monte_carlo_simulation

May 18, 2020

[1]:
```
%matplotlib inline

## Load packages
import os
import pickle
import datetime
import numpy as np
import pandas as pd
from numba import jit

from scipy import stats
from statsmodels.tsa import stattools
from statsmodels.tsa.stattools import acf, pacf, ARMA
from statsmodels.stats.stattools import durbin_watson
from statsmodels.tsa.seasonal import STL
from statsmodels.tsa.ar_model import AutoReg
import statsmodels.api as sm

from dateutil.relativedelta import relativedelta

import matplotlib.pyplot as plt
import matplotlib.dates as mdates
import pylab
import seaborn as sns
```

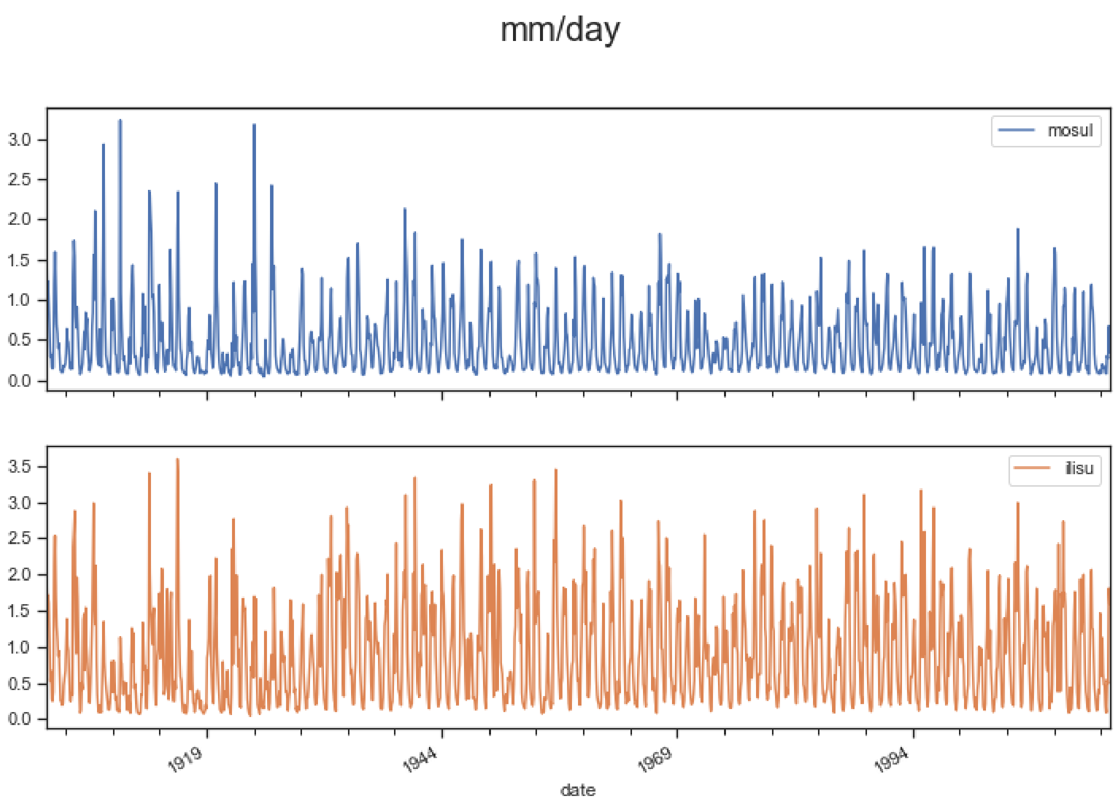
1 CCIWR class project

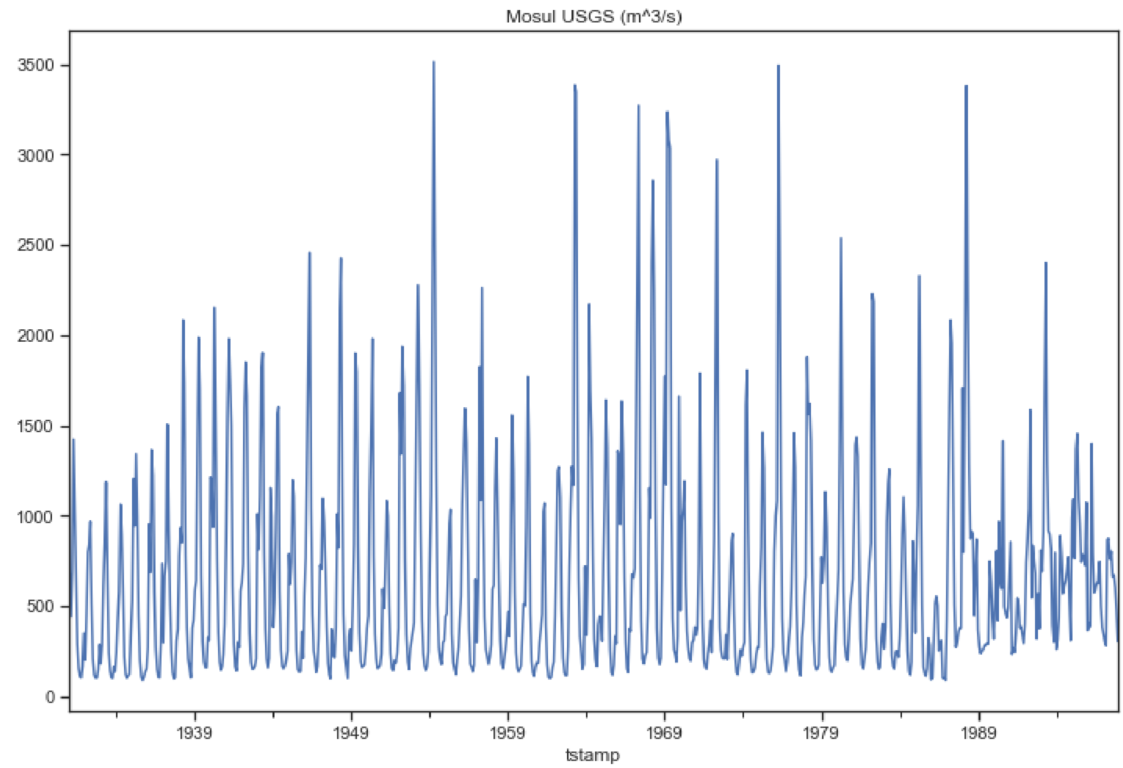
This code implements monte carlo simulation and prediction based on autoregressive processes.

[2]:
```
# Some plotting defaults
sns.set(style="ticks")
plt.rcParams.update({'font.size': 18})
plt.rcParams.update({'mathtext.default':'regular'})
plt.rcParams["figure.figsize"] = [12, 8]
```

[3]:
```python
# Load the data
df = pd.read_csv("data/GRUN/grun_data.csv", index_col='date', usecols=['date',␣
↪'mosul', 'ilisu'], parse_dates=True)
df.plot(subplots=True, title='mm/day')

# usgs data
mosul_usgs = pd.read_csv("data/USGS/MosulMonthly1931-1997.csv", sep=';',␣
↪index_col = 'year')
mosul_usgs = mosul_usgs.unstack().reset_index()
mosul_usgs = mosul_usgs.rename(columns={"level_0": "month", 0:␣
↪'discharge_cmps'})
mosul_usgs['day'] = np.repeat(1, len(mosul_usgs))
mosul_usgs['tstamp'] = pd.to_datetime(mosul_usgs[['day', 'month', 'year']],␣
↪format="%d%m%Y")
mosul_usgs = mosul_usgs.sort_values(by='tstamp').
↪set_index('tstamp')['discharge_cmps']
mosul_usgs = mosul_usgs.fillna(value=mosul_usgs.mean())

f, ax = plt.subplots()
mosul_usgs.plot(title="Mosul USGS (m^3/s)")
```

