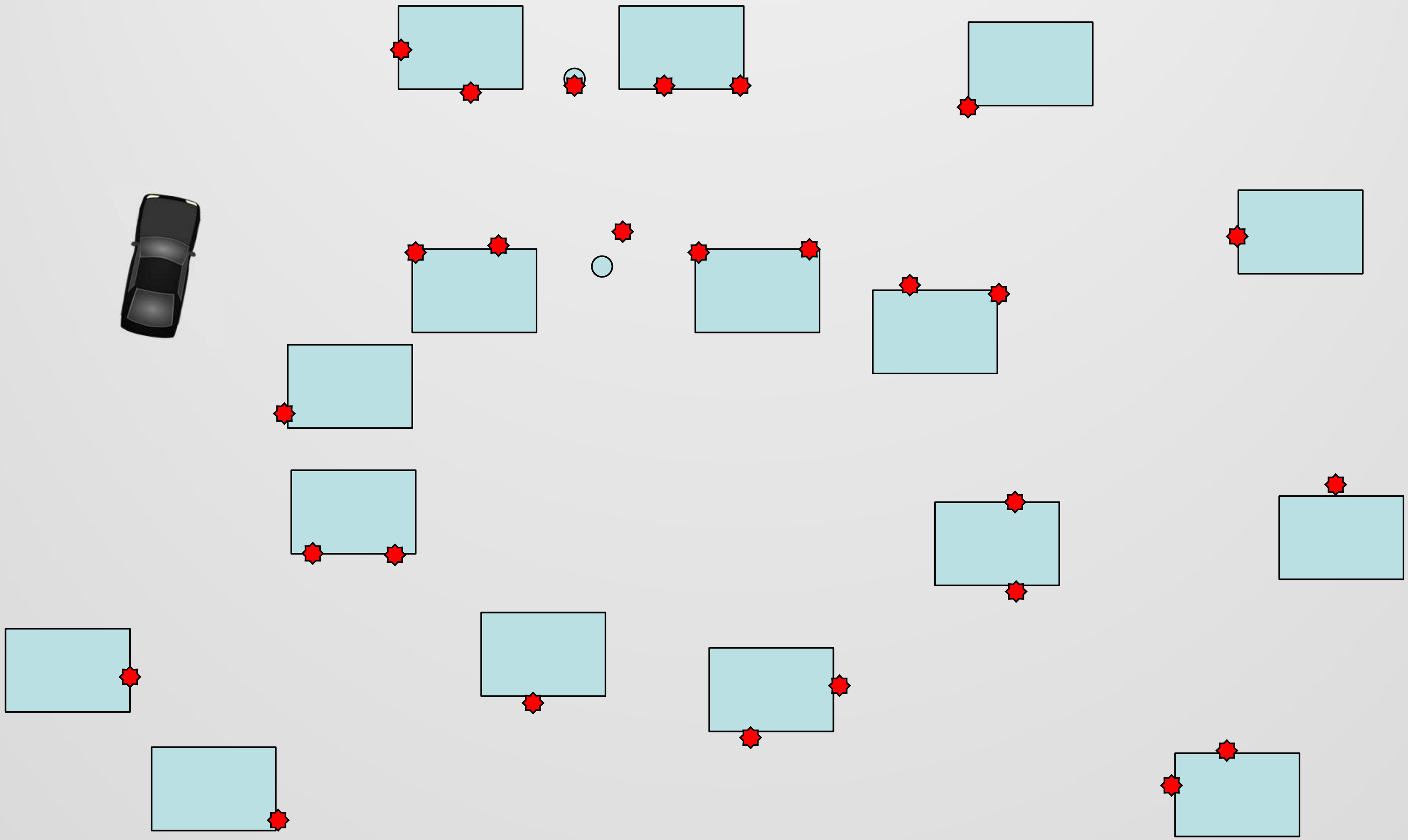
COMP0130: Robotic Vision and Navigation

Lecture 05B: Notation and the Big SLAM Equation

Goals

- Introduce the mathematical description and notation used to model the system
- Present the joint probabilistic model which is the cornerstone for SLAM

