COMP0130: ROBOT VISION AND NAVIGATION
Workshop 2: Aircraft Navigation using GNSS and Kalman Filtering
ANSWERS

Task 0: Theoretical Design Problem

a) $\mathbf{\Phi} = \begin{pmatrix} 1 & 0 & \tau_s & 0 & 0.5\tau_s^2 & 0 \\ 0 & 1 & 0 & \tau_s & 0 & 0.5\tau_s^2 \\ 0 & 0 & 1 & 0 & \tau_s & 0 \\ 0 & 0 & 0 & 1 & 0 & \tau_s \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$;

b) $\Delta\mathbf{r} = \begin{pmatrix} 1 & 0 & 0.5 & 0 & 0.5\times0.5^2 & 0 \\ 0 & 1 & 0 & 0.5 & 0 & 0.5\times0.5^2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$ m;

c) $\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$.

Task 1A: Basic Kalman Filter First Epoch

d) The propagated state estimates ($\hat{\mathbf{x}}_1^-$) are:

State	Meaning	Value	Unit
1	*x* position	2447203.0	m
2	*y* position	–5884122.0	m
3	*z* position	–284783.0	m
4	*x* velocity	184.0	m/s
5	*y* velocity	77.0	m/s
6	*z* velocity	0.0	m/s

e) The propagated error covariance matrix ($\mathbf{P}_1^-$) is:

126.6667	0	0	27.5000	0	0
0	126.6667	0	0	27.5000	0
0	0	126.6667	0	0	27.5000
27.5000	0	0	30.0000	0	0
0	27.5000	0	0	30.0000	0
0	0	27.5000	0	0	30.0000

i) The Kalman gain matrix ($\mathbf{K}_1$) is

0.953	0.000	0.000
0.000	0.953	0.000
0.000	0.000	0.953
0.207	0.000	0.000
0.000	0.207	0.000
0.000	0.000	0.207

j) The measurement innovation vector ($\delta\mathbf{z}_1^-$) is

Component	Value
x	1.038
y	−0.093
z	−0.3375

k) The updated state estimates ($\hat{\mathbf{x}}_1^+$) are.

State	Meaning	Value	Unit
1	*x* position	2447203.99	m
2	*y* position	−5884122.09	m
3	*z* position	−284783.32	m
4	*x* velocity	184.21	m/s
5	*y* velocity	76.98	m/s
6	*z* velocity	−0.07	m/s

l) The propagated error covariance matrix ($\mathbf{P}_1^-$) is:

5.9561	0	0	1.2931	0	0
0	5.9561	0	0	1.2931	0
0	0	5.9561	0	0	1.2931
1.2931	0	0	24.3103	0	0
0	1.2931	0	0	24.3103	0
0	0	1.2931	0	0	24.3103

Note that a large number of off-diagonal elements are zero because the initial uncertainty, system noise and measurement noise are all assumed to be the same for the *x*, *y* and *z* directions. Consequently, no correlations build up between the *x*, *y* and *z* direction state estimates. Thus, this particular Kalman filter effectively behaves as three separate parallel Kalman filters each operating in one spatial dimension.

Please see Task 1B below for the position and velocity solution.

Task 1B: Basic Kalman Filter Multiple Epochs

The position and velocity solution is as follows:

