COMP0130 Robot Vision and Navigation

3B: Multisensor Integrated Navigation

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Session Objectives

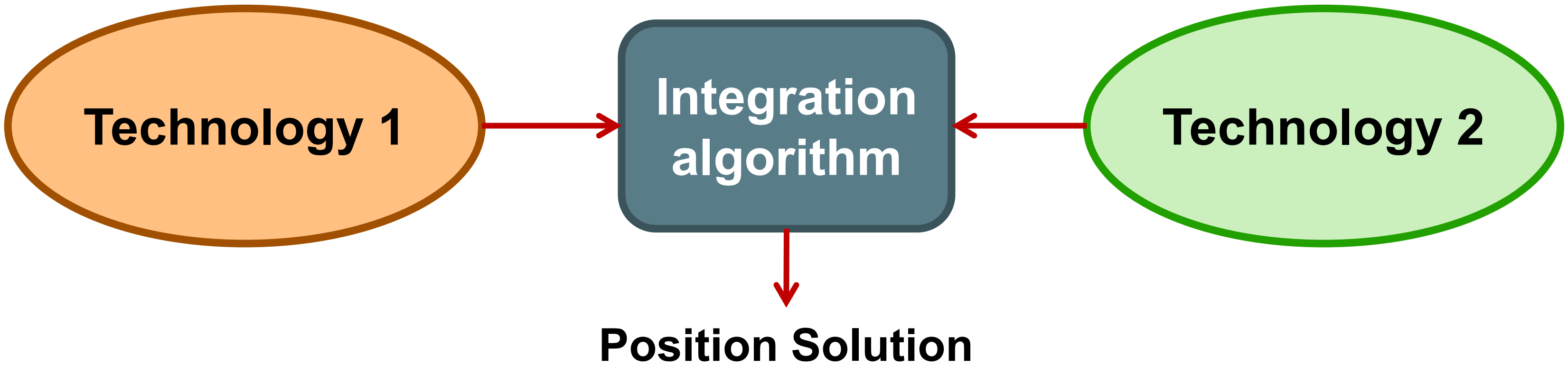
- Introduce the fundamentals and motivation of integrated navigation
- Show how a Kalman filter is used to integrate GNSS with Dead Reckoning technologies
- Show how this can be extended to calibrate various sensor errors

Contents

1. Introduction to Integrated Navigation
2. Basic Integration of GNSS with Dead Reckoning
3. Gyro-Magnetometer Integration
4. INS/GNSS Integration with Error Estimation

1. Introduction to Integrated Navigation

What is Integrated Navigation?



The combination of two or more *different* positioning technologies to obtain a better position solution

- Greater availability
- Greater reliability
- Greater accuracy

Note: Integration is *not* the combination of more than one GNSS

GPS, GLONASS, Galileo and Beidou are not *different* technologies

Why Integrate? - Availability

Radio Positioning (e.g. GNSS)

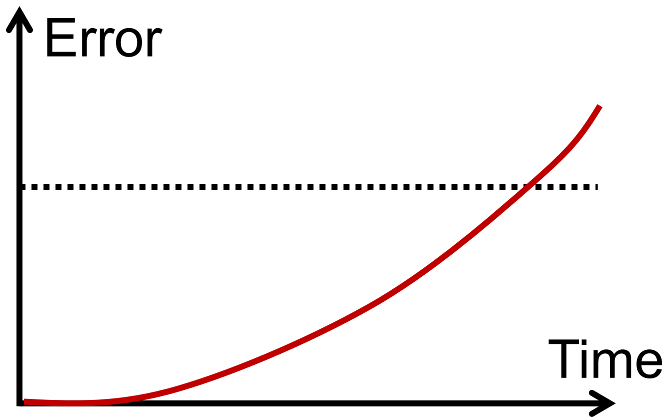
- Only works when a sufficient number of good signals are received
- This can never be guaranteed 100% of the time

Environmental Feature Matching

- Relies on the availability of distinct and identifiable features
- These are never available everywhere

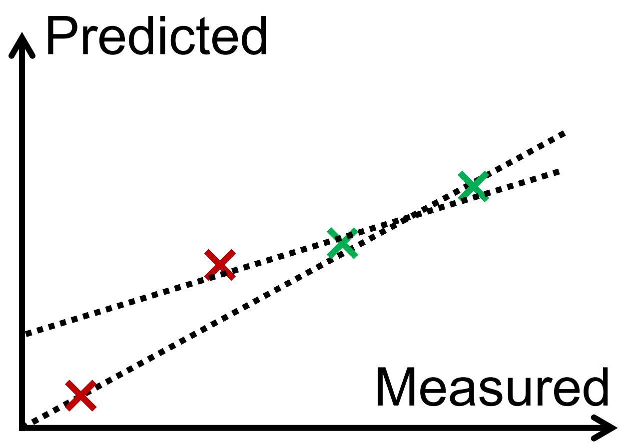
Dead Reckoning (e.g. INS)

- Position error grows with time
- At some point, it will exceed the performance specification

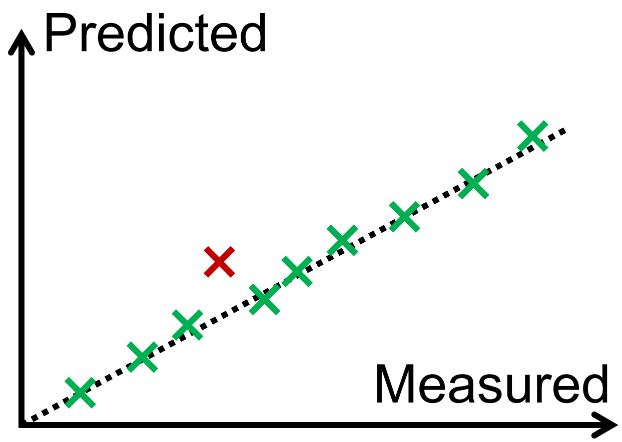


Why Integrate? - Reliability

The more information you have, the easier it is to detect faults



Which is the outlier?

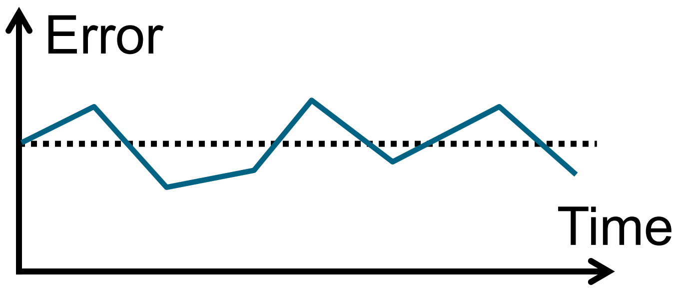


Outlier is clear

Why Integrate? - Accuracy

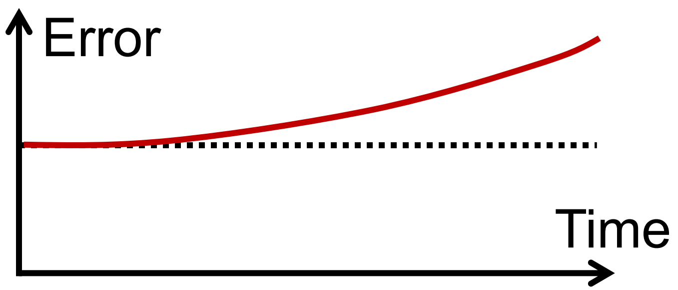
Some systems offer better long-term accuracy

- e.g. GNSS

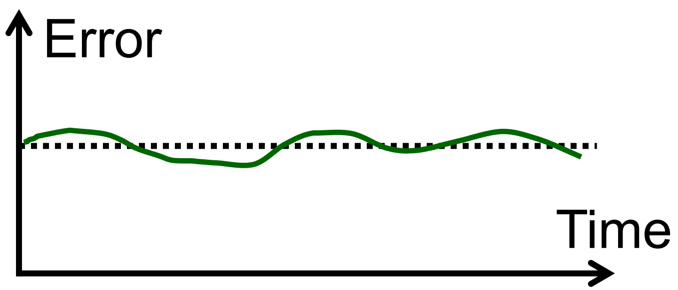


Other systems offer better short-term accuracy

- i.e. Changes in position
- e.g. INS



Best overall accuracy is obtained by integrating them



Multisensor Navigation

Dead Reckoning (DR)

- **Inertial navigation**
- **Wheel-speed odometry**
- **Magnetic heading**
- Visual odometry
- Doppler radar
- Doppler sonar
- Pedestrian dead reckoning

This session will focus on a limited number of technologies, but the methodology is widely applicable

Position Fixing

- **GNSS**
- Wi-Fi positioning
- Image matching
- Terrain referenced navigation
- Ultra-wideband (UWB)
- Phone-signal positioning
- Map matching
- Magnetic anomaly matching
- Gravity gradiometry
- Acoustic ranging

