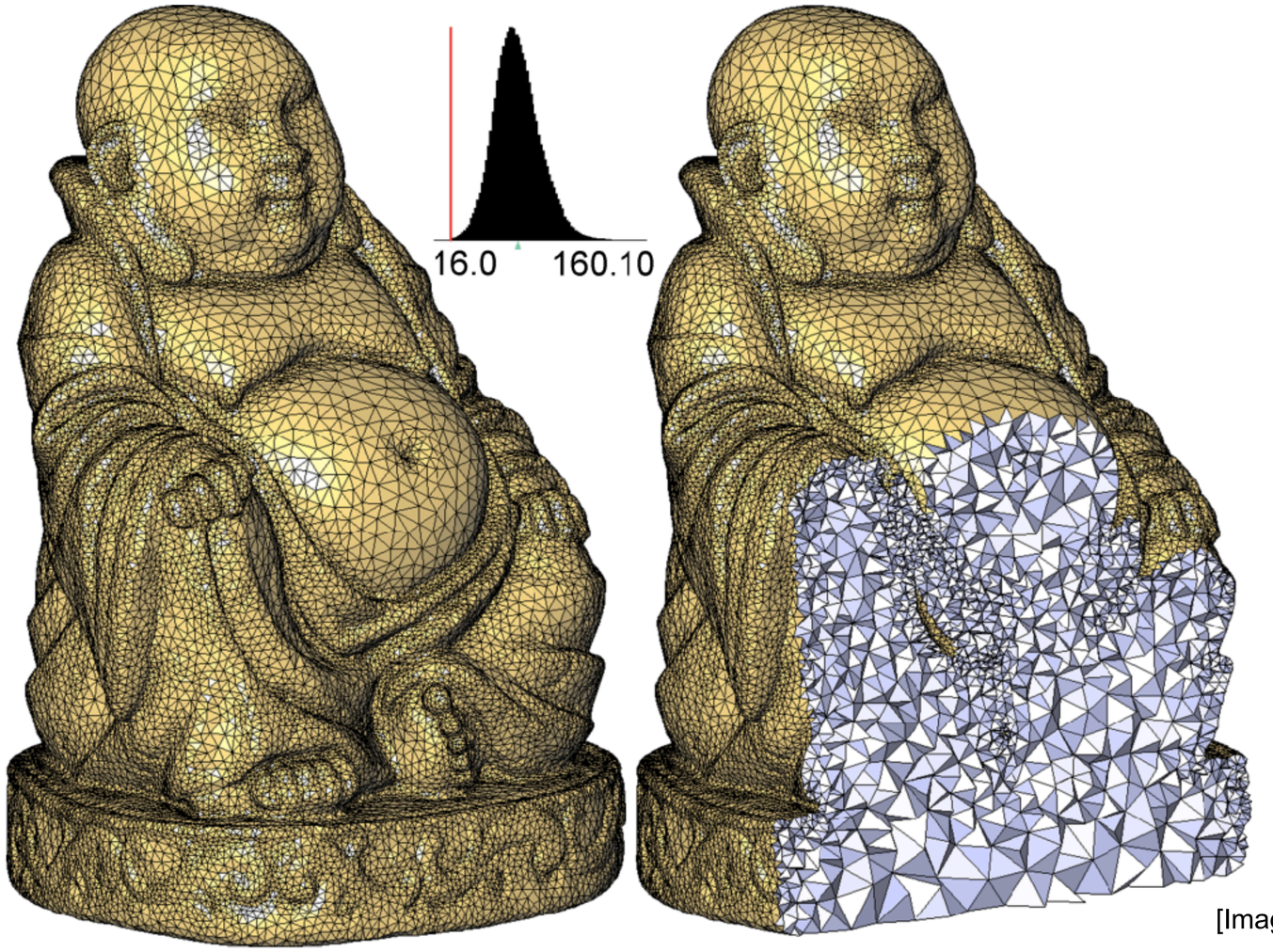
Winter 2022-23

Acquisition and Processing of 3D Geometry

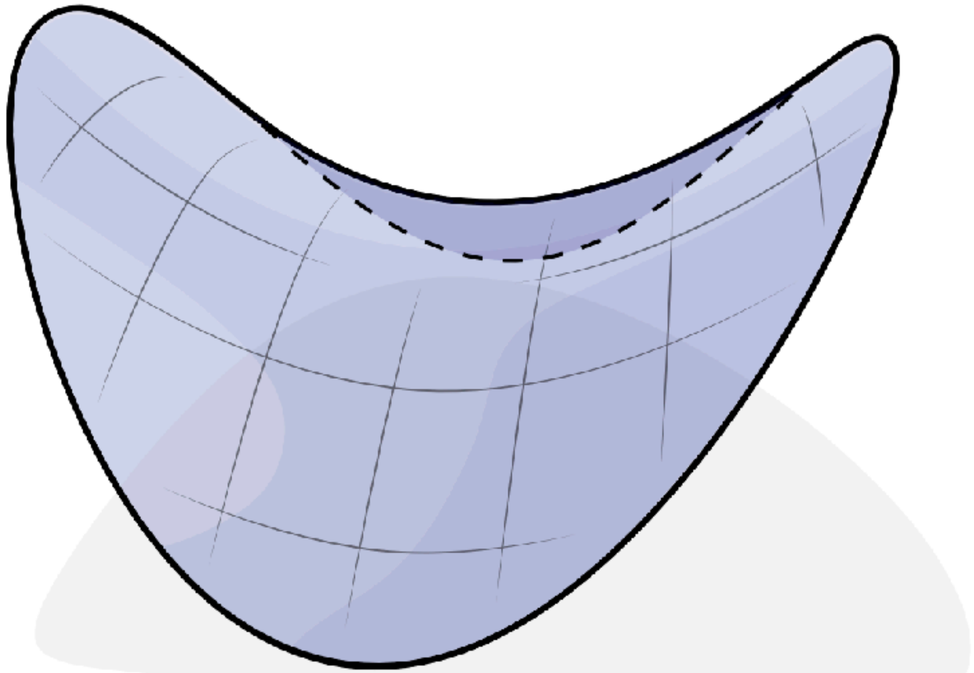
(COMP0119)

Niloy J. Mitra

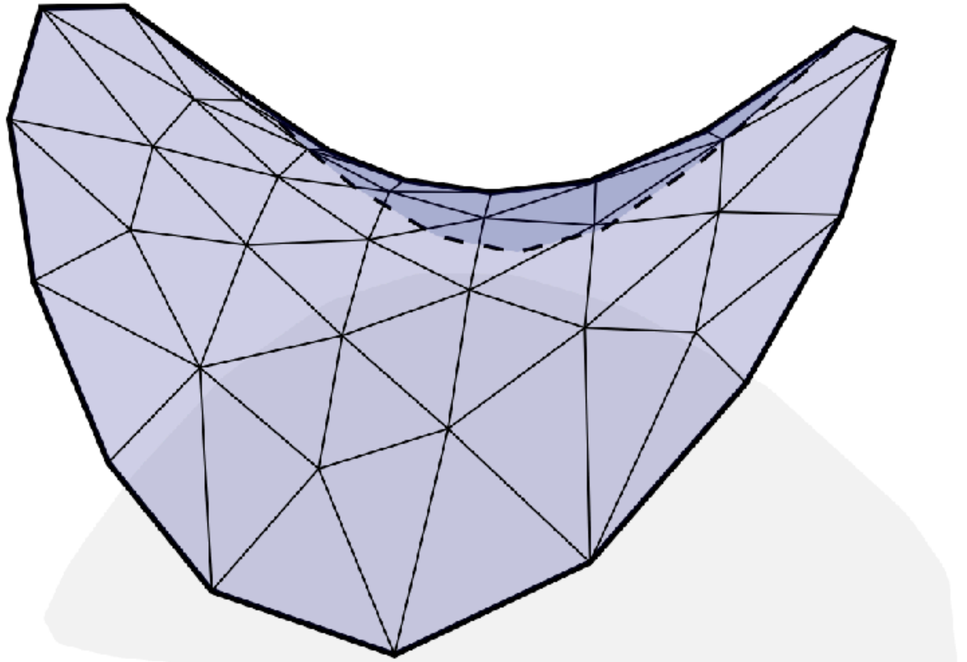


[Image source: Geometrica]

Continuous vs Discrete Representation



Continuous



Discrete

[Image from Keenan Crane]

Interactive Shape Modeling

Tools for design, editing and animation of digital shapes

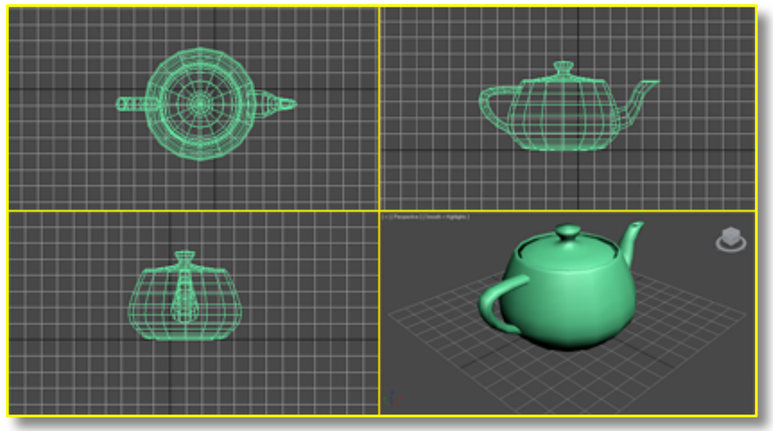
- **Interactive:** fast algorithms
- **Intuitive:** convenient interface + predictable outcome

Digital Shape Modeling

How do shapes find their way into computers?

- Geometric modeling is difficult

Humans have no direct “video out”

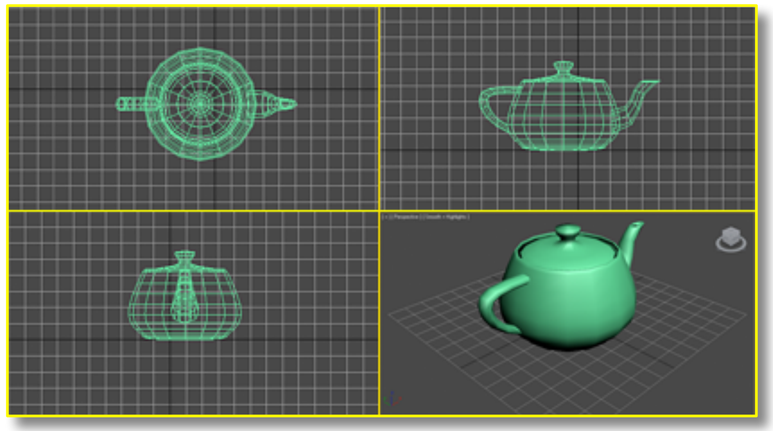


“Translation” from 2D to 3D is hard

Digital Shape Modeling

How do shapes find their way into computers?

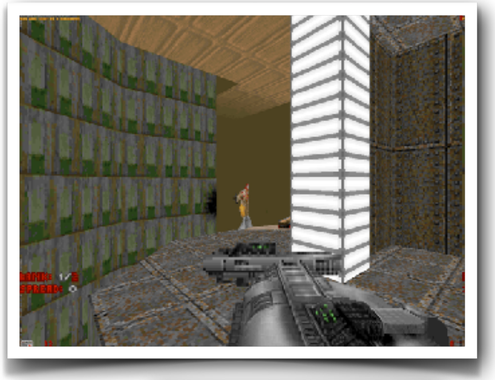
- Geometric modeling is difficult

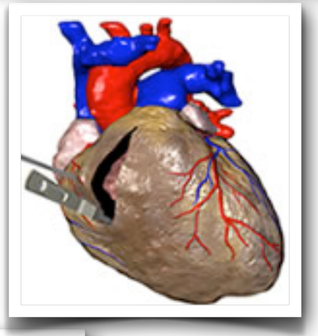
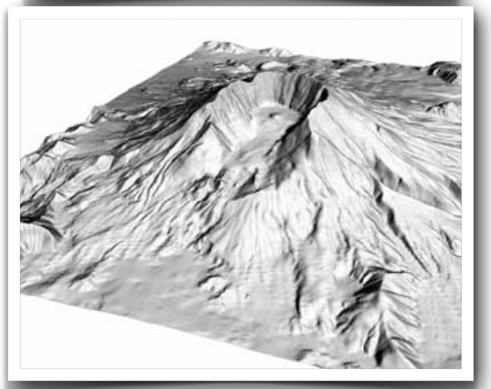
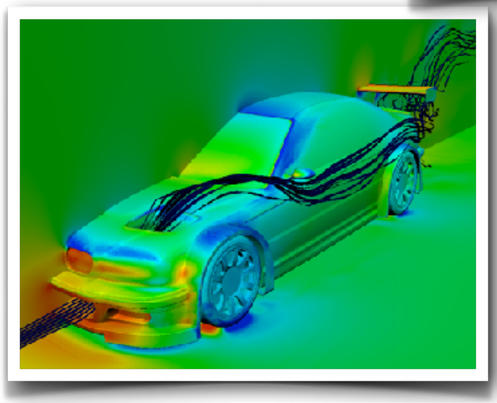


computation can compensate for lack of direct ability to convey visual information

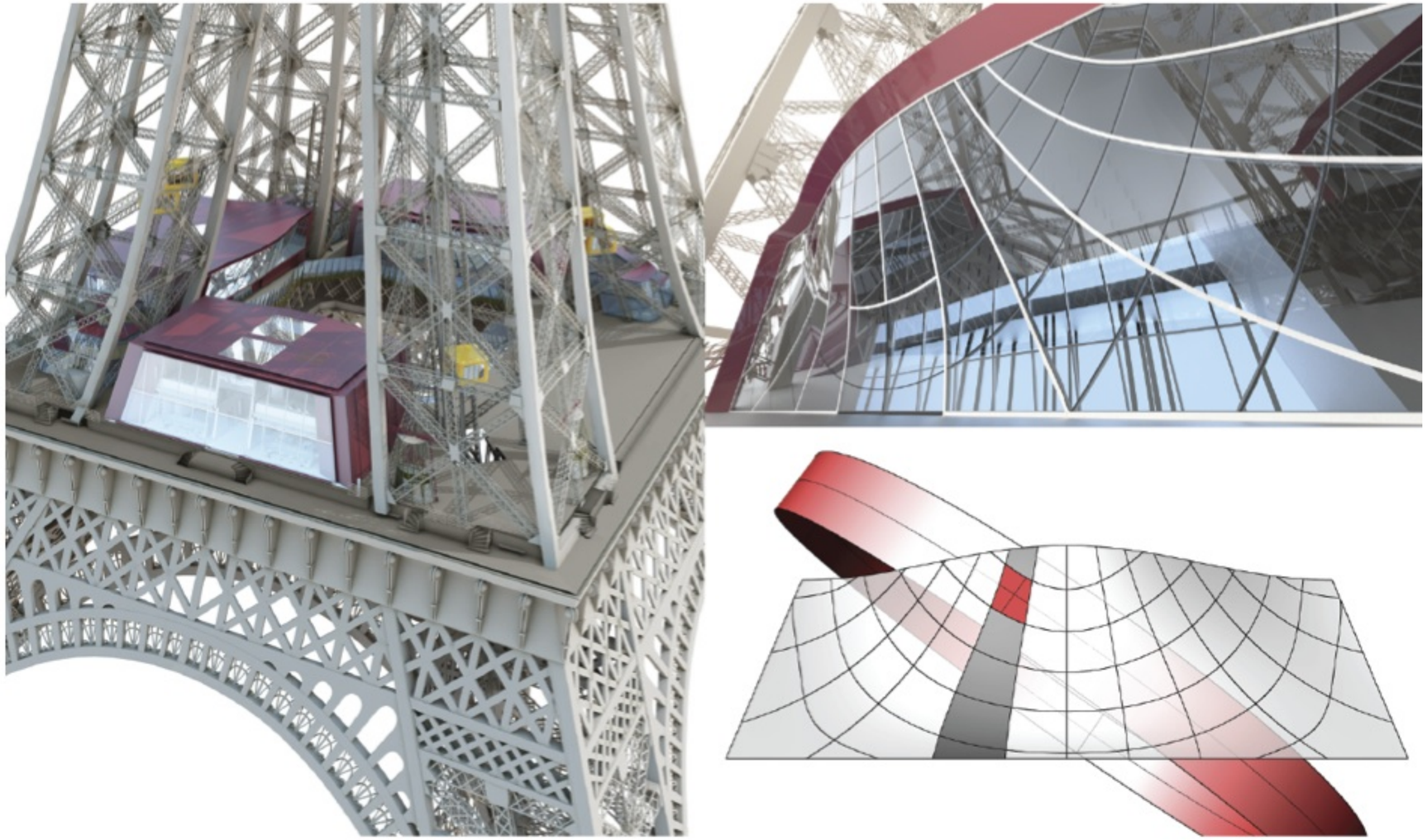
Application Areas

- Computer games
- Movie production
- Engineering
- Cultural Heritage
- Topography
- Architecture
- Medicine
- many more …




Form Finding (Collaboration w/ Evolute)



Digitizing Physical Models

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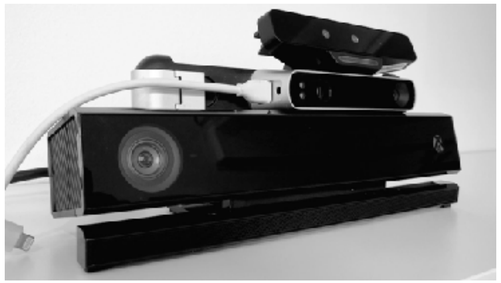
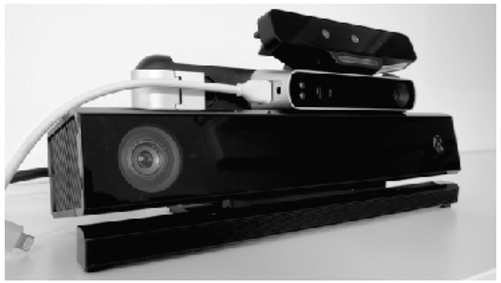


laser scanner

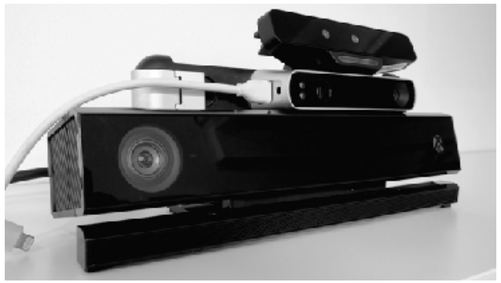
Digitizing Physical Models



laser scanner

3D geometry

Digitizing Physical Models



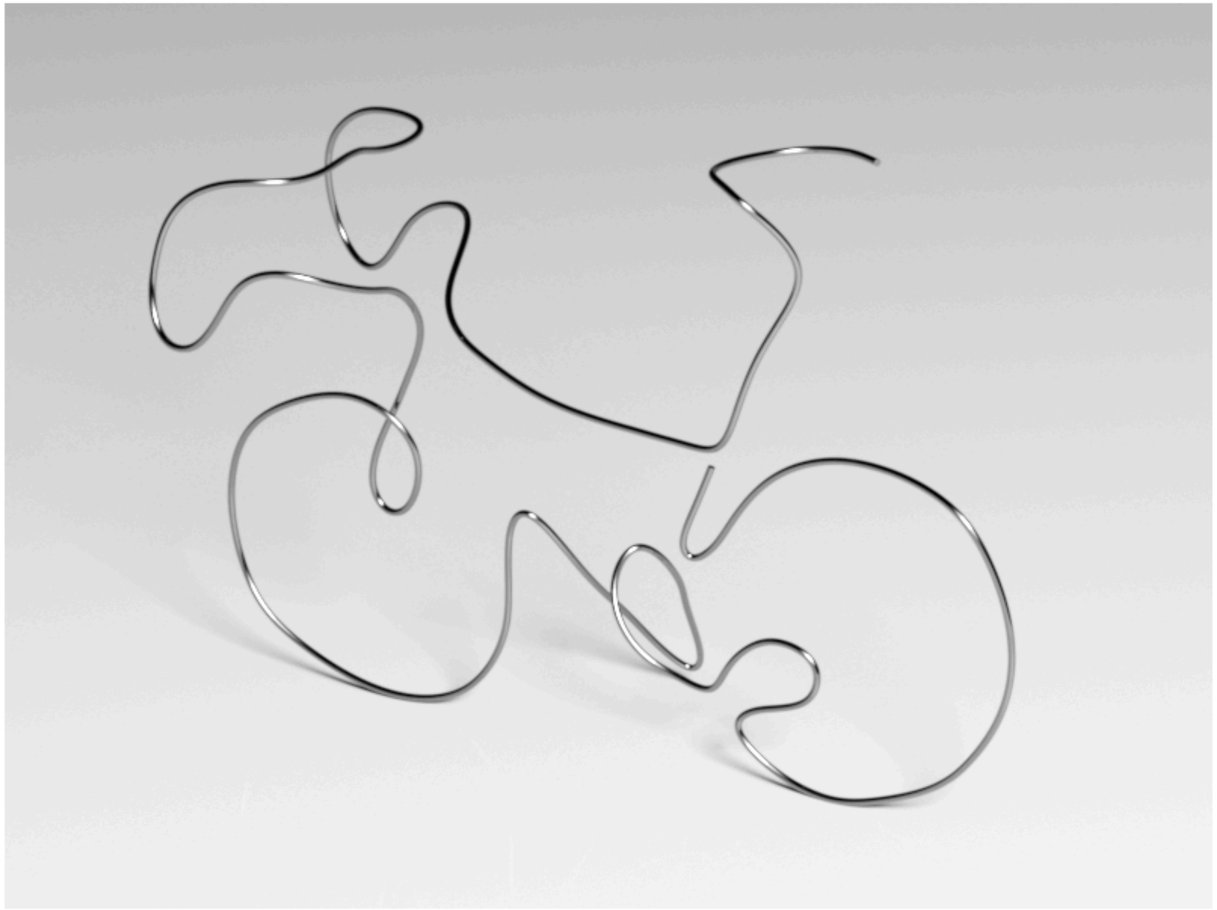
laser scanner

3D geometry

entertainment

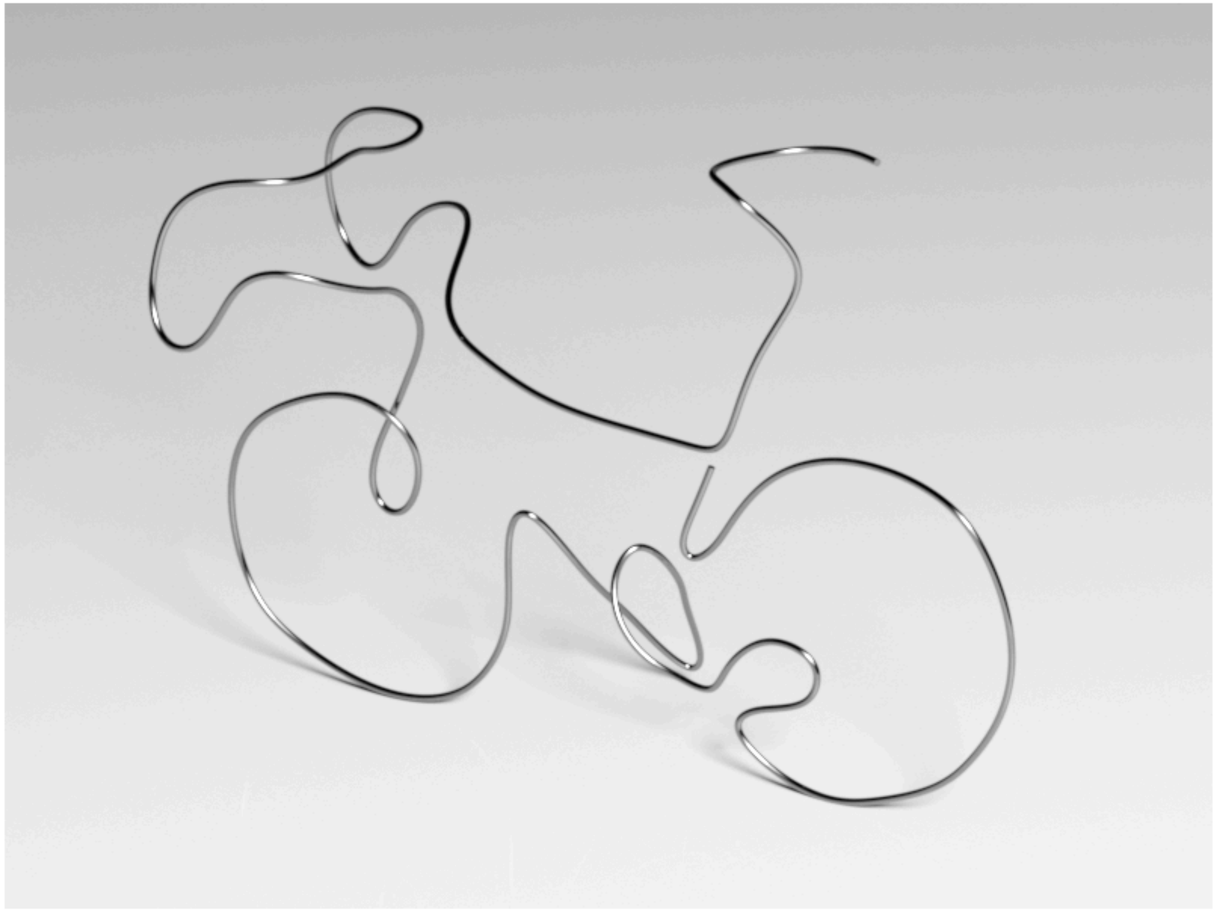
Scanning

BIKE



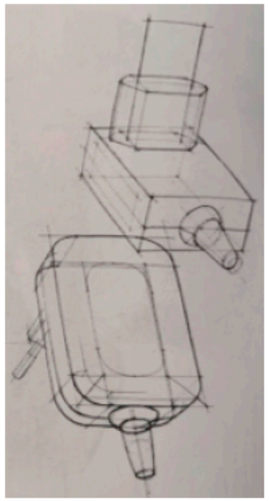
Scanning

BIKE



2D to 3D ‘Lifting’

(a) inspirational sketch



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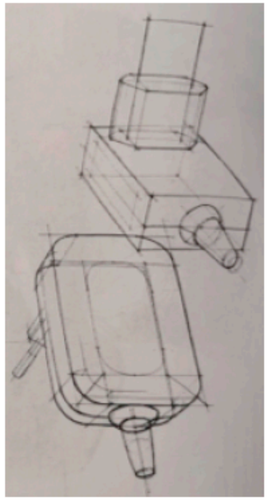
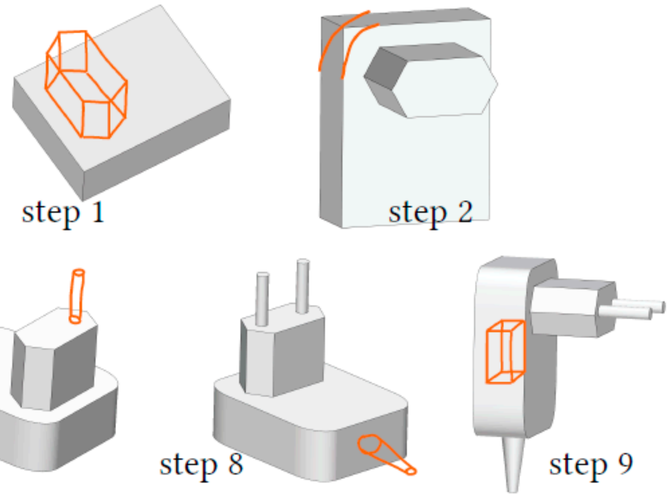


(b) sketching sequence using Sketch2CAD



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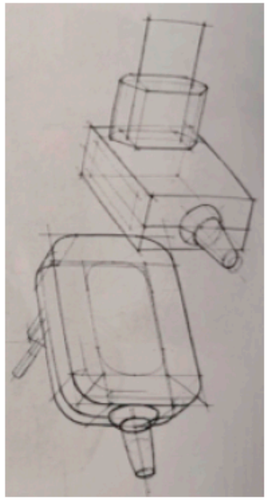

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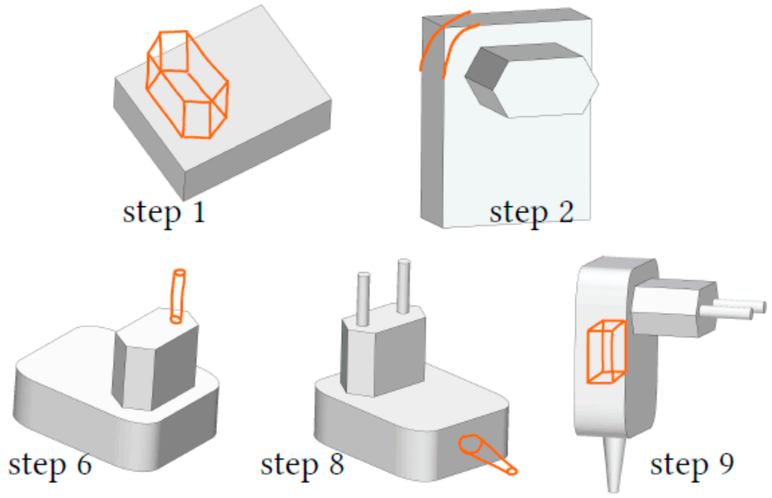

