Lab 3

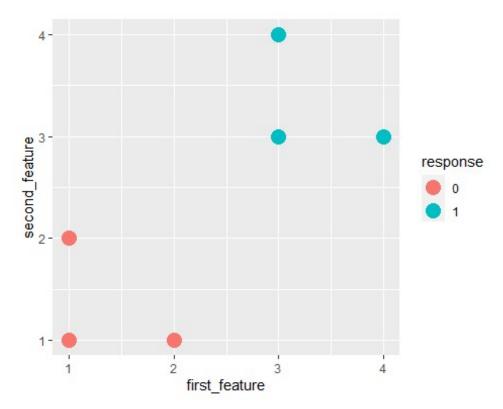
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11:59PM March 4, 2021

Support Vector Machine vs. Perceptron

We recreate the data from the previous lab and visualize it:

```
pacman::p_load(ggplot2)
Xy_simple = data.frame(
    response = factor(c(0, 0, 0, 1, 1, 1)), #nominal
    first_feature = c(1, 1, 2, 3, 3, 4), #continuous
    second_feature = c(1, 2, 1, 3, 4, 3) #continuous
)
simple_viz_obj = ggplot(Xy_simple, aes(x = first_feature, y = second_feature,
color = response)) +
    geom_point(size = 5)
simple_viz_obj
```

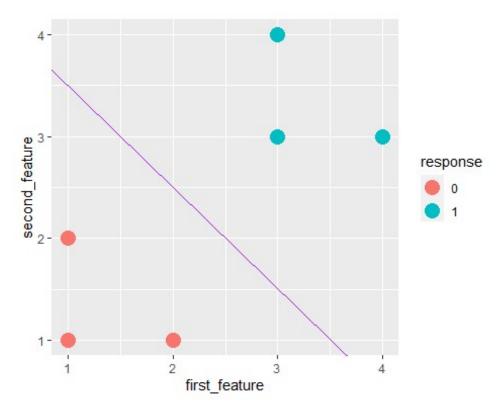


Use the e1071 package to fit an SVM model to the simple data. Use a formula to create the model, pass in the data frame, set kernel to be linear for the linear SVM and don't scale the covariates. Call the model object svm_model. Otherwise the remaining code won't work.

```
pacman::p_load(e1071)
svm_model = svm(
  formula = Xy_simple$response ~.,
  data = Xy_simple,
  kernel = "linear",
  scale = FALSE
)
```

and then use the following code to visualize the line in purple:

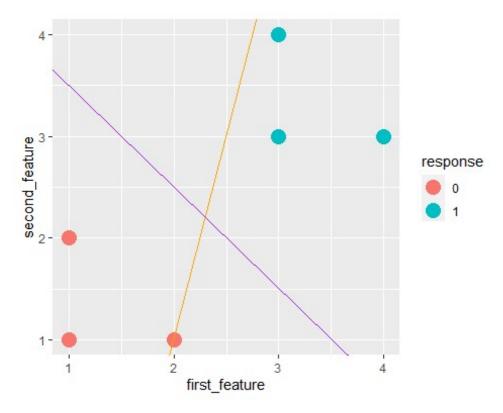
```
w_vec_simple_svm = c(
   svm_model$rho, #the b term
   -t(svm_model$coefs) %*% cbind(Xy_simple$first_feature,
Xy_simple$second_feature)[svm_model$index, ] # the other terms
)
simple_svm_line = geom_abline(
   intercept = -w_vec_simple_svm[1] / w_vec_simple_svm[3],
   slope = -w_vec_simple_svm[2] / w_vec_simple_svm[3],
   color = "purple")
simple_viz_obj + simple_svm_line
```



Source the perceptron_learning_algorithm function from lab 2. Then run the following to fit the perceptron and plot its line in orange with the SVM's line:

```
perceptron_learning_algorithm = function(Xinput, y_binary, MAX_ITER = 1000, w
= NULL){
```

```
Xinput = as.matrix(cbind(1,Xinput))
  p = ncol(Xinput)
  w = rep(0, p)
  r = nrow(Xinput)
  for (iter in 1 : MAX ITER){
    for (i in 1 : r) {
      x_i = Xinput[i, ]
      yhat_i = ifelse(sum(x_i * w) > 0, 1, 0)
      y_i = y_binary[i]
      for(j in 1:p){
        w[j] = w[j] + (y_i - yhat_i) * x_i[j]
      }
    }
  }
  W
}
w_vec_simple_per = perceptron_learning_algorithm(
  cbind(Xy_simple$first_feature, Xy_simple$second_feature),
  as.numeric(Xy_simple$response == 1)
simple_perceptron_line = geom_abline(
    intercept = -w_vec_simple_per[1] / w_vec_simple_per[3],
    slope = -w_vec_simple_per[2] / w_vec_simple_per[3],
    color = "orange")
simple_viz_obj + simple_perceptron_line + simple_svm_line
```



Is this SVM line a better fit than the perceptron?

Yes, this SVM line is a better fit than the perceptron, it is marginally acceptable.

Now write pseuocode for your own implementation of the linear support vector machine algorithm using the Vapnik objective function we discussed.

Note there are differences between this spec and the perceptron learning algorithm spec in question #1. You should figure out a way to respect the MAX_ITER argument value.

```
#' Support Vector Machine
#
#' This function implements the hinge-loss + maximum margin linear support
vector machine algorithm of Vladimir Vapnik (1963).
#'
#' @param Xinput
                      The training data features as an n x p matrix.
#' @param y_binary
                      The training data responses as a vector of Length n
consisting of only 0's and 1's.
#'@param MAX ITER The maximum number of iterations the algorithm
performs. Defaults to 5000.
#' @param Lambda
                     A scalar hyperparameter trading off margin of the
hyperplane versus average hinge loss.
                      The default value is 1.
                      The computed final parameter (weight) as a vector of
#' @return
length p + 1
linear_svm_learning_algorithm = function(Xinput, y_binary, MAX_ITER = 5000,
lambda = 0.1){
```

```
#Declaring the Sum of hinge error
 # SHE = 0
 # initialize a w vector
 # w <- () (This something like this?)</pre>
 #Iterate through the for-loop till the MAX_ITER
 # for x in MAX_ITER{
 #Iterate through each row
      for i in nrow(Xinput){
           #Adding the hinge error for each i to obtain total
 #
           SHE += max\{0, (1/2) - (y\_binary[i]-1/2)(w*Xinput[i]-b)\}
 #
 #
 # To maximize the margin
 # argmin{(SHE/n) + Lambda(distance of w)^2}
 # Returns w (the wedge/weights?)
 #
}
```

