COMP0130 Robot Vision and Navigation

1E: Quality Control

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Session Objectives

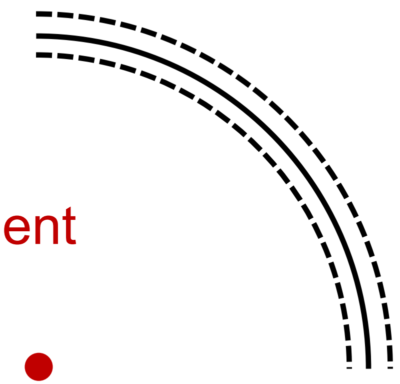
- Show how to classify and characterise measurement errors
- Introduce a methods for detecting faulty measurements

Quality Control

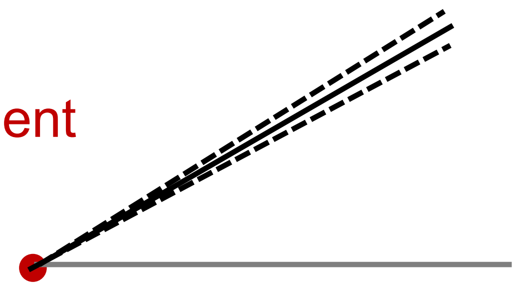
Measurement Errors

Measurements all have unknown errors, making them uncertain

Ranging measurement



Angle measurement



The total error can be due to a combination of different error sources:

Each error source can be a combination of

- Random error
- Systematic error
- Gross error

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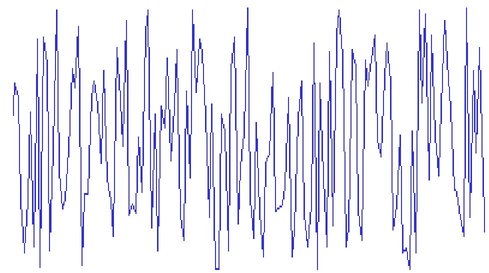
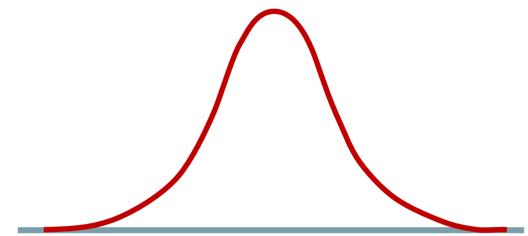
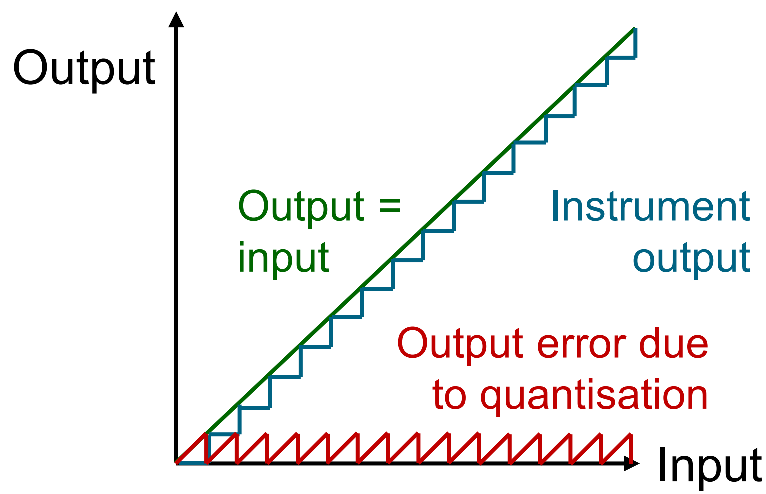
Random Errors

- Cannot be predicted
- Are different on every *raw* measurement
- Will usually have a known statistical distribution





Quantisation



Examples:

- Scale-reading errors for manual measurements, e.g. a tape measure
- Electronic measurement noise
- Quantisation/ rounding errors

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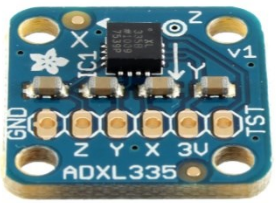
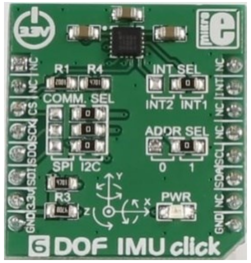
Systematic Errors