[3]: <matplotlib.axes._subplots.AxesSubplot at 0x123b69dd0>





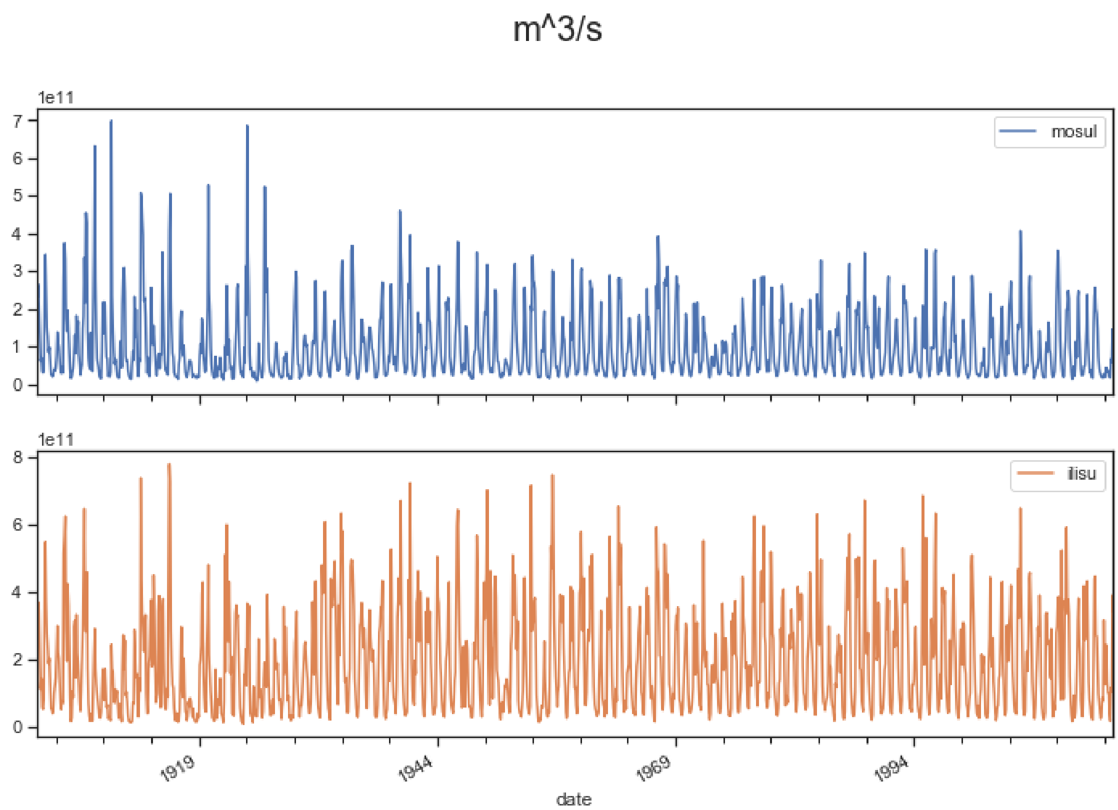
[4]:
```
df.mosul.corr(mosul_usgs)
```

[4]: 0.501333200777497

Convert mm/day to m^3/s https://www.researchgate.net/post/How_to_convert_discharge_m3_s_to_mm_of_

[5]:
```
# approximate pixel area in m^2
pixel_area = 50000*50000
day2seconds = 24*60*60

# mm/day --> m/day
m_per_day = df / 1000

# m/day --> m^3/day
cubicmeters_per_day = m_per_day * pixel_area

# m^3/day --> m^3/second
cubicmeters_per_second = cubicmeters_per_day * day2seconds
cubicmeters_per_second.plot(subplots=True, title='m^3/s')
```

```
[5]: array([<matplotlib.axes._subplots.AxesSubplot object at 0x10a515190>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x12628dcd0>],
       dtype=object)
```

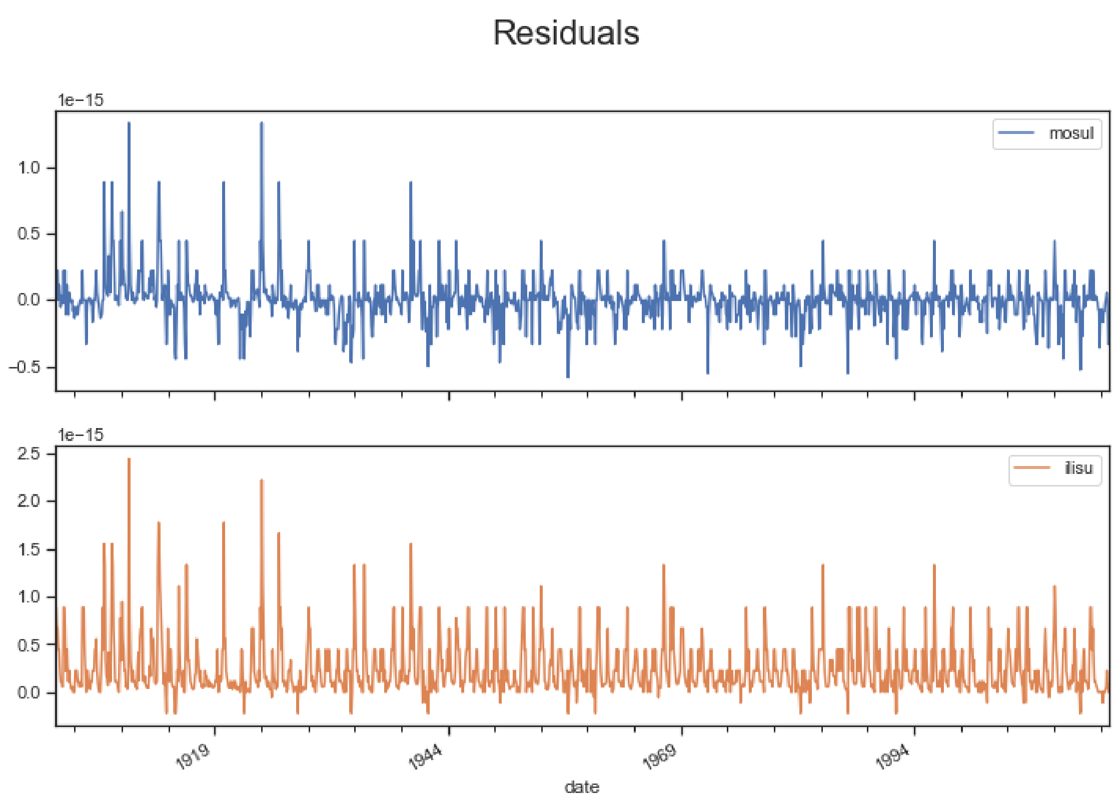


Find order of auto-regressive process (monthly sampling): AR(1) seems likely

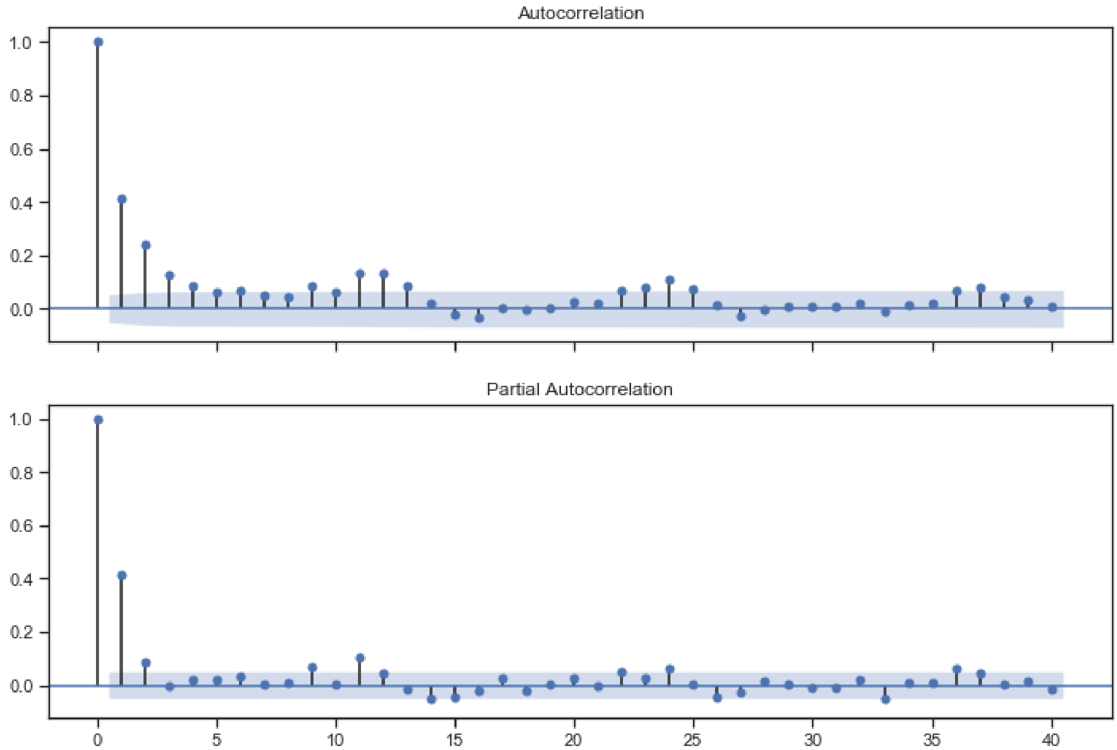
```python
[6]: def detrend(s):
         y = s.values
         x = list(range(1, len(s) + 1))
         x = np.reshape(x, (-1, 1))
         x = sm.add_constant(x)  # to add intercept"
         trd = sm.OLS(y, x).fit()  # fit regression"
         trend = trd.predict(x)  # compute trend line"
         return pd.Series(data=(y-trend), index=s.index)

     # detrend
     df_residuals = df.apply(detrend, axis=1)
     df_residuals.plot(subplots=True, title="Residuals")

     # acf and pacf
     fig, axes = plt.subplots(nrows=2, ncols=1, sharex=True, sharey=True)
```

```
x = sm.graphics.tsa.plot_acf(df_residuals.mosul.values.squeeze(), lags=40,␣
↪ax=axes[0])
y = sm.graphics.tsa.plot_pacf(df_residuals.mosul.values.squeeze(), lags=40,␣
↪ax=axes[1])
plt.suptitle("Mosul GRUN")


fig, axes = plt.subplots(nrows=2, ncols=1, sharex=True, sharey=True)
x = sm.graphics.tsa.plot_acf(df_residuals.ilisu.values.squeeze(), lags=40,␣
↪ax=axes[0])
y = sm.graphics.tsa.plot_pacf(df_residuals.ilisu.values.squeeze(), lags=40,␣
↪ax=axes[1])
plt.suptitle("Ilisu GRUN")

fig, axes = plt.subplots(nrows=2, ncols=1, sharex=True, sharey=True)
x = sm.graphics.tsa.plot_acf(mosul_usgs.values.squeeze(), lags=40, ax=axes[0])
y = sm.graphics.tsa.plot_pacf(mosul_usgs.values.squeeze(), lags=40, ax=axes[1])
plt.suptitle("Mosul USGS")
```