(c) inferred CAD instructions

`AddPolyhedron`: < plane 3, length 0.25 >
`BevelCorner`: < plane 3, corner 3 >
`BevelCorner`: < plane 3, corner 2 >
`BevelCorner`: < plane 3, corner 0 >
`BevelCorner`: < plane 3, corner 1 >
`AddSweepShape`: < plane 4, length 0.20 >
`AddSweepShape`: < plane 4, length 0.20 >
`AddSweepShape`: < plane 1, length 0.23 >
`SubtractPolyhedron`: < plane 12, length 0.03 >
`SubtractPolyhedron`: < plane 13, length 0.03 >

2D to 3D ‘Lifting’

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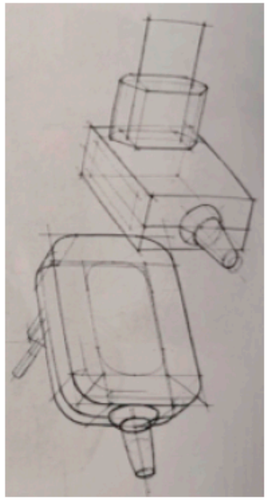

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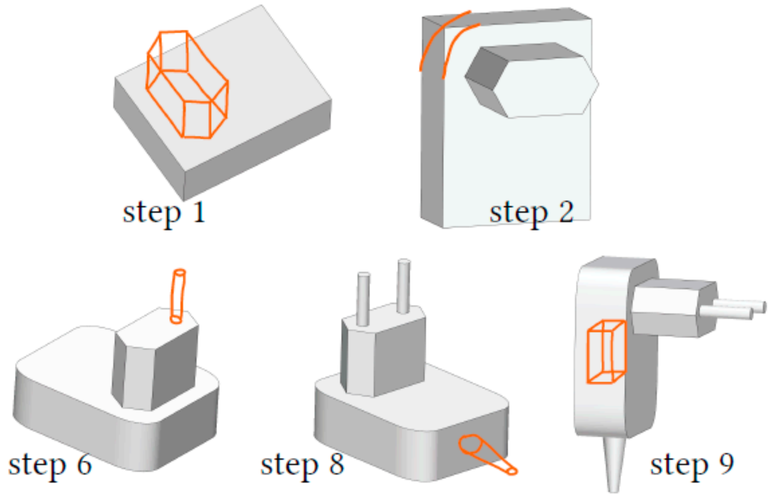

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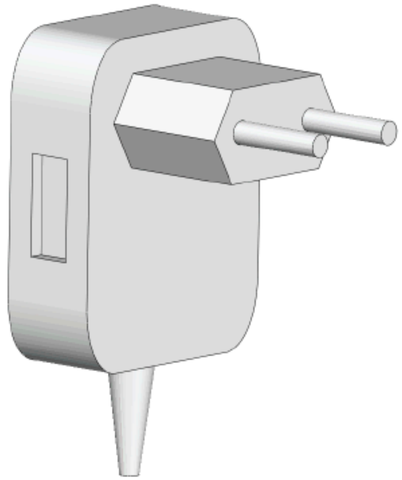
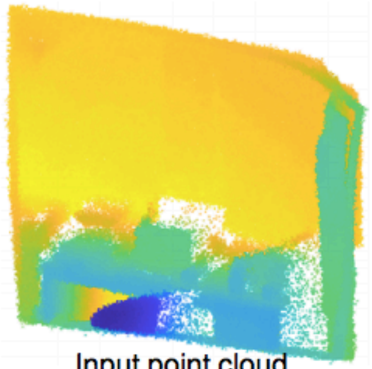
(d) CAD model

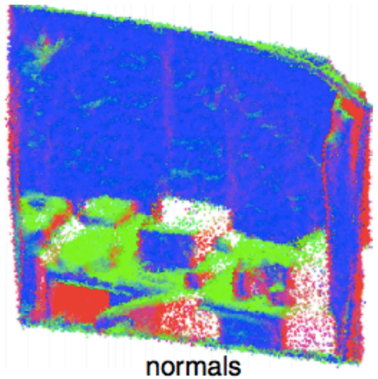


Estimating Local Properties

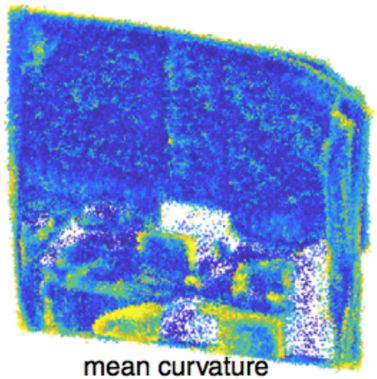
RGBD Image



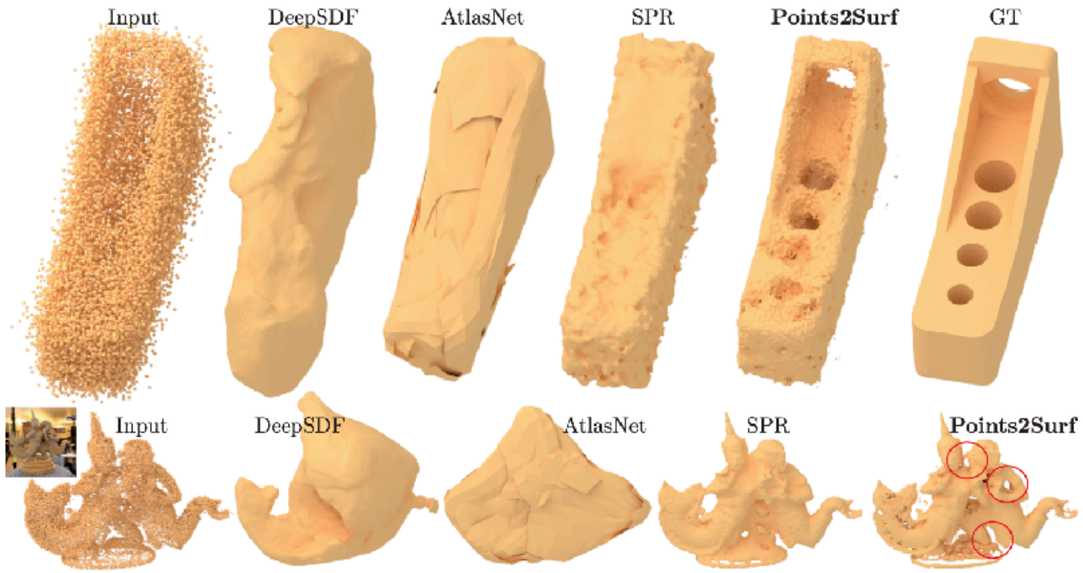
Input point cloud



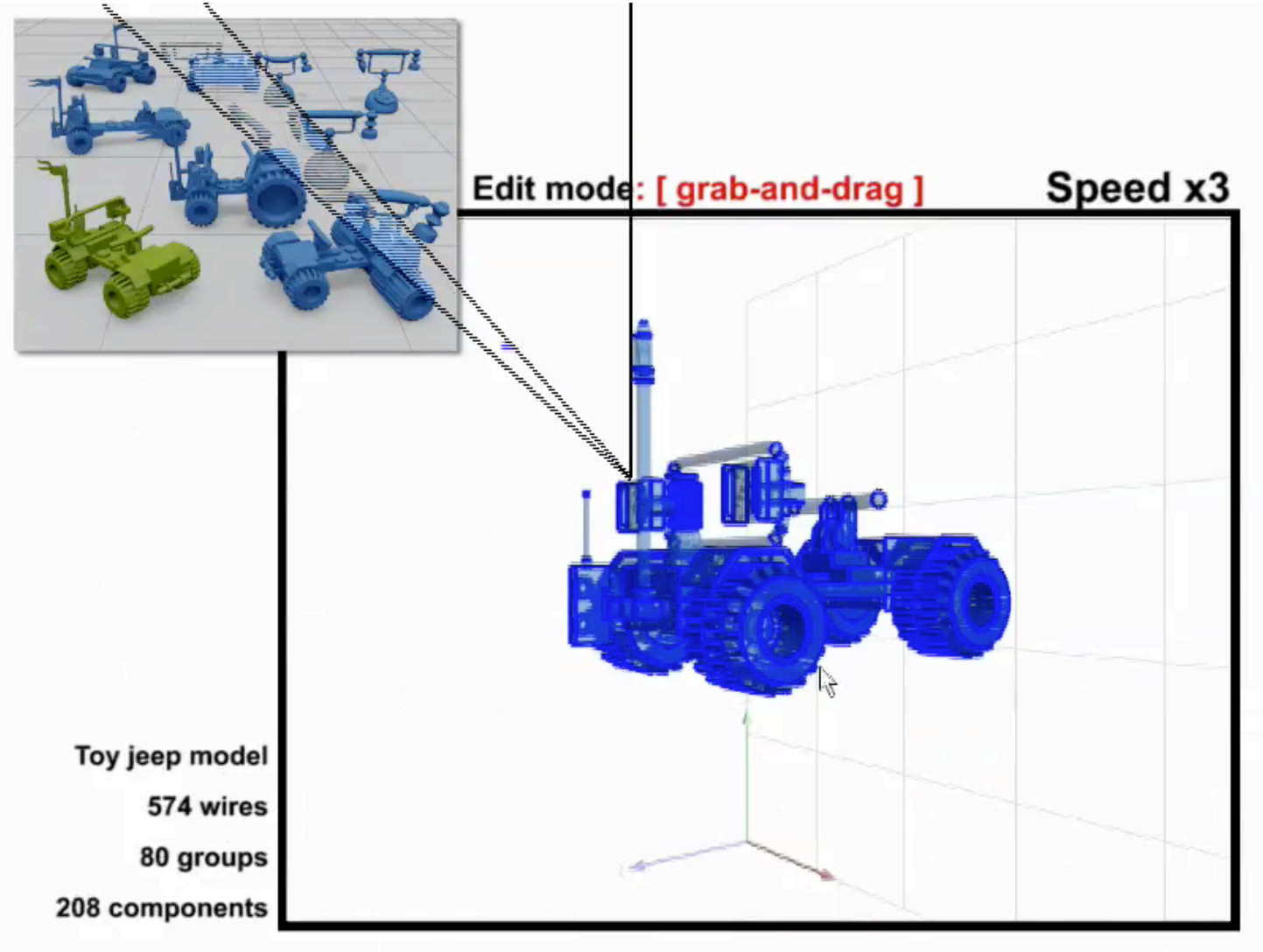
normals



mean curvature



Mesh Manipulation



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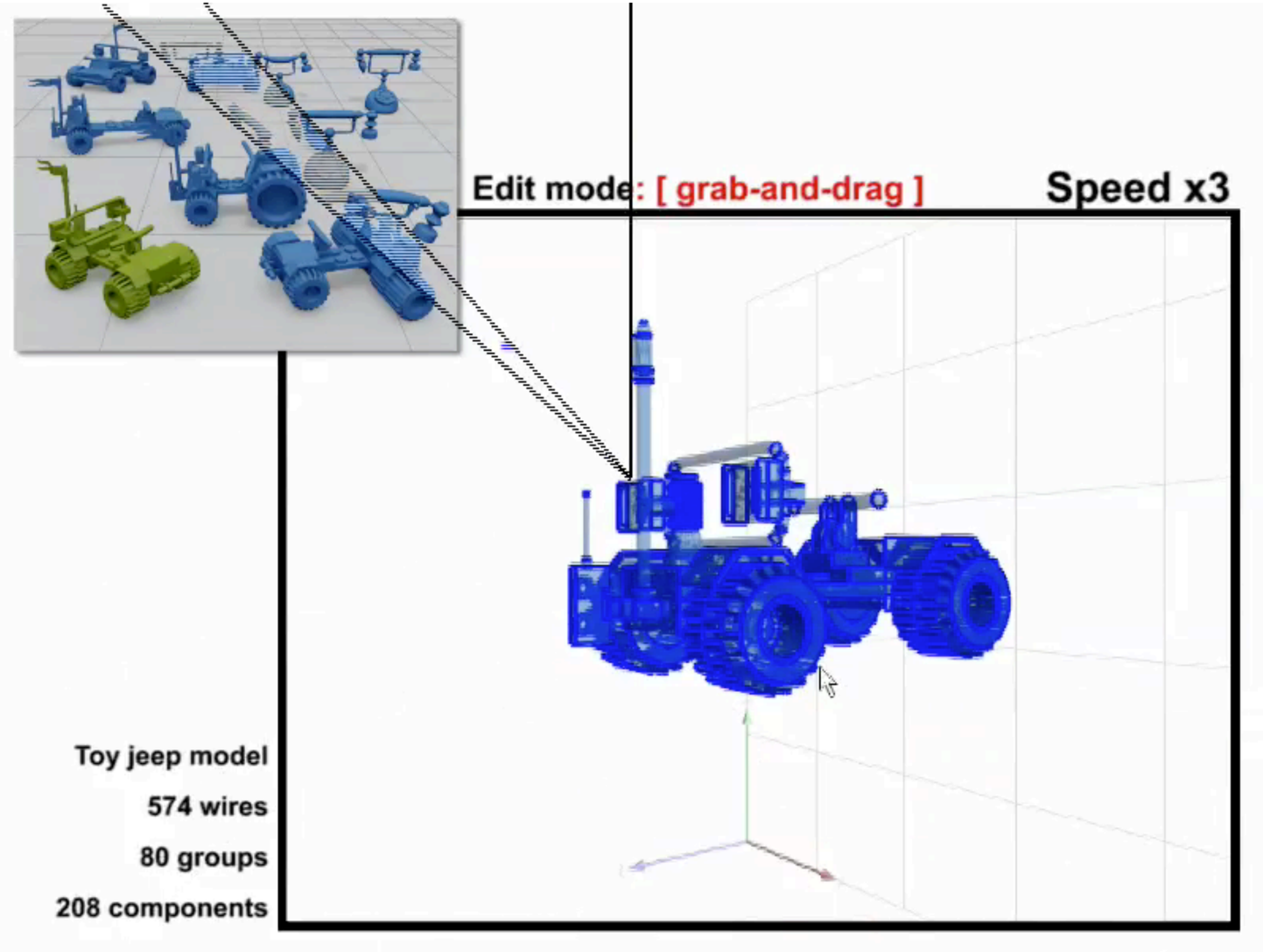
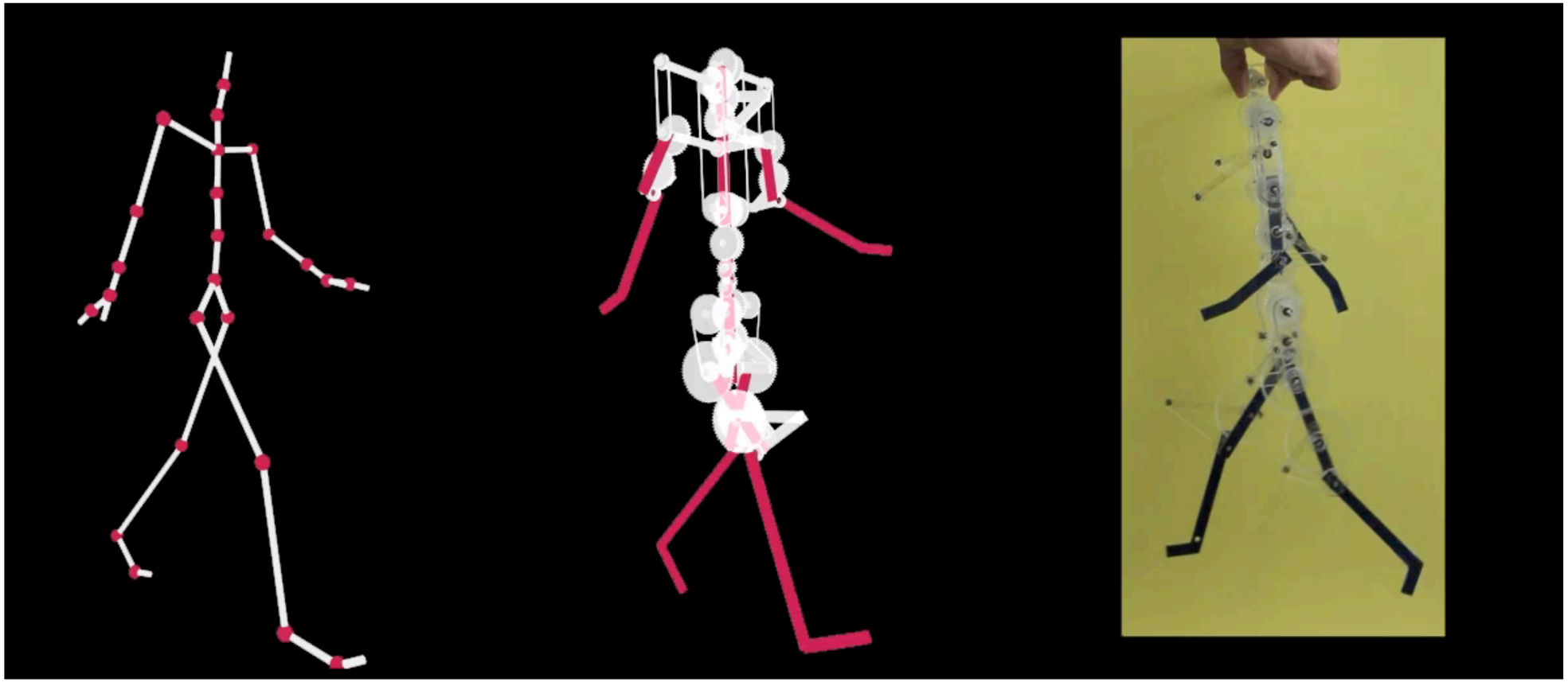
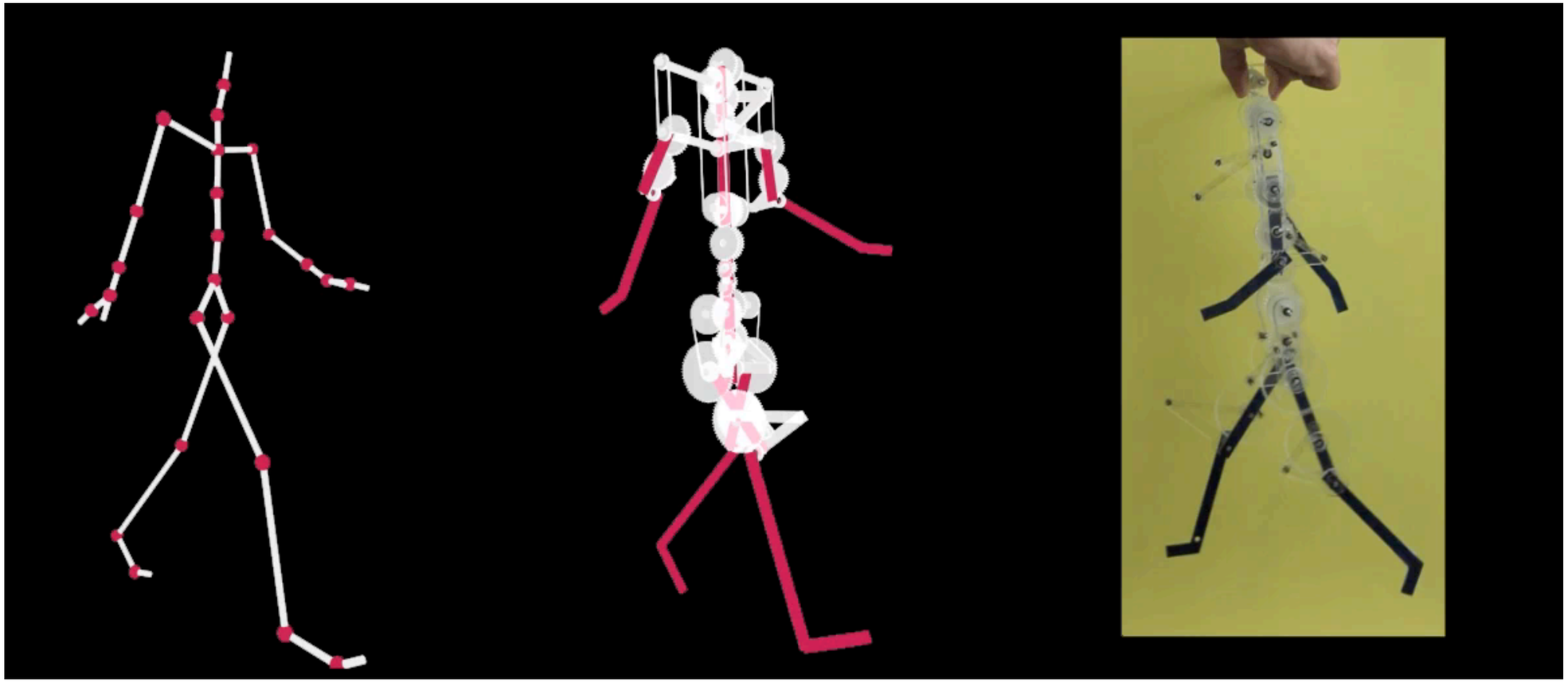


Motion to Geometry



Input Motion | Generated Automaton | Physical Prototype

Motion to Geometry



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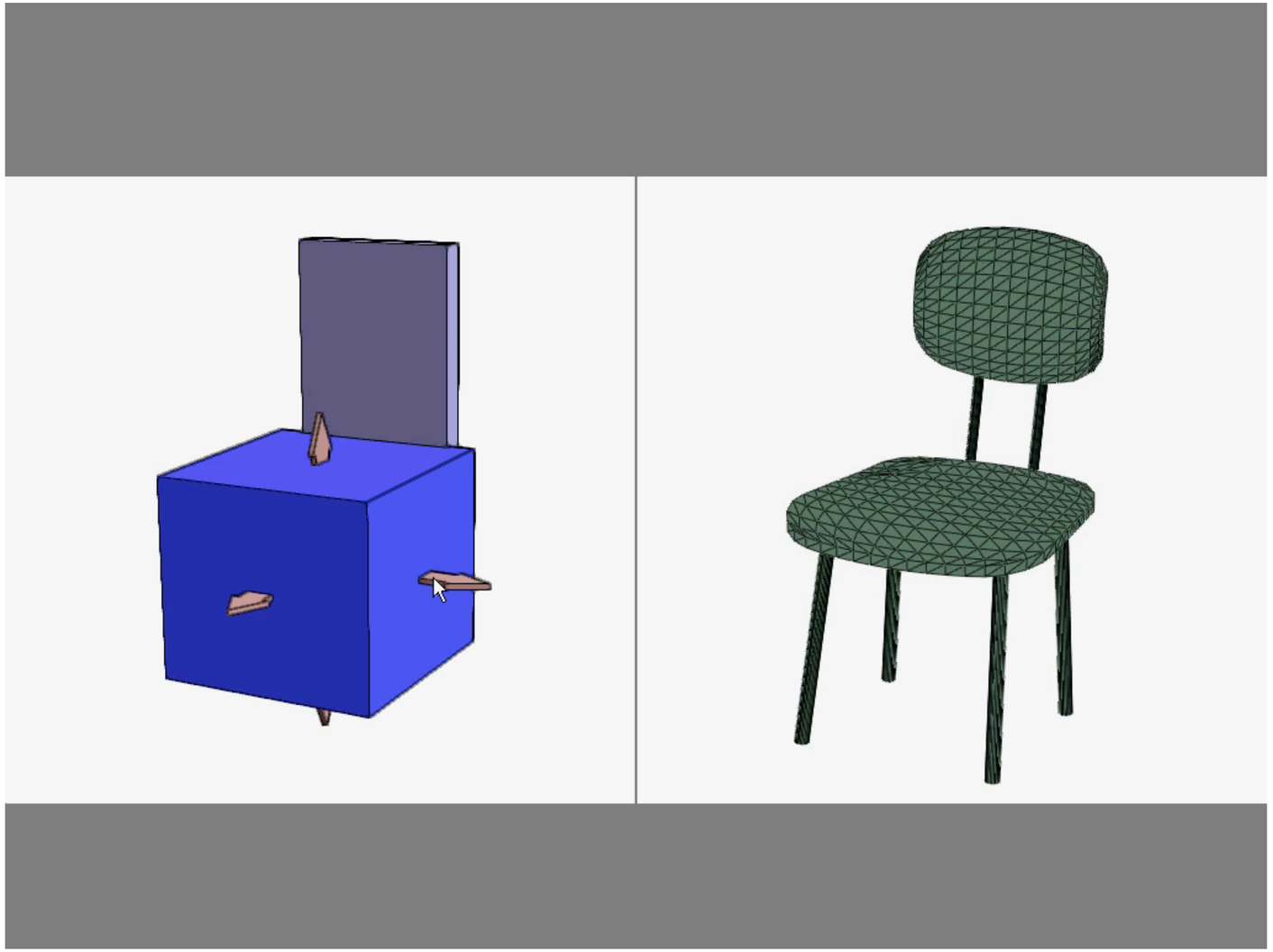
Exploring Data Collections



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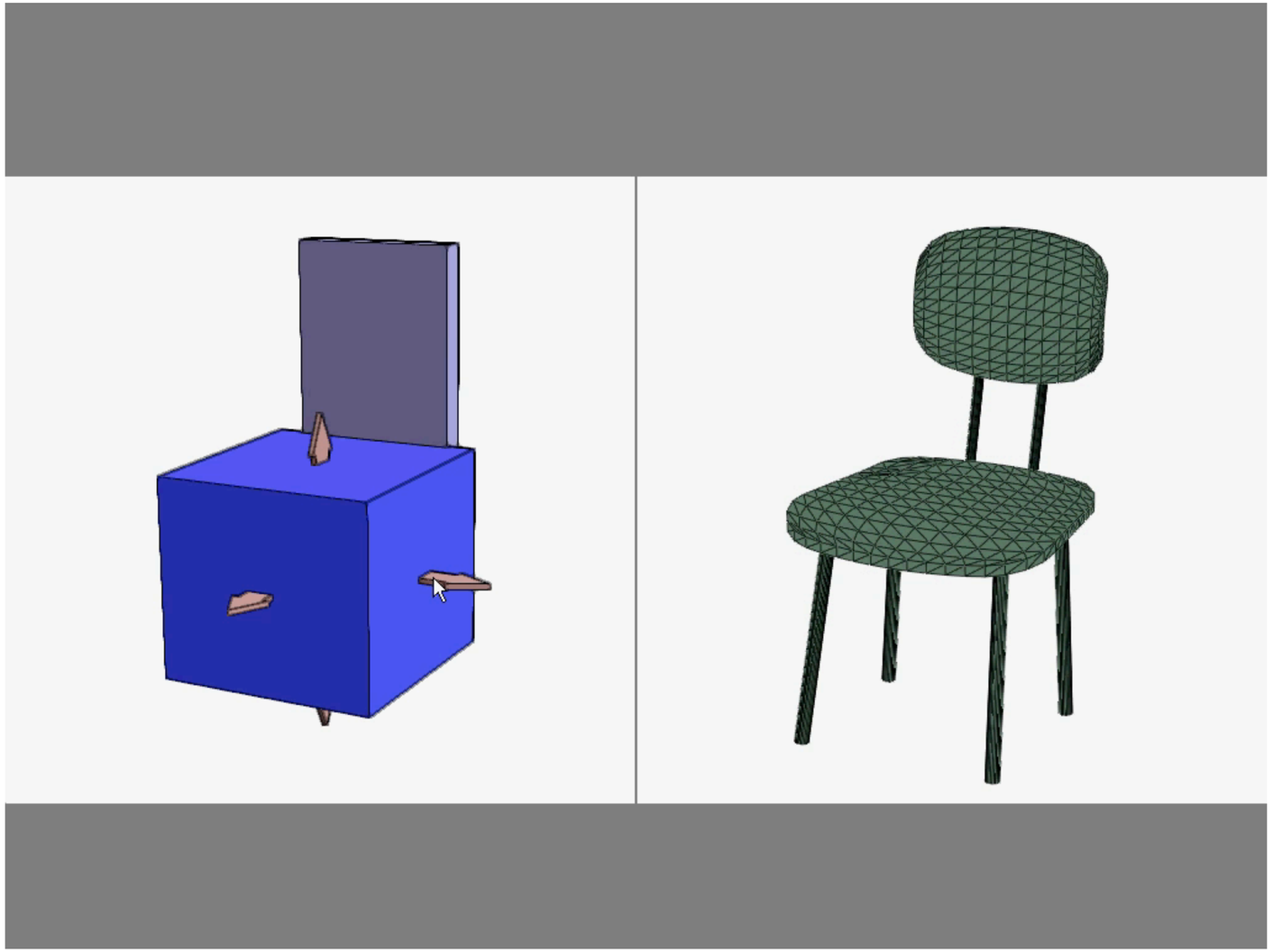
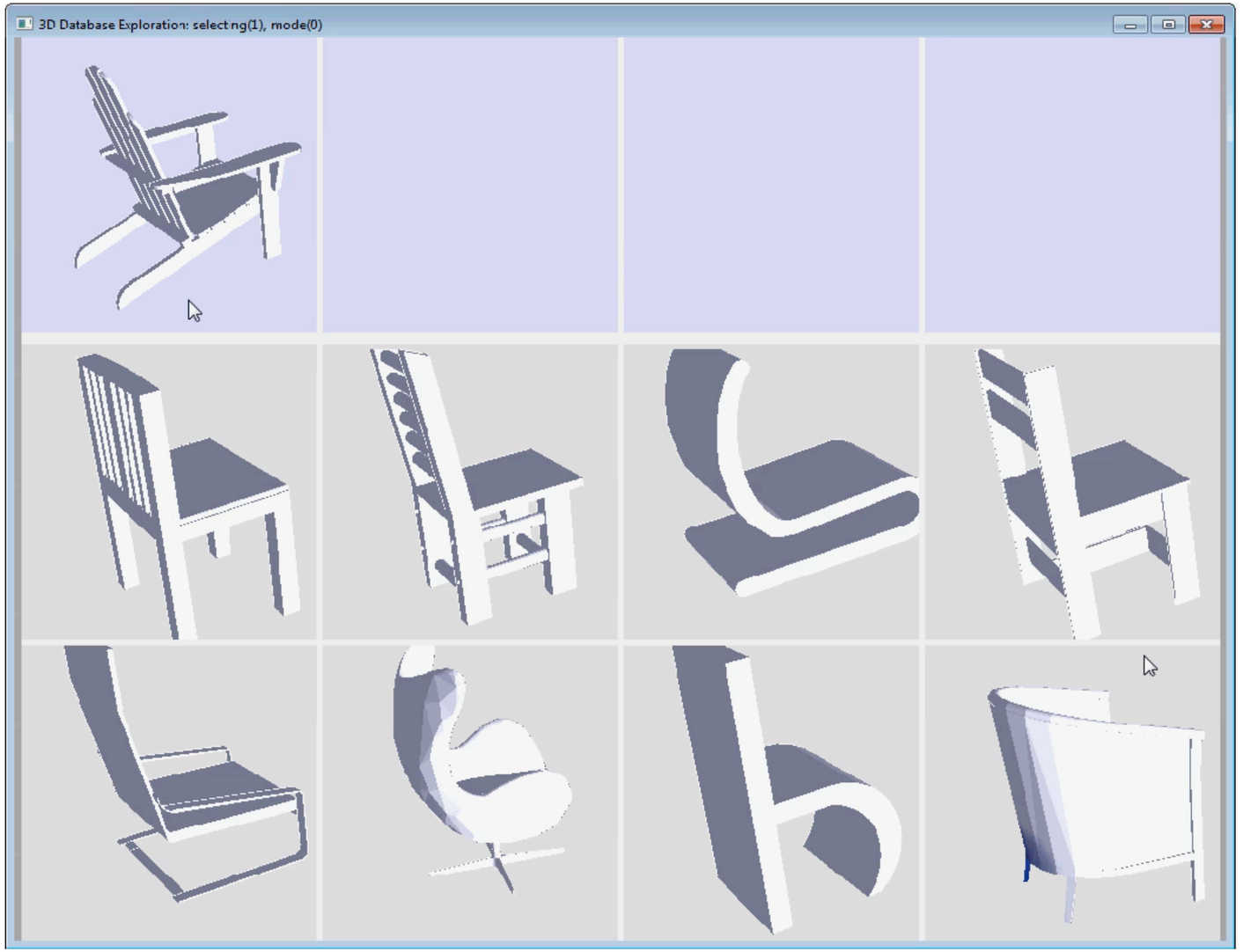