- Impact multiple measurements
- Systematic errors on one measurement can be predicted from those on other measurements affected by the same error source
- They can often be treated as extra states in the estimation problem
  - *This is known as State Augmentation* (see week 3)

Examples:

- Accelerometer and gyroscope biases (constant or slowly-varying errors)
- Wheel speed sensor, accelerometer and gyroscope scale-factor errors (proportional to the quantities measured)





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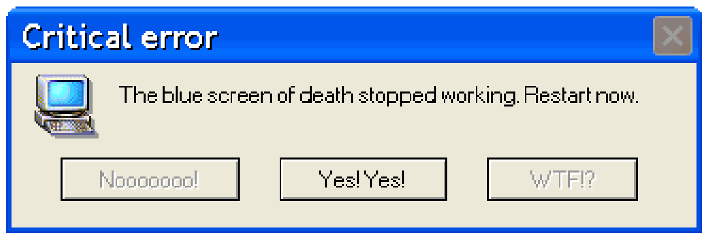
Gross Errors due to Faulty Measurements

A **gross error** is any error that is **much larger** than expected

It can be caused by **faulty measurements** or faulty estimation.

Examples:

Hardware fault

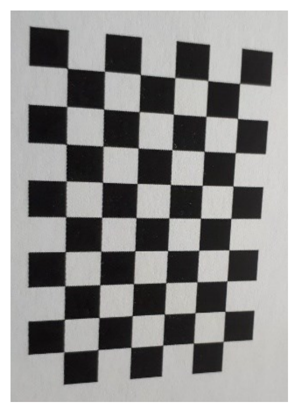


Software fault

Poor calibration

Human error

Interference



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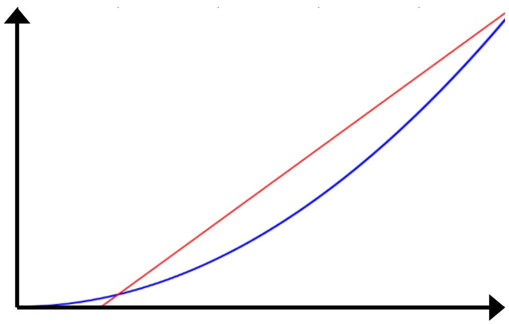
Gross Errors due to Faulty Estimation

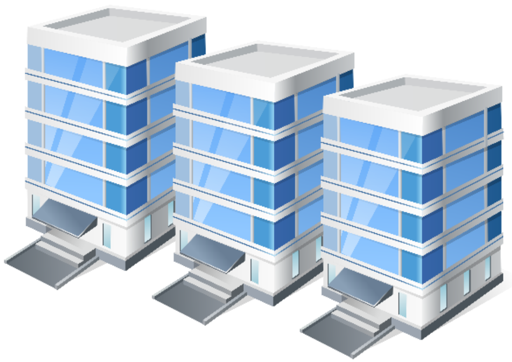
A **gross error** is any error that is **much larger** than expected

It can be caused by faulty measurements or **faulty estimation**.

Examples:



Incorrect or incomplete measurement model



Incorrect association, e.g. linking a measurement with the wrong object

Errors in the known parameters used in the estimation process

Poor stochastic model of the measurement error characteristics

Measurement Error Covariance Matrix

Expectation of square of the measurement error vector

Comprises variances and covariances of all of the measurement errors

Vector of measured values

$$\mathbf{C_z} = \mathrm{E}\left(\boldsymbol{\varepsilon\varepsilon}^{\mathrm{T}}\right) = \mathrm{E}\left[(\tilde{\mathbf{z}} - \mathbf{z})(\tilde{\mathbf{z}} - \mathbf{z})^{\mathrm{T}}\right] = \begin{pmatrix} \sigma_{z1}^2 & C_{z12} & \cdots & C_{zlm} \\ C_{z21} & \sigma_{z2}^2 & \cdots & C_{zlm} \\ \vdots & \vdots & \ddots & \vdots \\ C_{zm1} & C_{zm2} & \cdots & \sigma_{zm}^2 \end{pmatrix}$$