If you are enrolled in 342W the following is extra credit but if you're enrolled in 650, the following is required. Write the actual code. You may want to take a look at the optimx package. You can feel free to define another function (a "private" function) in this chunk if you wish. R has a way to create public and private functions, but I believe you need to create a package to do that (beyond the scope of this course).

```
#' This function implements the hinge-loss + maximum margin linear support
vector machine algorithm of Vladimir Vapnik (1963).
#' @param Xinput
                      The training data features as an n x p matrix.
#' @param y_binary
                     The training data responses as a vector of length n
consisting of only 0's and 1's.
#'@param MAX_ITER The maximum number of iterations the algorithm
performs. Defaults to 5000.
#'@param Lambda A scalar hyperparameter trading off margin of the
hyperplane versus average hinge loss.
                     The default value is 1.
#' @return
                     The computed final parameter (weight) as a vector of
Length p + 1
#linear svm learning algorithm = function(Xinput, y binary, MAX ITER = 5000,
Lambda = 0.1) \{
#}
```

If you wrote code (the extra credit), run your function using the defaults and plot it in brown vis-a-vis the previous model's line:

```
#svm_model_weights = linear_svm_learning_algorithm(Xinput, y_binary)
#my_svm_line = geom_abline(
# intercept = svm_model_weights[1] / svm_model_weights[3],#NOTE: negative
sign removed from intercept argument here
# slope = -svm_model_weights[2] / svm_model_weights[3],
# color = "brown")
#simple_viz_obj + my_svm_line
```

Is this the same as what the e1071 implementation returned? Why or why not?

TO-D0

We now move on to simple linear modeling using the ordinary least squares algorithm.

Let's quickly recreate the sample data set from practice lecture 7:

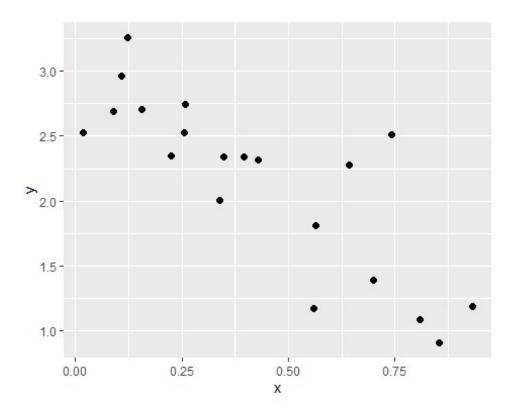
```
n = 20
x = runif(n)
beta_0 = 3
beta_1 = -2
```

Compute $h^*(x)$ as h_star_x, then draw \$\epsilon \sim N(0, 0.33^2)\$ asepsilon`, then compute \$\y\$.

```
h_star_x = beta_0 + beta_1 * x
epsilon = rnorm(n, 0, 0.33)
y = h_star_x + epsilon
```

Graph the data by running the following chunk:

```
pacman::p_load(ggplot2)
simple_df = data.frame(x = x, y = y)
simple_viz_obj = ggplot(simple_df, aes(x, y)) +
    geom_point(size = 2)
simple_viz_obj
```



Does this make sense given the values of $beta_0$ and $beta_1$?

Write a function my_simple_ols that takes in a vector x and vector y and returns a list that contains the b_0 (intercept), b_1 (slope), yhat (the predictions), e (the residuals), SSE, SST, MSE, RMSE and Rsq (for the R-squared metric). Internally, you can only use the functions sum and length and other basic arithmetic operations. You should throw errors if the inputs are non-numeric or not the same length. You should also name the class of the return value my_simple_ols_obj by using the class function as a setter. No need to create ROxygen documentation here.

```
my_simple_ols = function(x, y){
    n = length(y)

if(n != length(x)){
    stop("x and y need to be the same length")
}

if(class(x) != 'numeric' && class(x) != 'integer'){
    stop("x needs to be numeric.")
}

if(class(y) != 'numeric' && class(y) != 'integer'){
    stop("y needs to be numeric.")
}

if (n <= 2){</pre>
```

```
stop("n must be more than 2")
  }
  x bar = sum(x) / n
  y_bar = sum(y) / n
  b_1 = (sum(x * y) - n * x_bar * y_bar)/(sum(x ^ 2) - n * x_bar ^ 2)
  b_0 = y_bar - b_1 * x_bar
  yhat = b_0 + b_1 * x
  e = y - yhat
  SSE = sum(e ^ 2)
  SST = sum ((y - y_bar) ^ 2)
  MSE = SSE / (n - 2)
  RMSE = sqrt(MSE)
  Rsqrt = 1 - (SSE/SST)
  model = list(b_0 = b_0, b_1 = b_1, yhat = yhat, e = e, SSE = SSE, SST = b_1
SST, MSE = MSE, RMSE = RMSE, Rsqrt = Rsqrt)
  class(model) = "my simple ols obj"
  model
}
```

Verify your computations are correct for the vectors x and y from the first chunk using the 1m function in R:

```
lm_mod = lm(y ~ x)
my_simple_ols_mod = my_simple_ols(x,y)
#run the tests to ensure the function is up to spec
pacman::p_load(testthat)
expect_equal(my_simple_ols_mod$b_0, as.numeric(coef(lm_mod)[1]), tol = 1e-4)
expect_equal(my_simple_ols_mod$b_1, as.numeric(coef(lm_mod)[2]), tol = 1e-4)
expect_equal(my_simple_ols_mod$RMSE, summary(lm_mod)$sigma, tol = 1e-4)
expect_equal(my_simple_ols_mod$Rsq, summary(lm_mod)$r.squared, tol = 1e-4)
```

Verify that the average of the residuals is 0 using the expect_equal. Hint: use the syntax above.

```
mean(my_simple_ols_mod$e)
## [1] 9.993041e-17
expect_equal(mean(my_simple_ols_mod$e), 0)
```

Create the *X* matrix for this data example. Make sure it has the correct dimension.

```
X = cbind(1, x)
```

Use the model.matrix function to compute the matrix X and verify it is the same as your manual construction.

```
model.matrix(~x)
      (Intercept)
##
## 1
                1 0.10924993
## 2
                1 0.22476013
## 3
                1 0.56309345
## 4
                1 0.42804407
## 5
                1 0.80875295
## 6
                1 0.33911941
## 7
                1 0.15550835
## 8
                1 0.64240411
## 9
                1 0.56008416
## 10
                1 0.08981154
## 11
                1 0.39491582
## 12
                1 0.34775613
                1 0.25622114
## 13
## 14
                1 0.69927738
                1 0.25742601
## 15
## 16
                1 0.85305058
## 17
                1 0.74138863
## 18
                1 0.12251034
## 19
                1 0.01860284
## 20
                1 0.93226887
## attr(,"assign")
## [1] 0 1
```

Create a prediction method g that takes in a vector x_star and my_simple_ols_obj, an object of type my_simple_ols_obj and predicts y values for each entry in x_star.

```
g = function(my_simple_ols_obj, x_star){
  y_star = my_simple_ols_obj$b_0 + my_simple_ols_obj$b_1 * x_star
}
```

Use this function to verify that when predicting for the average x, you get the average y.

```
expect_equal(g(my_simple_ols_mod, mean(x)), mean(y))
```