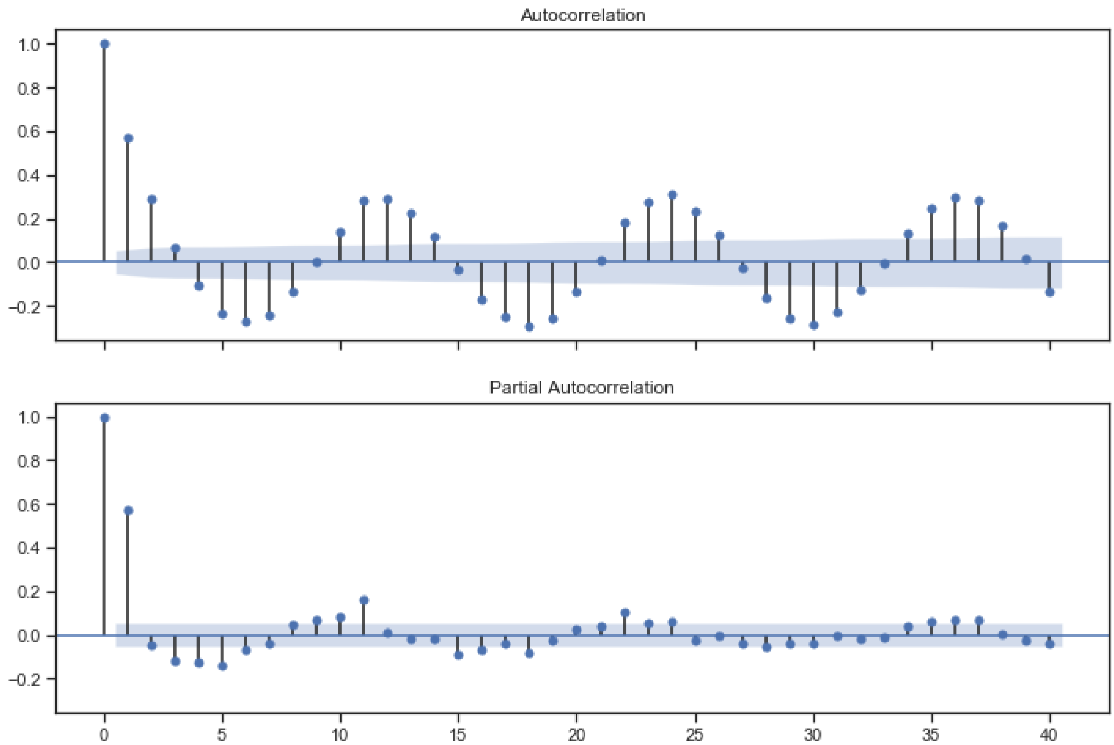
[6]: Text(0.5, 0.98, 'Mosul USGS')

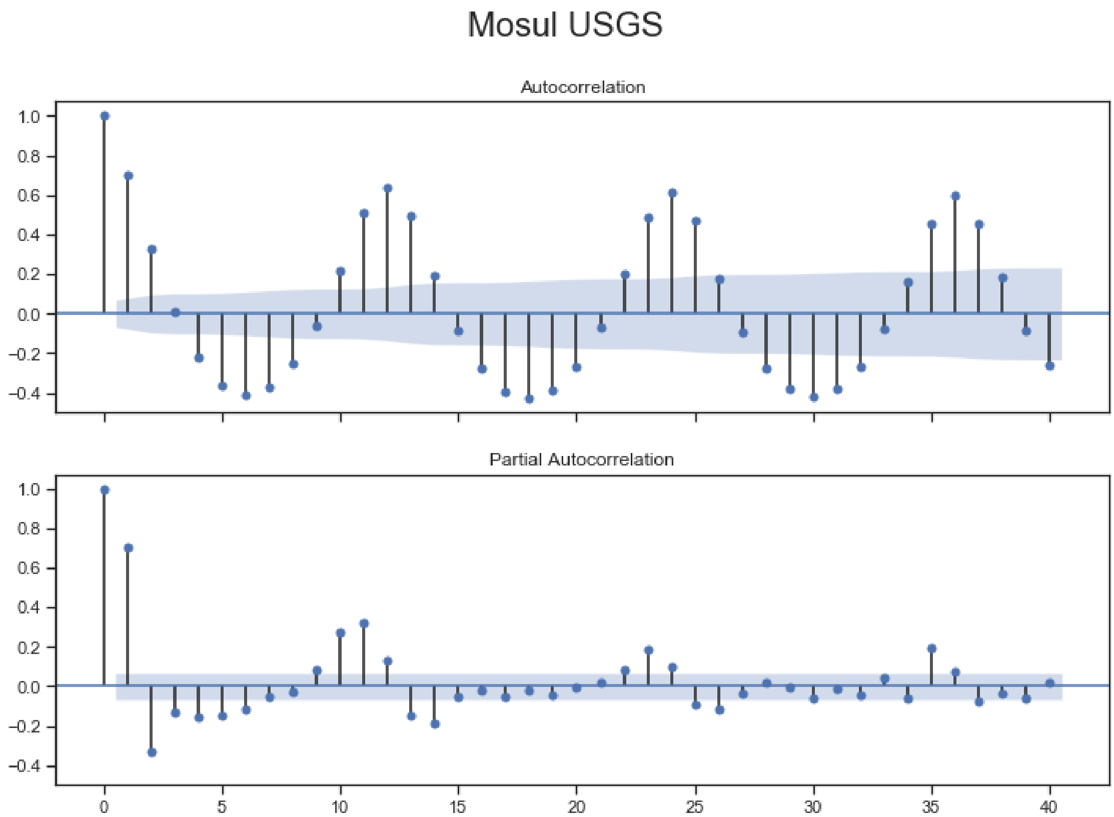


Mosul GRUN


Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
Partial Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
40

Ilisu GRUN


Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
Partial Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
5
10
15
20
25
30
35
40



An auto-regressive model of order 1 ("AR(1)" process) is described by

$X_t = c + \phi * X_{t-1} + \epsilon_t$.

It specifies that the output variable depends linearly on its own previous values lagged by t and on a stochastic term ϵ_t in the form of a stochastic difference equation. In the regression analysis above we made the iid assumptions. Since the data seem to follow an AR(1) process, the assumption of independence between residuals ϵ_t is flawed. This leads to a biased trend estimate.

