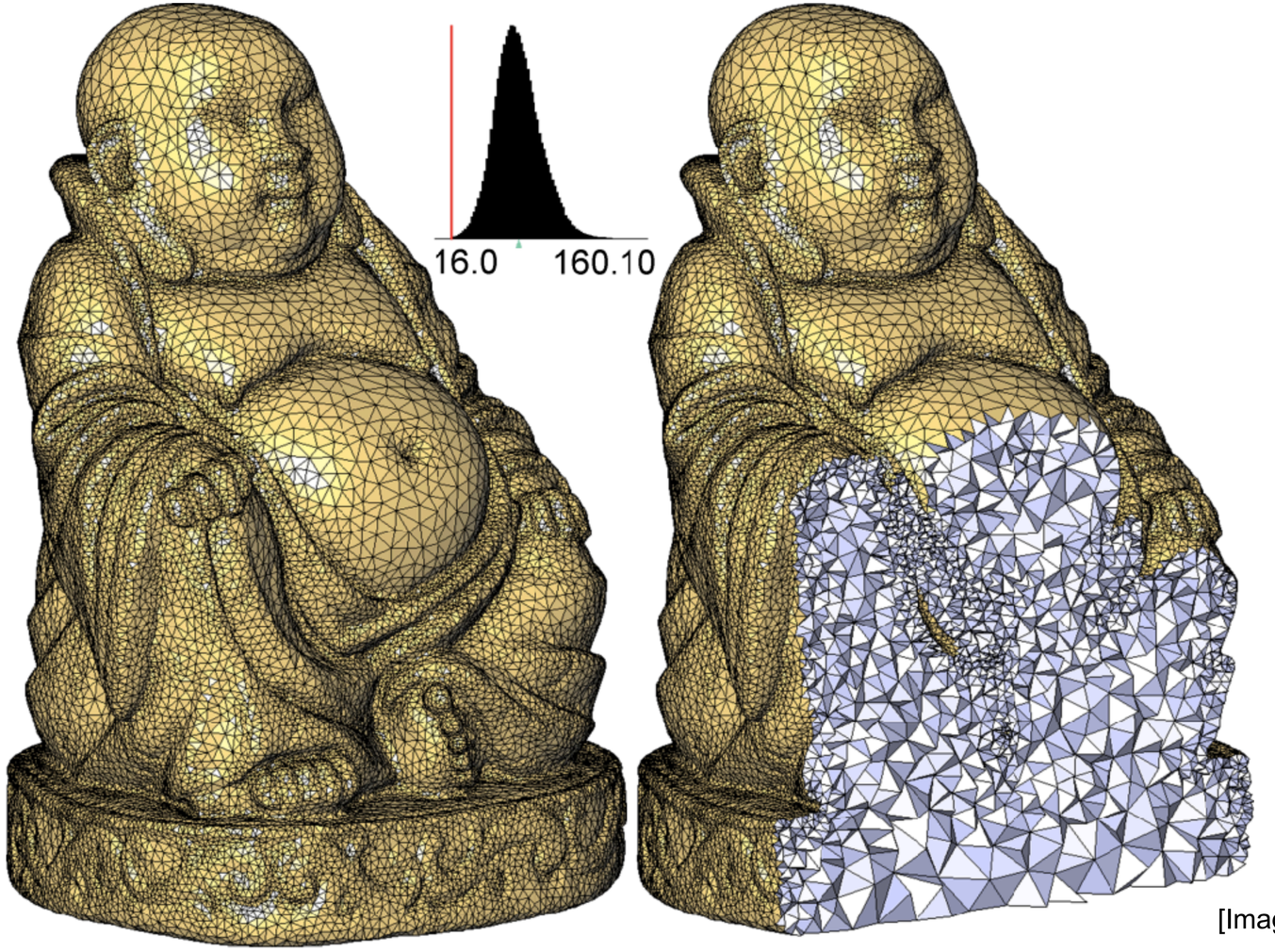
Winter 2022-23

# Acquisition and Processing of 3D Geometry

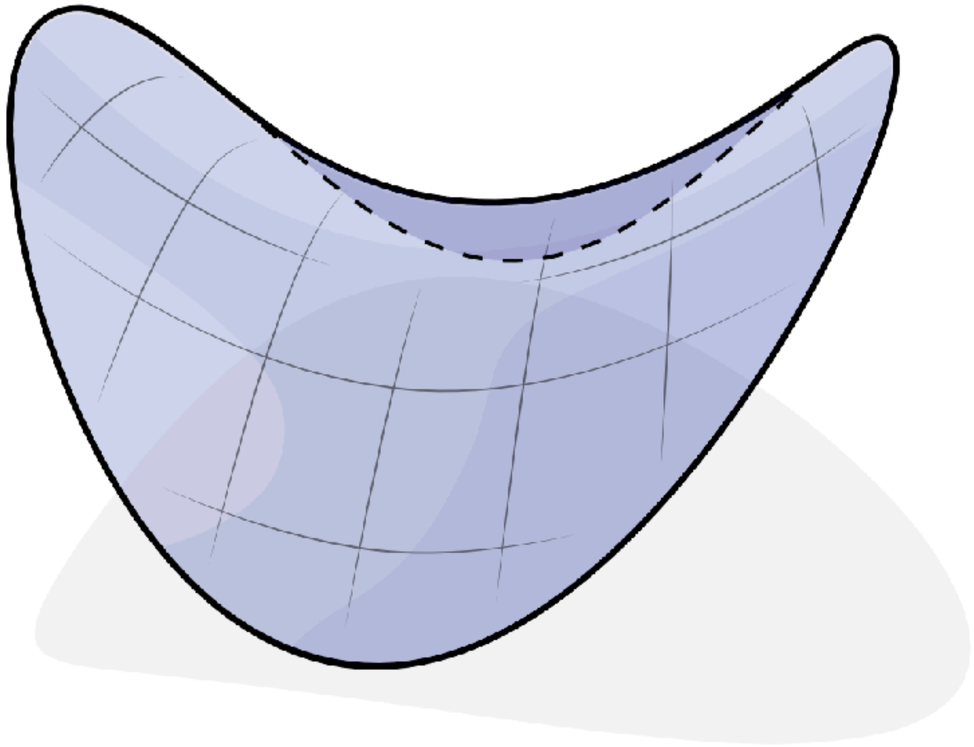
(COMP0119)

Niloy J. Mitra

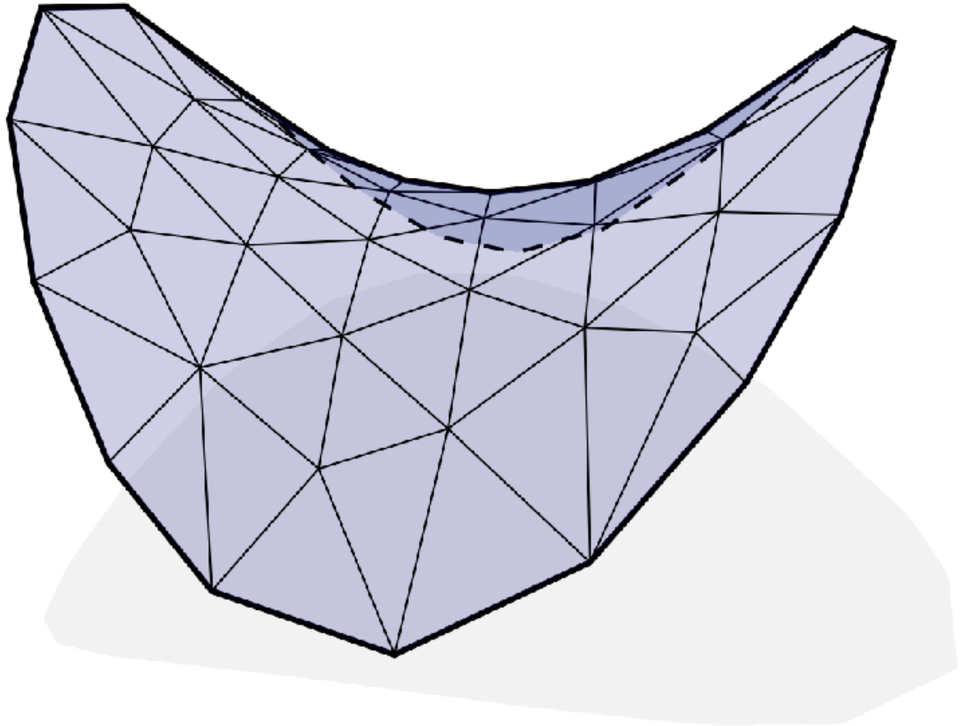



[Image source: Geometrica]

# Continuous vs Discrete Representation



Continuous



Discrete

[Image from Keenan Crane]

# Interactive Shape Modeling

Tools for design, editing and animation of digital shapes

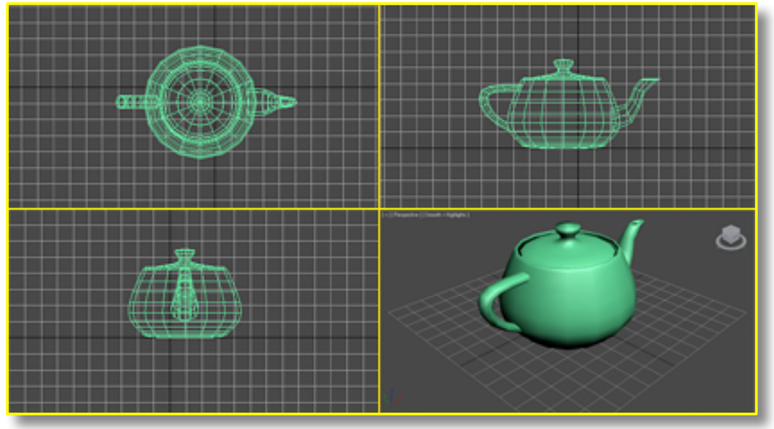
- **Interactive:** fast algorithms
- **Intuitive:** convenient interface + predictable outcome

# Digital Shape Modeling

How do shapes find their way into computers?

- Geometric modeling is difficult

Humans have no direct “video out”



“Translation” from 2D to 3D is hard

# Digital Shape Modeling

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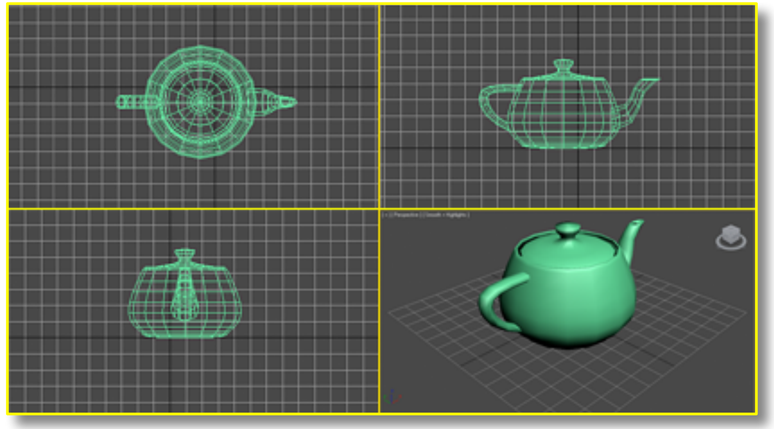
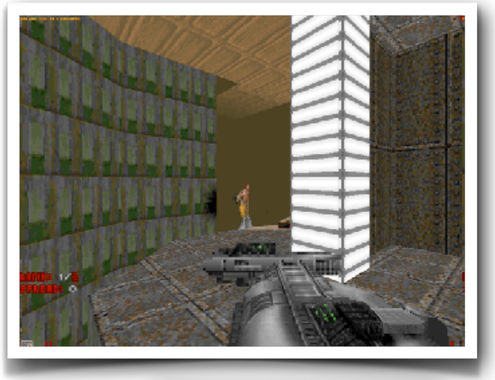
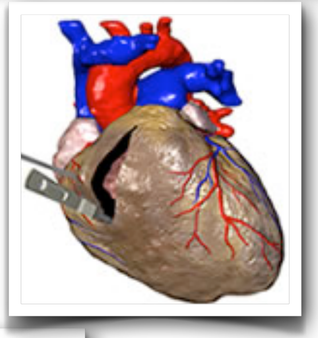
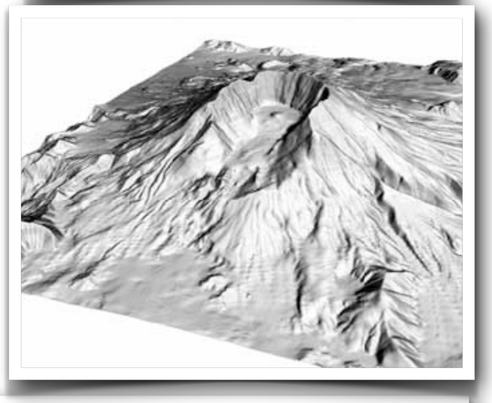
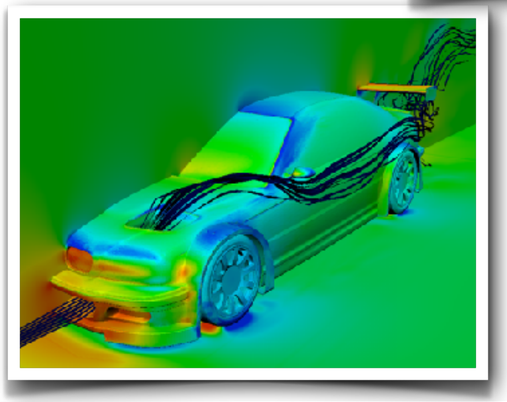


*computation can compensate for lack of direct ability to convey visual information*

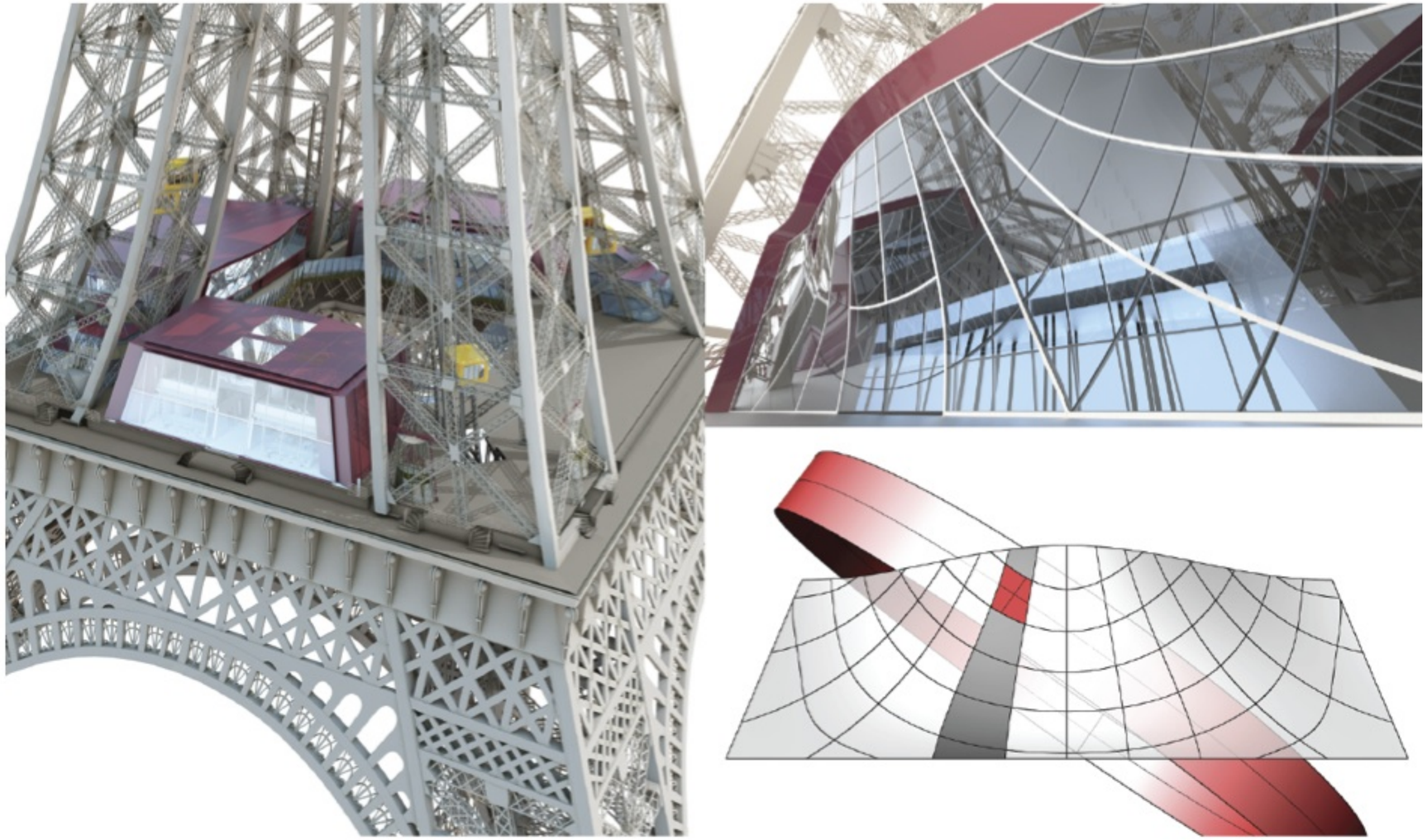
# Application Areas

- Computer games
- Movie production
- Engineering
- Cultural Heritage
- Topography
- Architecture
- Medicine
- many more …



# Form Finding (Collaboration w/ Evolute)



# Digitizing Physical Models

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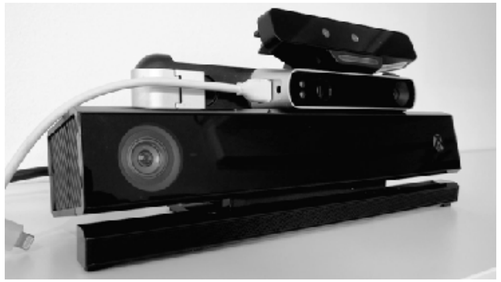

laser scanner

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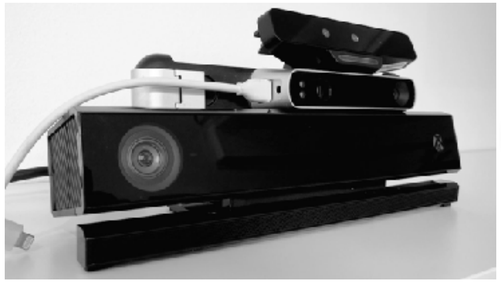

laser scanner

3D geometry

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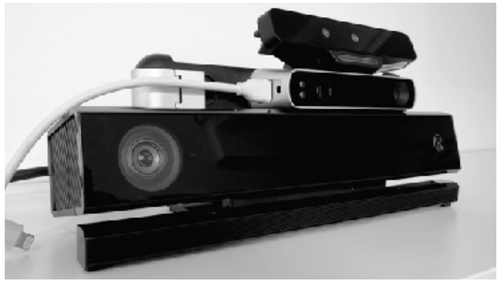


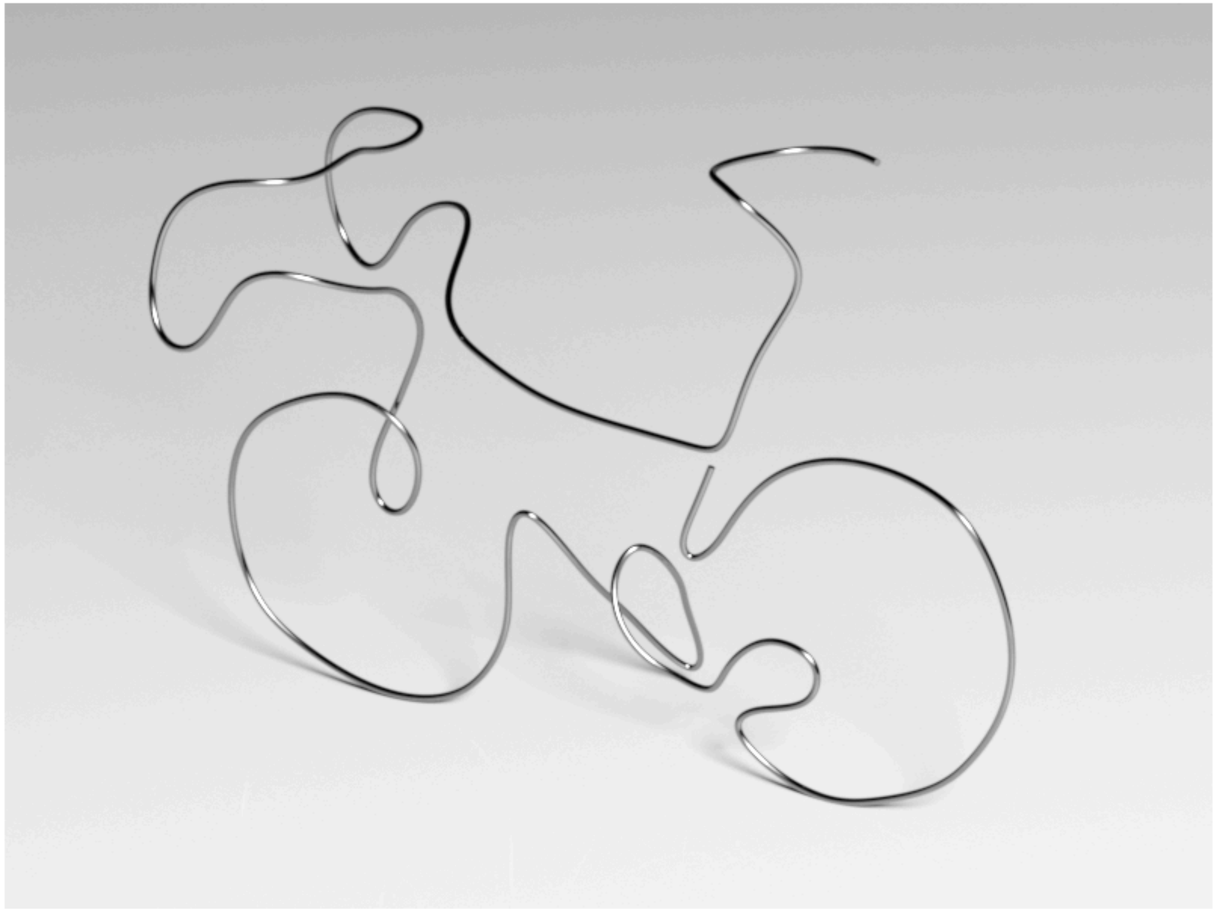
laser scanner

3D geometry

entertainment

# Scanning

BIKE



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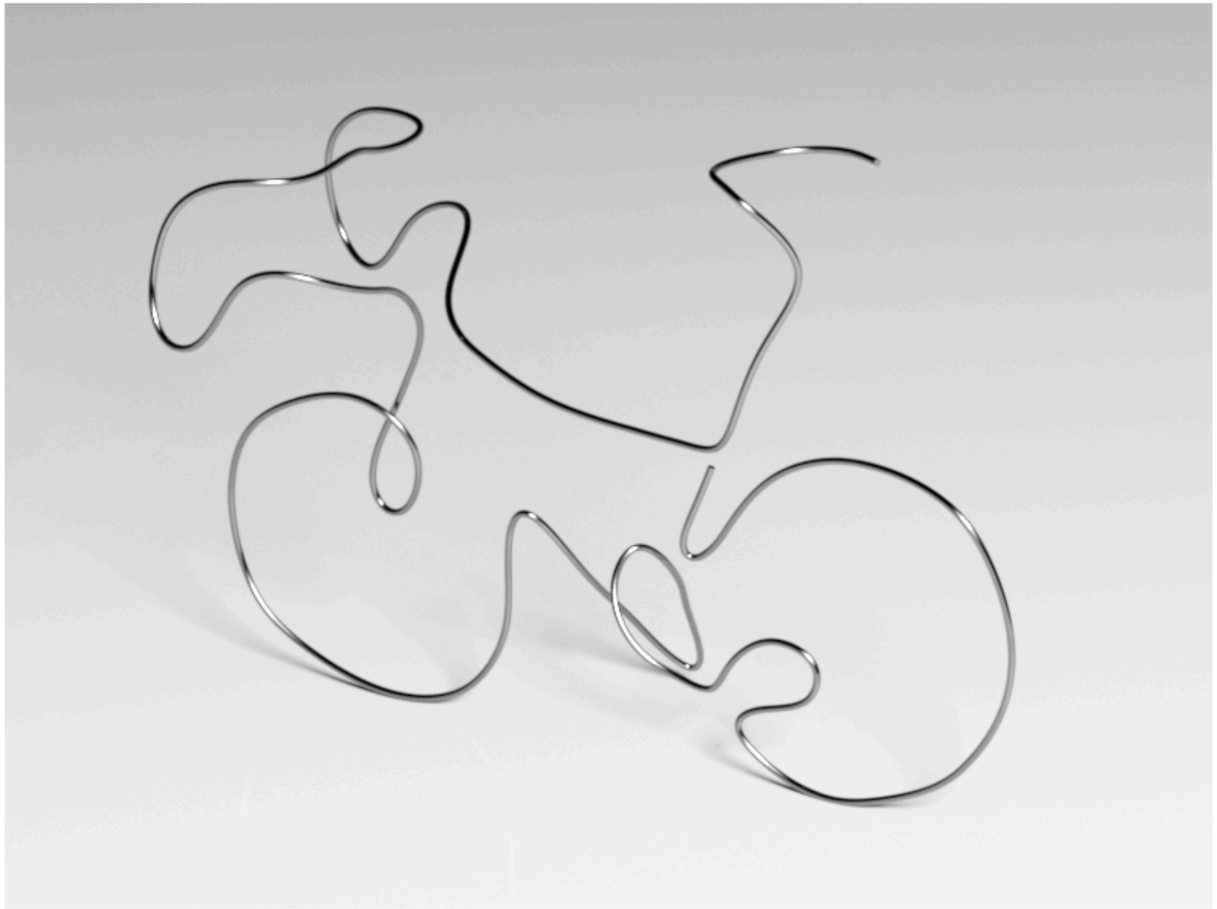
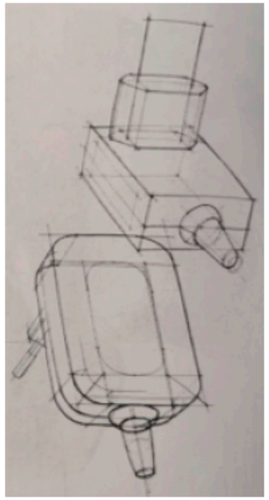


# 2D to 3D 'Lifting'

(a) inspirational sketch



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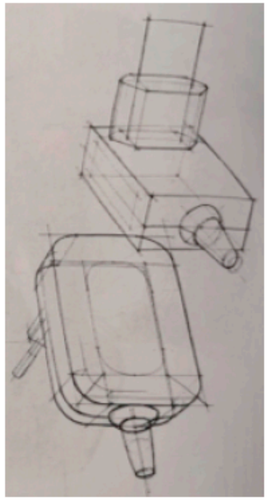
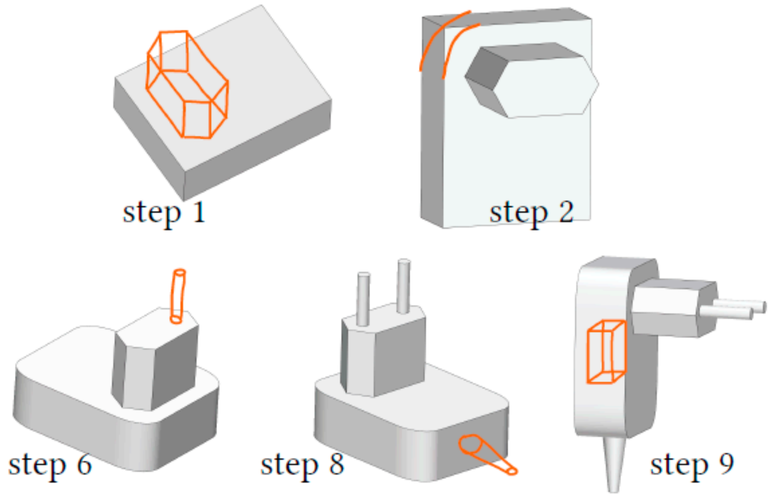


(b) sketching sequence using Sketch2CAD



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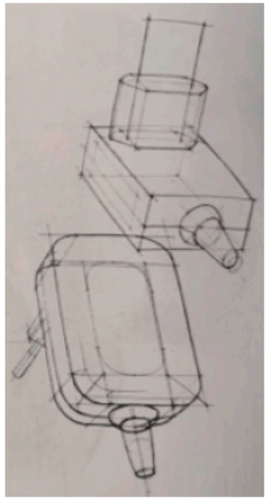

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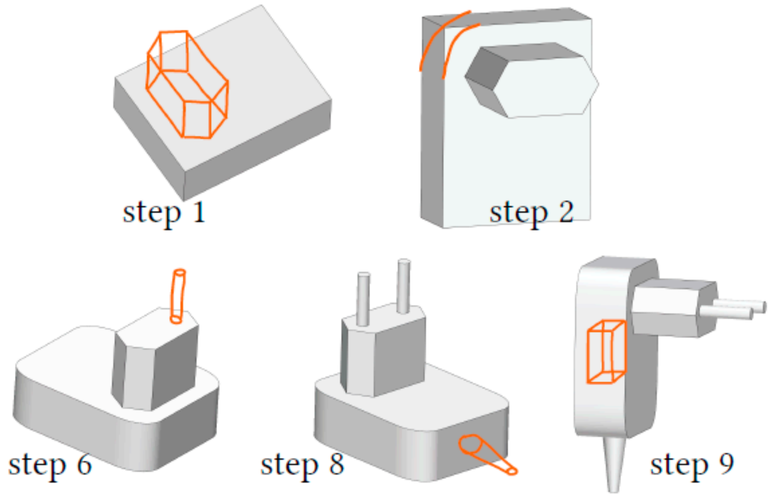


**(c) inferred CAD instructions**

```
AddPolyhedron: < plane 3, length 0.25 >
BevelCorner: < plane 3, corner 3 >
BevelCorner: < plane 3, corner 2 >
BevelCorner: < plane 3, corner 0 >
BevelCorner: < plane 3, corner 1 >
AddSweepShape: < plane 4, length 0.20 >
AddSweepShape: < plane 4, length 0.20 >
AddSweepShape: < plane 1, length 0.23 >
SubtractPolyhedron: < plane 12, length 0.03 >
SubtractPolyhedron: < plane 13, length 0.03 >
```