Exploring Model Collections



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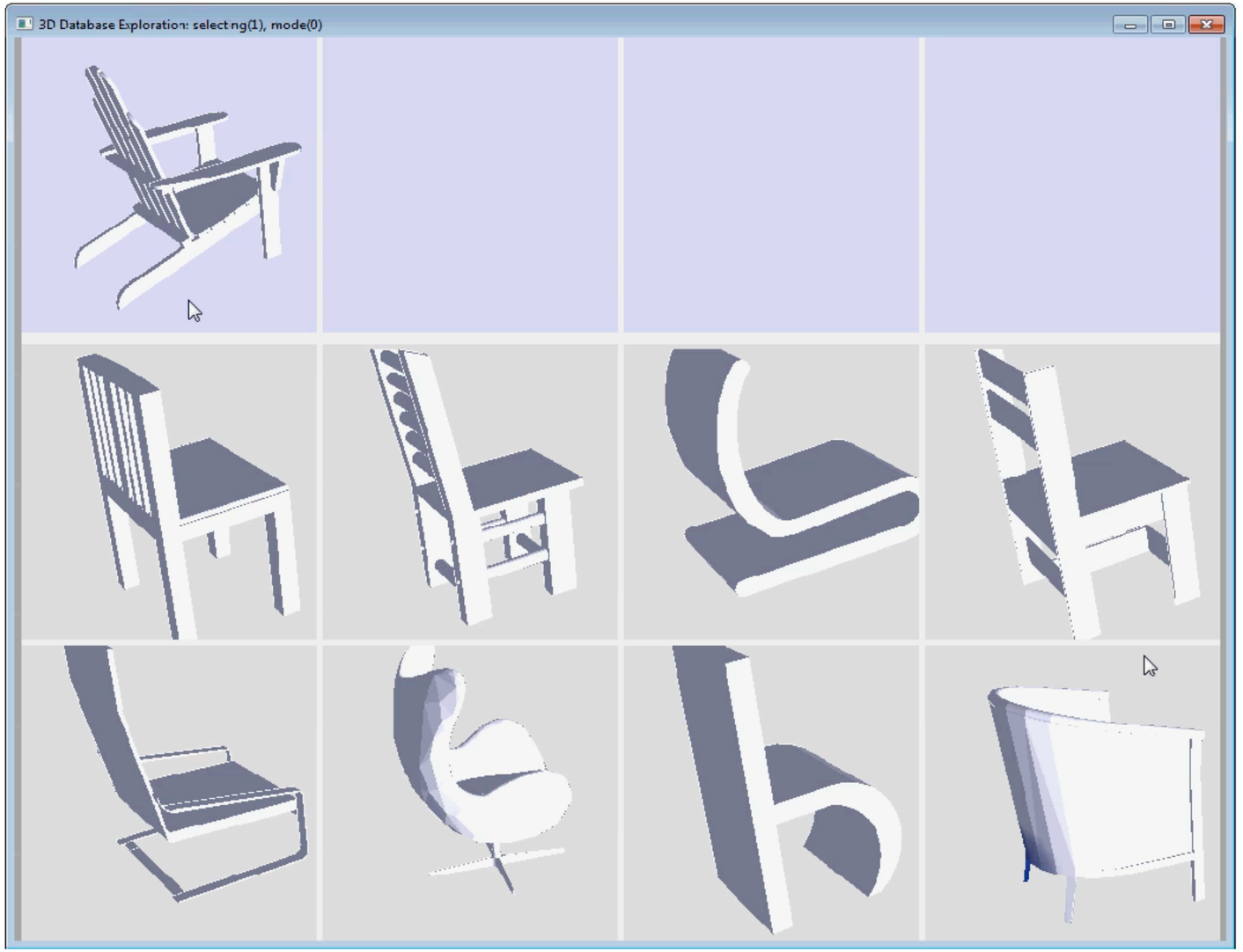
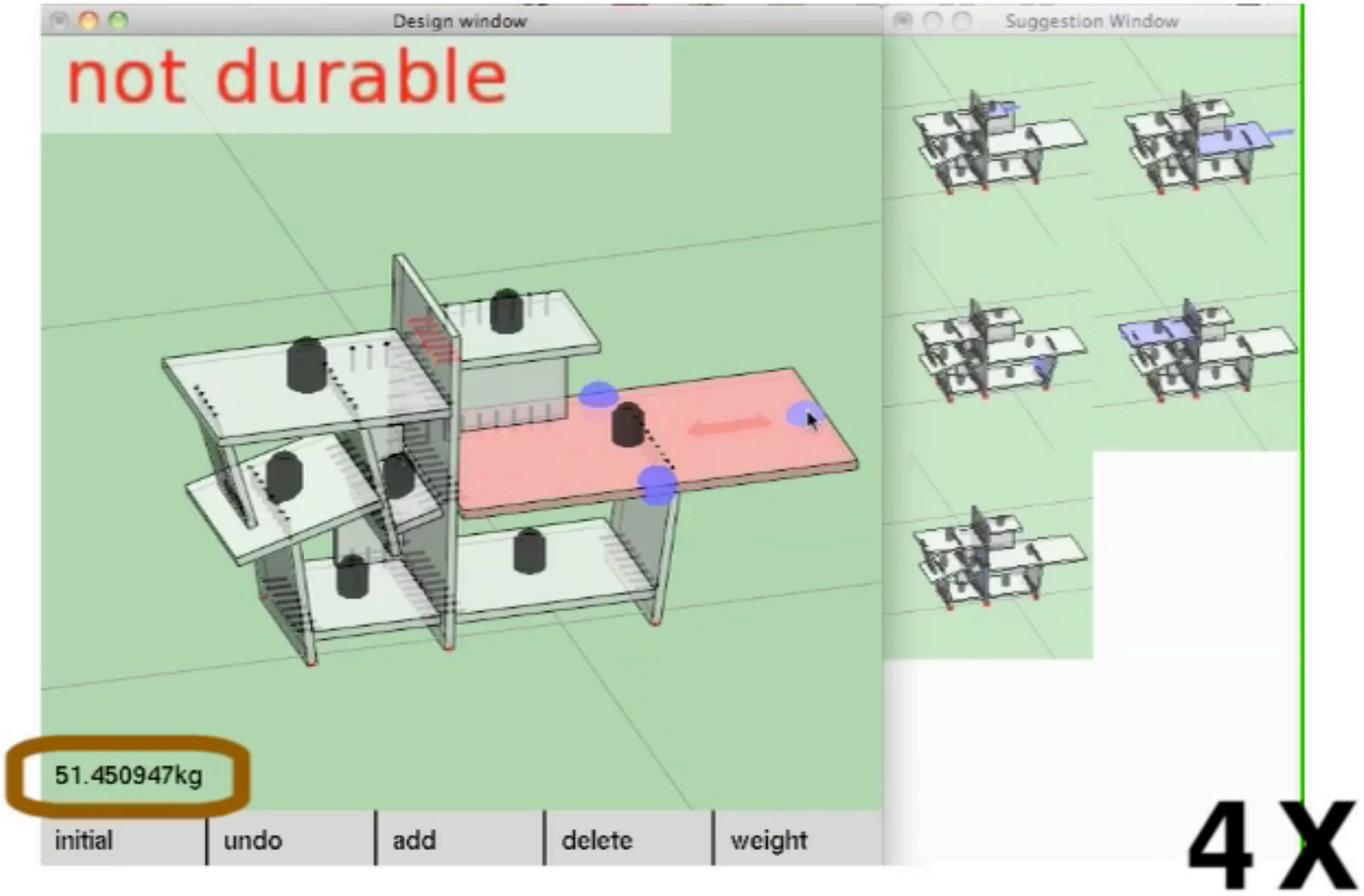


“Forms as Force Diagrams”



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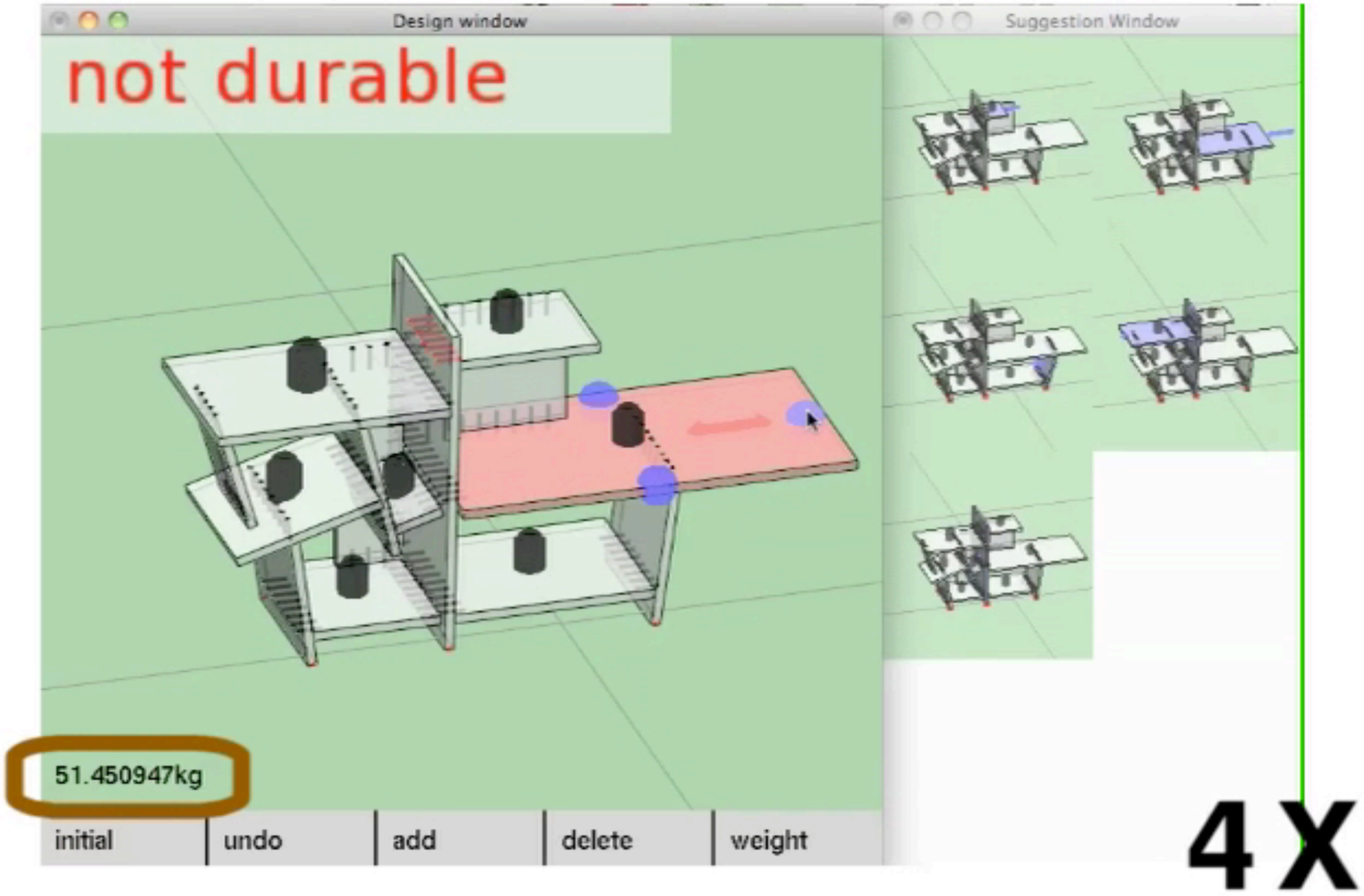
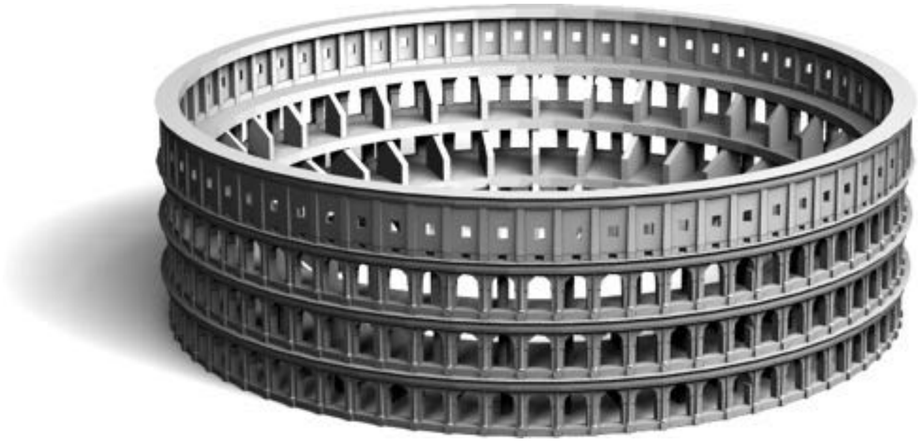


Detecting Regularities

low-level geometry → (structure+element) + variations



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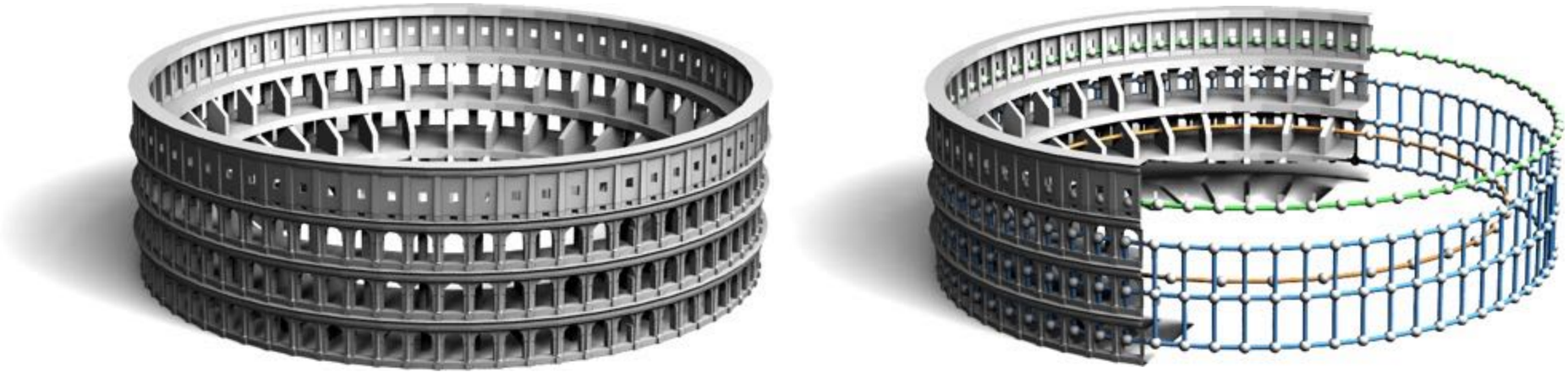

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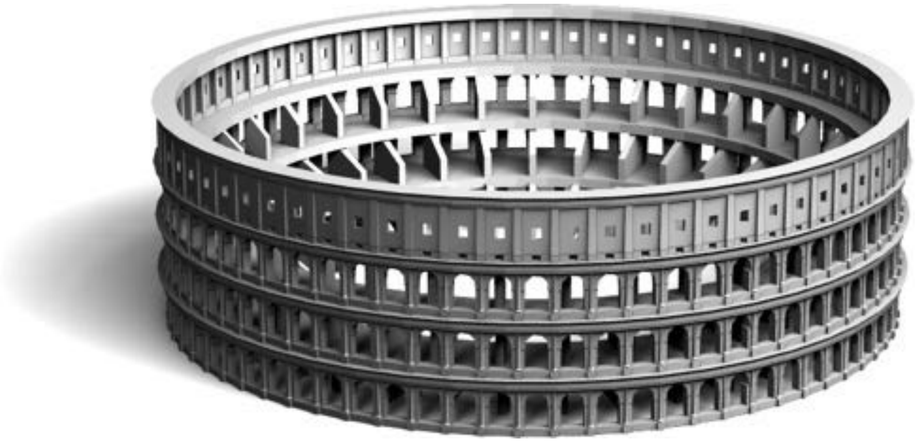

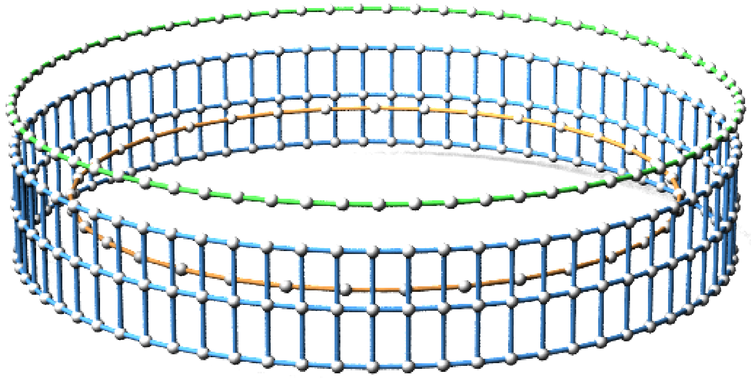

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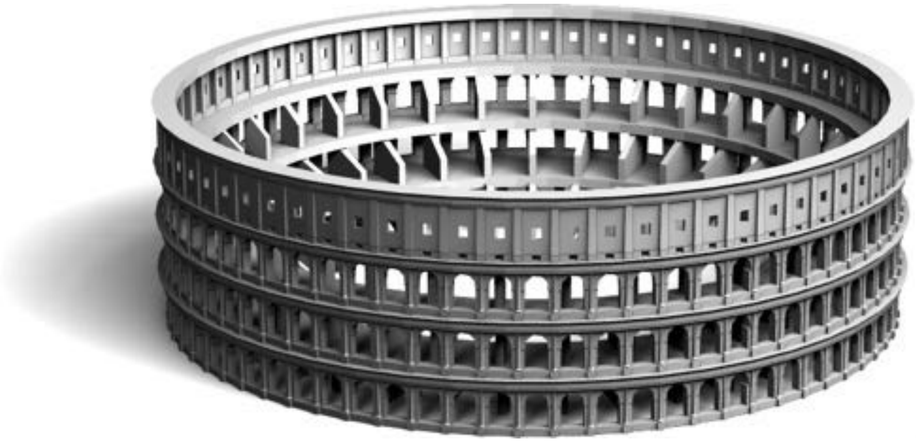
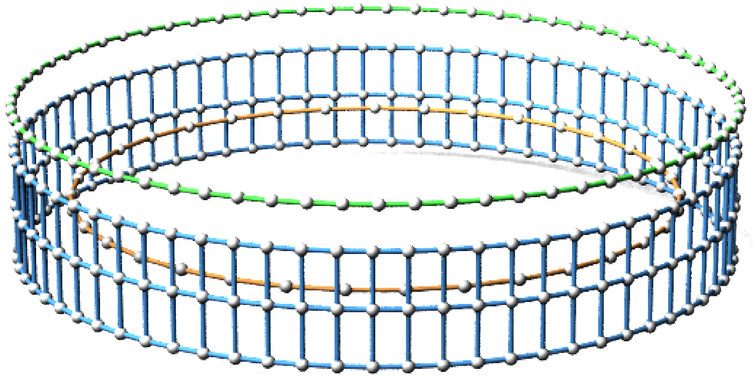


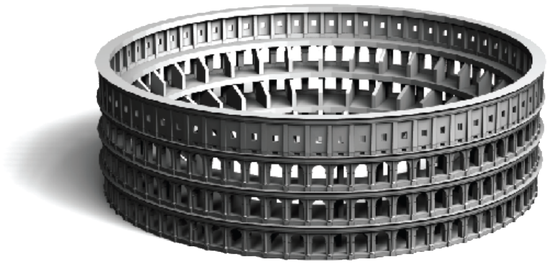


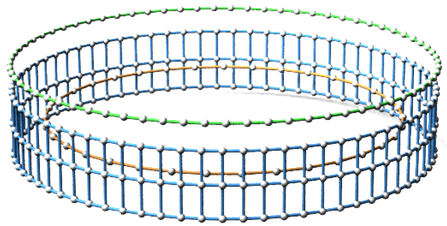
Rot: 72

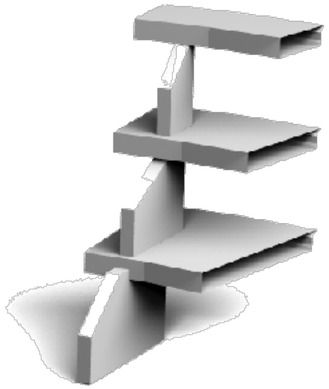
Rot x Trans: 72 x 3

Rot: 35

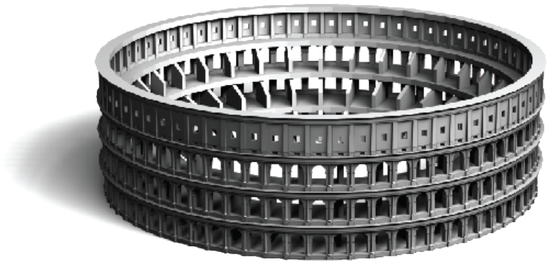
Find and Replace

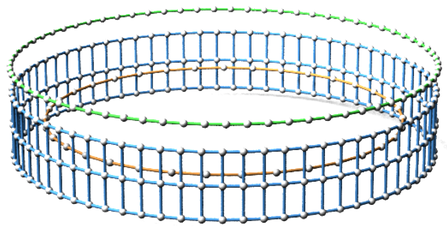


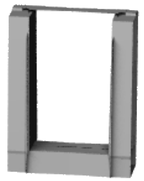


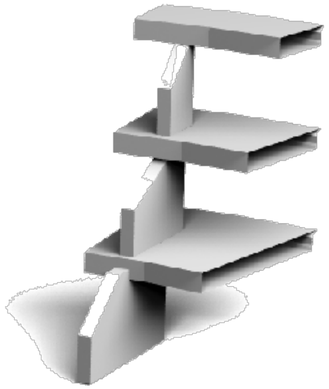


Find and Replace









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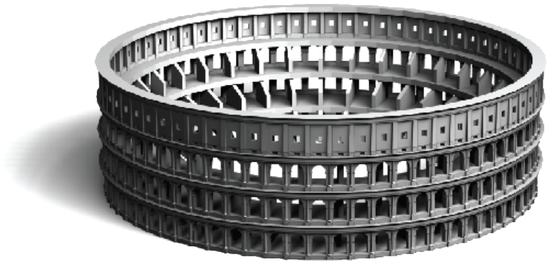

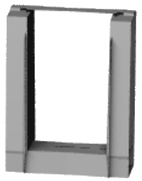

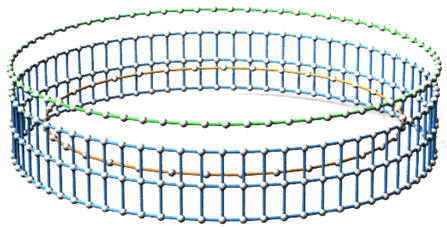

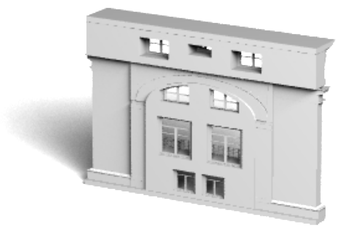

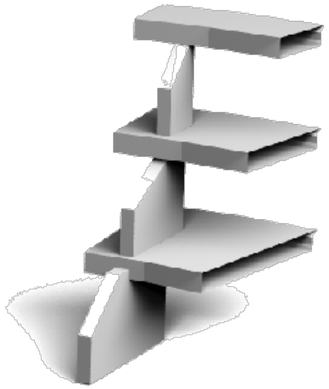

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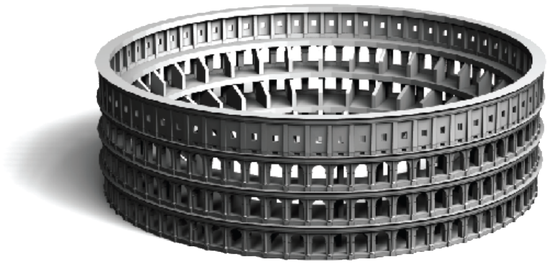

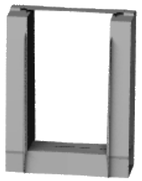

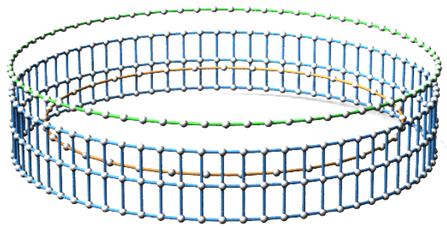

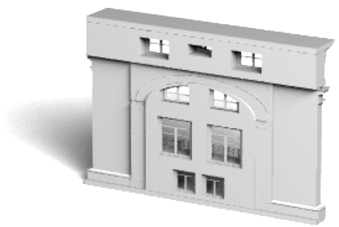

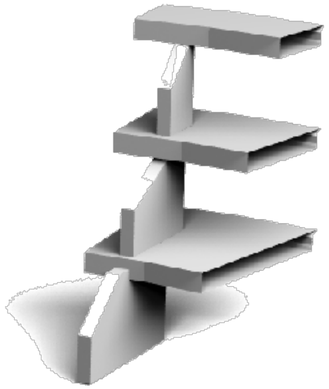

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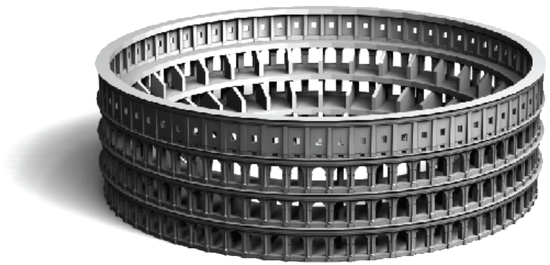