We need these 3 formulas:

1) $x_t = \beta_0 + \beta_1 * t + \epsilon_t$
2) $\epsilon_t = \epsilon_{t-1} * \alpha + w_t$
3) Combined: $x_t = \beta_0 + \beta_1 * t + x_{t-1} * \alpha + w_t$

[7]:
```
class TimeSeriesModel(object):
    def __init__(self, ts):
        assert type(ts) is pd.Series, "Class expects a pd.Series as input."
        self.ts = ts
        self.freq = self.ts.index.freq
        self.y = ts.values
        self.idx = ts.index.values # store datetime index
```

```python
        self.slen = ts.shape[0] # length of SI data
        self.x = np.arange(1, self.slen + 1, 1) # discrete vector

        # fix as column vectors
        self.x.shape = (self.slen, 1)
        self.y.shape = (self.slen, 1)

        # set seed
        np.random.seed(42)

    def fit(self, ar_order=1, ar_trend='n'):
        """fit the 4 model parameters"""
        # OLS
        self.ar_order = ar_order
        self.ar_trend = ar_trend
        xvec = np.reshape(self.x, (-1, 1))
        xvec = sm.add_constant(xvec)  # to add intercept
        ols = sm.OLS(self.y, xvec).fit()  # fit regression
        self.beta0 = ols.params[0] # OLS intercept
        self.beta1 = ols.params[1] # OLS slope
        self.trend_line = ols.predict(xvec) # trend line
        self.trend_line.shape = (self.slen, 1)
        self.residuals = self.y - self.trend_line # residuals

        # AR
        AR = AutoReg(endog=self.residuals, lags=self.ar_order, trend=self.
↪ar_trend) # fit AR process
        ARfit = AR.fit(cov_type="HC0") # robust SE
        html = ARfit.summary().as_html() # save model results
        self.sd = float(pd.read_html(html)[0].iloc[2,3]) # get sd
        self.alpha = float(pd.read_html(html)[1].iloc[1,1]) # get autocorrelation
        return self

    def monte_carlo(self, n=1):
        """Do n simulations."""
        self.out_shape = (self.slen, n)

        # trend matrix
        trend_mat = np.tile(self.trend_line, (1, n))

        # white noise matrix
        white_noise = np.random.normal(0, self.sd, size=self.out_shape)

        # red noise matrix: fill iteratively
        red_noise = np.empty_like(white_noise)
        for pos in np.arange(1, self.slen):
```

```
            red_noise[pos,:] = self.alpha * red_noise[pos-1,:] +␣
↪white_noise[pos,:]

        # compute sum
        xt = trend_mat + red_noise

        # to dataframe
        self.simulation = pd.DataFrame(data=xt, index=self.idx, columns=np.
↪arange(1, xt.shape[1] + 1))
        return self

    def plot(self, what='obs'):
        """Plot obs, sim or extrp"""
        if what not in ['obs', 'sim', 'extrp']:
            raise IOError("Data does not exist.")

        if what == 'obs':
            data = self.ts
        elif what == 'sim':
            data = self.simulation
        else:
            data = self.extrapolation

        # compute statistics
        if data.shape[1] > 1:
            self.quantiles = data.quantile(q=[0, 0.025, 0.25, 0.5, 0.75, 0.975,␣
↪1], axis=1).transpose()

            # plot simulation results
            # ------------------------------
            f, ax = plt.subplots(figsize=(10,5))
            # median
            self.quantiles[0.5].plot(ax=ax, color='blue', alpha=0.3)

            # observations
            ts_obs = pd.Series(data=np.ravel(self.y), index=self.idx)
            ts_obs.name = 'obs'
            ts_obs.plot(ax=ax, color='red', alpha=0.5)

            # min - max
            ax.fill_between(self.quantiles.index.values,
                            self.quantiles[0],
                            self.quantiles[1],
                            color='black', alpha=0.1)

            # 2.5% - 97.5%
            ax.fill_between(self.quantiles.index.values,
```

```
                            self.quantiles[0.025],
                            self.quantiles[0.975],
                            color='black', alpha=0.2)

            # 25% - 75%
            ax.fill_between(self.quantiles.index.values,
                            self.quantiles[0.25],
                            self.quantiles[0.75],
                            color='black', alpha=0.3)

            ax.set_ylabel("Discharge (mm/day)")

        else:
            f, ax = plt.subplots(figsize=(10,5))
            data.plot(ax=ax, color='blue', alpha=0.3)
            #self.ts.plot(ax=ax, color='red', alpha=0.3)
            ax.set_ylabel("Discharge (mm/day)")

        plt.legend()
        return f, ax


    def extrapolate(self, until, n=1):
        """Extrapolate time series into the future based on the fitted AR model.
↪"""

        # construct index
        last_obs = self.ts.index[-1]
        first_extrp = last_obs + relativedelta(months=1)
        new_idx = pd.date_range(first_extrp, until, freq='M')
        combined_idx = pd.date_range(self.ts.index[0], until, freq='M')

        # extrapolate trend
        new_len = len(new_idx) + self.slen
        new_x = np.arange(1, new_len + 1)[self.slen:]
        trd_extrp = self.beta0 + self.beta1 * new_x
        trd_extrp.shape = (len(trd_extrp), 1)
        self.slen_extrp = len(new_idx)
        out_shape_extrp = (self.slen_extrp, n)

        # simulate AR process
        trend_mat = np.tile(trd_extrp, (1, n))

        # extrapolate stochastic components
        white_noise = np.random.normal(0, self.sd, size=out_shape_extrp)

        # create (1, n) array of the last observations residual
```

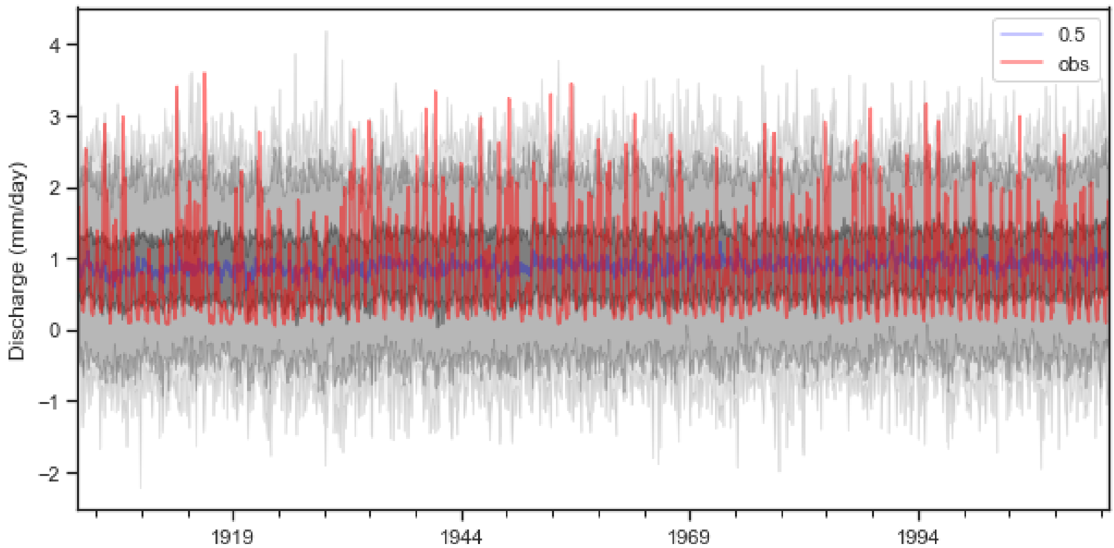
```
        resid_last_obs = self.residuals[0][0]
        last_obs_array = np.repeat(resid_last_obs, n)

        # create empty red noise matrix
        red_noise = np.empty_like(white_noise)

        # initialise AR process with the last observation
        red_noise = np.vstack((last_obs_array, red_noise))

        # red noise matrix: fill iteratively
        for pos in np.arange(1, self.slen_extrp):
            red_noise[pos,:] = self.alpha * red_noise[pos-1,:] +␣
↪white_noise[pos,:]

        # compute sum
        xt = trend_mat + red_noise[1:,]


        # combine with observations
        observations = np.tile(self.y, (1, n))
        combined_series = np.concatenate((observations, xt), axis=0)

        # to series
        self.extrapolation = pd.DataFrame(data=combined_series,
                                          index=combined_idx,
                                          columns=np.arange(1, combined_series.
↪shape[1] + 1))
        return self
```