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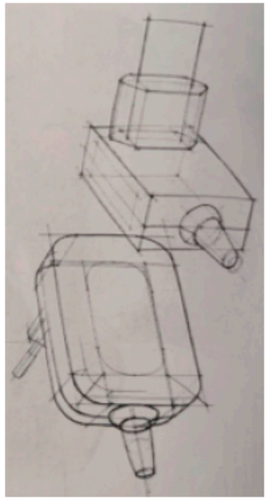

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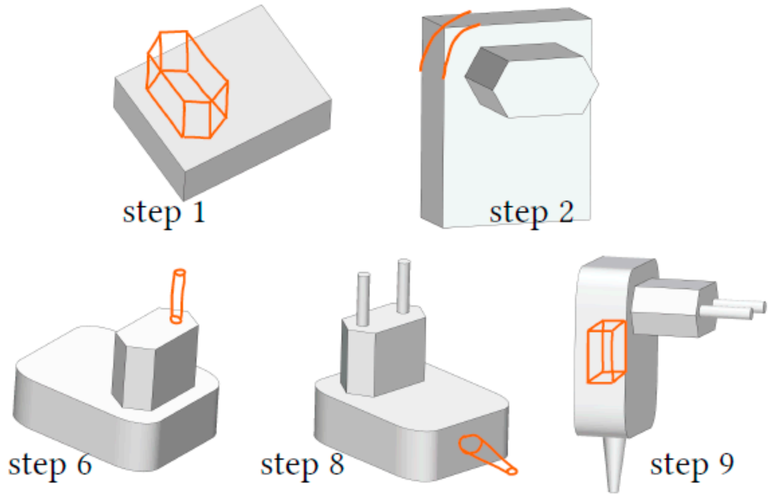


