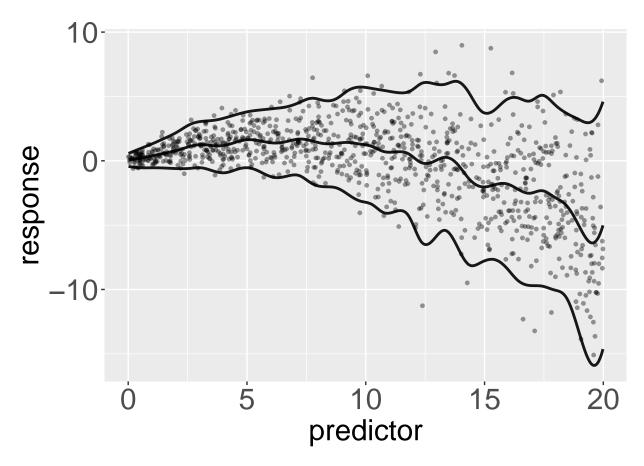
Appendix A: Example for package asp21bpsplines

```
require(asp21bpspline)
set.seed(2)
n = 1000
x = c(runif(n, 0, 20))
mean = -0.04*x^2 + 0.5* x
y = mean + rnorm(n, 0, 0.2 *x + 0.3)
m1 = list(x = x, z = x, y = y)
```

Consider you have the above data and want to fit a spline to it. With the asp21bpsplines package you would first call the spline function. The parameter m1 contains a list with the above structure, where x is the predictor for the mean, z is the predictor for the scale and y is the response. The parameter kn is short for knots and specifies the number of knots separately for the mean and the scale. In the same way the parameter stands for the order of the spline, p_order stands for the order of the smoothness penalty and smooth specifies the penalization strength.

```
##
## Call:
         spline(m = m1, kn = c(40, 40), order = c(3, 3), p_order = c(3, 3), p
##
##
                      3), smooth = c(1, 1)
##
## Location coefficients:
                            -0.01004
##
             [1]
                                                                0.05816
                                                                                                0.20045
                                                                                                                                 0.39816
                                                                                                                                                                 0.60367
                                                                                                                                                                                                  0.78701
                                                                                                                                                                                                                                  1.12899
             [8]
                                1.31770
                                                                1.13720
                                                                                                1.15841
                                                                                                                                 1.52651
                                                                                                                                                                  1.70267
                                                                                                                                                                                                  1.51014
                                                                                                                                                                                                                                  1.32741
##
          [15]
                                1.47338
                                                                1.76374
                                                                                                1.58687
                                                                                                                                 1.28502
                                                                                                                                                                  1.33392
                                                                                                                                                                                                  1.48259
                                                                                                                                                                                                                                  1.28549
##
          [22]
                                1.20726
                                                                0.59270
                                                                                                0.71117
                                                                                                                                 0.72973
                                                                                                                                                               -0.31139
                                                                                                                                                                                              -0.18020
                                                                                                                                                                                                                                  0.46701
         [29]
                             -0.06945
                                                                                                                                                                                              -2.21873
##
                                                             -1.55891
                                                                                              -2.16357
                                                                                                                              -1.88410
                                                                                                                                                              -1.63250
                                                                                                                                                                                                                               -2.70749
                             -2.08524
##
         [36]
                                                             -2.89393
                                                                                              -3.24740
                                                                                                                              -4.97449
                                                                                                                                                              -6.86876
                                                                                                                                                                                              -5.73026
                                                                                                                                                                                                                               -0.31415
##
## Scale coefficients:
##
            [1]
                             -2.037993
                                                                -1.291260
                                                                                                    -0.934212
                                                                                                                                       -0.736558
                                                                                                                                                                           -0.514893
                                                                                                                                                                                                              -0.327901
            [7]
                             -0.109484
                                                                -0.098214
                                                                                                       0.007503
                                                                                                                                          0.109413
                                                                                                                                                                              0.076077
                                                                                                                                                                                                                  0.102036
##
## [13]
                                0.227166
                                                                    0.335080
                                                                                                       0.312734
                                                                                                                                          0.329984
                                                                                                                                                                              0.562001
                                                                                                                                                                                                                  0.534393
        [19]
##
                                0.523903
                                                                    0.733961
                                                                                                       0.834234
                                                                                                                                          0.811065
                                                                                                                                                                              0.913872
                                                                                                                                                                                                                  0.848839
        [25]
                                0.830546
                                                                    1.204878
                                                                                                       1.152149
                                                                                                                                           1.004304
                                                                                                                                                                              1.205800
                                                                                                                                                                                                                  1.271068
## [31]
                                1.064764
                                                                    1.053703
                                                                                                       1.202539
                                                                                                                                           1.308075
                                                                                                                                                                               1.280613
                                                                                                                                                                                                                  1.350264
## [37]
                                1.302844
                                                                    1.259890
                                                                                                       1.440140
                                                                                                                                           1.583490
                                                                                                                                                                               1.613201
                                                                                                                                                                                                                  1.503776
plot(m)
```



The model estimates separate parameters for the location and the scale of the data. In this plot the middle line represents the prediction for the mean value and the outer lines represent the 95~% confidence interval for new predictions. However it seems that the predictions are overfitting a bit. One could now raise the values of the smoothing parameters in the call to the spline function. An alternative is a call to the MCMC function.

```
sample = mcmc.spline(m, it = 3000, burning = 1000, thinning = 10)
```