Vector of true values

Off-diagonal elements are covariances

Diagonal elements are variances

Covariance matrices are symmetric: $\mathbf{C_z}^{\mathrm{T}} = \mathbf{C_z}$

This sometimes called the **stochastic model** of the measurements

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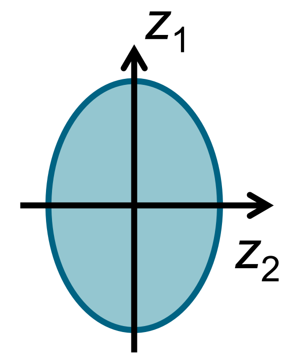
Independent and Correlated Errors

Independent errors affect only individual measurements

- They only contribute to the diagonals of the covariance matrix
- Off-diagonal elements due to independent errors are zero

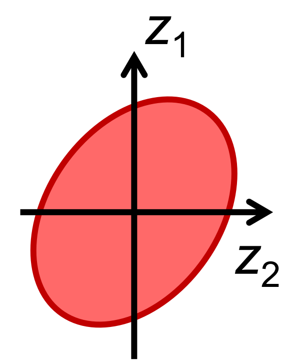
$$\begin{pmatrix} \sigma_{z1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{z2}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{zm}^2 \end{pmatrix}$$



Correlated errors affect multiple measurements

- They contribute to the diagonal and off-diagonal elements of the covariance matrix
- Some off-diagonal elements may still be zero

$$\begin{pmatrix} \sigma_{z1}^2 & C_{z12} & \cdots & C_{zlm} \\ C_{z21} & \sigma_{z2}^2 & \cdots & C_{z2m} \\ \vdots & \vdots & \ddots & \vdots \\ C_{zm1} & C_{zm2} & \cdots & \sigma_{zm}^2 \end{pmatrix}$$



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Raw and Derived Measurement Errors

Only the errors in raw measurements can be independent of each other

Consider a relative position fix (ΔE, ΔN, ΔU), *a derived measurement,* obtained from three raw measurements:

Range (r), Azimuth (ψ) and Elevation (θ), i.e. polar coordinates

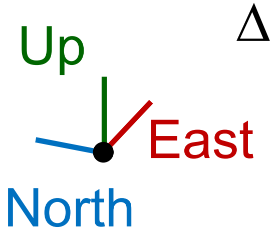
$$\Delta E = r \sin\theta \sin\psi$$
$$\Delta N = r \sin\theta \cos\psi$$
$$\Delta U = r \cos\theta$$

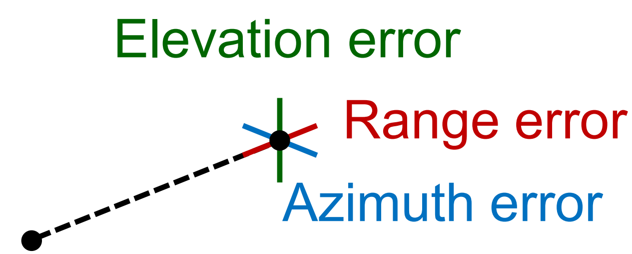
Azimuth error affects E and N position

Elevation error affects E, N, & U position

Range error affects E, N, & U position

See Moodle note on position fix uncertainty





The East, north and vertical position errors are thus correlated

Any measurements derived from the same raw measurements will have correlated errors

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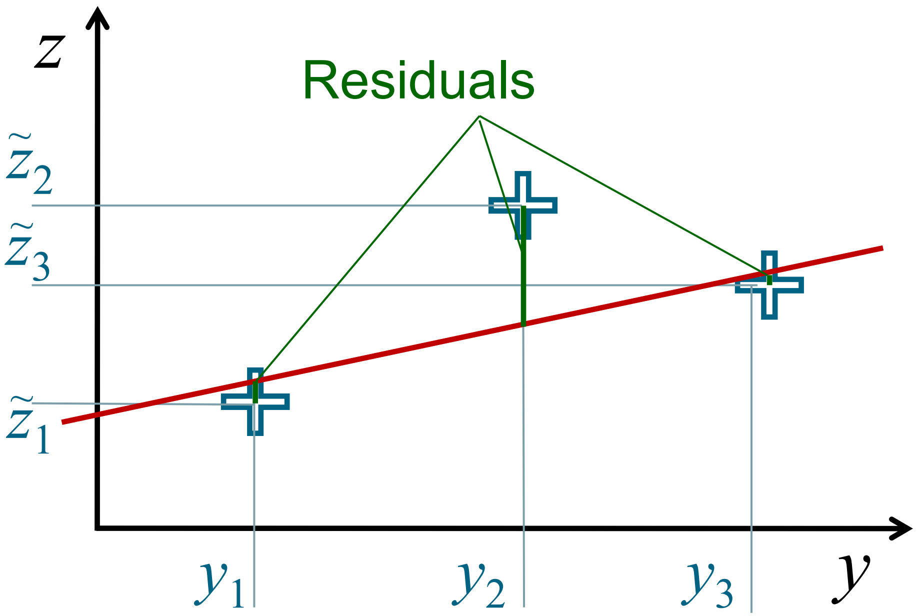
Detecting Faults using Residuals



