Computer Graphics (COMP0027) 2023/24

Simple Camera

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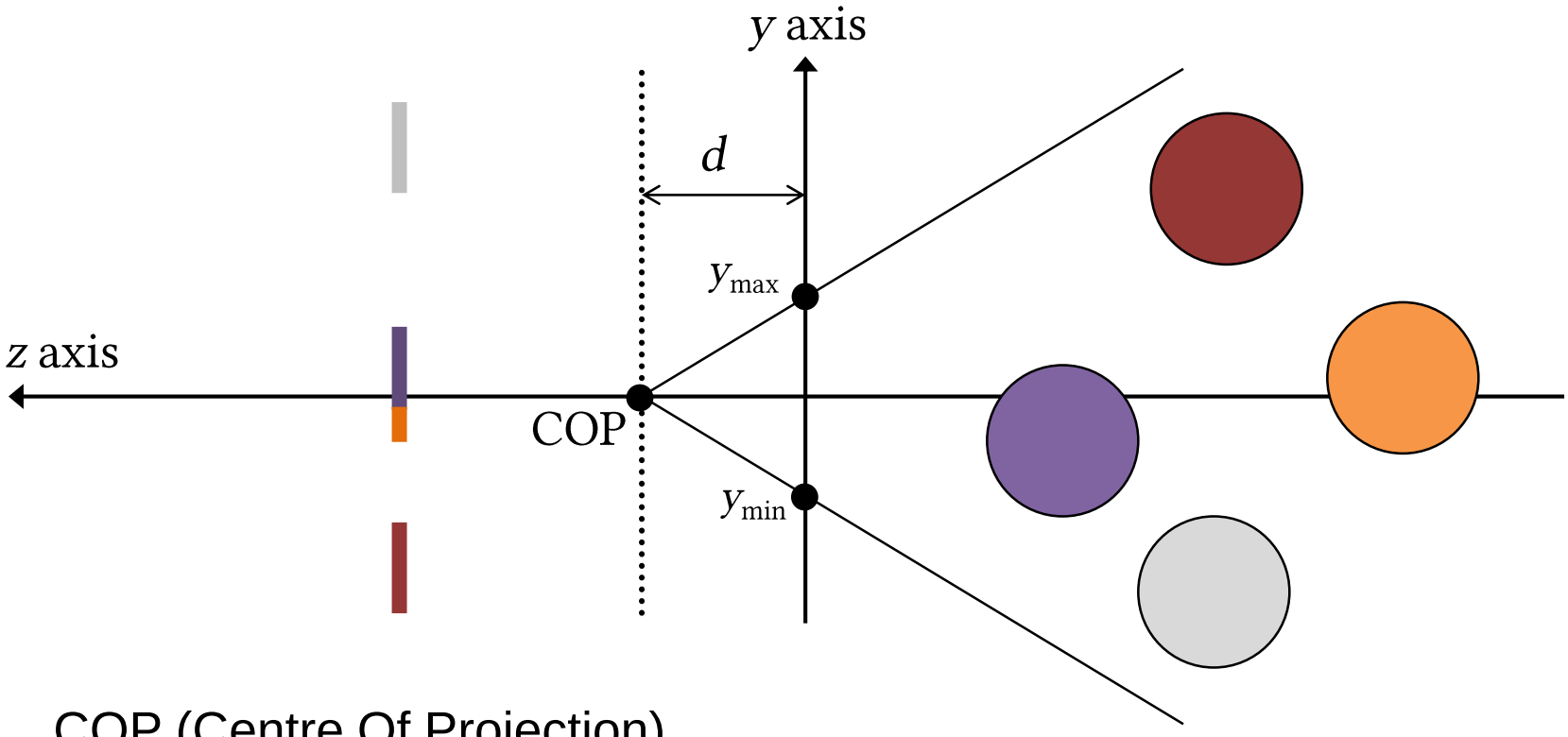
Overview