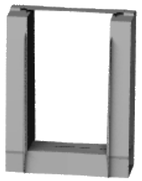

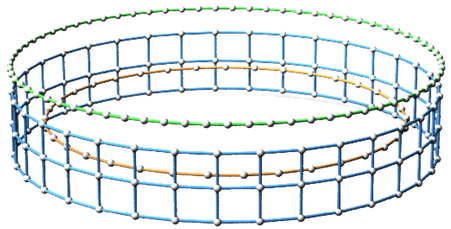

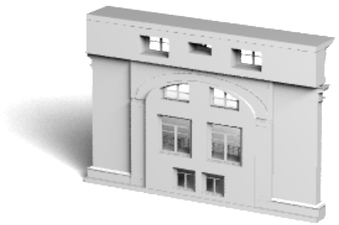

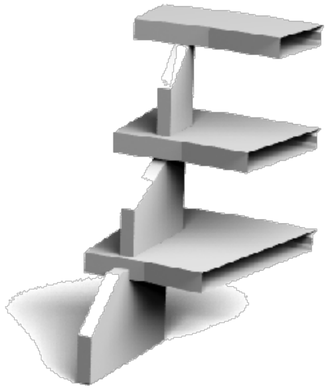

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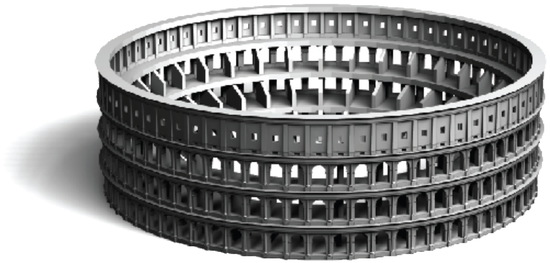
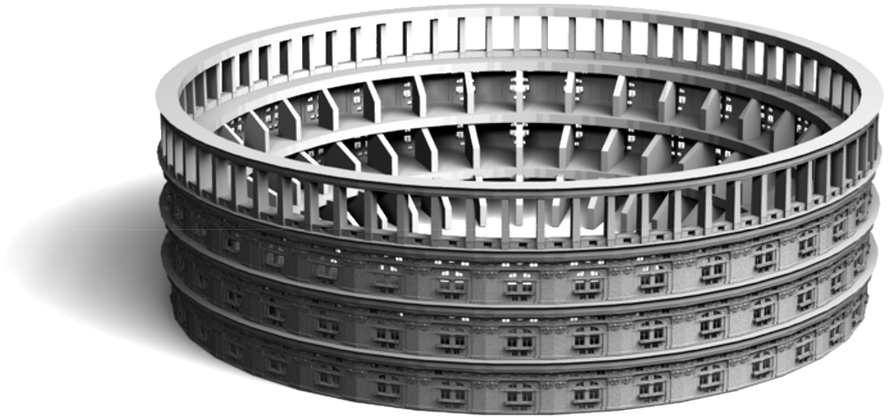
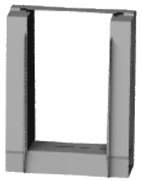
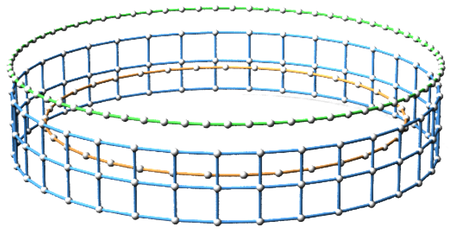
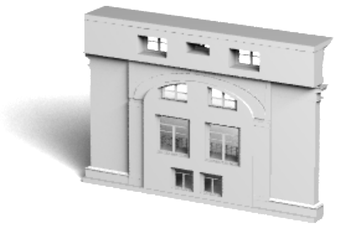
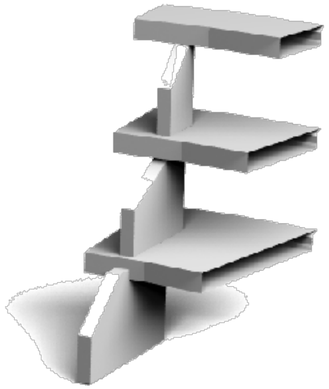




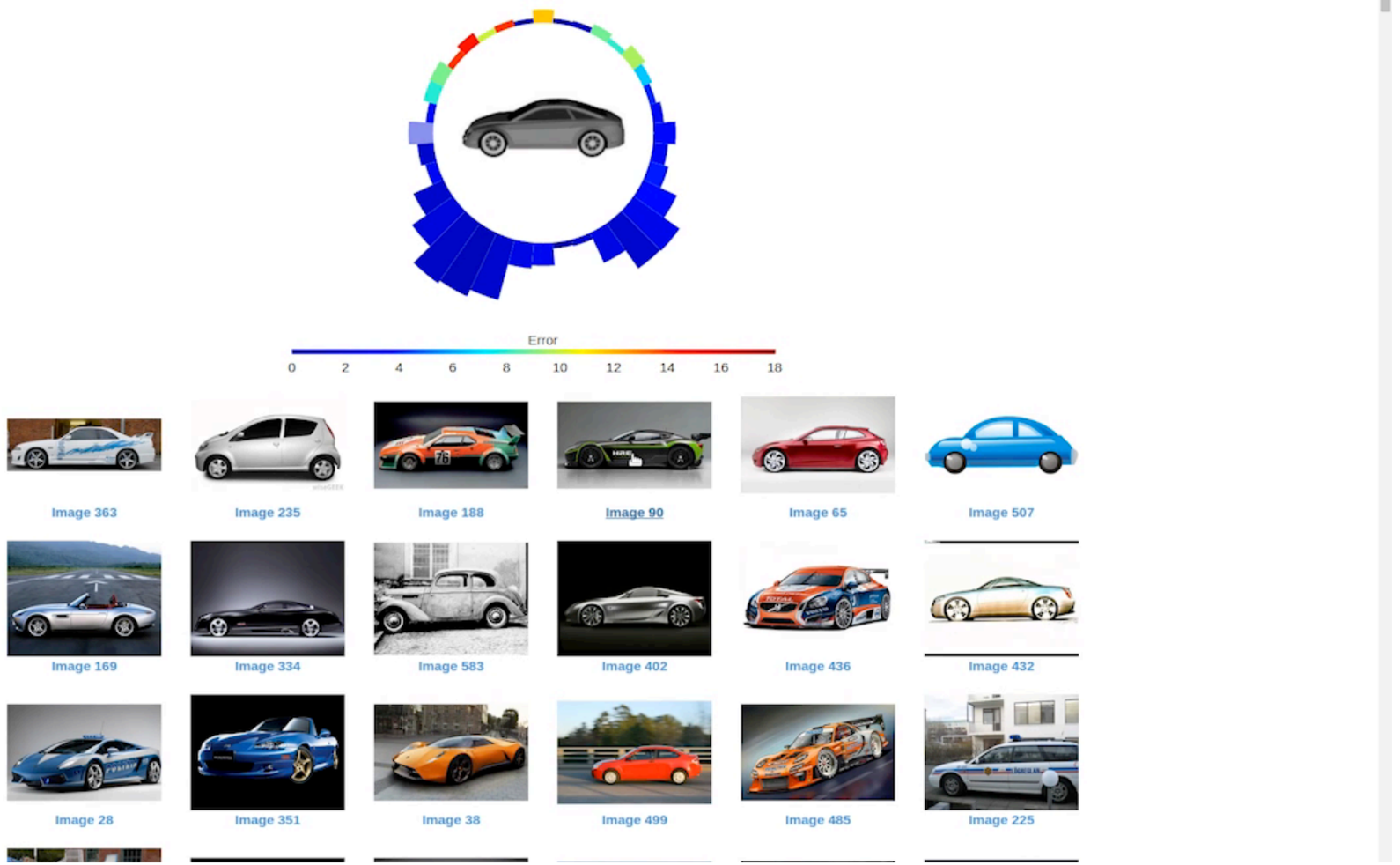




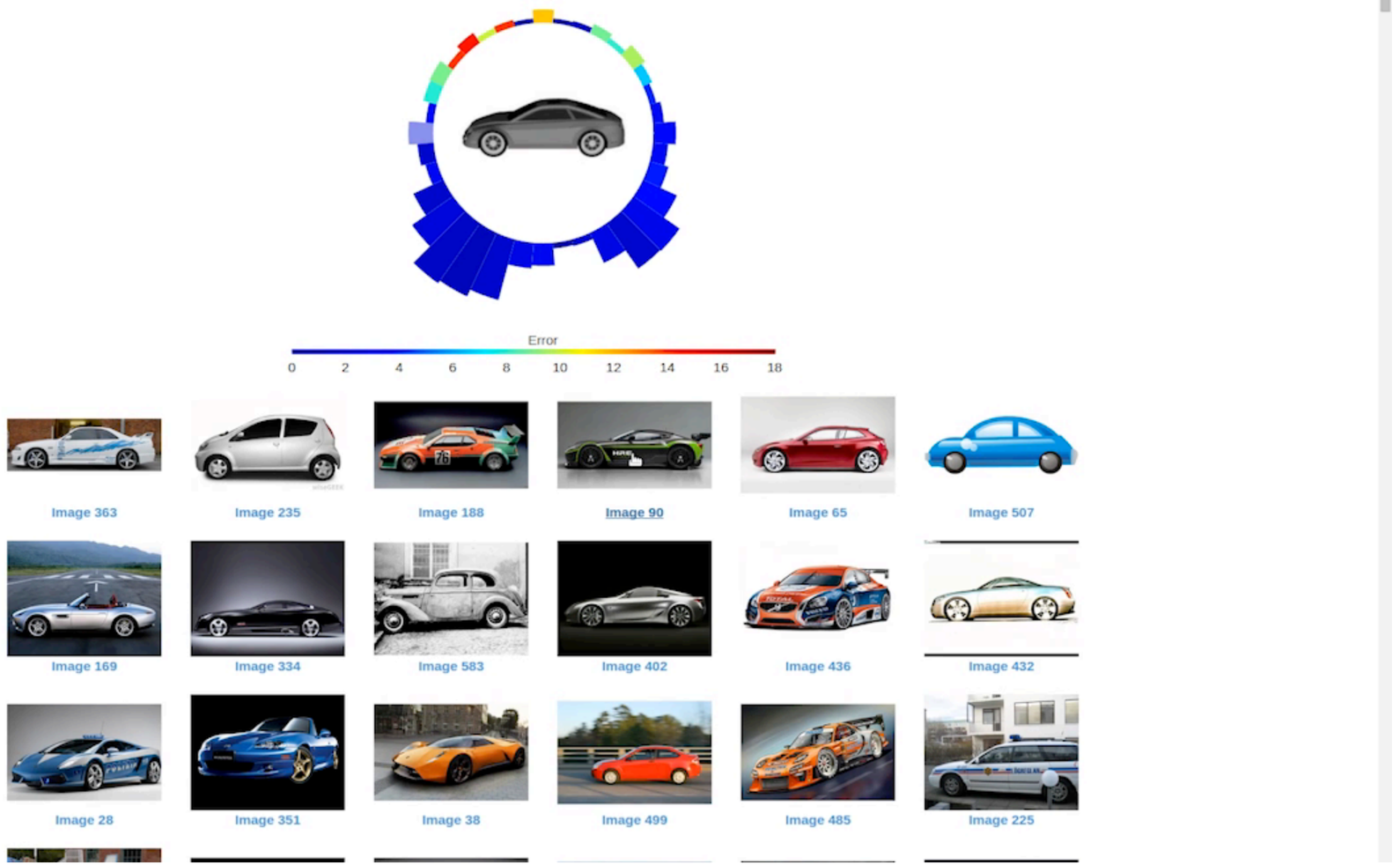




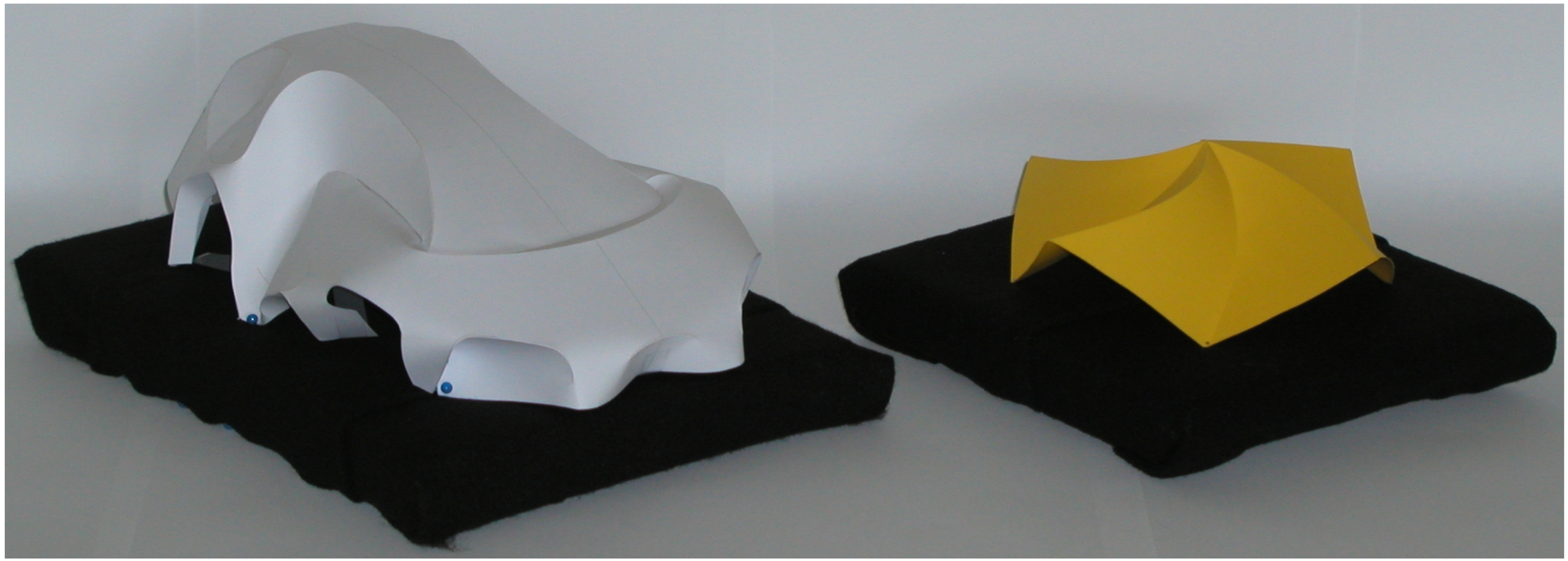
CrossLink: Images <> 3D Models



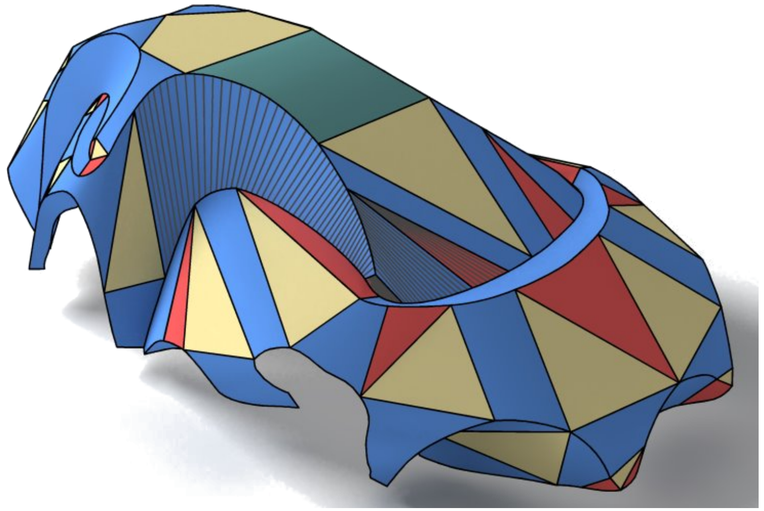
CrossLink: Images <> 3D Models



Geometry of Folds



Layout in 2D



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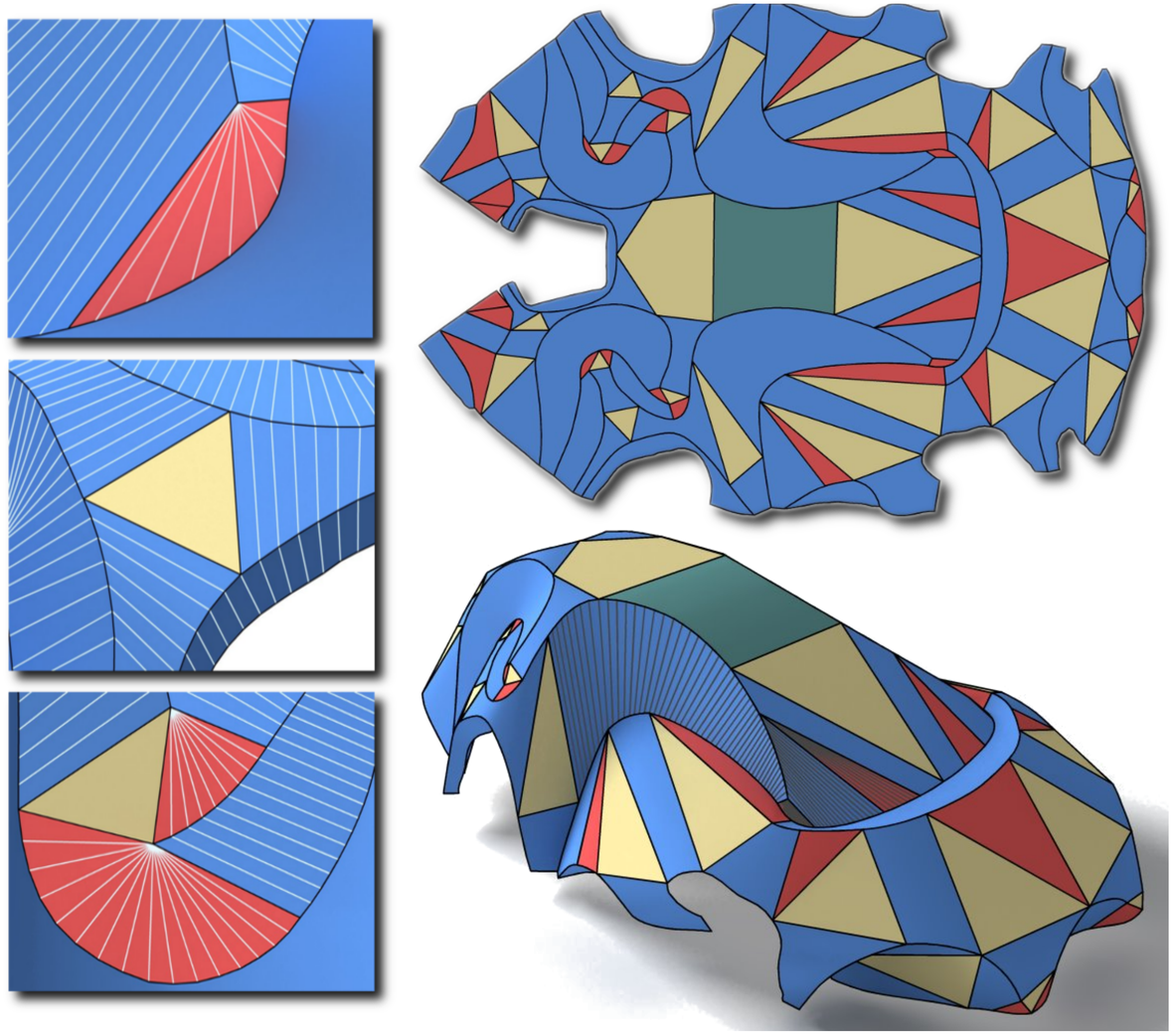

Folded Roof

Folded Roof

Folded Roof


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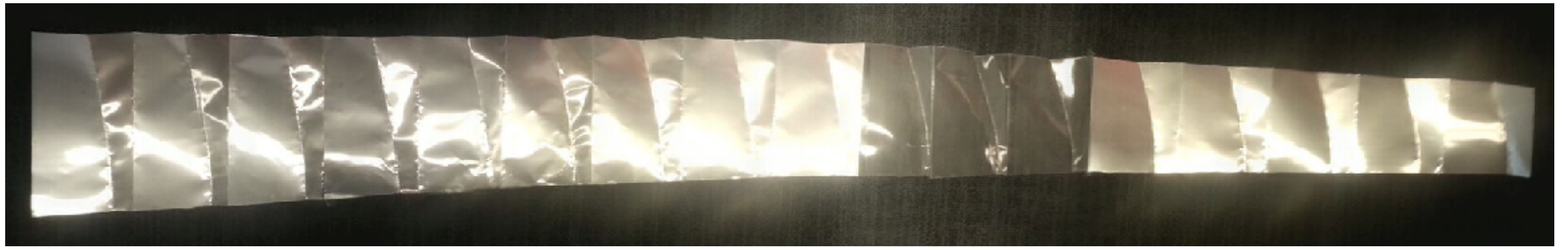
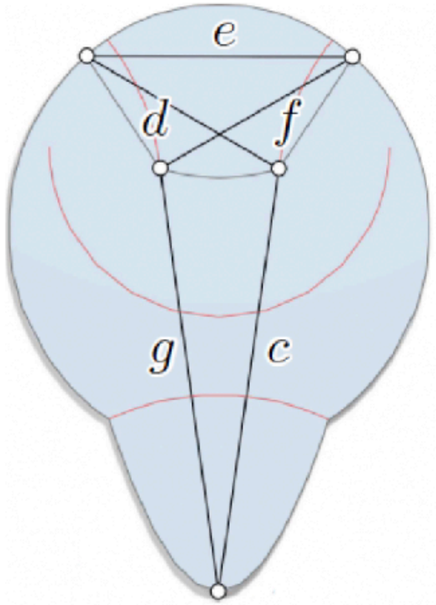
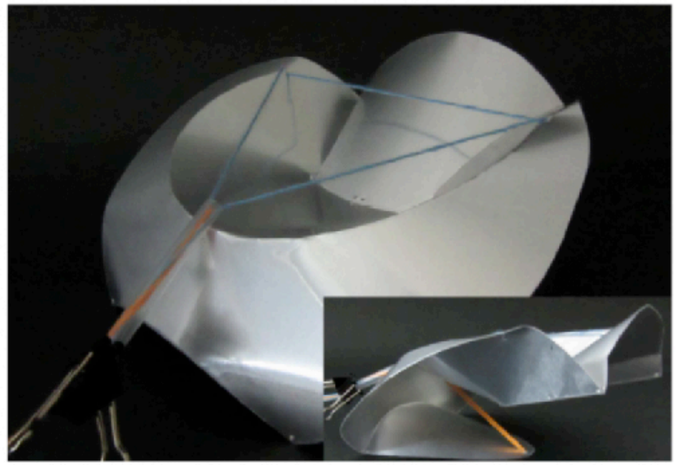


Concept Chair

Sequence: CONCEPT CHAIR





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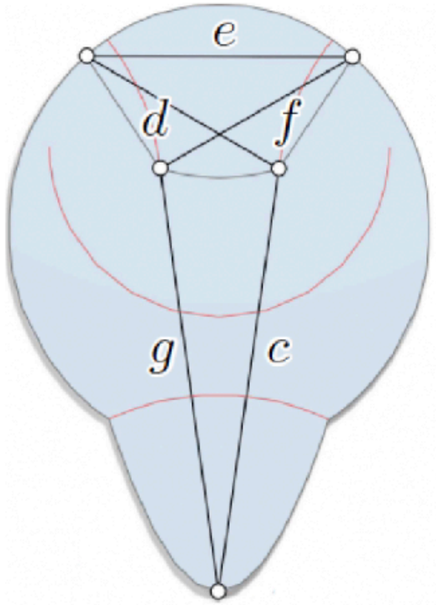

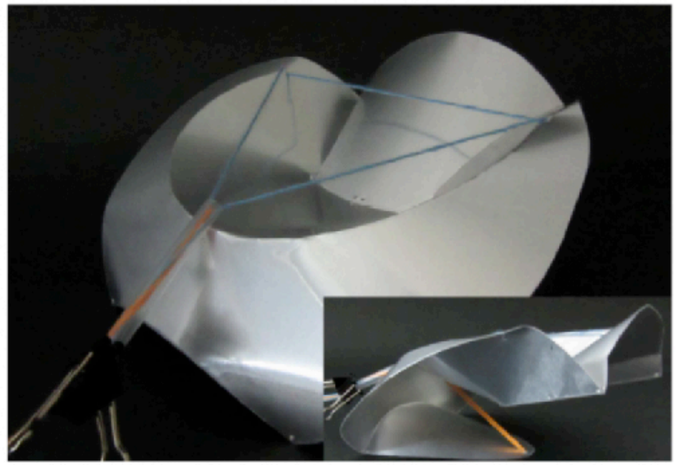

Overview

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Geometric Modeling

- techniques and algorithms for representing and processing geometric objects

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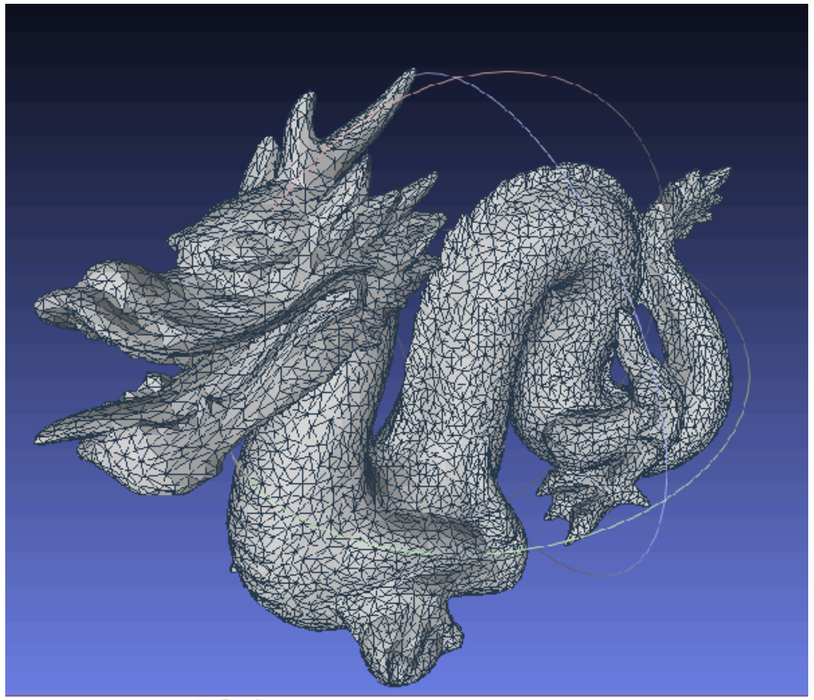
- techniques and algorithms for representing and processing geometric objects

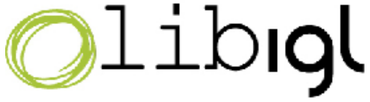
In this course, we will focus on **triangle meshes**

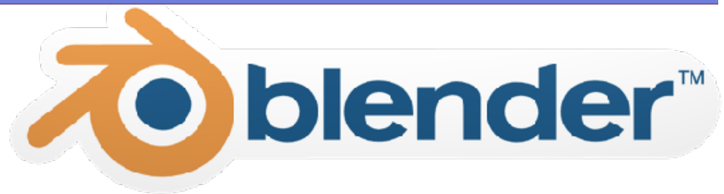
- **why** are triangle meshes a suitable representation for geometry processing?
- **what** are the central processing algorithms?
- **how** can they be implemented efficiently?