	Position			Velocity (m/s)		
Time (s)	Latitude (°)	Longitude (°)	Height (m)	North	East	Down
0	-2.575939	-67.417594	996.8	-0.08	199.65	0.33
1	-2.575945	-67.415797	999.7	-0.56	199.85	-2.30
2	-2.575960	-67.413991	998.5	-1.33	200.45	0.02
3	-2.575956	-67.412208	994.5	-0.20	199.17	2.44
4	-2.575955	-67.410410	994.3	-0.03	199.70	1.07
5	-2.575956	-67.408600	999.2	-0.09	200.71	-2.64
6	-2.575945	-67.406806	999.2	0.77	199.96	-1.04
7	-2.575942	-67.405025	996.5	0.43	198.78	1.27
8	-2.575950	-67.403215	996.5	-0.37	200.34	0.52
9	-2.575957	-67.401409	999.9	-0.58	200.72	-1.92
10	-2.575956	-67.399604	1002.5	-0.20	200.71	-2.39
20	-2.575942	-67.381650	997.6	-0.06	198.29	3.12
30	-2.577013	-67.363688	998.9	-20.97	198.66	1.77
40	-2.580300	-67.346023	995.1	-45.87	193.96	1.88
50	-2.585732	-67.328901	992.7	-69.23	186.40	3.73
60	-2.593211	-67.312531	999.5	-90.70	177.53	-1.12
70	-2.602677	-67.297204	996.3	-112.35	165.47	2.21
80	-2.613945	-67.283118	1002.4	-132.21	151.46	-2.64
90	-2.626613	-67.270299	998.9	-141.72	141.18	1.06
100	-2.639393	-67.257596	998.6	-141.71	140.85	-1.62
110	-2.652182	-67.244875	995.8	-141.58	141.85	-0.34
120	-2.664722	-67.231941	996.4	-133.27	148.95	-0.40
130	-2.675737	-67.217696	997.1	-114.18	163.25	1.42
140	-2.684913	-67.202195	1000.1	-92.45	177.85	0.26
150	-2.692138	-67.185722	998.9	-71.02	186.84	-0.26
160	-2.697257	-67.168473	996.7	-47.43	195.33	-1.49
170	-2.700217	-67.150749	998.7	-22.77	198.63	-0.31
180	-2.701025	-67.132779	1002.5	-1.04	201.26	-1.57

Task 2A: GNSS Kalman Filter First Epoch

d) The propagated state estimates ($\hat{\mathbf{x}}_1^-$) are:

State	Meaning	Value	Unit
1	*x* position	2447207.90	m
2	*y* position	−5884119.00	m
3	*z* position	−284784.40	m
4	*x* velocity	184.50	m/s
5	*y* velocity	76.90	m/s
6	*z* velocity	−0.10	m/s
7	clock offset	10001.00	m
8	clock drift	99.90	m/s

e) The propagated error covariance matrix ($\mathbf{P}_1^-$) is:

101.6767	0	0	2.5100	0	0	0	0
0	101.6767	0	0	2.5100	0	0	0
0	0	101.6767	0	0	2.5100	0	0
2.5100	0	0	5.0100	0	0	0	0
0	2.5100	0	0	5.0100	0	0	0
0	0	2.5100	0	0	5.0100	0	0
0	0	0	0	0	0	100.0333	0.0300
0	0	0	0	0	0	0.0300	0.0500

k) The Kalman gain matrix ($\mathbf{K}_1$) is

Columns 1 to 10:

0.142	-0.116	0.114	-0.061	-0.173	0.142	-0.120	0.113	-0.064	-0.175
0.084	-0.028	-0.053	0.225	-0.109	0.087	-0.027	-0.051	0.229	-0.109
-0.122	-0.146	-0.192	-0.091	-0.040	0.121	0.146	0.195	0.088	0.039
0.000	-0.000	0.000	-0.000	0.000	0.000	-0.000	0.000	-0.000	0.000
-0.000	0.000	-0.000	0.000	-0.000	-0.000	0.000	-0.000	0.000	-0.000
-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000	0.000	0.000
0.100	0.066	0.144	-0.014	0.080	0.101	0.066	0.146	-0.015	0.081
0.000	-0.000	0.000	-0.000	0.000	0.000	-0.000	0.000	-0.000	0.000

Columns 11 to 20:

0.106	-0.032	0.221	-0.241	-0.003	0.106	-0.035	0.223	-0.246	-0.004
-0.091	0.012	-0.262	0.326	-0.048	-0.090	0.015	-0.264	0.332	-0.048
-0.016	-0.019	-0.024	-0.014	-0.005	0.017	0.019	0.028	0.010	0.006
0.342	-0.181	0.530	-0.522	-0.186	0.340	-0.191	0.533	-0.534	-0.189
-0.082	-0.001	-0.544	0.848	-0.198	-0.077	0.006	-0.547	0.864	-0.197
-0.153	-0.185	-0.240	-0.119	-0.050	0.154	0.185	0.249	0.108	0.050
0.080	-0.012	0.193	-0.220	0.024	0.080	-0.014	0.196	-0.225	0.024
0.245	0.039	0.499	-0.428	0.121	0.245	0.036	0.506	-0.437	0.120

l) The measurement innovation vector ($\delta\mathbf{z}_1^-$) is

Satellite	Rows 1 to 10, Pseudo-range (m)	Rows 11 to 20, Pseudo-range rate (m/s)
4	–7.89	0.052
5	1.17	–0.131
9	–8.28	–0.032
14	–4.04	–0.024
15	0.77	–0.024
19	–5.85	0.111
20	–1.67	0.060
24	–1.29	0.133
29	–7.22	0.042
30	–0.93	–0.005

m) The updated state estimates ($\hat{\mathbf{x}}_1^+$) are.

