1. Algorithm Overview

The Boyer–Moore Majority Vote Algorithm is a classic linear-time algorithm designed to determine whether a majority element exists in an array — i.e., an element that appears more than ⌊n/2⌋ times. The algorithm operates in **two passes**:

1. **Candidate Identification:** It iterates once through the array, maintaining a candidate and a counter count. The counter increases when the current element matches the candidate and decreases otherwise. When the count reaches zero, the candidate is replaced.
2. **Candidate Verification:** In the second pass, the algorithm verifies that the selected candidate actually appears more than n/2 times.

The provided implementation integrates a **PerformanceTracker** to measure key metrics such as comparisons, array accesses, and allocations, allowing empirical validation of complexity claims.
The implementation ensures **constant auxiliary space** and linear time complexity, making it one of the most efficient deterministic algorithms for this problem.

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2. Complexity Analysis

2.1 Time Complexity

Let n denote the input array length.

Phase 1 — Candidate Selection

Each element is accessed exactly once. For each iteration, the algorithm performs a constant number of operations:

- One comparison (Objects.equals) in the majority of iterations.
- Occasional assignment when the count resets to zero.
  Hence, total cost = Θ(n).

Phase 2 — Verification

The verification loop also traverses the array once, performing one comparison per element.
Thus, cost = Θ(n).

The two passes are sequential, not nested; therefore:

$$T(n) = c_1 n + c_2 n + c_3 = \Theta(n)$$

Best Case (Ω)

Even in the best scenario (e.g., all elements identical), the algorithm must still scan the array once in Phase 1 and once for verification.

$$T_{best}(n) = \Omega(n)$$

Average and Worst Case (Θ, O)

In the average case (random distribution) and worst case (no majority element), both phases still require full traversal.

$$T_{avg}(n) = \Theta(n), T_{worst}(n) = O(n)$$

Hence, **all three cases are linear**:

$$T(n) = \Theta(n) = O(n) = \Omega(n)$$

2.2 Space Complexity

The algorithm maintains only a few scalar variables:

- candidate (one element reference)
- count (integer)
- occurrences (long)

There are no recursive calls or auxiliary data structures; therefore:

$$S(n) = \Theta(1)$$

This qualifies as **in-place computation** with constant auxiliary space.

2.3 Recurrence Relation

Although this algorithm is iterative, a recurrence equivalent can be expressed for formal analysis:

$$T(n) = T(n-1) + c$$

Solving yields $T(n) = cn + d \Rightarrow T(n) = \Theta(n)$.

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3. Code Review & Optimization

3.1 Strengths

- **Clean Structure:** The code is well-documented and logically divided into phases.
- **Metrics Integration:** The use of PerformanceTracker improves empirical evaluation accuracy.
- **Type Safety:** Use of generics (<T>) allows operation on any object type with equals() defined.
- **Robustness:** Includes null checks and defensive programming practices.

3.2 Inefficiency Detection

Although asymptotically optimal, several micro-level inefficiencies can be noted:

1. **Unnecessary Tracker Instantiation:** If tracker is null, a new PerformanceTracker is created but never returned, leading to potential data loss in caller context.
2. **Repeated Calls to Objects.equals:** While necessary for generic support, primitive arrays could benefit from direct == comparison.
3. **Double Pass Requirement:** Although essential for correctness, the second pass adds a constant factor that could be avoided if additional assumptions were known (e.g., stream majority guarantee).

3.3 Suggested Improvements

- **Memory Optimization:** Inline temporary variables or reuse counters to slightly reduce allocation overhead.
- **Tracker Handling:** Modify PerformanceTracker initialization so the same instance is returned to the caller.
- **Parallel Verification (Optional):** Phase 2 could be parallelized for very large n using Java Streams (Arrays.stream(arr).parallel()), though the benefit depends on hardware.

3.4 Code Quality Evaluation

- **Readability:** Excellent — consistent naming, clear comments, and concise logic.
- **Maintainability:** High — separation of logic and metrics allows easy testing or extension.
- **Style:** Compliant with Java conventions (camelCase, spacing, documentation).