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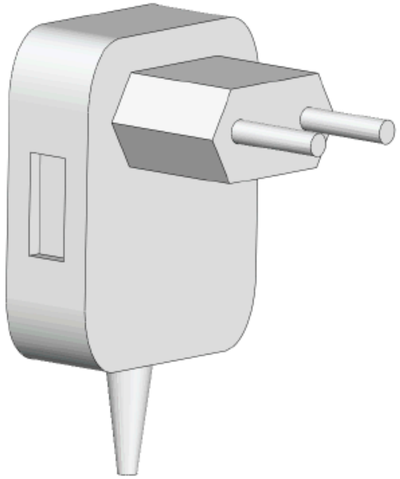
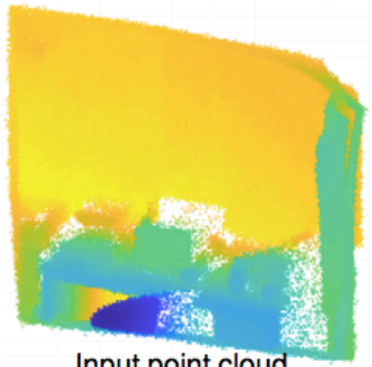
SubtractPolyhedron: < plane 12, length 0.03 >
SubtractPolyhedron: < plane 13, length 0.03 >
```

(d) CAD model

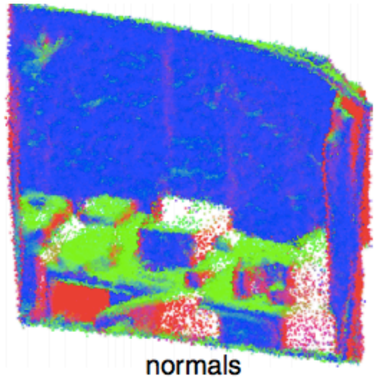


# Estimating Local Properties

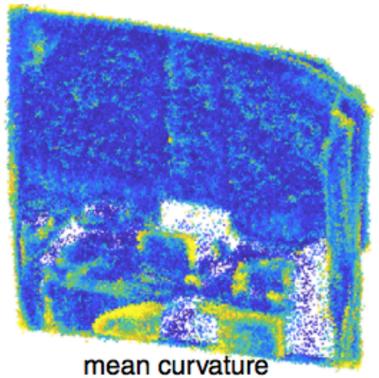
RGBD Image



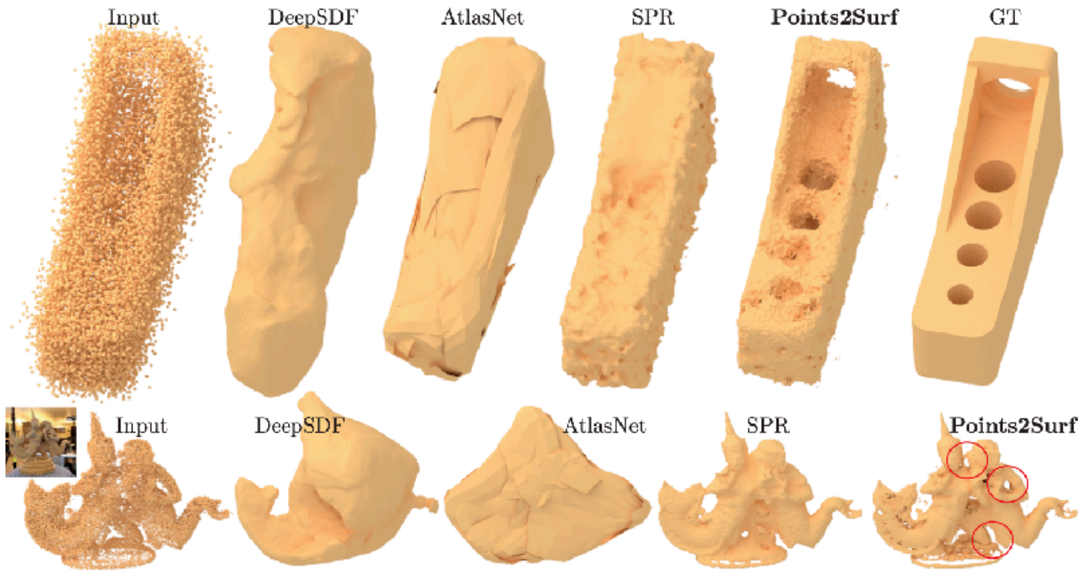
Input point cloud



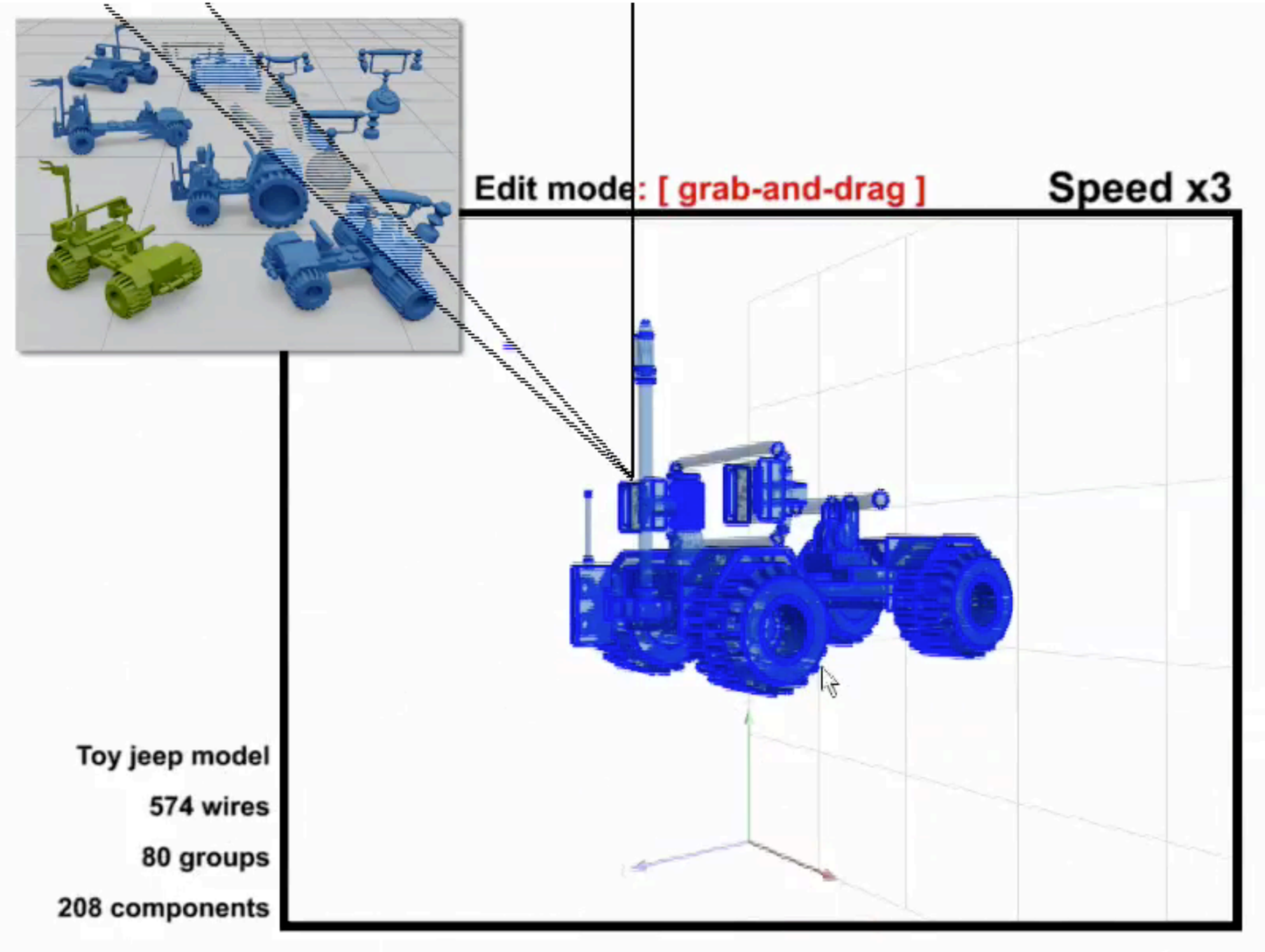
normals



mean curvature



# Mesh Manipulation



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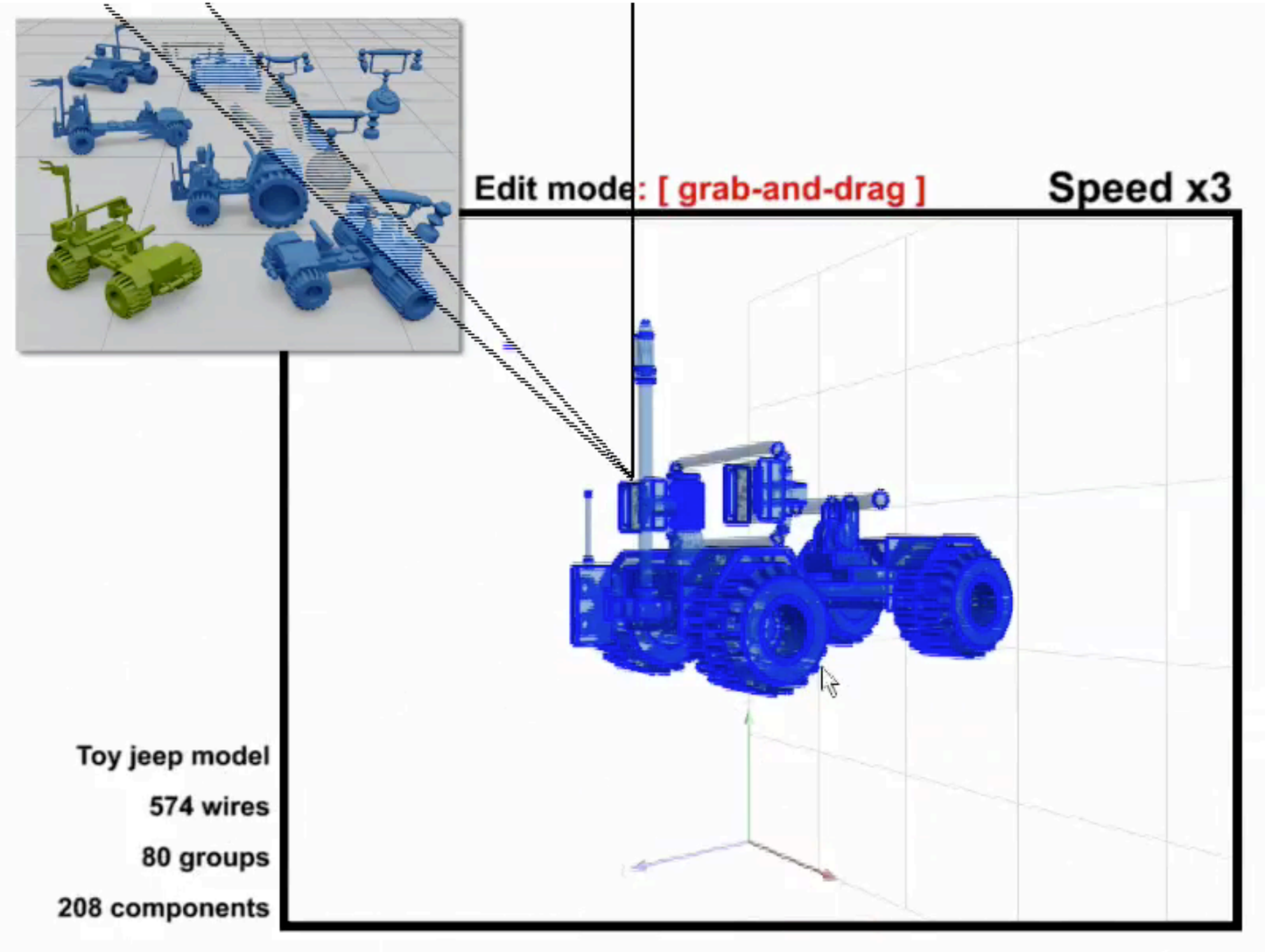
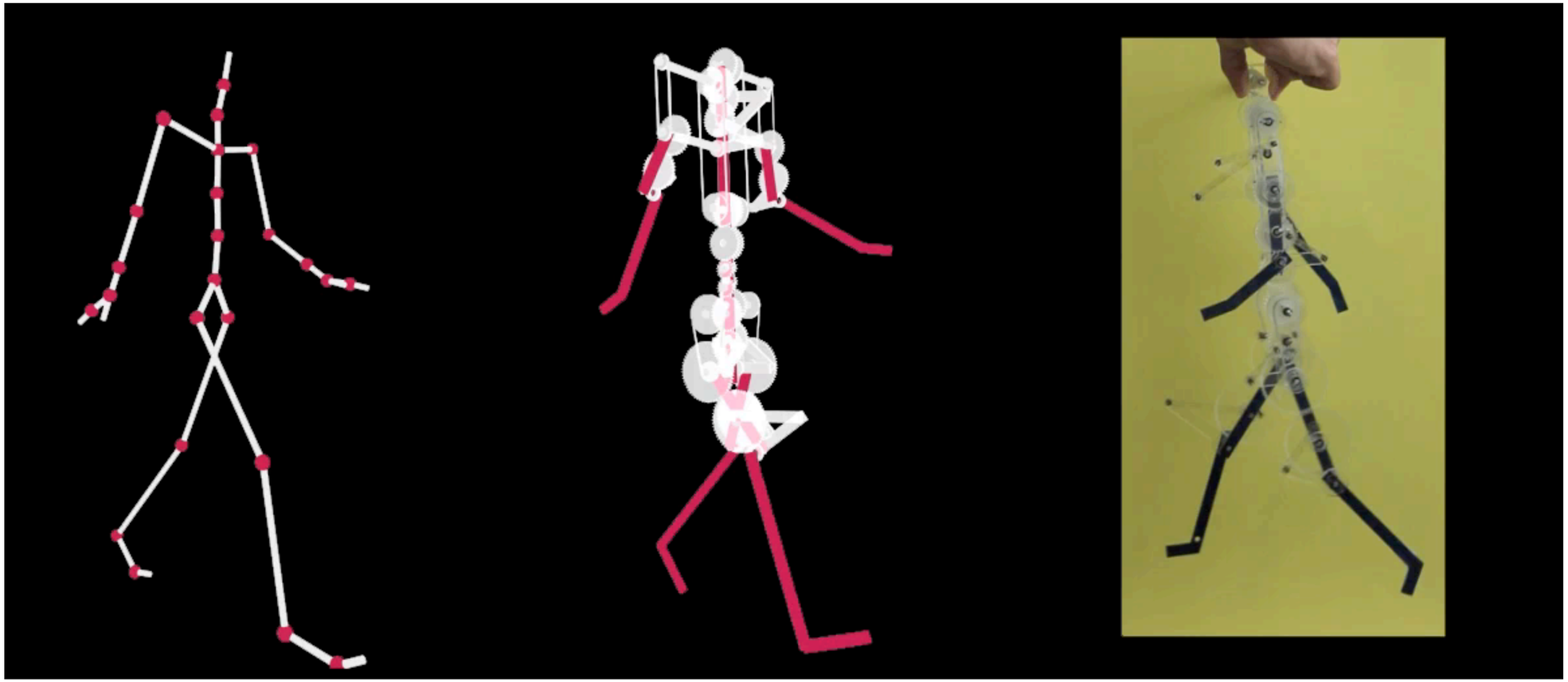


# Motion to Geometry



Input Motion | Generated Automaton | Physical Prototype

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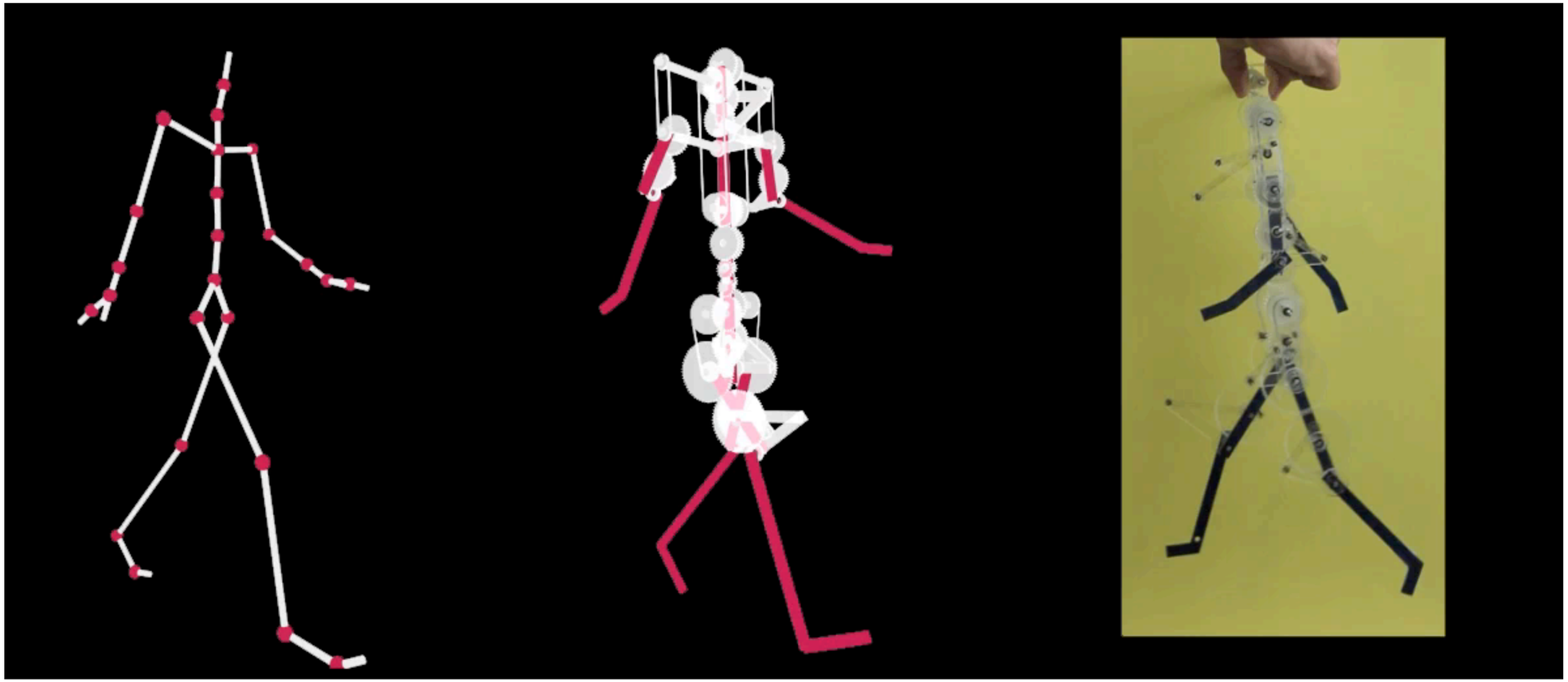

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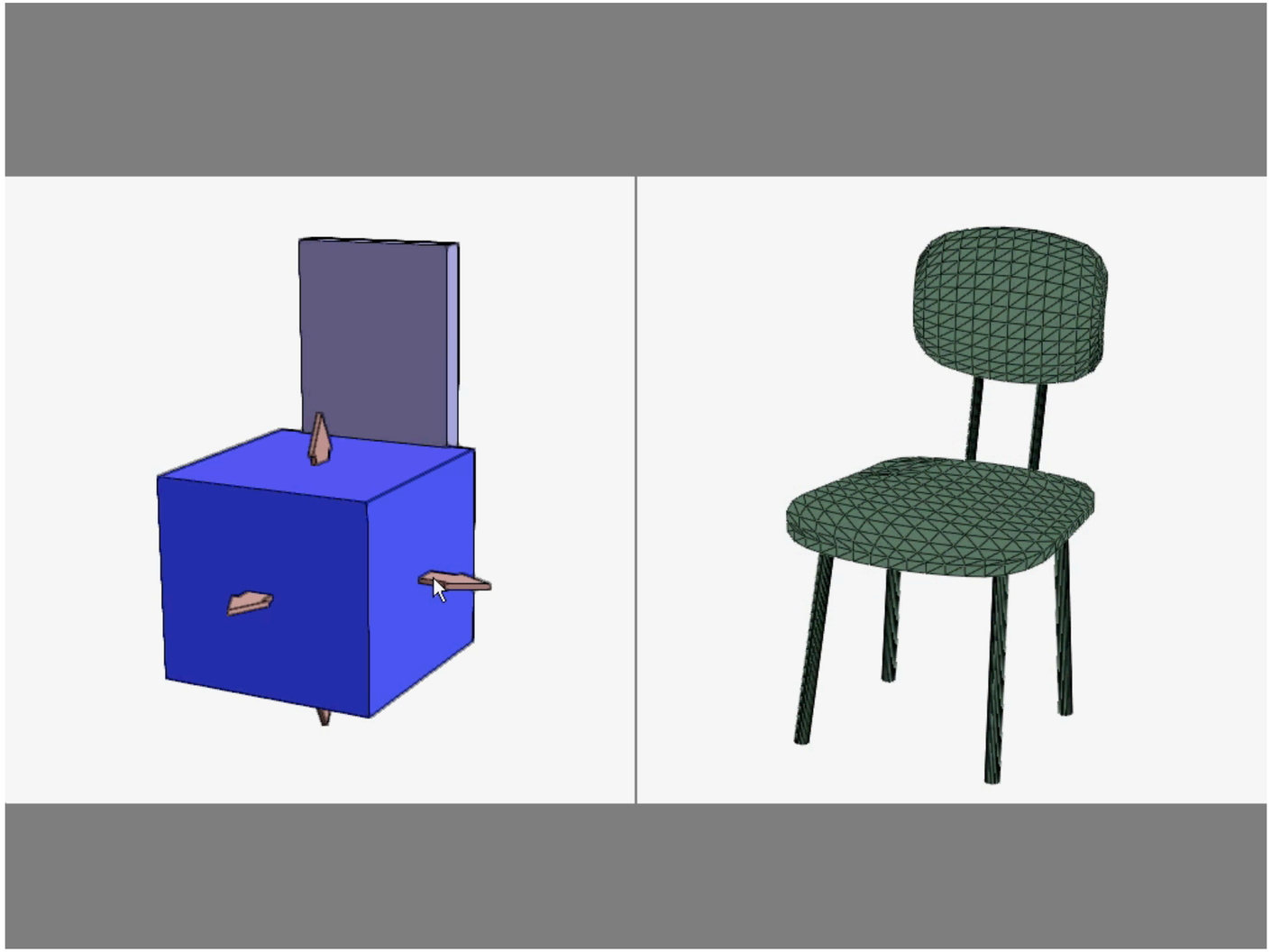
# Exploring Data Collections



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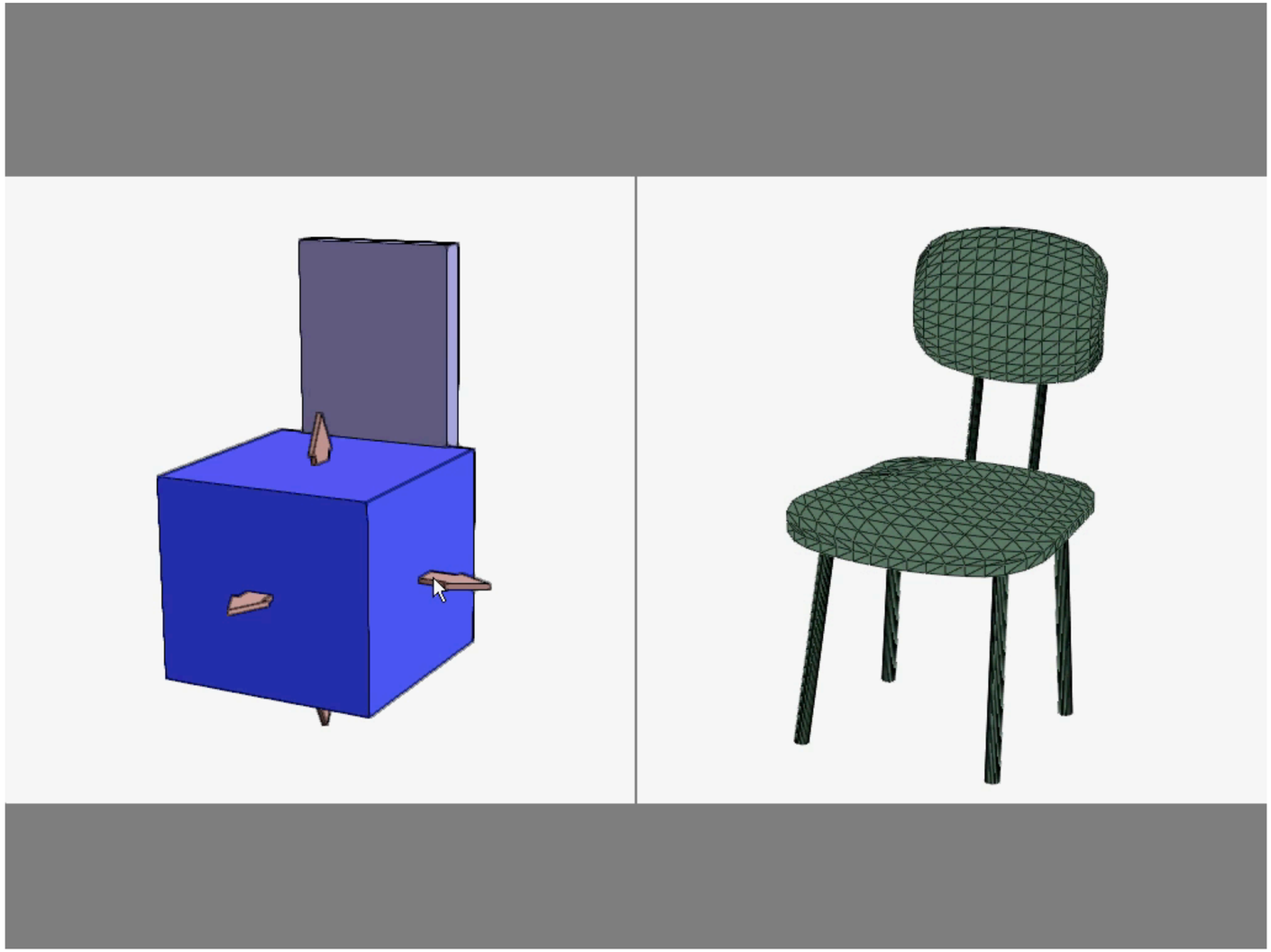

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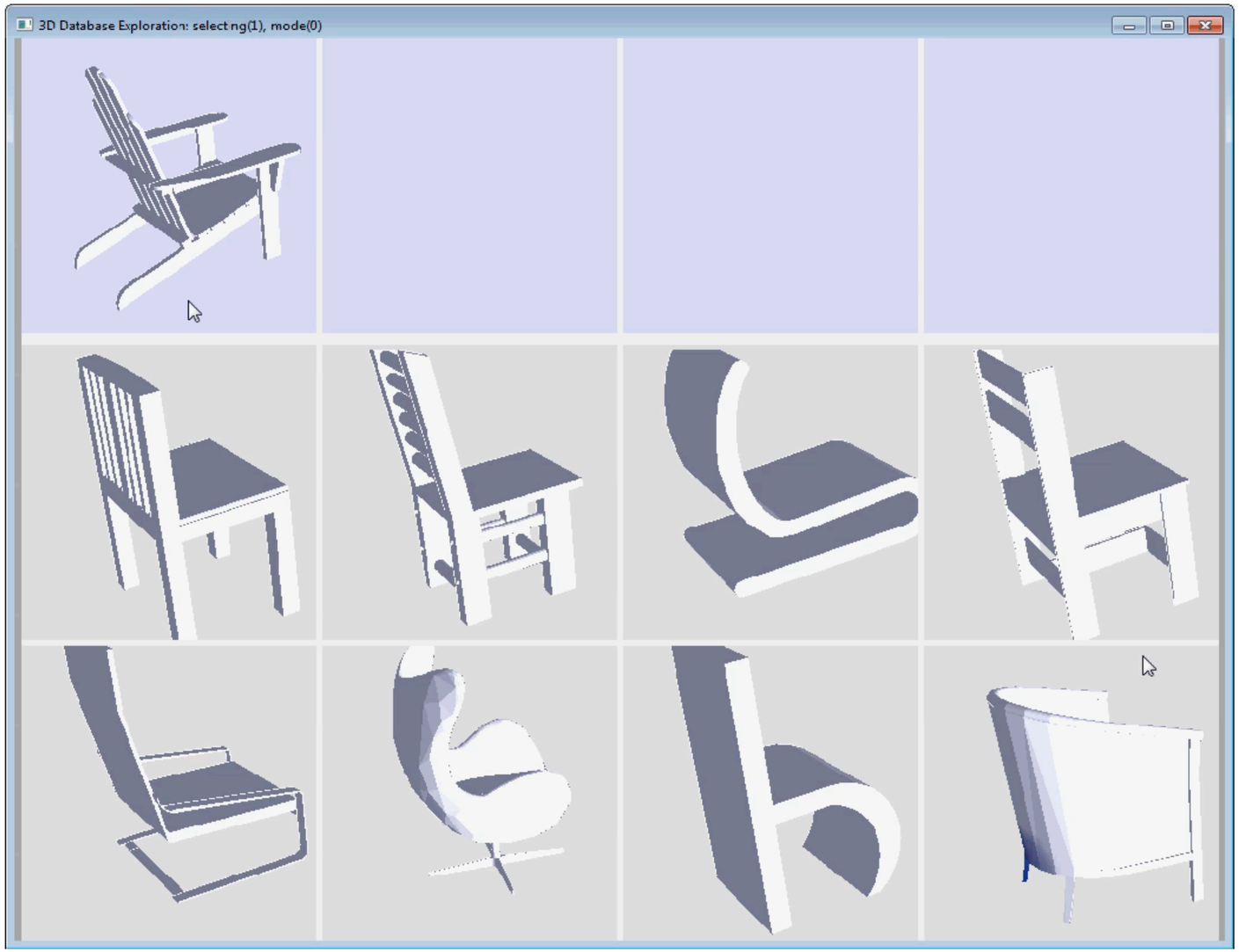

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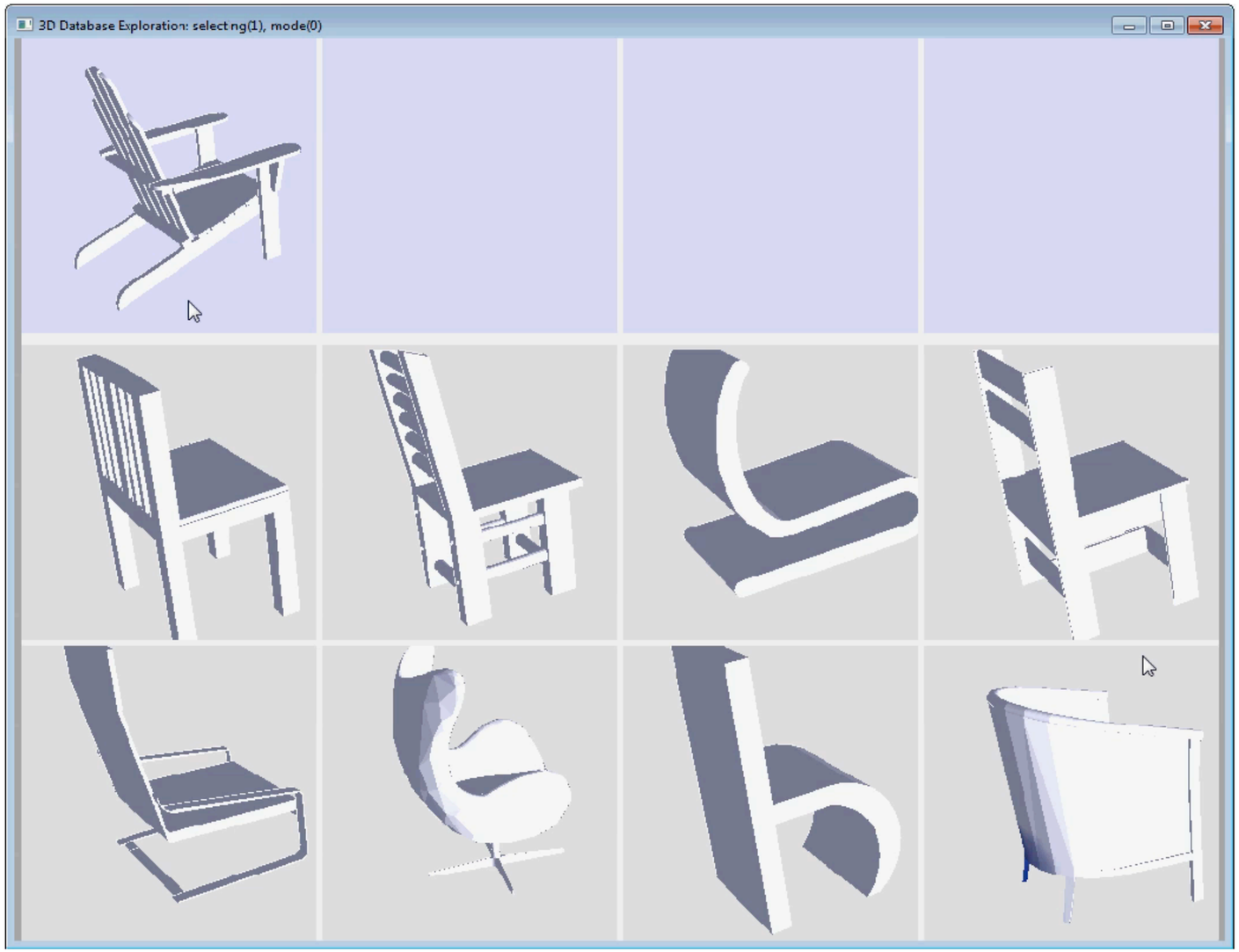
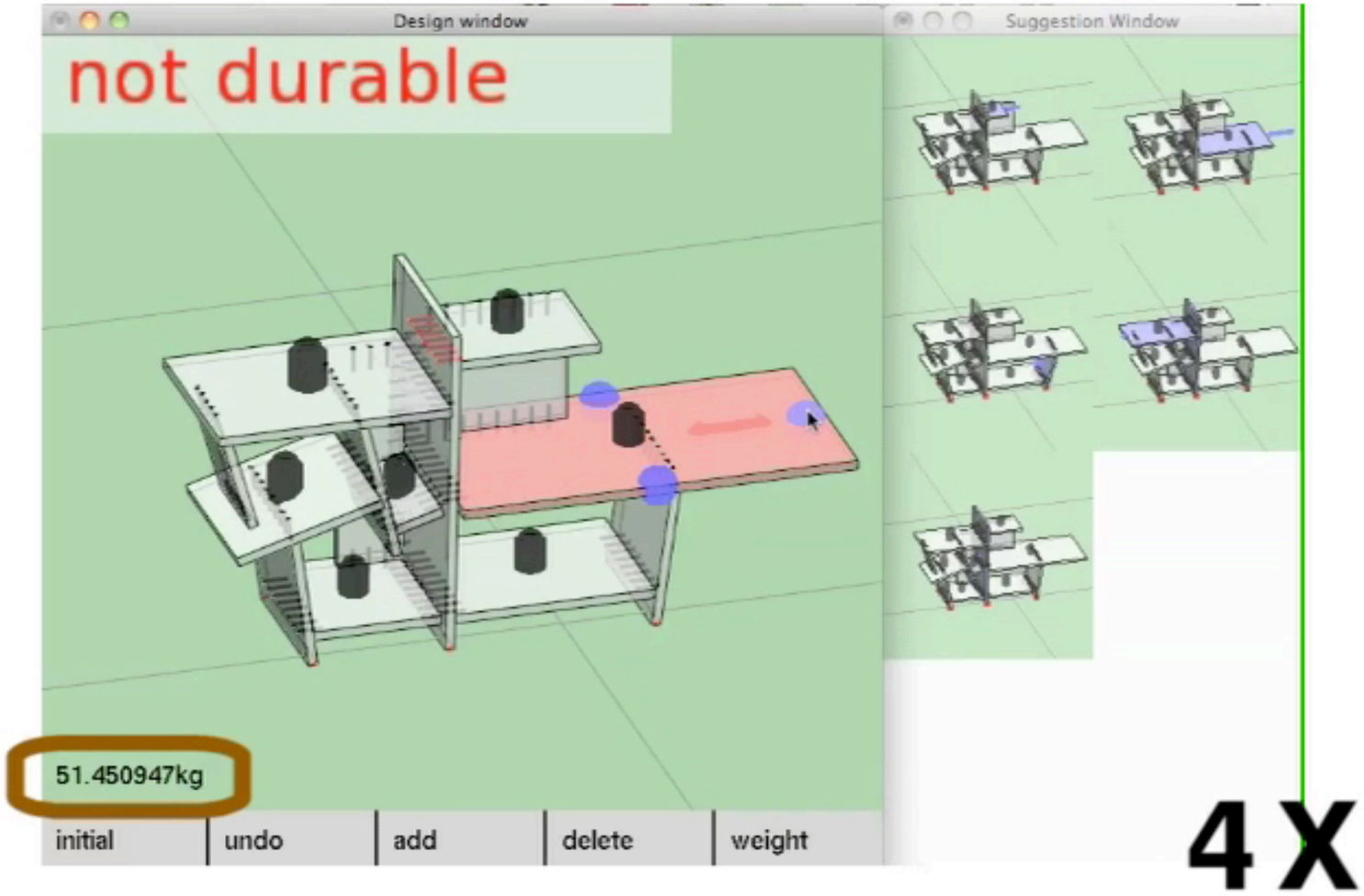


# “Forms as Force Diagrams”



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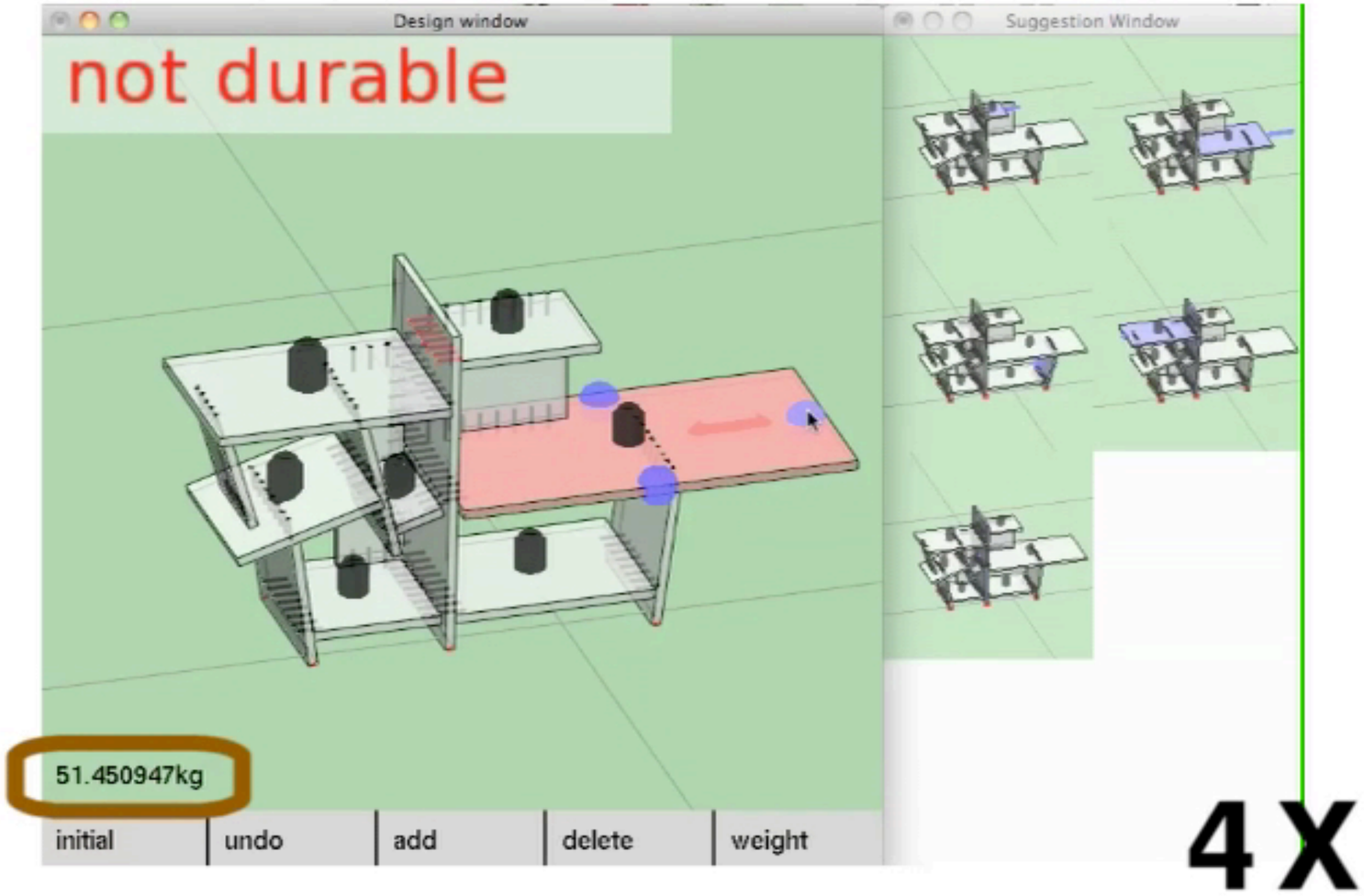
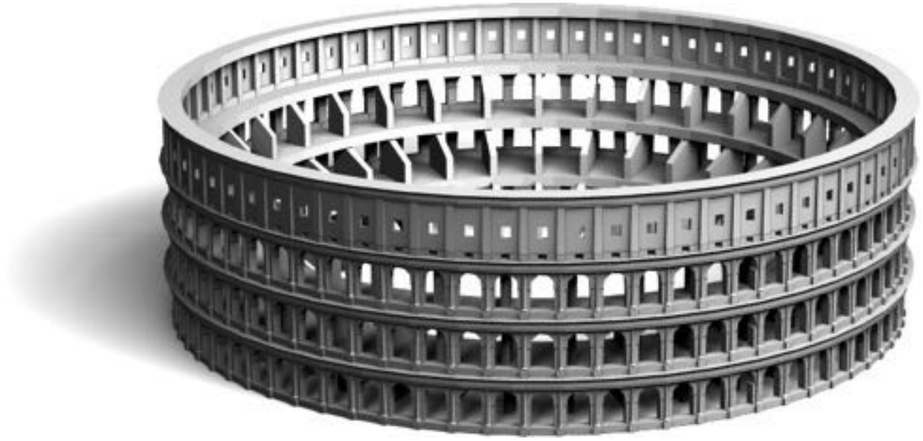


# Detecting Regularities

low-level geometry → (structure+element) + variations



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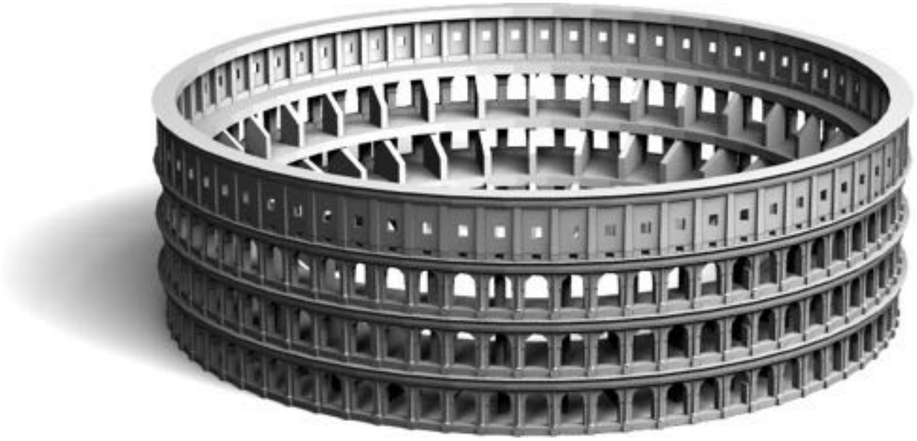

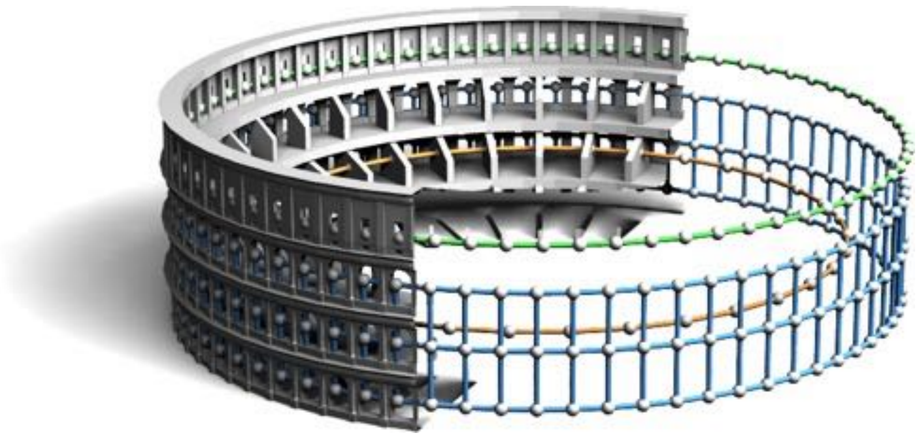

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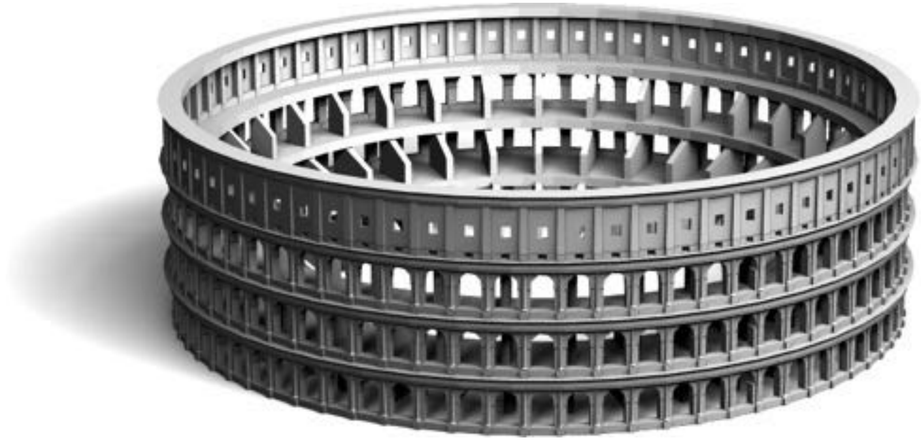

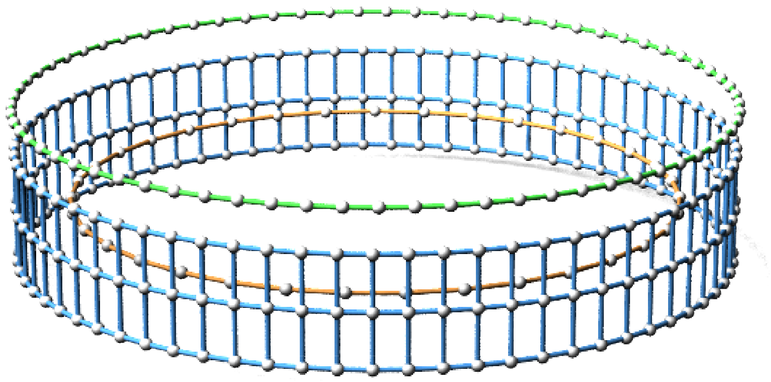

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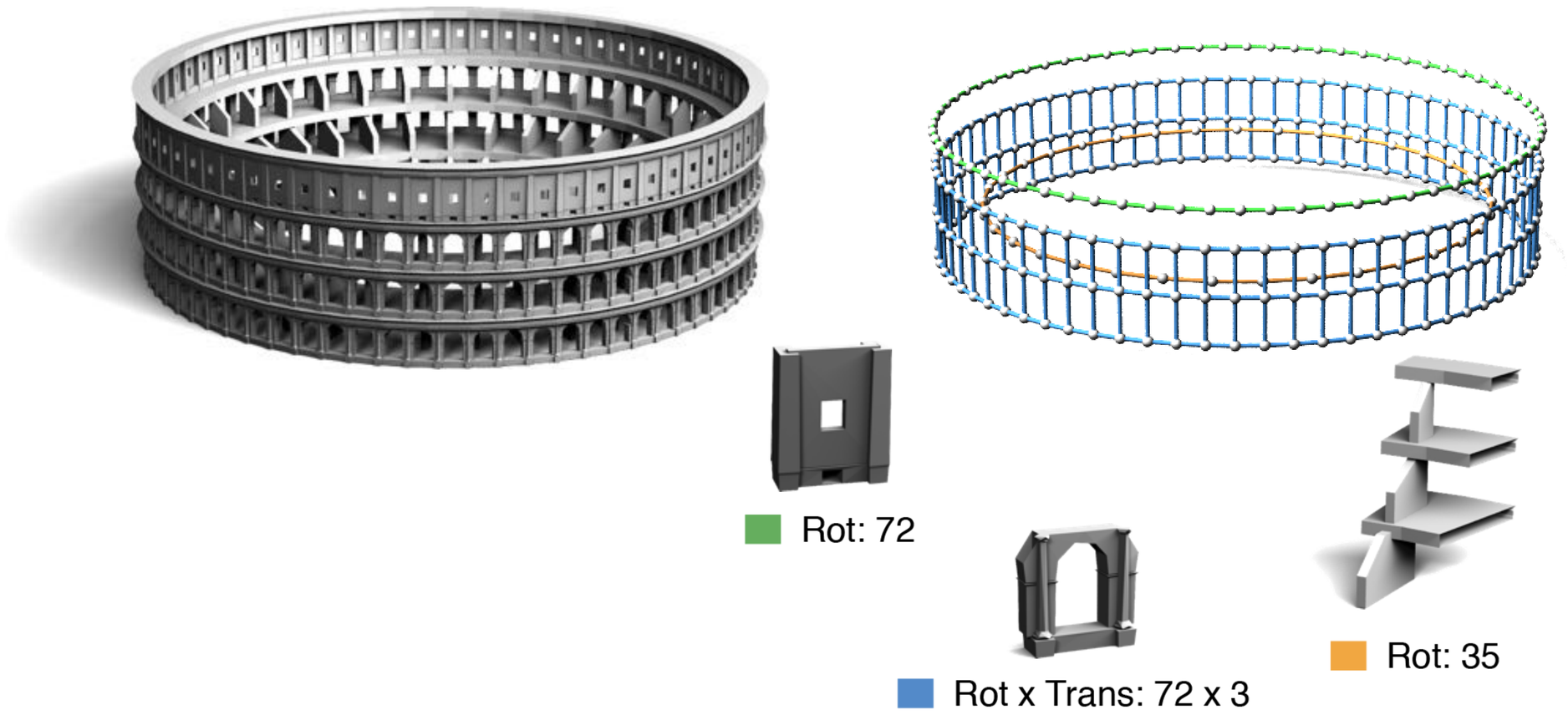
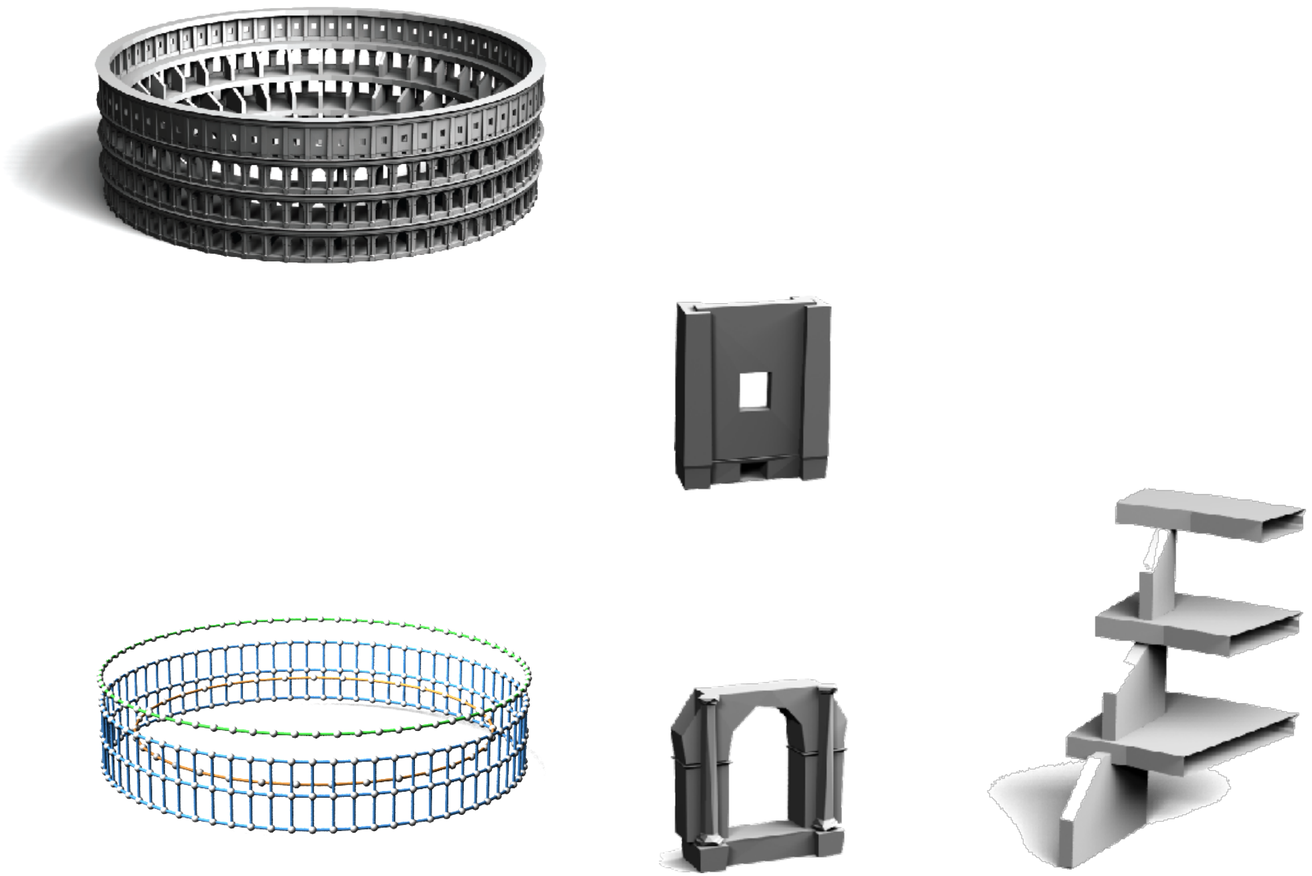


# Find and Replace



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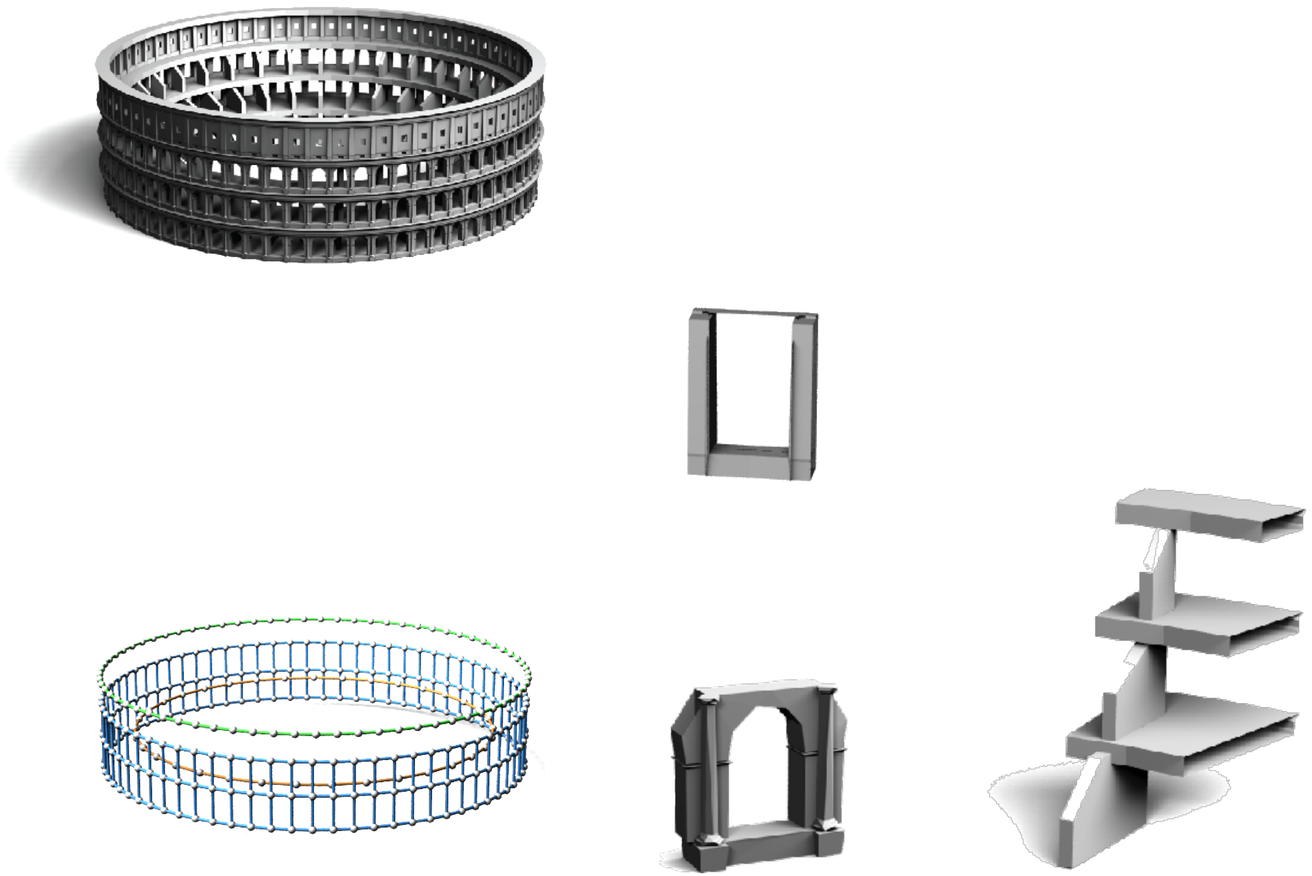

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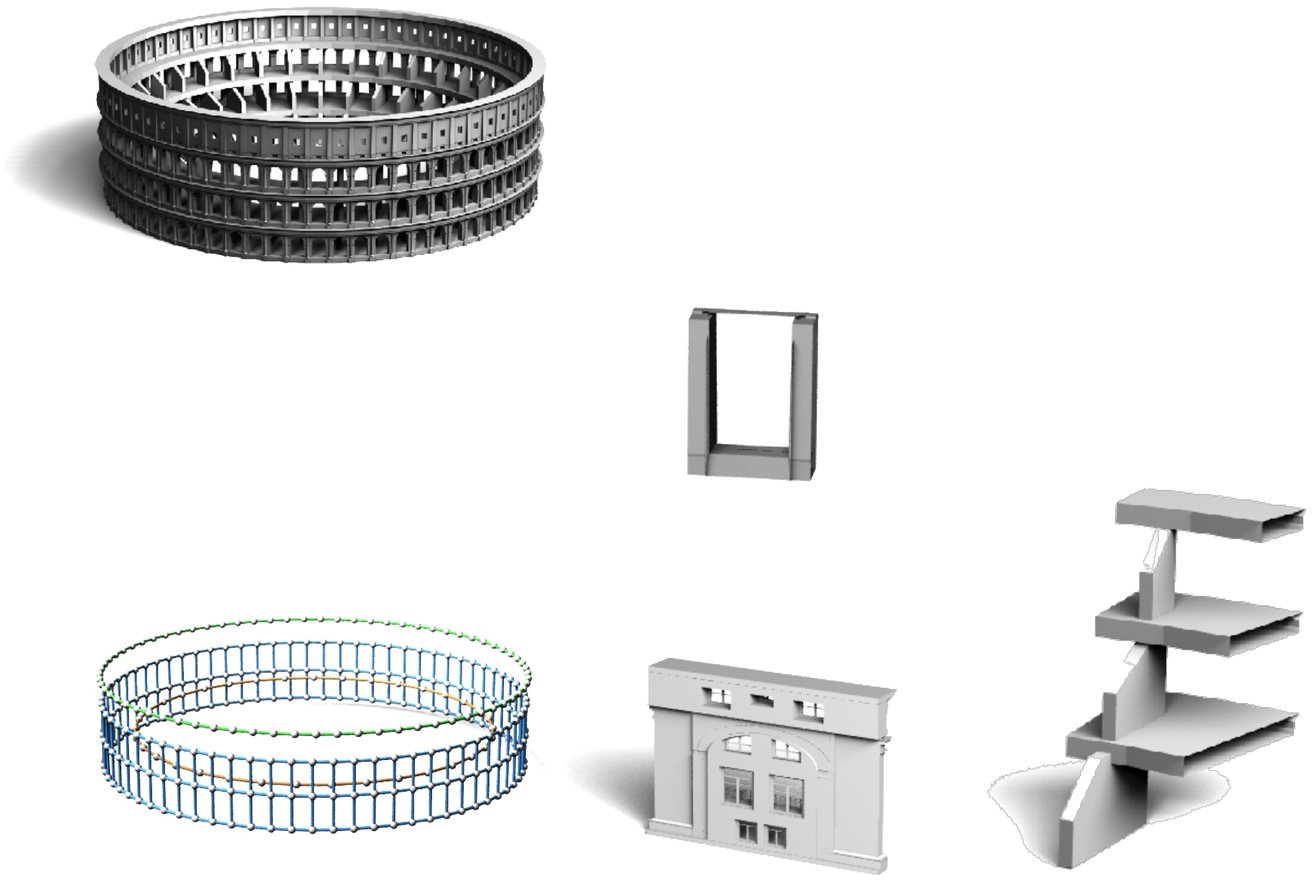

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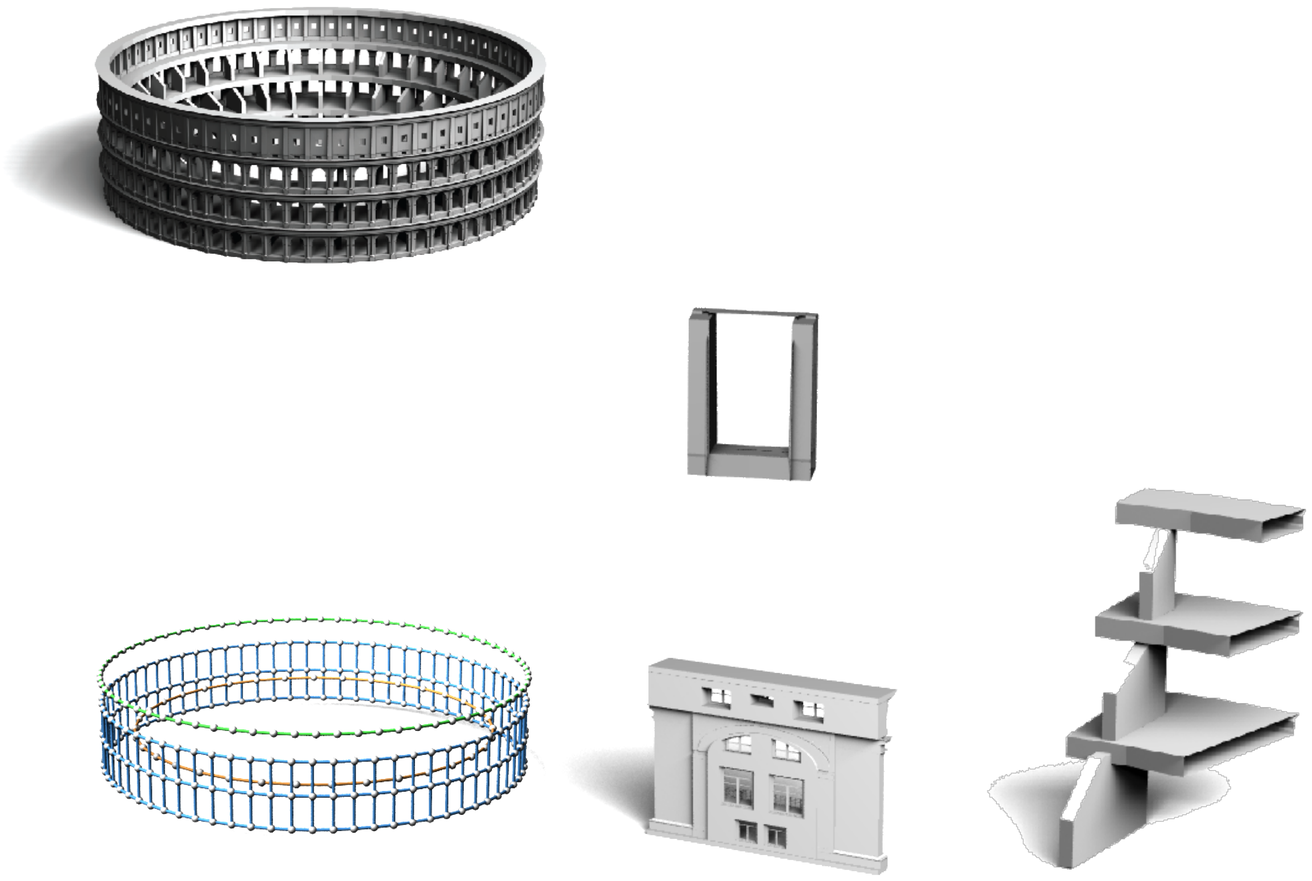

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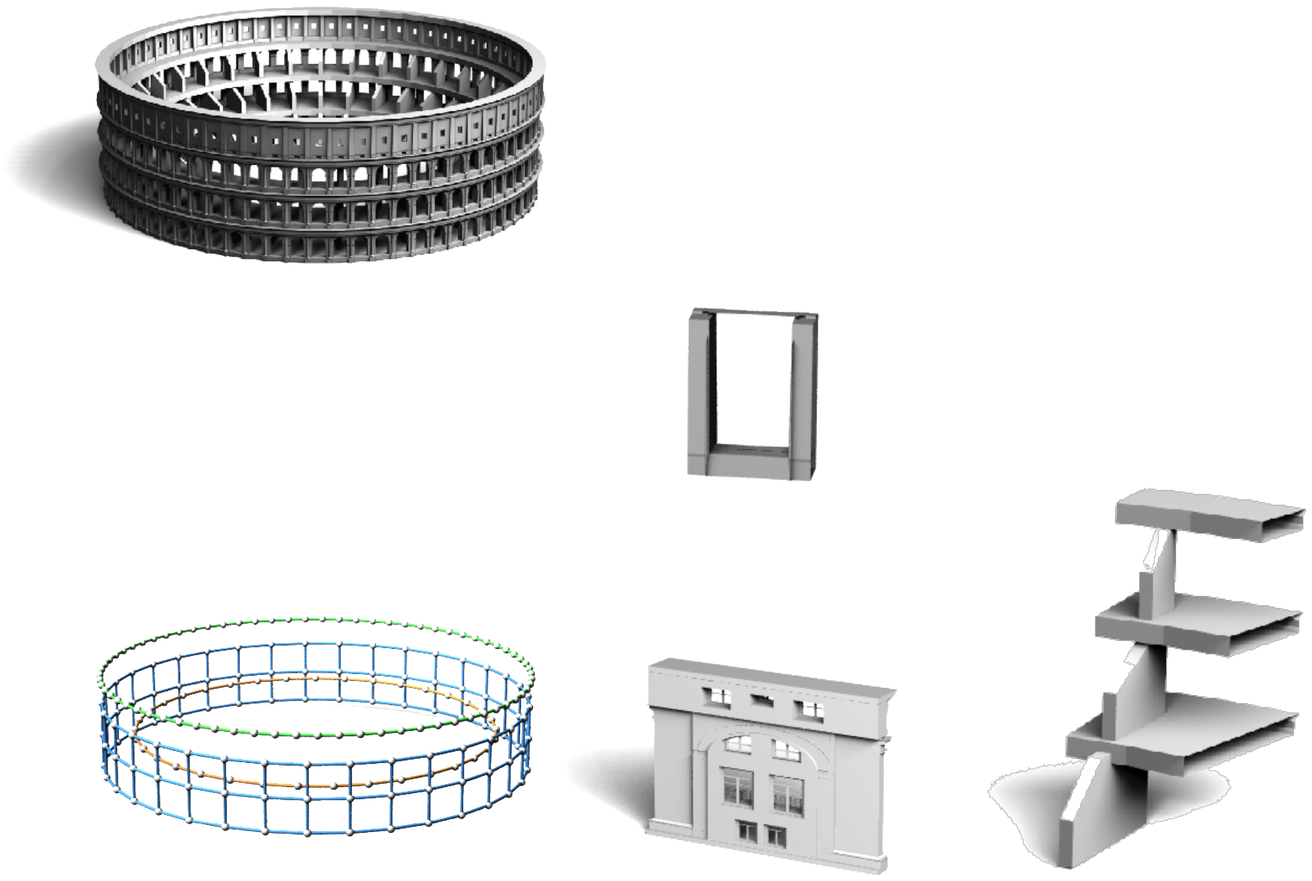

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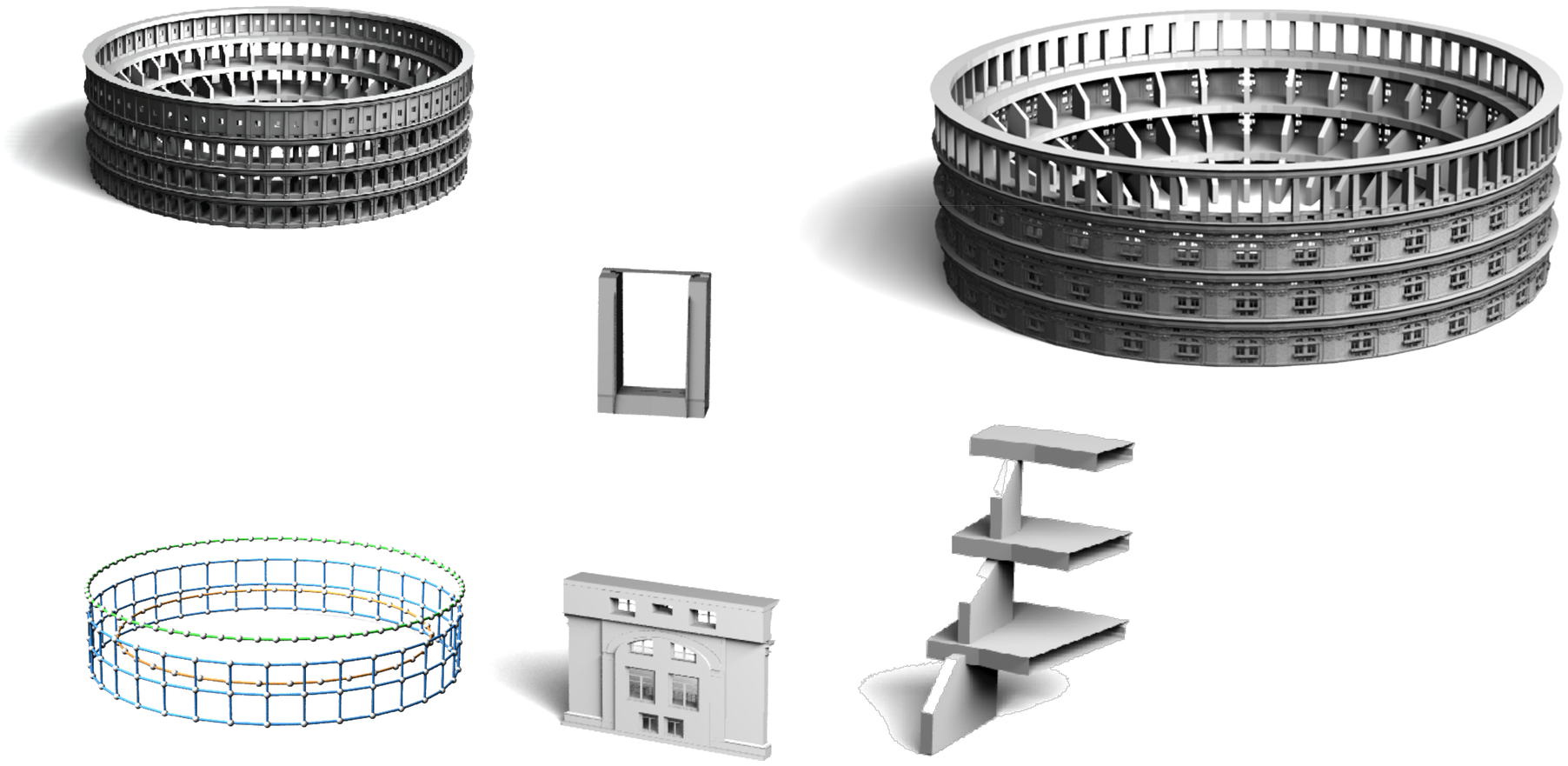
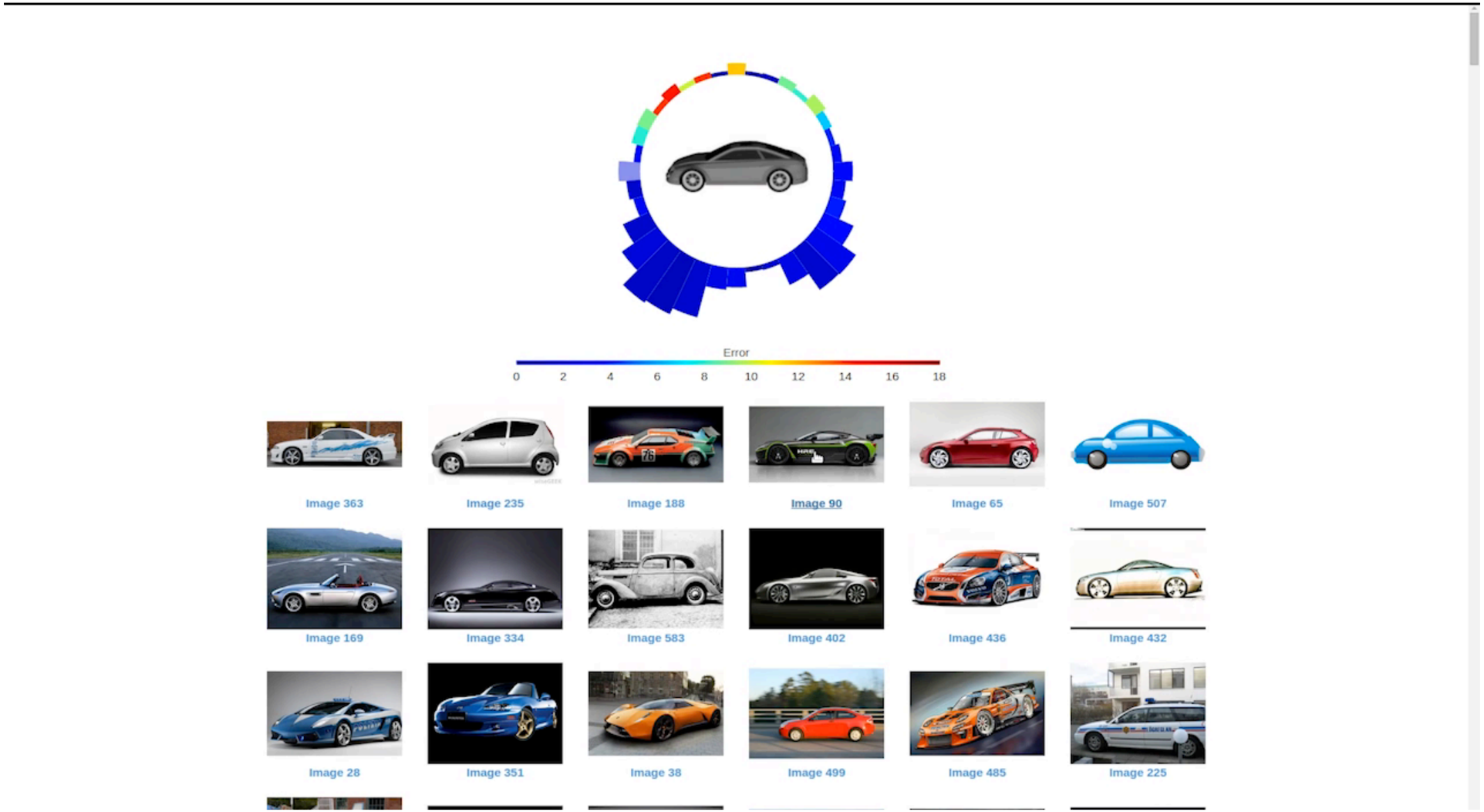
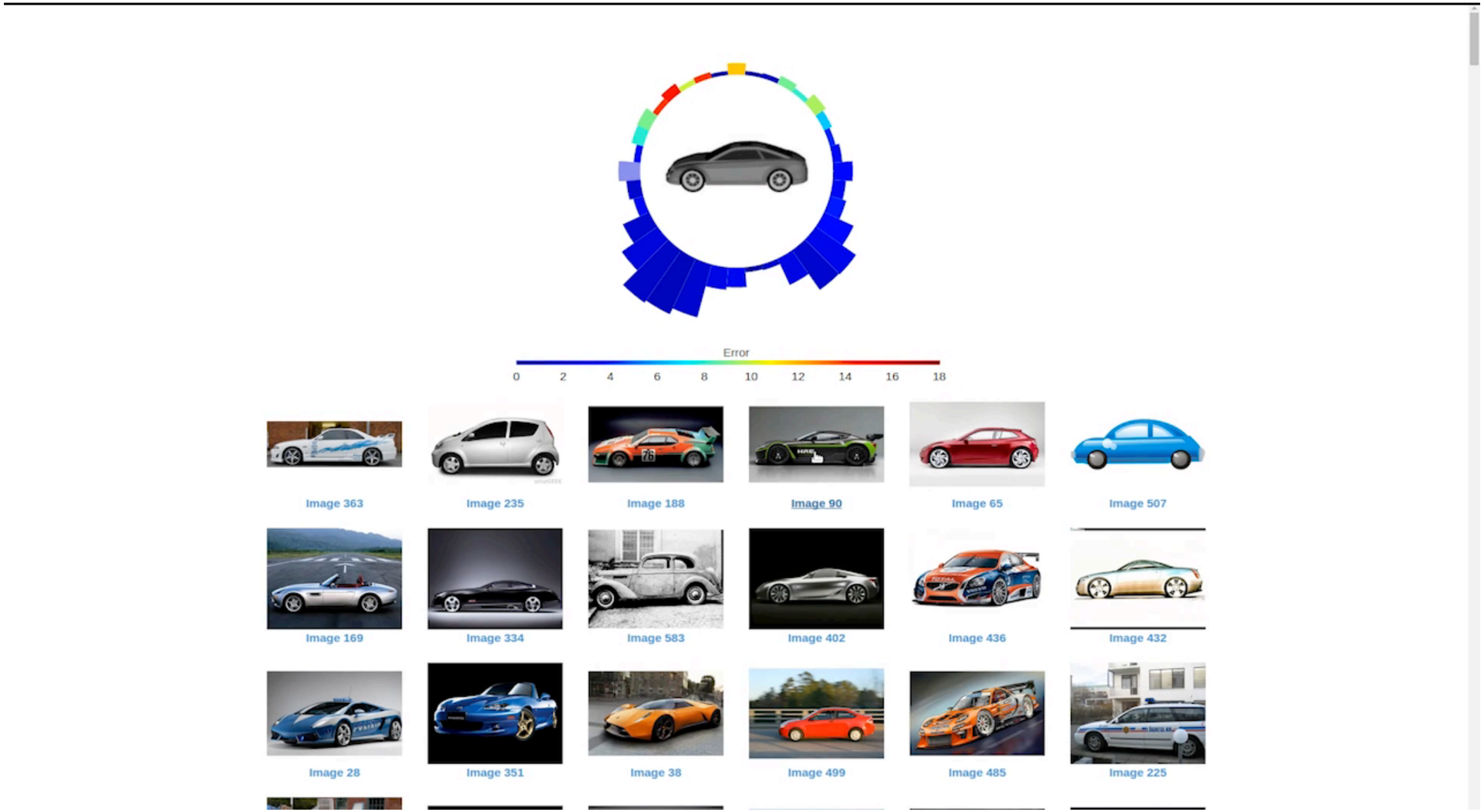


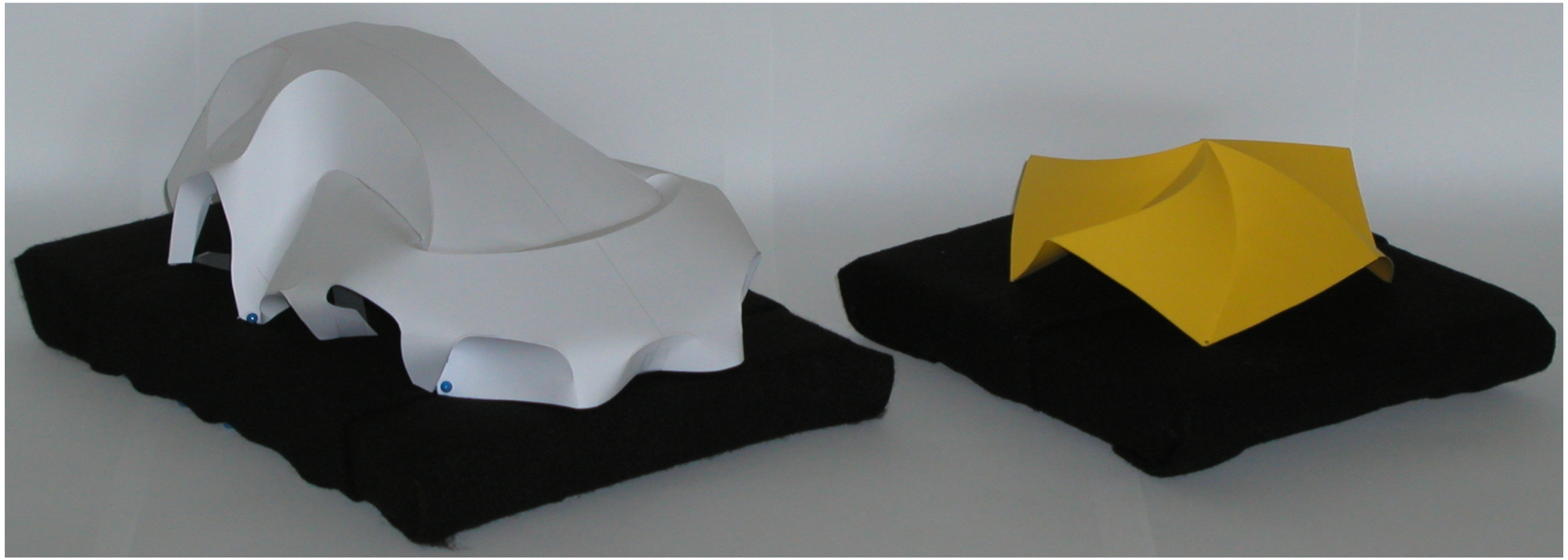
# CrossLink: Images <> 3D Models



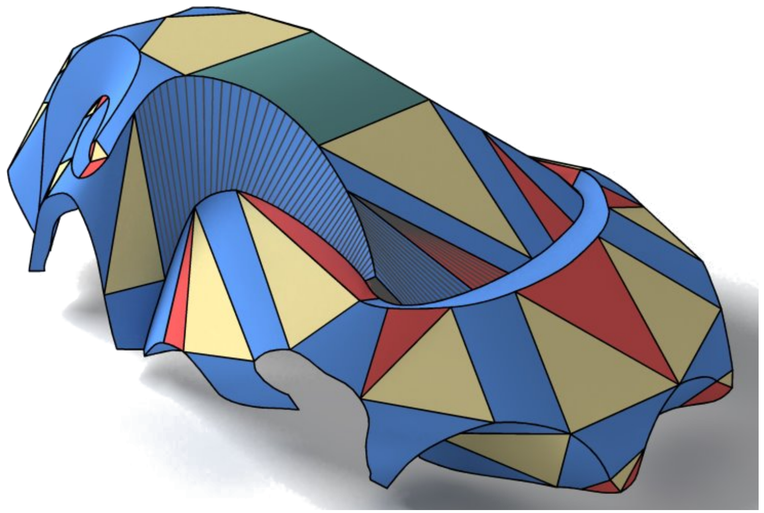
# CrossLink: Images <> 3D Models



# Geometry of Folds



# Layout in 2D



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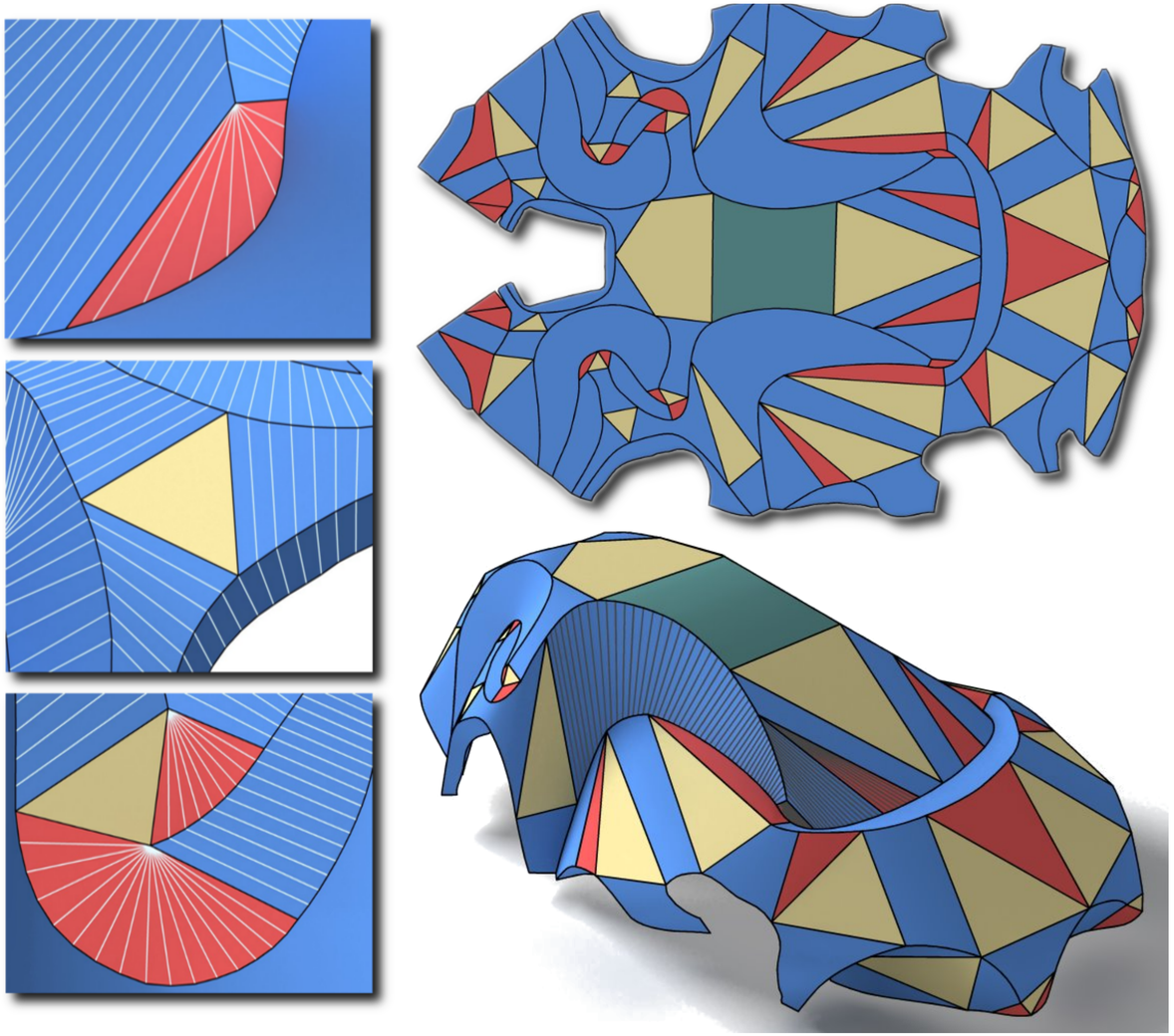