In class we spoke about error due to ignorance, misspecification error and estimation error. Show that as n grows, estimation error shrinks. Let us define an error metric that is the difference between b_0 and b_1 and β_0 and β_1 . How about $h = ||b - \beta||^2$ where the quantities are now the vectors of size two. Show as n increases, this shrinks.

```
beta_0 = 3
beta_1 = -2
beta = c(beta_0, beta_1)

ns = 10 ^ (1:8)

errors_in_betas = array(NA, length(ns))

for (i in 1 : length(ns)) {
```

```
n = ns[i]
x = runif(n)
h_star_x = beta_0 + beta_1 * x
epsilon = rnorm(n, mean = 0, sd = 0.33)
y = h_star_x + epsilon

mod = my_simple_ols(x,y)
b = c(mod$b_0, mod$b_1)

errors_in_betas[i] = sum((beta - b)^2)
}

log(errors_in_betas, 10)
## [1] -1.295662 -2.290760 -2.793365 -5.560141 -5.290644 -6.943108 -7.651440
## [8] -7.613632
```

We are now going to repeat one of the first linear model building exercises in history—that of Sir Francis Galton in 1886. First load up package HistData.

```
pacman::p_load(HistData)
```

In it, there is a dataset called Galton. Load it up.

```
data(Galton)
```

You now should have a data frame in your workspace called Galton. Summarize this data frame and write a few sentences about what you see. Make sure you report n, p and a bit about what the columns represent and how the data was measured. See the help file ?Galton. p is 1 and n is 928 the number of observations

```
pacman::p_load(skimr)
skim(Galton)
```

Data summary

Name Galton
Number of rows 928
Number of columns 2

Column type frequency:

numeric 2

Group variables None

Variable type: numeric

skim_variab	n_missin	complete_ra	mea						p10	
le	g	te	n	sd	p0	p25	p50	p75	0	hist
parent	0	1		1.7 9					73.0	
child	0	1	68.0 9			66. 2			73.7	_

In Galton's data, I see the number of rows and columns within the data. Within the data, in the skim part. I see the parent and the child in the row, in the columns, I see the mean (parent is 68.3, child is 68.1), sd and see the percentile, 0, 25, 50, 75 and 100. n is the number of observations, p is the number of independent variable in this case it is height.

Find the average height (include both parents and children in this computation).

```
avg_height = mean(c(Galton$parent, Galton$child))
avg_height
## [1] 68.19833
```

If you were predicting the child height from the parent height and you were using null model, what would the RMSE be of this model be?

```
n = nrow(Galton)
SST = sum((Galton$child - mean(Galton$child)) ^ 2)
sqrt(SST / (n - 1))
## [1] 2.517941
```

Note that in Math 241 you learned that the sample average is an estimate of the "mean", the population expected value of height. We will call the average the "mean" going forward since it is probably correct to the nearest tenth of an inch with this amount of data.

Run a linear model attempting to explain the childrens' height using the parents' height. Use 1m and use the R formula notation. Compute and report b_0 , b_1 , RMSE and R^2 .

```
mod = lm(child ~ parent, Galton)
b_0 = coef(mod)[1]
b_1 = coef(mod)[2]
summary(mod)$sigma
## [1] 2.238547
summary(mod)$r.squared
## [1] 0.2104629
```

Interpret all four quantities: b_0 , b_1 , RMSE and R^2 . Use the correct units of these metrics in your answer.

 b_0 = If x = 0, then the parent has no height that means the parent does not exist (can't have zero height, in inches) b_1 = If there is a one inch increase by the parent's height, the child

average height will increases by the .64 inches. R^2 = This is about 21% variance explained which is consider low to certain standards. RMSE = This is Plus or minus 4.5 (2.23(2)) inches which makes it a 9 inches range 95% of the time

How good is this model? How well does it predict? Discuss.

This model is pretty good because it only considers one feature which is height but 9 inches can be considered a large range because the confidence interval can be almost a foot(12 inches) off of the prediction.

It is reasonable to assume that parents and their children have the same height? Explain why this is reasonable using basic biology and common sense.

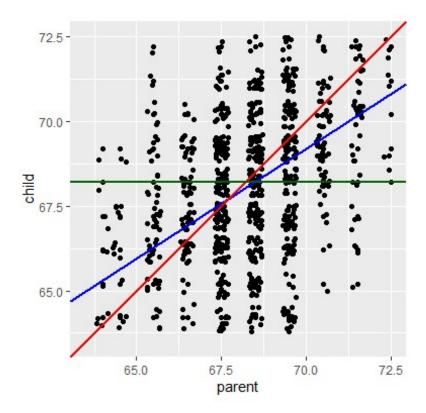
Yes is it reasonable to assume that parents and their children have the same height because children inherit both their parent's height genes which would have the similar height as their parents.

If they were to have the same height and any differences were just random noise with expectation 0, what would the values of β_0 and β_1 be?

The value of beta one is 1 and value of beta zero is 0.

Let's plot (a) the data in \mathbb{D} as black dots, (b) your least squares line defined by b_0 and b_1 in blue, (c) the theoretical line β_0 and β_1 if the parent-child height equality held in red and (d) the mean height in green.

```
pacman::p_load(ggplot2)
ggplot(Galton, aes(x = parent, y = child)) +
    geom_point() +
    geom_jitter() +
    geom_abline(intercept = b_0, slope = b_1, color = "blue", size = 1) +
    geom_abline(intercept = 0, slope = 1, color = "red", size = 1) +
    geom_abline(intercept = avg_height, slope = 0, color = "darkgreen", size =
1) +
    xlim(63.5, 72.5) +
    ylim(63.5, 72.5) +
    coord_equal(ratio = 1)
## Warning: Removed 76 rows containing missing values (geom_point).
## Warning: Removed 86 rows containing missing values (geom_point).
```



Fill in the following sentence:

Children of short parents became tall on average and children of tall parents became short on average.

Why did Galton call it "Regression towards mediocrity in hereditary stature" which was later shortened to "regression to the mean"?

Galton called it because if the tall parents had short children and short parents had tall parents, this led to a balance on height in the gene pool. The population by on height is regressing back to the mean.

Why should this effect be real?

This effect should be real because as time passes by, the height's mean for the human population stays around the same point due to biology. If someone is taller or shorter than average, the population will regress to the mean.

You now have unlocked the mystery. Why is it that when modeling with *y* continuous, everyone calls it "regression"? Write a better, more descriptive and appropriate name for building predictive models with *y* continuous.

The reason why everyone calls it "regression" is because we expected the parent/child relationship to be around 1 to 1. However after running the test and viewing the results, the height average of the children seem to have regressed compared to the height average of parent. The children height average turned out to be lower than the parents height average.

