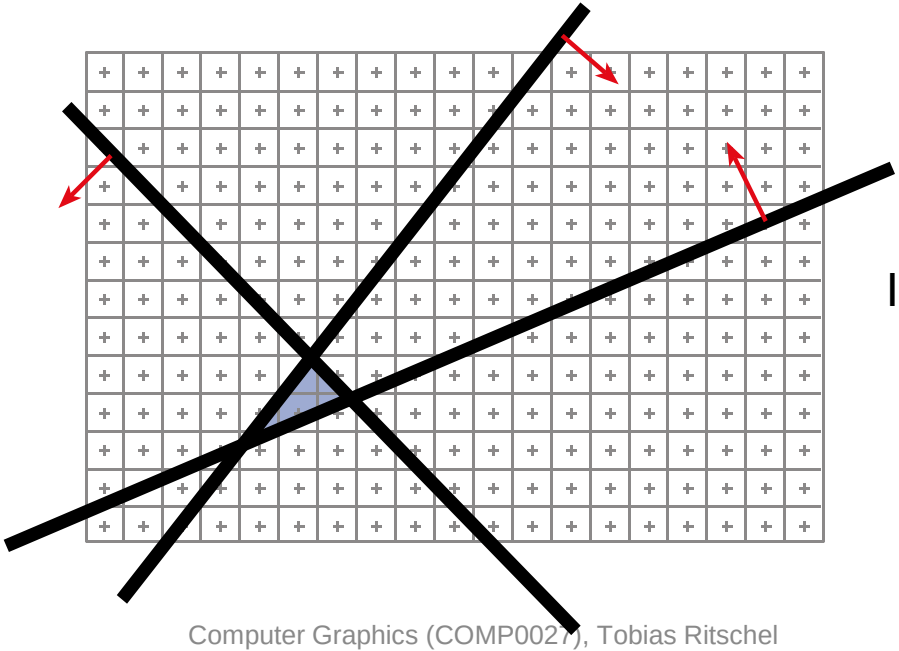
Brute Force Solution for Triangles

```
For all pixels i
 For all edges j
  Test if i is inside j
```



Brute Force Solution for Triangles

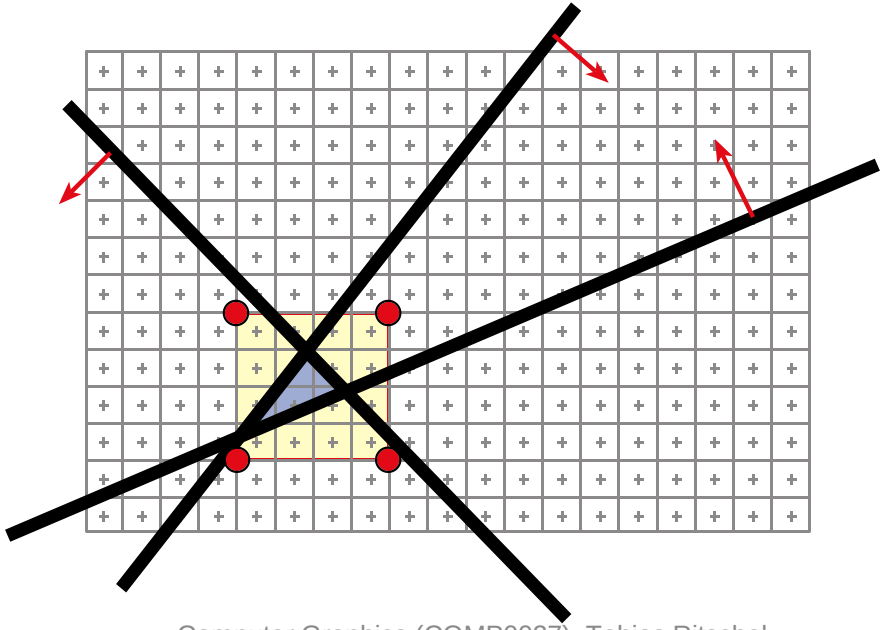
```
For all pixels i
 For all edges j
  Test if i is inside j
```



Problem?
If the triangle is small, a lot of useless computation!

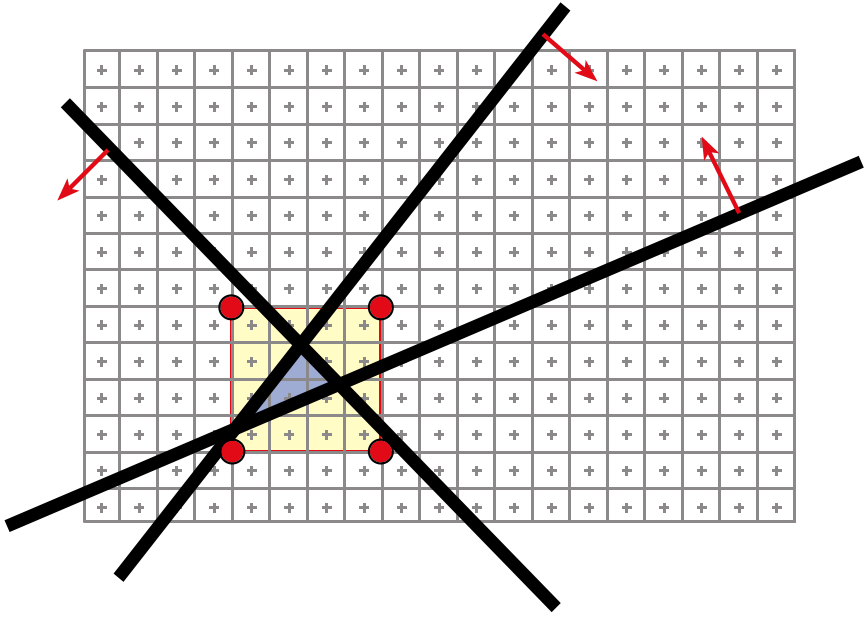
Brute Force Solution for Triangles

- Improvement: Compute only for the *screen bounding box* of the triangle
- How do we get such a bounding box?
  - $x_{\min}$, $x_{\max}$, $y_{\min}$, $y_{\max}$ of the triangle vertices