# Folded Roof

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UCL
UCL ENGINEERING
Change the world

UCL ENGINEERING
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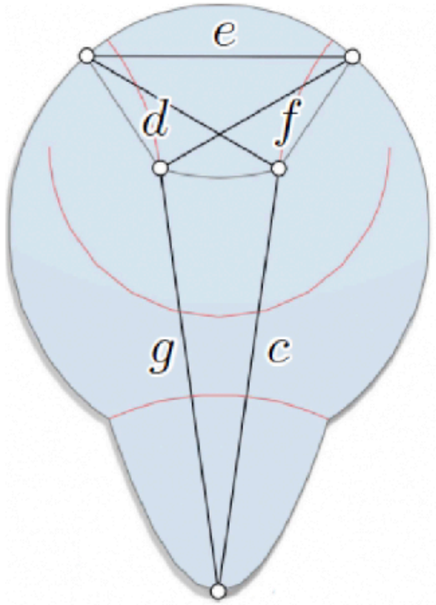
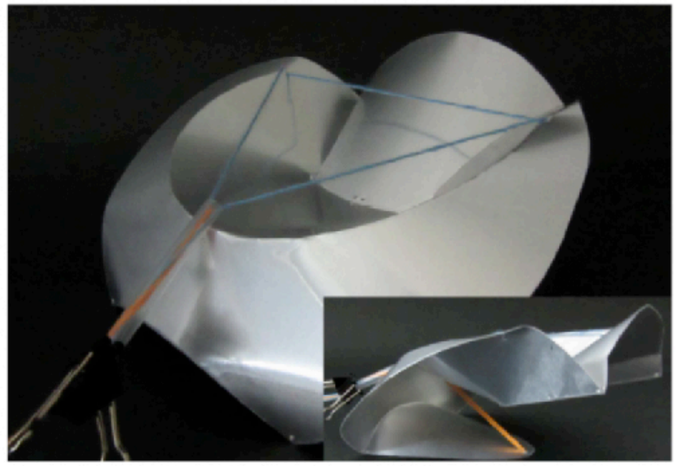
UCL

# Concept Chair

Sequence: CONCEPT CHAIR



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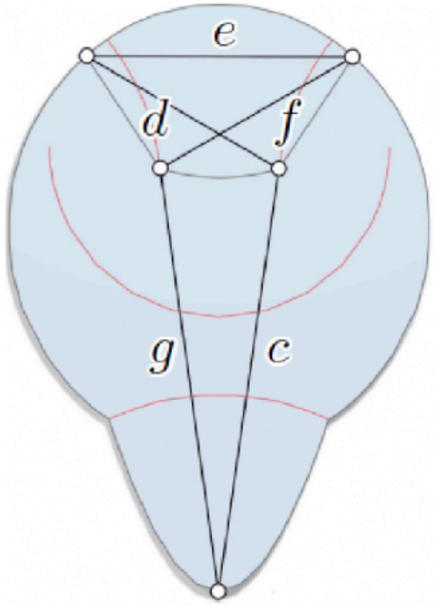


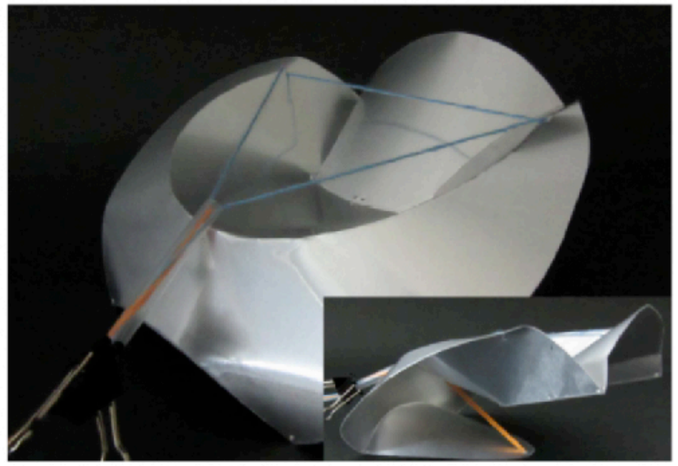

# Overview

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## Geometric Modeling

- techniques and algorithms for representing and processing geometric objects

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## Geometric Modeling

- techniques and algorithms for representing and processing geometric objects

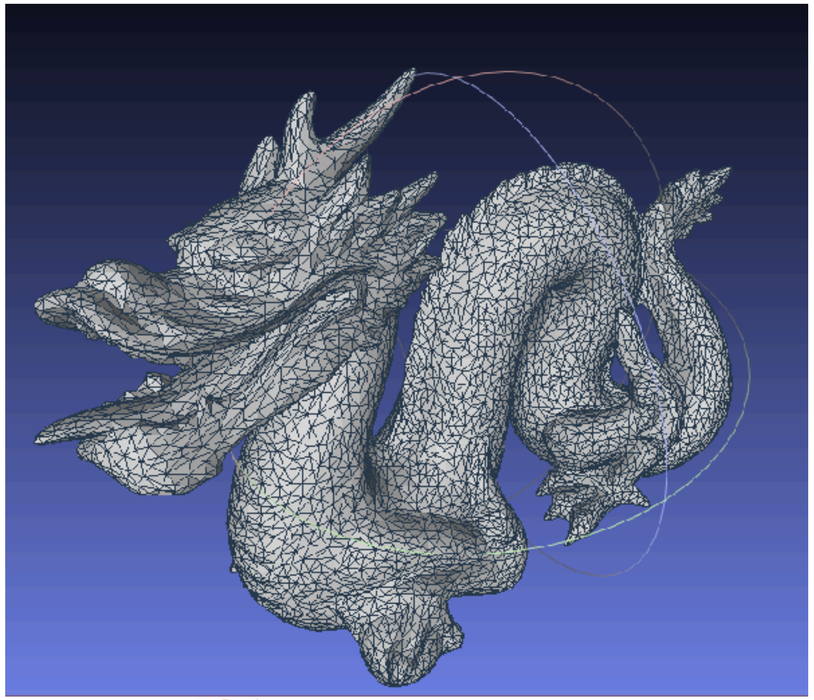
## In this course, we will focus on **triangle meshes**

- **why** are triangle meshes a suitable representation for geometry processing?
- **what** are the central processing algorithms?
- **how** can they be implemented efficiently?

# Looking at Meshes

Meshlab

MATLAB
The Language of Technical Computing



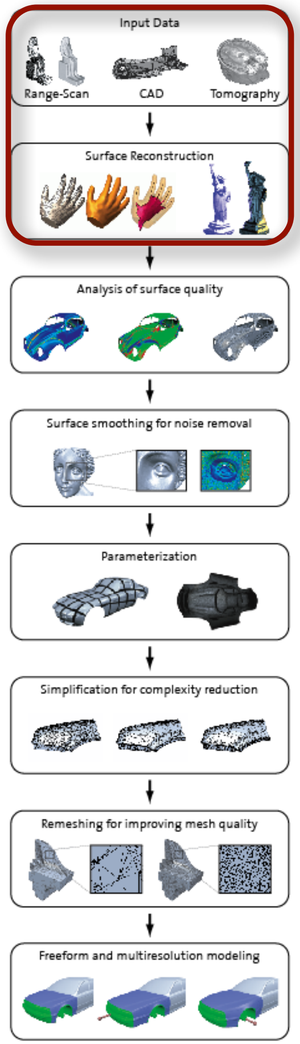
CGAL

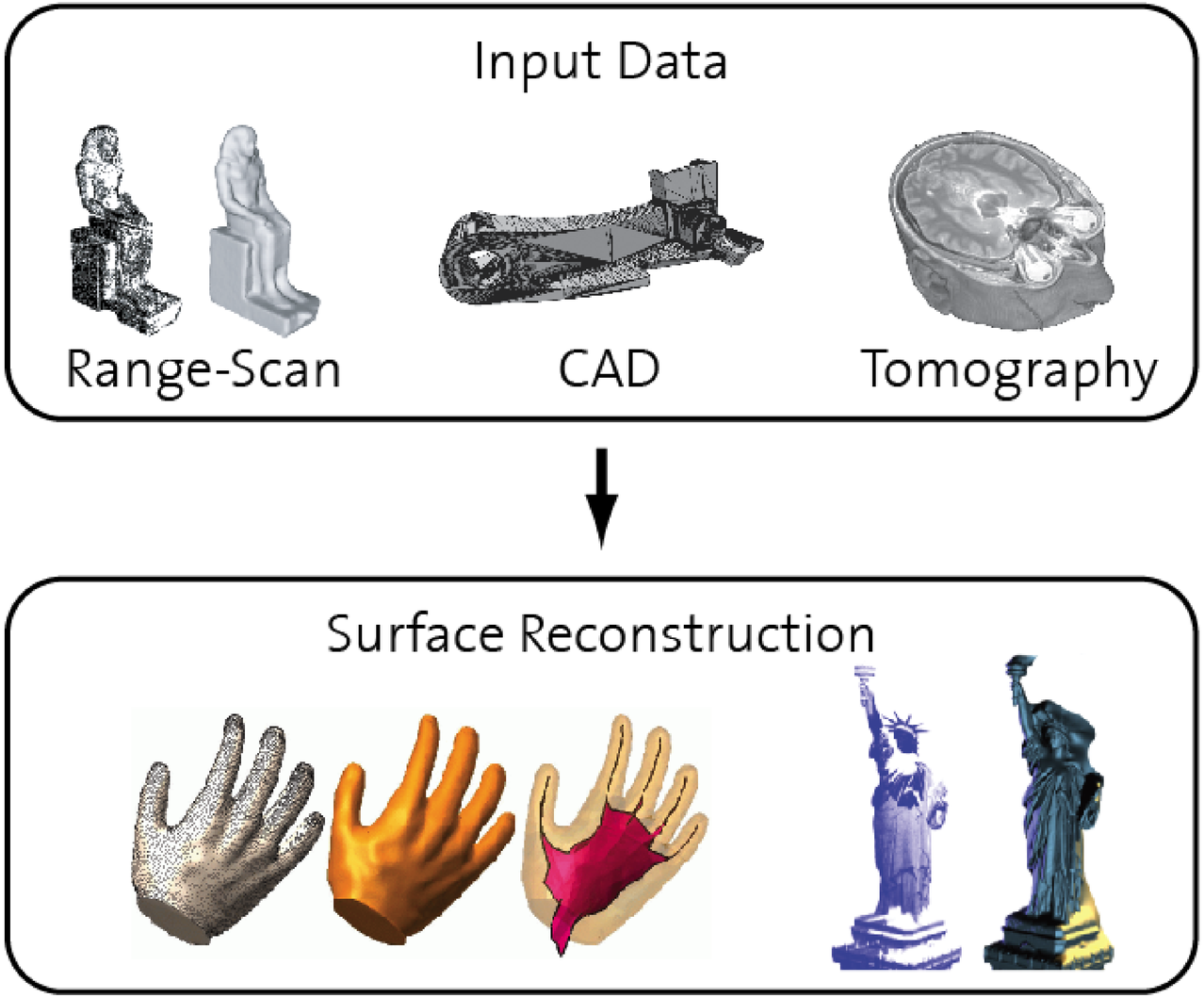
OpenMesh

libigl

blender™

# Geometry Processing Pipeline



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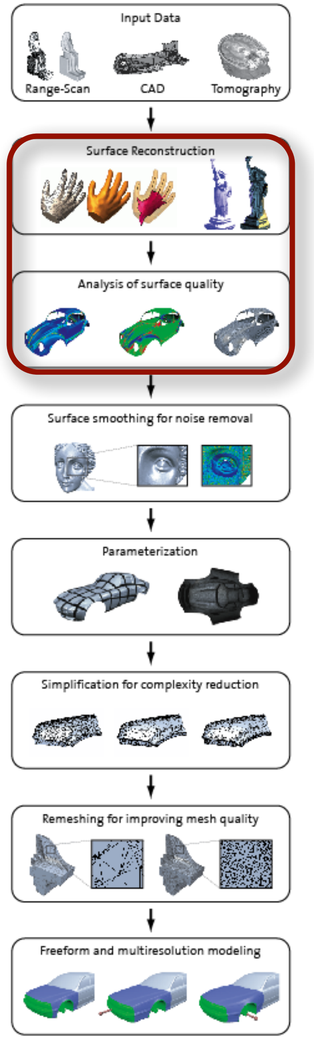


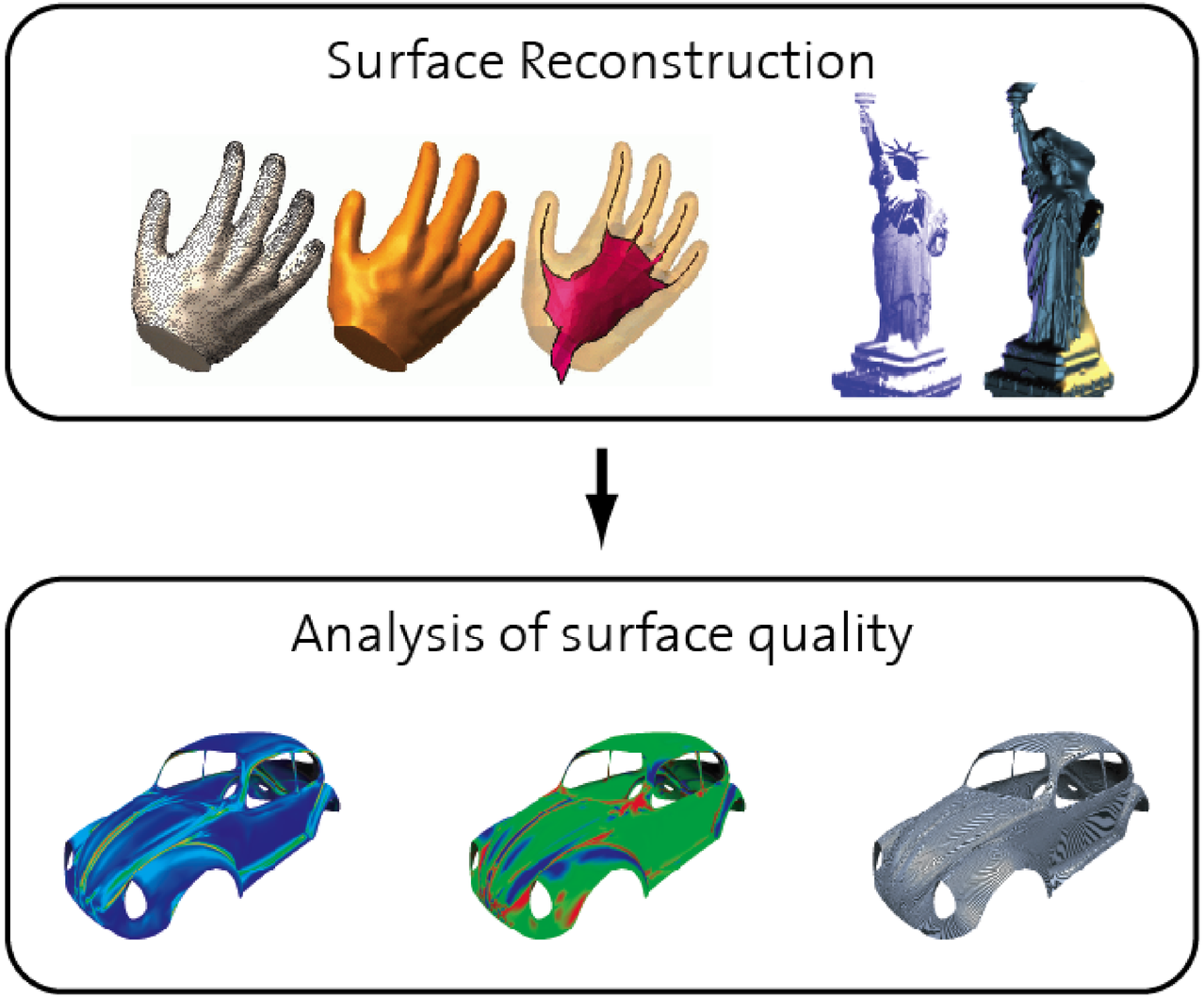

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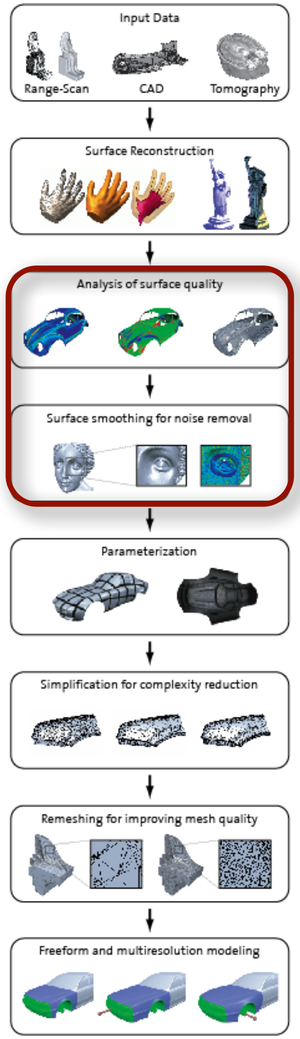


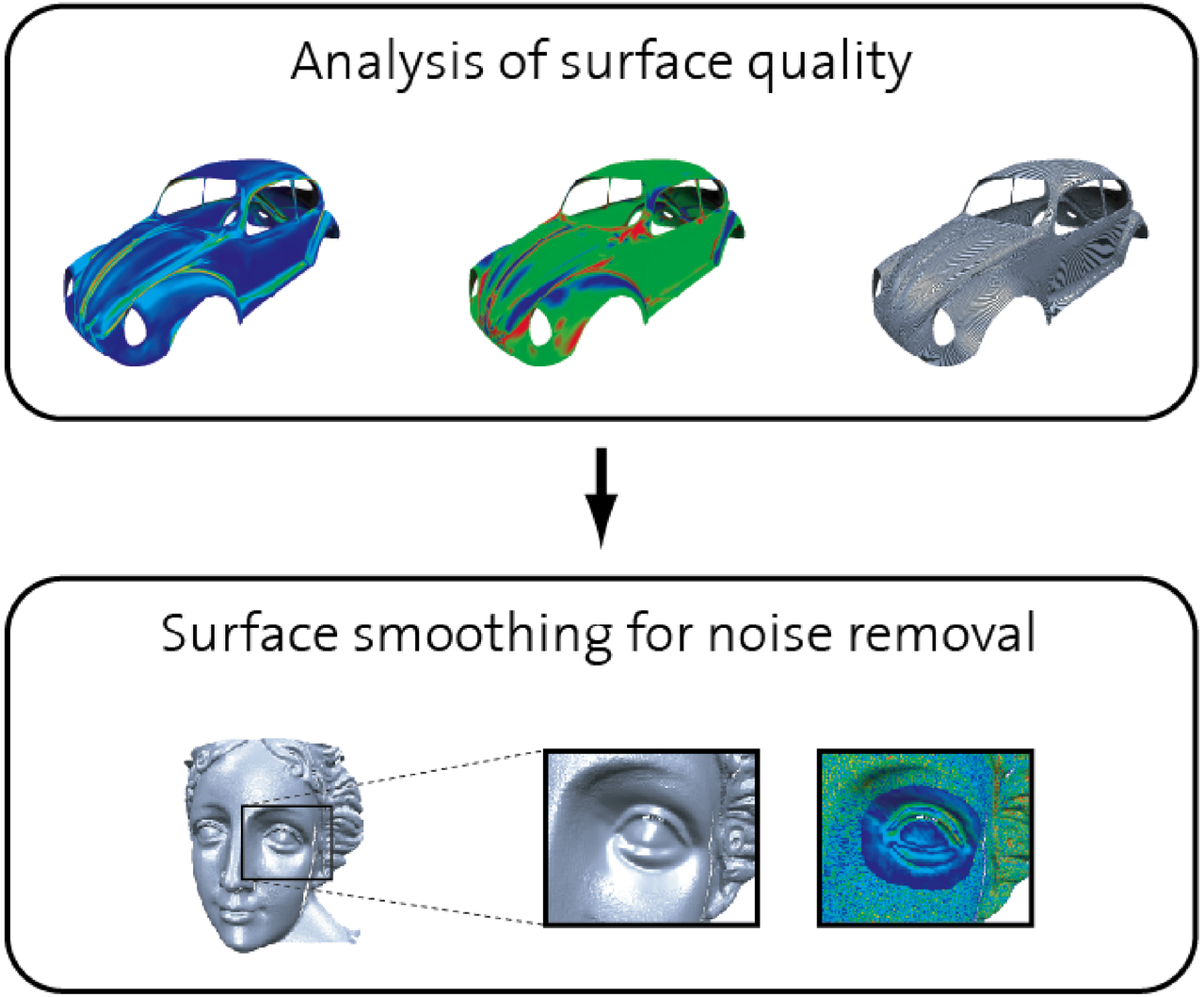

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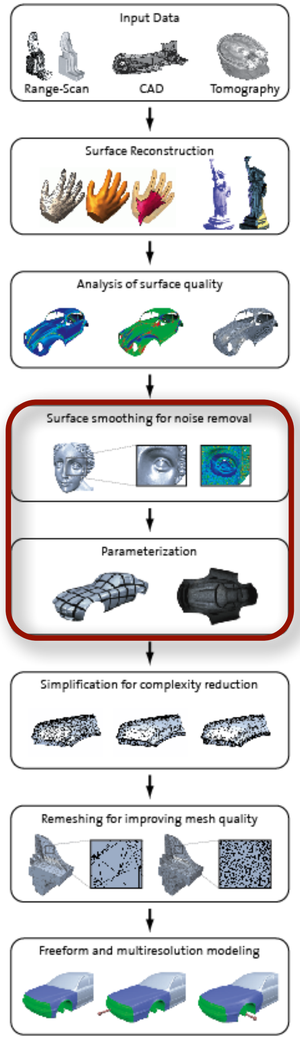
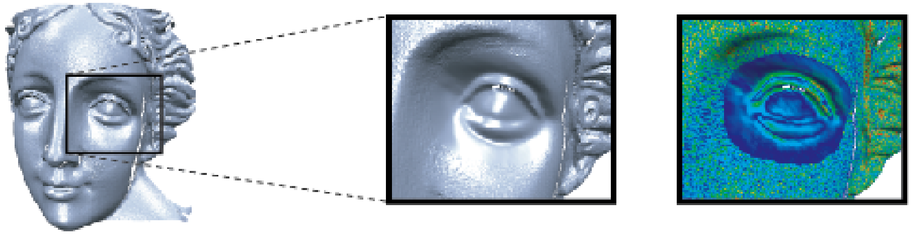
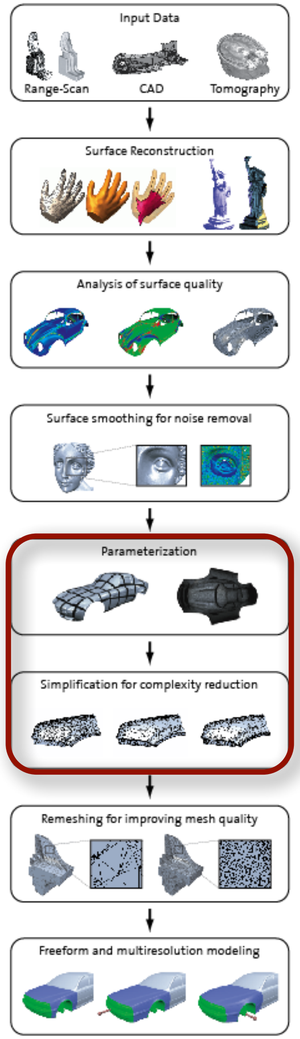


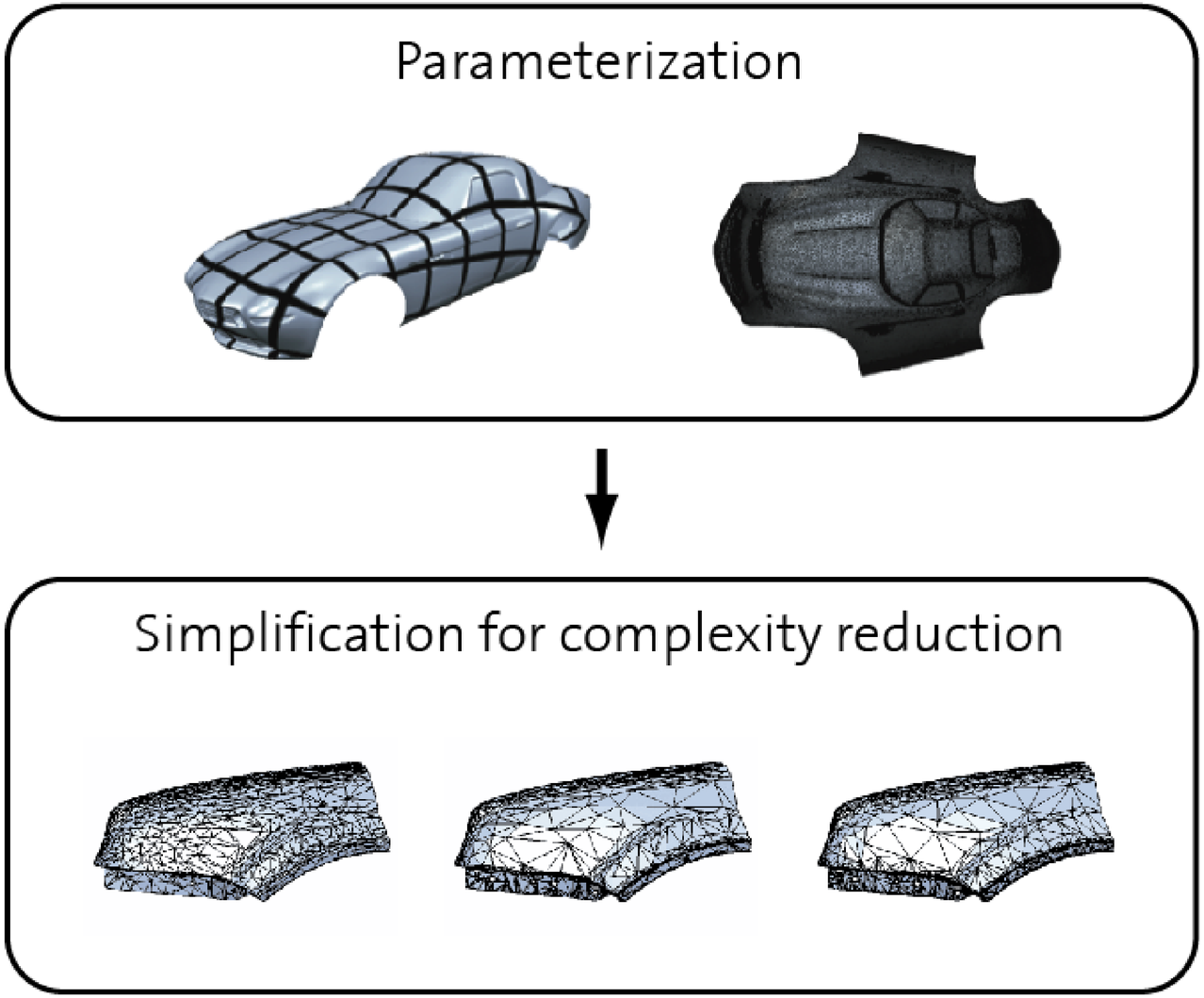

## Surface smoothing for noise removal



## Parameterization

# Geometry Processing Pipeline



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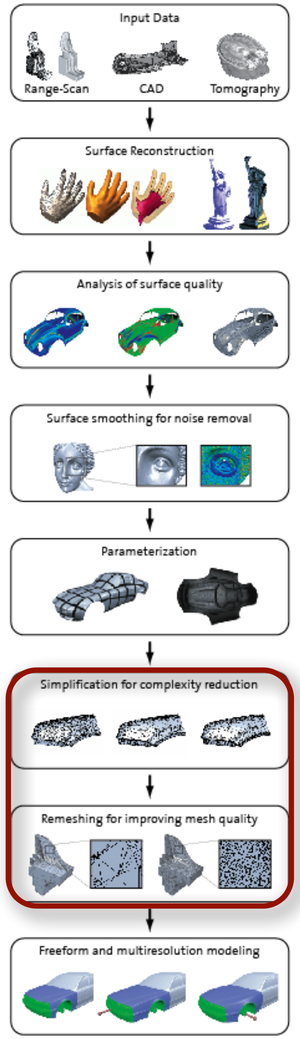


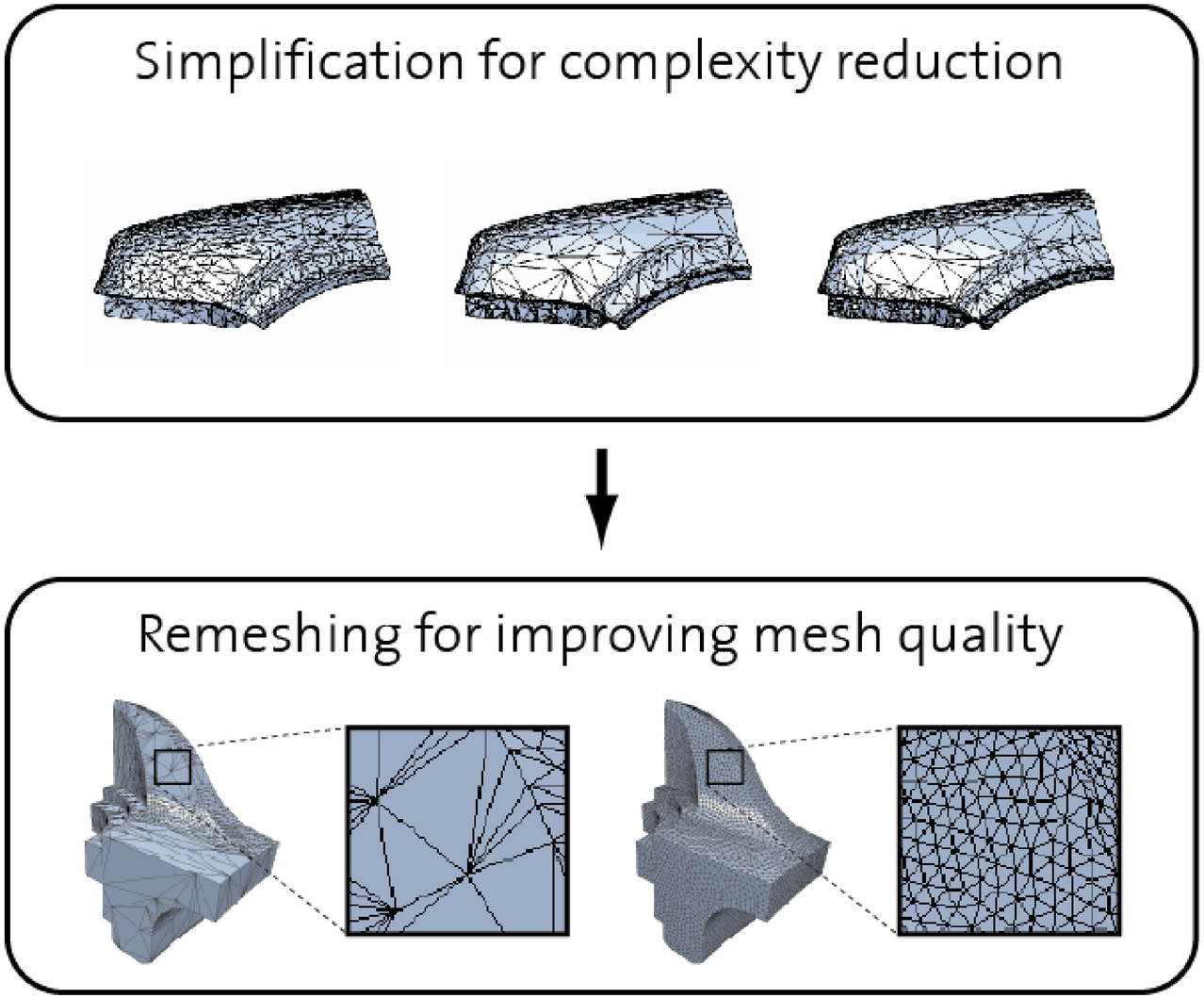

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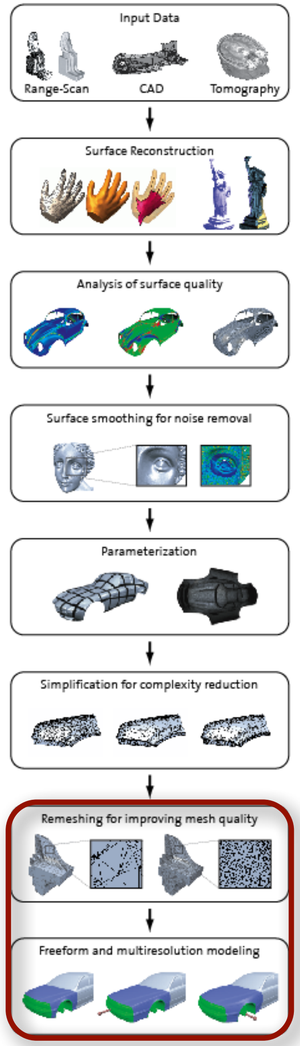


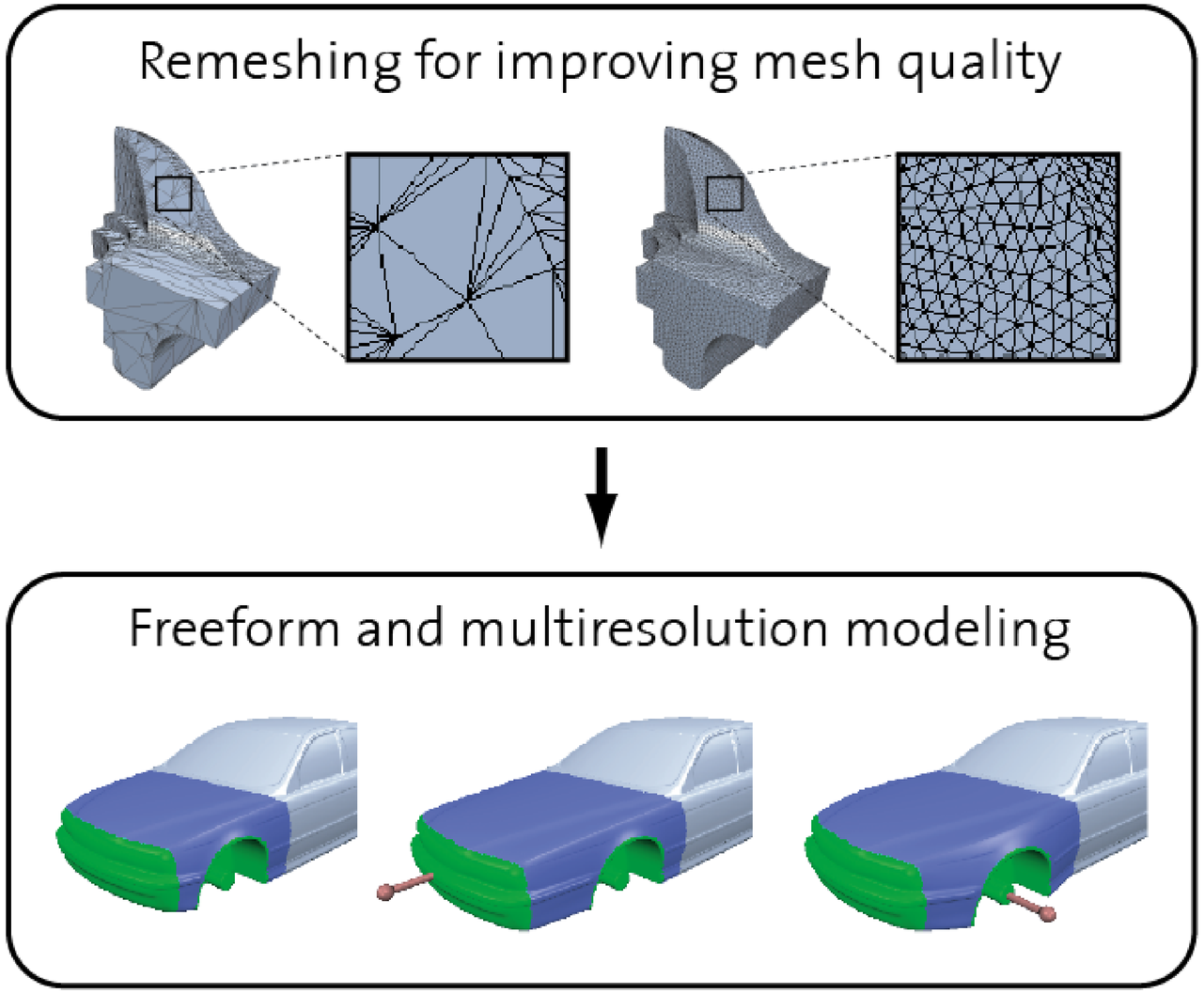

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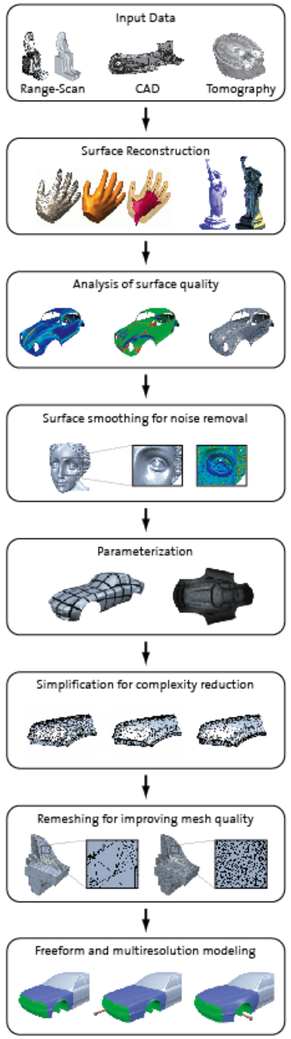

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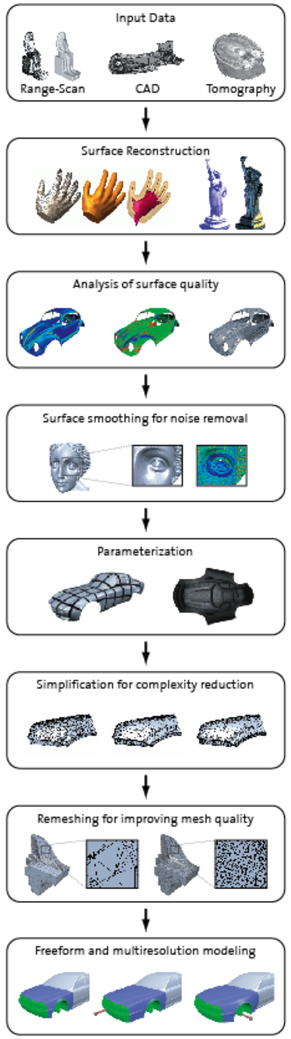

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- ***define*** and ***relate*** the basic concepts, tools, and algorithms in geometric modeling and digital geometry processing
- critically ***analyze*** and ***assess*** current research on surface representations and geometric modeling and apply the proposed methods in your own work
- ***design*** and ***implement*** individual components of a geometric modeling system

# Exercises

## Programming assignments

- based on OpenMesh/Meshlab/Trimesh/others