Benefits of Integrating GNSS with DR

Dead-reckoning sensors improve the robustness of GNSS

- DR bridges short-term GNSS outages, boosting continuity
- Some technologies, such as inertial navigation, increase the bandwidth of the position solution and smooth out the noise

GNSS integration enables useful navigation with low-cost DR sensors

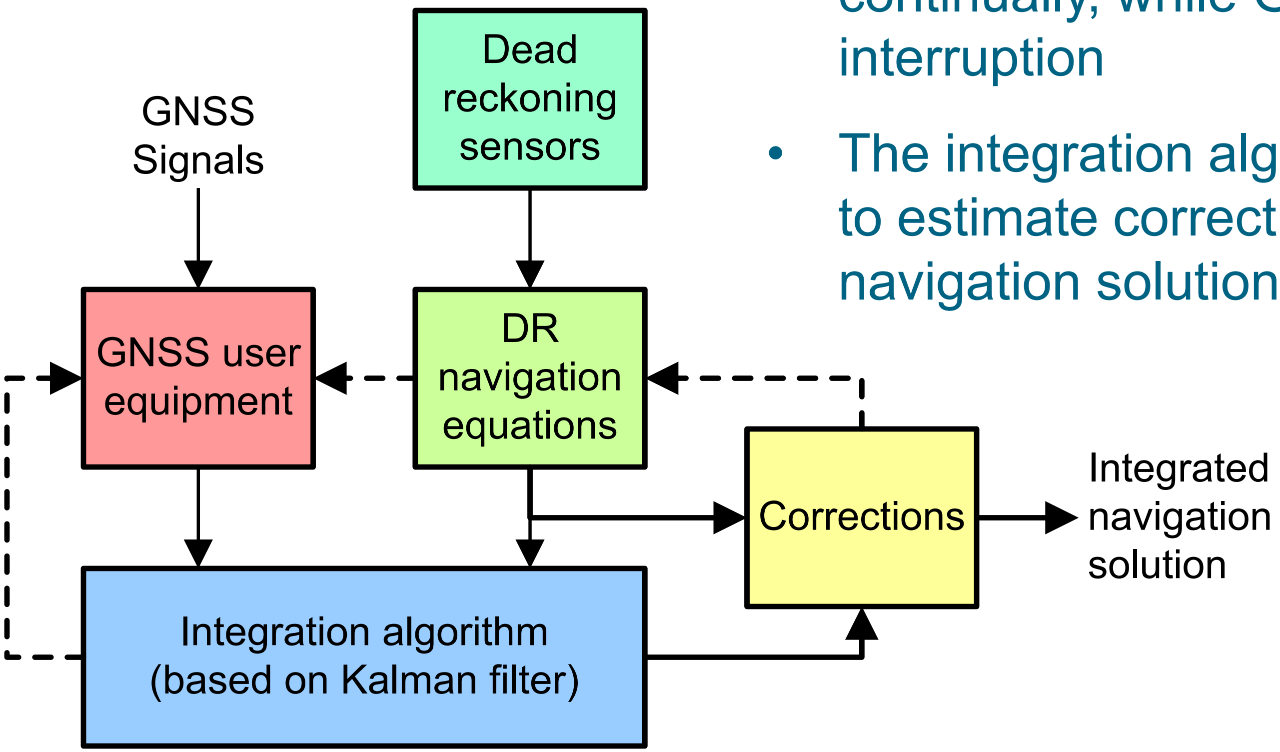
- GNSS prevents the DR position solution error growing with time
- GNSS aids calibration of the sensor systematic errors

Applications

- Navigation of all-types of autonomous and conventional vehicles
- Hydrographic surveying, mobile mapping, sensor stabilization (INS/GNSS)

Image: Applanix Corporation

Fundamentals of DR/GNSS Integration

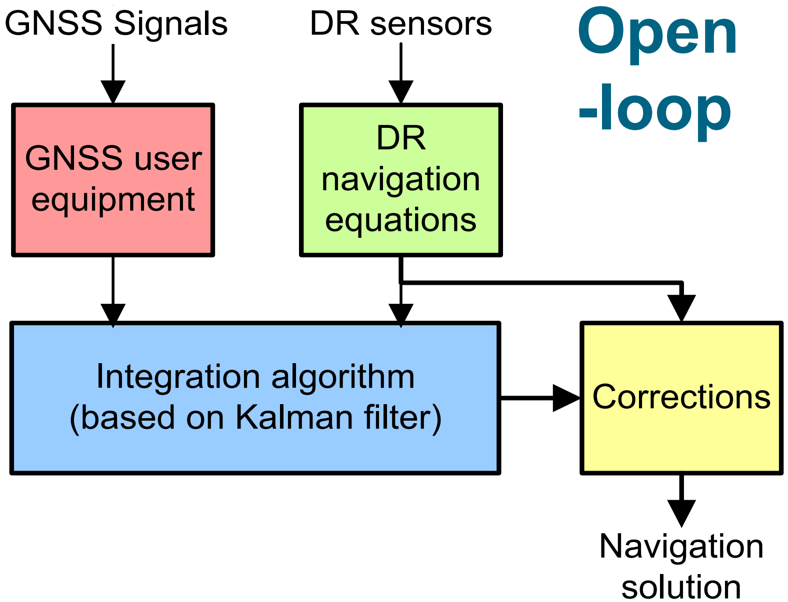


- Dead reckoning forms the integrated solution because DR operates continually, while GNSS is subject to interruption
- The integration algorithm uses GNSS to estimate corrections to the DR navigation solution

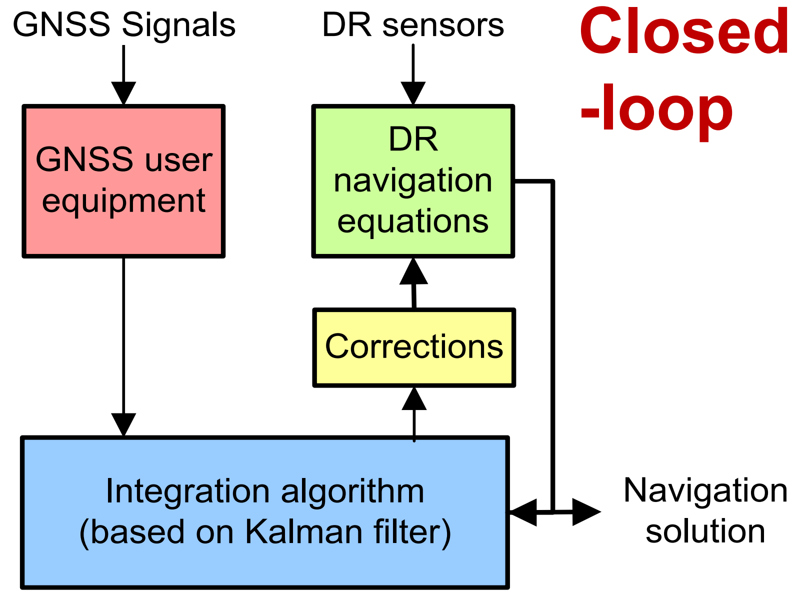
1. Introduction to Integrated Navigation