Looking at Meshes

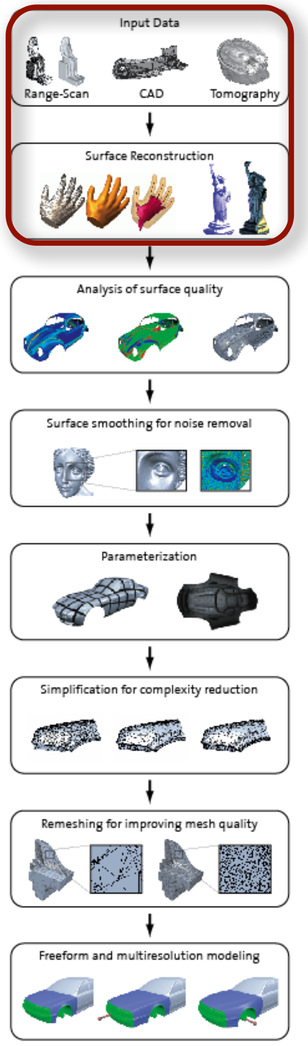


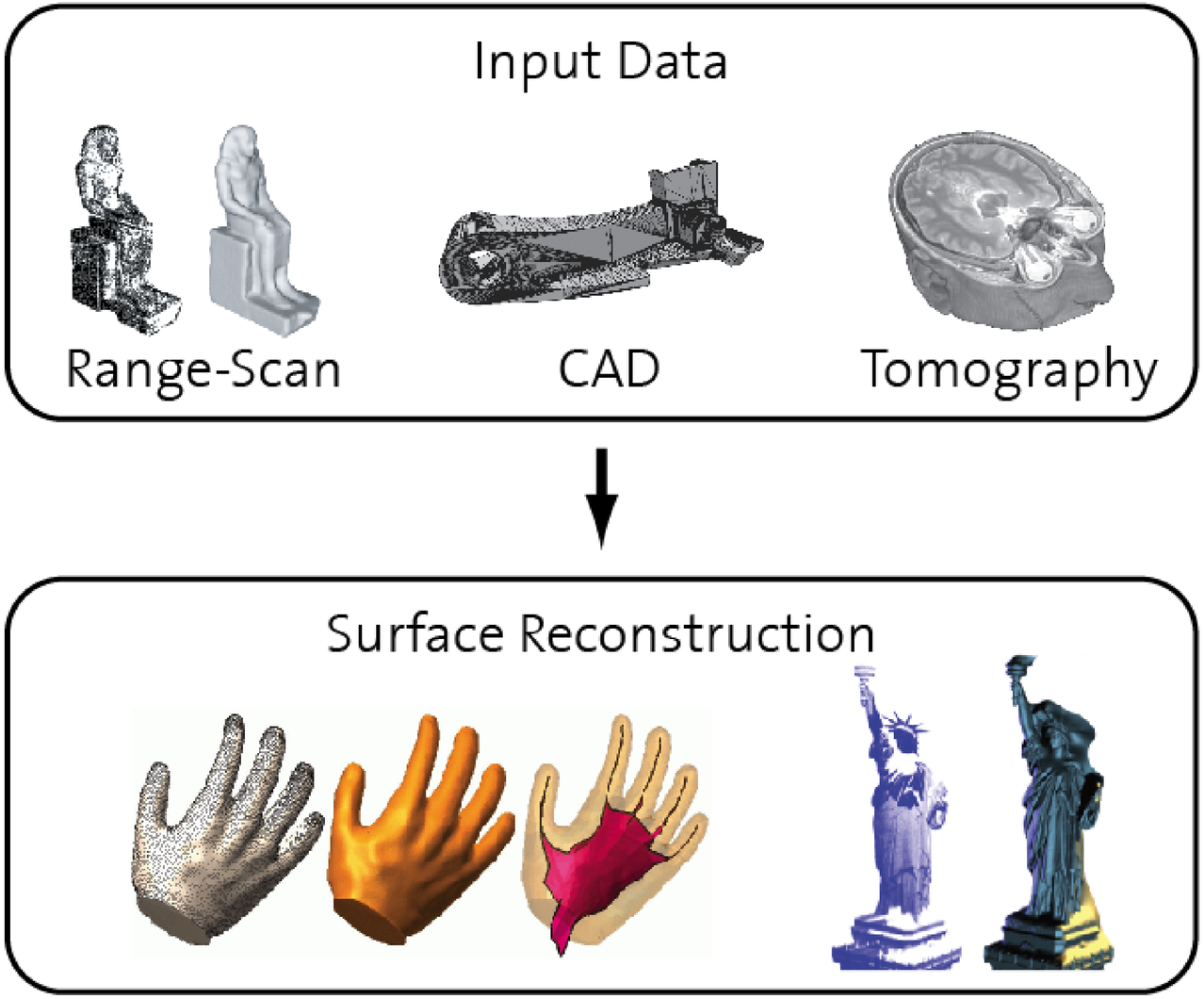
OpenMesh





Geometry Processing Pipeline





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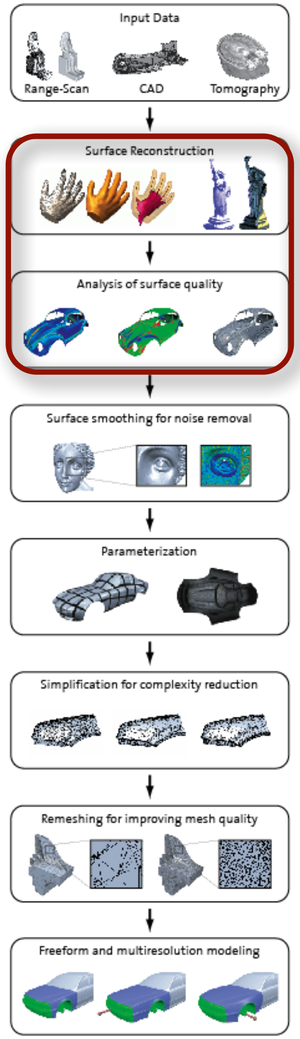

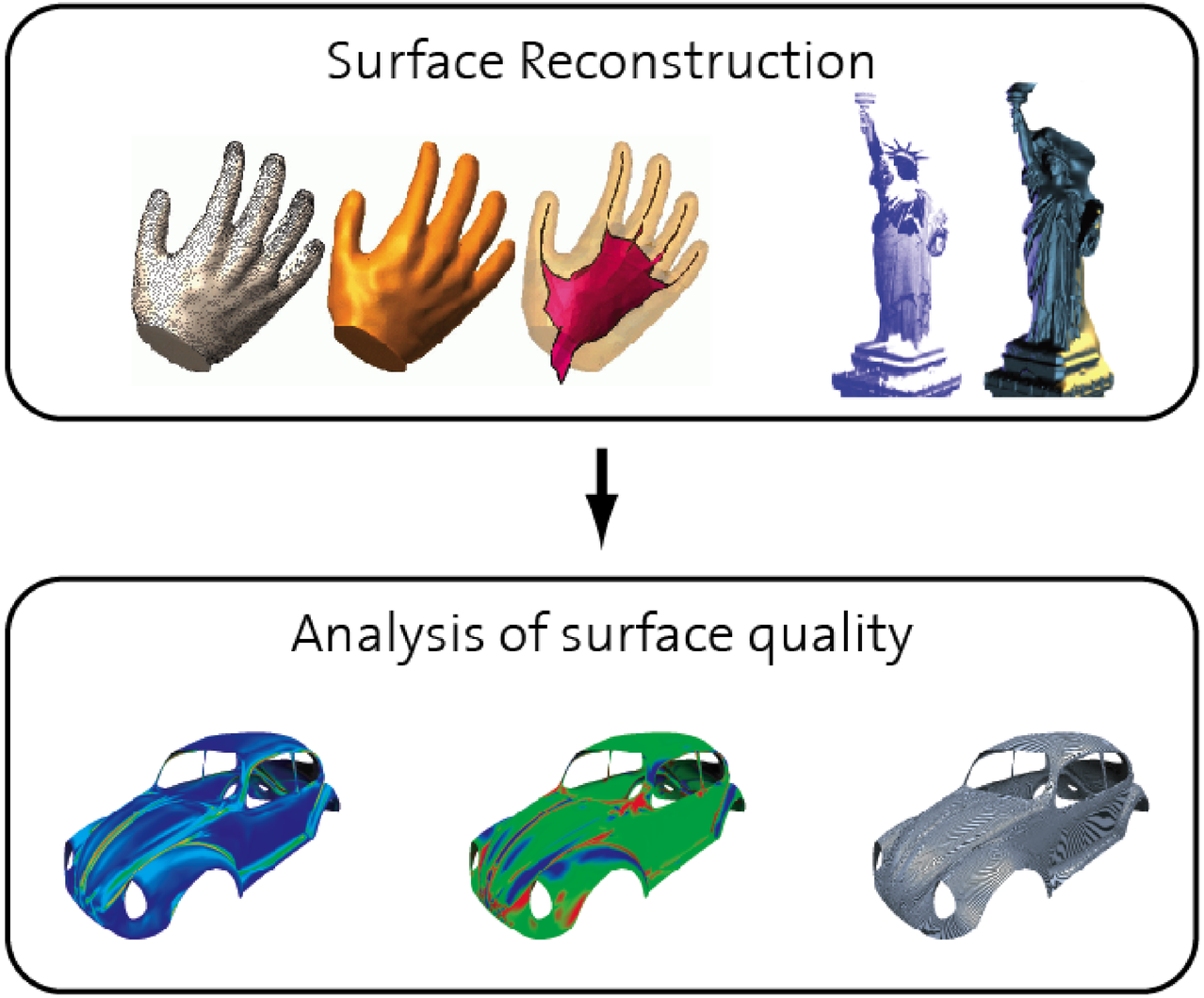

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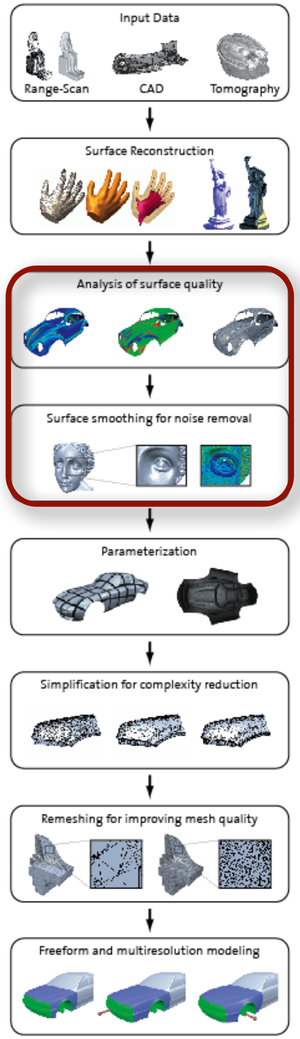

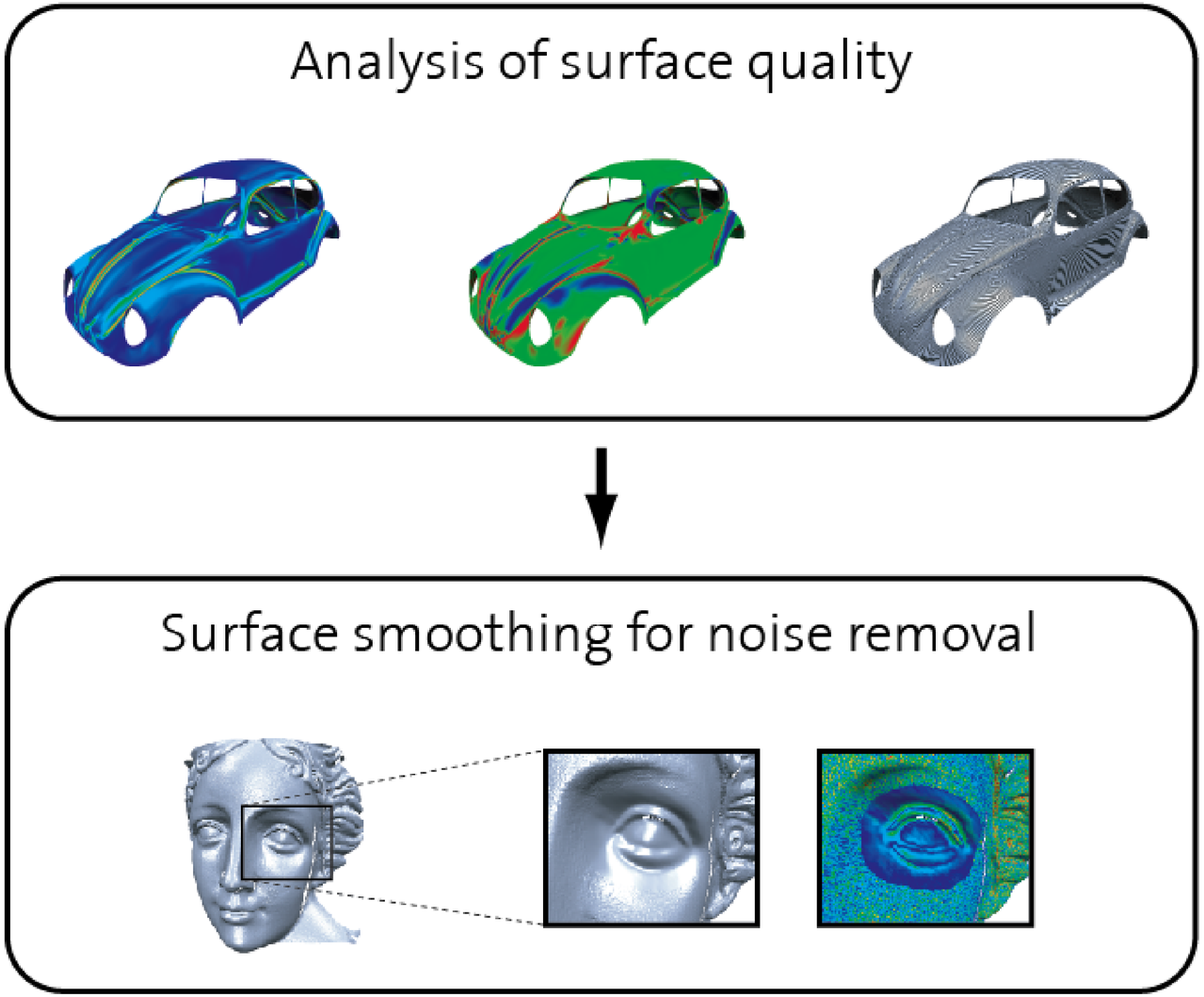

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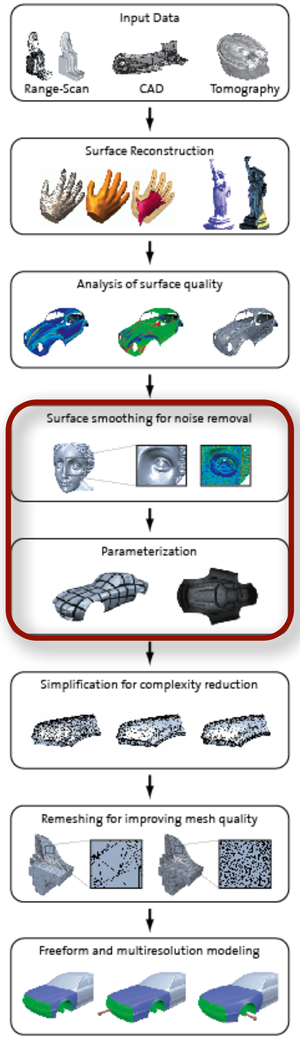

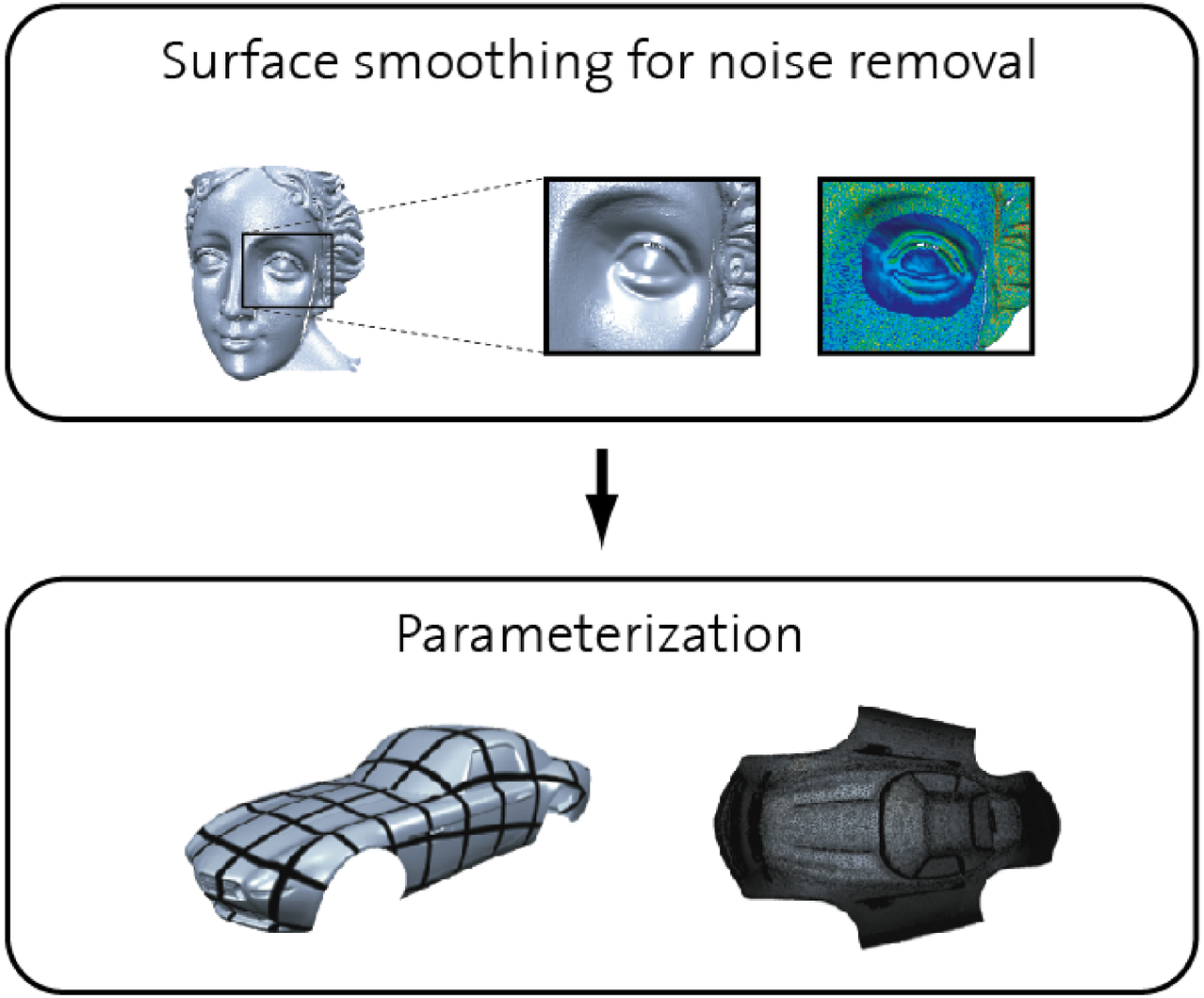

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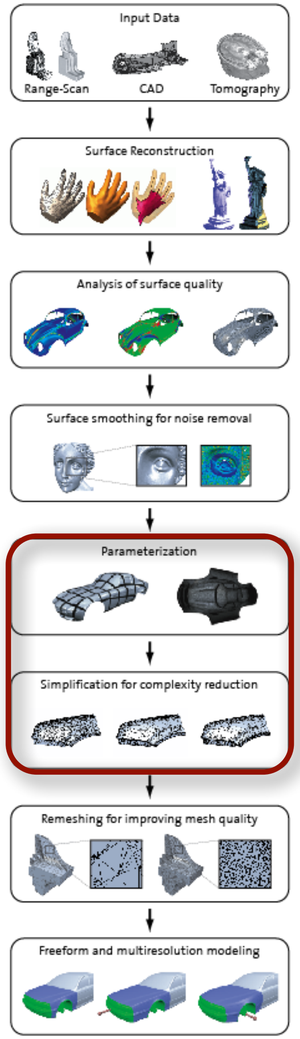

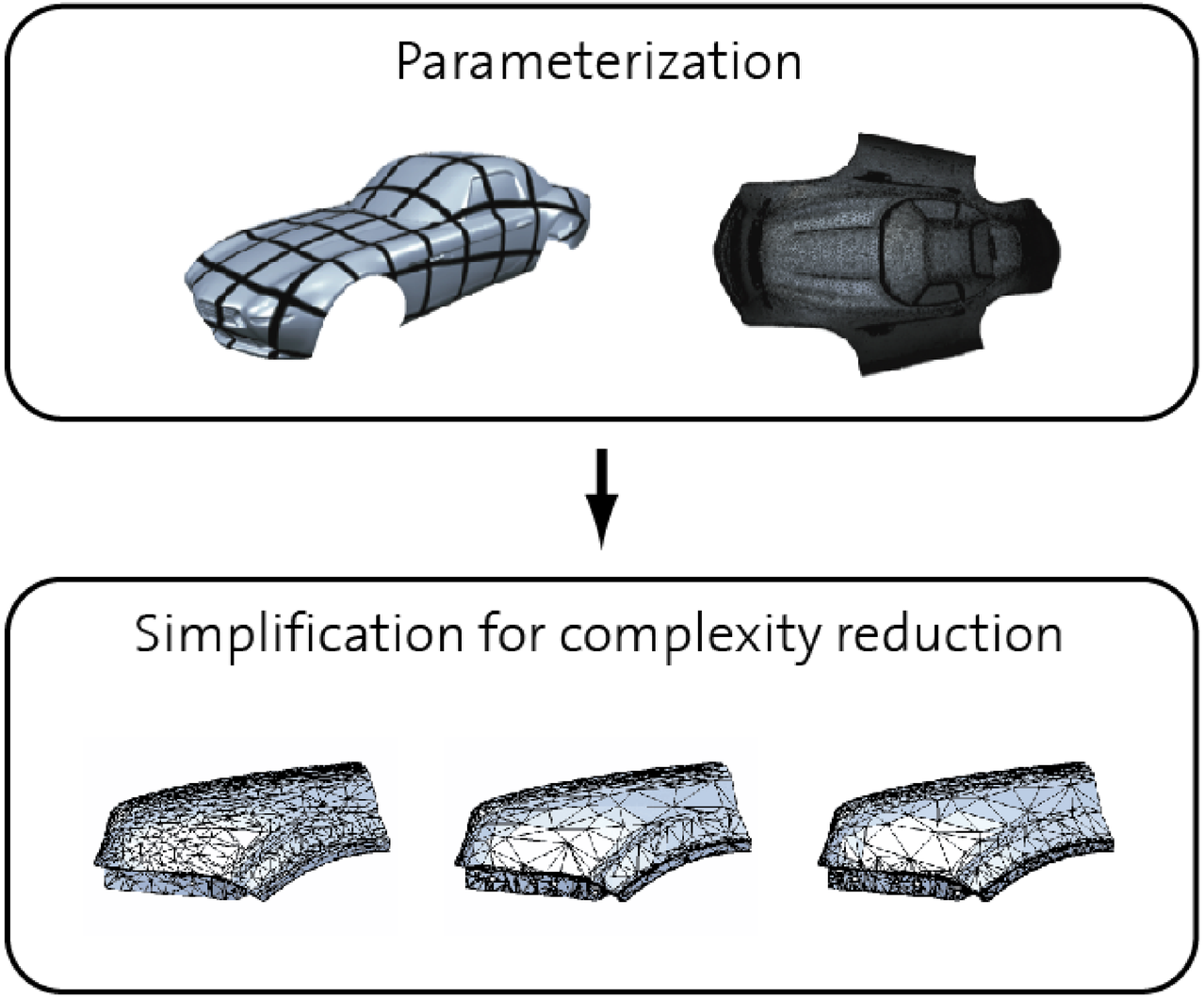

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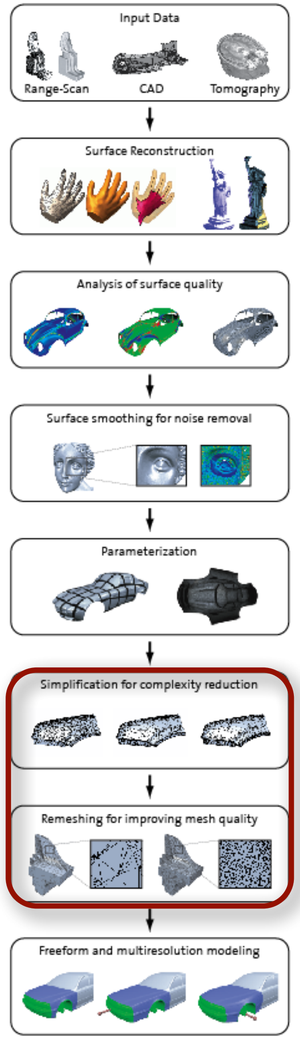

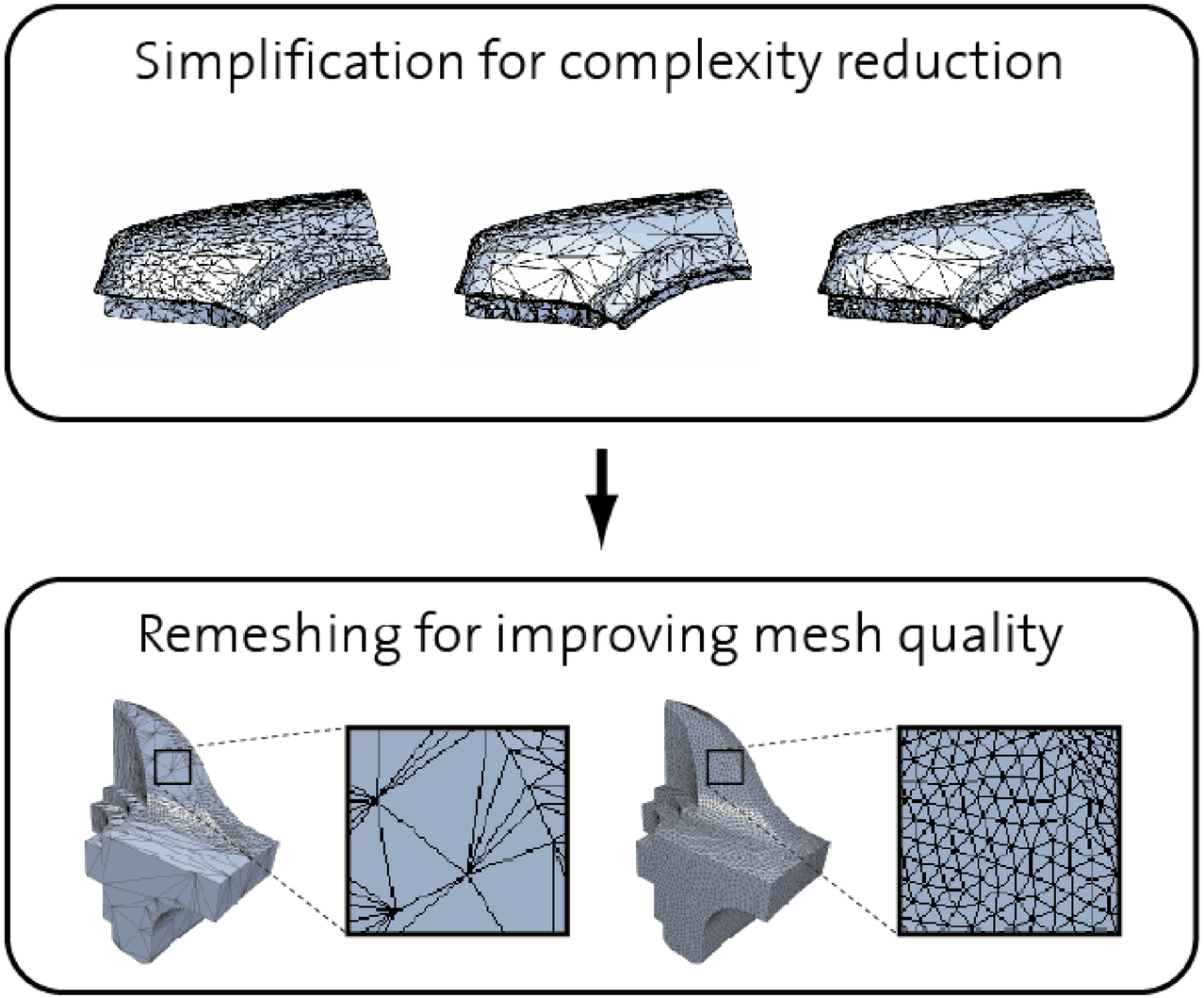

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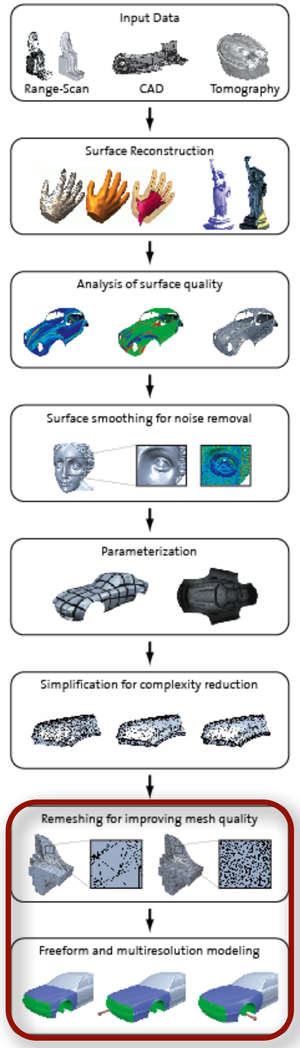
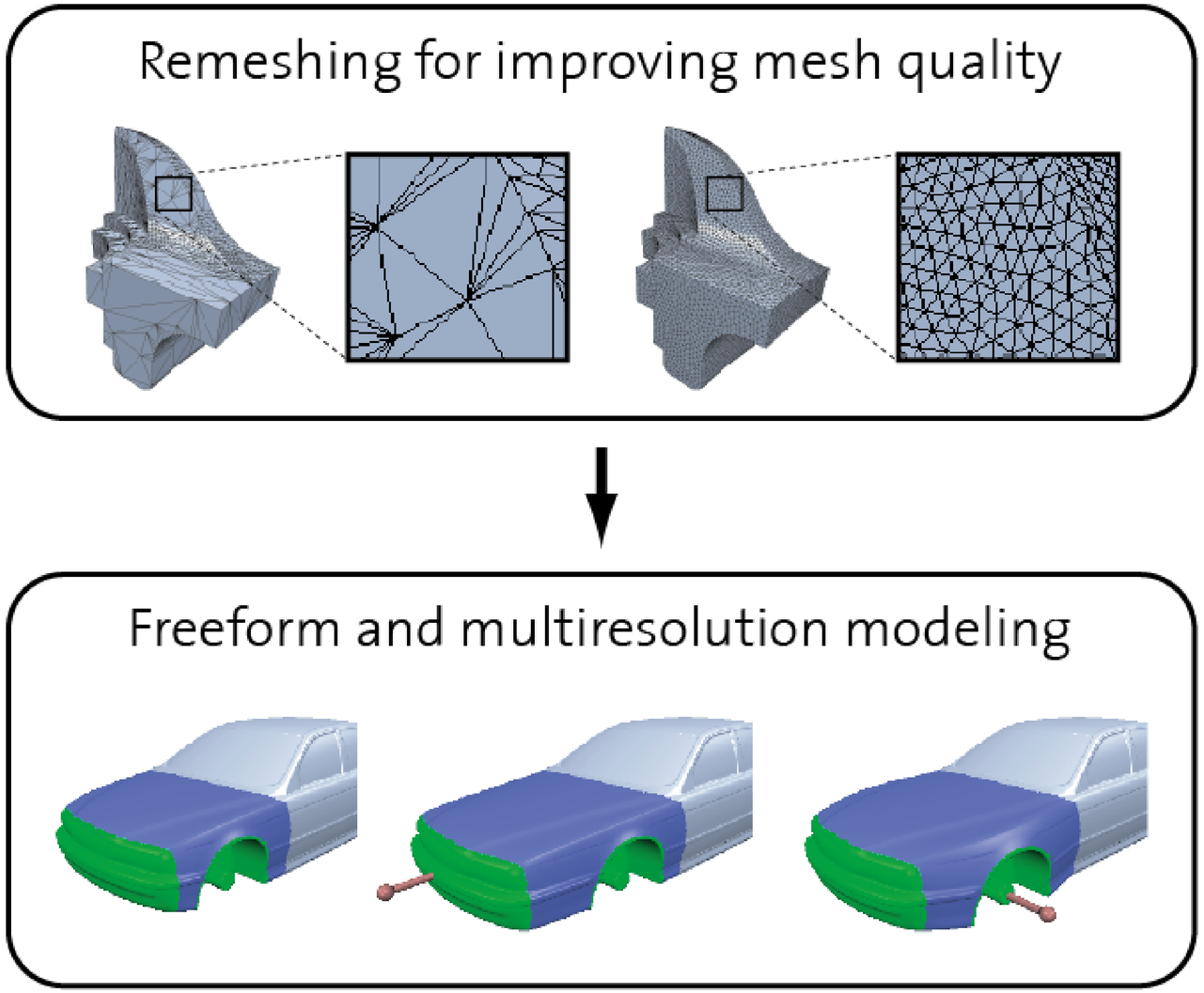
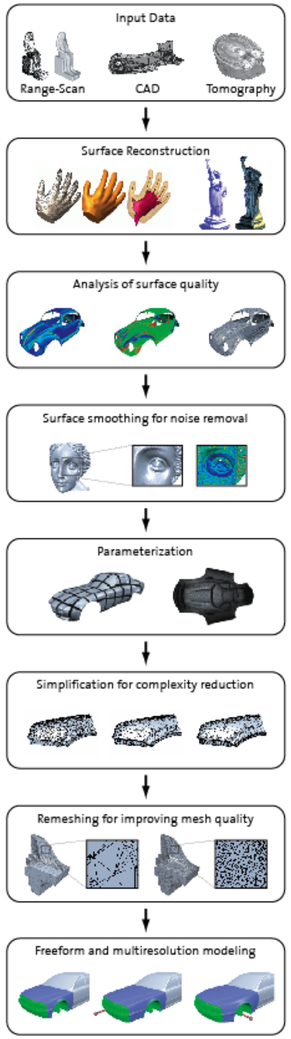




