Computer Graphics (COMP0027) 2022/23

用于创建平滑曲线的数学工具或函数

Spline surfaces

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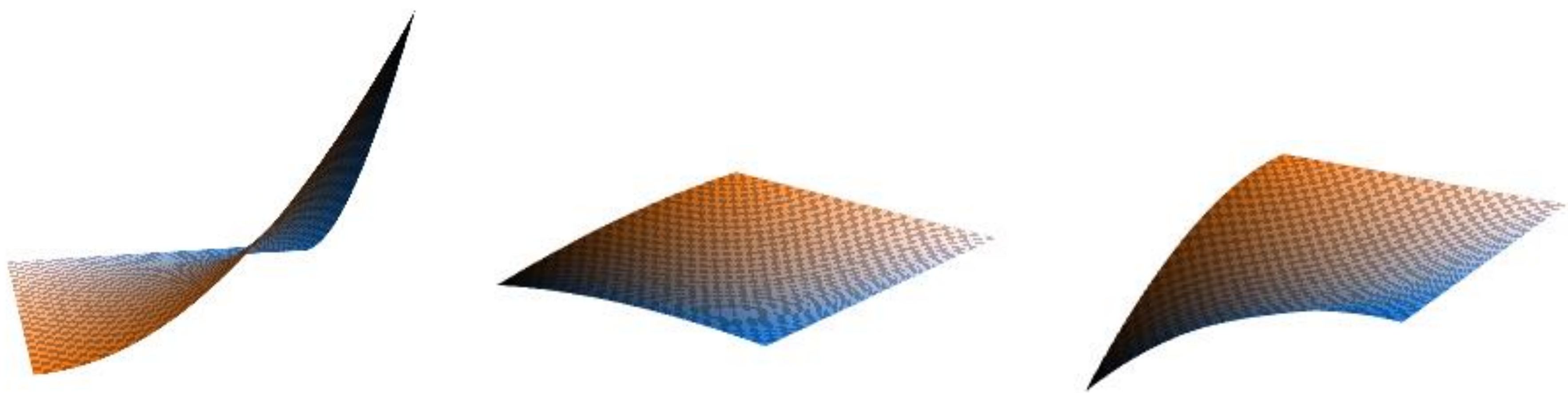
Remember: we want to remove facets

平坦切割面

Example `cg.cs.ucl.ac.uk`



Bezier Surfaces Introduction

- Constructing a surface relies very much on the ideas behind constructing curves
- Surfaces can be thought of as ‘Bezier curves in all directions’ across the surface
- Tensor products of Bezier curves
- Teapot most famous example
  - produced entirely by Bezier surfaces

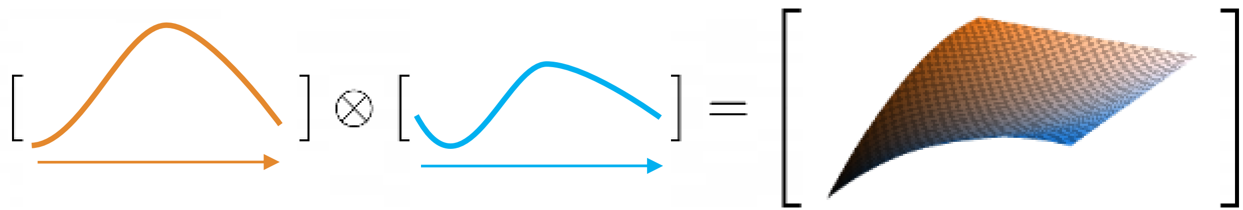
Tensor Product

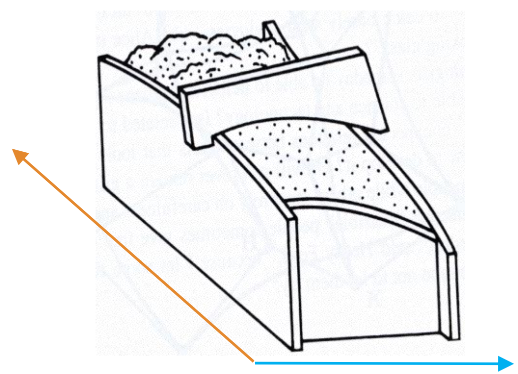
- Tensor product of two **vectors**

$$\mathbf{a} \otimes \mathbf{b} = \begin{bmatrix} a_1 & a_2 & a3 \end{bmatrix} \otimes \begin{bmatrix} b_1 & b_2 & b3 \end{bmatrix} = \begin{bmatrix} a_1 b_1 & a_2 b_1 & a3 b_1 \\ a_1 b_2 & a_2 b_2 & a3 b_2 \\ a_1 b_3 & a_2 b_3 & a3 b_3 \end{bmatrix}$$