1.1 Monte Carlo Simulation for Ilisu

[8]:
```
# run simulation
n_realisations = 100
model_ilisu = TimeSeriesModel(ts=df.ilisu)
model_ilisu = model_ilisu.fit(ar_order=1)
model_ilisu = model_ilisu.monte_carlo(n=n_realisations)
```

[9]:
```
model_ilisu.plot(what='sim')
```

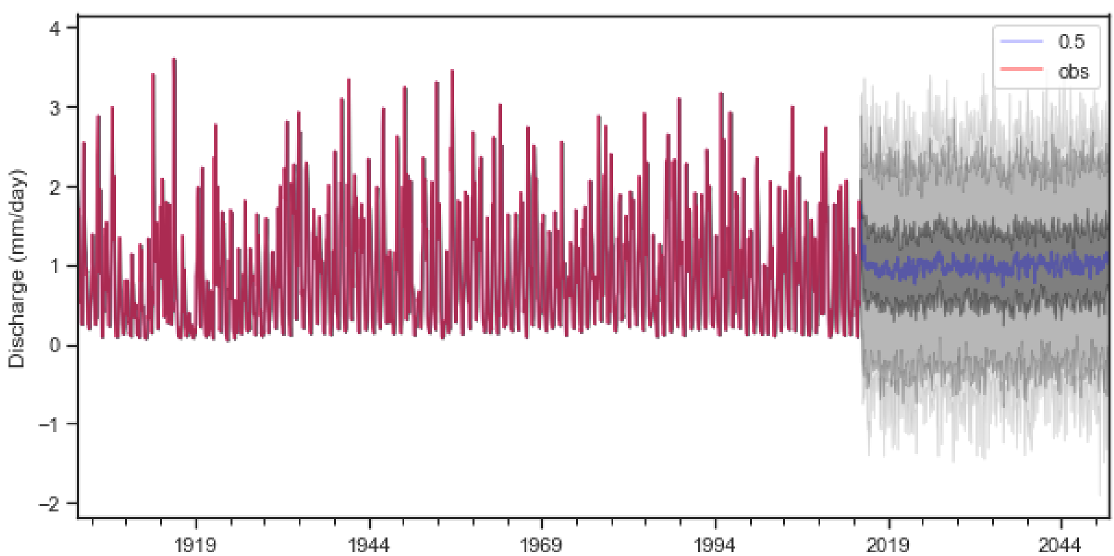
[9]:
```
(<Figure size 720x360 with 1 Axes>,
 <matplotlib.axes._subplots.AxesSubplot at 0x131ad03d0>)
```



extrapolate to ~ 2030. TODO: figure out where exactly to initialise the AR process (last obs, right?)

[10]:
```
model_ilisu = model_ilisu.extrapolate(until='2050-12-31', n=n_realisations)
model_ilisu.plot(what='extrp')
```

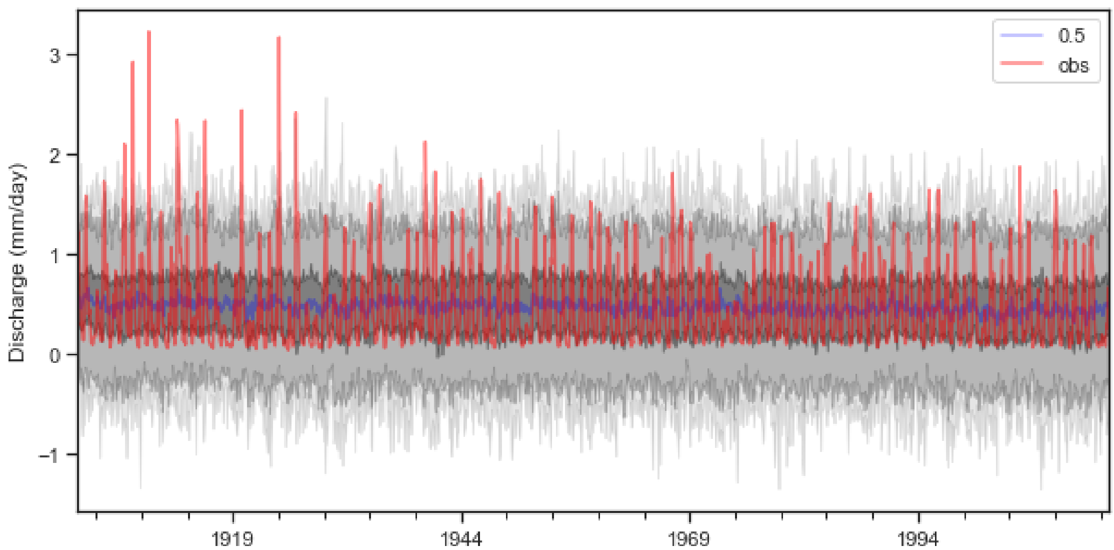
[10]:
```
(<Figure size 720x360 with 1 Axes>,
 <matplotlib.axes._subplots.AxesSubplot at 0x131ed6b10>)
```

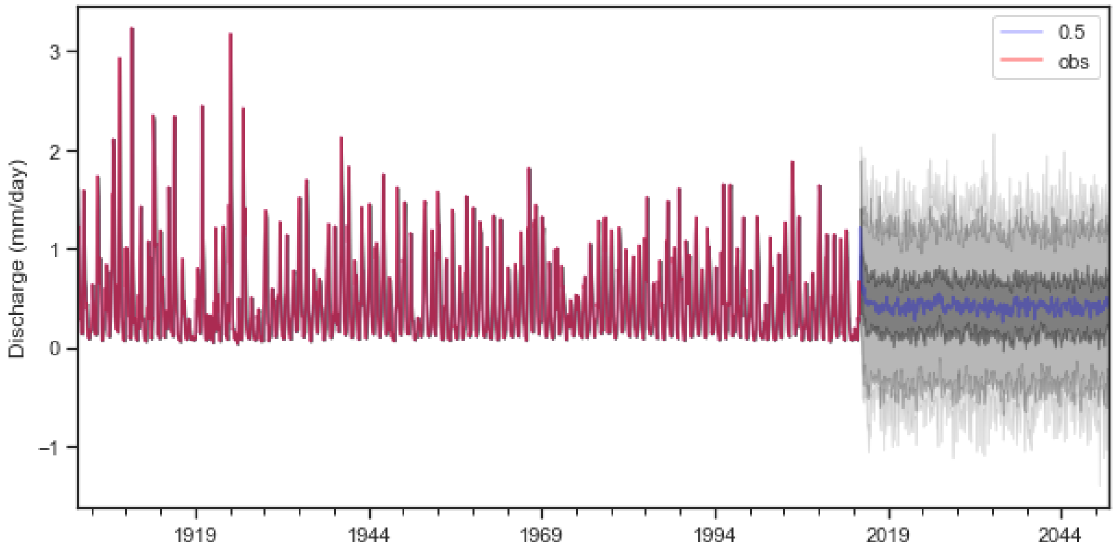


1.2 Monte Carlo Simulation for Mosul

[11]:
```
# run simulation
model_mosul = TimeSeriesModel(ts=df.mosul)
model_mosul = model_mosul.fit()
model_mosul = model_mosul.monte_carlo(n=n_realisations)

# plot simulations
model_mosul.plot(what='sim')

# extrapolate
model_mosul = model_mosul.extrapolate(until='2050-12-31', n=n_realisations)
model_mosul.plot(what='extrp')
```

[11]:
```
(<Figure size 720x360 with 1 Axes>,
 <matplotlib.axes._subplots.AxesSubplot at 0x126059050>)
```