Open and Closed-loop Correction

Open-loop



Closed-loop

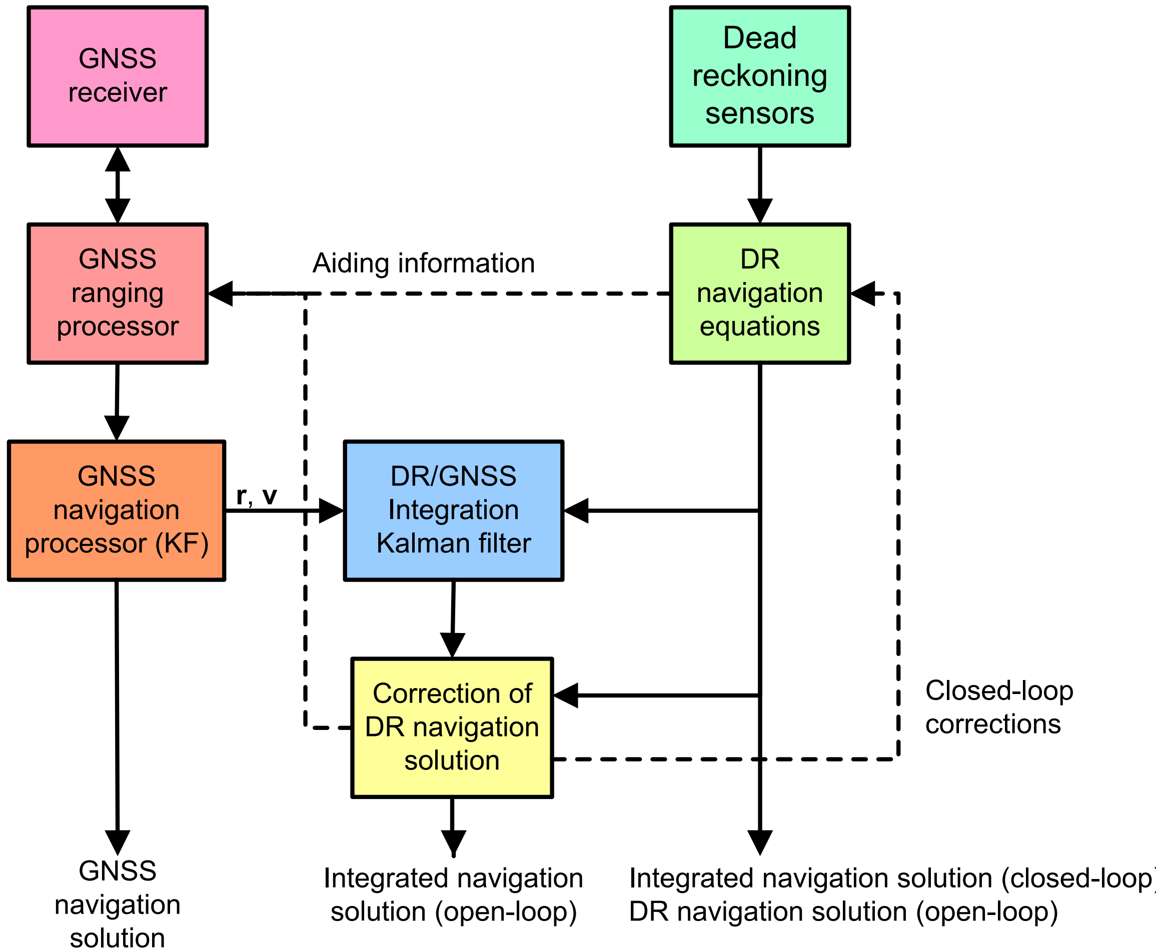


- Separate DR solution maintained
- Corrected by KF every epoch
- KF DR error estimates can grow large, causing linearisation problems

- Corrections fed back to correct the DR system every epoch
- KF DR error estimates are then zeroed, keeping them small

Loosely-coupled DR/GNSS Integration

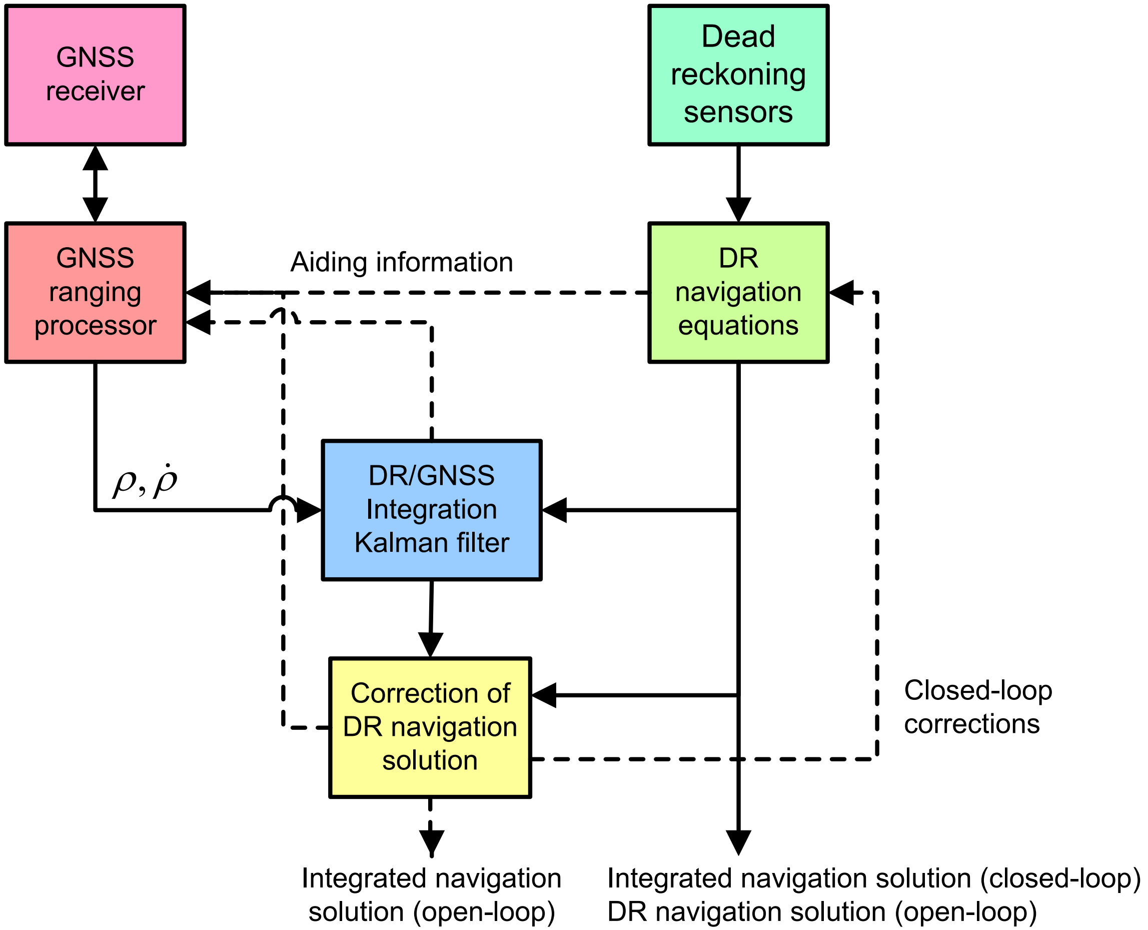
- Can work with any DR and GNSS user equipment
- Maintains stand-alone GNSS solution as back-up
- Needs at least 4 GNSS satellites
- Cascaded Kalman filters
- Low DR/GNSS filter gain to avoid instability
- Limits performance



1. Introduction to Integrated Navigation

Tightly-coupled DR/GNSS Integration

- Requires suitable GNSS user equipment with pseudo-range and range-rate (Doppler) outputs
- No Kalman filter cascade
- Optimal DR/GNSS filter gain
- Better performance than loosely-coupled
- Works with less than 4 GNSS satellites



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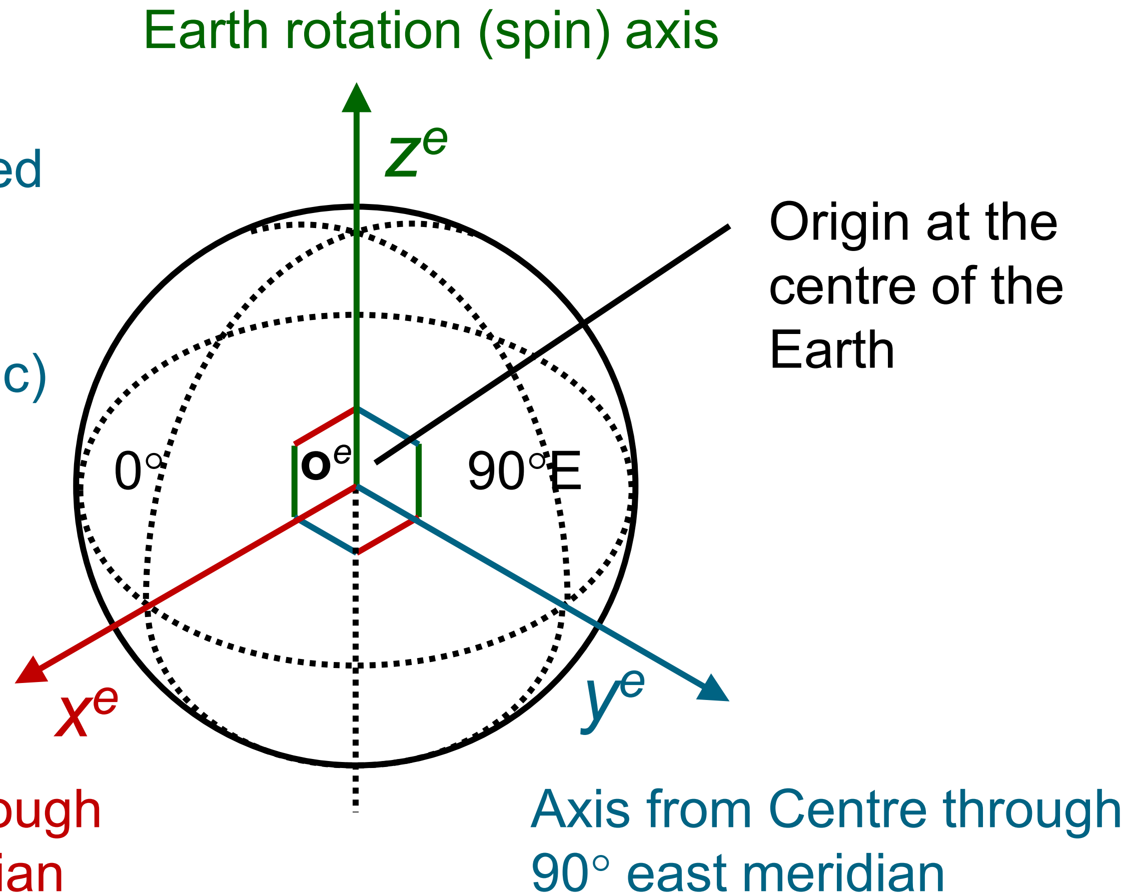
2. Basic Integration of GNSS with Dead Reckoning