Tensor Product

- Tensor product of two **functions**

$\mathbf{a} \otimes \mathbf{b} =$ 



Bicubic Bezier Patch

- Let

$$\mathbf{c}(t|\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$$

be a 1D spline at t through the control points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$

- Then the surface is

$$\begin{aligned}\mathbf{p}(s,t) = \mathbf{c}(s|&\mathbf{c}(t|\mathbf{p}_{00}, \mathbf{p}_{01}, \mathbf{p}_{02}, \mathbf{p}_{03}),\\ &\mathbf{c}(t|\mathbf{p}_{10}, \mathbf{p}_{11}, \mathbf{p}_{12}, \mathbf{p}_{13}),\\ &\mathbf{c}(t|\mathbf{p}_{20}, \mathbf{p}_{21}, \mathbf{p}_{22}, \mathbf{p}_{23}),\\ &\mathbf{c}(t|\mathbf{p}_{30}, \mathbf{p}_{31}, \mathbf{p}_{32}, \mathbf{p}_{33}))\end{aligned}$$

4 points
moving on
4 spline
in 3D space

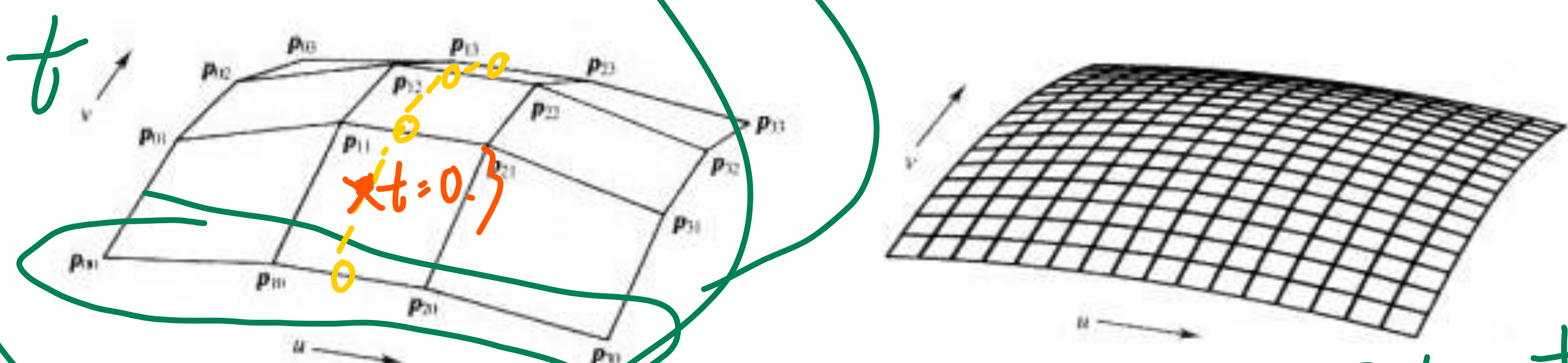
step 1 取 s

step 2 取 t

Bicubic Bezier Patch

$$
\begin{aligned}
\mathbf{p}(s,t) = \mathbf{c}(s|\, &\mathbf{c}(t|\mathbf{p}_{00},\mathbf{p}_{01},\mathbf{p}_{02},\mathbf{p}_{03}),\\
&\mathbf{c}(t|\mathbf{p}_{10},\mathbf{p}_{11},\mathbf{p}_{12},\mathbf{p}_{13}),\\
&\mathbf{c}(t|\mathbf{p}_{20},\mathbf{p}_{21},\mathbf{p}_{22},\mathbf{p}_{23}),\\
&\mathbf{c}(t|\mathbf{p}_{30},\mathbf{p}_{31},\mathbf{p}_{32},\mathbf{p}_{33}))
\end{aligned}
$$