You can now clear the workspace. Create a dataset \mathbb{D} which we call Xy such that the linear model as R^2 about 50% and RMSE approximately 1.

```
x = 1:5
y = x^25
Xy = data.frame(x = x, y = y)
model = lm(x~y)
summary(model)$r.squared
## [1] 0.5028341
summary(model)$sigma
## [1] 1.28733
```

Create a dataset $\mathbb D$ which we call Xy such that the linear model as $\mathbb R^2$ about 0% but x, y are clearly associated.

```
x = 1:400
y = x ^ 118
Xy = data.frame(x = x, y = y)
model = lm(x~y)
summary(model)$r.squared
## [1] 0.04978688
```

Extra credit: create a dataset $\mathbb D$ and a model that can give you R^2 arbitrarily close to 1 i.e. approximately 1 - epsilon but RMSE arbitrarily high i.e. approximately M.

```
epsilon = 0.01
M = 1000
#TO-DO
```

Write a function my_ols that takes in X, a matrix with with p columns representing the feature measurements for each of the n units, a vector of n responses y and returns a list that contains the b, the p+1-sized column vector of OLS coefficients, yhat (the vector of n predictions), e (the vector of n residuals), df for degrees of freedom of the model, SSE, SST, MSE, RMSE and Rsq (for the R-squared metric). Internally, you cannot use lm or any other package; it must be done manually. You should throw errors if the inputs are non-numeric or not the same length. Or if X is not otherwise suitable. You should also name the class of the return value my_ols by using the class function as a setter. No need to create ROxygen documentation here.

```
#From Office Hours

my_ols = function(X, y){

n = length(y)

if (!is.numeric(X) && !is.integer(X)) {
    stop("X is not numeric")
```

```
}
  X = cbind(rep(1, n), X)
  p = ncol(X)
  df = ncol(X)
  if (n != nrow(X)){
    stop("X rows and length of y need to be the same length.")
  if(class(y) !='numeric' && class(y) !='integer'){
   stop("y needs to be numeric.")
   }
  if(n<=ncol(X)+1){</pre>
    stop("n must be more than 2.")
  }
 y_bar = sum(y) / n
  b = solve(t(X) %*% X) %*% t(X) %*% y
 yhat = X %*% b
  e = y - yhat
  SSE = (t(e) %*% e)
  SST = t(y - y_bar) %*% (y - y_bar)
  MSE = SSE / (n-(p+1))
  RMSE = sqrt(MSE)
  RSQ = 1 - (SSE / SST)
  model = list(b = b, yhat = yhat, df = df, e = e, SSE = SSE, SST = SST, MSE
= MSE, RMSE = RMSE, RSQ = RSQ)
  class(model) = "my_ols_obj"
 model
}
```

Verify that the OLS coefficients for the Type of cars in the cars dataset gives you the same results as we did in class (i.e. the ybar's within group).

```
cars = MASS::Cars93
mod = lm(Price~Type, data=cars)
my_ols(as.numeric(data.matrix(data.frame((cars$Type)))), cars$Price)
```

```
## $b
          [,1]
##
##
     22.871020
## X -1.001939
##
## $yhat
##
             [,1]
   [1,] 18.86327
##
   [2,] 19.86520
   [3,] 21.86908
##
##
   [4,] 19.86520
   [5,] 19.86520
##
##
   [6,] 19.86520
##
   [7,] 20.86714
##
   [8,] 20.86714
  [9,] 19.86520
## [10,] 20.86714
## [11,] 19.86520
## [12,] 21.86908
## [13,] 21.86908
## [14,] 17.86133
## [15,] 19.86520
## [16,] 16.85939
## [17,] 16.85939
## [18,] 20.86714
## [19,] 17.86133
## [20,] 20.86714
## [21,] 21.86908
## [22,] 20.86714
## [23,] 18.86327
## [24,] 18.86327
## [25,] 21.86908
## [26,] 16.85939
## [27,] 19.86520
## [28,] 17.86133
## [29,] 18.86327
## [30,] 20.86714
## [31,] 18.86327
## [32,] 18.86327
## [33,] 21.86908
## [34,] 17.86133
## [35,] 17.86133
## [36,] 16.85939
## [37,] 19.86520
## [38,] 20.86714
## [39,] 18.86327
## [40,] 17.86133
## [41,] 17.86133
## [42,] 18.86327
## [43,] 21.86908
```

```
## [44,] 18.86327
## [45,] 18.86327
## [46,] 17.86133
## [47,] 19.86520
## [48,] 19.86520
## [49,] 19.86520
## [50,] 19.86520
## [51,] 19.86520
## [52,] 20.86714
## [53,] 18.86327
## [54,] 18.86327
## [55,] 21.86908
## [56,] 16.85939
## [57,] 17.86133
## [58,] 21.86908
## [59,] 19.86520
## [60,] 17.86133
## [61,] 19.86520
## [62,] 18.86327
## [63,] 19.86520
## [64,] 18.86327
## [65,] 21.86908
## [66,] 16.85939
## [67,] 19.86520
## [68,] 21.86908
## [69,] 19.86520
## [70,] 16.85939
## [71,] 20.86714
## [72,] 17.86133
## [73,] 18.86327
## [74,] 21.86908
## [75,] 17.86133
## [76,] 19.86520
## [77,] 20.86714
## [78,] 21.86908
## [79,] 18.86327
## [80,] 18.86327
## [81,] 18.86327
## [82,] 21.86908
## [83,] 18.86327
## [84,] 18.86327
## [85,] 17.86133
## [86,] 19.86520
## [87,] 16.85939
## [88,] 18.86327
## [89,] 16.85939
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## [93,] 19.86520
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cars
                                          Type Min.Price Price Max.Price MPG.city
##
       Manufacturer
                                Model
## 1
                              Integra
                                         Small
                                                     12.9
                                                            15.9
                                                                       18.8
                                                                                   25
               Acura
                                                                       38.7
## 2
               Acura
                               Legend Midsize
                                                     29.2
                                                            33.9
                                                                                   18
## 3
                Audi
                                   90 Compact
                                                     25.9
                                                            29.1
                                                                       32.3
                                                                                   20
## 4
                Audi
                                  100 Midsize
                                                     30.8
                                                            37.7
                                                                       44.6
                                                                                   19
## 5
                  BMW
                                                     23.7
                                                                       36.2
                                 535i Midsize
                                                            30.0
                                                                                   22
## 6
               Buick
                              Century Midsize
                                                     14.2
                                                            15.7
                                                                       17.3
                                                                                   22
## 7
                                                                       21.7
                                                                                   19
               Buick
                              LeSabre
                                                     19.9
                                                            20.8
                                         Large
## 8
               Buick
                          Roadmaster
                                         Large
                                                     22.6
                                                            23.7
                                                                       24.9
                                                                                   16
## 9
               Buick
                                                     26.3
                                                            26.3
                                                                       26.3
                                                                                   19
                              Riviera Midsize
## 10
            Cadillac
                              DeVille
                                         Large
                                                     33.0
                                                            34.7
                                                                       36.3
                                                                                   16
## 11
            Cadillac
                                                     37.5
                                                            40.1
                                                                       42.7
                              Seville Midsize
                                                                                   16
## 12
           Chevrolet
                            Cavalier Compact
                                                      8.5
                                                            13.4
                                                                       18.3
                                                                                   25
## 13
           Chevrolet
                              Corsica Compact
                                                     11.4
                                                            11.4
                                                                       11.4
                                                                                   25
## 14
           Chevrolet
                                                     13.4
                                                            15.1
                                                                       16.8
                                                                                   19
                               Camaro
                                        Sporty
## 15
                                                                                   21
           Chevrolet
                               Lumina Midsize
                                                     13.4
                                                            15.9
                                                                       18.4
## 16
           Chevrolet
                          Lumina APV
                                           Van
                                                     14.7
                                                            16.3
                                                                       18.0
                                                                                   18
## 17
           Chevrolet
                                                     14.7
                                                                                   15
                                Astro
                                           Van
                                                            16.6
                                                                       18.6
## 18
           Chevrolet
                              Caprice
                                         Large
                                                     18.0
                                                            18.8
                                                                       19.6
                                                                                   17
## 19
           Chevrolet
                            Corvette
                                        Sporty
                                                     34.6
                                                            38.0
                                                                       41.5
                                                                                   17
## 20
            Chrylser
                                                            18.4
                                                                       18.4
                                                                                   20
                            Concorde
                                         Large
                                                     18.4
## 21
            Chrysler
                              LeBaron Compact
                                                     14.5
                                                            15.8
                                                                       17.1
                                                                                   23
## 22
            Chrysler
                            Imperial
                                         Large
                                                     29.5
                                                            29.5
                                                                       29.5
                                                                                   20
## 23
                                                      7.9
                                                             9.2
                                                                                   29
               Dodge
                                 Colt
                                         Small 
                                                                       10.6
## 24
                                                                                   23
                               Shadow
                                         Small
                                                      8.4
                                                           11.3
                                                                       14.2
               Dodge
```