- cover the core stages of the geometry processing pipeline
- framework including mesh viewer will be provided (Python, C/C++)

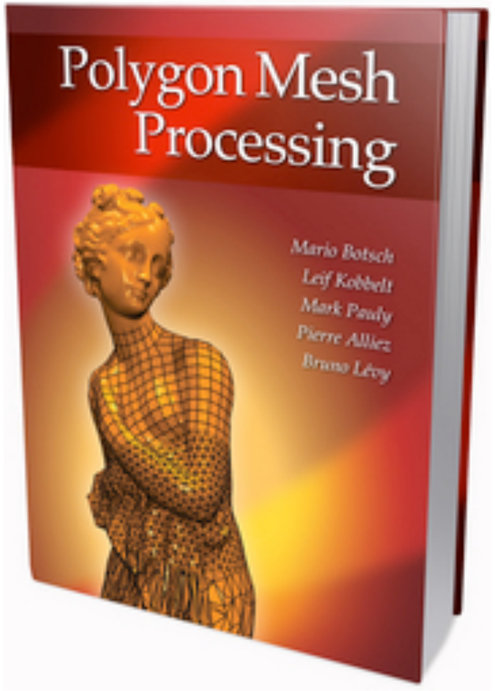
## Final project (groups of 2)

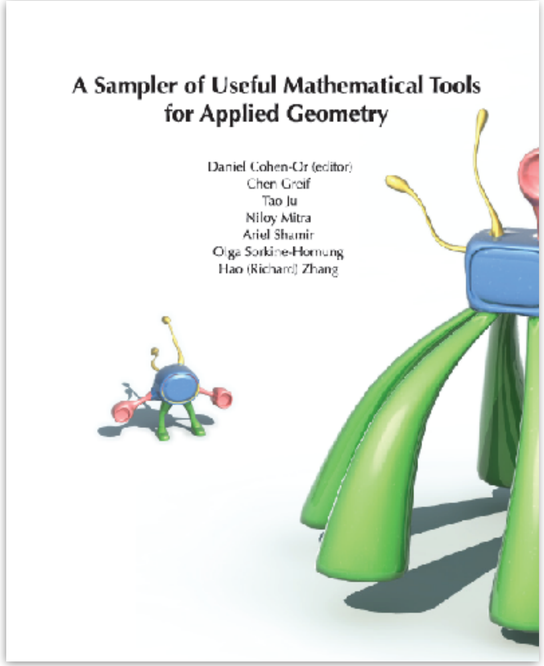
Integral part of the lecture

- important for achieving course objectives

# Literature

- Botsch, Kobbelt, Pauly, Alliez, Levy:
  ***Polygon Mesh Processing***, AK Peters, 2010
- Cohen-Or, Greif, Ju, Mitra, Shamir, Sorkine, Zhang:
  ***A Sampler of Useful Mathematical Tools for Applied Geometry***, CRC, 2015



# Outline

- Parametric Approximations
- Polygon Meshes
- Data structures

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- **Parametric Approximations**
- Polygon Meshes
- Data structures

# Parametric Representation

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- Surface is the **range** of a function

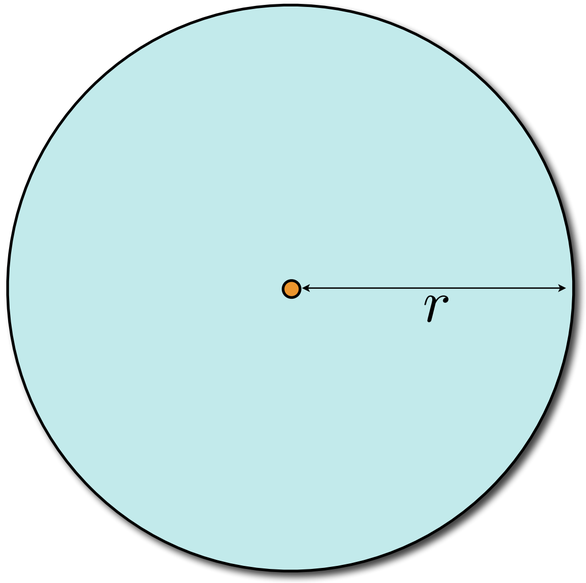
$$\mathbf{f} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3, \quad \mathcal{S}_\Omega = \mathbf{f}(\Omega)$$

$$\mathbf{c} : \Lambda \in \mathbb{R} \rightarrow \mathbb{R}^2, \quad \mathcal{C}_\Lambda = \mathbf{c}(\Lambda)$$

- 2D example: Circle

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} r\cos(t) \\ r\sin(t) \end{pmatrix}$$



# Parametric Representation

- Surface is the range of a function

$$\mathbf{f} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3, \quad \mathcal{S}_\Omega = \mathbf{f}(\Omega)$$

- 2D example: Island coast line

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} ??? \\ ??? \end{pmatrix}$$

# Piecewise Approximation

- Surface is the range of a function

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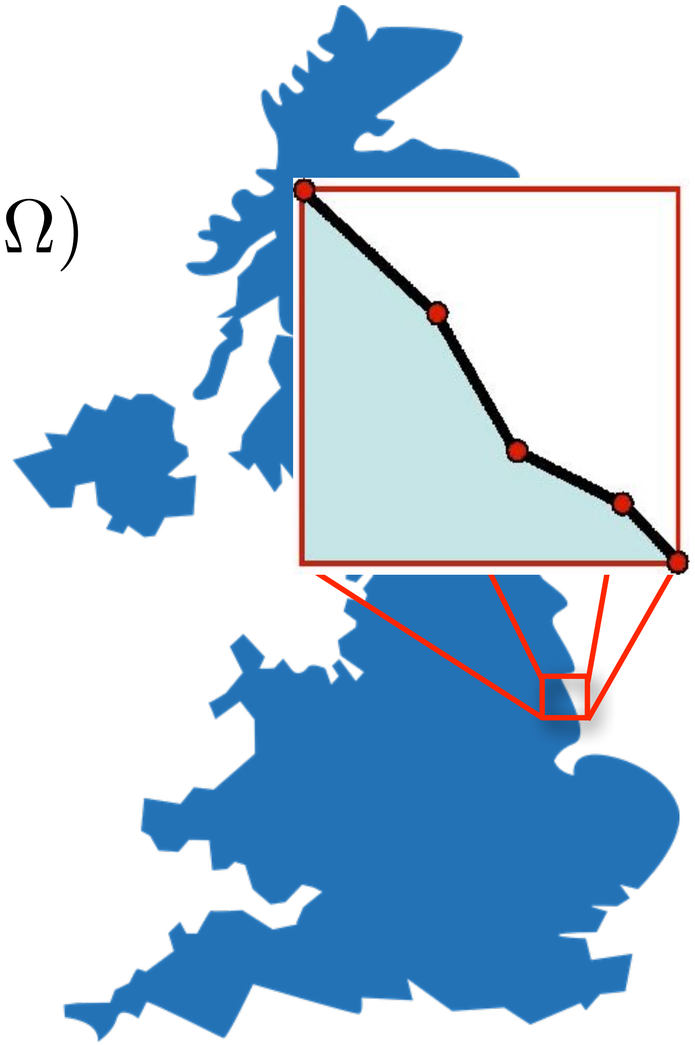

# Polynomial Approximation

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$$f(t) \;=\; \sum_{i=0}^{p} c_i \, t^i \;=\; \sum_{i=0}^{p} \tilde{c}_i \, \phi_i(t)$$

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- **Taylor expansion** up to degree *p*

$$g(h) \;=\; \sum_{i=0}^{p} \frac{1}{i!}\, g^{(i)}(0)\; h^i \;+\; O\big(h^{p+1}\big)$$

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$$g(h) \;=\; \sum_{i=0}^{p} \frac{1}{i!}\, g^{(i)}(0)\; h^i \;+\; O\big(h^{p+1}\big)$$

- Error for approximating a function *g* by polynomial *f*

$$f(t_i) = g(t_i)\,, \quad 0 \le t_0 < \cdots < t_p \le h$$

$$|f(t) - g(t)| \;\le\; \frac{1}{(p+1)!}\, \max f^{(p+1)} \prod_{i=0}^{p} (t - t_i) \;=\; O\Big(h^{(p+1)}\Big)$$

# Polynomial Approximation

- **Approximation error** is $O(h^{p+1})$

- **Improve** approximation quality by
  - increasing *p* ... higher order polynomials
  - decreasing *h* ... smaller / more segments

- Issues
  - smoothness of the target data $\max_t f^{p+1}(t)$
  - smoothness conditions between segments