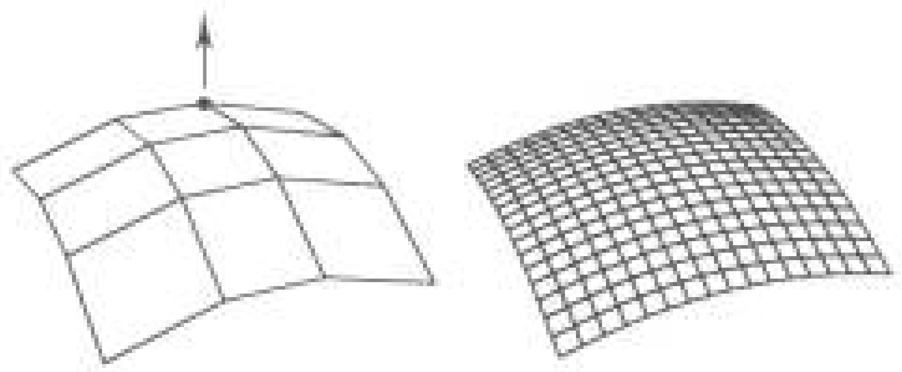
DEMO



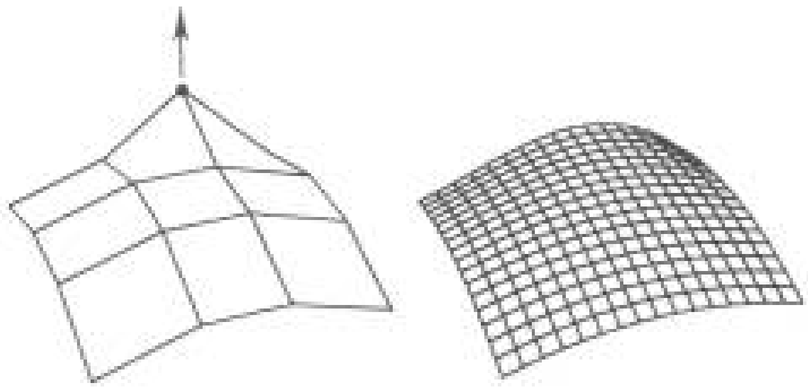
(a) (b)

(这里 s, t 可以直接用来在 texture 图上取)