SLAM



Mathematical Description

- Platform state
- Platform process model
- Landmark state
- Platform / landmark observation model

Platform State

- This is the state that the robot is in on a timestep-by-timestep basis
- For example, for a driven car with position and heading, the state might be

$$\mathbf{x}_k = \begin{bmatrix} \underbrace{x_k \quad y_k}_{} & \underbrace{\psi_k}_{} \end{bmatrix}^\top$$

Platform Process Model

- The platform state evolves according to the familiar Kalman filter-type of process model

$$\dot{\mathbf{x}}_k = \mathbf{f}\left[\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{v}_k\right]$$

Q

w

R

Map State

- The map is a set of N landmarks or features

$$\mathbf{m} = \{\mathbf{m}^1, \mathbf{m}^2, \cdots, \mathbf{m}^N\}$$

- Each landmark maintains its own state, such as its position

$$\mathbf{m}^i = \begin{bmatrix} u^i & v^i \end{bmatrix}^\top$$

Map State

- We made two innocent-looking assumptions

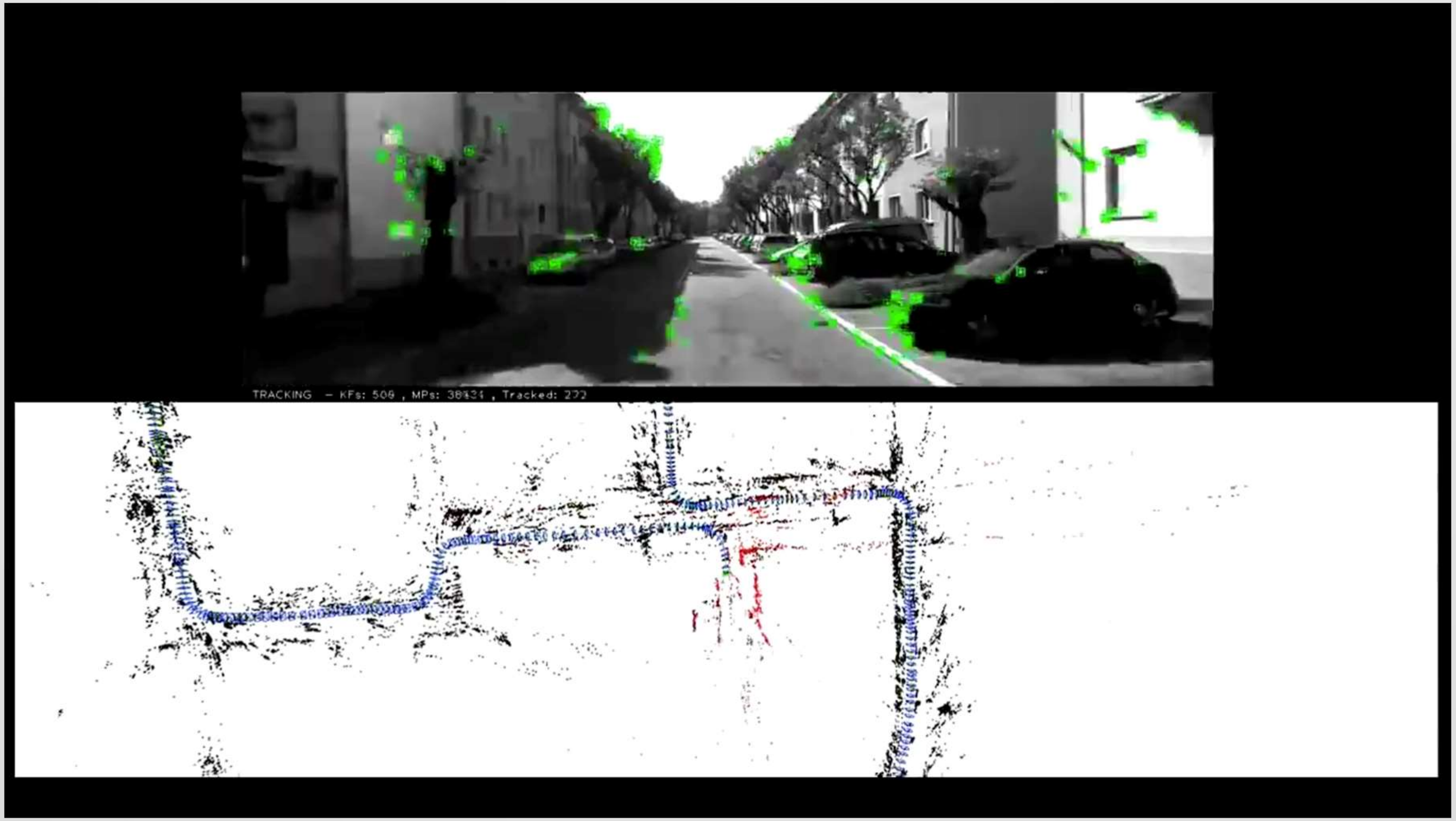
1. All the landmarks have a unique label associated with them
2. All landmarks are static and don't change over time