Geometry Processing Pipeline



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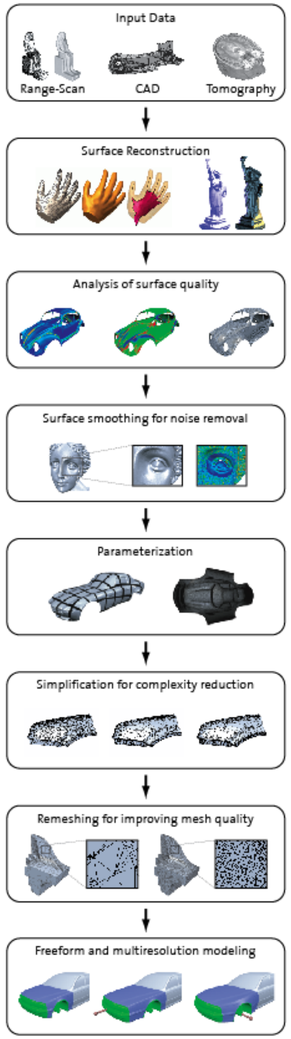

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- ***design*** and ***implement*** individual components of a geometric modeling system

Exercises

Programming assignments

- based on OpenMesh/Meshlab/Trimesh/others
- cover the core stages of the geometry processing pipeline
- framework including mesh viewer will be provided (Python, C/C++)

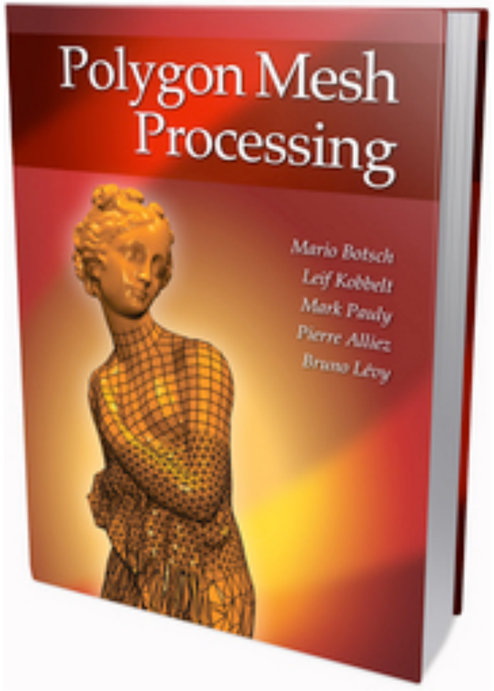
Final project (groups of 2)

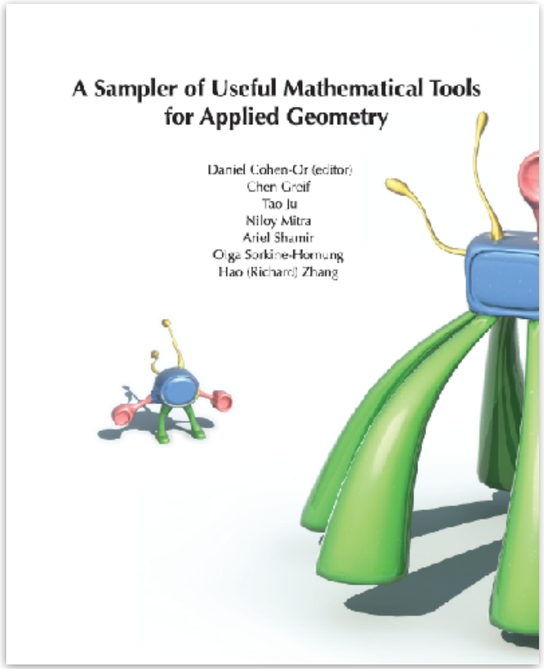
Integral part of the lecture

- important for achieving course objectives

Literature

- Botsch, Kobbelt, Pauly, Alliez, Levy:
  Polygon Mesh Processing, AK Peters, 2010
- Cohen-Or, Greif, Ju, Mitra, Shamir, Sorkine, Zhang:
  A Sampler of Useful Mathematical Tools for Applied Geometry, CRC, 2015





Outline

- Parametric Approximations
- Polygon Meshes
- Data structures

Outline

- **Parametric Approximations**
- Polygon Meshes
- Data structures

Parametric Representation

Parametric Representation

- Surface is the **range** of a function

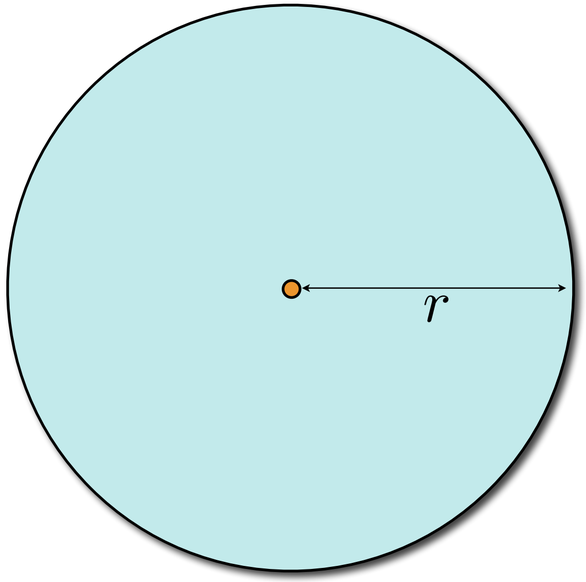
$$\mathbf{f} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3, \quad \mathcal{S}_\Omega = \mathbf{f}(\Omega)$$

$$\mathbf{c} : \Lambda \in \mathbb{R} \rightarrow \mathbb{R}^2, \quad \mathcal{C}_\Lambda = \mathbf{c}(\Lambda)$$

- 2D example: Circle

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} r\cos(t) \\ r\sin(t) \end{pmatrix}$$



Parametric Representation

- Surface is the range of a function

$$\mathbf{f} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3, \quad \mathcal{S}_\Omega = \mathbf{f}(\Omega)$$

- 2D example: Island coast line

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} ??? \\ ??? \end{pmatrix}$$

Piecewise Approximation

- Surface is the range of a function

$$\mathbf{f} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3, \quad \mathcal{S}_\Omega = \mathbf{f}(\Omega)$$

- 2D example: Island coast line

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathbb{R}^2$$

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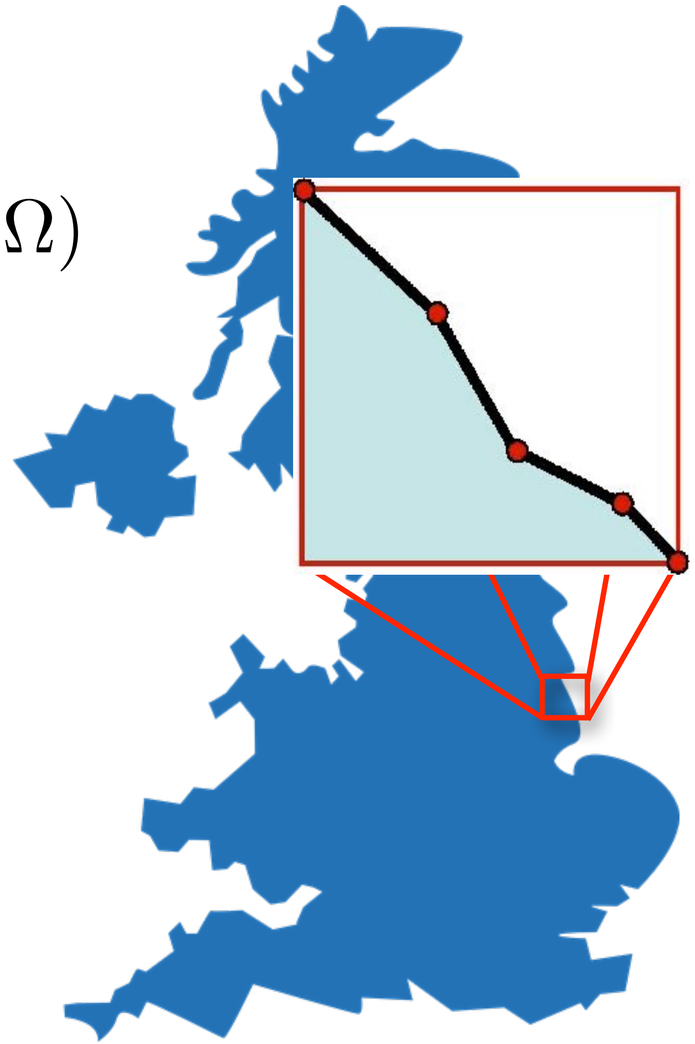

Polynomial Approximation

Polynomial Approximation

- Polynomials are computable functions

$$f(t) = \sum_{i=0}^{p} c_i \, t^i = \sum_{i=0}^{p} \tilde{c}_i \, \phi_i(t)$$

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- **Taylor expansion** up to degree *p*

$$g(h) \;=\; \sum_{i=0}^{p} \frac{1}{i!}\, g^{(i)}(0)\; h^i \;+\; O\big(h^{p+1}\big)$$

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- Error for approximating a function *g* by polynomial *f*

$$f(t_i) = g(t_i)\,, \quad 0 \le t_0 < \cdots < t_p \le h$$

$$|f(t) - g(t)| \;\le\; \frac{1}{(p+1)!}\, \max f^{(p+1)} \prod_{i=0}^{p} (t - t_i) \;=\; O\Big(h^{(p+1)}\Big)$$

Polynomial Approximation

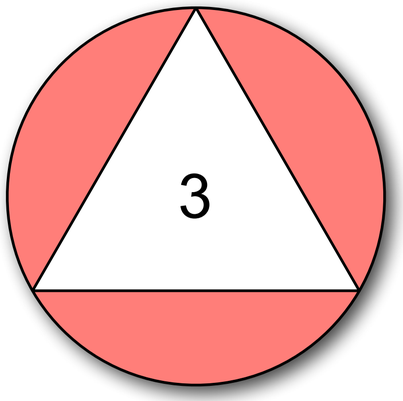
- **Approximation error** is $O(h^{p+1})$

- **Improve** approximation quality by
  - increasing *p* ... higher order polynomials
  - decreasing *h* ... smaller / more segments

- Issues
  - _ smoothness of the target data $\max_t f^{p+1}(t)$
  - smoothness conditions between segments

Polygonal Meshes

- **Polygonal meshes** are a good compromise
  - Piecewise linear approximation → error is $O(h^2)$



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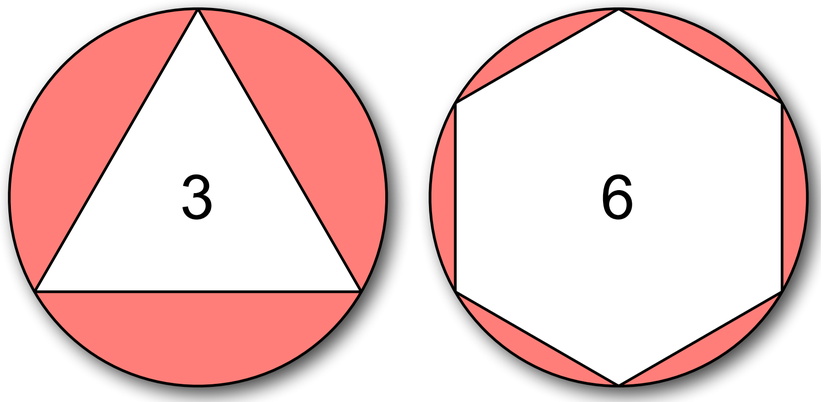

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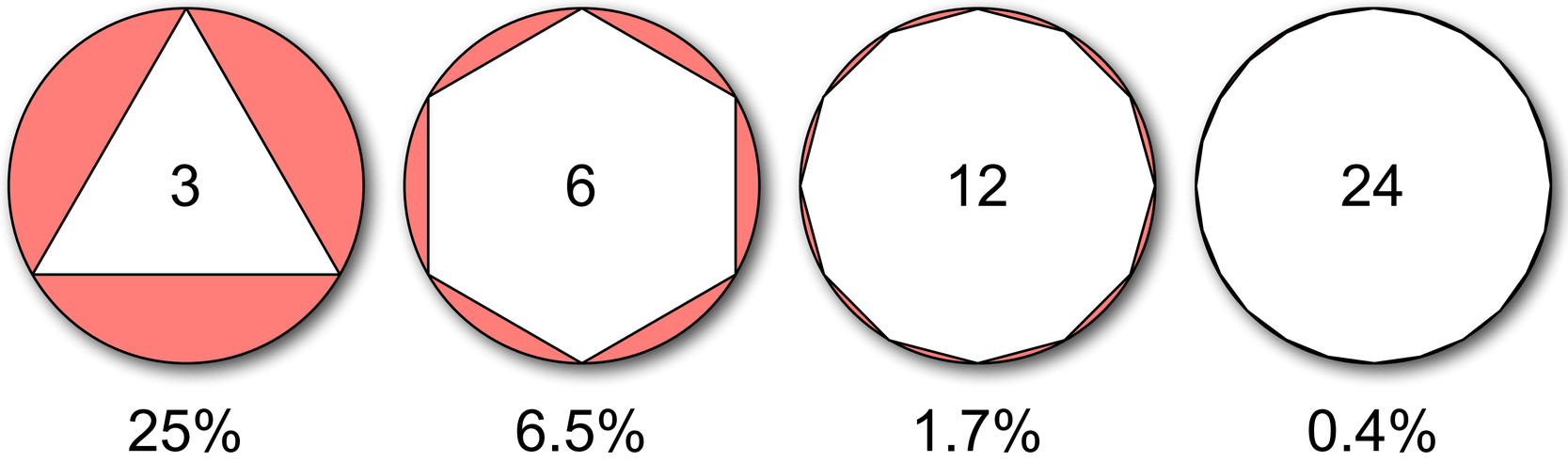

Polygonal Meshes

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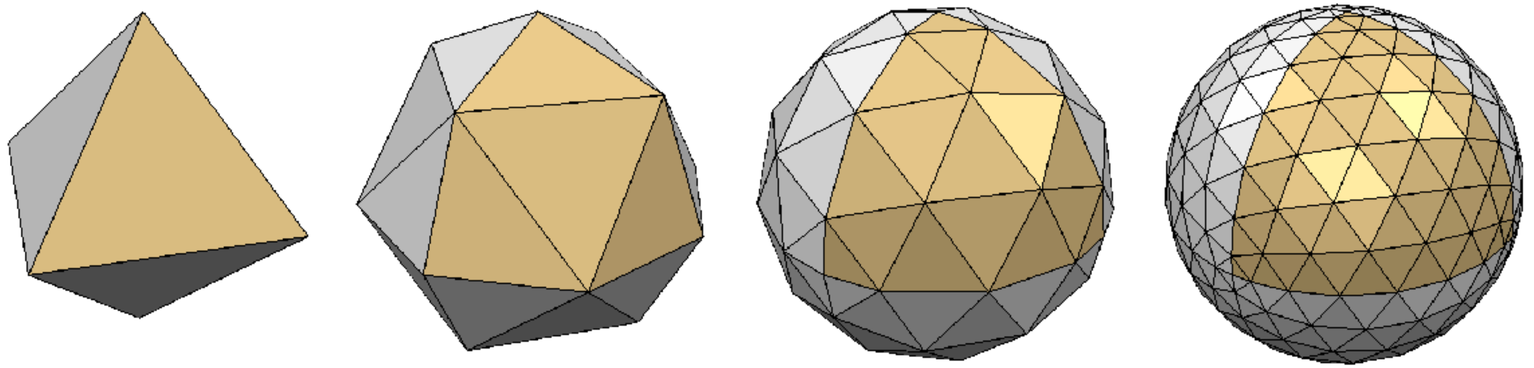
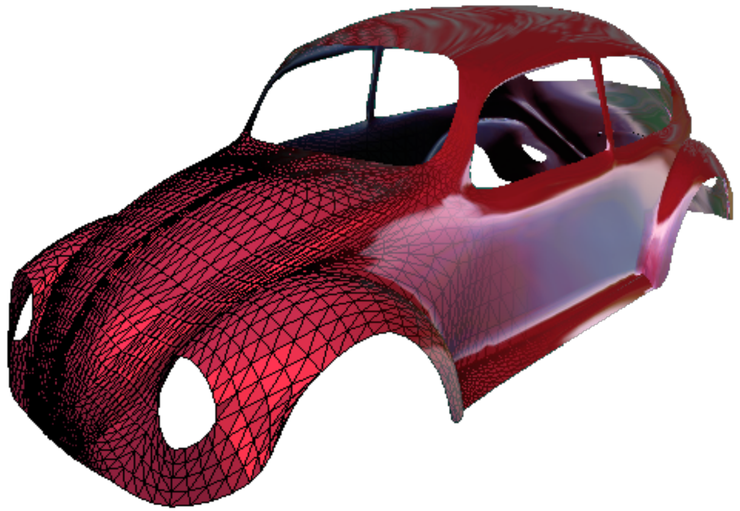
  - Error inverse proportional to #faces



Polygonal Meshes

- Polygonal meshes are a good compromise
  - Piecewise linear approximation → error is $O(h^2)$
  - Error inverse proportional to #faces
  - Arbitrary topology surfaces



Polygonal Meshes

- Polygonal meshes are a good compromise
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  - Error inverse proportional to #faces
  - Arbitrary topology surfaces
  - Piecewise smooth surfaces

Polygonal Meshes

- Polygonal meshes are a good compromise
  - Piecewise linear approximation → error is $O(h^2)$
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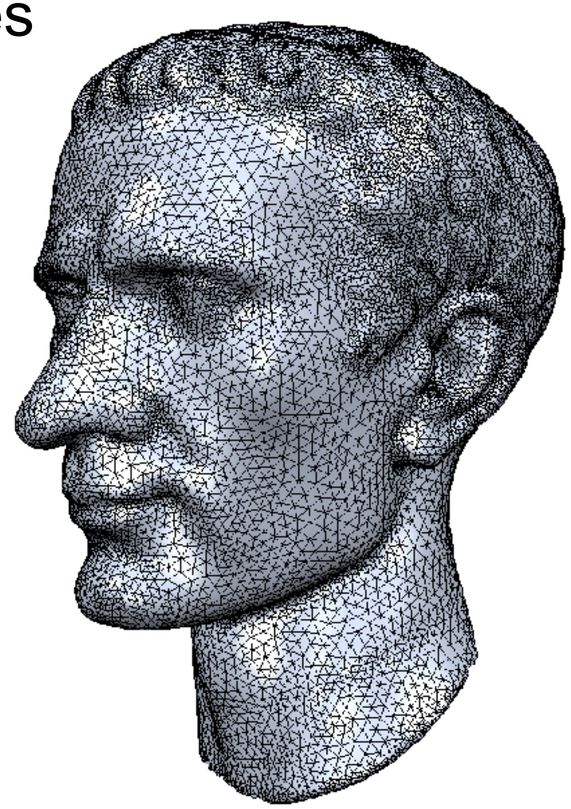
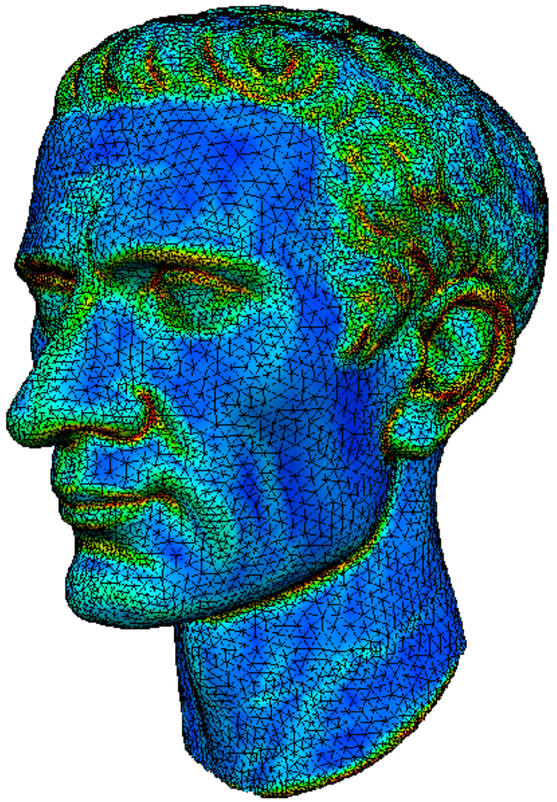
  - Arbitrary topology surfaces
  - Piecewise smooth surfaces
  - Curvature adaptive sampling



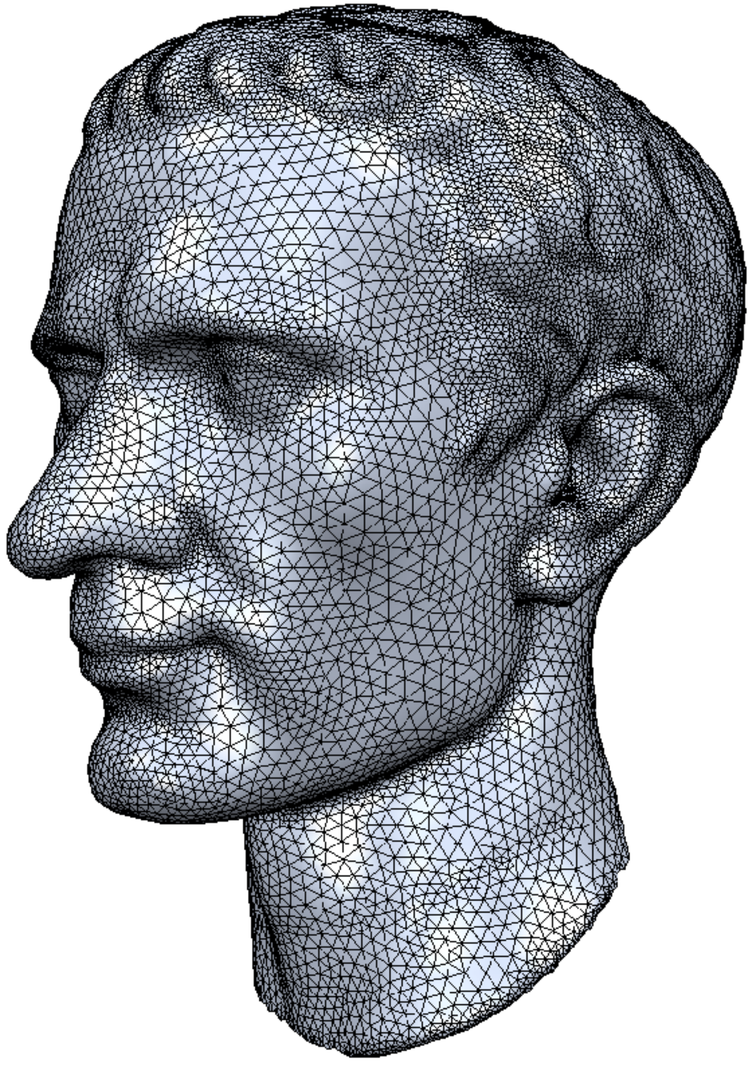


Polygonal Meshes

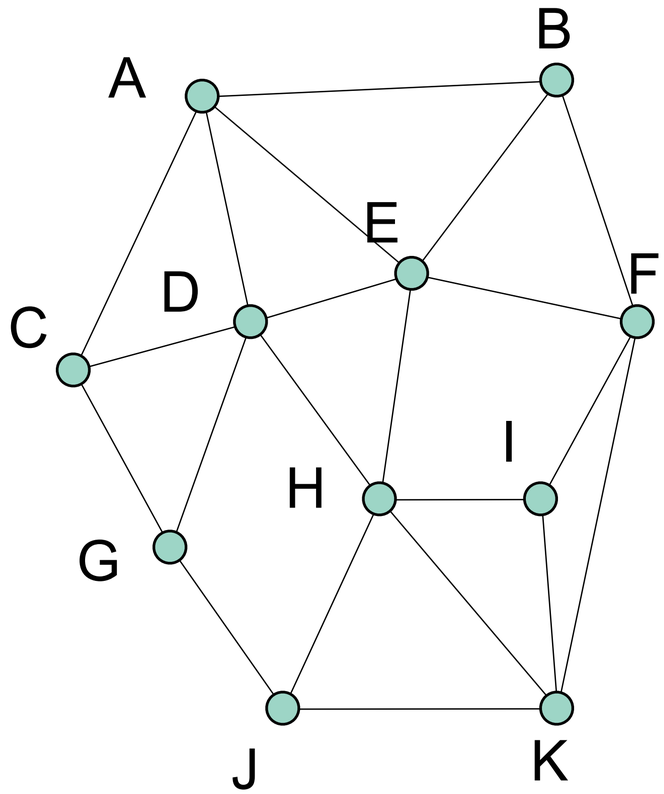
- Polygonal meshes are a good compromise
  - Piecewise linear approximation → error is $O(h^2)$
  - Error inverse proportional to #faces
  - Arbitrary topology surfaces
  - Piecewise smooth surfaces
  - Curvature adaptive sampling
  - Efficient GPU-based rendering

Outline

- Parametric Approximations
- **Polygon Meshes**
- Data structures

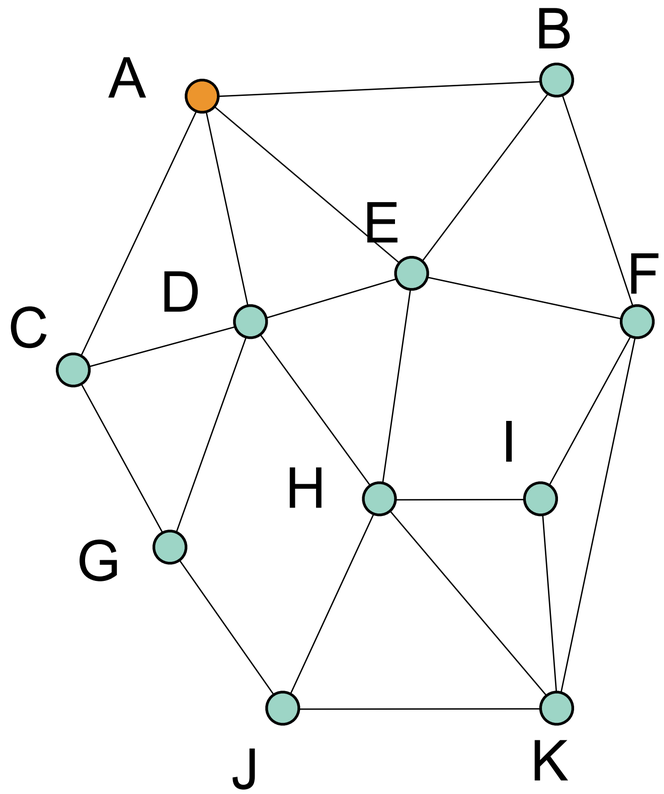


Graph Definitions



Graph $\{V, E\}$

Graph Definitions



Graph $\{V, E\}$

Vertices $V = \{A, B, C, \ldots, K\}$

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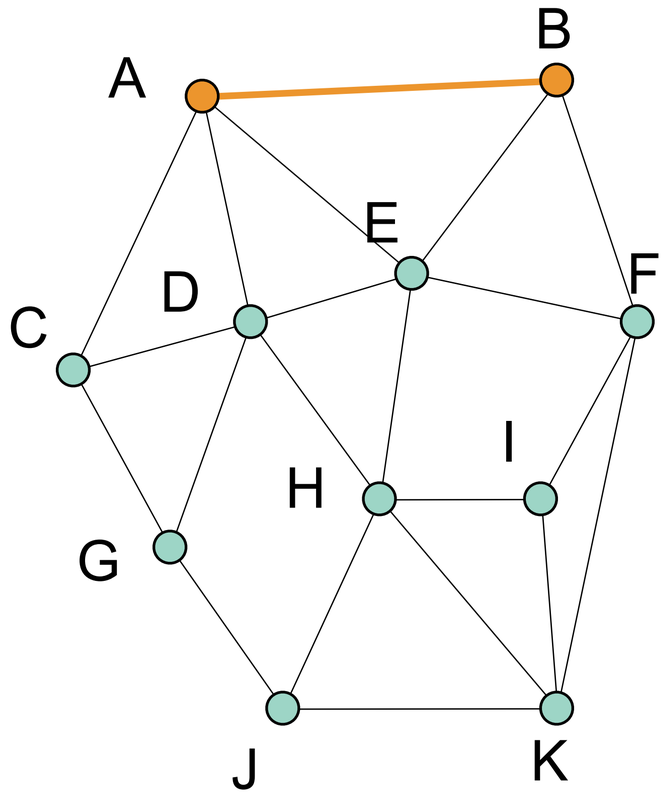