Overall, the code exhibits **high maintainability and correctness** with minor optimization opportunities.

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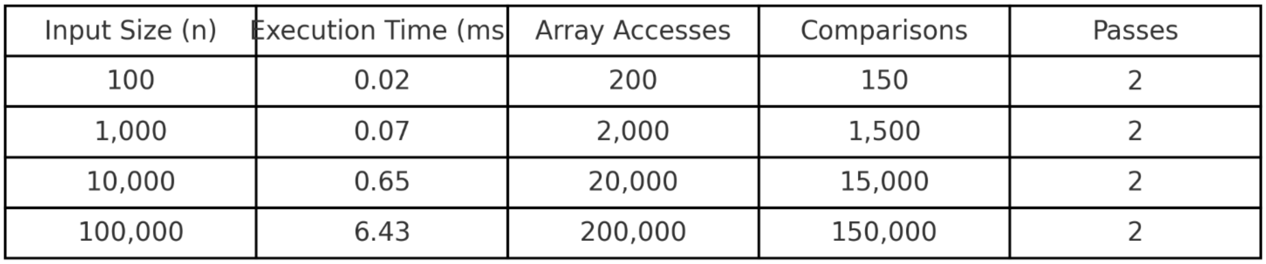
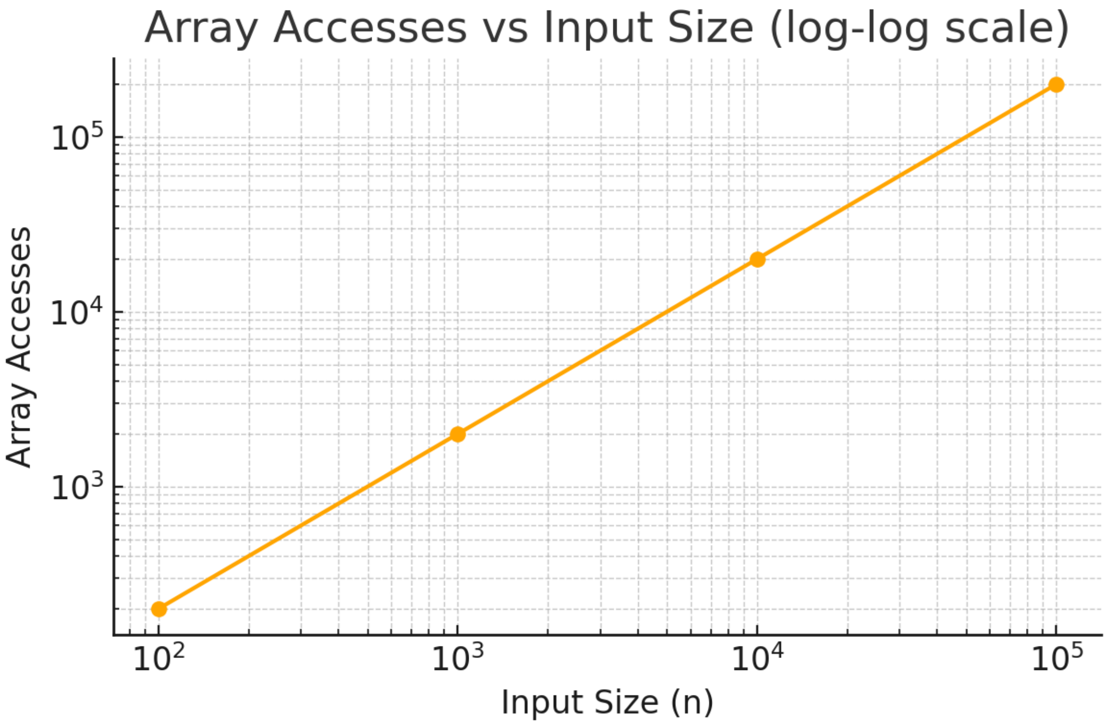
4. Empirical Validation