A simple straight-line example

Residuals show how well the solution, $z = h(\mathbf{x}, y)$, fits the measurements

Residuals can be used to identify a number of potential problems

- Poor stochastic model
- Poor measurement model
- Faulty measurements

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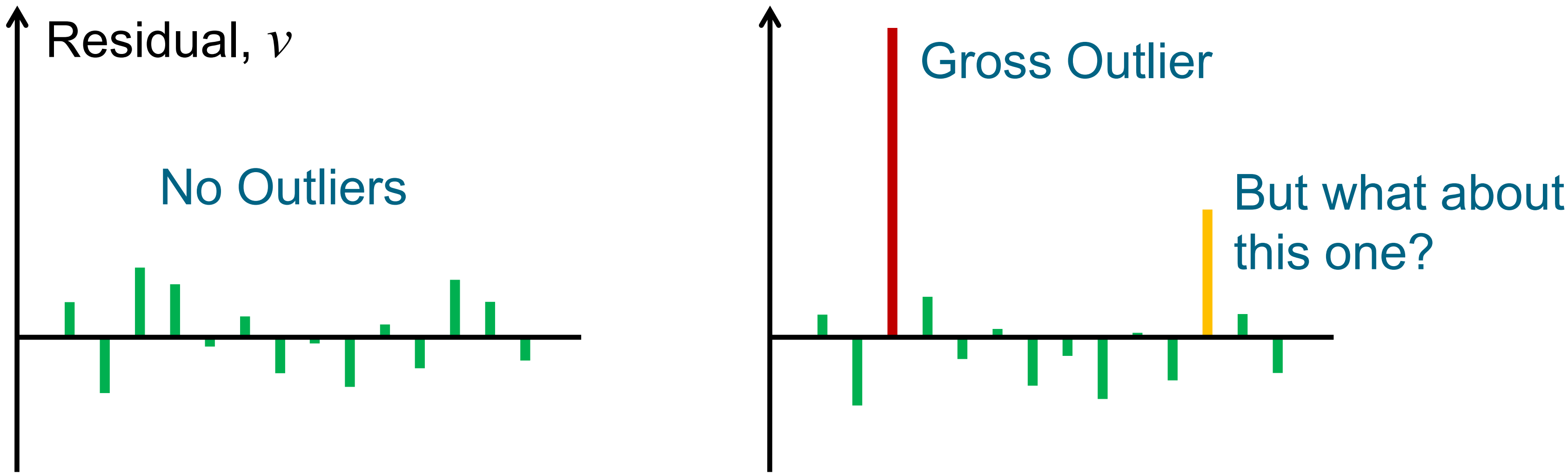
Effect of Measurement Errors on Residuals

Errors in individual measurements or their measurement models, also known as "Gross Outliers", can be identified from the residuals



Some measurements are more precise than others

∴ Normalised residuals can be a more reliable indicator:

$$w_i = \frac{v_i}{\sigma_{vi}}$$

Root diagonal of $\mathbf{C}_\mathbf{v}$

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Residuals Covariance Matrix

This describes the statistical properties of the residuals and is defined similarly to the other covariance matrices

$$\mathbf{C_v} = \mathrm{E}\left(\mathbf{v}\mathbf{v}^{\mathrm{T}}\right) = \begin{pmatrix} \sigma_{v1}^2 & C_{v12} & \cdots \\ C_{v21} & \sigma_{v2}^2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

