50.039 Theory and Practice of Deep Learning Homework 2

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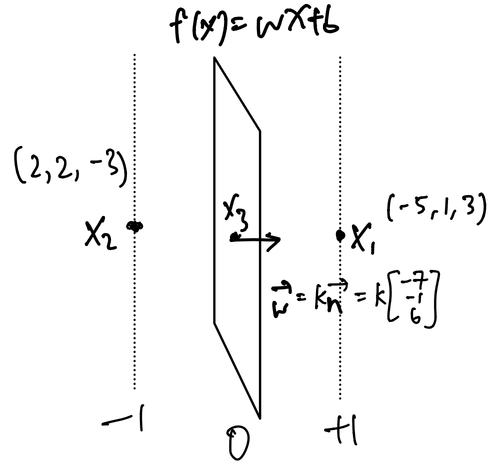
1) <u>Hyperplane</u>



Let X_3 be the middle point between X_1 and X_2

$$X_3 = \frac{X_1 + X_2}{2} = (-1.5, 1.5, 0)$$

A possible orthogonal vector can be:

$$\vec{n} = X_1 - X_2 = \begin{bmatrix}-7\\-1\\6\end{bmatrix}$$

Since $\vec{w}$ is parallel to $\vec{n}$, we can express

$$\vec{w} = k\vec{n} = k\begin{bmatrix}-7\\-1\\6\end{bmatrix} \quad \text{where } k \text{ is a scalar}$$

$$f(x) = \vec{w}\cdot\vec{x} + b = k\vec{n}\cdot\vec{x} + b$$

$$X_3: \; -(1.5)(-7k) + 1.5(-k) + b = 0 \Rightarrow 9k + b = 0$$

$$X_1: \; -5(-7k) + 1(-k) + 3(6k) + b = 1 \Rightarrow 52k + b = 1$$

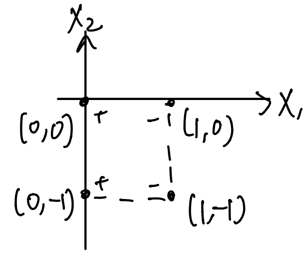
$$43k = 1 \Rightarrow k = \frac{1}{43}, \; b = -\frac{9}{43}, \; \vec{w} = \frac{1}{43}\begin{bmatrix}-7\\-1\\6\end{bmatrix}$$

We can substitute $\vec{w}$ and b to check if X_2 is predicted correctly

$$X_2: \; 2\left(\frac{-7}{43}\right) + 2\left(\frac{-1}{43}\right) - 3\left(\frac{6}{43}\right) - \frac{9}{43} = -1$$

Thus, one linear classifier is: $f(x) = \frac{1}{43}\begin{bmatrix}-7\\-1\\6\end{bmatrix}x - \frac{9}{43}$ //

2) Basic NN



$$z = w_1 x_1 + w_2 x_2 - w_0$$

$$(0,0): \quad g(0 + 0 - w_0) = +1 \Rightarrow w_0 \le -1, \text{ set } w_0 = -1$$

$$(0,-1): \quad g(0 + w_2(-1) \cancel{- w_0}^{+1}) = +1 \Rightarrow w_2 \le -1$$

$$(1,0): \quad g(w_1 + 0 \cancel{- w_0}^{+1}) = -1 \Rightarrow w_1 \le -2$$

$$(1,-1): \quad g(w_1 - w_2 \cancel{- w_0}^{+1}) = -1 \Rightarrow |w_1| \geq |w_2| - 2$$

one possible solution: $w_1 = -100$, $w_2 = -10$, $w_0 = -1$ //

3) Logistic Regression

3.1) $$s(a) = \frac{e^a}{1+e^a} = \frac{1}{e^{-a}+1}$$

$$1 - s(a) = \frac{1+e^a}{1+e^a} - \frac{e^a}{1+e^a} = \frac{1}{1+e^a}$$

$$\log(s(a)) = -\log(e^{-a}+1)$$

$$\frac{\partial \log(s(a))}{\partial a} = -\frac{1}{e^{-a}+1} \cdot (-e^{-a}) = \frac{e^{-a}}{1+e^{-a}}$$

$$= \frac{1}{1+e^a} = 1 - s(a)$$ //

$$\log(1-s(a)) = -\log(1+e^a)$$

$$\frac{\partial \log(1-s(a))}{\partial a} = -\frac{1}{1+e^a} \cdot (e^a) = -s(a)$$ //

3.2) $L = (-1) \cdot \sum_{i=1}^{n} y_i \log(h(x_i)) + (1-y_i) \log(1-h(x_i))$

$$= (-1) \cdot \sum_{i=1}^{n} y_i \log(s(w \cdot x_i)) + (1-y_i) \log(1 - s(w \cdot x_i))$$

$$\nabla_w L = (-1) \cdot \sum_{i=1}^{n} y_i (1 - s(w \cdot x_i)) x_i + (1-y_i)(-s(w \cdot x_i)) x_i$$ (using results from above)

$$= -\sum_{i=1}^{n} x_i \left(y_i - y_i s(w \cdot x_i) - s(w \cdot x_i) + y_i s(w \cdot x_i) \right)$$

$$= \sum_{i=1}^{n} x_i (s(w \cdot x_i) - y_i)$$

$$= \sum_{i=1}^{n} x_i (h(x_i) - y_i)$$

4) Back-Propagation

$\frac{\partial L}{\partial n_2}$ and $\frac{\partial L}{\partial n_3}$ are trivial

$$\frac{\partial L}{\partial n_4} = \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4}$$

$$\frac{\partial L}{\partial n_5} = \frac{\partial L}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5}$$

$$= \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5}$$

$$\frac{\partial L}{\partial n_6} = \frac{\partial L}{n_4} \cdot \frac{\partial n_4}{\partial n_6} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_6}$$

$$= \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_6} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_6} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_6}$$

$$\frac{\partial L}{\partial n_7} = \frac{\partial L}{\partial n_5} \cdot \frac{\partial n_5}{\partial n_7} + \frac{\partial L}{\partial n_6} \cdot \frac{\partial n_6}{\partial n_7}$$

$$= \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5} \cdot \frac{\partial n_5}{\partial n_7} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5} \cdot \frac{\partial n_5}{\partial n_7}$$

$$+ \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_6} \cdot \frac{\partial n_6}{\partial n_7} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_6} \cdot \frac{\partial n_6}{\partial n_7}$$

$$+ \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_6} \cdot \frac{\partial n_6}{\partial n_7} //$$

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial n_7} \cdot \frac{\partial n_7}{\partial x}$$

$$= \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5} \cdot \frac{\partial n_5}{\partial n_7} \cdot \frac{\partial n_7}{\partial x} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5} \cdot \frac{\partial n_5}{\partial n_7} \cdot \frac{\partial n_7}{\partial x}$$

$$+ \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_6} \cdot \frac{\partial n_6}{\partial n_7} \cdot \frac{\partial n_7}{\partial x} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_6} \cdot \frac{\partial n_6}{\partial n_7} \cdot \frac{\partial n_7}{\partial x}$$

$$+ \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_6} \cdot \frac{\partial n_6}{\partial n_7} \cdot \frac{\partial n_7}{\partial x}$$

//

5) Some Einsum

a) torch.einsum('ijk, i → jk', [A, b]) //

b) torch.einsum('ijkl → ik', [A]) //

c) torch.einsum('ijkl → ki', [A]) //

d) torch.einsum('ijk, ijk → j', [A, A]) //

e) torch.einsum('de, ef, fl → dl', [A, G.T, B]) //