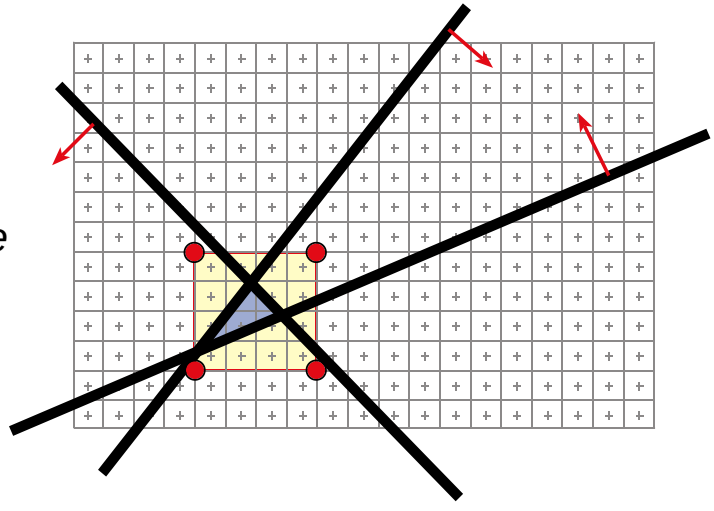
Rasterisation on Graphics Cards

- Triangles are usually very small
  - Setup cost are becoming more troublesome
- Brute force is tractable



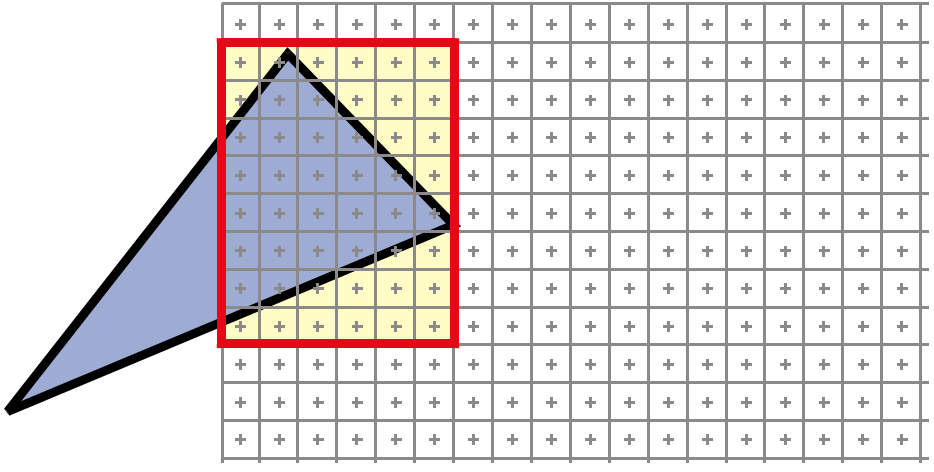
Rasterisation on Graphics Cards

```
For every triangle
 ComputeProjection
 Compute bbox, clip bbox to screen limits
 Compute line equations
 For all pixels in bbox
  If all line equations > 0  // Pixel [x,y] in triangle
   framebuffer[x,y] = color;
```



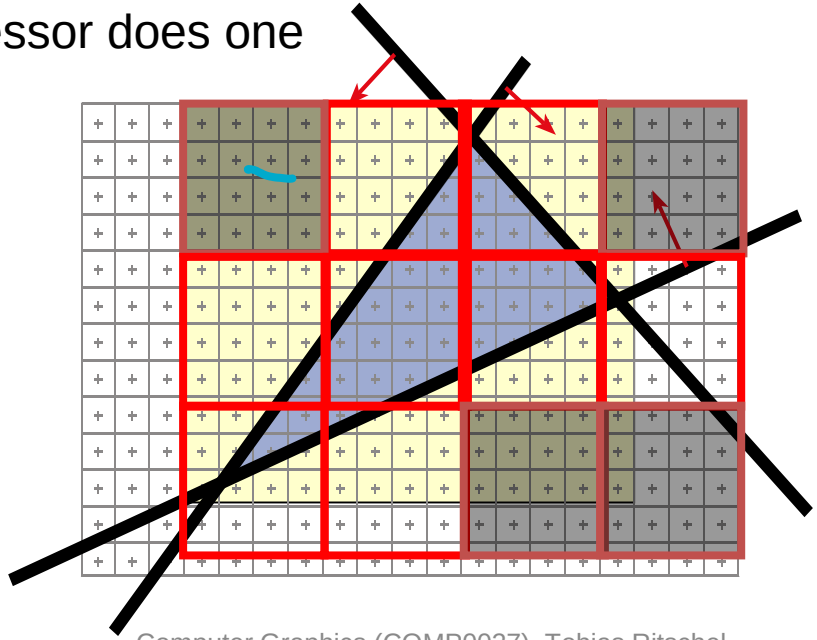
Rasterisation on Graphics Cards

Note that Bbox clipping is trivial, unlike triangle clipping



Tiling

- Subdivide BBox into smaller tiles
  - Early rejection of tiles
  - Memory access coherence
  - Each micrprocessor does one



Recap

- Moving away from ray-tracing to projection
- How to
  - Project a point
  - Clip a polygon
  - Cull a polygon
  - Fill the interior (rasterise)
- We'll spend next lectures tidying up the remaining problems!