##	25	Dodge	Spirit	Compact	11.9	13.3	14.7	22
##	26	Dodge	Caravan	Van	13.6	19.0	24.4	17
##	27	Dodge	Dynasty	Midsize	14.8	15.6	16.4	21
##	28	Dodge	Stealth	Sporty	18.5	25.8	33.1	18
##	29	Eagle	Summit	Small	7.9	12.2	16.5	29
##	30	Eagle	Vision	Large	17.5	19.3	21.2	20
##	31	Ford	Festiva	Small	6.9	7.4	7.9	31
##	32	Ford	Escort	Small	8.4	10.1	11.9	23
##	33	Ford	Tempo	Compact	10.4	11.3	12.2	22
##	34	Ford	Mustang	Sporty	10.8	15.9	21.0	22
##	35	Ford	Probe	Sporty	12.8	14.0	15.2	24
##	36	Ford	Aerostar	Van	14.5	19.9	25.3	15
##	37	Ford	Taurus	Midsize	15.6	20.2	24.8	21
##	38	Ford	Crown_Victoria	Large	20.1	20.9	21.7	18
##	39	Geo	_ Metro	Small	6.7	8.4	10.0	46
##	40	Geo	Storm	Sporty	11.5	12.5	13.5	30
##	41	Honda	Prelude	Sporty	17.0	19.8	22.7	24
##	42	Honda	Civic	Small	8.4	12.1	15.8	42
##	43	Honda	Accord	Compact	13.8	17.5	21.2	24
##	44	Hyundai	Excel	Small	6.8	8.0	9.2	29
##	45	Hyundai	Elantra	Small	9.0	10.0	11.0	22
##	46	Hyundai	Scoupe	Sporty	9.1	10.0	11.0	26
##	47	Hyundai	•	Midsize	12.4	13.9	15.3	20
##	48	Infiniti		Midsize	45.4	47.9	50.4	17
##	49	Lexus		Midsize	27.5	28.0	28.4	18
	50	Lexus		Midsize	34.7	35.2	35.6	18
	51	Lincoln	Continental		33.3	34.3	35.3	17
	52	Lincoln	Town Car	Large	34.4	36.1	37.8	18
	53	Mazda	323	Small	7.4	8.3	9.1	29
##	54	Mazda	Protege	Small	10.9	11.6	12.3	28
	55	Mazda	_	Compact	14.3	16.5	18.7	26
	56	Mazda	MPV	Van	16.6	19.1	21.7	18
	57	Mazda	RX-7	Sporty	32.5	32.5	32.5	17
		Mercedes-Benz		Compact	29.0	31.9	34.9	20
		Mercedes-Benz		Midsize	43.8	61.9	80.0	19
	60	Mercury	Capri		13.3	14.1	15.0	23
	61	Mercury		Midsize	14.9	14.9	14.9	19
	62	Mitsubishi	Mirage	Small	7.7	10.3	12.9	29
	63	Mitsubishi	Diamante		22.4	26.1	29.9	18
	64	Nissan	Sentra	Small	8.7	11.8	14.9	29
	65	Nissan		Compact	13.0	15.7	18.3	24
	66	Nissan	Quest	Van	16.7	19.1	21.5	17
	67	Nissan		Midsize	21.0	21.5	22.0	21
	68	Oldsmobile		Compact	13.0	13.5	14.0	24
	69	Oldsmobile	Cutlass_Ciera	-	14.2	16.3	18.4	23
	70	Oldsmobile	Silhouette	Van	19.5	19.5	19.5	18
	71	Oldsmobile	Eighty-Eight	Large	19.5	20.7	21.9	19
	72	Plymouth	Laser	Sporty	11.4	14.4	17.4	23
	73	Pontiac	LeMans	Small	8.2	9.0	9.9	31
	74	Pontiac		Compact	9.4	11.1	12.8	23
тπ	<i>,</i> +	ronciac	Julibil u	Compact	J.4	TT.T	12.0	23

## 7E D	ontiac	Firebird	Cnonty	1/ 0	17 7	21 /	10
	ontiac ontiac				17.7	21.4 21.6	19
		Grand_Prix			18.5		19
		Bonneville	Large		24.4	29.4	19
## 78	Saab		Compact		28.7	37.1	20
	Saturn	SL			11.1	12.9	28
	Subaru	Justy		7.3	8.4	9.5	33
	Subaru	Loyale			10.9	11.3	25
## 82	Subaru	Legacy	Compact	16.3	19.5	22.7	23
## 83	Suzuki	Swift	Small	7.3	8.6	10.0	39
## 84	Toyota	Tercel	Small	7.8	9.8	11.8	32
## 85	Toyota	Celica	Sporty	14.2	18.4	22.6	25
	Toyota	Camry	Midsize	15.2	18.2	21.2	22
	Toyota	Previa	Van		22.7	26.6	18
	swagen	Fox		8.7	9.1	9.5	25
	swagen	Eurovan	Van		19.7	22.7	17
	swagen		Compact		20.0	22.4	21
	swagen						18
	•	Corrado			23.3	23.7	
## 92	Volvo		Compact		22.7	23.5	21
## 93	Volvo		Midsize		26.7	28.5	20
## MPG.hig	hway	AirBa	gs DriveTrain	Cylind	ers Engine	eSize	
Horsepower							
## 1	31	No	ne Front		4	1.8	
140							
## 2	25 Driver	& Passeng	er Front		6	3.2	
200		J					
## 3	26	Driver on	ly Front		6	2.8	
172	-		,		-		
## 4	26 Driver	& Passeng	er Front		6	2.8	
172	20 011101	a rasseng			Ü	2.0	
## 5	30	Driver on	ly Rear		4	3.5	
208	50	DI IVEI OII.	Ly iteal		4	٠.٥	
	21	Dudwan an	1 Fman+		4	2 2	
## 6	31	Driver on	ly Front		4	2.2	
110						2.0	
## 7	28	Driver on	ly Front		6	3.8	
170							
## 8	25	Driver on	ly Rear		6	5.7	
180							
## 9	27	Driver on	ly Front		6	3.8	
170							
## 10	25	Driver on	ly Front		8	4.9	
200			•				
## 11	25 Driver	& Passeng	er Front		8	4.6	
295							
## 12	36	No	ne Front		4	2.2	
110	50	140	.c ironc				
## 13	34	Driver on	ly Front		4	2.2	
	34	pi.tvei. oii.	ry Front		4	۷.۷	
110	20 0	0 D	n D			2.4	
## 14	28 Driver	& Passeng	er Rear		6	3.4	
160	20		_				
## 15	29	No	ne Front		4	2.2	