State	Meaning	Value	Unit
1	*x* position	2447205.71	m
2	*y* position	−5884122.25	m
3	*z* position	−284783.54	m
4	*x* velocity	184.62	m/s
5	*y* velocity	76.85	m/s
6	*z* velocity	−0.01	m/s
7	clock offset	9998.39	m
8	clock drift	99.98	m/s

n) The propagated error covariance matrix ($\mathbf{P}_1^-$) is:

40.4771	-20.0719	0.0752	0.0015	-0.0018	0.0000	19.8331	0.0013
-20.0719	49.4142	-0.1788	-0.0018	0.0023	-0.0000	-24.9340	-0.0016
0.0752	-0.1788	21.0216	0.0000	-0.0000	0.0001	0.2416	0.0000
0.0015	-0.0018	0.0000	0.0039	-0.0039	0.0000	0.0013	0.0029
-0.0018	0.0023	-0.0000	-0.0039	0.0057	-0.0000	-0.0016	-0.0036
0.0000	-0.0000	0.0001	0.0000	-0.0000	0.0007	0.0000	0.0000
19.8331	-24.9340	0.2416	0.0013	-0.0016	0.0000	24.5875	0.0011
0.0013	-0.0016	0.0000	0.0029	-0.0036	0.0000	0.0011	0.0027

Please see Task 2B below for the position and velocity solution.

Task 2B: GNSS Kalman Filter Multiple Epochs

The position and velocity solution is as follows:

	Position			Velocity (m/s)		
Time (s)	Latitude (°)	Longitude (°)	Height (m)	North	East	Down
0	-2.575940	-67.417580	997.6	-0.01	199.97	0.07
1	-2.575943	-67.415789	998.3	0.01	200.03	-0.06
2	-2.575949	-67.413990	998.3	0.02	200.02	-0.05
3	-2.575949	-67.412195	997.8	0.00	200.00	0.01
4	-2.575950	-67.410398	997.6	0.01	199.97	0.02
5	-2.575951	-67.408600	997.9	0.00	199.99	0.00
6	-2.575949	-67.406803	998.0	0.01	200.00	0.00
7	-2.575948	-67.405008	997.8	0.00	200.00	-0.03
8	-2.575949	-67.403210	997.7	-0.01	200.01	-0.01
9	-2.575951	-67.401412	998.0	-0.01	200.00	-0.01
10	-2.575951	-67.399613	998.3	0.00	199.99	0.02
20	-2.575947	-67.381629	998.9	0.00	200.00	0.01
30	-2.577031	-67.363692	998.6	-24.16	198.55	-0.03
40	-2.580310	-67.346018	998.9	-48.29	194.08	0.01
50	-2.585740	-67.328881	999.1	-71.64	186.70	0.03
60	-2.593235	-67.312530	998.7	-93.95	176.55	-0.02
70	-2.602689	-67.297210	998.6	-114.82	163.75	-0.05

	Position			Velocity (m/s)		
Time (s)	Latitude (°)	Longitude (°)	Height (m)	North	East	Down
80	-2.613956	-67.283153	998.3	-133.99	148.45	0.02
90	-2.626603	-67.270305	998.3	-141.41	141.42	0.00
100	-2.639390	-67.257585	998.6	-141.41	141.44	-0.10
110	-2.652177	-67.244873	998.7	-141.44	141.42	-0.02
120	-2.664714	-67.231920	998.8	-131.42	150.79	-0.04
130	-2.675728	-67.217674	999.3	-111.97	165.71	0.03
140	-2.684909	-67.202194	999.1	-90.84	178.17	-0.01
150	-2.692121	-67.185716	999.2	-68.38	187.96	-0.04
160	-2.697247	-67.168481	998.6	-44.88	194.92	-0.02
170	-2.700215	-67.150752	999.0	-20.71	198.93	0.00
180	-2.701015	-67.132797	999.0	0.00	200.00	0.01