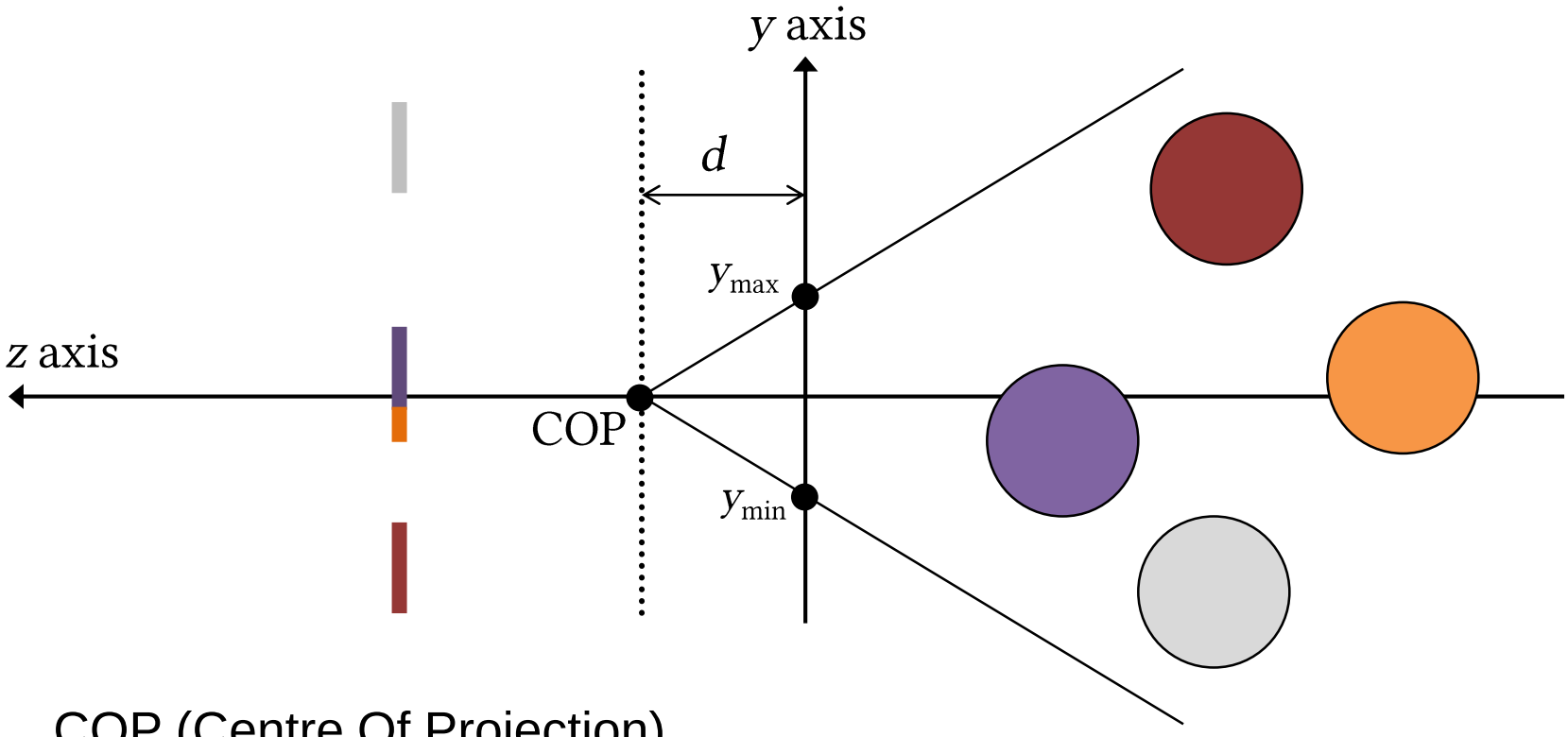
- The simplest rendering possible
- Some spheres only
- Simple Camera
  COP (Centre Of Projection) on z
- Looking along the $-z$ axis

Simple Camera (Cross Section)