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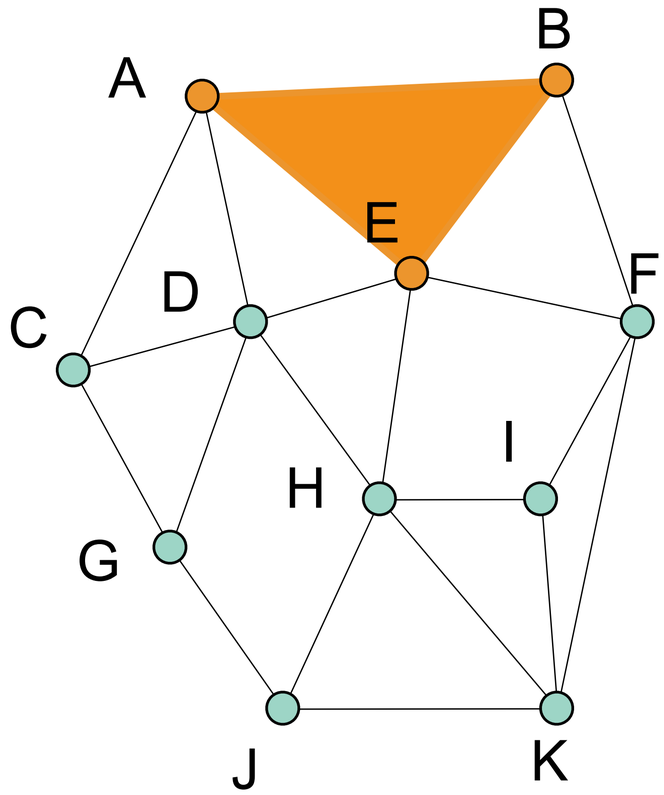


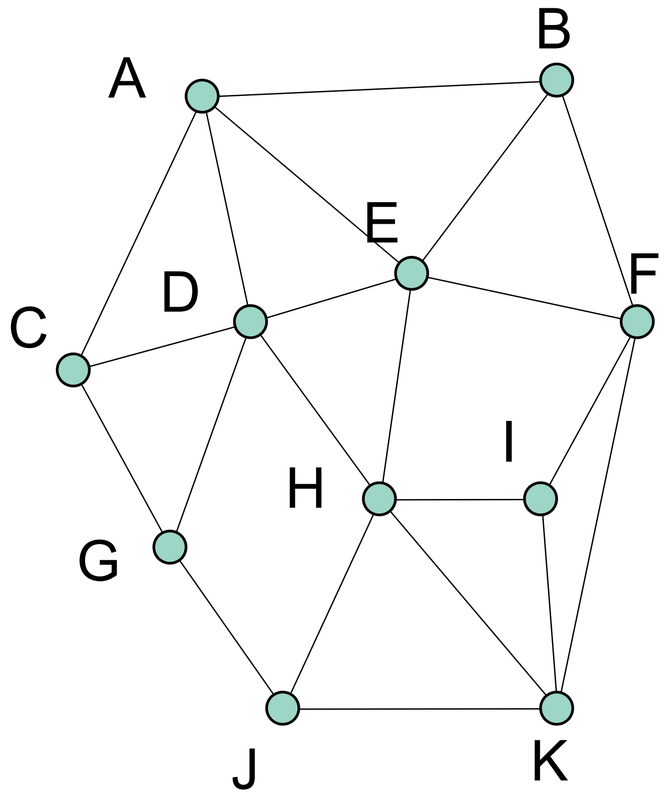
Graph $\{V, E\}$

Vertices $V = \{A, B, C, \ldots, K\}$

Edges $E = \{(AB), (AE), (CD), \ldots\}$

Faces $F = \{(ABE), (EBF), (EFIH), \ldots\}$

Graph Definitions



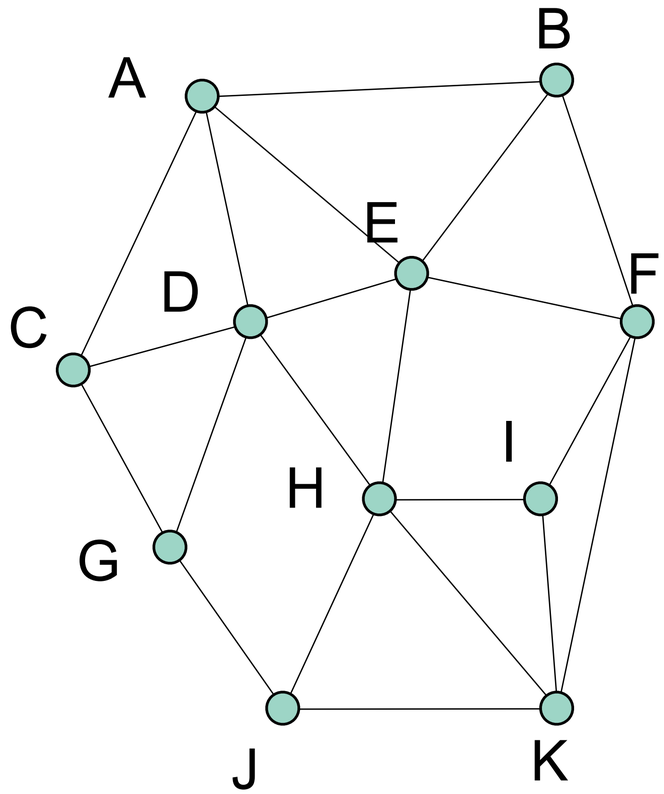
Vertex degree or **valence**:
number of incident edges.

deg(A) = 4

deg(E) = 5

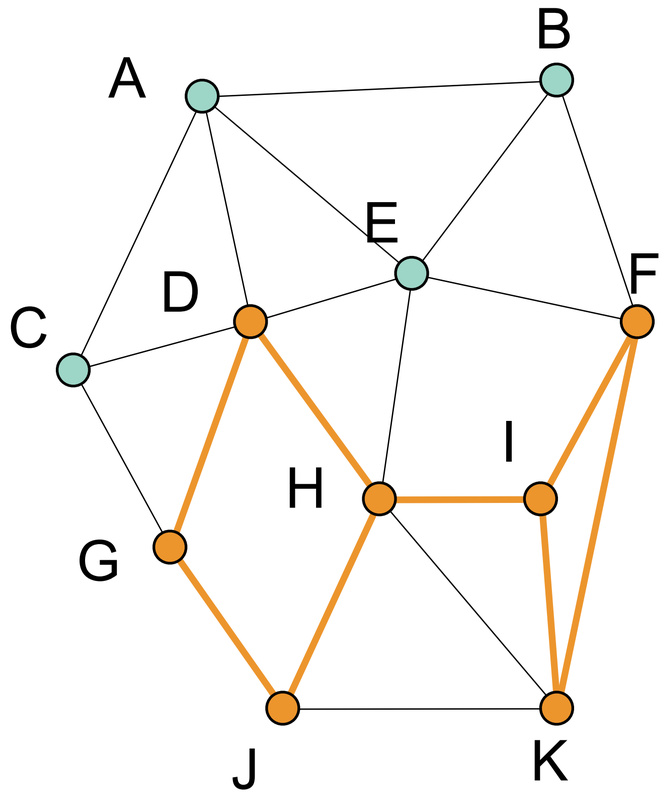
For triangle mesh, **average** valence is 6.

Connectivity



Connected:
Path of edges connecting every two vertices.

Connectivity



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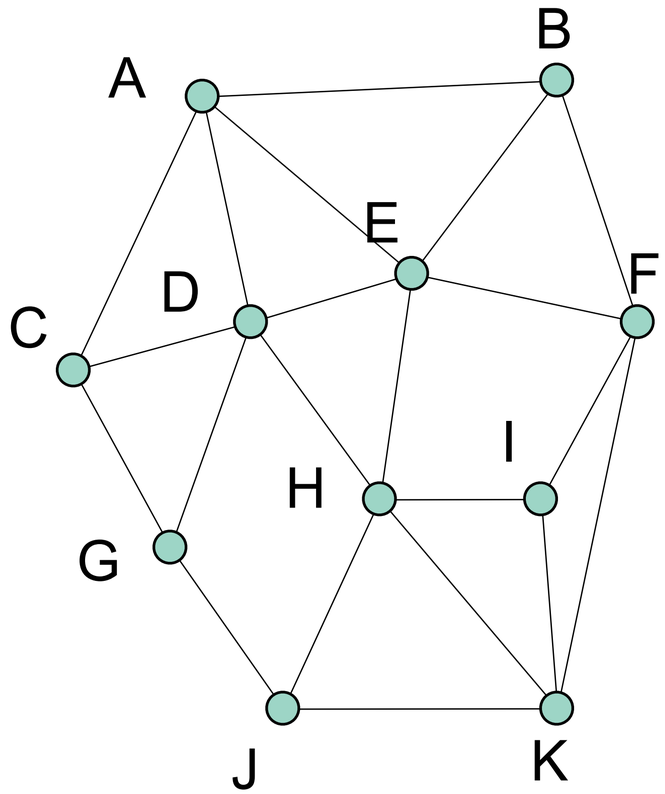
Connectivity



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Maximally connected subgraph.

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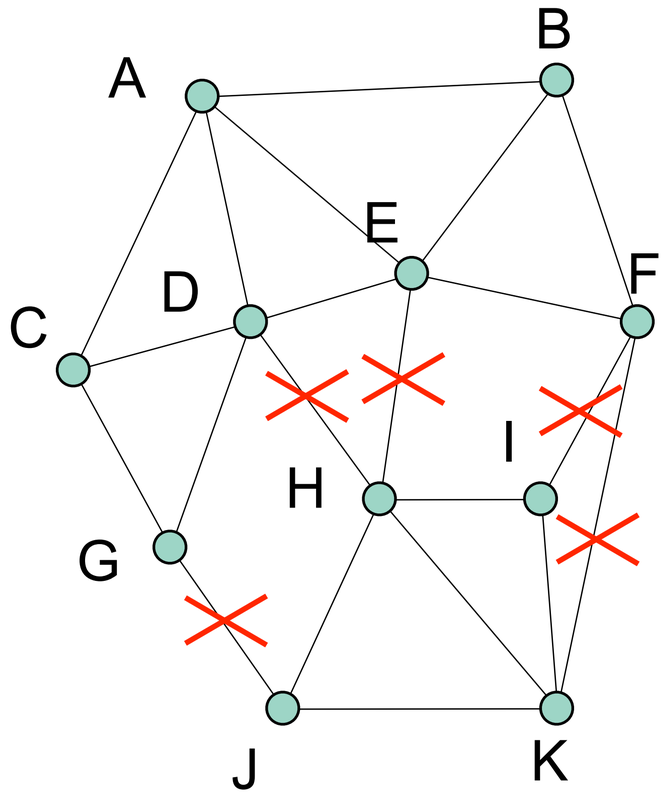

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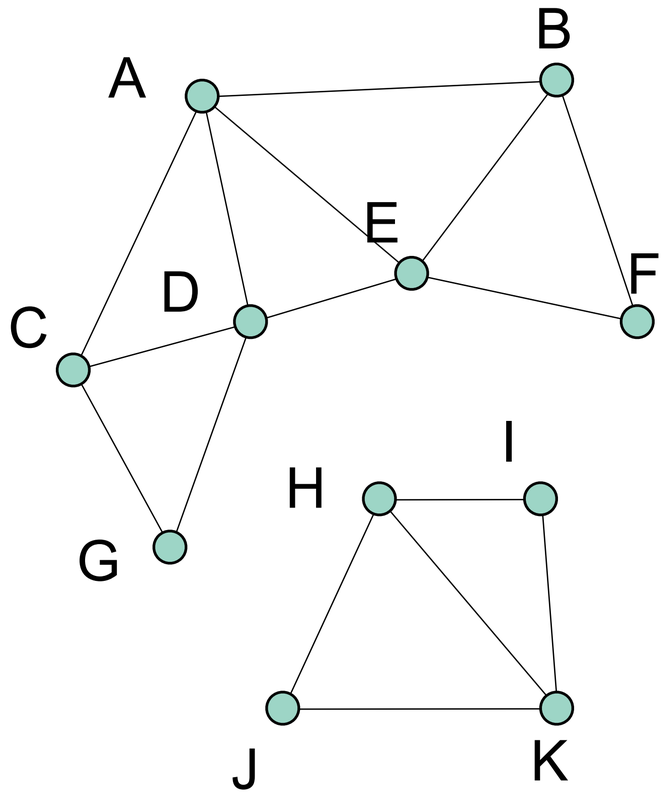

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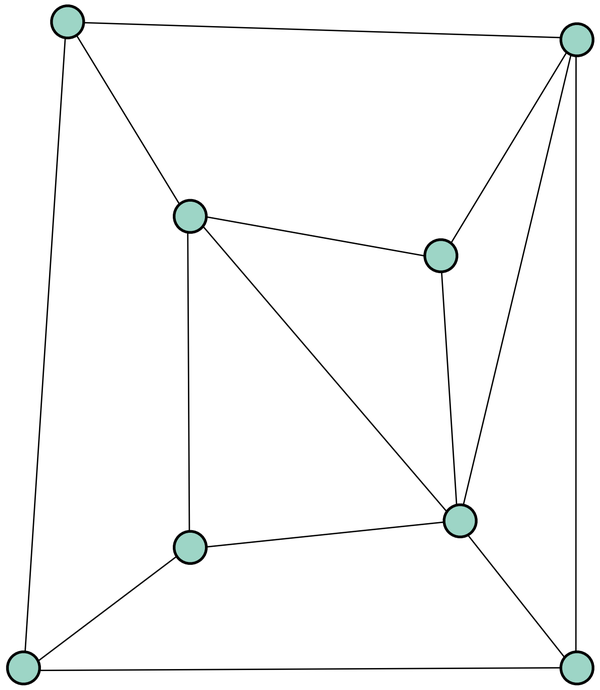
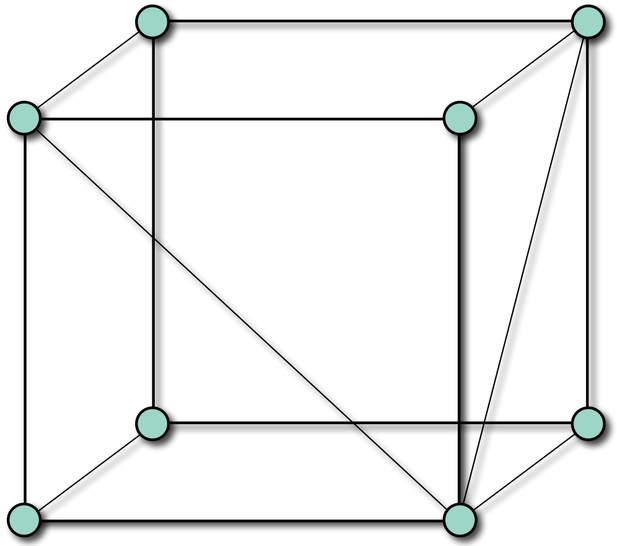
Graph Embedding

Embedding: Graph is embedded in $\mathbb{R}^d$, if each vertex is assigned a position in $\mathbb{R}^d$.



Embedded in $\mathbb{R}^2$



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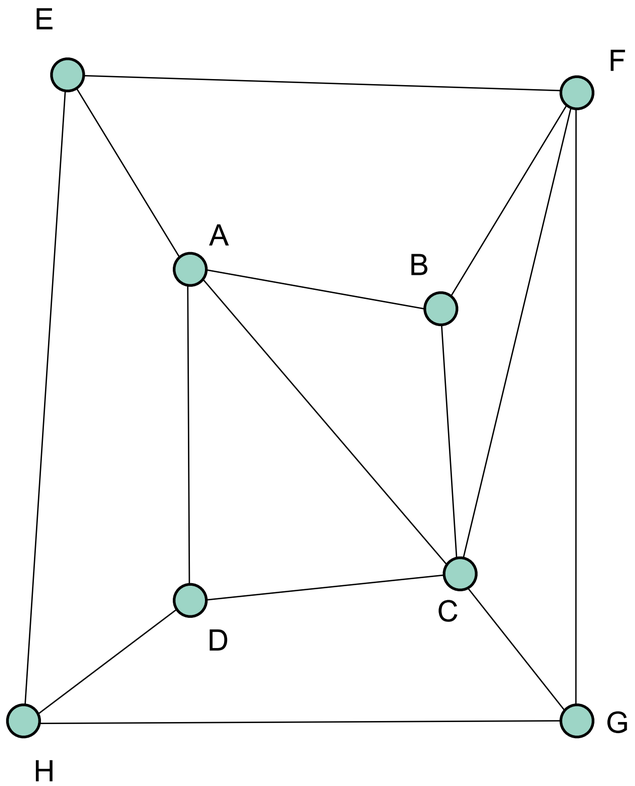

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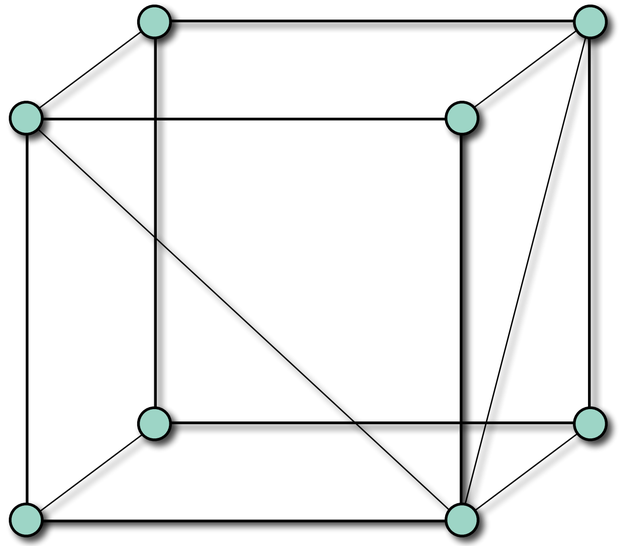

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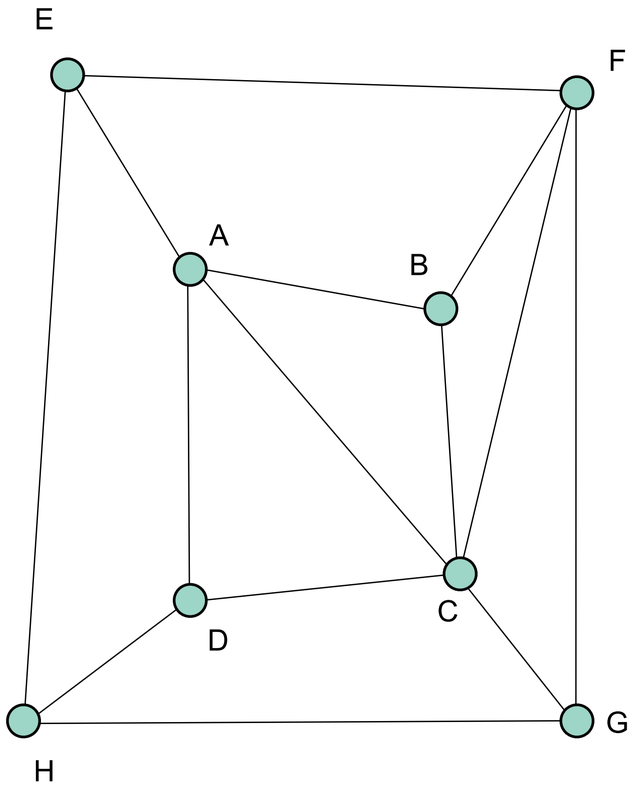

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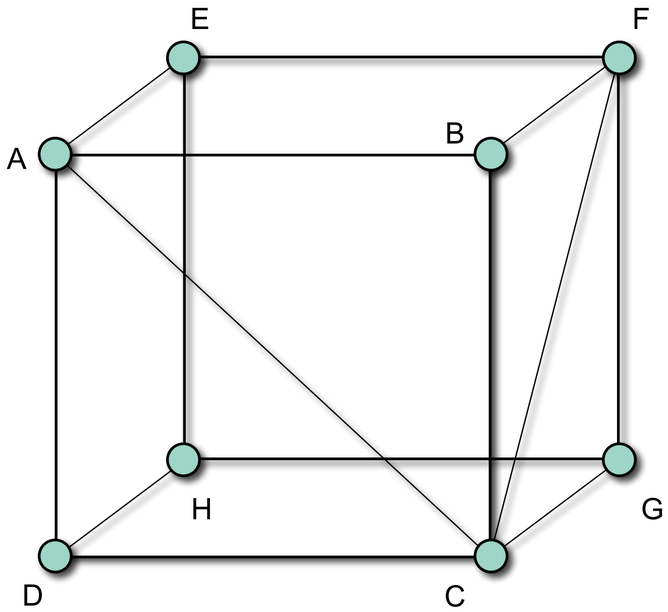

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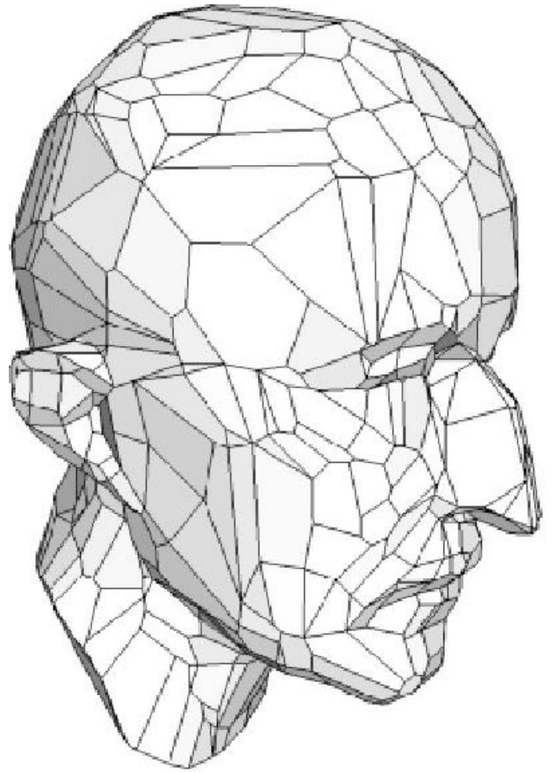
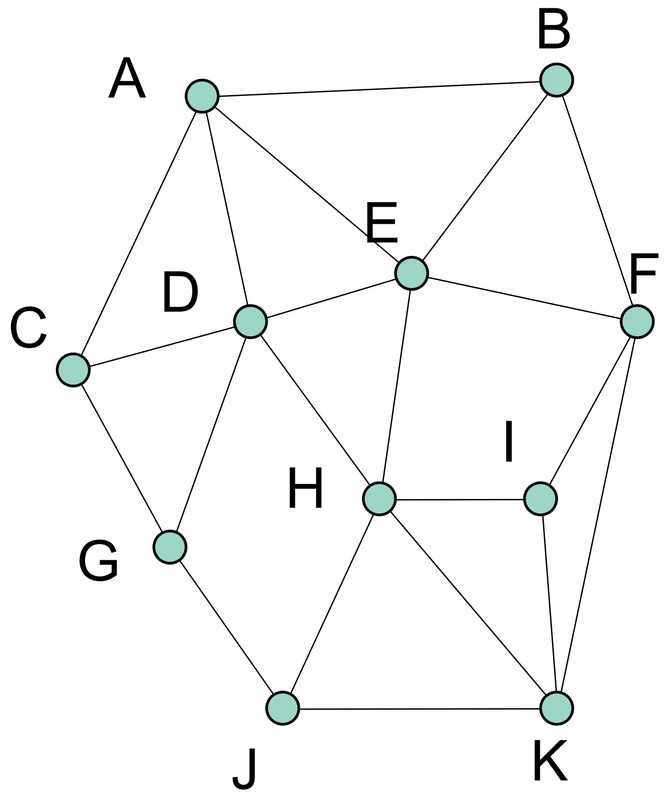


Embedded in $\mathbb{R}^3$

Triangulation



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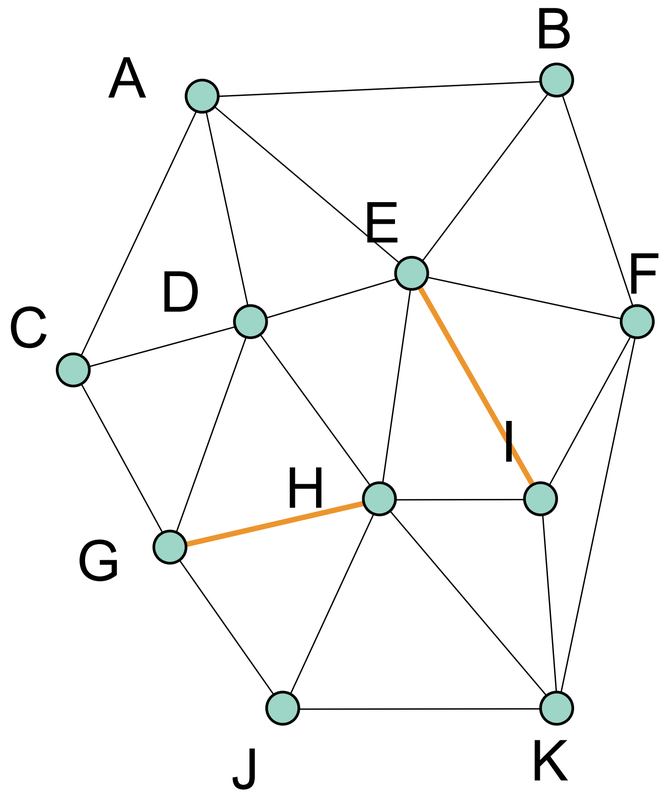

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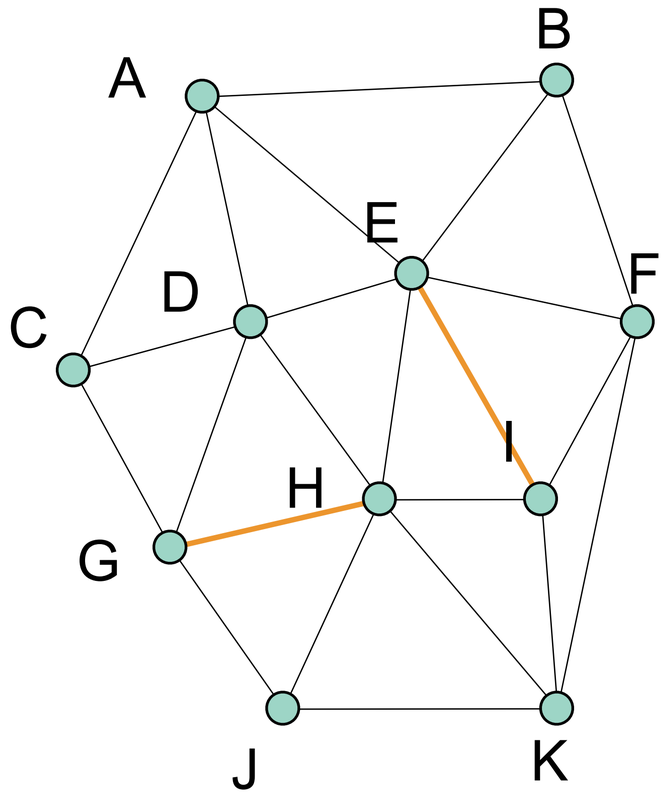
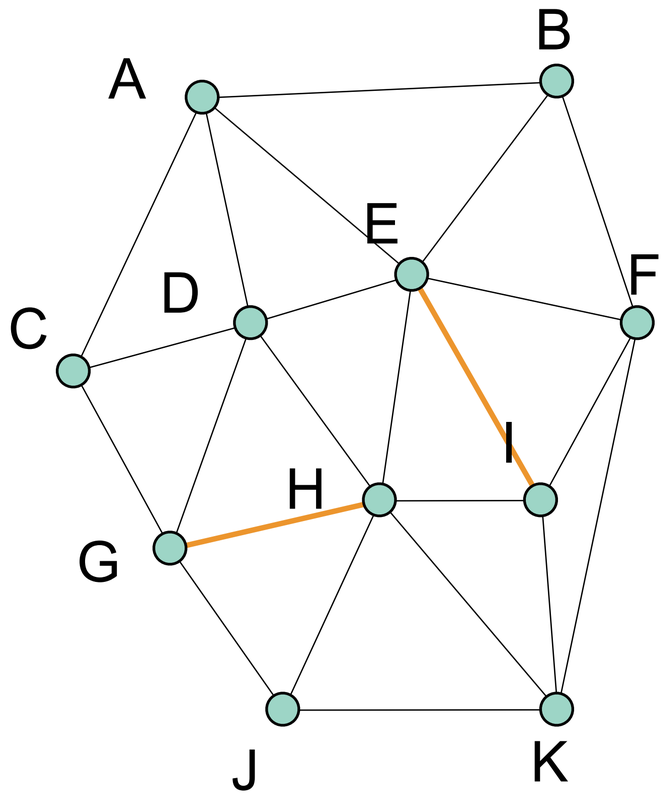


Triangulation: Graph where every face is a triangle.