For linear problems

$$\begin{aligned} \mathbf{v} &= \mathbf{H}\hat{\mathbf{x}}^{+} - \tilde{\mathbf{z}} \\ &= \mathbf{H}\left(\mathbf{x}+\mathbf{e}\right) - \left(\mathbf{z}+\boldsymbol{\varepsilon}\right) \end{aligned}$$

$\boldsymbol{\varepsilon}$ = measurement error

$\mathbf{e}$ = state estimate error

$$\mathbf{e} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{C_z}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{C_z}^{-1}\boldsymbol{\varepsilon}$$

and $\mathbf{z} = \mathbf{H}\mathbf{x}$, so:

$$\begin{aligned} \mathbf{v} &= \mathbf{H}\mathbf{e} - \boldsymbol{\varepsilon} \\ &= \left(\mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{C_z}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{C_z}^{-1} - \mathbf{I}\right)\boldsymbol{\varepsilon} \end{aligned}$$

This also applies to nonlinear problems

Following Derivation 3 on Moodle,

$$\begin{aligned} \mathbf{C_v} &= \mathbf{C_z} - \mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{C_z}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}} \\ &= \mathbf{C_z} - \mathbf{H}\mathbf{C_x}\mathbf{H}^{\mathrm{T}} \end{aligned}$$

Outlier Detection using Normalised Residuals

1. Calculate Normalised Residuals

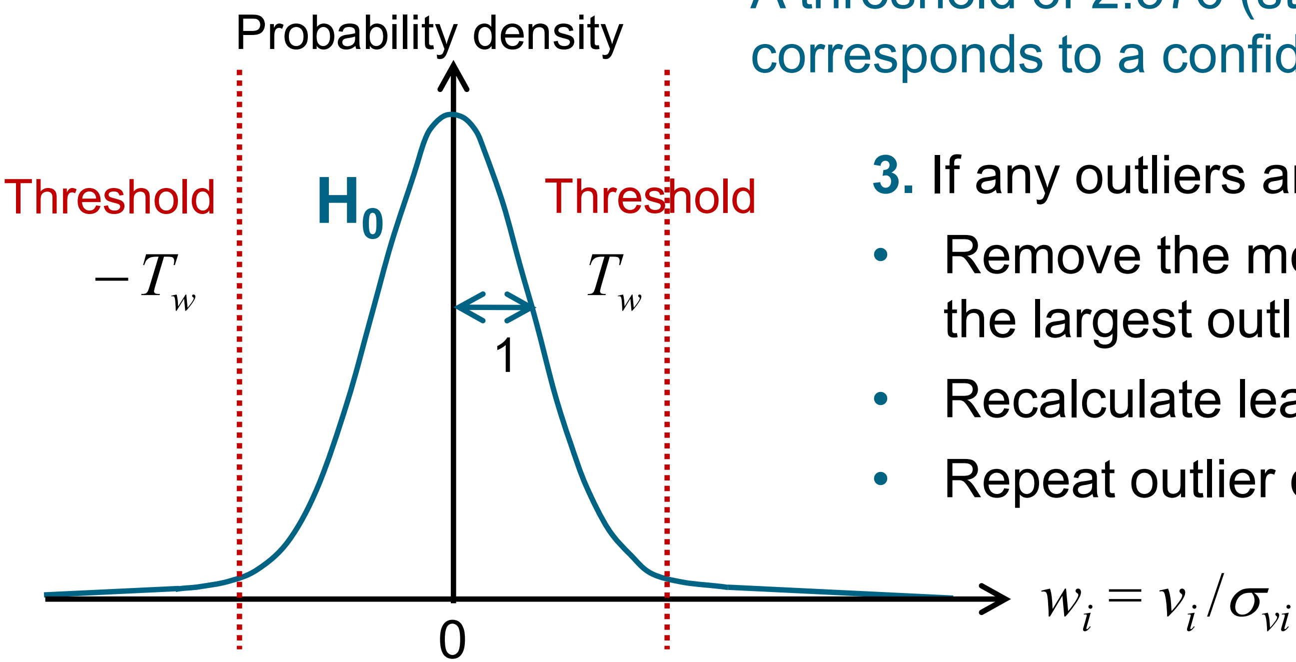
$$w_i = \frac{v_i}{\sigma_{vi}}$$

2. Compare them with a threshold

$|w_i| \le T_w$: No fault assumed (Hypothesis H_0)