ECEF Cartesian Coordinates

GNSS Positioning algorithms normally use

an Earth-centred Earth-fixed (ECEF) frame

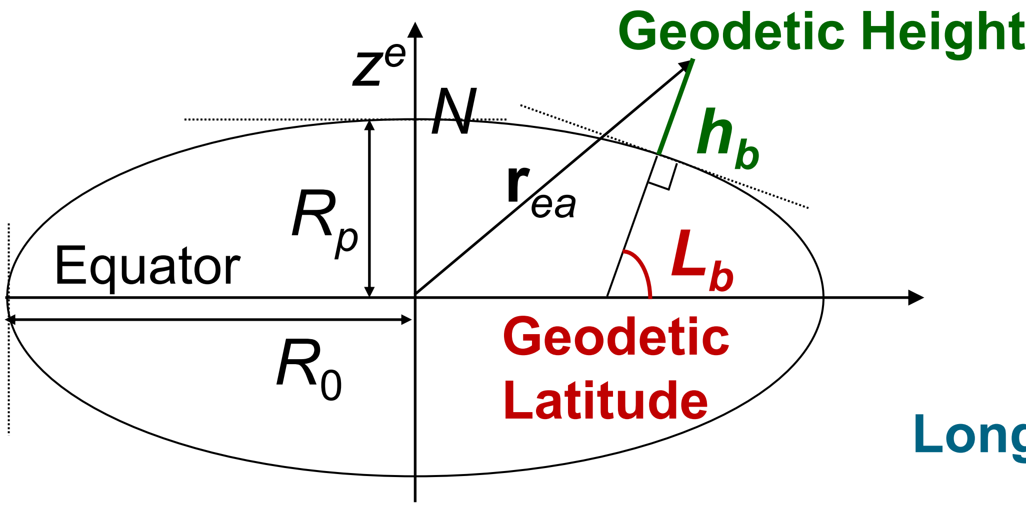
with Cartesian position coordinates (i.e., geocentric)

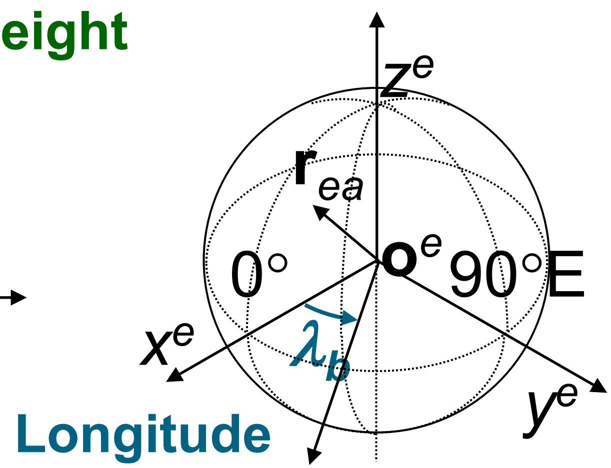


Latitude, longitude and height (1)

ECEF-referenced position has limited practical use

Latitude, longitude and height are more useful

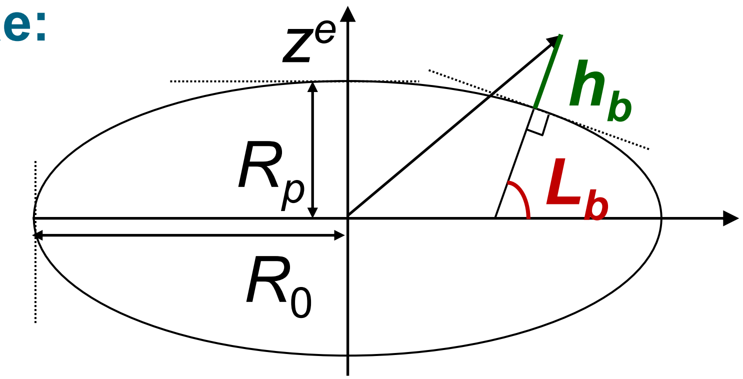


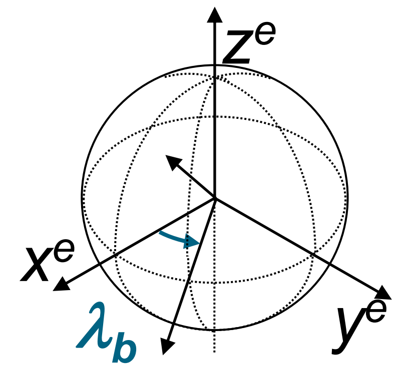


Projection of $\mathbf{r}_{eb}$ on to equatorial plane

Latitude, longitude and height (2)

Conversion formulae:





$$\tan \lambda_b = \frac{y^e_{eb}}{x^e_{eb}}$$

$$\tan \zeta_b = \frac{z^e_{eb}}{\sqrt{1-e^2}\sqrt{x^{e\,2}_{eb} + y^{e\,2}_{eb}}}$$

$$R_E = \frac{R_0}{\sqrt{1-e^2 \sin^2 L_b}}$$

$$\tan L_b \approx \frac{z^e_{eb}\sqrt{1-e^2} + e^2 R_0 \sin^3 \zeta_b}{\sqrt{1-e^2}\left(\sqrt{x^{e\,2}_{eb} + y^{e\,2}_{eb}} - e^2 R_0 \cos^3 \zeta_b\right)}$$

$$h_b = \frac{\sqrt{x^{e\,2}_{eb} + y^{e\,2}_{eb}}}{\cos L_b} - R_E$$

WGS84 datum: R_0 = 6,378,137.0 m e = 0.0818191908425

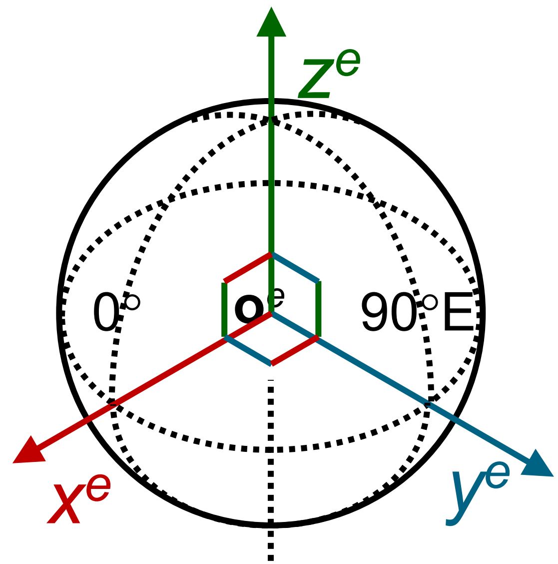
2. Basic Integration of GNSS with Dead Reckoning

Latitude, longitude and height (3)

Converting back to Cartesian Position

$$x_{eb}^{e} = \left(R_E + h_b\right)\cos L_b \cos \lambda_b$$
$$y_{eb}^{e} = \left(R_E + h_b\right)\cos L_b \sin \lambda_b$$
$$z_{eb}^{e} = \left[\left(1 - e^2\right)R_E + h_b\right]\sin L_b$$

$$R_E = \frac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}}$$



This is exact

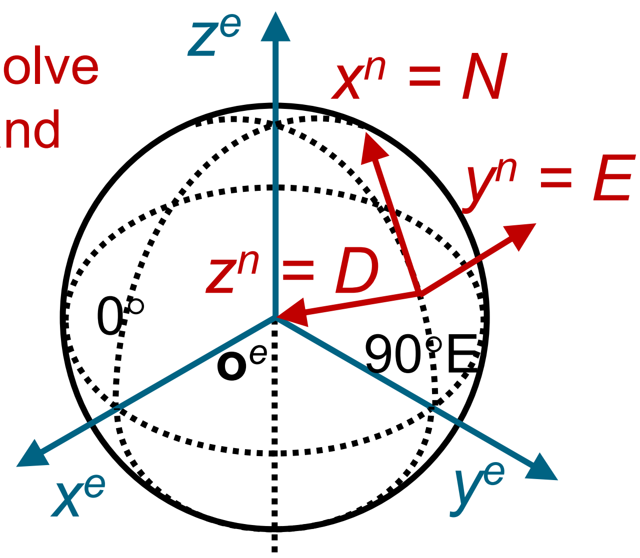
WGS84 datum: R_0 = 6,378,137.0 m e = 0.0818191908425