Labelling Landmarks

$m = \{m^{(1)}, m^{(2)}, \ldots, m^{(N)}\}$

- Each landmark has some label associated with it
- This can become very complicated when you don't know how many landmarks are out there, there are multiple robots, etc.
- We are going to use a simple integer counter which increases by 1 each time

Landmarks Are Static?



From https://www.youtube.com/watch?v=8DISRmsO2YQ
(00:57.5-00:62.5 seconds)

Landmarks Are Static

Fixed
Real
World

Varying
Estimate

Describing Sensor Observations

- At time k, the robot observes a set of M_k landmarks,

$$\mathbf{Z}_k = \left\{ \underline{\mathbf{z}_k^1, \ \mathbf{z}_k^2, \ \ldots, \ \mathbf{z}_k^{M_k}} \right\}$$

- Each observation comes from a landmark
- The mapping index is

$$I_k = \left\{ i_k^1, \ i_k^2, \ \ldots, \ i_k^{M_k} \right\}$$

Sensor Observations

$$\mathbf{Z}_k = \left\{ \mathbf{z}_k^1, \ \mathbf{z}_k^2, \ \ldots, \ \mathbf{z}_k^{M_k} \right\}$$

$$I_k = \left\{ i_k^1, \ i_k^2, \ \ldots, \ i_k^{M_k} \right\}$$

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Process and Observation Models

- The observation model for the jth landmark is

$Z_k = (\text{———} \}$

$$\mathbf{z}_k^j = \mathbf{h}\left[\mathbf{x}_k, \mathbf{m}^{i_j}, \mathbf{w}_k^j\right]$$

$v \quad w$

Probabilistic Equation for SLAM

- The expression for the probabilistic formulation is

$$f\left(\mathbf{x}_k, \mathbf{m} \middle| \mathbf{Z}_{0:k}, \mathbf{U}_{0:k}, \mathbf{x}_0\right)$$

Current pose of the platform

Map

Set of all observations

$\mathbf{Z}_{0:k} = \{\mathbf{Z}_0, \ldots, \mathbf{Z}_k\}$

Set of all control inputs

$\mathbf{U}_{0:k} = \{\mathbf{u}_0, \ldots, \mathbf{u}_k\}$

Initial conditions

So… How Do We Implement SLAM?

- We have the problem that we want to estimate the joint distribution of the pose of the platform and the landmarks in one go
- This is an estimation algorithm which takes in a set of observations and control inputs to estimate a state vector
- We have already seen a solution – the Kalman filter

SLAM