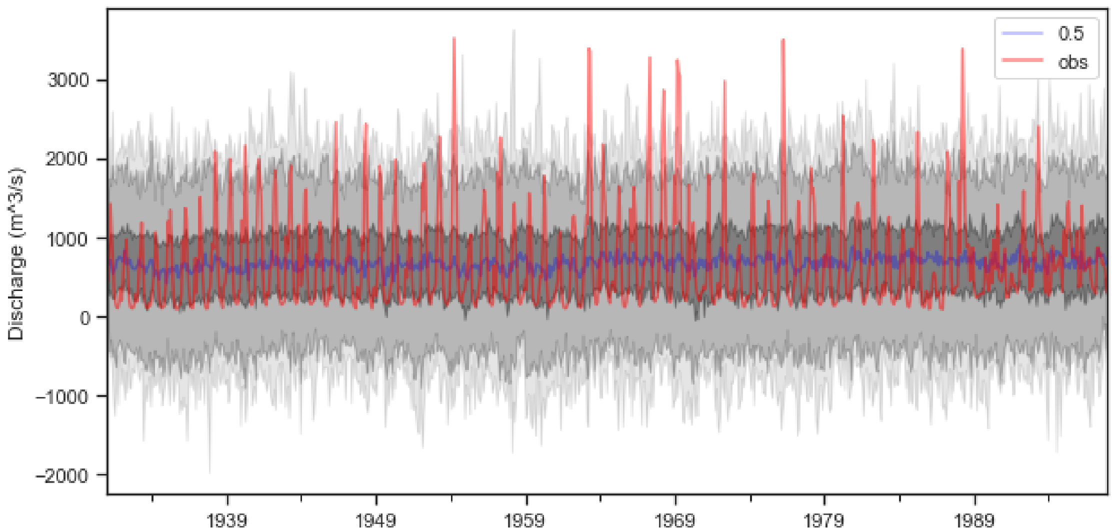


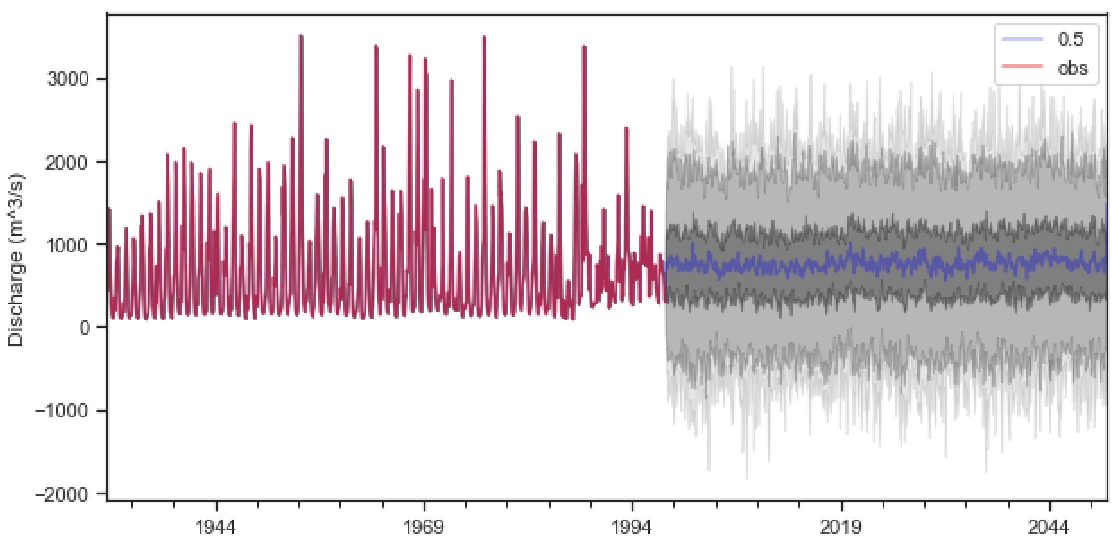
Calculate quantiles of the simulation distributions

2 Monte carlo simulation for Mosul using USGS data

[12]:
```
model_mosul_usgs = TimeSeriesModel(ts=mosul_usgs)
model_mosul_usgs = model_mosul_usgs.fit(ar_order=1)
model_mosul_usgs = model_mosul_usgs.monte_carlo(n=100)

# plot historical simulation
f, ax = model_mosul_usgs.plot(what='sim')
ax.set_ylabel("Discharge (m^3/s)")

# extrapolate
model_mosul_usgs = model_mosul_usgs.extrapolate(until='2050-12-31',␣
 ↪n=n_realisations)
f, ax2 = model_mosul_usgs.plot(what='extrp')
ax2.set_ylabel("Discharge (m^3/s)")

# linear trend (m^3/s)
model_mosul_usgs.beta1
```

[12]: 0.11546249324677546





Store 100 simulations each

[13]:
```
model_ilisu.extrapolation.to_csv("data/monte_carlo/extrapolation_ilisu.csv")
model_mosul.extrapolation.to_csv("data/monte_carlo/extrapolation_mosul.csv")
model_mosul_usgs.extrapolation.to_csv("data/monte_carlo/
 ↪extrapolation_mosul_usgs.csv")

model_ilisu.simulation.to_csv("data/monte_carlo/simulation_ilisu.csv")
model_mosul.simulation.to_csv("data/monte_carlo/simulation_mosul.csv")
model_mosul_usgs.simulation.to_csv("data/monte_carlo/simulation_mosul_usgs.csv")
```

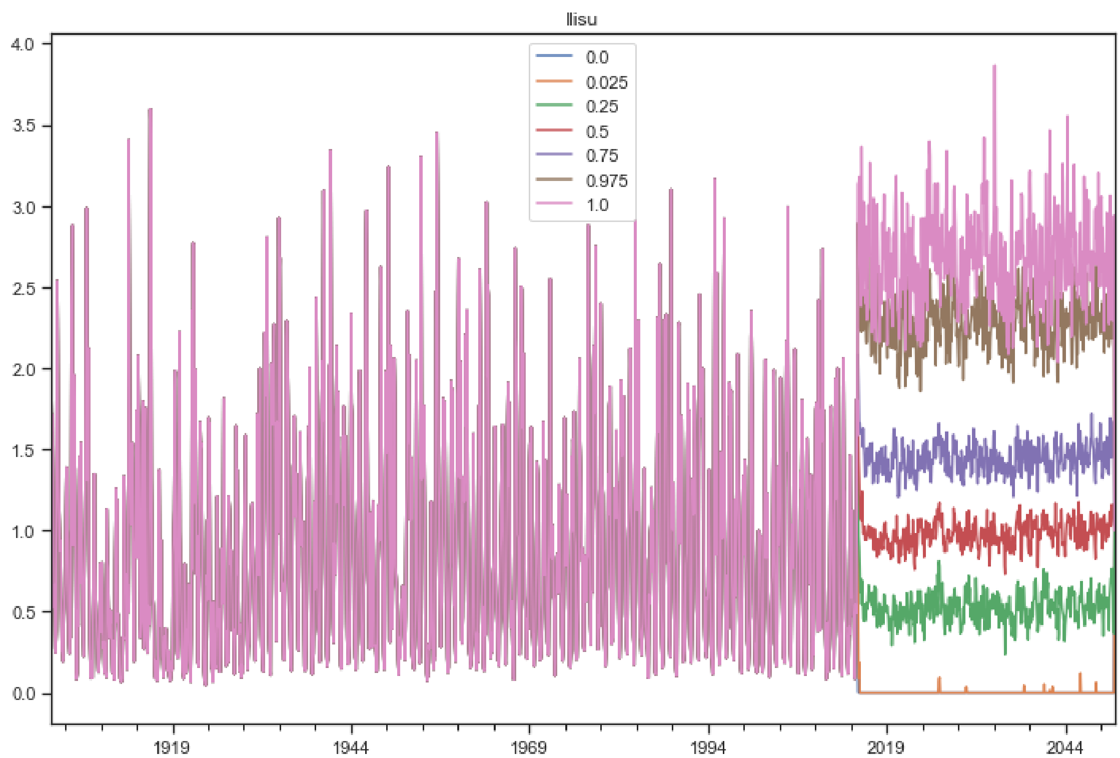
[14]:
```
quantiles = [0., 0.025, 0.25, 0.5, 0.75, 0.975, 1.]
quantiles_ilisu = model_ilisu.extrapolation.quantile(q=quantiles, axis=1).T
quantiles_mosul = model_mosul.extrapolation.quantile(q=quantiles, axis=1).T
quantiles_mosul_usgs = model_mosul_usgs.extrapolation.quantile(q=quantiles,␣
↪axis=1).T
```