Converting Velocity between ECEF & NED

GNSS algorithms typically resolve velocity along ECEF axes

DR algorithms typically resolve velocity along north, east and down axes

Matlab functions for converting position and velocity are available on Moodle



ECEF *to* NED

$$\mathbf{v}_{eb}^{n} = \mathbf{C}_e^n \mathbf{v}_{eb}^{e}$$

$$\mathbf{C}_e^n = \begin{pmatrix} -\sin L\cos\lambda & -\sin L\sin\lambda & \cos L \\ -\sin\lambda & \cos\lambda & 0 \\ -\cos L\cos\lambda & -\cos L\sin\lambda & -\sin L \end{pmatrix}$$

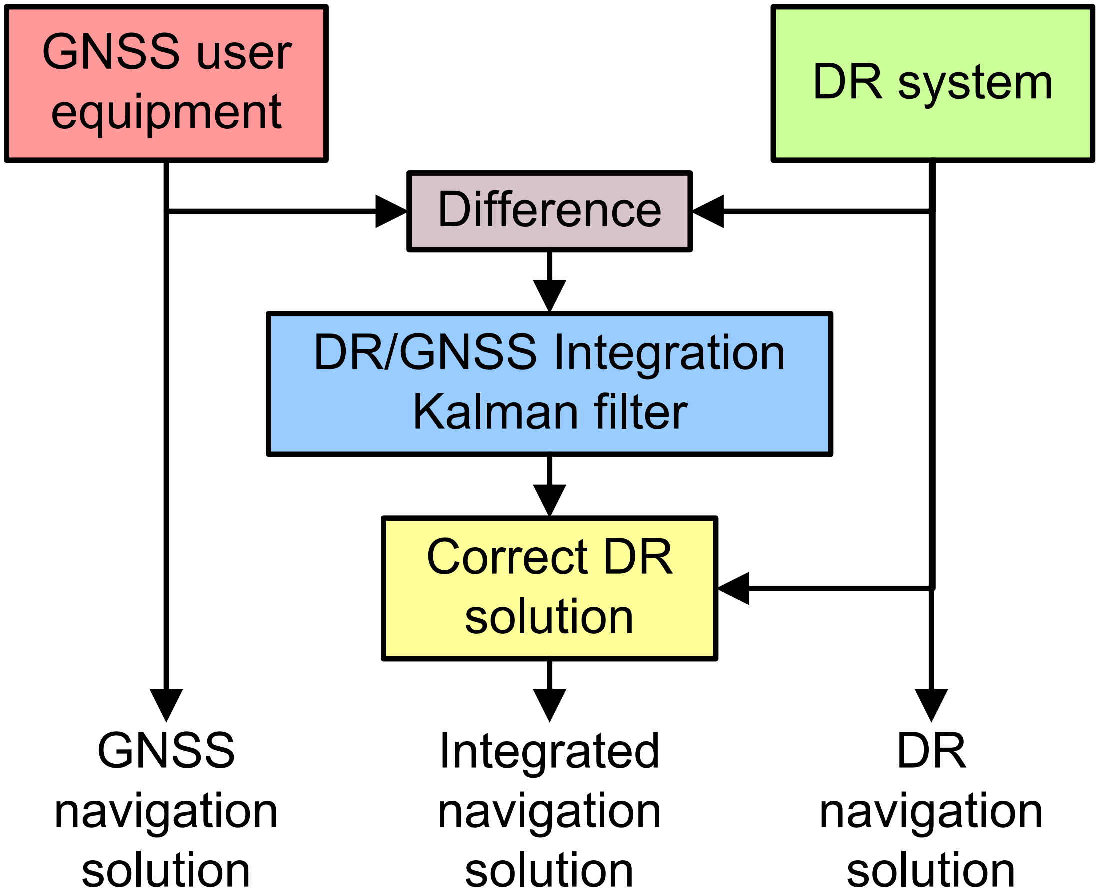
NED *to* ECEF

$$\mathbf{v}_{eb}^{e} = \mathbf{C}_n^e \mathbf{v}_{eb}^{n}$$

$$\mathbf{C}_n^e = \begin{pmatrix} -\sin L\cos\lambda & -\sin\lambda & -\cos L\cos\lambda \\ -\sin L\sin\lambda & \cos\lambda & -\cos L\sin\lambda \\ \cos L & 0 & -\sin L \end{pmatrix}$$

Basic Loosely-coupled Open-loop Integration

- Separate GNSS and dead reckoning solutions are computed
- Kalman filter estimates DR position and velocity errors (only)
- Corrected DR navigation solution is the integrated solution
- ECEF and NED implementations



2. Basic Integration of GNSS with Dead Reckoning

The Kalman Filter

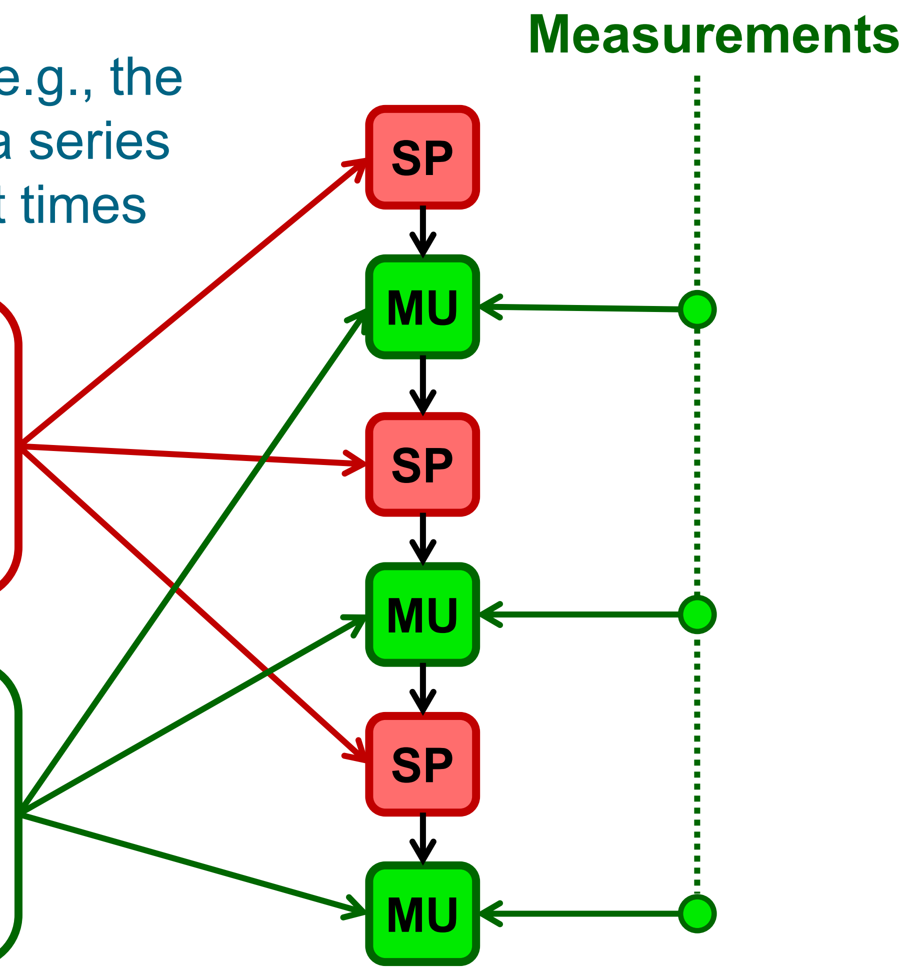
Estimates states that vary in time, e.g., the position of a moving object, using a series of measurements made at different times

State Propagation

Uses a dynamic model to predict the state estimates forward in time

Measurement Update

Uses measurements to update the state estimates



ECEF Implementation: Kalman Filter States

6 states are estimated

- 3D DR Velocity error (ECEF)
  - x^e component
  - y^e component
  - z^e component
- 3D DR Position error (ECEF)
  - x^e component
  - y^e component
  - z^e component

$$\mathbf{x} = \begin{pmatrix} \delta\mathbf{v}^e_{eb} \\ \delta\mathbf{r}^e_{eb} \end{pmatrix} = \begin{pmatrix} \delta v^e_{eb,x} \\ \delta v^e_{eb,y} \\ \delta v^e_{eb,z} \\ \delta x^e_{eb} \\ \delta y^e_{eb} \\ \delta z^e_{eb} \end{pmatrix}$$