110 ## 16	23	N	lone	Front	6	3.8
170	20			41.15		4.2
## 17 165	20	N	lone	4WD	6	4.3
## 18	26	Driver o	only	Rear	8	5.0
170 ## 19	25	Driver o	only	Rear	8	5.7
300 ## 20	28 Driver	& Passen	iger	Front	6	3.3
153 ## 21	28 Driver	9 Daccon	agon.	Front	4	3.0
141	20 DIJIVEL	Q Passen	igei.	Fronc	4	3.0
## 22 147	26	Driver o	only	Front	6	3.3
## 23 92	33	N	lone	Front	4	1.5
## 24	29	Driver o	only	Front	4	2.2
93 ## 25	27	Driver o	only	Front	4	2.5
100 ## 26	21	Driver o	only	4WD	6	3.0
142			•		_	
## 27 100	27	Driver o	only	Front	4	2.5
## 28	24	Driver o	nly	4WD	6	3.0
300 ## 29	33	N	lone	Front	4	1.5
92						
## 30 214	28 Driver	& Passen	iger	Front	6	3.5
## 31 63	33	N	lone	Front	4	1.3
## 32 127	30	N	lone	Front	4	1.8
## 33	27	N	lone	Front	4	2.3
96 ## 34	29	Driver o	only	Rear	4	2.3
105 ## 35	30	Driver o	only	Front	4	2.0
115 ## 36	20	Driver o	nlv	4WD	6	3.0
145	20	DI IVEL O	літу	4WD	U	3.0
## 37 140	30	Driver o	only	Front	6	3.0
## 38	26	Driver o	only	Rear	8	4.6
190 ## 39	50	N	lone	Front	3	1.0
55						
## 40	36	Driver o	nity	Front	4	1.6

90 ## 41	31 Driver	^ & Passenger	Front	4	2.3	
160				_	4 =	
## 42 102	46	Driver only	Front	4	1.5	
## 43	31 Driver	r & Passenger	Front	4	2.2	
140 ## 44	33	None	Front	4	1.5	
81						
## 45 124	29	None	Front	4	1.8	
## 46	34	None	Front	4	1.5	
92 ## 47	27	None	Front	4	2.0	
128	22	Duivon only	Daan	0	4 5	
## 48 278	22	Driver only	Rear	8	4.5	
## 49	24	Driver only	Front	6	3.0	
185 ## 50	23 Driver	^ & Passenger	Rear	6	3.0	
225	26 Duine	. 0 D	Format	_	2.0	
## 51 160	26 Driver	^ & Passenger	Front	6	3.8	
## 52	26 Driver	~ & Passenger	Rear	8	4.6	
210 ## 53	37	None	Front	4	1.6	
82	57	None	110110	7	1.0	
## 54	36	None	Front	4	1.8	
103 ## 55	34	Driver only	Front	4	2.5	
164		•				
## 56 155	24	None	4WD	6	3.0	
## 57	25	Driver only	Rear	rotary	1.3	
255	20	Dudana anla	D	4	2.2	
## 58 130	29	Driver only	Rear	4	2.3	
## 59	25 Driver	^ & Passenger	Rear	6	3.2	
217 ## 60	26	Driver only	Front	4	1.6	
100		·				
## 61 140	26	None	Rear	6	3.8	
## 62	33	None	Front	4	1.5	
92 ## 63	24	Driver only	Front	6	3.0	
202	22	Dudanas	F t	4	1.6	
## 64 110	33	Driver only	Front	4	1.6	
## 65	30	Driver only	Front	4	2.4	

150 ## 66	23	None	e Front	6	3.0	
151 ## 67	26	Driver only	y Front	6	3.0	
160	20	Di Iver Oniy	TTOILE	U	3.0	
## 68 155	31	None	Front	4	2.3	
## 69	31	Driver only	Front	4	2.2	
110 ## 70	23	None	e Front	6	3.8	
170						
## 71 170	28	Driver only	y Front	6	3.8	
## 72	30	None	4WD	4	1.8	
92 ## 73	41	None	. Front	4	1.6	
74 ## 74	31	None	e Front	4	2.0	
110 ## 75	28 Driver	`& Passenger	Rear	6	3.4	
160 ## 76	27	None	. Front	6	3.4	
200						
## 77 170	28 Driver	` & Passenger	r Front	6	3.8	
## 78	26	Driver only	Front	4	2.1	
140 ## 79	38	Driver only	Front	4	1.9	
85 ## 80	37	None	4WD	3	1.2	
73						
## 81 90	30	None	e 4WD	4	1.8	
## 82	30	Driver only	4WD	4	2.2	
130 ## 83	43	None	. Front	3	1.3	
70 ## 84	37	Driver only	Front	4	1.5	
82 ## 85	32	Driver only	Front	4	2.2	
135 ## 86	29	Driver only	Front	4	2.2	
130 ## 87	22	Driver only	4WD	4	2.4	
138						
## 88 81	33	None	e Front	4	1.8	
## 89 109	21	None	e Front	5	2.5	
## 90	30	None	Front	4	2.0	