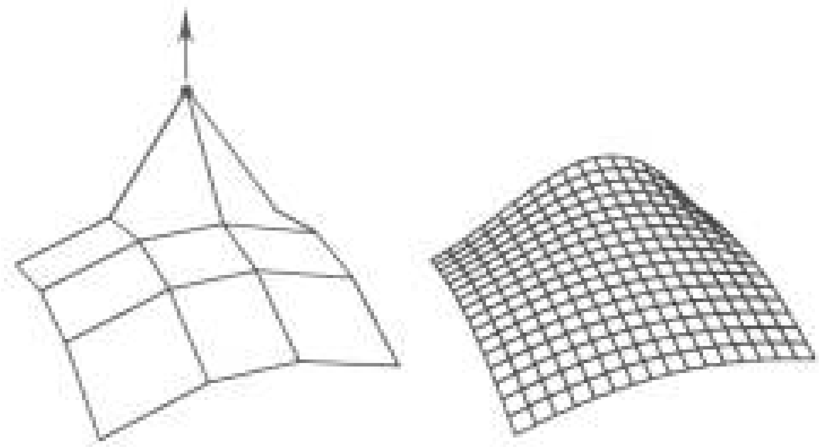
Editing Bicubic Bezier Patches



Editing Bicubic Bezier Patches

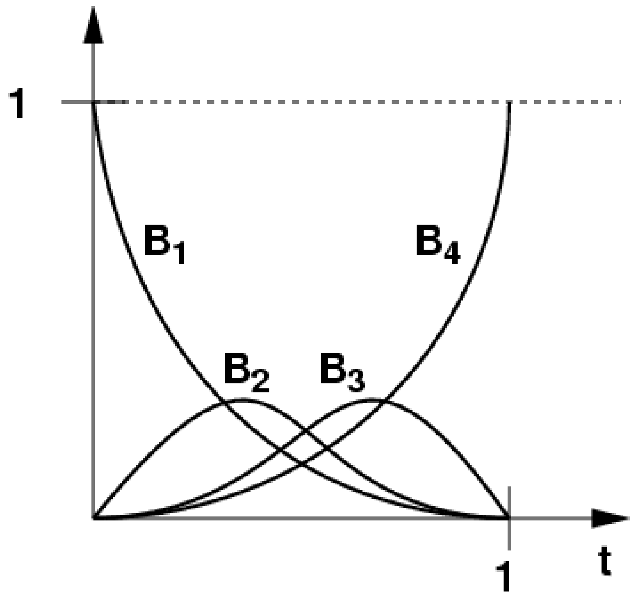


Editing Bicubic Bezier Patches

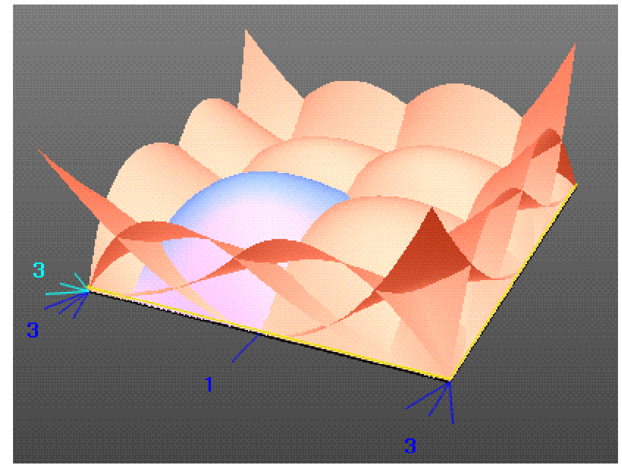


still smooth

Editing Bicubic Bezier Patches



1D Basis Functions

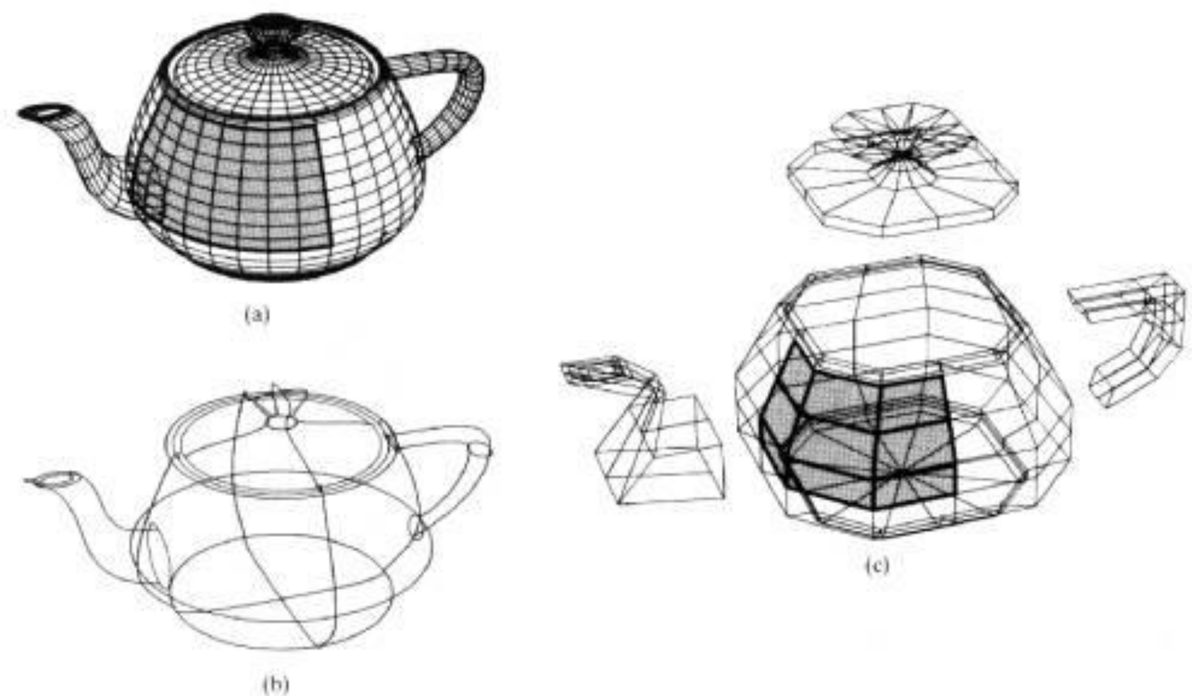


2D Basis Functions

在LG网上有
Demo实现

Patch Modelling

- Original Teapot specified with Bezier Patches



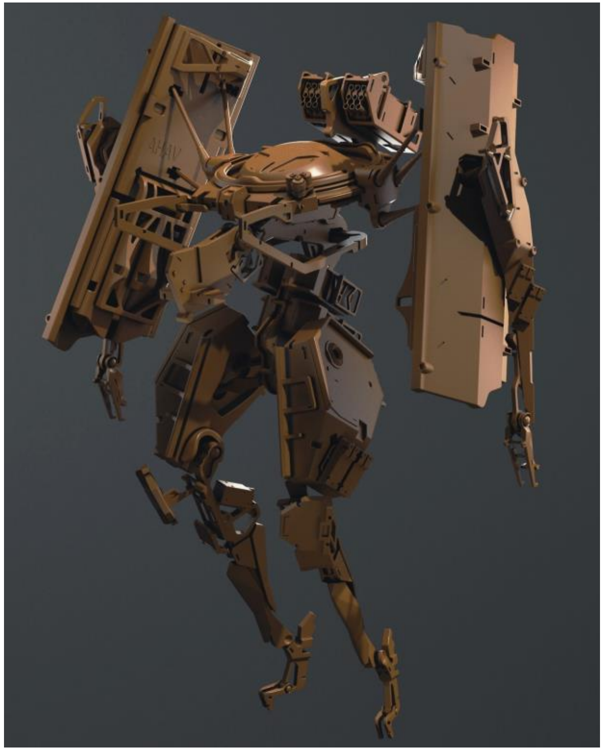
Alternative Splines Surfaces

- You can make surfaces from B-Splines in a similar way