2. Basic Integration of GNSS with Dead Reckoning

ECEF Step 1: Calculate Transition Matrix

The DR position error is the integral of the DR velocity error:

$$\delta\mathbf{v}^e_{eb,k} = \delta\mathbf{v}^e_{eb,k-1}$$
$$\delta\mathbf{r}^e_{eb,k} = \delta\mathbf{r}^e_{eb,k-1} + \tau_s \delta\mathbf{v}^e_{eb,k-1}$$

τ_s = time interval

In matrix-vector form:

$$\begin{pmatrix} \delta\mathbf{v}^e_{eb,k} \\ \delta\mathbf{r}^e_{eb,k} \end{pmatrix} = \begin{pmatrix} \mathbf{I}_3 & \mathbf{0}_3 \\ \tau_s\mathbf{I}_3 & \mathbf{I}_3 \end{pmatrix} \begin{pmatrix} \delta\mathbf{v}^e_{eb,k-1} \\ \delta\mathbf{r}^e_{eb,k-1} \end{pmatrix}$$

Transition matrix:

$$\mathbf{x}_k = \mathbf{\Phi}_{k-1}\mathbf{x}_{k-1} \Rightarrow$$

Old velocity error

Old position error

New velocity error

New position error

$$\mathbf{\Phi}_{k-1} = \begin{pmatrix} \mathbf{I}_3 & \mathbf{0}_3 \\ \tau_s\mathbf{I}_3 & \mathbf{I}_3 \end{pmatrix}$$

2. Basic Integration of GNSS with Dead Reckoning

ECEF Step 2: System Noise Covariance

System noise represents the unknown changes in the states over time

- Here, it comprises the change in the DR velocity error over time
- This will depend on the DR technology in use – different error models suit inertial navigation, wheel speed odometry etc
- For this generic algorithm, velocity error is modelled as a random walk with power spectral density (PSD) S_{DR}
- The velocity error random walk is also integrated onto the position error through the deterministic system model

$$\mathbf{Q}_{k-1} = \begin{pmatrix} S_{DR}\tau_s\mathbf{I}_3 & \frac{1}{2}S_{DR}\tau_s^2\mathbf{I}_3 \\ \frac{1}{2}S_{DR}\tau_s^2\mathbf{I}_3 & \frac{1}{3}S_{DR}\tau_s^3\mathbf{I}_3 \end{pmatrix}$$

2. Basic Integration of GNSS with Dead Reckoning

ECEF Steps 3 & 4: Propagate State & Covariance

Propagate state vector estimate:

$$\hat{\mathbf{x}}_k^- = \mathbf{\Phi}_{k-1}\hat{\mathbf{x}}_{k-1}^+$$

Known changes over time

Propagate error covariance matrix:

$$\mathbf{P}_k^- = \mathbf{\Phi}_{k-1}\mathbf{P}_{k-1}^+\mathbf{\Phi}_{k-1}^{\mathrm{T}} + \mathbf{Q}_{k-1}$$

Unknown changes over time

Using

$$\mathbf{\Phi}_{k-1} = \begin{pmatrix} \mathbf{I}_3 & \mathbf{0}_3 \\ \tau_s \mathbf{I}_3 & \mathbf{I}_3 \end{pmatrix} \qquad \mathbf{Q}_{k-1} = \begin{pmatrix} S_{DR}\tau_s \mathbf{I}_3 & \frac{1}{2} S_{DR}\tau_s^2 \mathbf{I}_3 \\ \frac{1}{2} S_{DR}\tau_s^2 \mathbf{I}_3 & \frac{1}{3} S_{DR}\tau_s^3 \mathbf{I}_3 \end{pmatrix}$$

2. Basic Integration of GNSS with Dead Reckoning

ECEF Step 5: Calculate Measurement Matrix

Kalman filter measurements comprise:

- GNSS 3D position solution – Dead reckoning 3D position solution
- GNSS 3D velocity solution – Dead reckoning 3D velocity solution

Measurement matrix

$$\mathbf{H}_k = \begin{pmatrix} \mathbf{0}_3 & -\mathbf{I}_3 \\ -\mathbf{I}_3 & \mathbf{0}_3 \end{pmatrix}_k$$

Position measurements row

Velocity measurements row

DR Velocity error column

DR Position error column

2. Basic Integration of GNSS with Dead Reckoning

ECEF Step 6: Measurement Noise Covariance

Kalman filter measurements comprise:

- GNSS 3D position solution – Dead reckoning 3D position solution
- GNSS 3D velocity solution – Dead reckoning 3D velocity solution

The dead reckoning errors are estimated by the Kalman filter

∴ Measurement noise comprises only the GNSS errors

Measurement noise covariance matrix

$$\mathbf{R}_k \approx \begin{pmatrix} \boxed{\sigma_{Gr}^2}\mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \boxed{\sigma_{Gv}^2}\mathbf{I}_3 \end{pmatrix}_k$$

Variance per axis of GNSS position noise

Variance per axis of GNSS velocity noise

2. Basic Integration of GNSS with Dead Reckoning

Step 7: Calculate Kalman Gain Matrix

The **Kalman Gain matrix**, $\mathbf{K}_k$, is used to determine the weighting of the measurement vector in updating the state estimates

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^{\mathrm{T}} \left(\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k \right)^{-1}$$

Matrix inversion

Qualitatively...

$$\text{Transformation from measurement domain to state domain} \times \frac{\text{State variance in measurement domain}}{\text{State variance in measurement domain} + \text{Measurement variance}}$$

2. Basic Integration of GNSS with Dead Reckoning

ECEF Step 8: Measurement Innovation

Kalman filter measurements comprise:

GNSS position solution

DR position solution

$$\tilde{\mathbf{z}}_k = \begin{pmatrix} \boxed{\mathbf{r}^{e,G}_{eb,k}} - \boxed{\mathbf{r}^{e,D}_{eb,k}} \\ \boxed{\mathbf{v}^{e,G}_{eb,k}} - \boxed{\mathbf{v}^{e,D}_{eb,k}} \end{pmatrix}$$

GNSS velocity solution

DR velocity solution

Measurement innovation is thus:

$$\begin{aligned}
\delta\mathbf{z}^-_k = \tilde{\mathbf{z}}_k - \mathbf{H}_k\hat{\mathbf{x}}^-_k \quad &= \begin{pmatrix} \mathbf{r}^{e,G}_{eb,k} - \mathbf{r}^{e,D}_{eb,k} \\ \mathbf{v}^{e,G}_{eb,k} - \mathbf{v}^{e,D}_{eb,k} \end{pmatrix} - \begin{pmatrix} \mathbf{0}_3 & -\mathbf{I}_3 \\ -\mathbf{I}_3 & \mathbf{0}_3 \end{pmatrix} \begin{pmatrix} \delta\hat{\mathbf{v}}^{e-}_{eb,k} \\ \delta\hat{\mathbf{r}}^{e-}_{eb,k} \end{pmatrix} \\
&= \begin{pmatrix} \mathbf{r}^{e,G}_{eb,k} - \mathbf{r}^{e,D}_{eb,k} + \delta\hat{\mathbf{r}}^{e-}_{eb,k} \\ \mathbf{v}^{e,G}_{eb,k} - \mathbf{v}^{e,D}_{eb,k} + \delta\hat{\mathbf{v}}^{e-}_{eb,k} \end{pmatrix}
\end{aligned}$$

Steps 9 and 10: Measurement Update

Update state vector estimate

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k \delta\mathbf{z}_k^-$$

Update error covariance matrix

$$\mathbf{P}_k^+ = \left(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k\right) \mathbf{P}_k^-$$

NED Implementation: Kalman Filter States

6 states are estimated

- 3D DR Velocity error (NED)
  - north component
  - east component
  - down component
- 3D DR Position error:
  - *latitude error*
  - *longitude error*
  - *height error*

$$\mathbf{x} = \begin{pmatrix} \delta\mathbf{v}_{eb}^{n} \\ \delta\mathbf{p}_{b} \end{pmatrix} = \begin{pmatrix} \delta v_{eb,N}^{n} \\ \delta v_{eb,E}^{n} \\ \delta v_{eb,D}^{n} \\ \delta L_{b} \\ \delta\lambda_{b} \\ \delta h_{b} \end{pmatrix}$$

NED Step 1: Calculate Transition Matrix

The DR position error is the integral of the DR velocity error:

$$\mathbf{\Phi}_{k-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \dfrac{\tau_s}{R_N + h_b} & 0 & 0 & 1 & 0 & 0 \\ 0 & \dfrac{\tau_s}{(R_E + h_b)\cos L_b} & 0 & 0 & 1 & 0 \\ 0 & 0 & -\tau_s & 0 & 0 & 1 \end{pmatrix}_{k-1} \qquad \mathbf{x} = \begin{pmatrix} \delta v^n_{eb,N} \\ \delta v^n_{eb,E} \\ \delta v^n_{eb,D} \\ \delta L_b \\ \delta \lambda_b \\ \delta h_b \end{pmatrix}$$