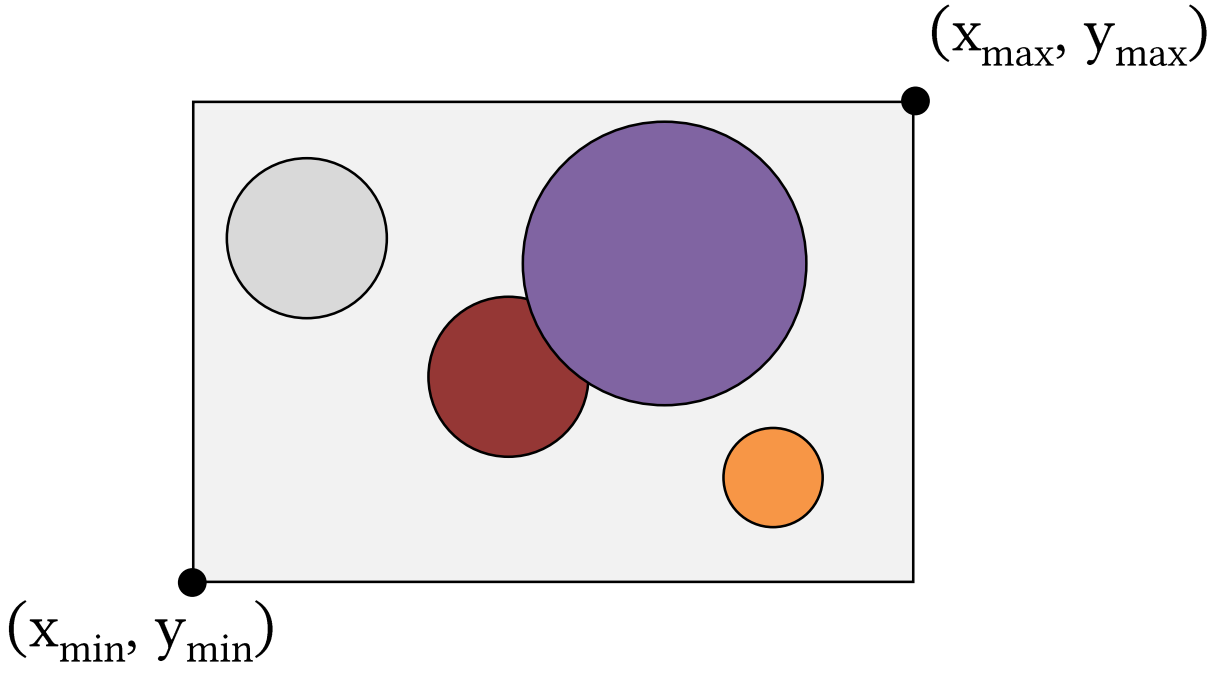
COP (Centre Of Projection)

Simple Camera (Cross Section)



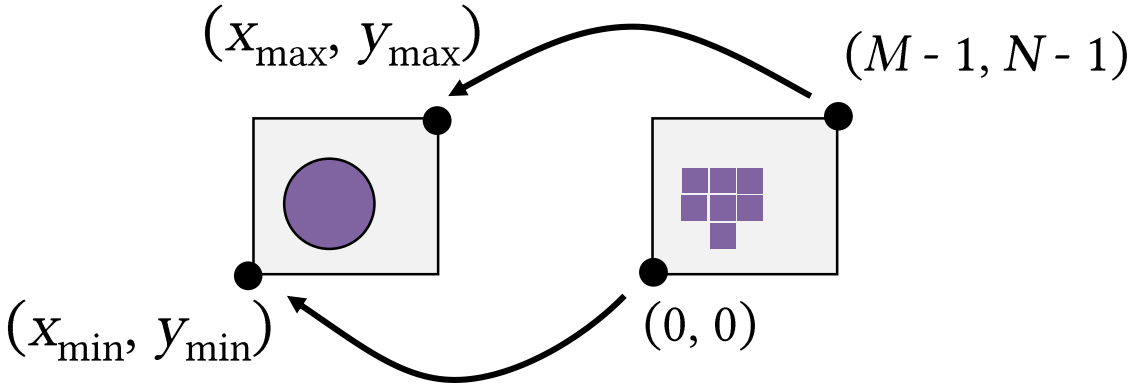
COP (Centre Of Projection)

View From the Camera



Forming the Rays

- Map screen pixels (M by N pixel window) to points in camera view plane



Forming the Rays

- Consider pixel (i, j)
- It corresponds to a rectangle
  - $w = (x_{max} - x_{min}) / M$
  - $h = (y_{max} - y_{min}) / N$
- Our ray goes through the **centre** of the pixel
- Thus the ray goes through the 3D point

$$(x_{min} + w\,(i + 0.5),\ y_{min} + h\,(j + 0.5),\ 0)$$

Forming the Rays

- Thus the ray from the COP through pixel i, j is defined by

$$\mathbf{r}(t) =$$
$$(x(t), y(t), z(t)) =$$
$$(p_{\mathrm{x}} + td_{\mathrm{x}}, p_{\mathrm{z}} + td_{\mathrm{z}}, p_{\mathrm{z}} + td_{\mathrm{z}}) =$$
$$(t(x_{\min} + w(i + \tfrac{1}{2})), t(y_{\min} + h(j + \tfrac{1}{2})), d - td)$$

Ray Casting

- Line-primitive intersection
- Simples variant:
  Line-sphere intersection
- Substitute the ray equation into the sphere equation and solve for t !