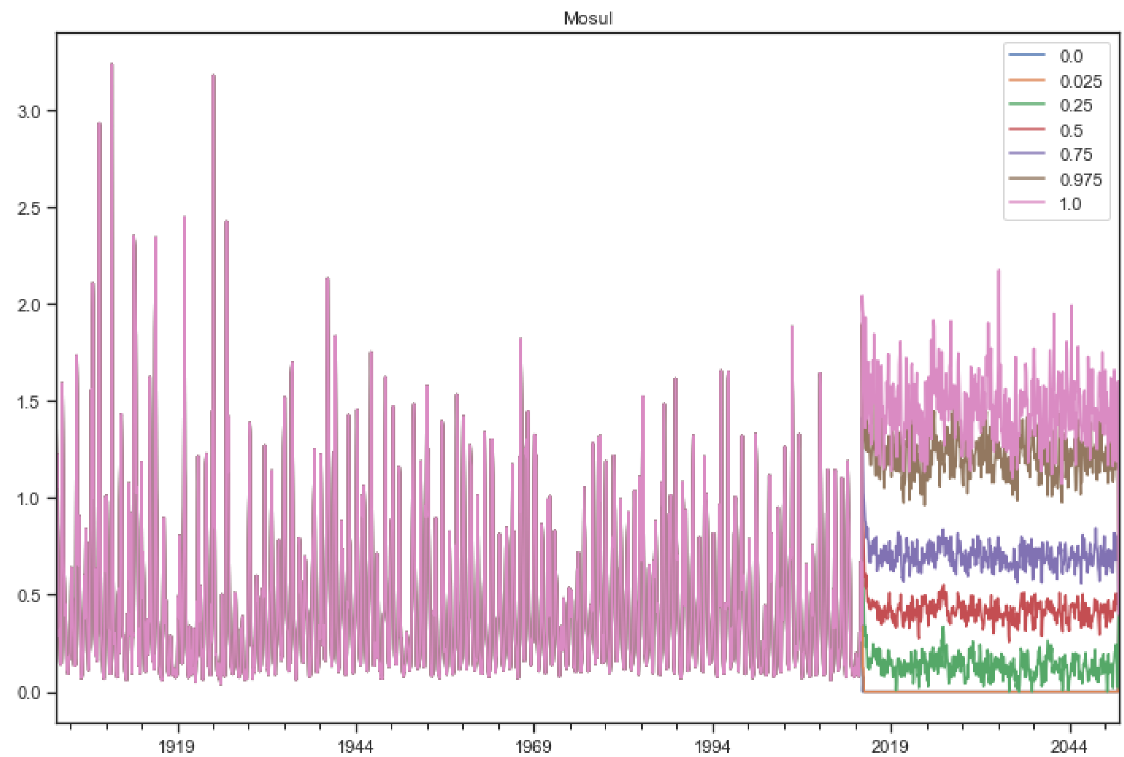
Negative discharge doesn't make a whole lot of sense. Set values < 0 to 0.

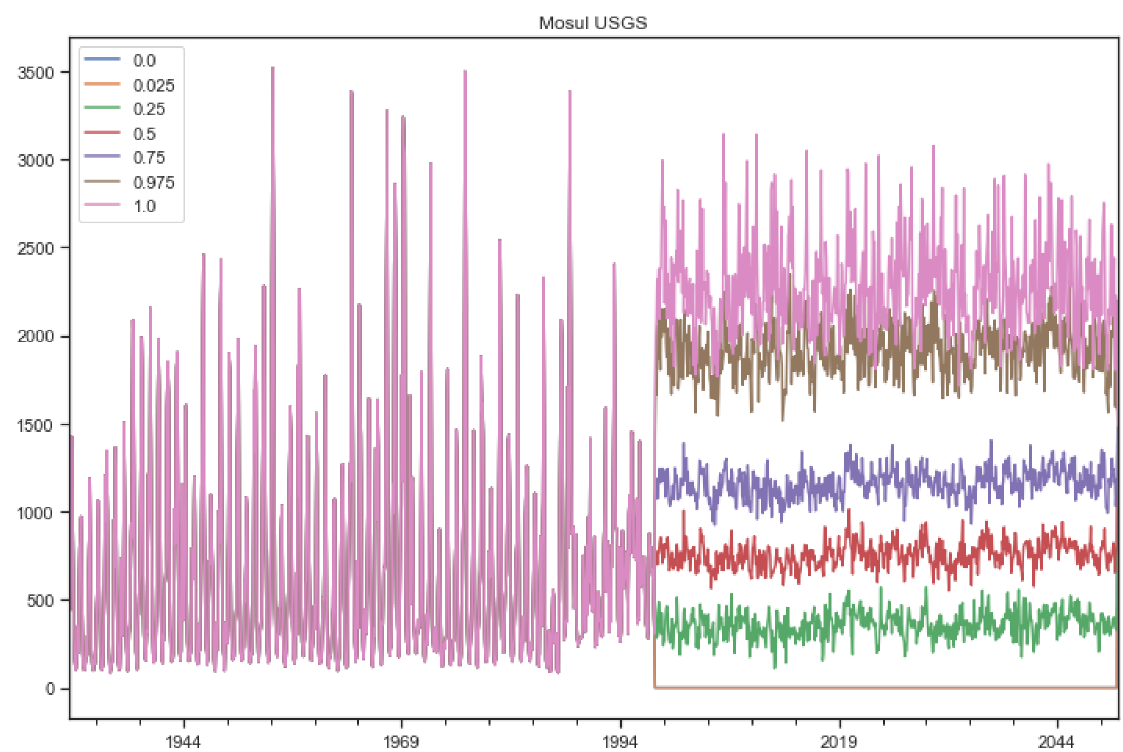
[15]:
```
quantiles_ilisu = quantiles_ilisu.where(quantiles_ilisu > 0., 0.)
quantiles_mosul = quantiles_mosul.where(quantiles_mosul > 0., 0.)
quantiles_mosul_usgs = quantiles_mosul_usgs.where(quantiles_mosul_usgs > 0., 0.)
```

[16]:
```
quantiles_ilisu.plot(title="Ilisu")
quantiles_mosul.plot(title="Mosul")
quantiles_mosul_usgs.plot(title="Mosul USGS")
```

[16]: <matplotlib.axes._subplots.AxesSubplot at 0x134e3c250>




Mosul
0.0
0.025
0.25
0.5
0.75
0.975
1.0
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1919
1944
1969
1994
2019
2044


Mosul USGS
0.0
0.025
0.25
0.5
0.75
0.975
1.0
3500
3000
2500
2000
1500
1000
500
0
1944
1969
1994
2019
2044

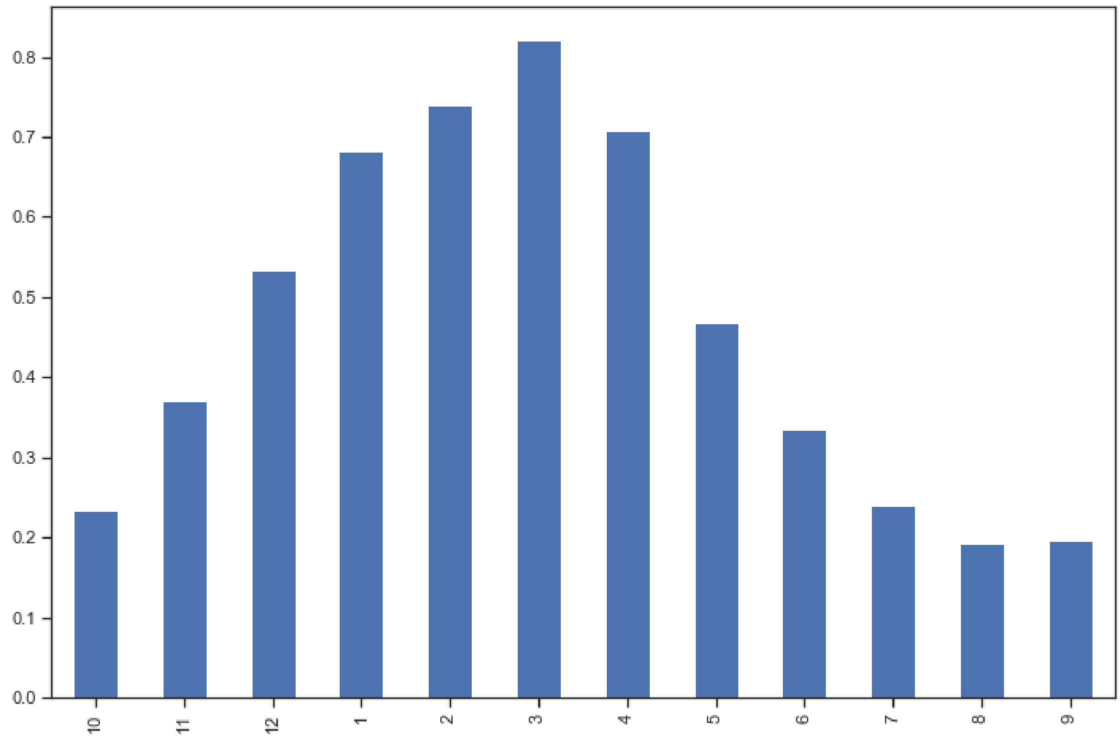
[17]:
```
quantiles_ilisu.to_csv("data/monte_carlo/quantiles_ilisu.csv")
quantiles_mosul.to_csv("data/monte_carlo/quantiles_mosul.csv")
quantiles_mosul_usgs.to_csv("data/monte_carlo/quantiles_mosul_usgs.csv")
```