- A particular types of B-Spline generalisation, Non-uniform rational Basis spline (NURBS) surfaces are particularly common

Geri's game by Pixar

Spline domain

- Splines are defined to map from (0,1)^2 to 3D
- Not every 3D shape can be represented like this
  - Homeomorphism
  - You cannot take every shape and map it to (0,1)^2
  - Counter example seen right

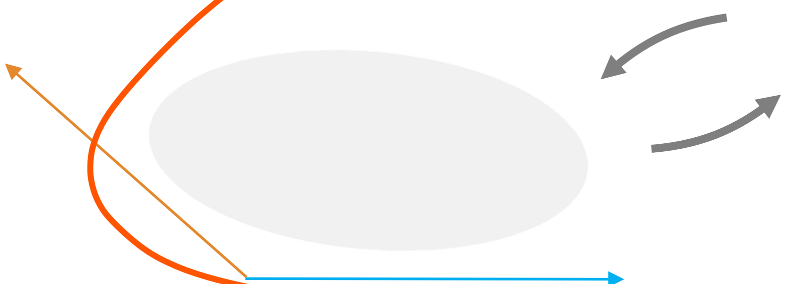


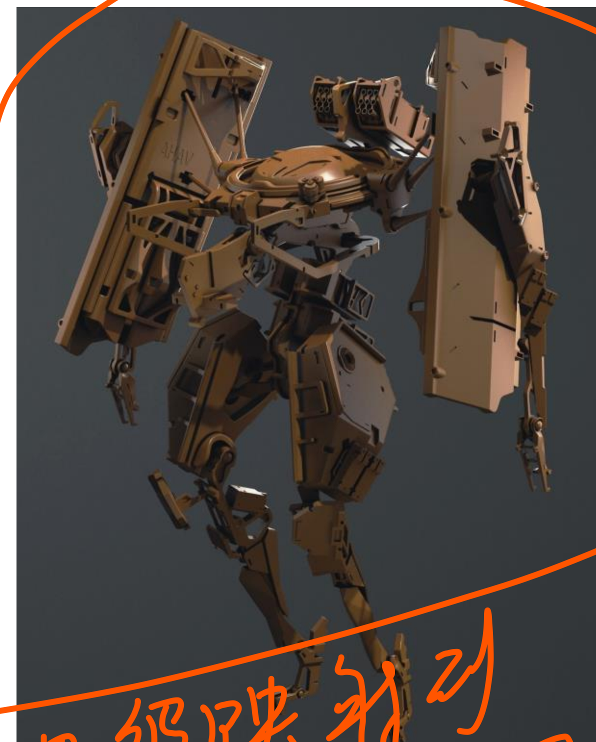
Splines on non-disk topology

一种拓扑结构

- Non-disk topology

No, sorry!