What is a 3D sphere?

Set of points (x, y, z), where distance to P is r

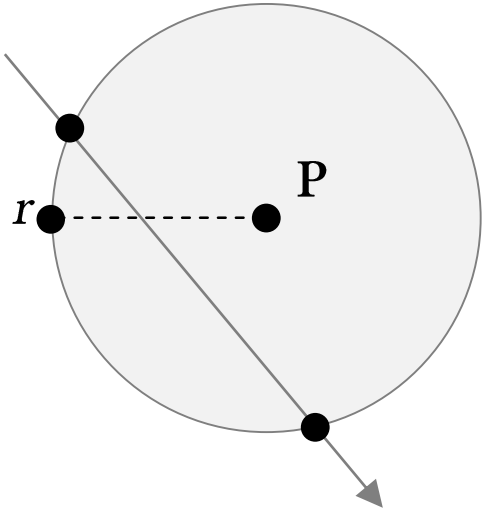
$$\sqrt{(x^2 + y^2 + z^2)} = r$$

$$x^2 + y^2 + z^2 = r^2$$

$$x(t)^2 + y(t)^2 + z(t)^2 = r^2$$

...

$$a\,t^2 + 2\,b\,t + c = 0$$



Formal Derivation

$$(\mathbf{p} + t\mathbf{d})^2 = r^2$$

$$(p_x + td_x)^2 + (p_y + td_y)^2 + (p_z + td_z)^2 = r^2$$

$$(p_x + td_x)^2 + \ldots = r^2$$

$$p_x^2 + 2p_x td_x + (td_x)^2 \ldots = r^2$$

$$(p_x^2 - r^2)t^0 + (2p_x d_x)t^1 + d_x^2 t^2 \ldots = 0$$

$$(d_x^2 + d_y^2 + d_z^2)t^2 + 2(p_x d_x + p_y d_y + p_z d_z)t^1 + (p_x^2 + p_y^2 + p_z^2 - r^2)t^0 = 0$$

$$at^2 + bt^1 + ct^0 = 0$$

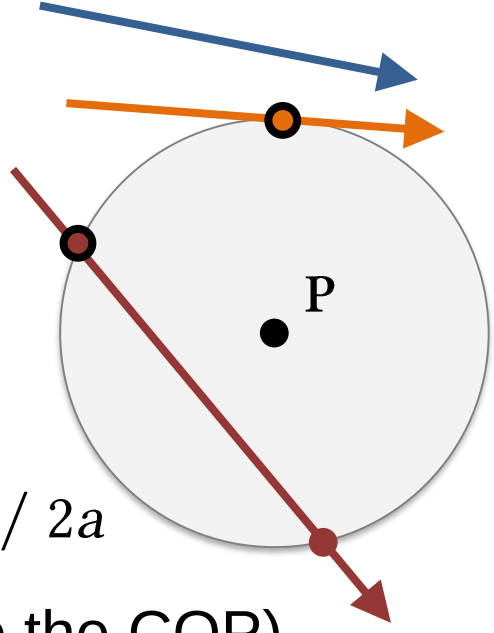
$$< \mathbf{d}, \mathbf{d} > t^2 + 2 < \mathbf{p}, \mathbf{d} > t^1 + (< \mathbf{p}, \mathbf{p} > -r^2)t^0 = 0$$