$|w_i| > T_w$: Fault/ gross outlier assumed (Hypothesis H_1)

A threshold of 2.576 (standard deviations) corresponds to a confidence interval of 99%



3. If any outliers are identified…

- Remove the measurement with the largest outlier
- Recalculate least-squares solution
- Repeat outlier detection

Hypothesis Testing

Outlier detection is an example of hypothesis testing

$$w_i = \frac{v_i}{\sigma_{vi}}$$

$|w_i| \le T_w$: Hypothesis H_0 (no fault assumed)

$|w_i| > T_w$: Hypothesis H_1 (fault/ gross outlier assumed)

Test Outcome	H_0 Hypothesis True (No fault present)	H_1 Hypothesis True (Fault present)
H_0 (No fault detected)	Probability: **$1 - \alpha$** **Correct Outcome**	Probability: β **Type II Error**
H_1 (fault detected)	Probability: α **Type I Error**	Probability: **$1 - \beta$** **Correct Outcome**

α is the level of significance (false alarm probability)

β is the missed detection probability

$1 - \alpha$ is the confidence level

$1 - \beta$ is the power of the test

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Null and Alternate Hypotheses

Null Hypothesis: $\mathrm{H_0}$

There is no fault or gross measurement error

$$\mathbf{H_0} : \tilde{z}_i = z_i + \varepsilon_i$$

Random error

Measurement

True value of measured parameter

Normalised residual, $w_i = v_i / \sigma_{vi}$:

- Mean: 0
- Standard deviation: 1

Alternate Hypothesis: $\mathrm{H_1}$

There is a fault or gross measurement error

$$\mathbf{H}_1 : \tilde{z}_i = z_i + \varepsilon_i + \Delta_i$$

Gross error

Normalised residual, $w_i = v_i / \sigma_{vi}$:

- Mean: $\Delta_i \, \sigma_{vi} / \sigma_{zi}^2$
- Standard deviation: 1

(see Section 5.4.1.3 and Appendix 2 of the Paul Cross working paper on Moodle for the derivation)

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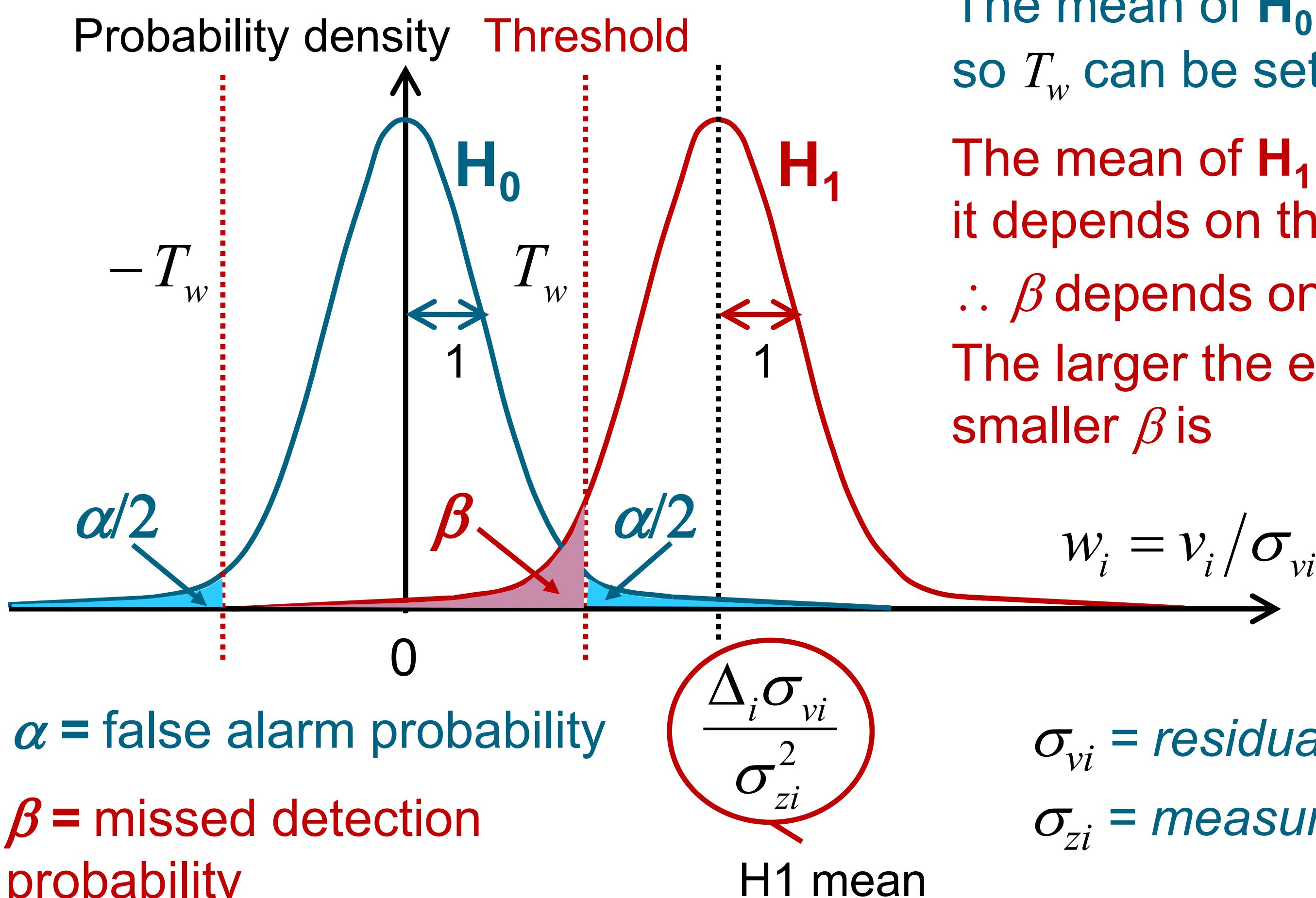
Null and Alternate Hypothesis Distributions