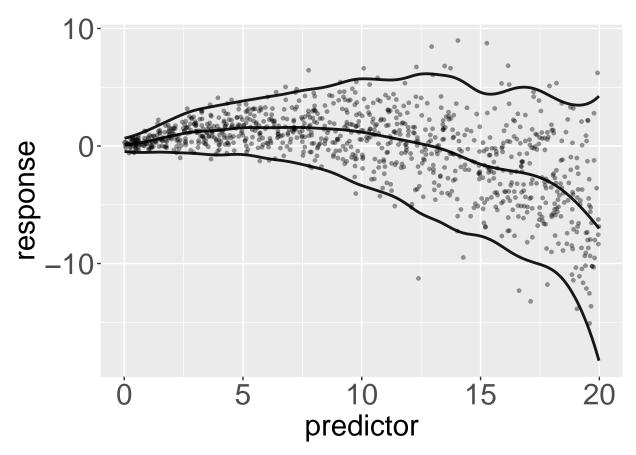
As the mcmc function samples directly from the posterior, the result is a sample for the beta and gamma parameters as well as for the smoothing values. The prediction with this sample is in result much smoother:

print(sample)

```
##
## Call:
## mcmc.spline(m = m, it = 3000, burning = 1000, thinning = 10)
##
##
  Posterior mean location coefficients:
          0.07013
                     0.08235
                                0.19628
                                           0.41081
                                                      0.67372
##
    [1]
                                                                 0.88701
                                                                            1.10408
##
    [8]
          1.24236
                     1.27852
                                1.36592
                                           1.49846
                                                      1.57784
                                                                 1.56632
                                                                            1.55068
   [15]
          1.57373
                                           1.50472
##
                     1.59465
                                1.56635
                                                      1.45330
                                                                 1.38649
                                                                            1.26112
##
   [22]
          1.09661
                     0.89301
                                0.68837
                                           0.47791
                                                      0.26807
                                                                 0.05046
                                                                           -0.21123
   [29]
         -0.58268
                    -1.04506
                                          -1.77933
                                                     -1.98965
                                                                           -2.32080
##
                               -1.47314
                                                                -2.14580
##
   [36]
         -2.60126
                    -3.05563
                               -3.72793
                                          -4.64300
                                                     -5.74116
                                                                -6.98388
                                                                           -8.38905
##
```

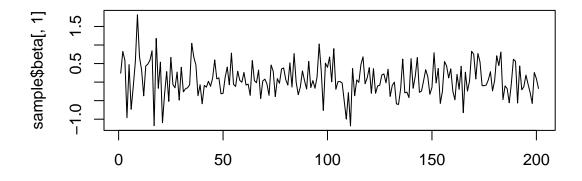
```
## Posterior mean scale coefficients:
##
    [1]
         -1.61617
                    -1.23520
                              -0.94355
                                         -0.70916
                                                    -0.49745
                                                              -0.29903
                                                                         -0.14274
         -0.04028
    [8]
                     0.03505
                               0.09194
                                          0.11207
                                                     0.16372
                                                               0.23756
                                                                          0.31270
## [15]
          0.35964
                     0.43009
                               0.50132
                                          0.55247
                                                     0.62782
                                                               0.72897
                                                                          0.82257
##
   [22]
          0.85963
                     0.88756
                               0.91642
                                          0.98529
                                                     1.09457
                                                               1.13974
                                                                          1.16014
## [29]
          1.20051
                     1.19204
                                          1.13640
                                                     1.20382
                                                                          1.32647
                               1.13349
                                                               1.28541
## [36]
          1.33085
                     1.31596
                               1.33539
                                          1.41842
                                                     1.55369
                                                               1.73943
                                                                          1.97703
```

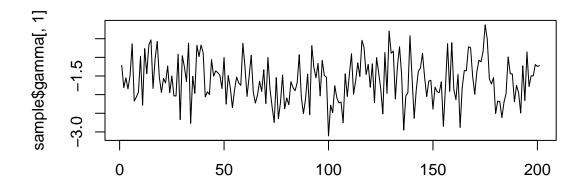
```
plot(sample, m)
```



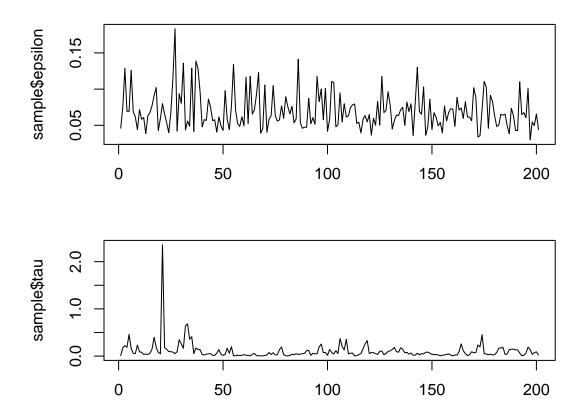
To investigate properties of the sample we can now look at the random walk for different parameters.

```
par(mfrow = c(2, 1))
par(mar = c(3, 5, 2, 4))
index = seq(1, length(sample$beta[,1]))
plot(index, sample$beta[,1], type = "l", xlab = "")
plot(index, sample$gamma[,1], type = "l", xlab = "")
```





```
plot(index, sample$epsilon, type = "1", xlab = "")
plot(index, sample$tau, type = "1")
```



For the sampling of gamma it is important that enough proposals are accepted in order to see movement in the random walk. This seems to be the case. However, it might be that consecutive iterations are correlated, since consecutive values tend to be near to each other. We now could choose a higher thinning value or alternatively set a custom stepsize for the sampling of the gamma values for the MCMC function to alleviate this issue.