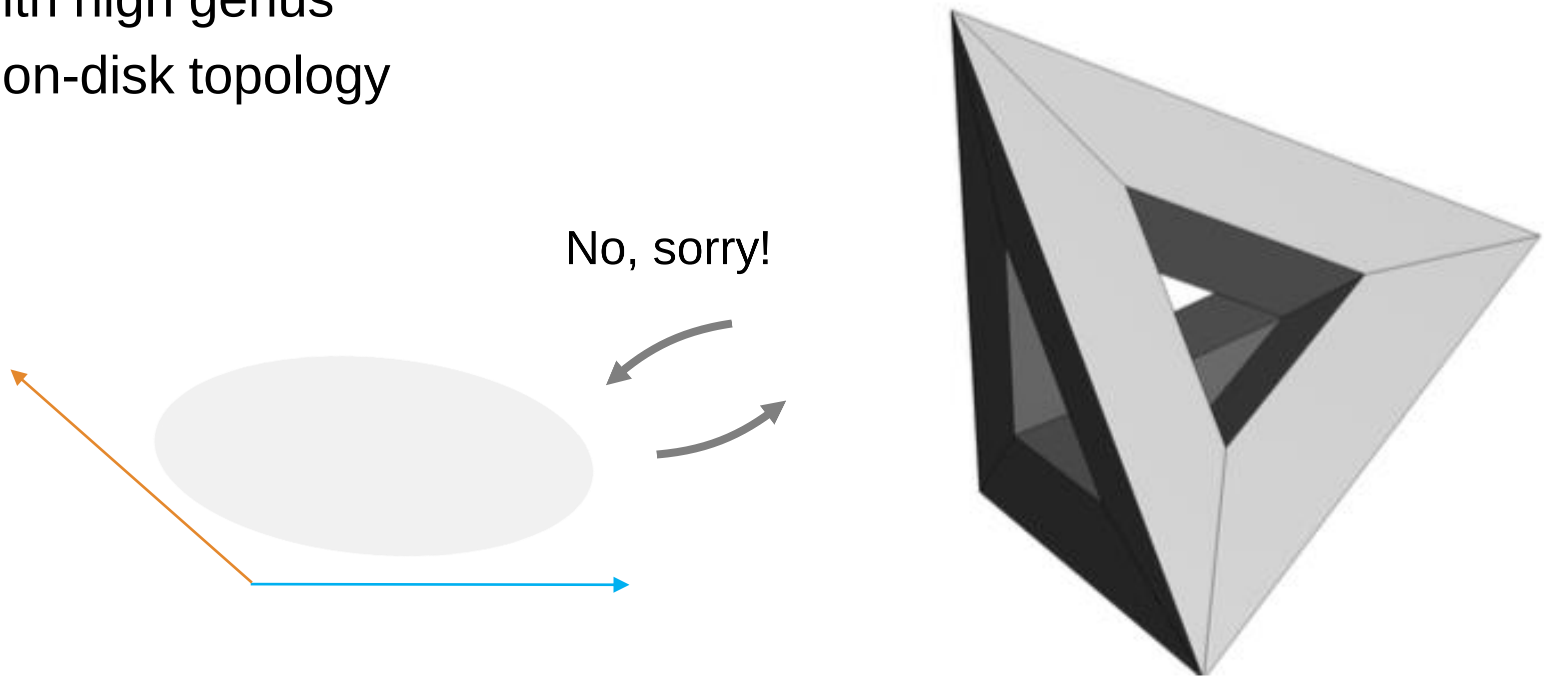
定义

不能简单地级映射到

一个平面圆盘 disk

Splines on non-disk topology

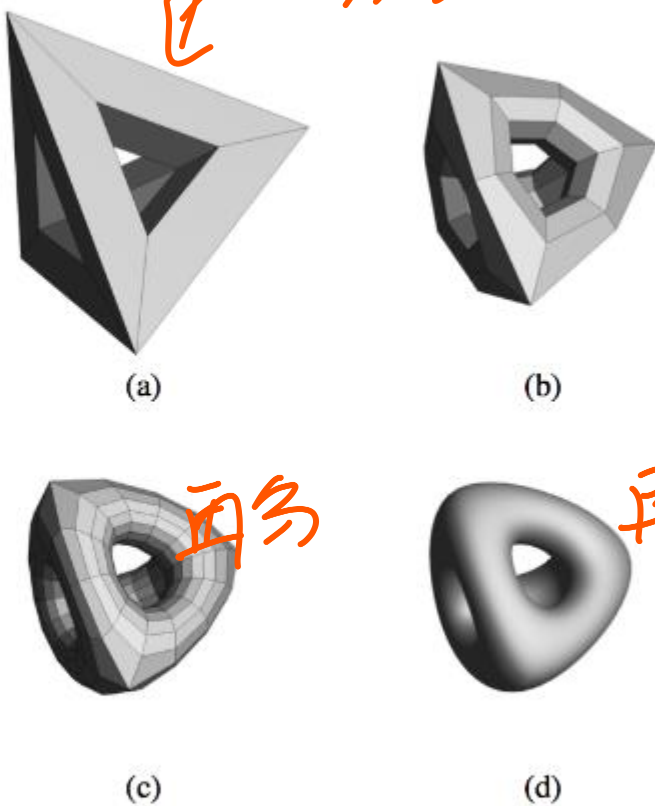
- Lets consider a simple case with high genus
- Non-disk topology