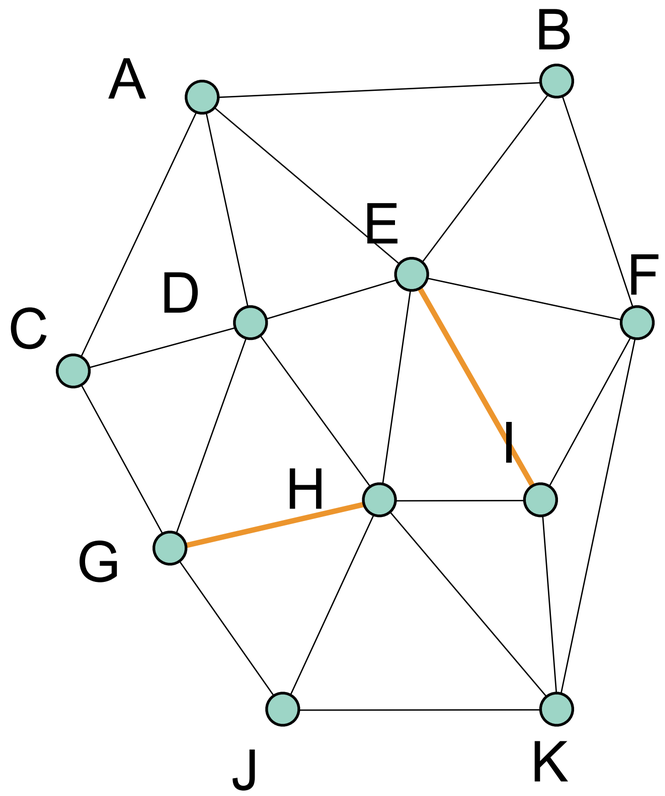
Triangulation



Triangulation: Graph where every face is a triangle.

Why...?

Triangulation

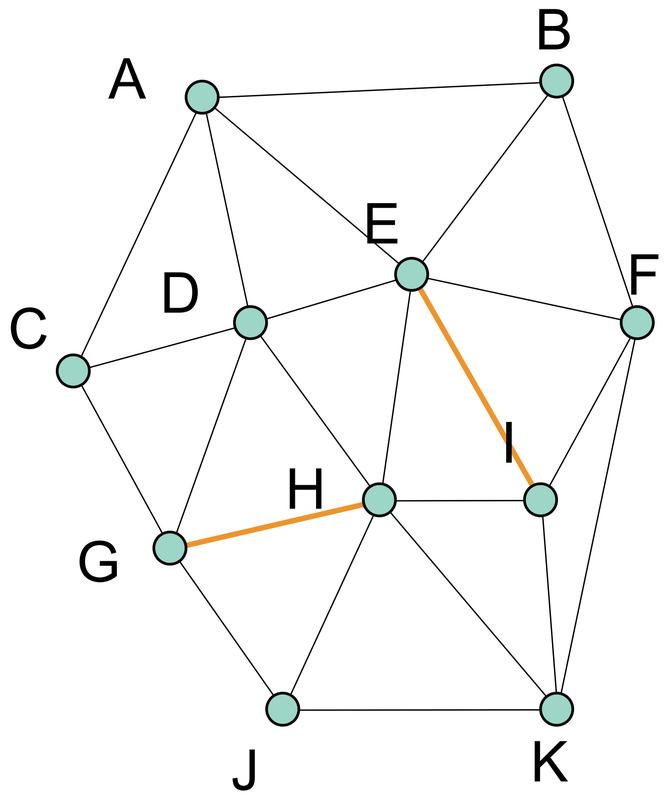


Triangulation: Graph where every face is a triangle.

Why...?

➡ simplifies data structures

Triangulation



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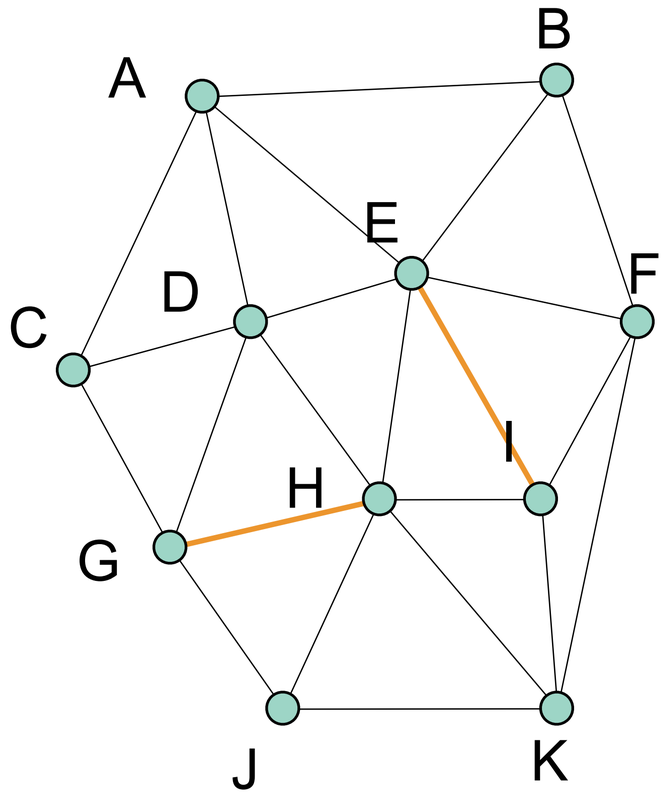
Triangulation



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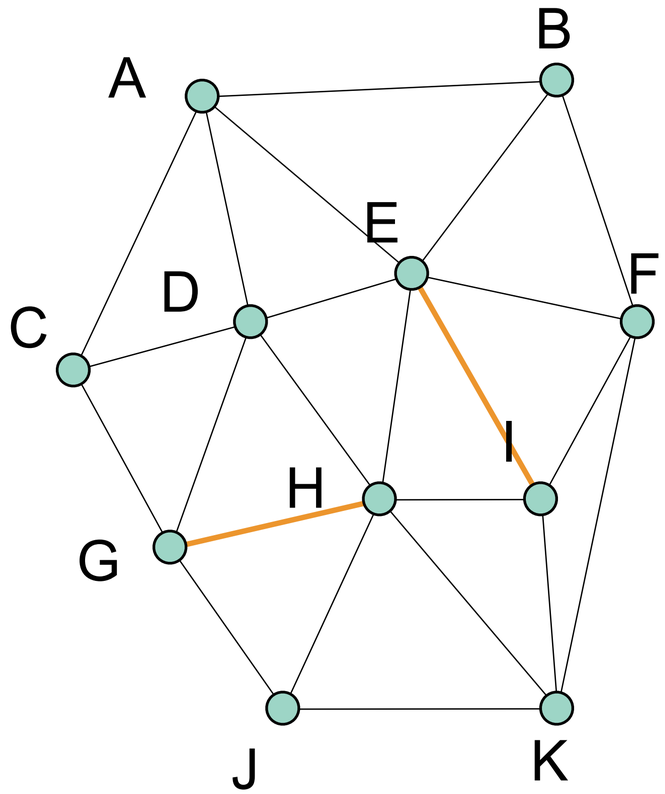
Triangulation



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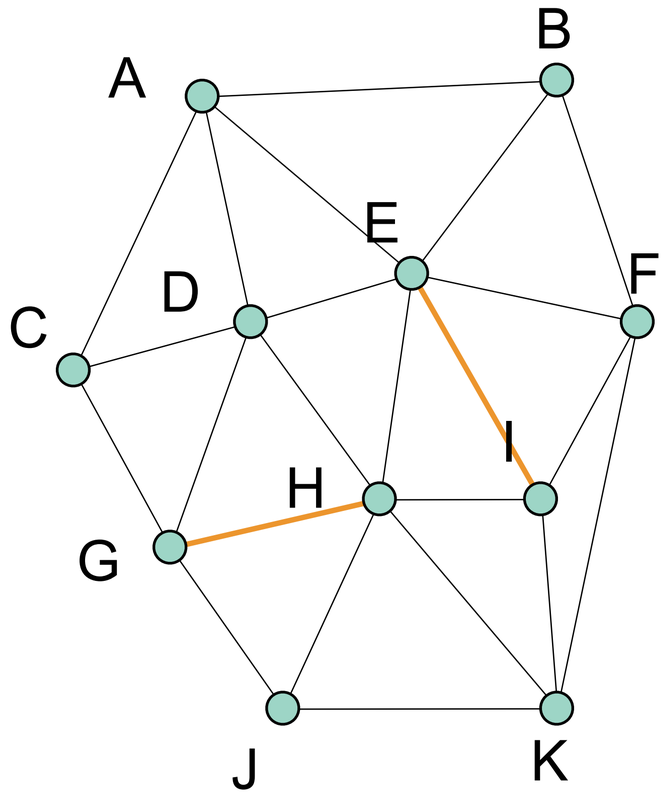

Triangulation: Graph where every face is a triangle.

Why...?

- ➡ simplifies data structures
- ➡ simplifies rendering
- ➡ simplifies algorithms
- ➡ by definition, triangle is **planar**
- ➡ any polygon can be triangulated (not necessarily uniquely)

Outline

- Parametric Approximations
- Polygon Meshes
- **Data structures**

Mesh Data Structures

- How to store **geometry** & **connectivity (topology)**?
- Compact storage
  - File formats
- Efficient algorithms on meshes
  - Identify time-critical operations
  - All vertices/edges of a face
  - All **incident/neighboring** vertices/edges/faces of a vertex

Data Structures

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  - **Geometry**: 3D coordinates

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    - e.g., normal, color, texture coordinate
    - Per vertex, per face, per edge
  - **Connectivity (topology)**
    - What is adjacent to what

Data Structures

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    - Which faces are adjacent to face #7?
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    - Remove/add a vertex/face
    - Vertex split, edge collapse

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  - Time to perform an operation
  - Space complexity
  - Redundancy
  - Time to update changes in geometry and/or topology
  - *Is this differentiable?*

Face Set (STL)

- `Face list`:
  - 3 positions

Triangles		
x_{11} y_{11} z_{11}	x_{12} y_{12} z_{12}	x_{13} y_{13} z_{13}
x_{21} y_{21} z_{21}	x_{22} y_{22} z_{22}	x_{23} y_{23} z_{23}
...	...	...
x_{F1} y_{F1} z_{F1}	x_{F2} y_{F2} z_{F2}	x_{F3} y_{F3} z_{F3}

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x_{F1} y_{F1} z_{F1}	x_{F2} y_{F2} z_{F2}	x_{F3} y_{F3} z_{F3}

36 B/f = 72 B/v

no connectivity!

Shared Vertex (OBJ, OFF)

- `Indexed Face List`
  - Vertex: position
  - Face: vertex indices

Vertices
x_1 y_1 z_1
...
x_V y_V z_V

Triangles
i_{11} i_{12} i_{13}
...
...
...
...
i_{F1} i_{F2} i_{F3}

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- `Indexed Face List`
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Vertices
$x_1\ y_1\ z_1$
...
$x_V\ y_V\ z_V$

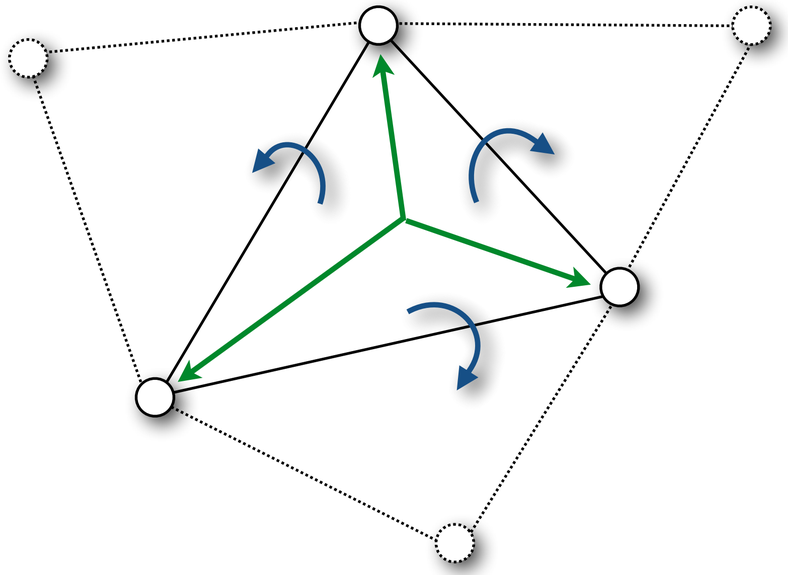
Triangles
$i_{11}\ i_{12}\ i_{13}$
...
...
...
...
$i_{F1}\ i_{F2}\ i_{F3}$

12 B/v + 12 B/f = 36 B/v

no explicit neighborhood information

Face-Based Connectivity

- Vertex:
  - position
  - 1 face
- Face:
  - 3 vertices
  - 3 face neighbors



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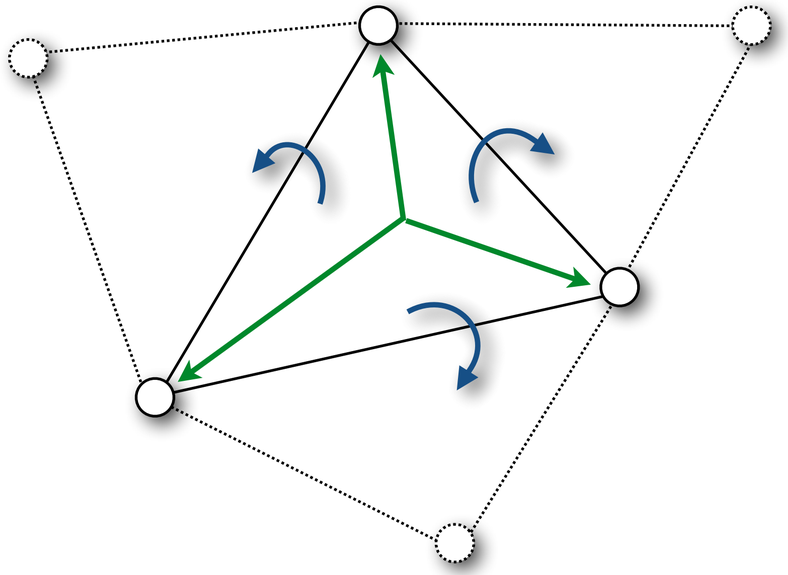
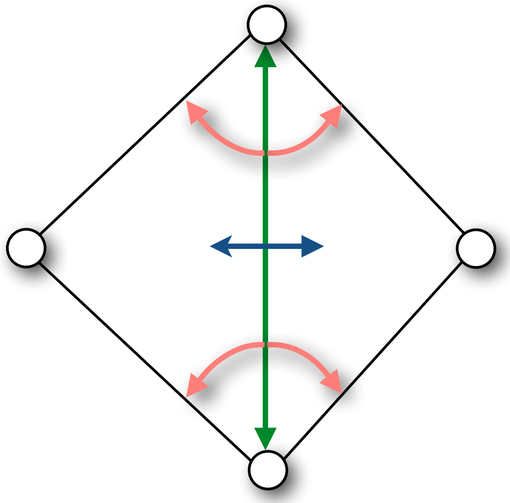


64 B/v

no edges!

Edge-Based Connectivity

- Vertex
  - position
  - 1 edge
- Edge
  - 2 vertices
  - 2 faces
  - 4 edges
- Face
  - 1 edge



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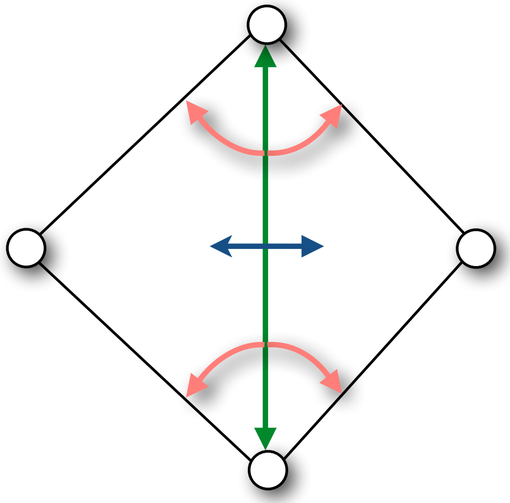
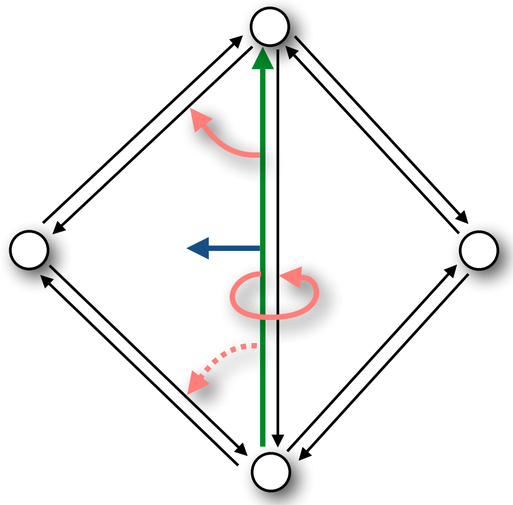


120 B/v

edge orientation?

Halfedge-Based Connectivity

- Vertex
  - position
  - 1 halfedge
- Halfedge
  - 1 vertex
  - 1 face
  - 1, 2, or 3 halfedges
- Face
  - 1 halfedge



Halfedge-Based Connectivity

- Vertex
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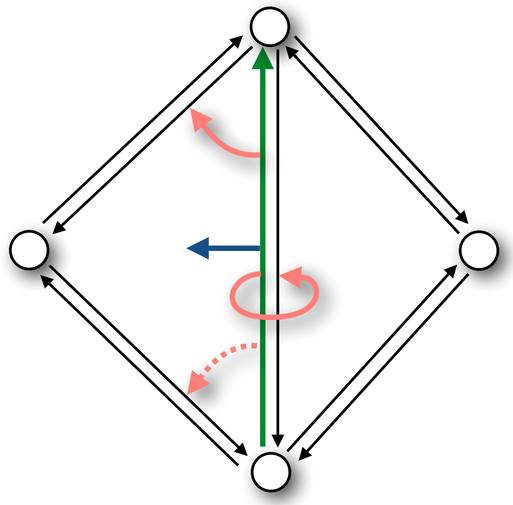
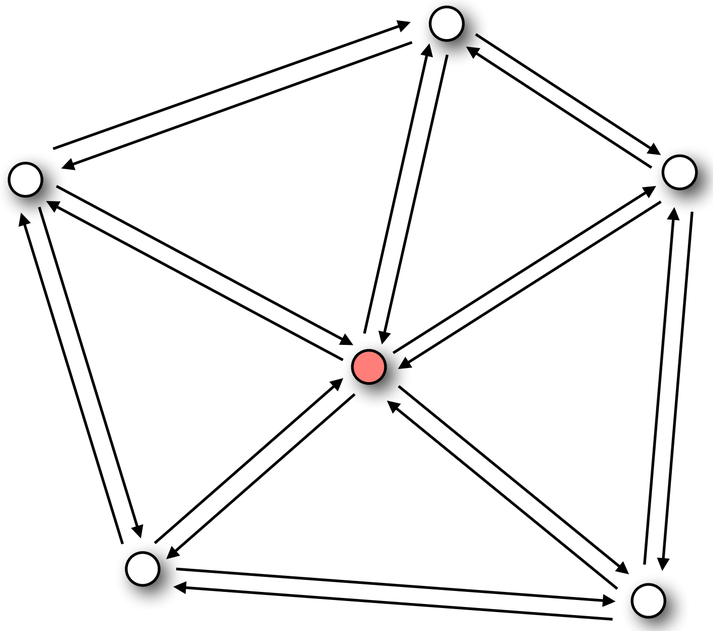


96 to 144 B/v

no case distinctions
during traversal

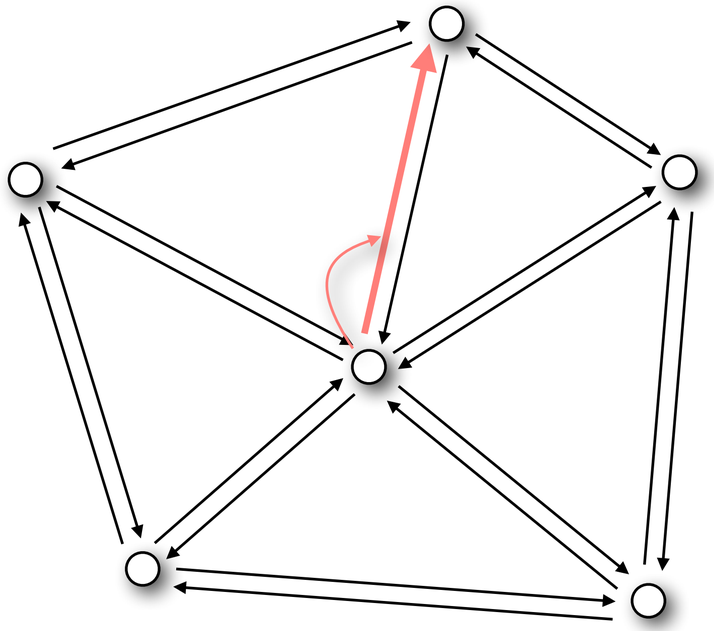
One-Ring Traversal

1. Start at vertex



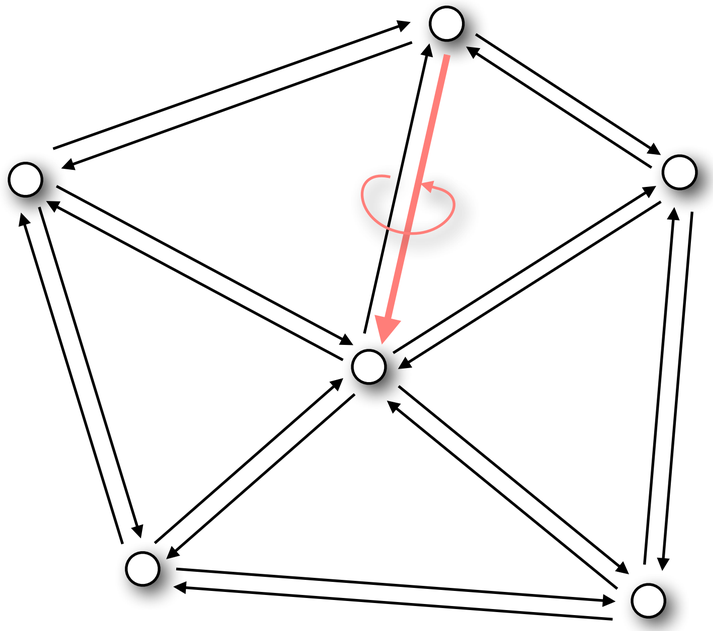
One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge



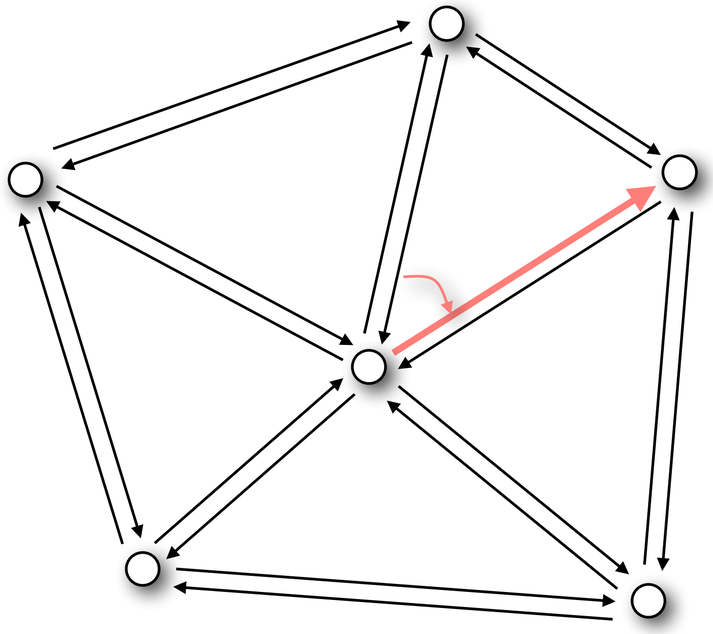
One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge



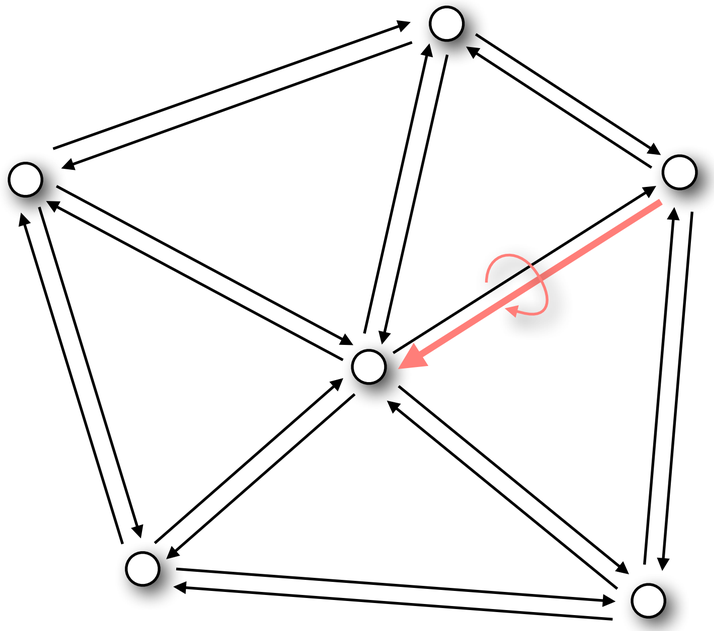
One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge
4. Next halfedge



One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge
4. Next halfedge
5. Opposite



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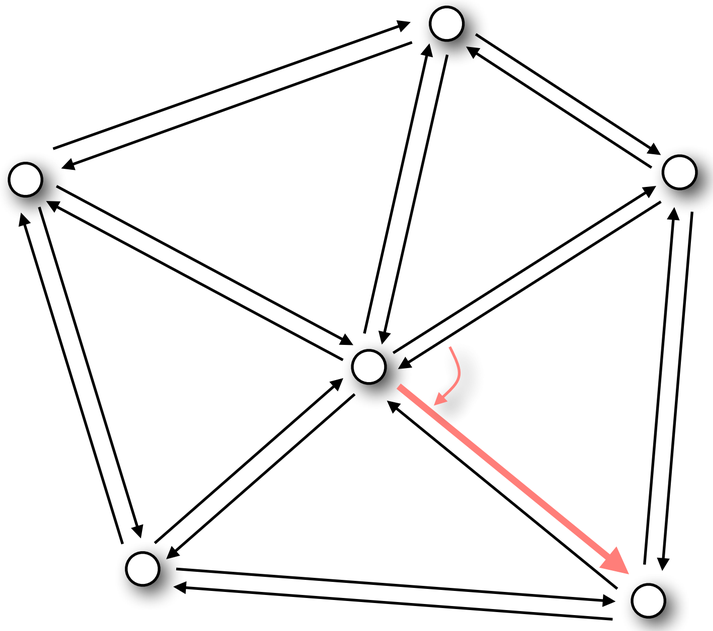
3. Opposite halfedge
4. Next halfedge
5. Opposite
6. Next
7. ...

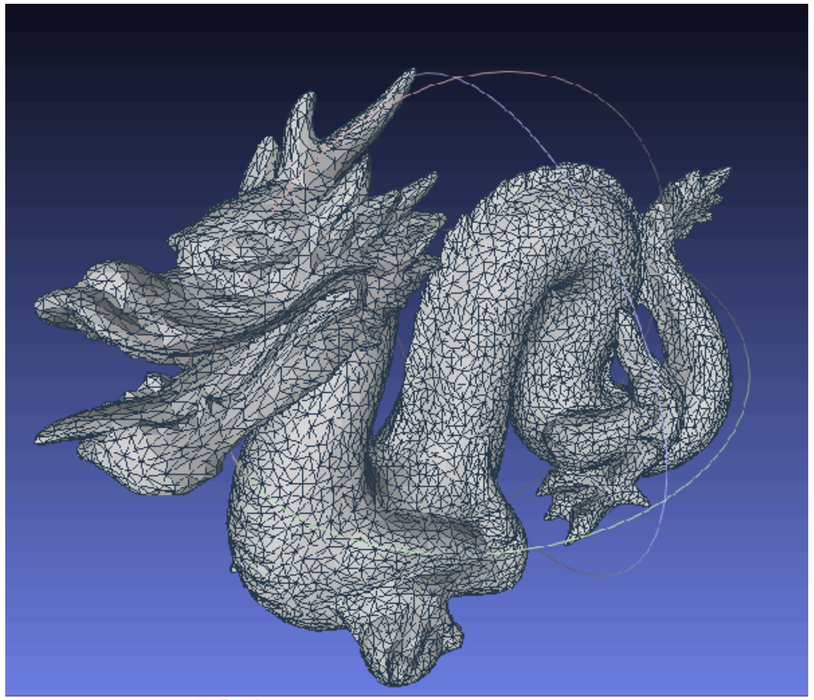


Halfedge-Based Libraries

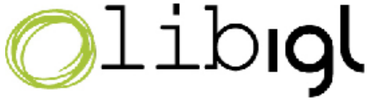
- CGAL
  - www.cgal.org
  - Computational geometry
  - Free for non-commercial use

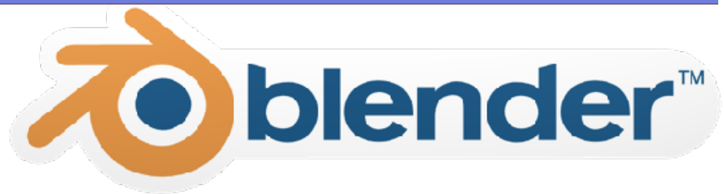
- OpenMesh
  - www.openmesh.org
  - Mesh processing
  - Free, LGPL licence

Looking at Meshes



OpenMesh





Literature

- Book #1: Chapters 2,3;
- Book #2: Chapters 1,2
- Kettner, *Using generic programming for designing a data structure for polyhedral surfaces*, Symp. on Comp. Geom., 1998
- Campagna et al., *Directed Edges - A Scalable Representation for Triangle Meshes*, Journal of Graphics Tools 4(3), 1998
- Botsch et al., *OpenMesh - A generic and efficient polygon mesh data structure*, OpenSG Symp. 2002