134				
## 91	25	None	Front 6	2.8
178				
## 92	28	Driver only	Rear 4	2.3
114		,		
## 93	28 Driver	& Passenger	Front 5	2.4
168		O		
	RPM Rev.per.mile M	Man.trans.avail	Fuel.tank.capacity	Passengers Length
	300 . 2890	Yes	13.2	5 177
## 2 55	500 2335	Yes	18.0	5 195
## 3 55	500 2280	Yes	16.9	5 180
## 4 55	500 2535	Yes	21.1	6 193
## 5 57	700 2545	Yes	21.1	4 186
## 6 52	200 2565	No	16.4	6 189
## 7 48	300 1570	No	18.0	6 200
## 8 46	900 1320	No	23.0	6 216
## 9 48	300 1690	No	18.8	5 198
## 10 41	L00 1510	No	18.0	6 206
## 11 60		No	20.0	5 204
## 12 52		Yes	15.2	
## 13 52		Yes	15.6	5 184
## 14 46		Yes	15.5	4 193
## 15 52		No	16.5	6 198
## 16 48		No	20.0	7 178
## 17 46		No	27.0	8 194
## 18 42		No	23.0	6 214
## 19 56		Yes	20.0	2 179
## 20 53		No	18.0	6 203
## 21 56		No	16.0	6 183
## 22 48		No	16.0	6 203
## 23 66 ## 24 48		Yes	13.2	5 174 5 172
## 25 48		Yes Yes	14.0	5 172 6 181
## 26 56		No	16.0 20.0	7 175
## 27 48		No	16.0	
## 28 66		Yes	19.8	4 180
## 29 66		Yes	13.2	5 174
## 30 58		No	18.0	6 202
## 31 56		Yes	10.0	4 141
## 32 65		Yes	13.2	5 171
## 33 42		Yes	15.9	5 177
## 34 46		Yes	15.4	
## 35 55		Yes	15.5	4 179
## 36 48	300 2080	Yes	21.0	7 176
## 37 48		No	16.0	5 192
## 38 42	200 1415	No	20.0	6 212
## 39 57	700 3755	Yes	10.6	4 151
## 40 54	100 3250	Yes	12.4	4 164
## 41 58		Yes	15.9	4 175
## 42 59	900 2650	Yes	11.9	4 173

## 43 5600	2610	Yes	17.0	4	185
## 44 5500	2710	Yes	11.9	5	168
## 45 6000	2745	Yes	13.7	5	172
## 46 5550	2540	Yes	11.9	4	166
## 47 6000	2335	Yes	17.2	5	184
## 48 6000	1955	No	22.5	5	200
## 49 5200	2325	Yes	18.5	5	188
## 50 6000	2510	Yes	20.6	4	191
## 51 4400	1835		18.4	6	205
		No No			
	1840	No	20.0	6	219
## 53 5000	2370	Yes	13.2	4	164
## 54 5500	2220	Yes	14.5	5	172
## 55 5600	2505	Yes	15.5	5	184
## 56 5000	2240	No	19.6	7	190
## 57 6500	2325	Yes	20.0	2	169
## 58 5100	2425	Yes	14.5	5	175
## 59 5500	2220	No	18.5	5	187
## 60 5750	2475	Yes	11.1	4	166
## 61 3800	1730	No	18.0	5	199
## 62 6000	2505	Yes	13.2	5	172
## 63 6000	2210	No	19.0	5	190
## 64 6000	2435	Yes	13.2	5	170
## 65 5600	2130	Yes	15.9	5	181
## 66 4800	2065	No	20.0	7	190
## 67 5200	2045	No	18.5	5	188
## 68 6000	2380	No	15.2	5	188
## 69 5200	2565	No	16.5	5	190
## 70 4800	1690	No	20.0	7	194
## 71 4800	1570	No	18.0	6	201
## 72 5000	2360	Yes	15.9	4	173
## 73 5600	3130	Yes	13.2	4	177
## 74 5200	2665	Yes	15.2	5	181
## 75 4600	1805	Yes	15.5	4	196
## 76 5000	1890	Yes	16.5	5	195
## 77 4800	1565	No	18.0	6	177
## 78 6000	2910	Yes	18.0	5	184
## 79 5000	2145	Yes	12.8	5	176
## 80 5600	2875	Yes	9.2	4	146
## 81 5200	3375	Yes	15.9	5	175
## 82 5600	2330	Yes	15.9	5	179
## 83 6000	3360	Yes	10.6	4	161
## 84 5200	3505	Yes	11.9	5	162
## 85 5400	2405	Yes	15.9	4	174
## 86 5400	2340	Yes	18.5	5	188
## 87 5000	2546 2515	Yes	19.8	5 7	187
## 88 5500	2515 2550	Yes	12.4	4	163
## 89 4500					187
## 90 5800	2915	Yes	21.1	7	
	2685	Yes	18.5	5	180
## 91 5800	2385	Yes	18.5	4	159
## 92 5400	2215	Yes	15.8	5	190

##	93	6200	231	10	Yes	19.3		184
##					Rear.seat.room			Origin
	1	102	68	37	26.5	11	_	non-USA
##	2	115	71	38	30.0	15		non-USA
##	3	102	67	37	28.0	14		non-USA
	4	106	70	37	31.0	17		non-USA
##	5	109	69	39	27.0	13		non-USA
	6	105	69	41	28.0	16	2880	USA
##	7	111	74	42	30.5	17	3470	USA
##	8	116	78	45	30.5	21	4105	USA
##	9	108	73	41	26.5	14	3495	USA
##	10	114	73	43	35.0	18	3620	USA
##	11	111	74	44	31.0	14	3935	USA
##	12	101	66	38	25.0	13	2490	USA
##	13	103	68	39	26.0	14	2785	USA
##	14	101	74	43	25.0	13	3240	USA
##	15	108	71	40	28.5	16	3195	USA
##	16	110	74	44	30.5	NA	3715	USA
##	17	111	78	42	33.5	NA	4025	USA
##	18	116	77	42	29.5	20	3910	USA
##	19	96	74	43	NA	NA	3380	USA
##	20	113	74	40	31.0	15	3515	USA
##	21	104	68	41	30.5	14	3085	USA
##	22	110	69	44	36.0	17	3570	USA
##	23	98	66	32	26.5	11	2270	USA
##	24	97	67	38	26.5	13	2670	USA
##	25	104	68	39	30.5	14	2970	USA
##	26	112	72	42	26.5	NA	3705	USA
##	27	105	69	42	30.5	16	3080	USA
##	28	97	72	40	20.0	11	3805	USA
##	29	98	66	36	26.5	11	2295	USA
##	30	113	74	40	30.0	15	3490	USA
##	31	90	63	33	26.0	12	1845	USA
	32	98	67	36	28.0	12	2530	USA
##		100	68	39	27.5	13	2690	USA
##		101	68	40	24.0	12	2850	USA
	35	103	70	38	23.0	18	2710	USA
## ##	36	119 106	72 71	45 40	30.0	NA 18	3735	USA USA
##		114	71 78	40	27.5 30.0	21	3325 3950	USA
##		93	63	34	27.5	10		non-USA
##		93 97	67	37	24.5	10		non-USA
##		100	70	39	23.5	8		non-USA
##		100	67	36	28.0	12		non-USA
##		103	67	41	28.0	14		non-USA
##		94	63	35	26.0	11		non-USA
##		98	66	36	28.0	12		non-USA
##		94	64	34	23.5	9		non-USA
##		104	69	41	31.0	14		non-USA
##		113	72	42	29.0	15		non-USA
ит	70	110	, _	72	25.0	1.0	-1000	1.011 03A