τ_s = time interval

R_N = meridian radius of curvature

R_E = transverse radius of curvature

$$R_N = \frac{R_0\left(1-e^2\right)}{\left(1-e^2\sin^2 L_b\right)^{3/2}} \qquad R_E = \frac{R_0}{\sqrt{1-e^2\sin^2 L_b}}$$

2. Basic Integration of GNSS with Dead Reckoning

NED Step 2: System Noise Covariance

- Velocity error is modelled as a random walk with power spectral density (PSD) S_{DR}
- The velocity error random walk is also integrated onto the position error through the deterministic system model

$$\mathbf{Q}_{k-1} = \begin{pmatrix} S_{DR}\tau_s & 0 & 0 & \frac{1}{2}\frac{S_{DR}\tau_s^2}{R_N+h_b} & 0 & 0 \\ 0 & S_{DR}\tau_s & 0 & 0 & \frac{1}{2}\frac{S_{DR}\tau_s^2}{\left(R_E+h_b\right)\cos L_b} & 0 \\ 0 & 0 & S_{DR}\tau_s & 0 & 0 & -\frac{1}{2}S_{DR}\tau_s^2 \\ \frac{1}{2}\frac{S_{DR}\tau_s^2}{R_N+h_b} & 0 & 0 & \frac{1}{3}\frac{S_{DR}\tau_s^3}{\left(R_N+h_b\right)^2} & 0 & 0 \\ 0 & \frac{1}{2}\frac{S_{DR}\tau_s^2}{\left(R_E+h_b\right)\cos L_b} & 0 & 0 & \frac{1}{3}\frac{S_{DR}\tau_s^3}{\left(R_E+h_b\right)^2\cos^2 L_b} & 0 \\ 0 & 0 & -\frac{1}{2}S_{DR}\tau_s^2 & 0 & 0 & \frac{1}{3}S_{DR}\tau_s^3 \end{pmatrix}_{k-1}$$

NED Step 5: Calculate Measurement Matrix

Kalman filter measurements comprise:

- GNSS latitude solution – Dead reckoning latitude solution
- GNSS longitude solution – Dead reckoning longitude solution
- GNSS height solution – Dead reckoning height solution
- GNSS 3D velocity solution – Dead reckoning 3D velocity solution

Measurement matrix

$$\begin{array}{cc} & \\ \mathbf{H}_k = \begin{pmatrix} \mathbf{0}_3 & -\mathbf{I}_3 \\ -\mathbf{I}_3 & \mathbf{0}_3 \end{pmatrix}_k & \begin{array}{l} \leftarrow \text{Position measurements row} \\ \leftarrow \text{Velocity measurements row} \end{array} \end{array}$$

DR Velocity error column

DR Position error column

2. Basic Integration of GNSS with Dead Reckoning

NED Step 6: Measurement Noise Covariance

Variance per axis of GNSS position noise (m²)

$$\mathbf{R}_k = \begin{pmatrix} \frac{\boxed{\sigma_{Gr}^2}}{\left(R_N + h_b\right)^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sigma_{Gr}^2}{\left(R_E + h_b\right)^2 \cos^2 L_b} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{Gr}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{Gv}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{Gv}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boxed{\sigma_{Gv}^2} \end{pmatrix}$$

Variance per axis of GNSS velocity noise (m²/s²)

2. Basic Integration of GNSS with Dead Reckoning

NED Step 8: Measurement Innovation

Kalman filter measurements comprise:

GNSS position solution

GNSS velocity solution

$$\tilde{\mathbf{z}}_k = \begin{pmatrix} L_{b,k}^G - L_{b,k}^D \\ \lambda_{b,k}^G - \lambda_{b,k}^D \\ h_{b,k}^G - h_{b,k}^D \\ \mathbf{v}_{eb,k}^{n,G} - \mathbf{v}_{eb,k}^{n,D} \end{pmatrix}$$

DR position solution

DR velocity solution

Measurement innovation is thus:

$$\delta\mathbf{z}_k^- = \tilde{\mathbf{z}}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^- = \begin{pmatrix} L_{b,k}^G - L_{b,k}^D + \delta\hat{L}_{b,k}^- \\ \lambda_{b,k}^G - \lambda_{b,k}^D + \delta\hat{\lambda}_{b,k}^- \\ h_{b,k}^G - h_{b,k}^D + \delta\hat{h}_{b,k}^- \\ \mathbf{v}_{eb,k}^{n,G} - \mathbf{v}_{eb,k}^{n,D} + \delta\hat{\mathbf{v}}_{eb,k}^{n-} \end{pmatrix}$$

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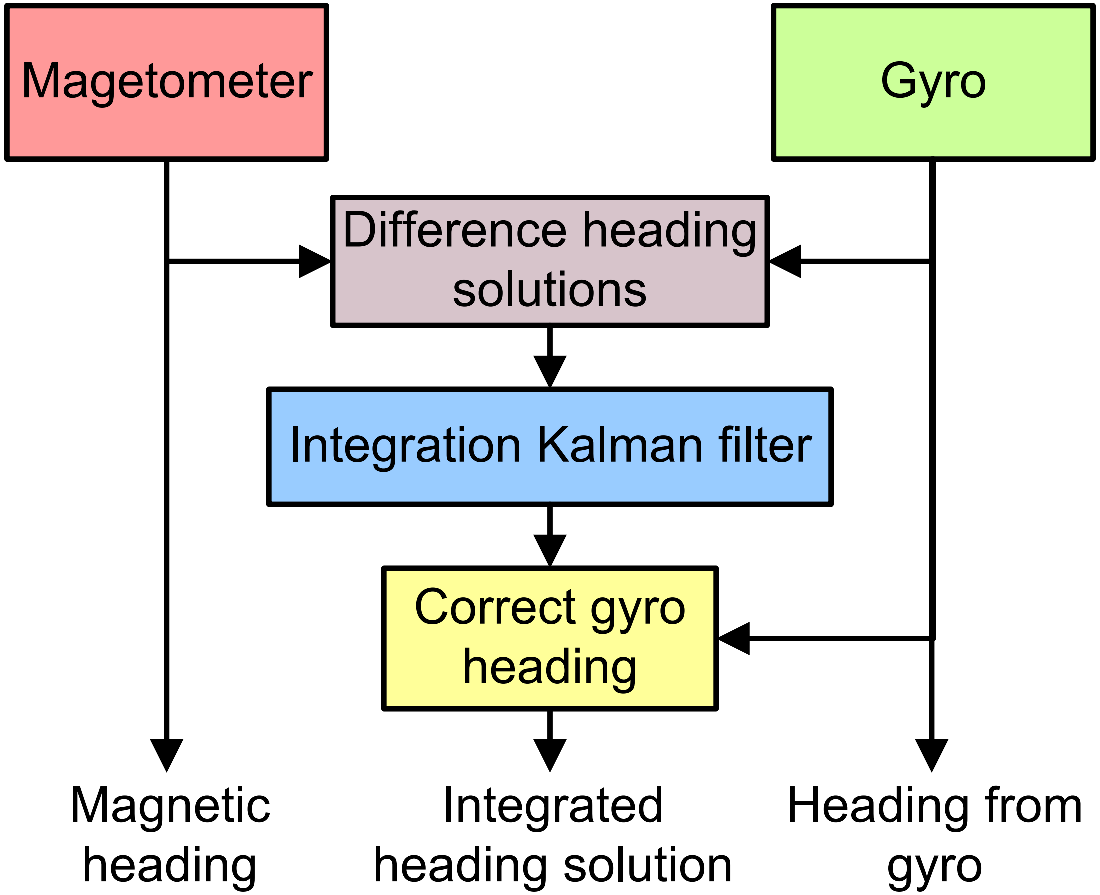
3. Gyro-Magnetometer Integration

Basic Open-loop Integration Architecture

- Separate magnetic and gyro-derived heading solutions are computed
- Kalman filter estimates gyro-derived heading error and also gyro bias
- Corrected gyro-derived heading is the integrated solution



3. Gyro-Magnetometer Integration

Gyro-Magnetometer Kalman Filter States

2 states are estimated

- Gyro-derived heading error
- Gyro bias

$$\mathbf{x} = \begin{pmatrix} \delta\psi^{g} \\ b_{g} \end{pmatrix}$$

3. Gyro-Magnetometer Integration

Gyro-Magnetometer System Model

The heading error is the integral of the gyro bias

$$\begin{pmatrix} \delta\psi_k^g \\ b_{g,k} \end{pmatrix} = \begin{pmatrix} 1 & \tau_s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \delta\psi_{k-1}^g \\ b_{g,k-1} \end{pmatrix} \qquad \Rightarrow \mathbf{\Phi}_{k-1} = \begin{pmatrix} 1 & \tau_s \\ 0 & 1 \end{pmatrix}$$