方法：(不再用 Basis function了 而是 ↓)

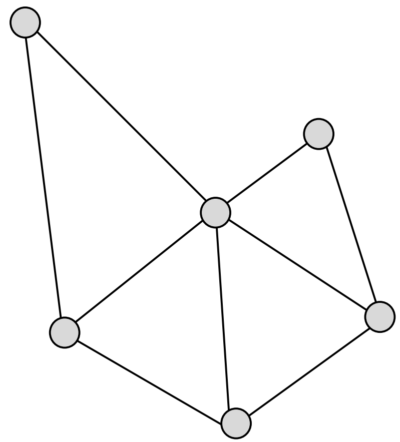
Iterative splitting

分成3块

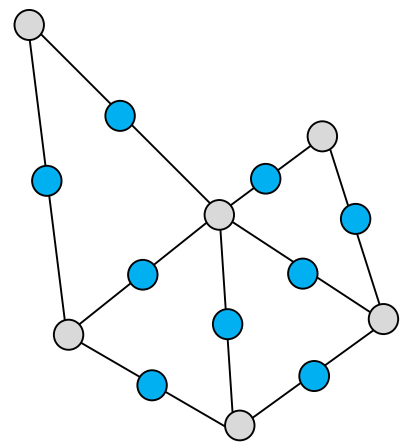


2D区分

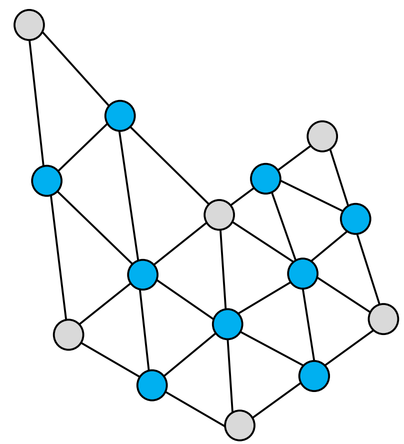
Best to think 2D again



Step 1: Split edges

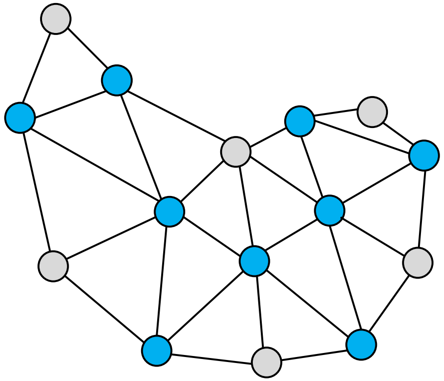


Step 2: Re-topologize

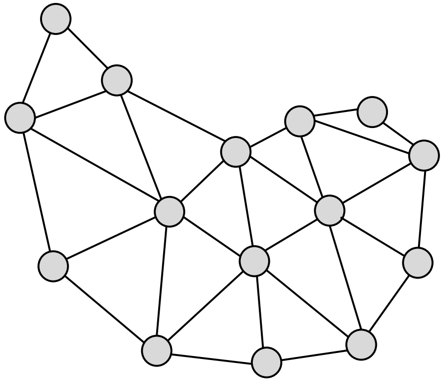


Step 3: Relax

(average with neighbours)