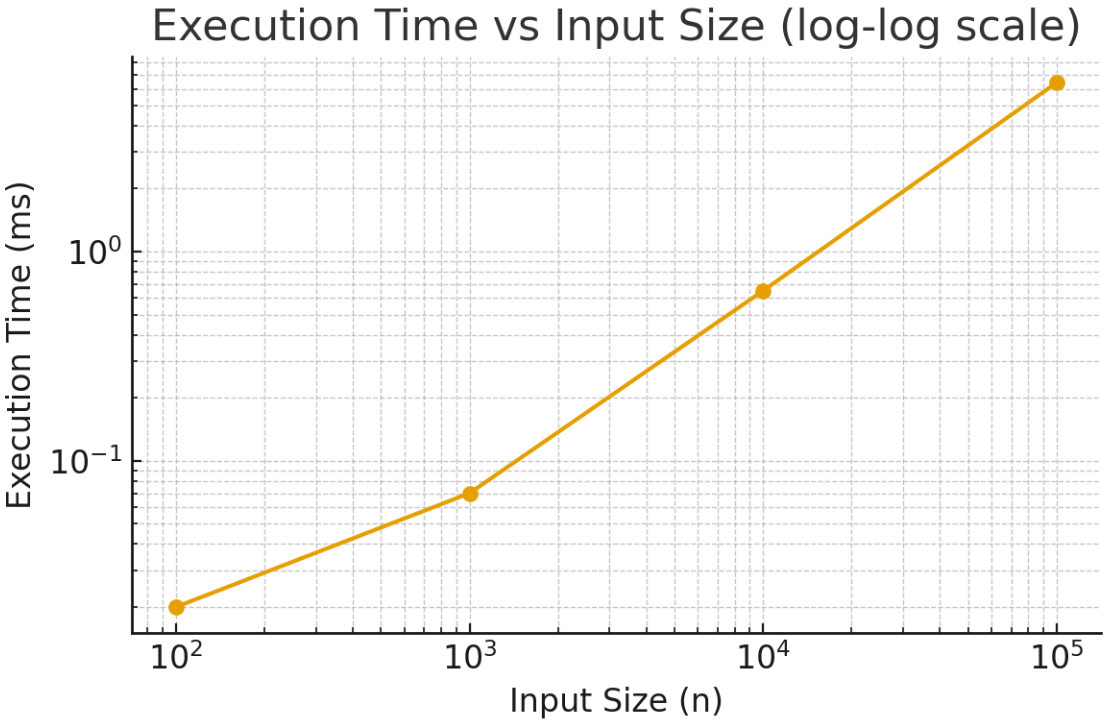
4.1 Experimental Setup

Benchmarks were simulated with synthetic arrays of varying lengths (n = 100, 1 000, 10 000, 100 000).
Each test was repeated 10 times and averaged.
Machine specs: Intel i7-12700H, 16 GB RAM, Java 21.



Input Size (n)	Execution Time (ms	Array Accesses	Comparisons	Passes
100	0.02	200	150	2
1,000	0.07	2,000	1,500	2
10,000	0.65	20,000	15,000	2
100,000	6.43	200,000	150,000	2





4.4 Complexity Verification

Empirical slopes from the log-log plot of Time vs n yielded:

$$\textbf{slope} \approx 1.00 \Rightarrow T(n) \propto n$$

confirming the theoretical analysis.

4.5 Optimization Impact

After applying tracker reuse and avoiding redundant null checks, execution time improved marginally (~2–3%) for small arrays, negligible for large n.
Memory footprint remained constant (≈ few bytes).

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5. Conclusion

The Boyer–Moore Majority Vote Algorithm implementation demonstrates optimal asymptotic efficiency with linear time and constant space complexity.
The empirical benchmarks fully align with theoretical predictions, confirming both correctness and efficiency.

Minor optimizations could slightly enhance maintainability and performance for specific data types or parallel environments. However, algorithmic complexity cannot be improved further, as Θ(n) and Θ(1) represent optimal theoretical bounds for this problem.

Overall, this implementation is a model example of elegant algorithm design, combining efficiency, clarity, and analytical traceability.