System noise represents the unknown changes in the states over time

- Gyro random noise with power spectral density (PSD) S_{rg}
- Gyro bias variation with PSD S_{bgd}

$$\mathbf{Q}_{k-1} = \begin{pmatrix} S_{rg}\tau_s + \frac{1}{3} S_{bgd}\tau_s^3 & \frac{1}{2} S_{bgd}\tau_s^2 \\ \frac{1}{2} S_{bgd}\tau_s^2 & S_{bgd}\tau_s \end{pmatrix}$$

τ_s = time interval

Gyro-Magnetometer Measurement Model

Kalman filter measurements comprise:

Magnetic heading solution

Gyro-derived heading solution

$$\tilde{z}_k = \boxed{\psi_k^M} - \boxed{\psi_k^g}$$

Measurement matrix:

$$\mathbf{H}_k = \begin{pmatrix} -1 & 0 \end{pmatrix}_k$$

Gyro heading error column

Gyro bias column

Measurement innovation is :

$$\delta z_k^- = \tilde{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^- = \left(\psi_k^M - \psi_k^g \right) - \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} \delta\hat{\psi}_k^g \\ \hat{b}_{g,k} \end{pmatrix} = \left(\psi_k^M - \psi_k^g + \delta\hat{\psi}_k^g \right)$$

Meas. noise covariance, $R_k = \sigma_M^2$, magnetic heading noise variance

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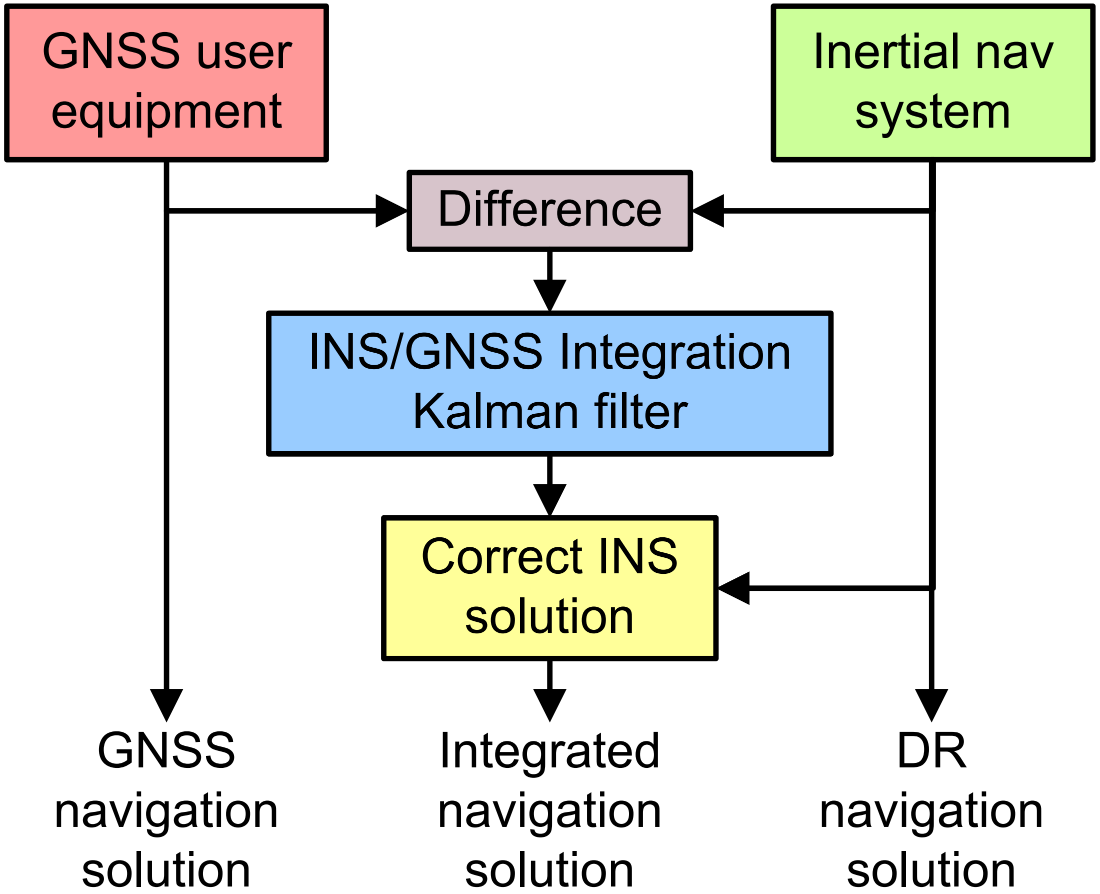
4. INS/GNSS Integration with Error Estimation

Loosely-coupled Open-loop INS/GNSS Integration

- Separate GNSS and INS solutions are computed
- Kalman filter estimates INS position, velocity and attitude errors + the accelerometer and gyro biases
- Corrected INS navigation solution is the integrated solution
- ECEF frame



INS/GNSS Kalman Filter States

An Earth-centred Earth-fixed (ECEF) frame is used in this example

- 3D Attitude error (small angle ECEF)
- 3D Velocity error (ECEF)
- 3D Position error (ECEF)
- 3 × Accelerometer bias (sensor body frame)
- 3 × Gyro bias (sensor body frame)

$$\mathbf{x} = \begin{pmatrix} \delta\boldsymbol{\psi}_{eb}^{e} \\ \delta\mathbf{v}_{eb}^{e} \\ \delta\mathbf{r}_{eb}^{e} \\ \mathbf{b}_{a} \\ \mathbf{b}_{g} \end{pmatrix}$$

4. INS/GNSS Integration with Error Estimation

INS/GNSS Kalman Filter State Propagation

Known variation in the states over time

Position error varies:

- as the integral of the velocity error

Velocity error varies:

- as the integral of the accelerometer bias
- due to gravity modelling errors resulting from the position error
- due to specific force frame transformation errors resulting from the attitude error
- due to Earth rotation

Attitude error varies:

- as the integral of the gyro bias
- due to Earth rotation

4. INS/GNSS Integration with Error Estimation

INS/GNSS KF Transition Matrix

$$\boldsymbol{\Phi}_{k-1} \approx \begin{bmatrix} \mathbf{I}_3 - \boldsymbol{\Omega}_{ie}^e \tau_s & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \tilde{\mathbf{C}}_b^e \tau_s \\ \mathbf{F}_{21} \tau_s & \mathbf{I}_3 - 2\boldsymbol{\Omega}_{ie}^e \tau_s & \mathbf{F}_{23} \tau_s & \tilde{\mathbf{C}}_b^e \tau_s & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{I}_3 \tau_s & \mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 \end{bmatrix}$$

$$\mathbf{F}_{21} = \left[-\left(\tilde{\mathbf{C}}_b^e \tilde{\mathbf{f}}_{ib}^b \right) \wedge \right], \qquad \mathbf{F}_{23} = -\frac{2 \hat{\boldsymbol{\gamma}}_{ib}^e}{r_{eS}^e (\tilde{L}_b)} \frac{\tilde{\mathbf{r}}_{eb}^{e\,\mathrm{T}}}{\left| \tilde{\mathbf{r}}_{eb}^e \right|}$$

τ_s = time interval

Further details in Workshop 3 instructions and book (linked on Moodle)

4. INS/GNSS Integration with Error Estimation

INS/GNSS KF System Noise Covariance

Velocity error uncertainty:

- Increases due to accelerometer random noise, PSD = S_{ra}

Attitude error uncertainty:

- Increases due to gyro random noise, PSD = S_{rg}

Accelerometer bias uncertainty:

- Increases due to variation in the sensor biases over time, PSD = S_{bad}

Gyro bias uncertainty:

- Increases due to variation in the sensor biases over time, PSD = S_{bgd}

$$\mathbf{Q}_{k-1} \approx \begin{pmatrix} S_{rg}\tau_s\mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & S_{ra}\tau_s\mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & S_{bad}\tau_s\mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & S_{bgd}\tau_s\mathbf{I}_3 \end{pmatrix}$$

4. INS/GNSS Integration with Error Estimation

INS/GNSS Measurement Update

Kalman filter measurements comprise:

- GNSS 3D position solution – inertial 3D position solution
- GNSS 3D velocity solution – inertial 3D velocity solution

Measurement matrix

$$\mathbf{H}_k \approx \begin{pmatrix} \mathbf{0}_3 & \mathbf{0}_3 & -\mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & -\mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \end{pmatrix}_k \begin{array}{l} \leftarrow \text{Position measurements row} \\ \leftarrow \text{Velocity measurements row} \end{array}$$

Velocity error column

Position error column

Other errors are estimated through their correlation with the position and velocity errors, represented by the **P** matrix

The measurement noise covariance and measurement innovation are for the same as the basic ECEF DR/GNSS filter from earlier