# Polygonal Meshes

- **Polygonal meshes** are a good compromise
  - Piecewise linear approximation → error is $O(h^2)$

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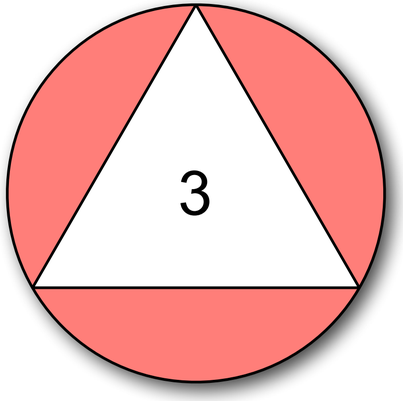


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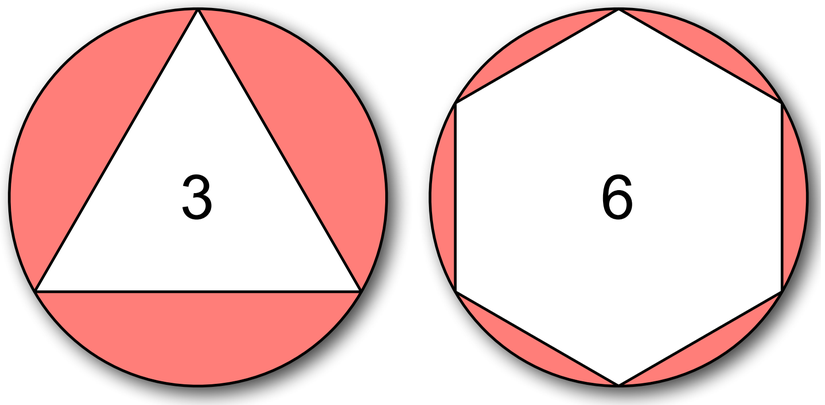

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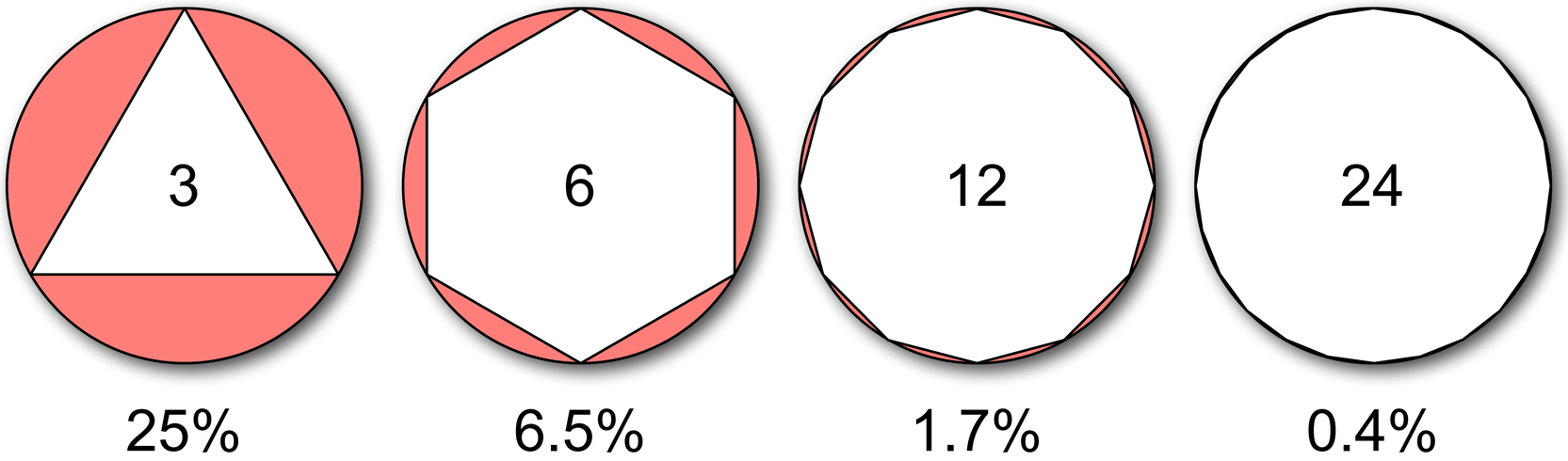

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- Polygonal meshes are a good compromise
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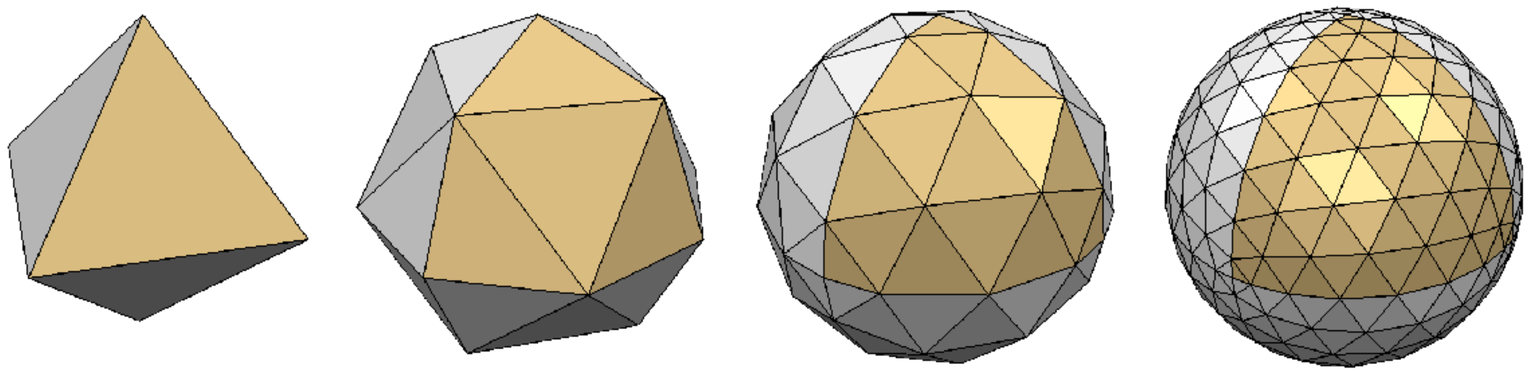

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- Polygonal meshes are a good compromise
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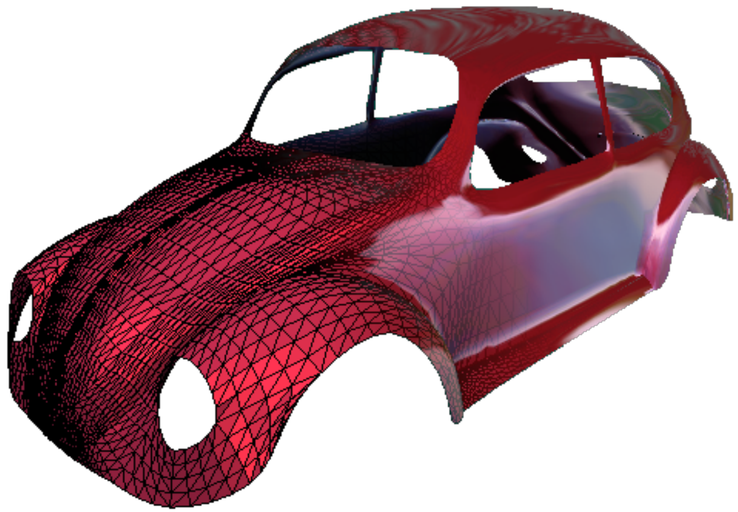
  - Arbitrary topology surfaces



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  - Piecewise smooth surfaces

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  - Piecewise linear approximation → error is $O(h^2)$
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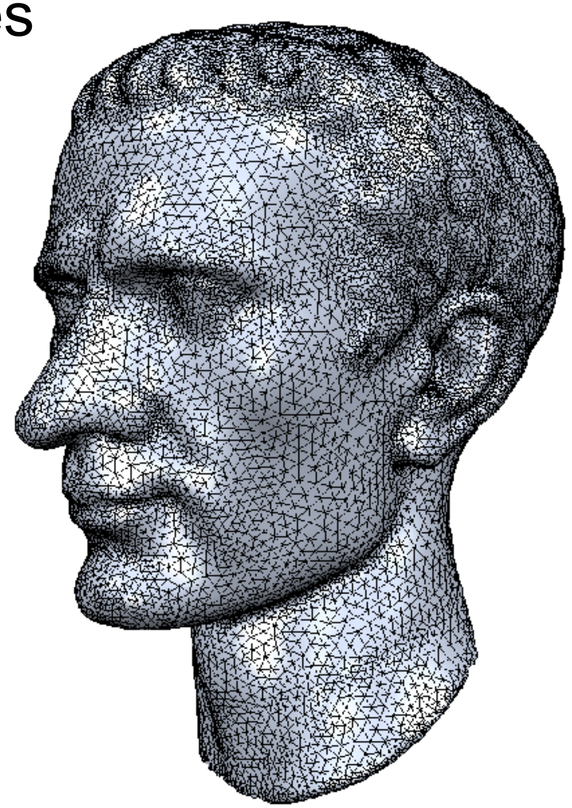
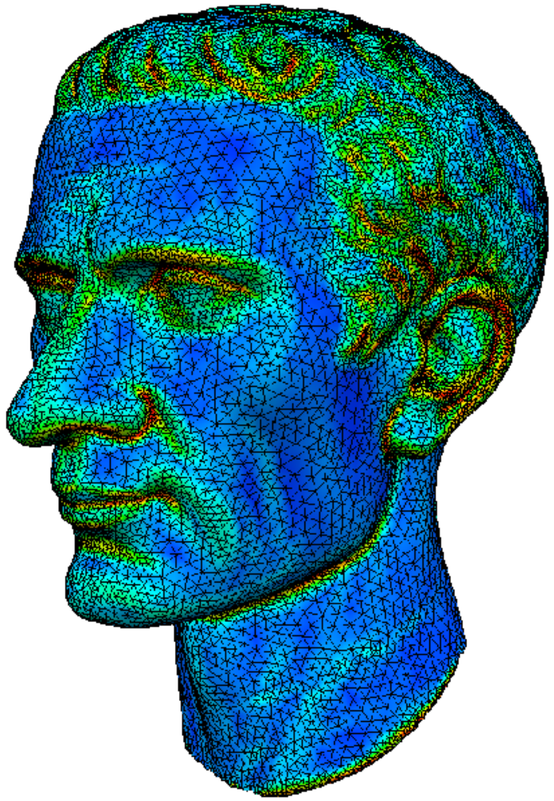
  - Arbitrary topology surfaces
  - Piecewise smooth surfaces
  - Curvature adaptive sampling





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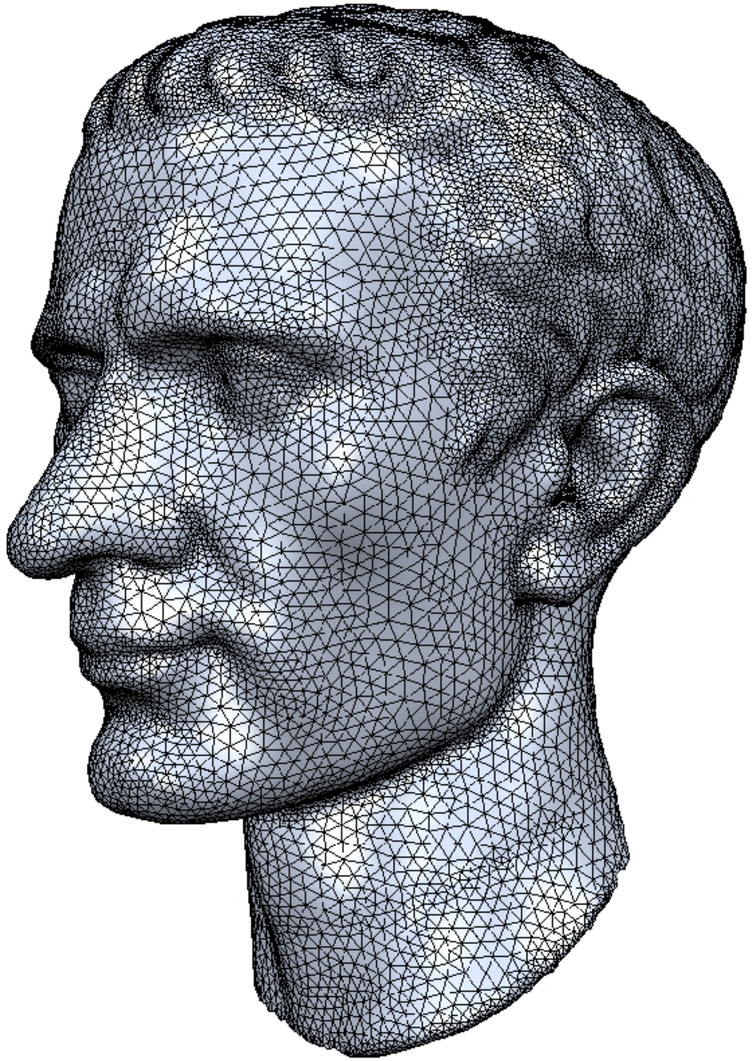
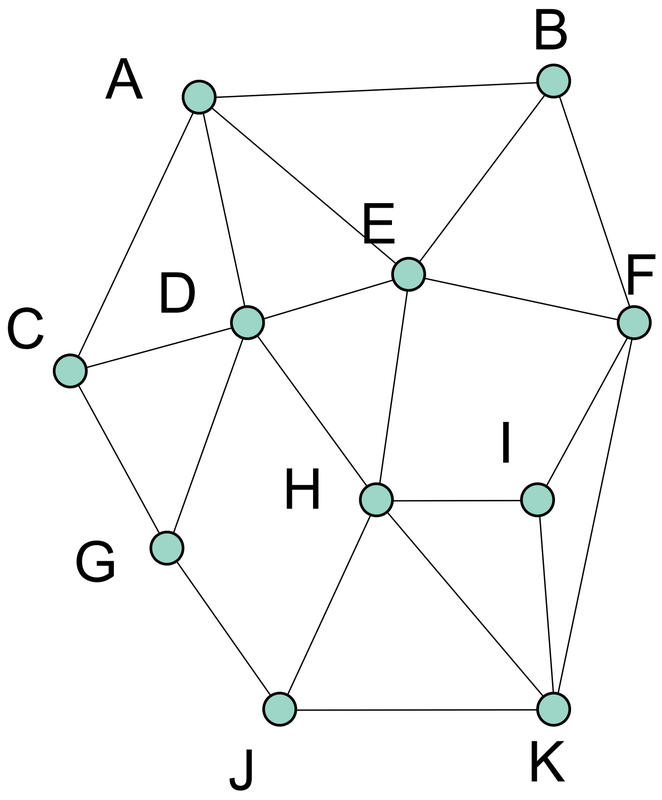
  - Arbitrary topology surfaces
  - Piecewise smooth surfaces
  - Curvature adaptive sampling
  - Efficient GPU-based rendering

# Outline

- Parametric Approximations
- **Polygon Meshes**
- Data structures

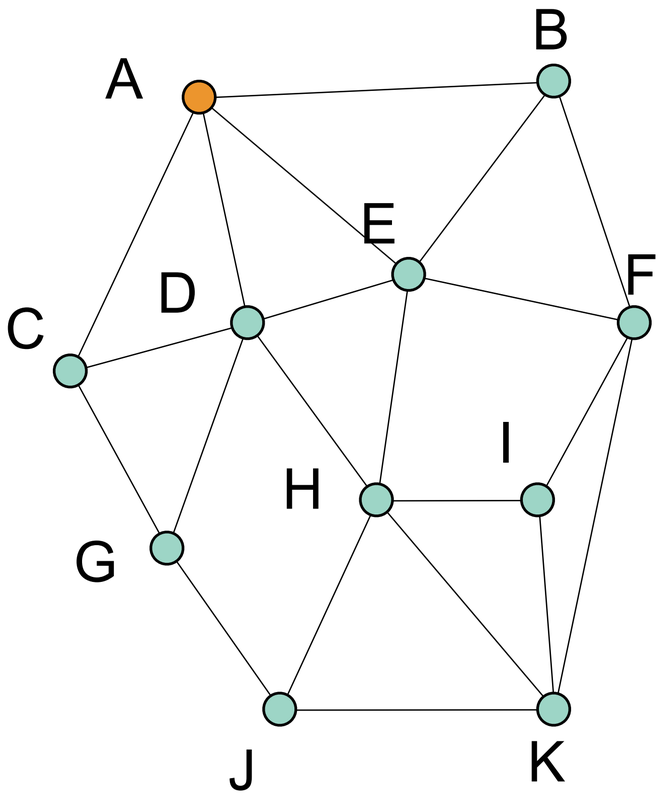


# Graph Definitions




Graph $\{V, E\}$

# Graph Definitions




Graph $\{V, E\}$

Vertices $V = \{A, B, C, \ldots, K\}$

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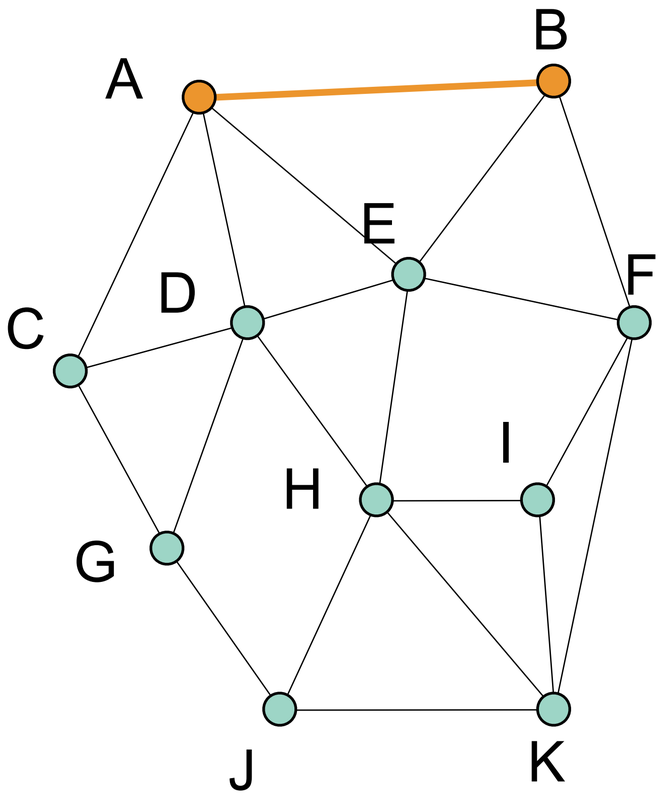
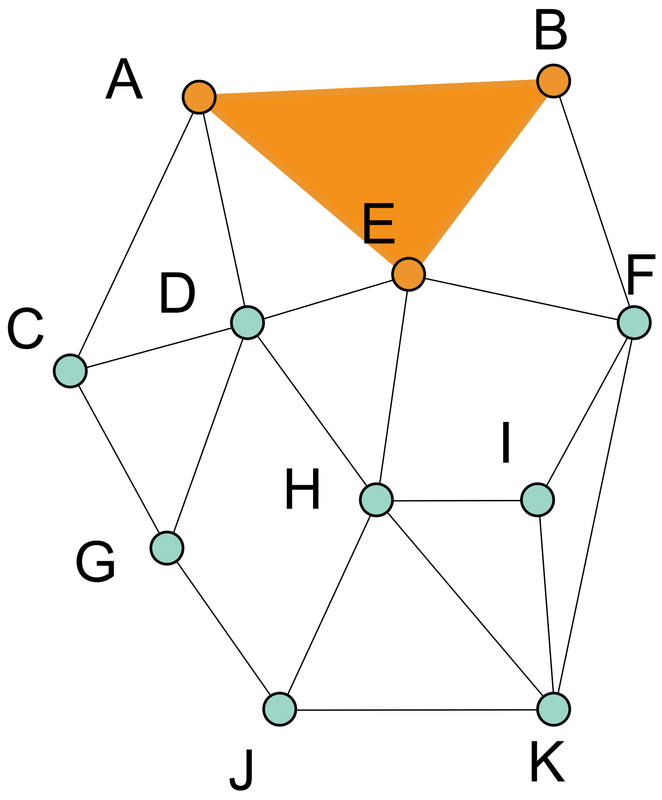



Graph $\{V, E\}$

Vertices $V = \{A, B, C, \ldots, K\}$

Edges $E = \{(AB), (AE), (CD), \ldots\}$

# Graph Definitions



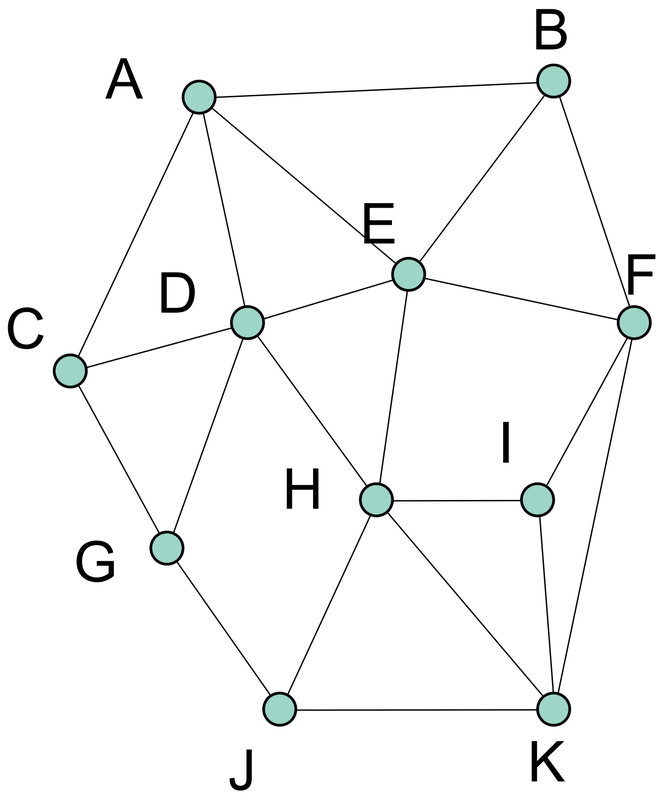

Graph $\{V, E\}$

Vertices $V = \{A, B, C, \ldots, K\}$

Edges $E = \{(AB), (AE), (CD), \ldots\}$

Faces $F = \{(ABE), (EBF), (EFIH), \ldots\}$

# Graph Definitions



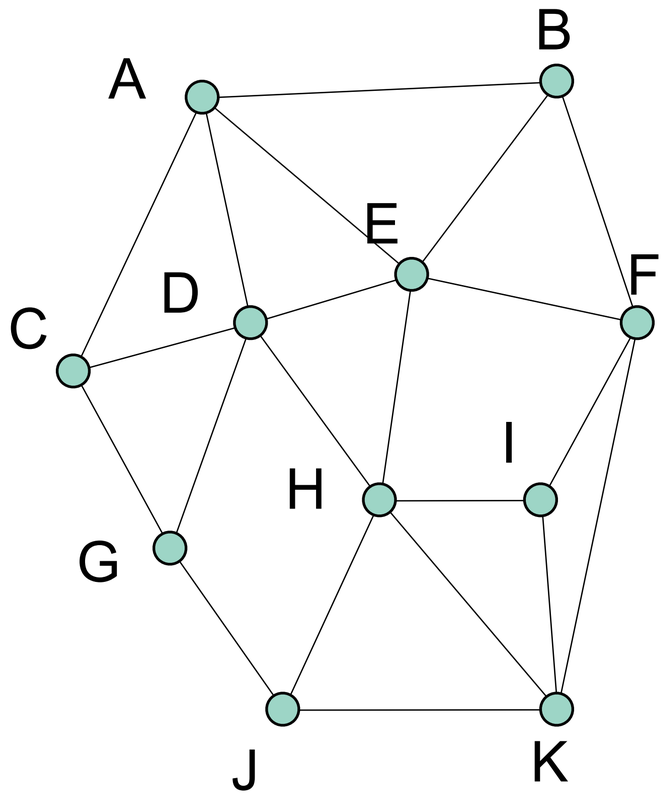

**Vertex degree** or **valence**:
number of incident edges.

deg(A) = 4

deg(E) = 5

For triangle mesh, **average** valence is 6.

# Connectivity

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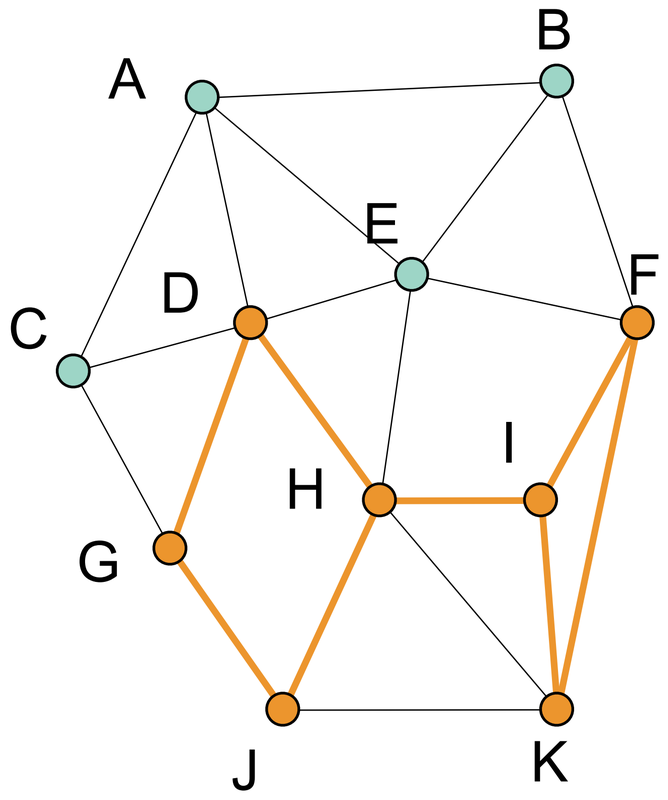


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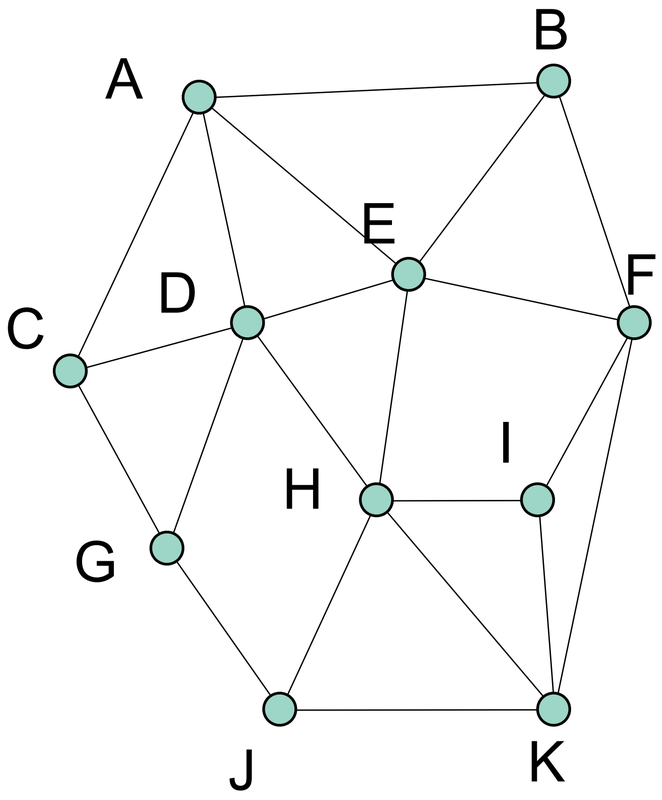


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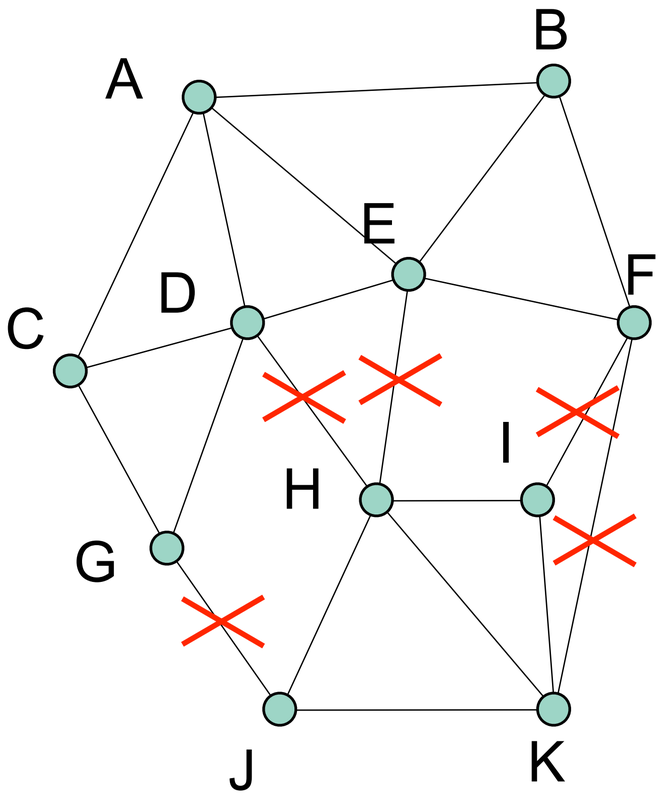


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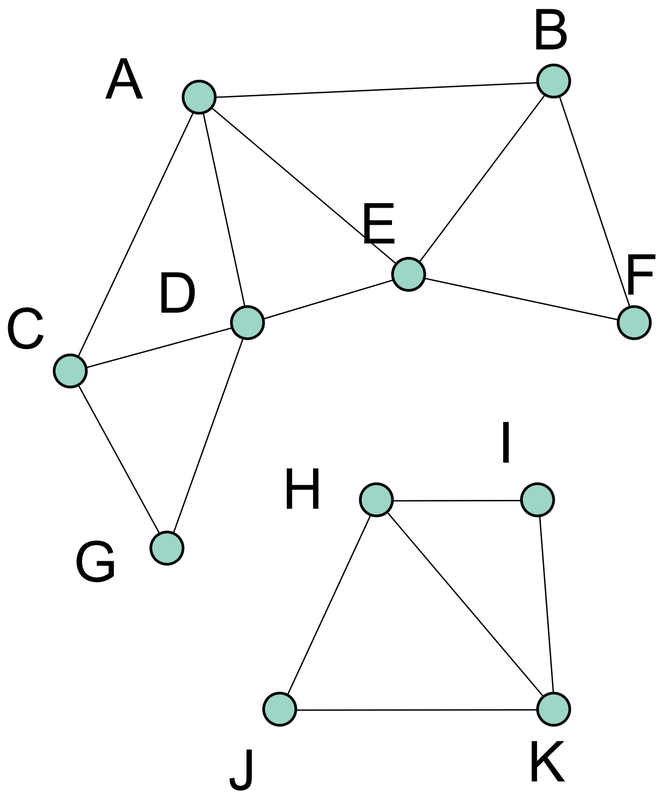