##	49	103	70	40	27.5	14	3510 non-USA
##	50	106	71	39	25.0	9	3515 non-USA
##	51	109	73	42	30.0	19	3695 USA
##	52	117	77	45	31.5	22	4055 USA
##	53	97	66	34	27.0	16	2325 non-USA
##	54	98	66	36	26.5	13	2440 non-USA
	55	103	69	40	29.5	14	2970 non-USA
##	56	110	72	39	27.5	NA	3735 non-USA
##	57	96	69	37	NA	NA	2895 non-USA
##	58	105	67	34	26.0	12	2920 non-USA
##	59	110	69	37	27.0	15	3525 non-USA
##	60	95	65	36	19.0	6	2450 USA
##	61	113	73	38	28.0	15	3610 USA
##	62	98	67	36	26.0	11	2295 non-USA
##	63	107	70	43	27.5	14	3730 non-USA
##	64	96	66	33	26.0	12	2545 non-USA
##	65	103	67	40	28.5	14	3050 non-USA
##	66	112	74	41	27.0	NA	4100 non-USA
##	67	104	69	41	28.5	14	3200 non-USA
##	68	103	67	39	28.0	14	2910 USA
##	69	105	70	42	28.0	16	2890 USA
##	70	110	74	44	30.5	NA	3715 USA
##	71	111	74	42	31.5	17	3470 USA
##	72	97	67	39	24.5	8	2640 USA
##	73	99	66	35	25.5	17	2350 USA
##	74	101	66	39	25.0	13	2575 USA
##	75	101	75	43	25.0	13	3240 USA
##	76	108	72	41	28.5	16	3450 USA
##	77	111	74	43	30.5	18	3495 USA
##	78	99	67	37	26.5	14	2775 non-USA
##	79	102	68	40	26.5	12	2495 USA
##	80	90	60	32	23.5	10	2045 non-USA
##	81	97	65	35	27.5	15	2490 non-USA
##	82	102	67	37	27.0	14	3085 non-USA
##	83	93	63	34	27.5	10	1965 non-USA
##	84	94	65	36	24.0	11	2055 non-USA
##	85	99	69	39	23.0	13	2950 non-USA
##	86	103	70	38	28.5	15	3030 non-USA
##	87	113	71	41	35.0	NA	3785 non-USA
##	88	93	63	34	26.0	10	2240 non-USA
##	89	115	72	38	34.0	NA	3960 non-USA
##	90	103	67	35	31.5	14	2985 non-USA
##	91	97	66	36	26.0	15	2810 non-USA
##	92	104	67	37	29.5	14	2985 non-USA
##	93	105	69	38	30.0	15	3245 non-USA
##			Make				
##	1		Acura Integra				
##			Acura Legend				
##			Audi 90				
##			Audi 100				

```
## 5
                       BMW 535i
## 6
                  Buick Century
## 7
                  Buick LeSabre
## 8
               Buick Roadmaster
## 9
                  Buick Riviera
## 10
               Cadillac DeVille
## 11
               Cadillac Seville
## 12
             Chevrolet Cavalier
## 13
              Chevrolet Corsica
## 14
               Chevrolet Camaro
## 15
               Chevrolet Lumina
## 16
          Chevrolet Lumina APV
## 17
                Chevrolet Astro
## 18
             Chevrolet Caprice
## 19
            Chevrolet Corvette
## 20
             Chrylser Concorde
## 21
               Chrysler LeBaron
## 22
              Chrysler Imperial
## 23
                     Dodge Colt
## 24
                   Dodge Shadow
## 25
                   Dodge Spirit
## 26
                  Dodge Caravan
## 27
                  Dodge Dynasty
## 28
                  Dodge Stealth
## 29
                   Eagle Summit
## 30
                   Eagle Vision
## 31
                   Ford Festiva
## 32
                    Ford Escort
## 33
                     Ford Tempo
## 34
                   Ford Mustang
## 35
                     Ford Probe
## 36
                  Ford Aerostar
## 37
                    Ford Taurus
## 38
            Ford Crown Victoria
## 39
                      Geo Metro
## 40
                      Geo Storm
## 41
                  Honda Prelude
## 42
                    Honda Civic
## 43
                   Honda Accord
## 44
                  Hyundai Excel
## 45
                Hyundai Elantra
## 46
                 Hyundai Scoupe
## 47
                 Hyundai Sonata
## 48
                   Infiniti Q45
## 49
                    Lexus ES300
## 50
                    Lexus SC300
## 51
           Lincoln Continental
## 52
               Lincoln Town Car
## 53
                      Mazda 323
## 54
                  Mazda Protege
```

```
## 55
                      Mazda 626
## 56
                      Mazda MPV
                     Mazda RX-7
## 57
## 58
            Mercedes-Benz 190E
## 59
            Mercedes-Benz 300E
## 60
                  Mercury Capri
## 61
                 Mercury Cougar
## 62
             Mitsubishi Mirage
## 63
           Mitsubishi Diamante
## 64
                  Nissan Sentra
## 65
                  Nissan Altima
                   Nissan Quest
## 66
## 67
                  Nissan Maxima
## 68
            Oldsmobile Achieva
## 69 Oldsmobile Cutlass_Ciera
         Oldsmobile Silhouette
## 70
## 71
       Oldsmobile Eighty-Eight
## 72
                 Plymouth Laser
## 73
                 Pontiac LeMans
## 74
                Pontiac Sunbird
## 75
               Pontiac Firebird
## 76
            Pontiac Grand Prix
## 77
            Pontiac Bonneville
## 78
                       Saab 900
## 79
                      Saturn SL
## 80
                   Subaru Justy
## 81
                  Subaru Loyale
## 82
                  Subaru Legacy
## 83
                   Suzuki Swift
## 84
                  Toyota Tercel
## 85
                  Toyota Celica
## 86
                   Toyota Camry
## 87
                  Toyota Previa
## 88
                 Volkswagen Fox
            Volkswagen Eurovan
## 89
             Volkswagen Passat
## 90
## 91
            Volkswagen Corrado
## 92
                      Volvo 240
## 93
                      Volvo 850
```

Create a prediction method g that takes in a vector x_{star} and the dataset $\mathbb D$ i.e. X and Y and returns the OLS predictions. Let Y be a matrix with with p columns representing the feature measurements for each of the n units

```
#From Office Hours

g = function(x_star, X, y){
  b = my_ols(X, y)$b
  x_star = c(1,x_star)
  x_star %*% b
```

```
}
X = model.matrix(~Type,cars)[, 2:6]
head(X)
     TypeLarge TypeMidsize TypeSmall TypeSporty TypeVan
##
## 1
            0
                         0
                                   1
                                              0
## 2
            0
                         1
                                   0
                                                      0
                                              0
## 3
            0
                         0
                                   0
                                                      0
                         1
                                              0
                                                      0
## 4
            0
## 5
                                                      0
                         1
            0
                         1
                                   0
                                                      0
## 6
g(X[1,], X, cars$Price)
##
          [,1]
## [1,] 10.16667
t(c(1,X[1,])) %*% my_ols(X, cars$Price)$b
##
            [,1]
## [1,] 10.16667
X[1,]
    TypeLarge TypeMidsize TypeSmall TypeSporty
##
                                                       TypeVan
##
                                     1
predict(mod, cars[1,])
##
## 10.16667
```