The mean of $\mathbf{H_0}$ is known (zero), so T_w can be set to fix α

The mean of $\mathbf{H_1}$ is unknown – it depends on the gross error

$\therefore\ \beta$ depends on Δ

The larger the error, the smaller β is

Normalised residual (unit variance)

α = false alarm probability

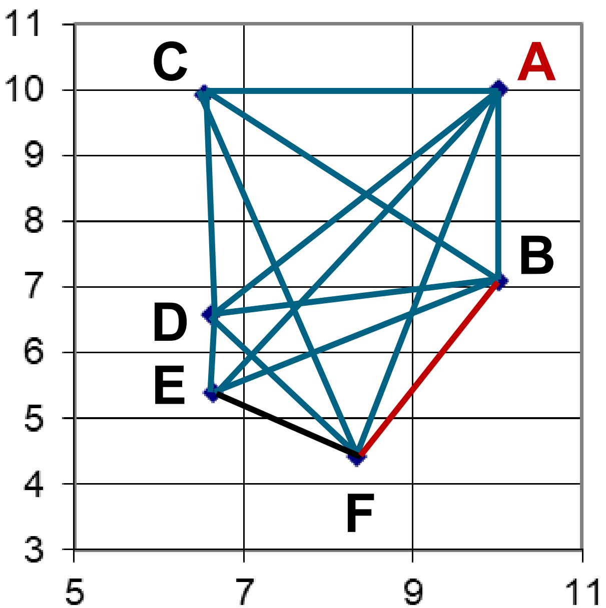
β = missed detection probability

σ_{vi} = *residual SD*

σ_{zi} = *measurement error SD*

Example 4A: Total Station Positioning

Measurement 13 is the range from E to F, but is processed with the measurement model for the range from B to F



Normalised residuals

Meas.	w	Meas.	w
1: r_{AB}	4.622	8: r_{BE}	-0.749
2: r_{AC}	-1.131	9: r_{CD}	-0.345
3: r_{AD}	1.652	10: r_{CF}	-0.644
4: r_{AE}	0.749	11: r_{DE}	-0.749
5: r_{AF}	-6.400	12: r_{DF}	2.296
6: r_{BC}	1.434	13: r_{EF}	6.830
7: r_{BD}	-3.126	14: ψ_{AB}	N/A

Four normalised residuals are large, but the one with the faulty measurement model is largest

See *RVN Least-Squares Examples.xlsx* on Moodle