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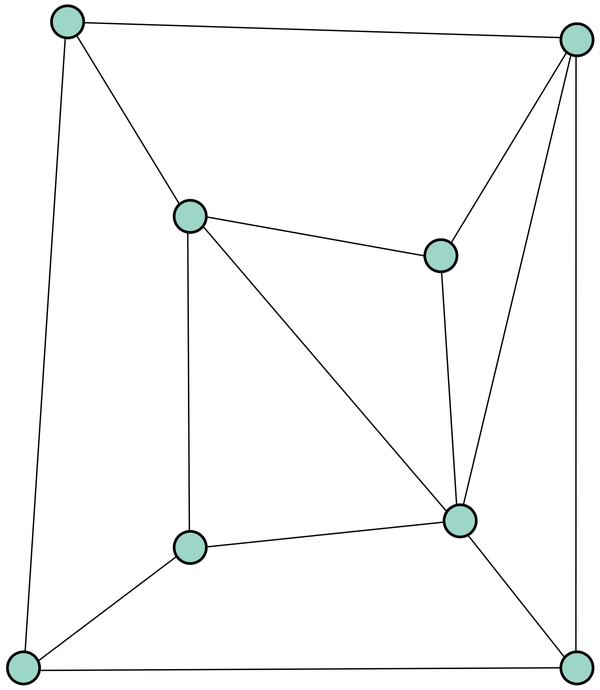
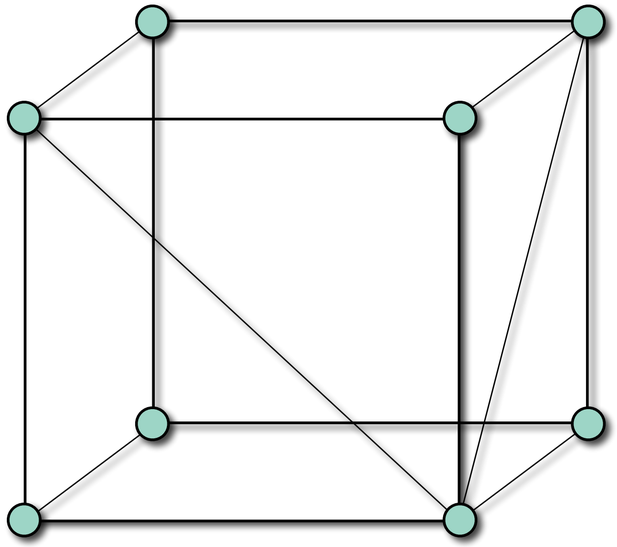
**Embedding**: Graph is embedded in $\mathbb{R}^d$, if each vertex is assigned a position in $\mathbb{R}^d$.



Embedded in $\mathbb{R}^2$



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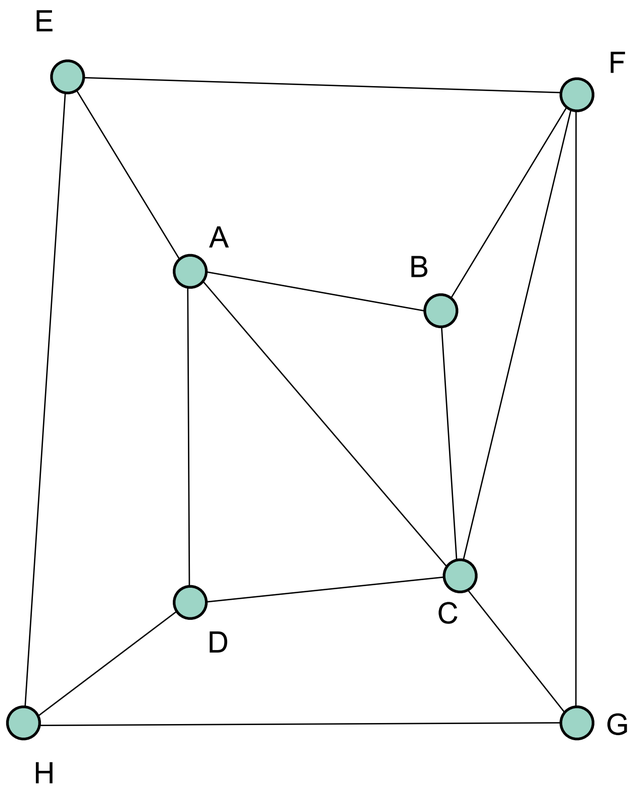


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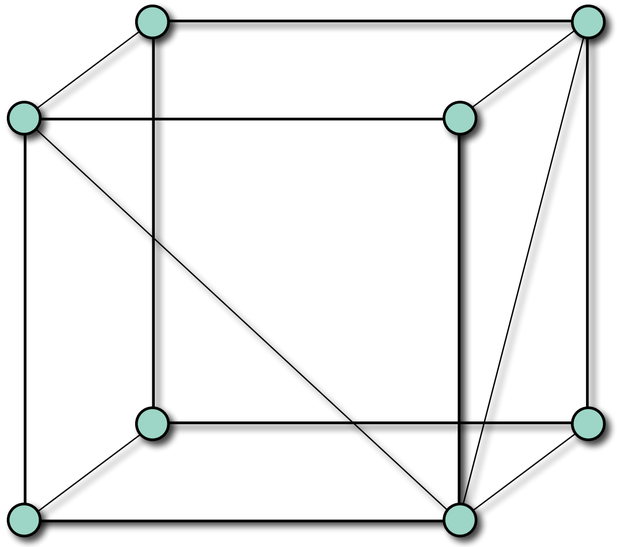

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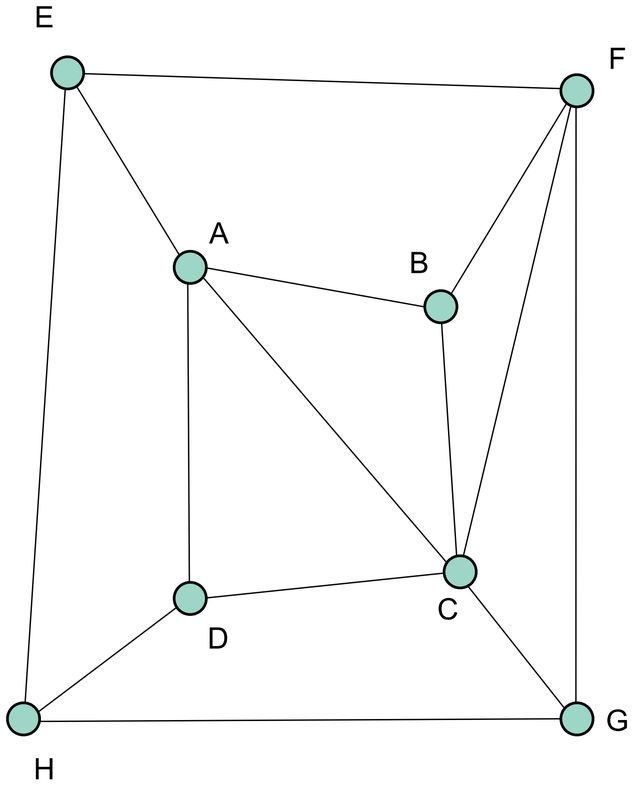


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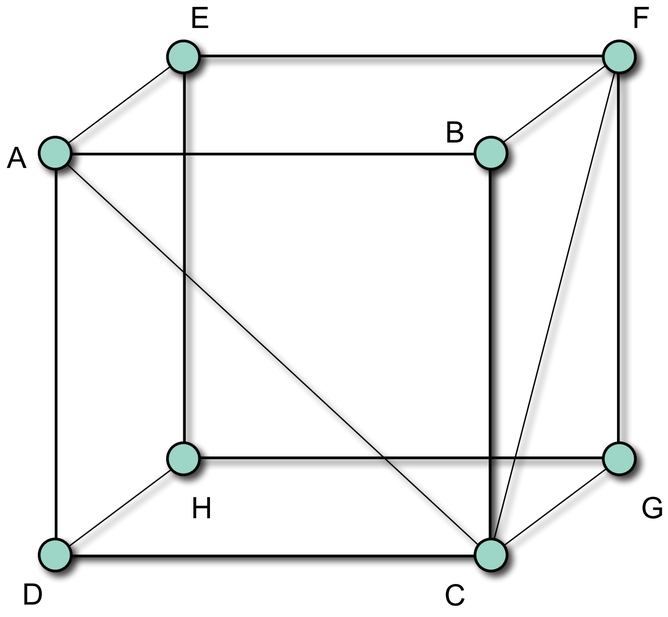


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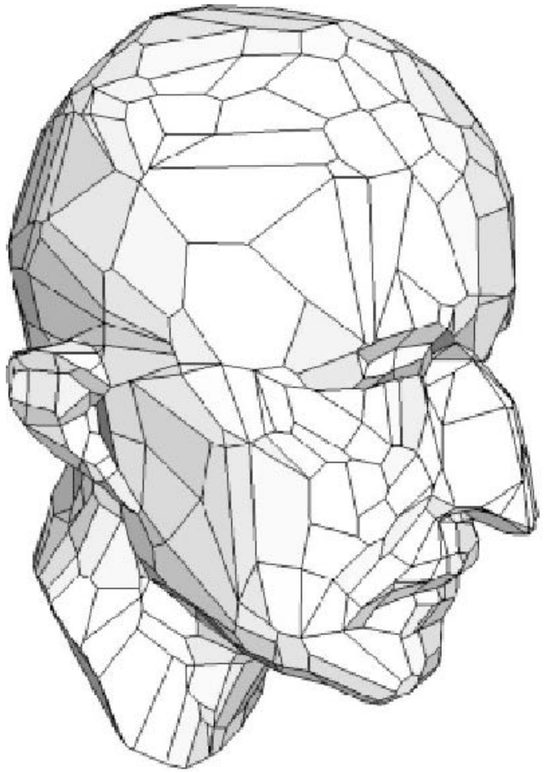

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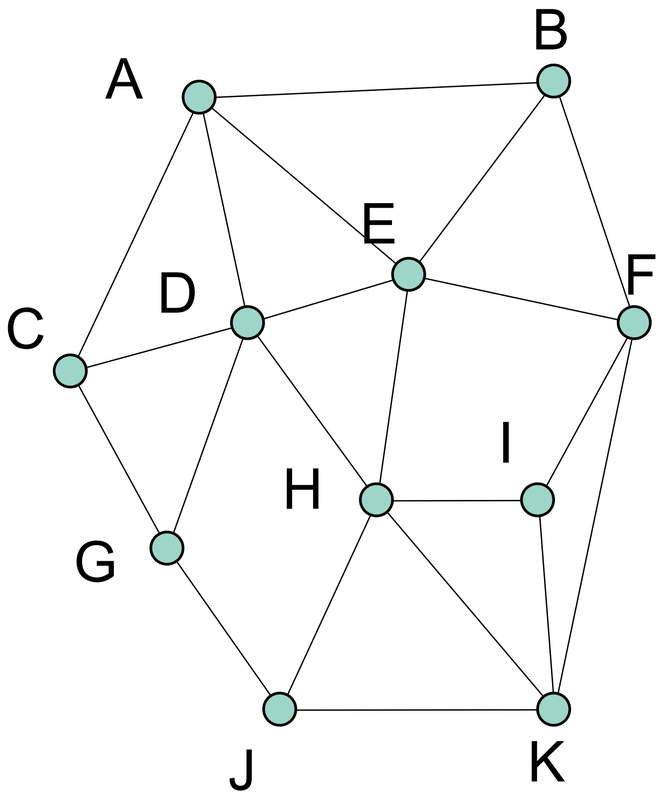
# Triangulation



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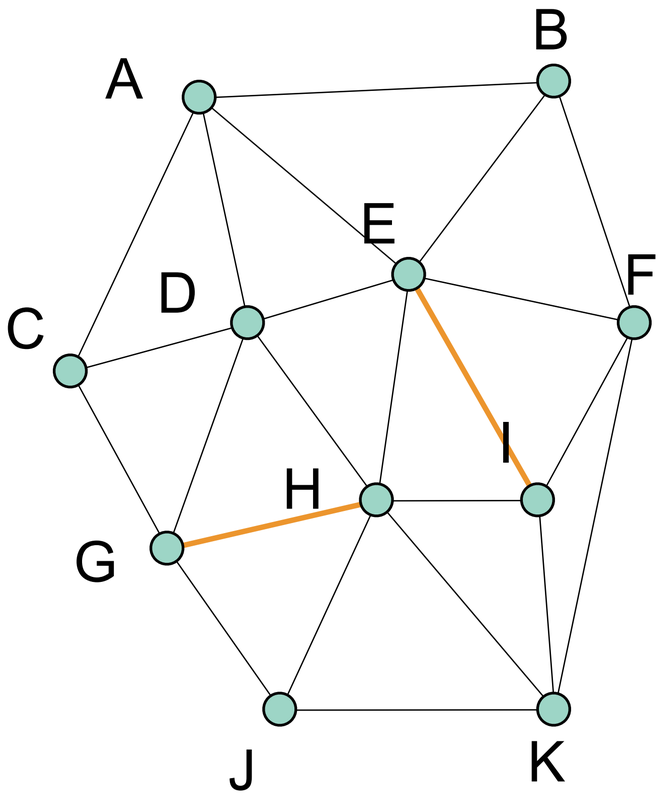

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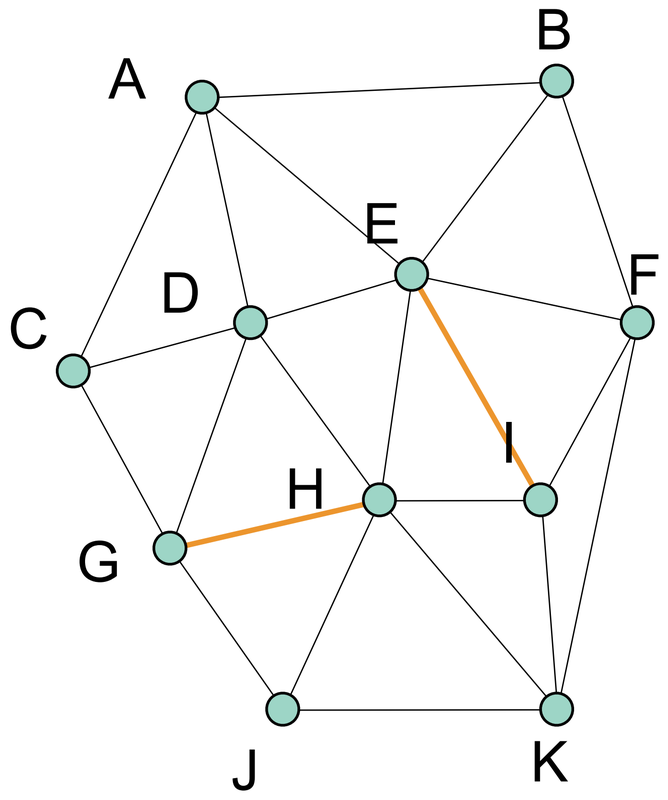


**Triangulation**: Graph where every face is a triangle.

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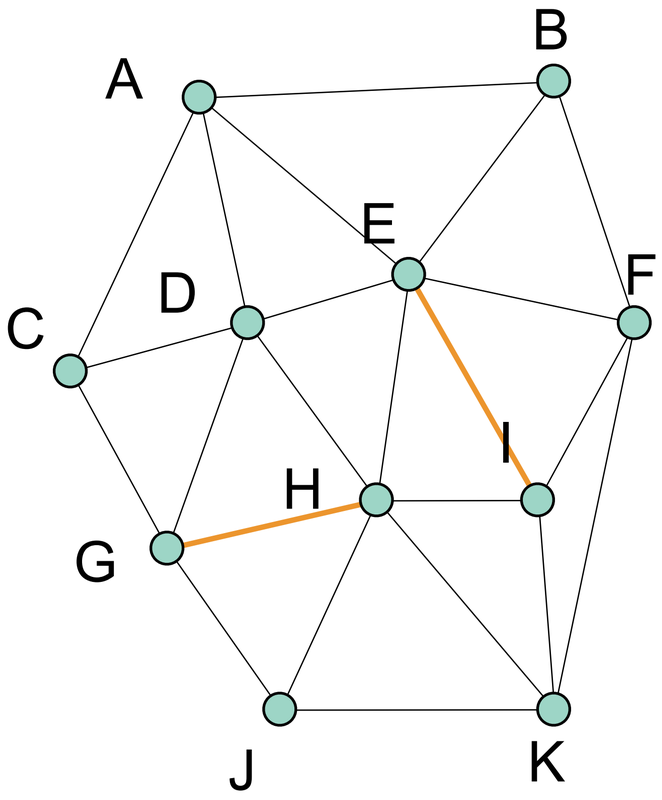


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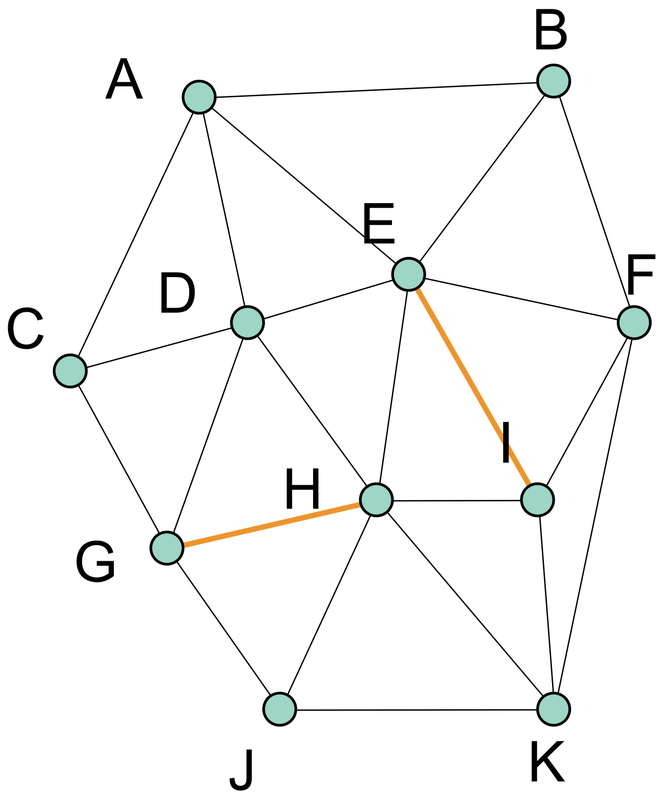


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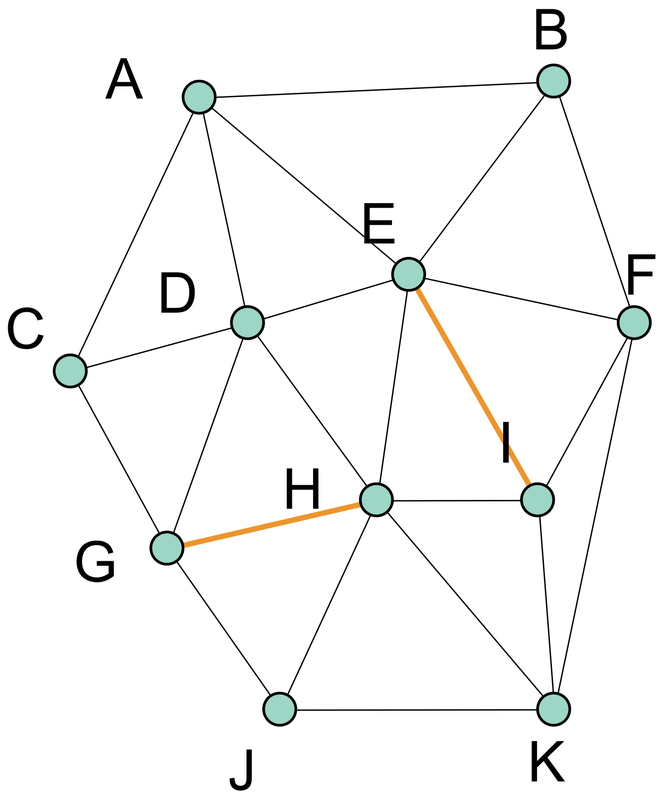


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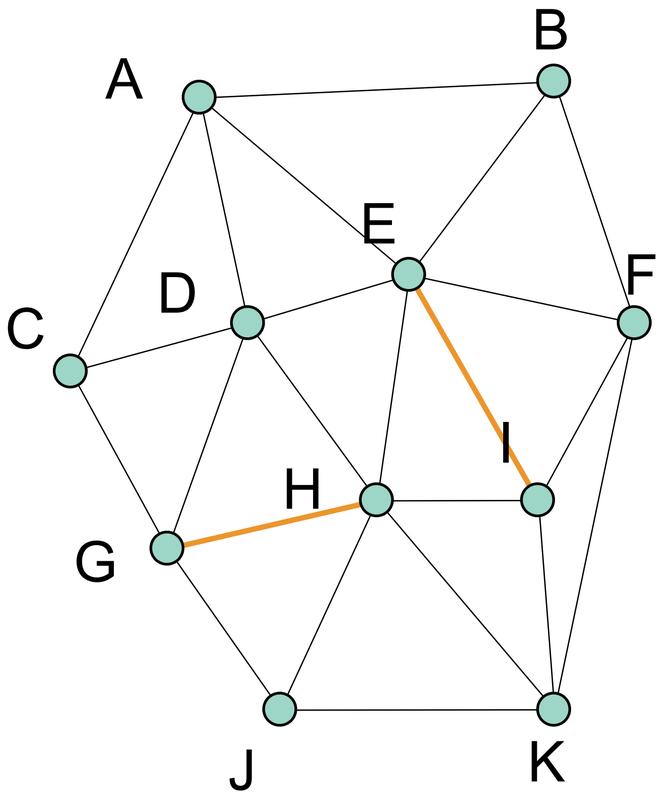


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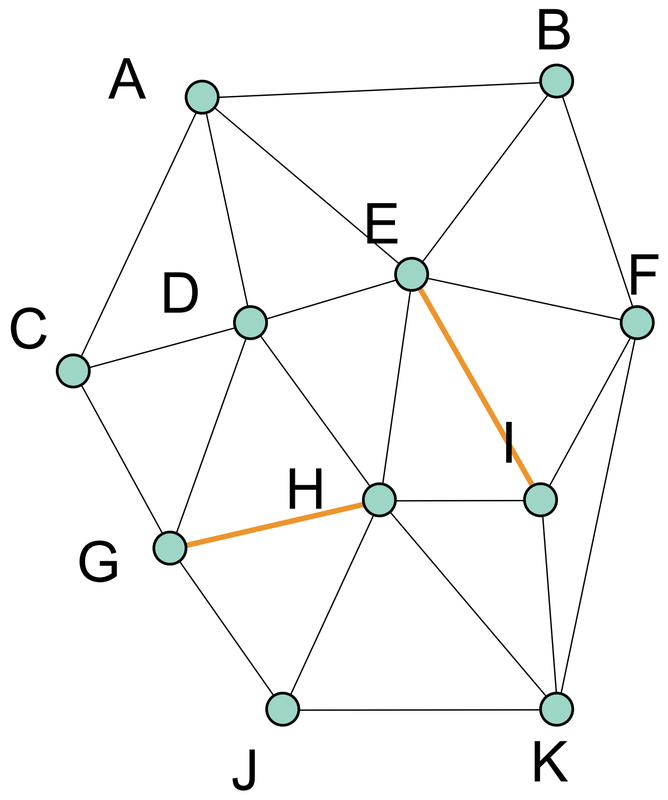


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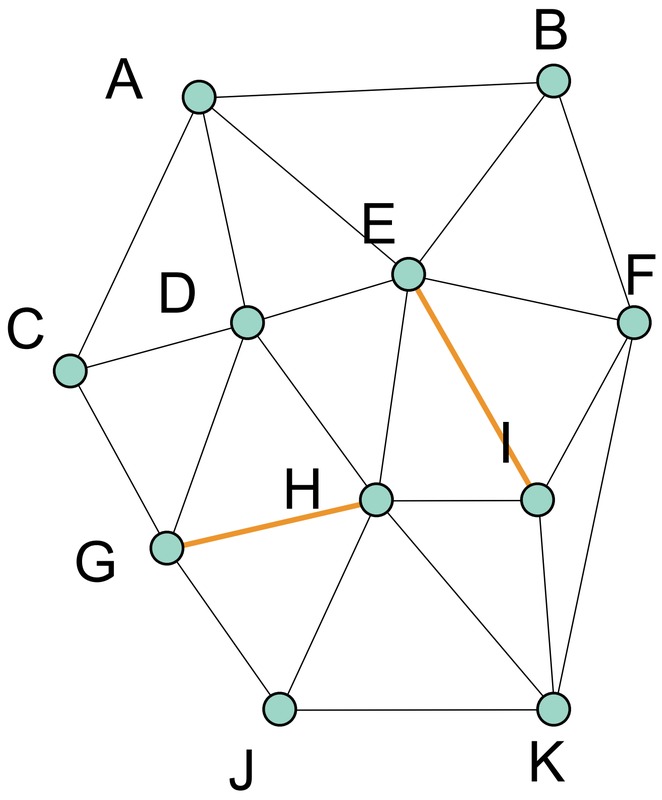


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- ➡ by definition, triangle is **planar**
- ➡ any polygon can be triangulated (not necessarily uniquely)

# Outline

- Parametric Approximations
- Polygon Meshes
- **Data structures**

# Mesh Data Structures

- How to store **geometry** & **connectivity (topology)**?
- Compact storage
  - File formats
- Efficient algorithms on meshes
  - Identify time-critical operations
  - All vertices/edges of a face
  - All **incident/neighboring** vertices/edges/faces of a vertex

# Data Structures

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    - What is adjacent to what

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  - *Is this differentiable?*

# Face Set (STL)

- `Face list:`
  - 3 positions

| Triangles | | |
|---|---|---|
| $x_{11}$ $y_{11}$ $z_{11}$ | $x_{12}$ $y_{12}$ $z_{12}$ | $x_{13}$ $y_{13}$ $z_{13}$ |
| $x_{21}$ $y_{21}$ $z_{21}$ | $x_{22}$ $y_{22}$ $z_{22}$ | $x_{23}$ $y_{23}$ $z_{23}$ |
| ... | ... | ... |
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36 B/f = 72 B/v

no connectivity!

# Shared Vertex (OBJ, OFF)

- `Indexed Face List`
  - Vertex: position