[18]:
```
usgs_data = pd.read_csv("data/usgs/MosulMean1931-1997.csv")
usgs_mosul = usgs_data['MeanDis'][:-1]
usgs_mosul
```

[18]:
```
0      553.93
1      739.28
2     1076.22
3     1629.11
4     1478.86
5      717.72
6      364.34
7      265.16
8      217.93
9      241.40
10     318.32
11     431.77
Name: MeanDis, dtype: float64
```

[19]:
```
fig, ax = plt.subplots()

climatology_mosul = model_mosul.extrapolation.groupby(model_mosul.extrapolation.
 ↪index.month).mean()
climatology_mosul.name = "Mosul"
climatology_mosul = climatology_mosul.mean(axis=1)

climatology_mosul_usgs = model_mosul_usgs.extrapolation.
 ↪groupby(model_mosul_usgs.extrapolation.index.month).mean()
climatology_mosul_usgs.name = "Mosul USGS"
climatology_mosul_usgs = climatology_mosul_usgs.mean(axis=1)

x = np.roll(climatology_mosul.index.values, 3)
y = np.roll(climatology_mosul.values, 3)

mosul_reordered = pd.Series(index=x, data=y)
mosul_reordered.plot.bar()


#climatology_ilisu = extrapolation_ilisu.groupby(extrapolation_mosul.index.
 ↪month).mean()
#climatology_ilisu.name = "Ilisu"
#climatology_ilisu.mean(axis=1).plot(ax=ax)
```

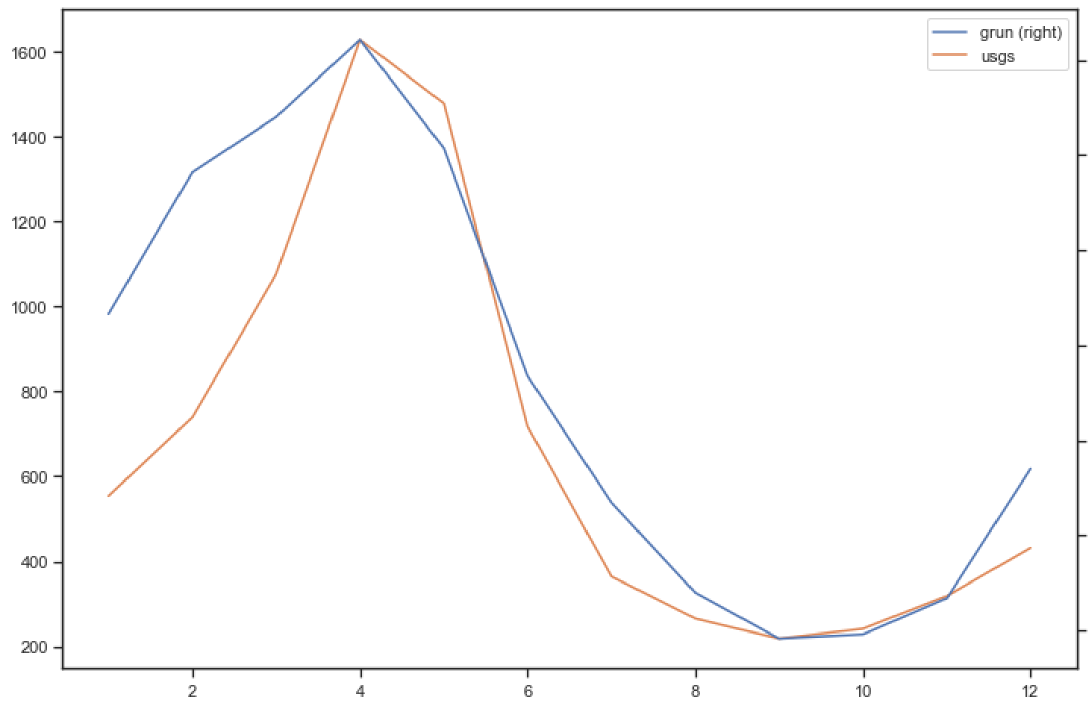
[19]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x12e99bd90>
```

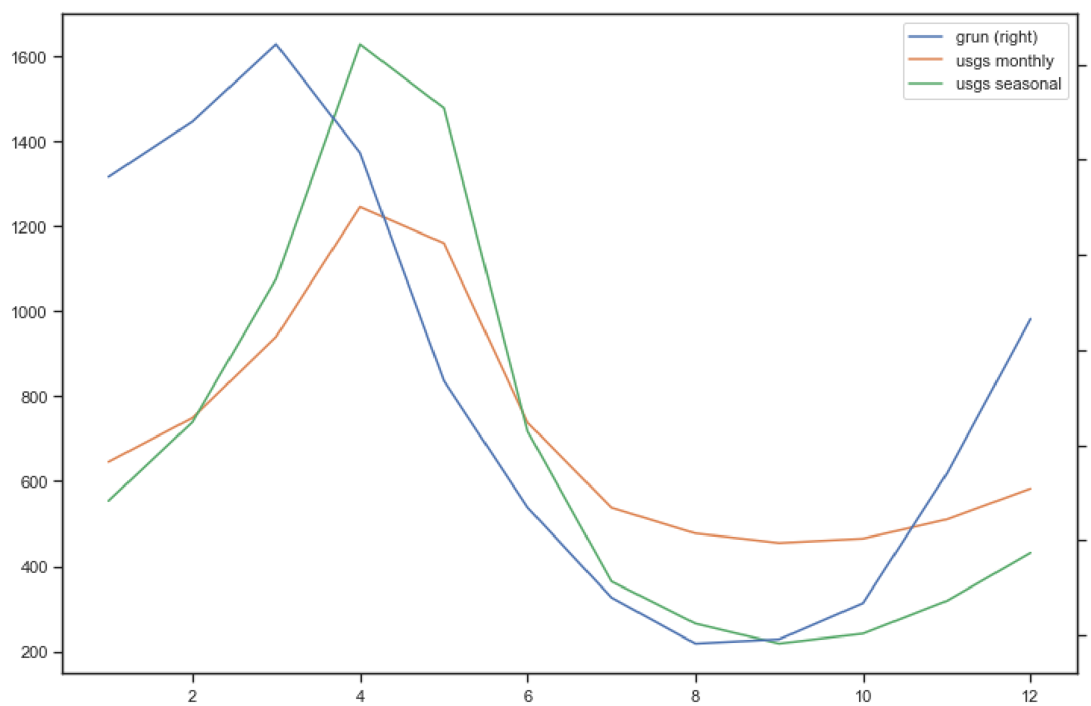


we see the impact of the trend in the extrapolations on the runoff regime, as it shifts towards winter

[20]:
```
climatology_mosul.values
usgs_mosul.values
index = np.arange(1, len(usgs_mosul.values) + 1)

# 1 month rolled
merged = pd.DataFrame(index=index,
            data={'grun': np.roll(climatology_mosul.values, 1),
                  'usgs': pd.to_numeric(usgs_mosul.values)})

merged.plot(secondary_y='grun')

# original
merged = pd.DataFrame(index=index,
            data={'grun': climatology_mosul.values,
                  'usgs monthly': climatology_mosul_usgs.values,
                  'usgs seasonal': pd.to_numeric(usgs_mosul.values)})

merged.plot(secondary_y='grun')
```

[20]: <matplotlib.axes._subplots.AxesSubplot at 0x133381d90>


grun (right)
usgs
1600
1400
1200
1000
800
600
400
200
2
4
6
8
10
12
0.8
0.7
0.6
0.5
0.4
0.3
0.2


grun (right)
usgs monthly
usgs seasonal
1600
1400
1200
1000
800
600
400
200
2
4
6
8
10
12
0.8
0.7
0.6
0.5
0.4
0.3
0.2