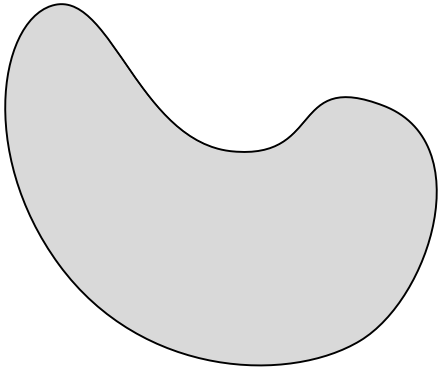
Done



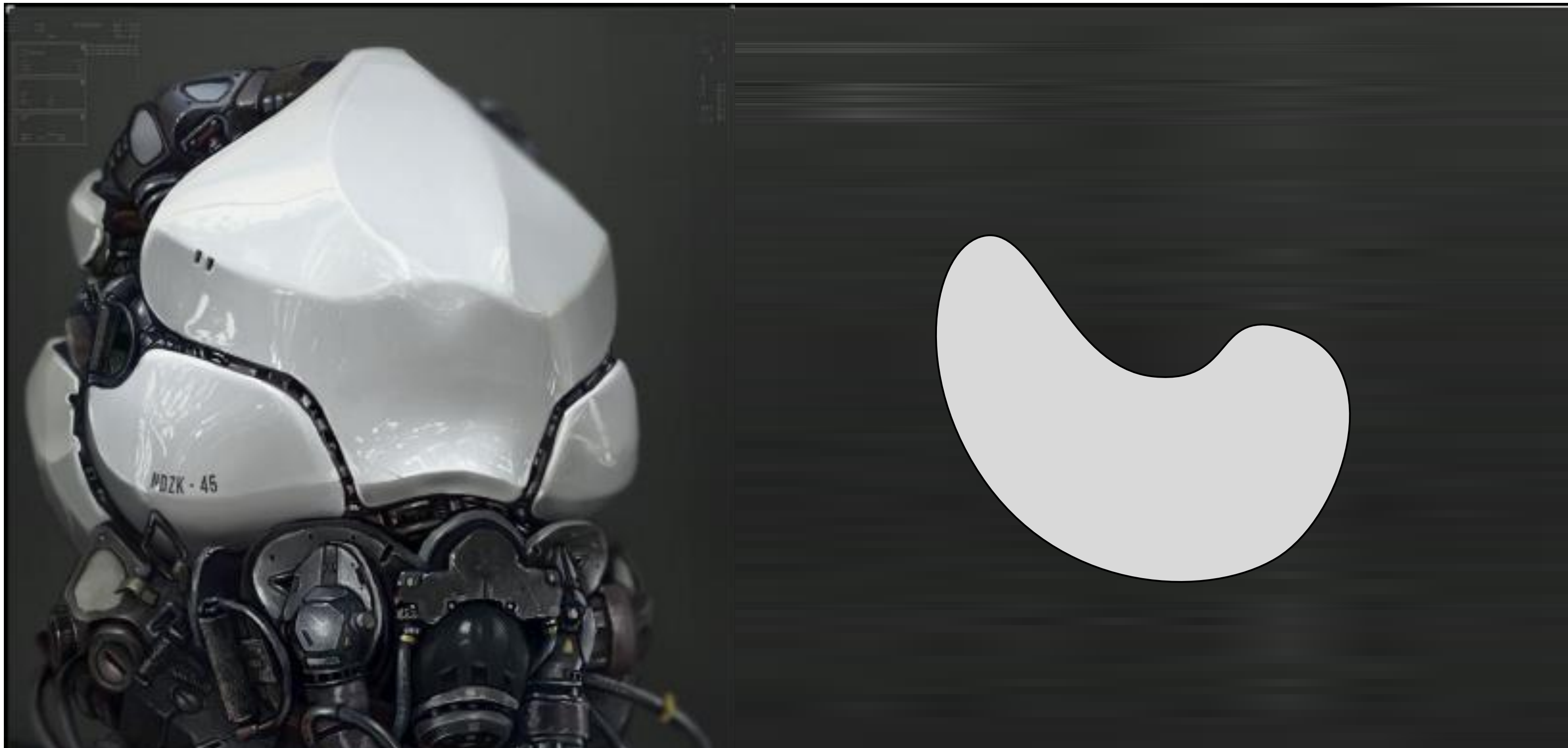
Does not look like much, but ..

… repeat forever …

Does not look like much, but ..



.. Is the key to high-quality 3D geometry



Conclusions

- Surfaces are a simple extension to curves
- Really just a tensor-product between two curves
  - One curve gets extruded along the other
- Subdivision surfaces are another way of generating curves
  - Particularly amenable to GPU implementation!