  - Face: vertex indices

| Vertices |
|---|
| $x_1$ $y_1$ $z_1$ |
| ... |
| $x_V$ $y_V$ $z_V$ |

| Triangles |
|---|
| $i_{11}$ $i_{12}$ $i_{13}$ |
| ... |
| ... |
| ... |
| ... |
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| ... |
| ... |
| ... |
| ... |
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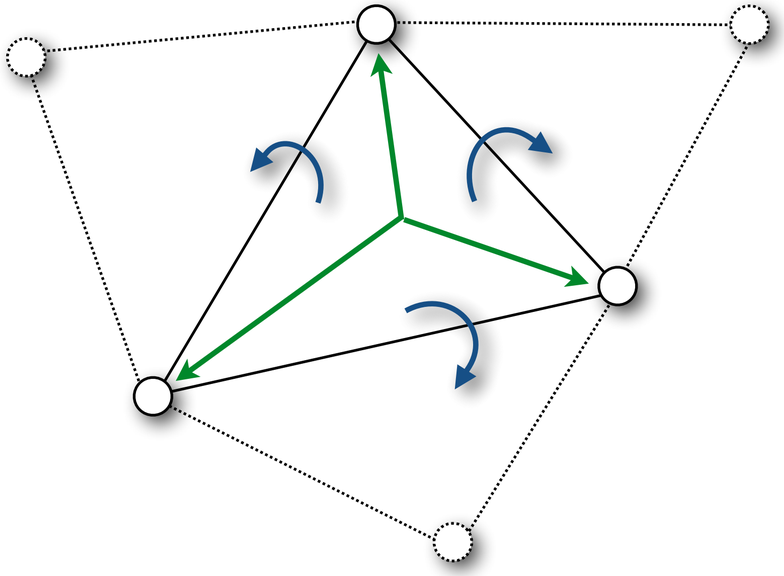
12 B/v + 12 B/f = 36 B/v

no explicit neighborhood information

# Face-Based Connectivity

- Vertex:
  - position
  - 1 face
- Face:
  - 3 vertices
  - 3 face neighbors



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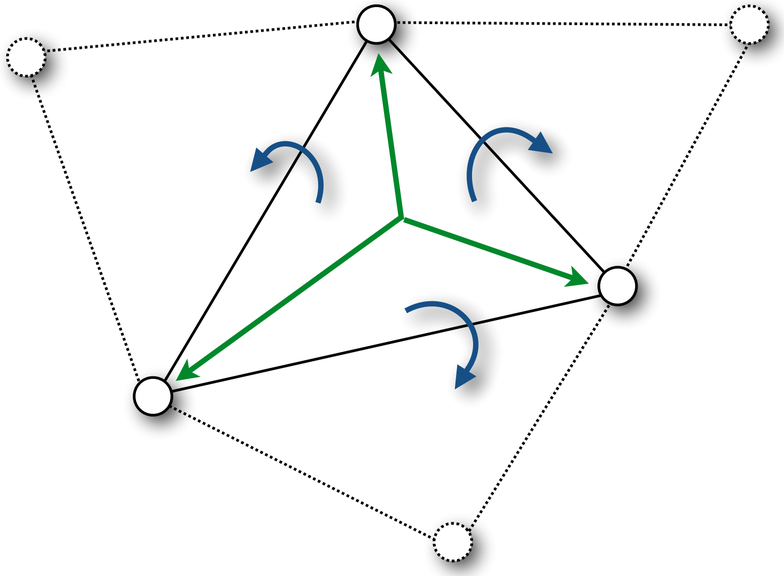
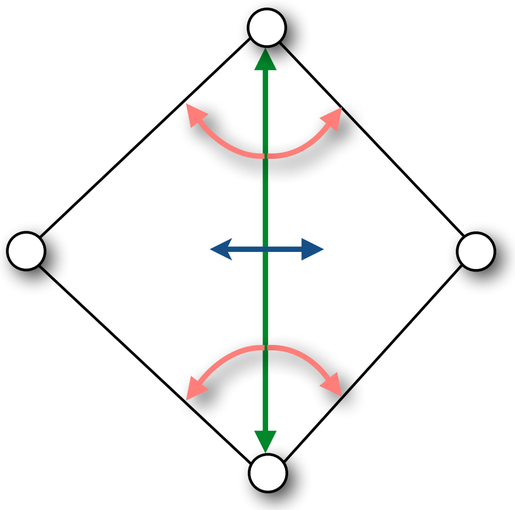


64 B/v

no edges!

# Edge-Based Connectivity

- Vertex
  - position
  - 1 edge
- Edge
  - 2 vertices
  - 2 faces
  - 4 edges
- Face
  - 1 edge



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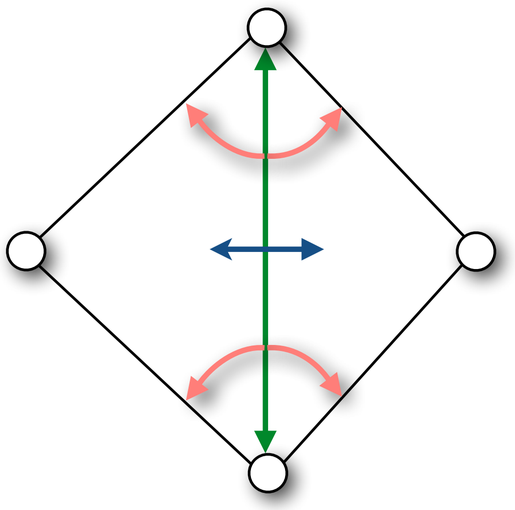
- Edge
  - 2 vertices
  - 2 faces
  - 4 edges
- Face
  - 1 edge

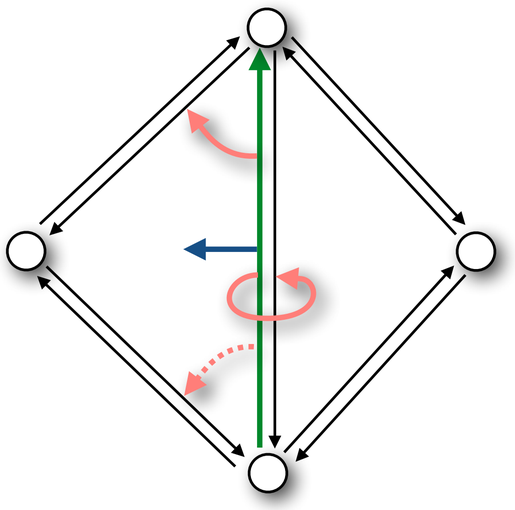


120 B/v

edge orientation?

# Halfedge-Based Connectivity

- Vertex
  - position
  - 1 halfedge
- Halfedge
  - 1 vertex
  - 1 face
  - 1, 2, or 3 halfedges
- Face
  - 1 halfedge



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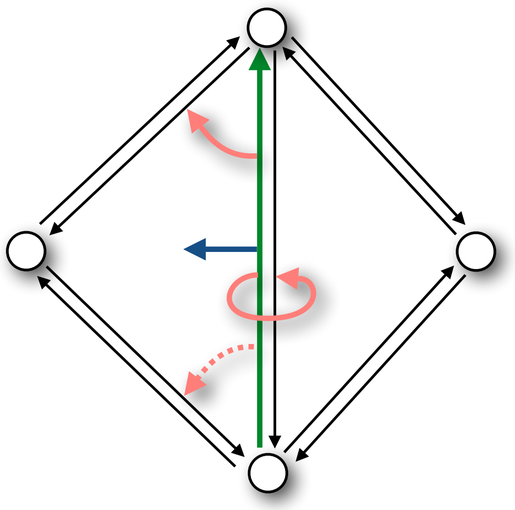
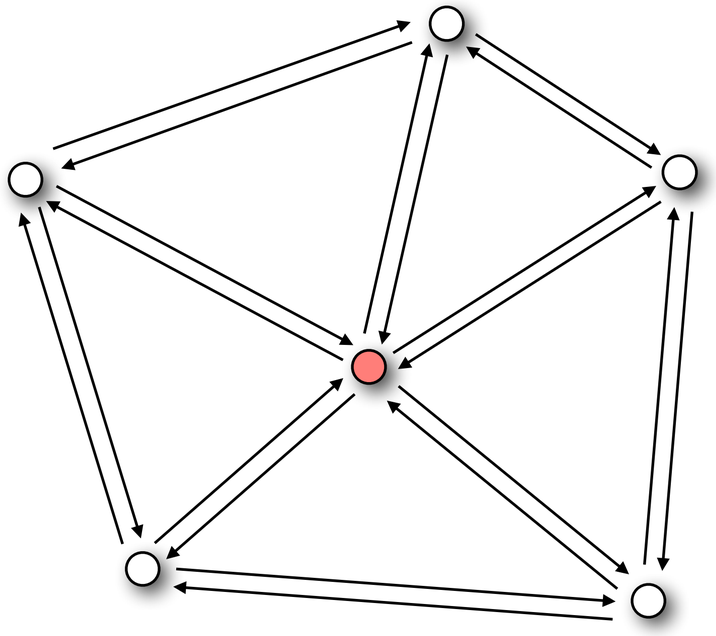


96 to 144 B/v

no case distinctions during traversal

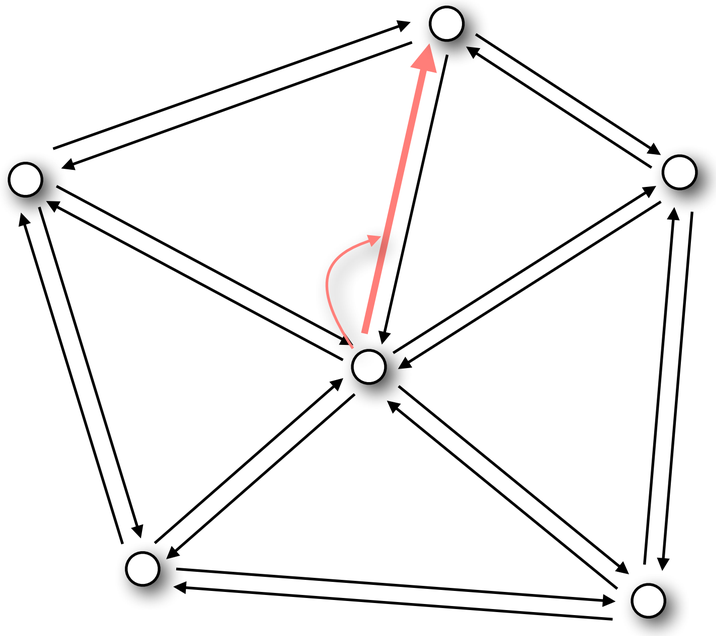
# One-Ring Traversal

1. Start at vertex



# One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge



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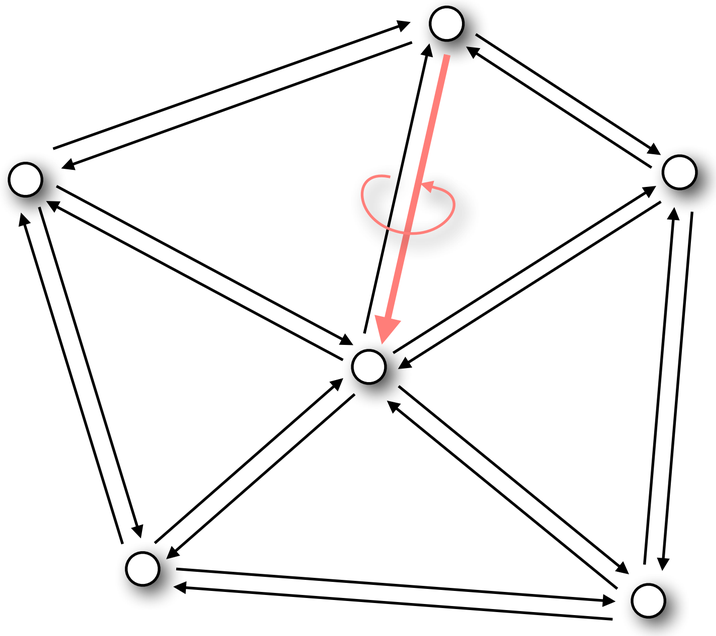
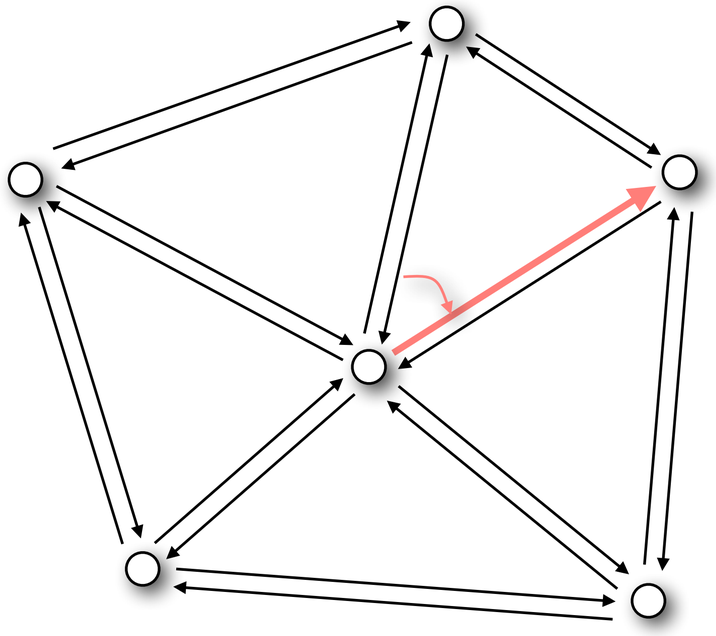
3. Opposite halfedge



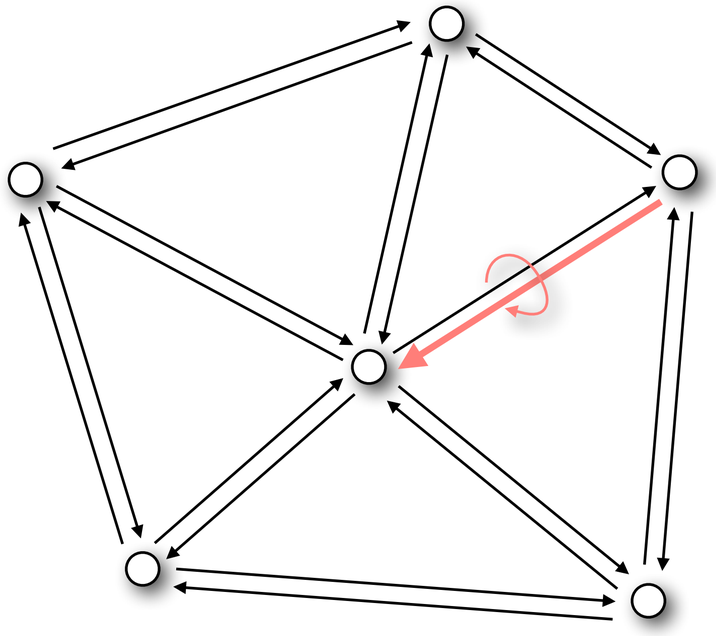
# One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge
4. Next halfedge



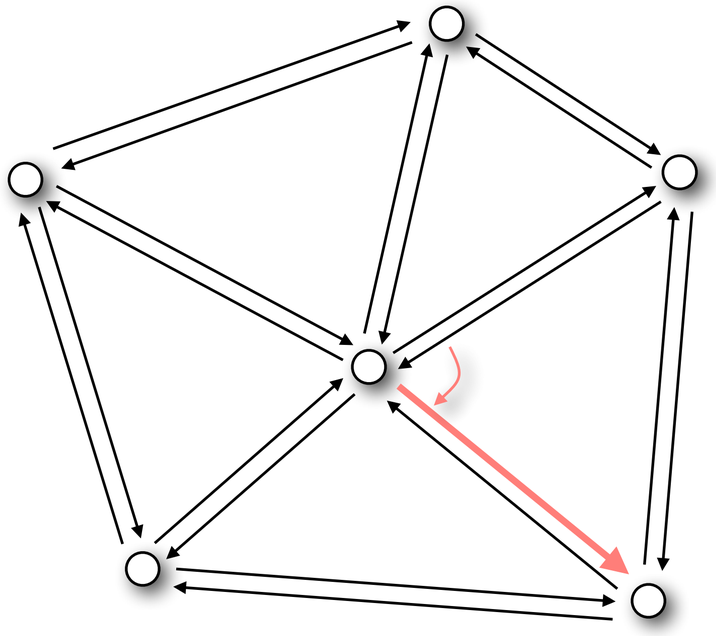
# One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge
4. Next halfedge
5. Opposite



# One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge
4. Next halfedge
5. Opposite
6. Next
7. ...



# Halfedge-Based Libraries

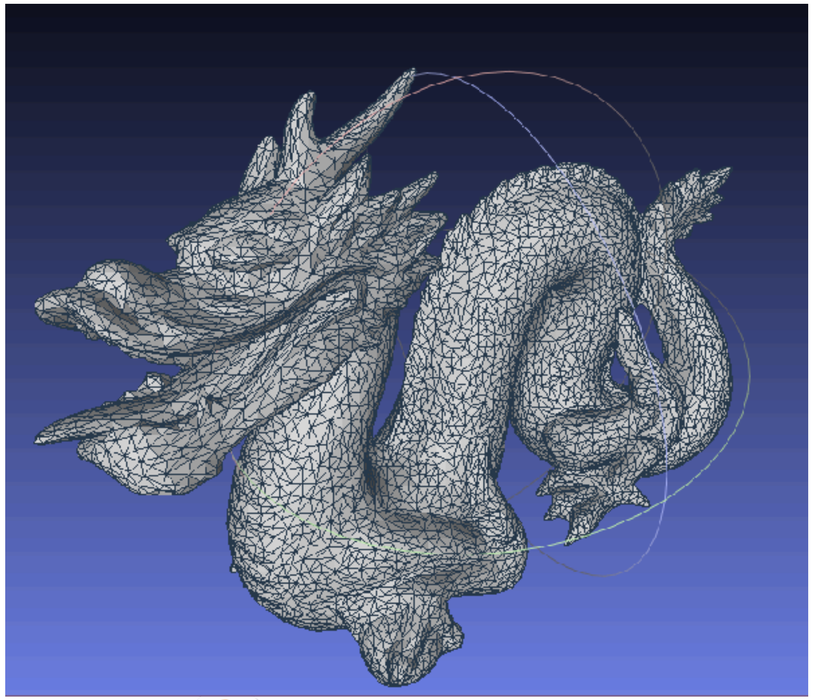
- CGAL
  - www.cgal.org
  - Computational geometry
  - Free for non-commercial use

- OpenMesh
  - www.openmesh.org
  - Mesh processing
  - Free, LGPL licence

# Looking at Meshes

Meshlab

MATLAB
The Language of Technical Computing



CGAL

OpenMesh

libigl

blender™

# Literature

- Book #1: Chapters 2,3;
- Book #2: Chapters 1,2
- Kettner, *Using generic programming for designing a data structure for polyhedral surfaces*, Symp. on Comp. Geom., 1998
- Campagna et al., *Directed Edges - A Scalable Representation for Triangle Meshes*, Journal of Graphics Tools 4(3), 1998
- Botsch et al., *OpenMesh - A generic and efficient polygon mesh data structure*, OpenSG Symp. 2002