Sphere intersection

- If $b^2 - 4a\ c < 0$
  ray doesn't intersect the sphere
- If $b^2 - 4a\ c = 0$
  ray tangential to the sphere
- If $b^2 - 4a\ c > 0$
  two intersections given by

$$t = (-b \pm \sqrt{(b^2 - 4\ a\ c)})\ /\ 2a$$

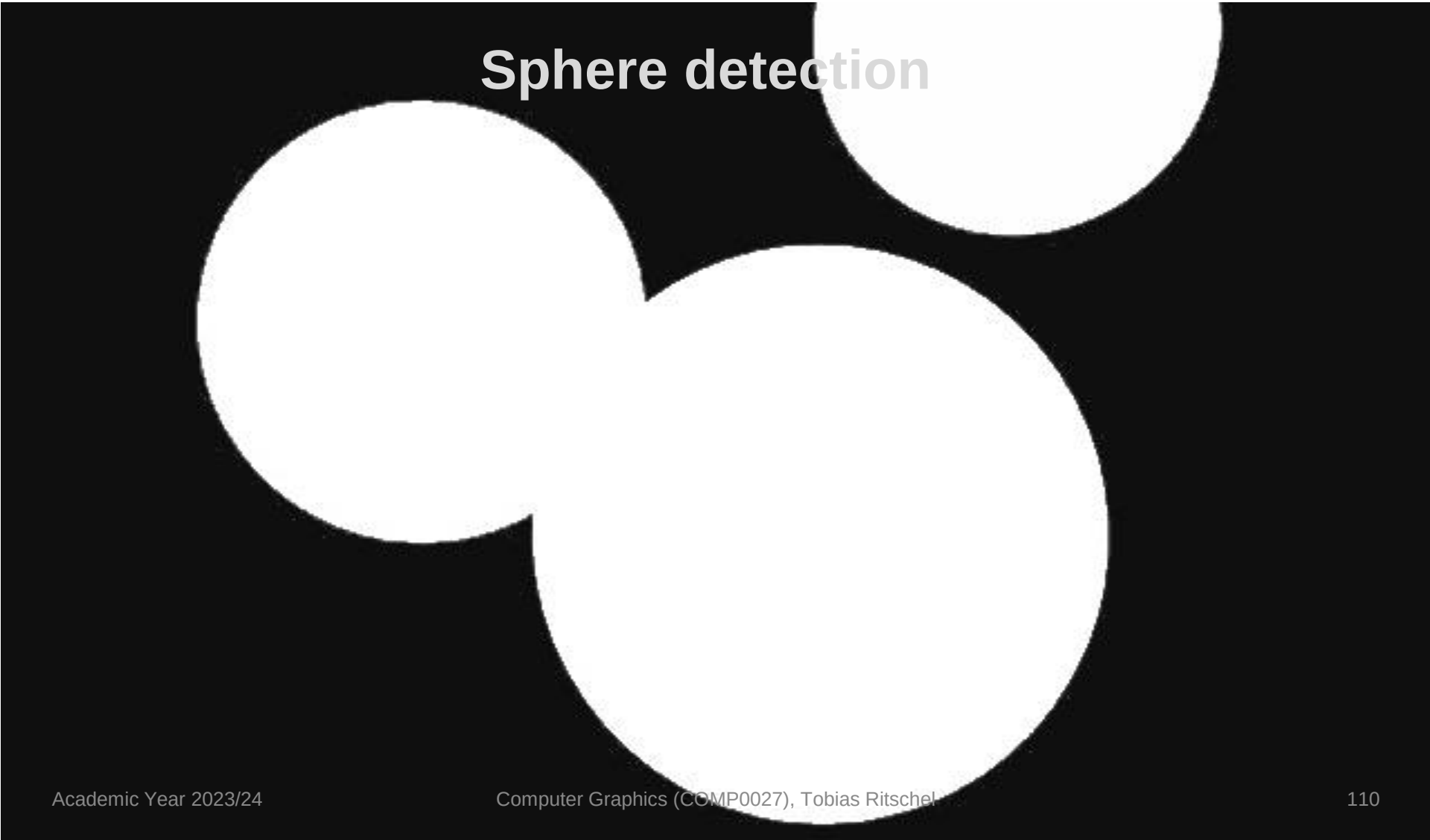
Choose the lowest value root (closer to the COP)



Ray Casting

- Intersection of sphere and line (general case)
  - Sphere is centred at (P_x, P_y, P_z)
  - Translate the start of the ray by $(-P_x, -P_y, -P_z)$
  - Proceed as before

Sphere detection



Conclusions

- We can now draw images
  - Forming rays from the camera
  - Intersecting those rays with objects (spheres) in the scene
- But
  - No colour – merely binary detection operation
  - Camera is static - at the moment we must move the objects in front of the camera to be able to see them
  - Need more interesting scenes!